Modified bow-tie analysis

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Traditional bow-tie analysis can produce a geometric factor and en effective energy for an energy channel. Here I propose an alternative method that can hopefully produce an effective energy range instead. Let us assume that the response function, $R(E)$, is such that it has a well-defined lower energy, E_1, but a high-energy tail. Let us also assume that the energy spectrum is a power law, $I(E) = AE^{-\gamma}$, up to a maximum energy, E_∞, and zero afterwards. (This is simply to limit the numerical integrals.) Thus, the counting rate of the channel can be obtained as

$$C = \int_0^{E_\infty} AE^{-\gamma} R(E)\,\mathrm{d}E.$$

Now, let us seek the optimal energy range and the corresponding geometric factor of the channel. Assume, that the range is $E \in [E_1, E_2]$, where $E_2 = E_1 + \Delta E$ is a parameter we want to optimize. The mean intensity in this range is

$$\bar{I} = \frac{1}{\Delta E} \int_{E_1}^{E_2} AE^{-\gamma}\mathrm{d}E.$$

We want to equate this to the counting rate divided by $G\Delta E$, where G is the geometric factor of the channel, i.e.,

$$\begin{aligned} \bar{I} &= \frac{C}{G\,\Delta E} \\ \Rightarrow\ G &= \frac{\int_{E_1}^{E_\infty} E^{-\gamma} R(E)\,\mathrm{d}E}{\int_{E_1}^{E_2} E^{-\gamma}\mathrm{d}E} \\ \Rightarrow\ G_\gamma(E_2; E_1) &= (\gamma - 1)\frac{\int_0^{E_\infty} E^{-\gamma} R(E)\,\mathrm{d}E}{E_1^{1-\gamma} - E_2^{1-\gamma}}. \end{aligned}$$

Like in the traditional bow-tie analysis, we will now plot the value of $G_\gamma(E_2; E_1)$ for different values of γ and then choose the point E_2 from the plot where the difference between the minimum and the maximum $G_\gamma(E_2)$ is smallest as the effective upper limit of the channel, E_2.

As an example, consider a response function given as the blue curve in Figure 1 below it starts from $E_1 = 0.11$ MeV and has a shape pretty typical of electron measurements, i.e., a peak followed by a long high-energy tail. Here, $E_\infty =$

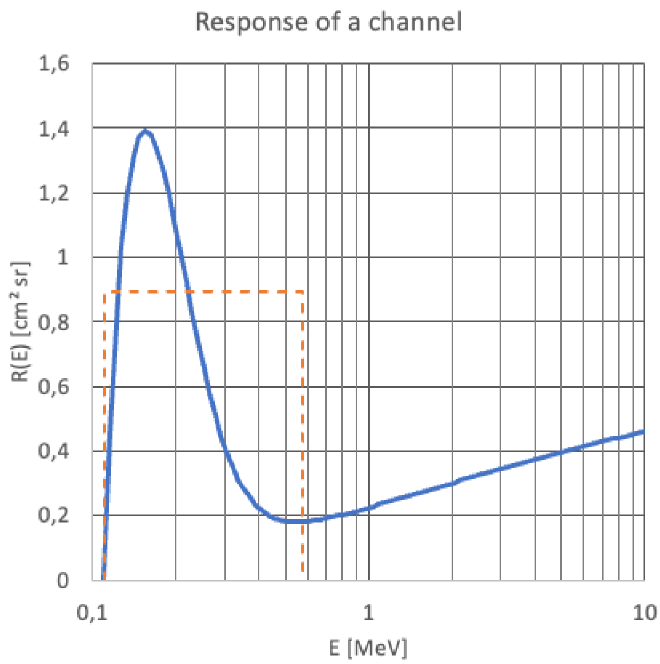

Figure 1: Response function of a fictional channel (blue curve). An equivalent box-car response, determined through a modified bow-tie analysis, is plotted with an orange dashed line.

10 MeV. The bow-tie analysis performed according to the method presented above is provided in Figure 2. The values of E_2 and the corresponding G are 0.58 MeV ($\pm 5\%$) and 0.90 cm^2 sr ($\pm 3\%$). The equivalent box-car channel is plotted in Figure 1 with the orange dashed line. Here I quote the error for the energy from the energy grid used in the analysis (log-spaced, $E_2^{(k+1)} = 1.05\, E_2^{(k)}$) and use the bow-tie spread as the error of the geometric factor. In a more accurate analysis, one could simply use a denser grid to minimize the error in E_2.

Although here we took the lower-energy limit of the channel as the first point where the response function is non-zero, this is not a requirement for the method. We could also choose another level, like a certain percentage of the maximum (10%, 50%, etc.) to be first-exceeded and put E_1 there. The method should still give a result. However, the result will necessarily be quite heavily dependent on especially the lowest γ value in the analysis, and although a minimum of δ is found, it needs to be understood that it could still be very dependent on the range chosen. The same thing, however, limits the validity of the traditional bow-tie analysis as well.

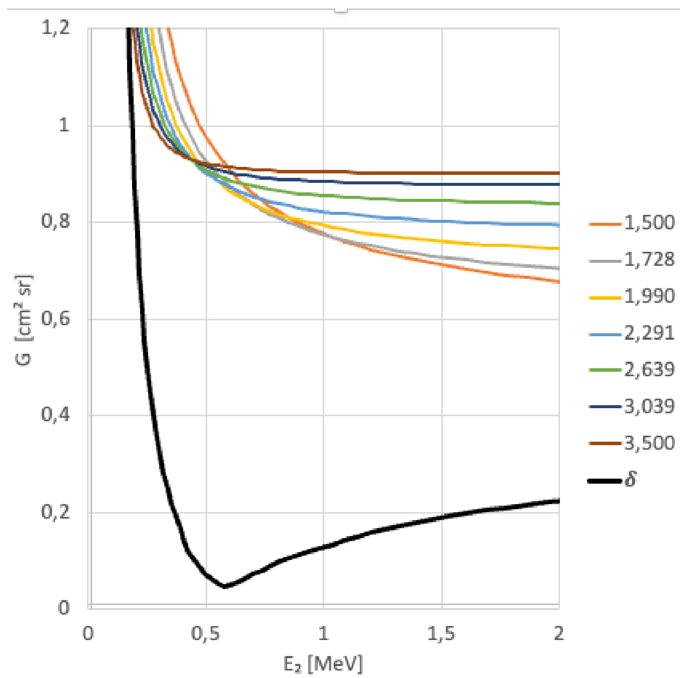

Figure 2: Bow-tie analysis of a fictional channel. Numbers in the legend denote the (log-spaced) values of γ, and δ denotes the difference between the highest and the lowest value of $G_\gamma(E_2)$ at each energy E_2.