LOCAL BARYCENTRIC COORDINATES

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Course: Graphics

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ABSTRACT

BARYCENTRIC COORDINATES YIELD A POWERFUL AND YET SIMPLE PARADIGM TO INTERPOLATE DATA VALUES ON POLYHEDRAL DOMAINS. LBC IS A FAST DEFORMATIONS OF A MODEL.

Affine combination

$$\sum_{i=1}^{n} \alpha_i \cdot x_i = \alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_n x_n, \qquad \sum_{i=1}^{n} \alpha_i = 1.$$

ABSTRACT

A LOCAL CHANGE IN THE VALUE AT A SINGLE CONTROL POINT WILL CREATE A GLOBAL CHANGE BY PROPAGATION INTO THE WHOLE DOMAIN.

Given a real vector space X together with a convex, real-valued function defined on a convex subset $\mathcal{X}$ of X

$$f : \mathcal{X} \to \mathbb{R}; \forall x_1, x_2 \in \mathcal{X}, \forall t \in [0,1] : \qquad f(tx_1 + (1-t)x_2) \leq tf(x_1) + (1-t)f(x_2),$$

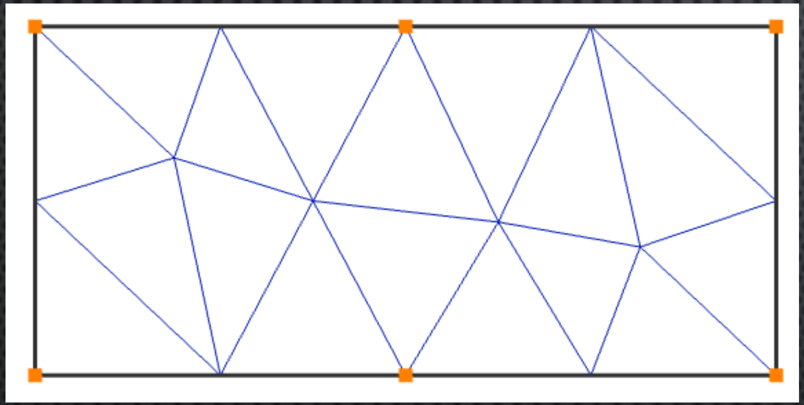
the problem is to find any point x^* in $\mathcal{X}$ for which the number $f(x)$ is smallest, i.e., a point x^* such that

$$f(x^*) \leq f(x) \text{ for all } x \in \mathcal{X}.$$

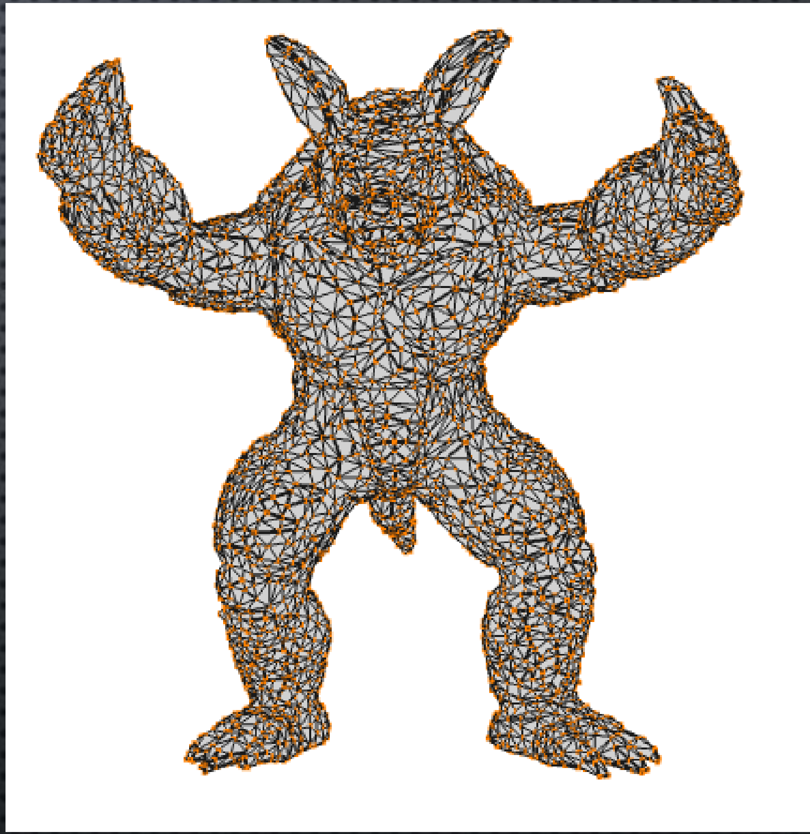
INTRODUCTION

- LBC ARE COMPUTED BY MINIMIZING A TARGET FUNCTIONAL BASED ON TOTAL VARIATION, SUBJECT TO A SET OF CONSTRAINTS THAT ENSURE DESIRED PROPERTIES.
- THE RESULTING IS LOCAL, MEANING THAT EACH CONTROL POINT ONLY INFLUENCES A NEARBY REGION.
- AS BENEFIT, LBC INDUCE LOWER COMPUTATIONAL COST FOR APPLICATIONS SUCH AS CAGE-BASED DEFORMATION.

BENEFITS



VS



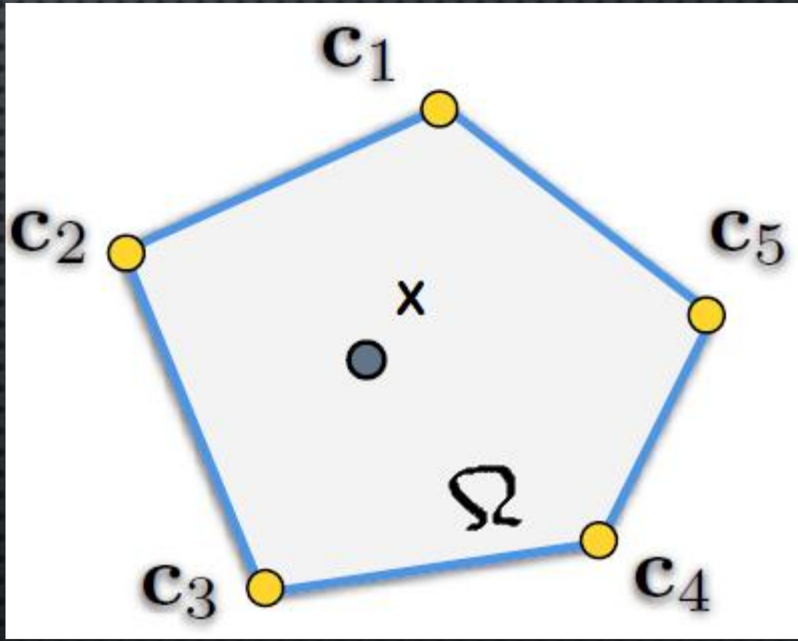
STORAGE REQUIREMENTS AS WELL AS COMPUTATIONAL COST.

OBJECTIVE

Ω: DOMAIN BOUNDED BY THE CAGE. FIND $\omega_i : \Omega \rightarrow \mathrm{IR}$

FOR EACH C_i THEN

$$f(x) = \sum_{i=1}^{n} w_i(x)\ f(C_i)$$



WHERE $[w_i(x),\ \ldots, w_n(x)]$ IS A SET OF GENERALIZED BARYCENTRIC COORDINATES OF $x \in \Omega$.

FOR TE QUALITY OF THE INTERPOLATION, WE NEED SOME PROPERTIES:

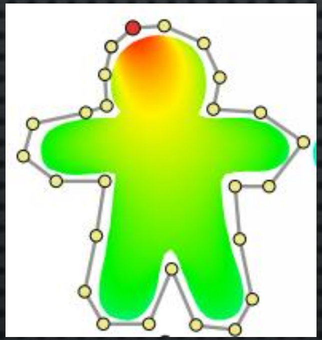
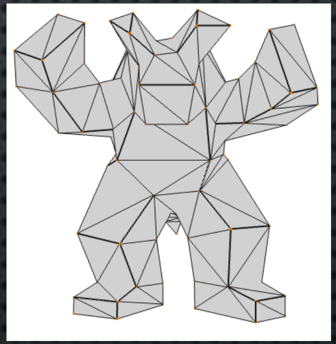
OBJECTIVE

PROPERTIES:

1. *Reproduction*: $\sum_{i=1}^{n} w_i(\mathbf{x})\mathbf{c}_i = \mathbf{x},\ \forall \mathbf{x} \in \Omega$;
2. *Partition of unity*: $\sum_{i=1}^{n} w_i(\mathbf{x}) = 1$;
3. *Non-negativity*: $w_i(\mathbf{x}) \geq 0\ \ \forall i$;
4. *Lagrange property*: $w_i(\mathbf{c}_j) = \begin{cases} 0, & \text{if } i \neq j, \\ 1, & \text{otherwise;} \end{cases}$
5. *Linearity*: functions $\{w_i\}$ are linear on cage edges and faces;
6. *Smoothness*: functions $\{w_i\}$ vary smoothly on Ω;
7. *Locality*: a control point only influences its nearby regions, and a point $\mathbf{x} \in \Omega$ is influenced by a small number of control points, i.e., the vector $[w_1(\mathbf{x}), \ldots, w_n(\mathbf{x})]$ is *sparse*.

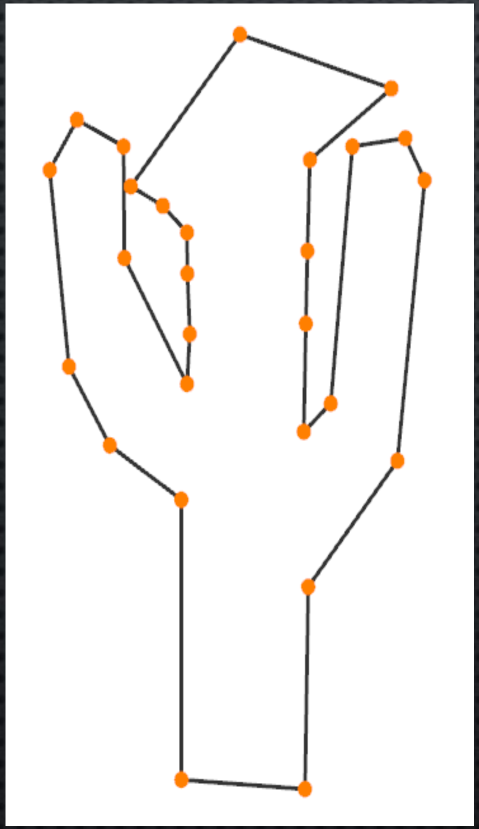
CONTRIBUTION

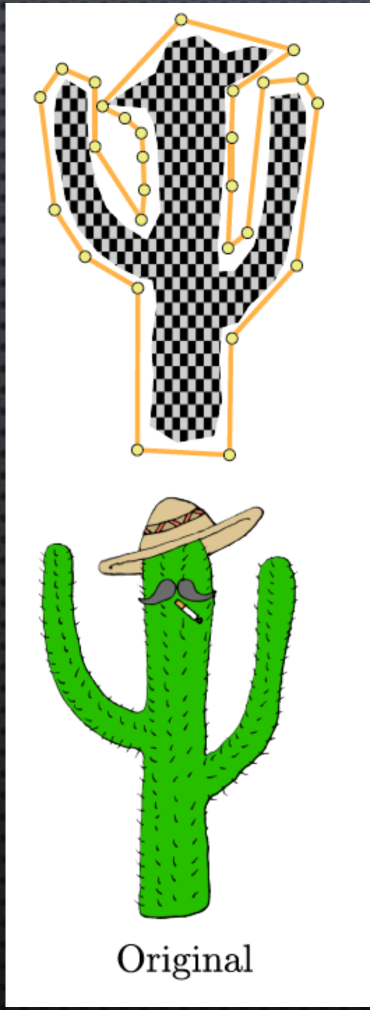
OpenGL → OpenGL Modern

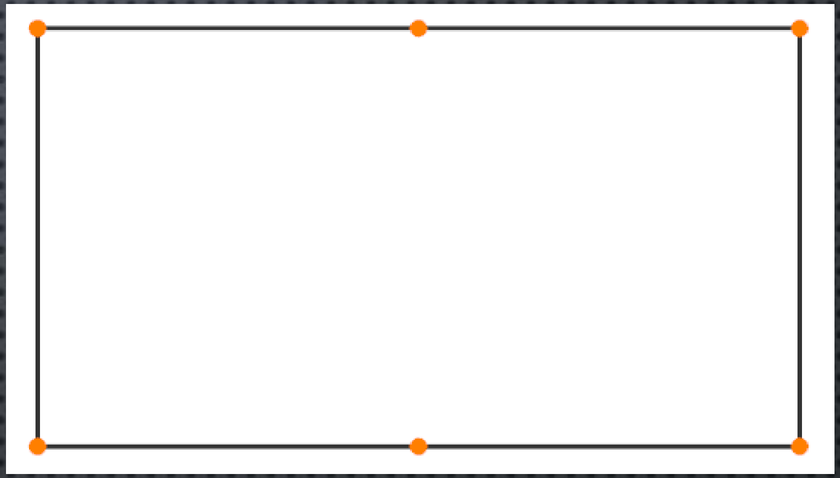
 → 

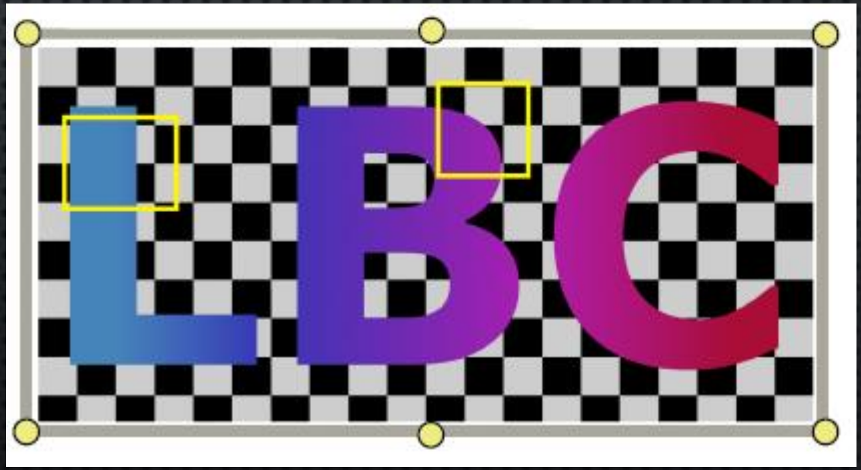
OpenMP → NVIDIA CUDA

DATA

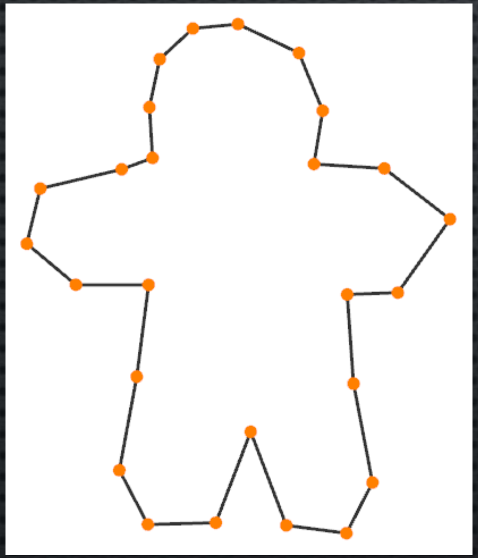


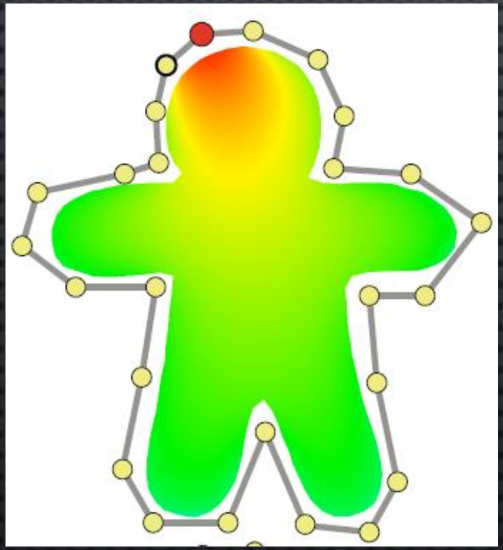


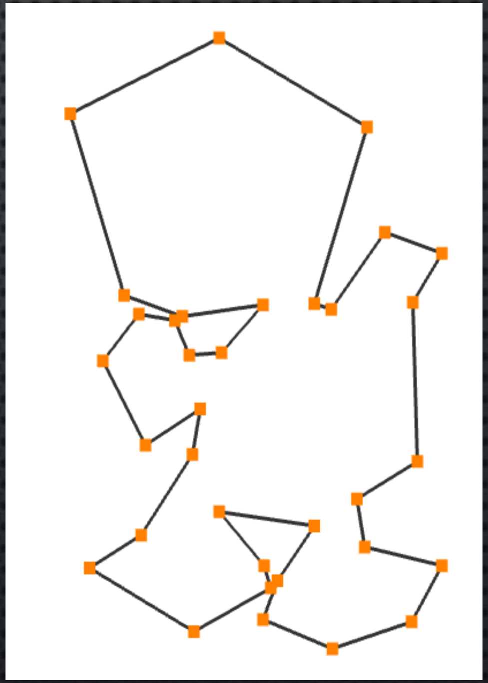




DATA

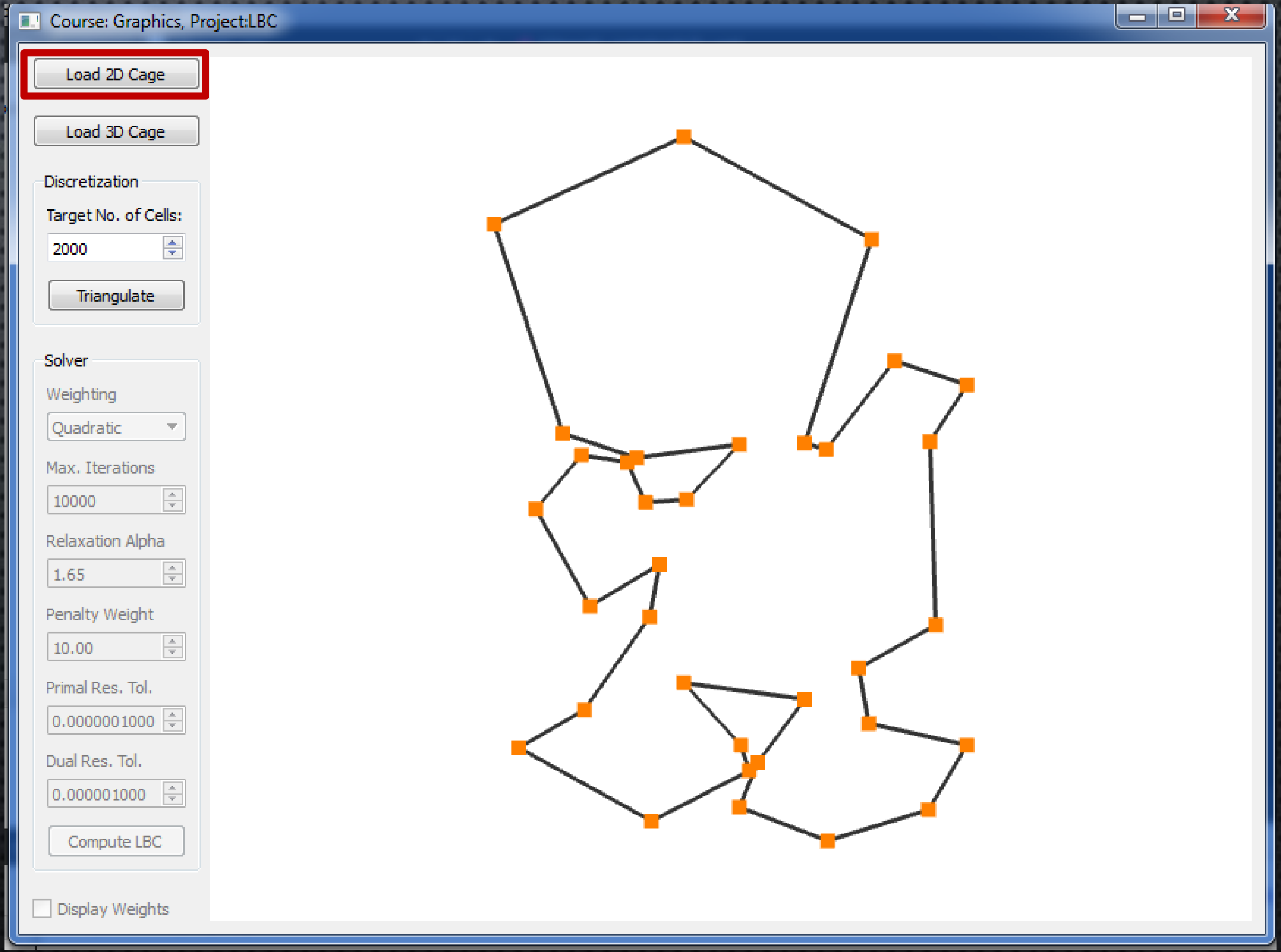






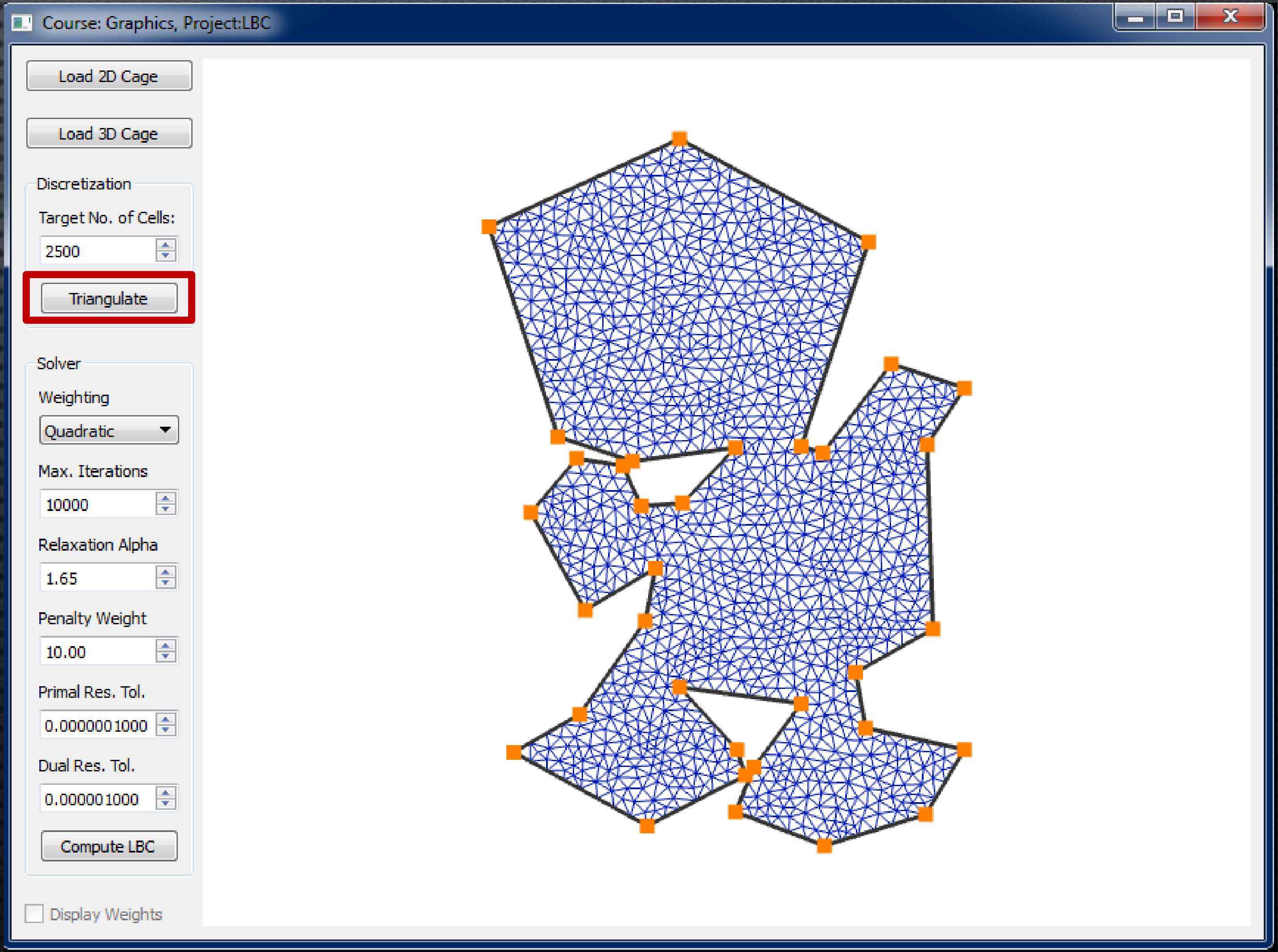
STEPS – 2D

LOAD A MODEL
(CAGE-DEFORMATION)



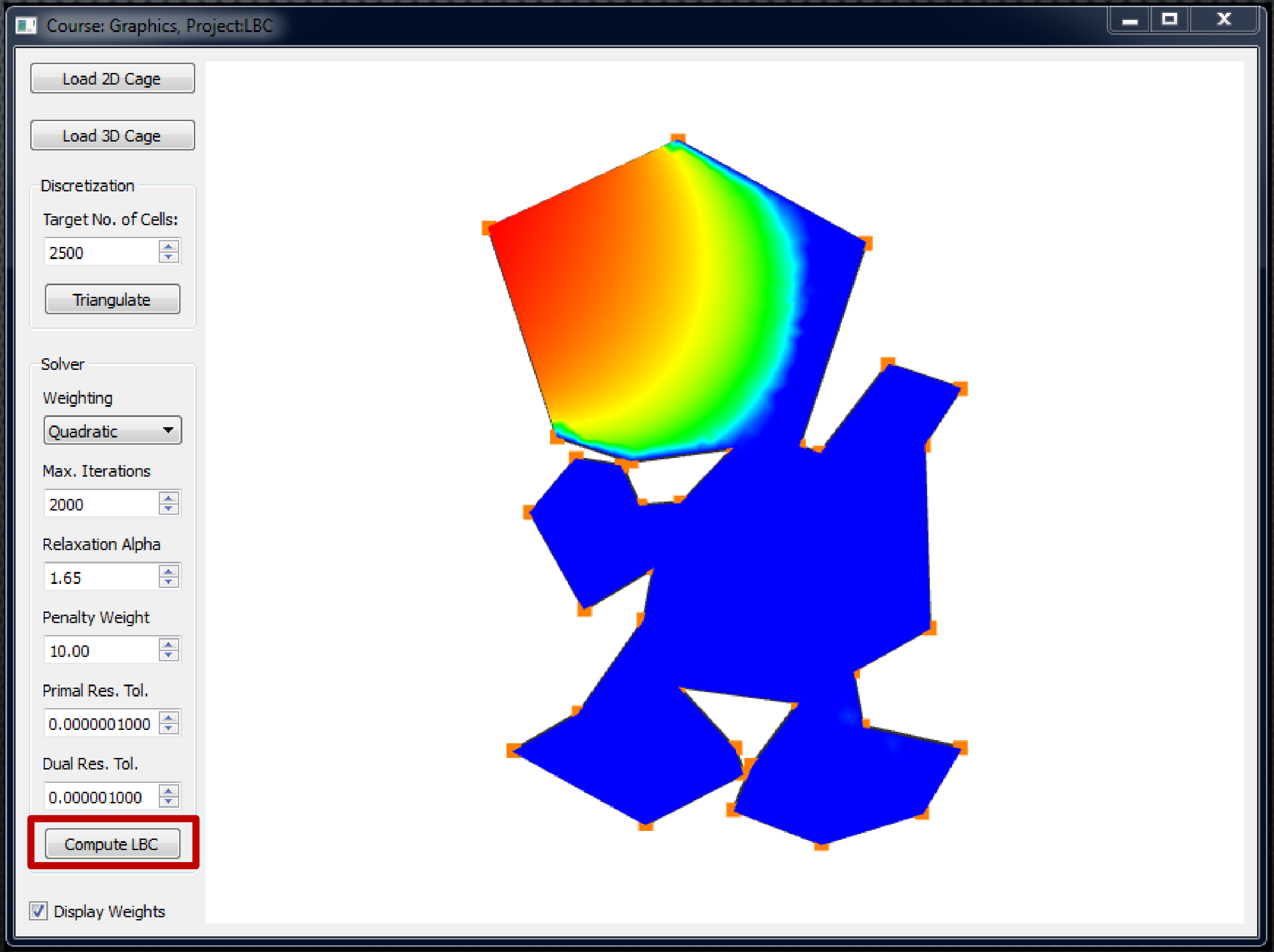
STEPS – 2D

CREATE A DELAUNAY TRIANGULATION FOR THE INSIDE OF THE BOX.

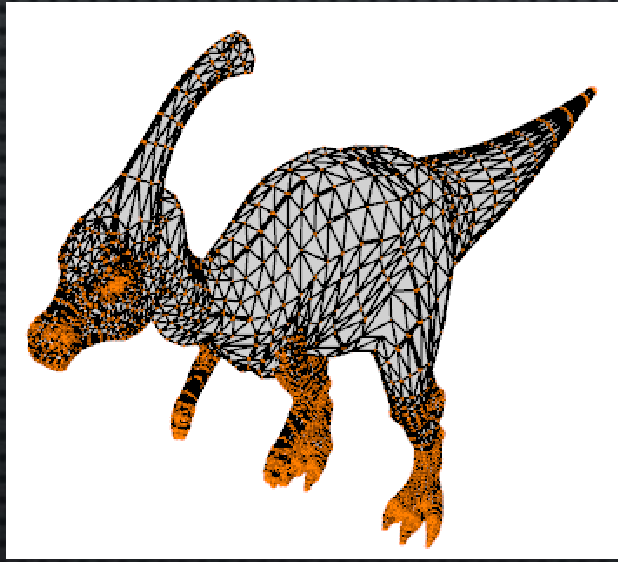


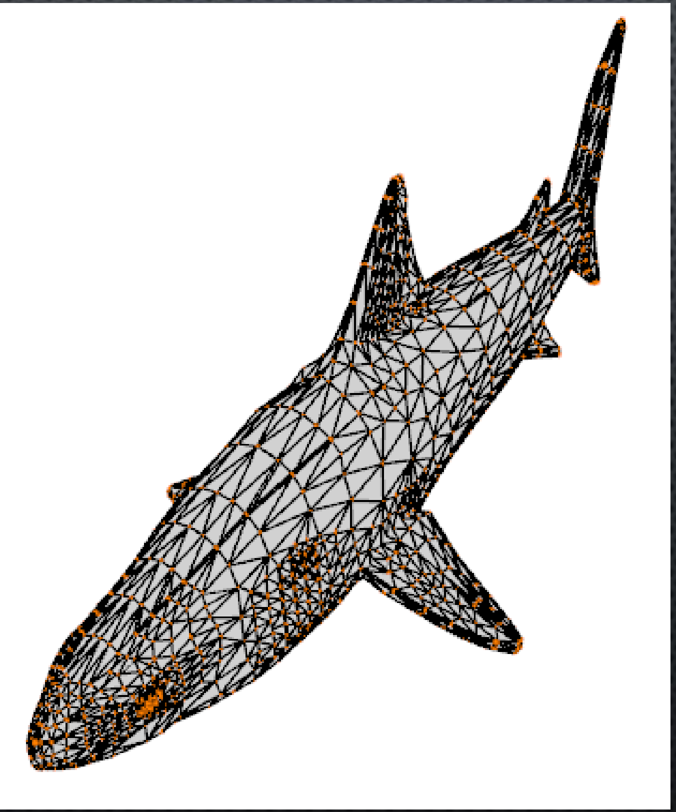
STEPS – 2D

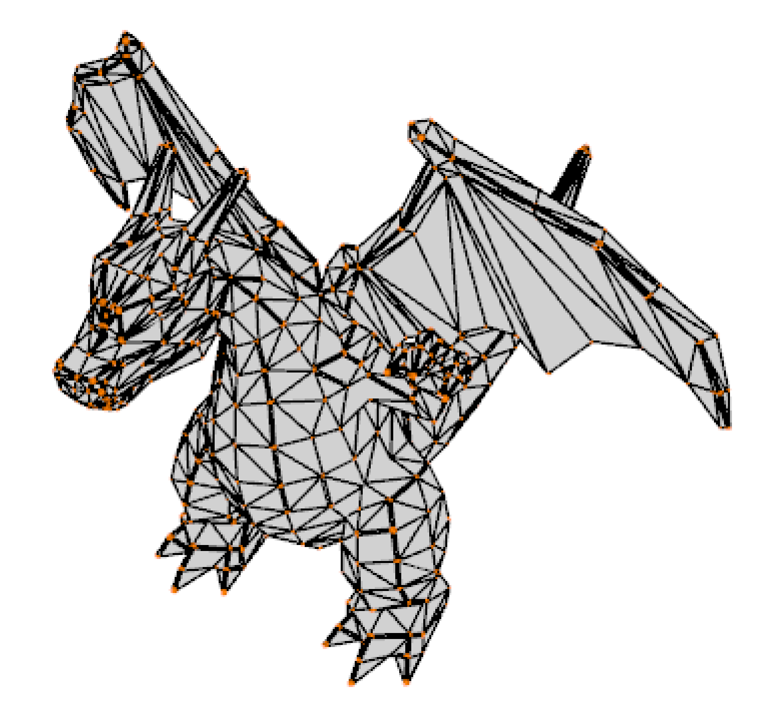
APPLY THE LOCAL BARYCENTRIC SOLVER METHOD TO GENERATE DISTORTIONS ON EACH CONTROL POINT.



PROBLEM 3D – NO CAGE DATA

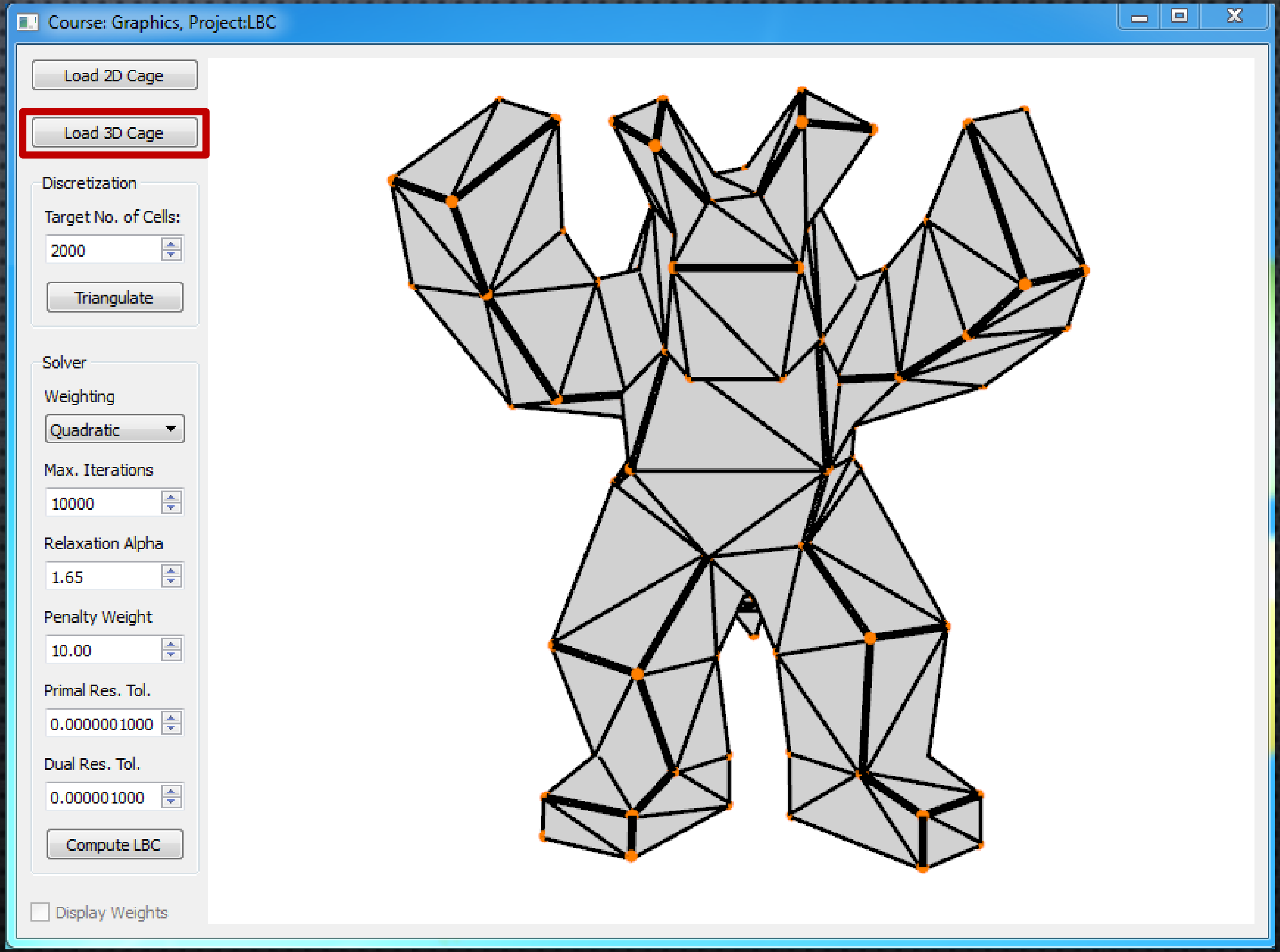






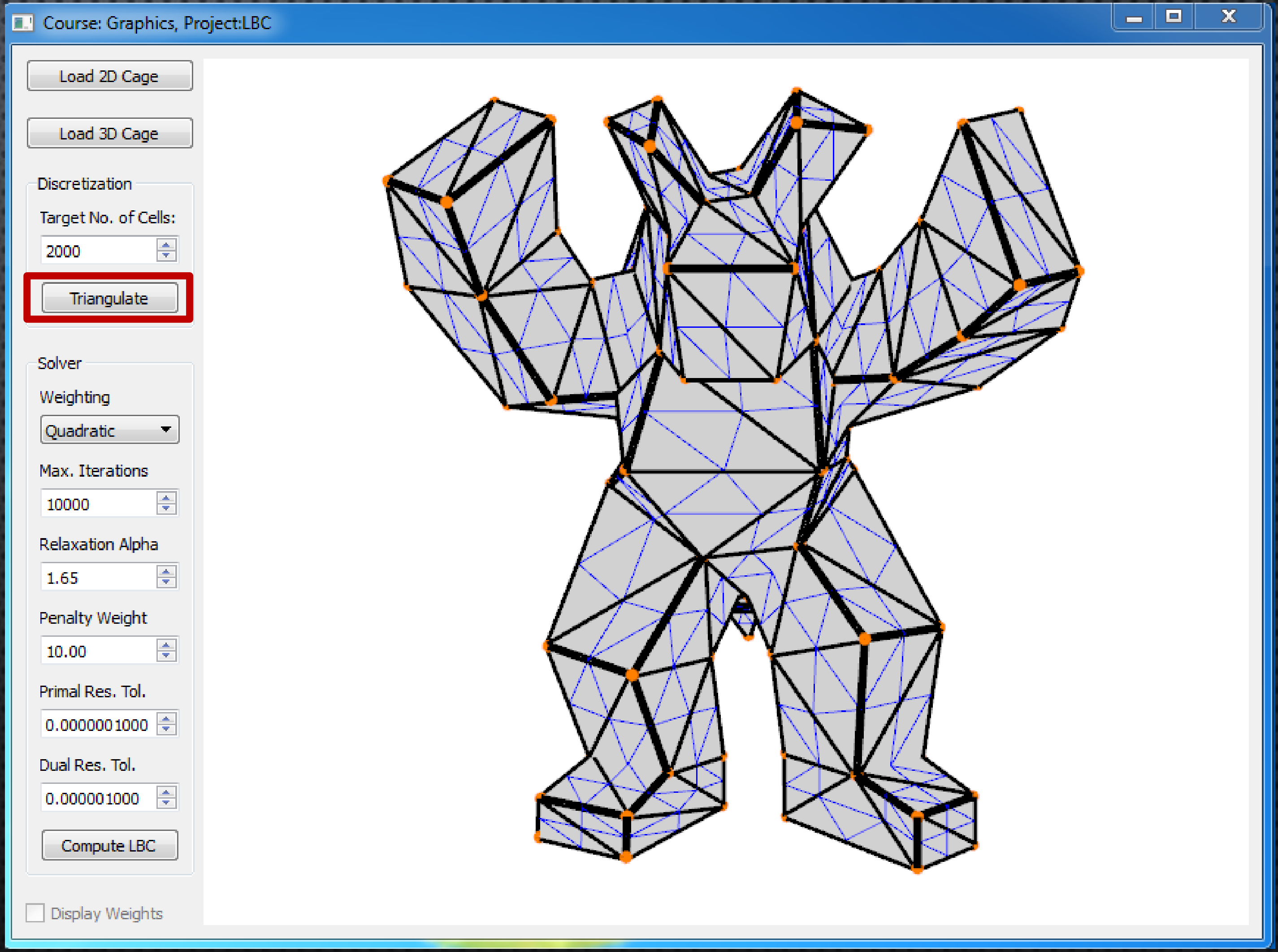
STEPS – 3D

LOAD A MODEL (CAGE-DEFORMATION)



STEPS – 2D

CREATE A DELAUNAY TRIANGULATION FOR THE INSIDE OF THE BOX.



COMPILATION

- Language: C++ 11
- Framework: Qt 5.2.0 MinGW 32 bit
- External Libraries:
  - EIGEN: For Linear Algebra operations (openMP)
  - TRIANGLE: For Delaunay Triangulation (2D)

CONCLUSIONS

- THE CODE WAS INITIALLY IMPLEMENTED **ONLY** FOR CAGE-BASE IN **2D**.
- THE FIRST CONTRIBUTION WAS TO USE MODERN OPENGL TO RENDER MODELS, EITHER IN 2D.
- THE SECOND CONTRIBUTION WAS TO LOAD MODELS IN 3D, BUT WITH THE ABSENCE OF THEIR BOXES, BECAUSE THE CODE WORKED ON IT.
- THANKS TO THE AUTHORS I WAS ABLE TO LOAD A MODEL SHOWN IN THE PAPER, THEN FOR A TIME IT WAS NOT POSSIBLE TO IMPLEMENT THE LBC ALGORITHM FOR 3D BOXES, IT WOULD REMAIN AS FUTURE WORK.

THANKS