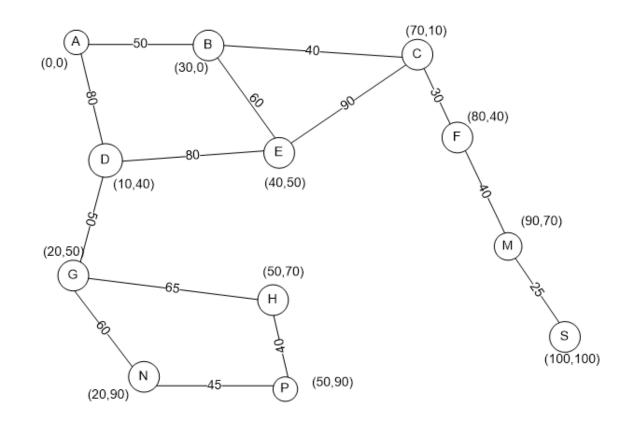
Exercise 2

Suppose two friends live in different cities.

In each turn: move both friends to an adjacent city (time = distance)

The friend who arrives first must wait until the other arrives before the next turn can begin.

We want the two friends to meet as soon as possible.



(a) Write a detailed formulation for this search problem

- State space: $\{(i,j) \ \forall (i,j) \in V \times V\}$
- Successor function: $(i,j) \rightarrow \{(x,y) \text{ with } (i,x) \in E, (j,y) \in E\}$
- Goal: $\{(i,i) \ \forall i \in V\}$
- Step cost function: $\max(d(i,x), d(j,y))$

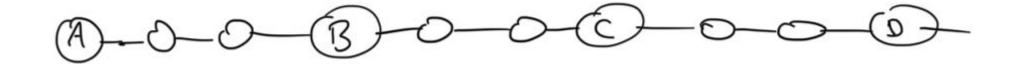
(b) Are there any fully connected (not an island) maps for which there is no solution?

Yes, since both have to move in each turn



with (A,B) being the starting state

or more generally: any chain with odd spacing



Starting states: (A,B), (A,D), (B,C), ...

(c) Let D(i,j) be the aerial distance between city i and j. Which of the following heuristic functions are admissible?

- D(i,j)
- 2 * D(i,j)
- D(i,j) / 2

Reminder: a heuristic function is admissible iff:

 $\forall n: h(n) \leq h^*(n)$ with

 $h^*(n)$ being the real costs

I.e.: A heuristic function shall not overestimate the costs

(c) Let D(i,j) be the aerial line between city i and j. Which of the following heuristic functions are admissible?

• D(i,j)

- not admissible. Counterexample:
- (A) 3 3 (B)

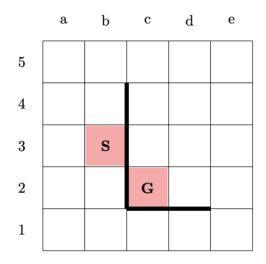
- 2 * D(i,j)
- → not admissible (even worse)

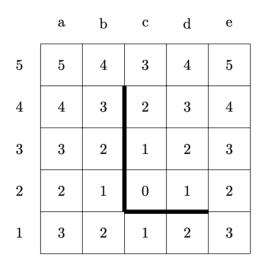
• D(i,j) / 2

→ admissible (heuristic shall not overestimate)

Exercise 3

A mobile robot wants to determine the shortest path from cell S (start) to cell G (goal). The robot can move horizontally and vertically, as long as it is not separated from the current cell by a wall (thick black line). Each movement has a uniform cost of 1. The left figure shows the initial state, the right figure the heuristic values.





	a	b	\mathbf{c}	d	e
5					
4					
3		S			
2			G		
1					

	\mathbf{a}	b	\mathbf{c}	d	e
5	5	4	3	4	5
4	4	3	2	3	4
3	3	2	1	2	3
2	2	1	0	1	2
1	3	2	1	2	3

Node	f = g + h	expanded

	\mathbf{a}	b	\mathbf{c}	d	e
5					
4					
3		s			
2			G		
1					

	\mathbf{a}	b	\mathbf{c}	d	e
5	5	4	3	4	5
4	4	3	2	3	4
3	3	2	1	2	3
2	2	1	0	1	2
1	3	2	1	2	3

Node	f = g + h	expanded
В3		

	a	b	\mathbf{c}	d	e
5					
4					
3		s			
2			G		
1					

	\mathbf{a}	b	\mathbf{c}	d	e
5	5	4	3	4	5
4	4	3	2	3	4
3	3	2	1	2	3
2	2	1	0	1	2
1	3	2	1	2	3

Node	f = g + h	expanded
В3	0+2=2	

	a	b	\mathbf{c}	d	e
5					
4					
3		s			
2			G		
1					

	a	b	\mathbf{c}	d	e
5	5	4	3	4	5
4	4	3	2	3	4
3	3	2	1	2	3
2	2	1	0	1	2
1	3	2	1	2	3

Node	f = g + h	expanded
В3	0+2=2	✓
B4		
А3		
B2		

	\mathbf{a}	b	С	d	е
5					
4					
3		S			
2			G		
1					

	a	b	c	d	e
5	5	4	3	4	5
4	4	3	2	3	4
3	3	2	1	2	3
2	2	1	0	1	2
1	3	2	1	2	3

Node	f = g + h	expanded
В3	0+2=2	✓
B4	1+3=4	
А3	1+3=4	
B2	1+1=2	\checkmark
A2	2+2=4	
B1	2+2=4	

	a	b	c	d	е
5					
4					
3		s			
2			G		
1					

	a	b	\mathbf{c}	d	e
5	5	4	3	4	5
4	4	3	2	3	4
3	3	2	1	2	3
2	2	1	0	1	2
1	3	2	1	2	3

Node	f = g + h	expanded
В3	0+2=2	✓
B4	1+3=4	✓
A3	1+3=4	
B2	1+1=2	✓
A2	2+2=4	
B1	2+2=4	
B5	2+4=6	
A4	2+4=6	

	\mathbf{a}	b	\mathbf{c}	d	e
5					
4					
3		s			
2			G		
1					

	\mathbf{a}	b	\mathbf{c}	d	e
5	5	4	3	4	5
4	4	3	2	3	4
3	3	2	1	2	3
2	2	1	0	1	2
1	3	2	1	2	3

Node	f = g + h	expanded
В3	0+2=2	✓
B4	1+3=4	\checkmark
A3	1+3=4	✓
B2	1+1=2	\checkmark
A2	2+2=4	
B1	2+2=4	
B5	2+4=6	
A4	2+4=6	

	a	b	\mathbf{c}	d	e
5					
4					
3		S			
2			G		
1					

	a	b	\mathbf{c}	d	e
5	5	4	3	4	5
4	4	3	2	3	4
3	3	2	1	2	3
2	2	1	0	1	2
1	3	2	1	2	3

Node	f = g + h	expanded
В3	0+2=2	✓
B4	1+3=4	✓
А3	1+3=4	✓
B2	1+1=2	✓
A2	2+2=4	
B1	2+2=4	\checkmark
B5	2+4=6	
A4	2+4=6	
A1	3+3=6	
C1	3+1=4	

	a	b	\mathbf{c}	d	e
5					
4					
3		S			
2			G		
1					

	a	b	\mathbf{c}	d	e
5	5	4	3	4	5
4	4	3	2	3	4
3	3	2	1	2	3
2	2	1	0	1	2
1	3	2	1	2	3

Node	f = g + h	expanded
В3	0+2=2	✓
B4	1+3=4	✓
А3	1+3=4	✓
B2	1+1=2	✓
A2	2+2=4	✓
B1	2+2=4	✓
B5	2+4=6	
A4	2+4=6	
A1	3+3=6	
C1	3+1=4	

	a	b	\mathbf{c}	d	e
5					
4					
3		S			
2			G		
1					

	a	b	\mathbf{c}	d	e
5	5	4	3	4	5
4	4	3	2	3	4
3	3	2	1	2	3
2	2	1	0	1	2
1	3	2	1	2	3

Node	f = g + h	expanded
В3	0+2=2	✓
B4	1+3=4	✓
A3	1+3=4	✓
B2	1+1=2	✓
A2	2+2=4	✓
B1	2+2=4	✓
B5	2+4=6	
A4	2+4=6	
A1	3+3=6	
C1	3+1=4	✓
D1	4+2=6	

	\mathbf{a}	b	С	d	e
5					
4					
3		S			
2			G		
1					

	a	b	\mathbf{c}	d	e
5	5	4	3	4	5
4	4	3	2	3	4
3	3	2	1	2	3
2	2	1	0	1	2
1	3	2	1	2	3

Node	f = g + h	expanded
	***	***
B5	2+4=6	
A4	2+4=6	
A1	3+3=6	
C1	3+1=4	✓
D1	4+2=6	

	\mathbf{a}	b	\mathbf{c}	d	e
5					
4					
3		s			
2			G		
1					

	\mathbf{a}	b	\mathbf{c}	d	e
5	5	4	3	4	5
4	4	3	2	3	4
3	3	2	1	2	3
2	2	1	0	1	2
1	3	2	1	2	3

Node	f = g + h	expanded
***	***	•••
B5	2+4=6	✓
A4	2+4=6	
A1	3+3=6	
C1	3+1=4	✓
D1	4+2=6	
A5	3+5=8	
C5	3+3=6	

	\mathbf{a}	b	\mathbf{c}	d	e
5					
4					
3		s			
2			G		
1					

	\mathbf{a}	b	\mathbf{c}	d	e
5	5	4	3	4	5
4	4	3	2	3	4
3	3	2	1	2	3
2	2	1	0	1	2
1	3	2	1	2	3

Node	f = g + h	expanded
***	•••	***
B5	2+4=6	\checkmark
A4	2+4=6	✓
A1	3+3=6	
C1	3+1=4	✓
D1	4+2=6	
A5	3+5=8	
C5	3+3=6	

	\mathbf{a}	b	c	d	e
5					
4					
3		s			
2			G		
1					

	\mathbf{a}	b	\mathbf{c}	d	e
5	5	4	3	4	5
4	4	3	2	3	4
3	3	2	1	2	3
2	2	1	0	1	2
1	3	2	1	2	3

Node	f = g + h	expanded
	•••	***
B5	2+4=6	\checkmark
A4	2+4=6	✓
A1	3+3=6	✓
C1	3+1=4	✓
D1	4+2=6	
A5	3+5=8	
C5	3+3=6	

	a	b	\mathbf{c}	d	e
5					
4					
3		s			
2			G		
1					

	\mathbf{a}	b	\mathbf{c}	d	e
5	5	4	3	4	5
4	4	3	2	3	4
3	3	2	1	2	3
2	2	1	0	1	2
1	3	2	1	2	3

Node	f = g + h	expanded
		•••
B5	2+4=6	\checkmark
A4	2+4=6	✓
A1	3+3=6	\checkmark
C1	3+1=4	✓
D1	4+2=6	\checkmark
A5	3+5=8	
C5	3+3=6	
E1	5+3=8	

	\mathbf{a}	b	\mathbf{c}	d	e
5					
4					
3		s			
2			G		
1					

	\mathbf{a}	b	\mathbf{c}	d	e
5	5	4	3	4	5
4	4	3	2	3	4
3	3	2	1	2	3
2	2	1	0	1	2
1	3	2	1	2	3

Node	f = g + h	expanded
•••		
B5	2+4=6	✓
A4	2+4=6	✓
A1	3+3=6	✓
C1	3+1=4	✓
D1	4+2=6	✓
A5	3+5=8	✓
C5	3+3=6	
E1	5+3=8	

	a	b	\mathbf{c}	d	e
5					
4					
3		S			
2			G		
1					

	a	b	\mathbf{c}	d	e
5	5	4	3	4	5
4	4	3	2	3	4
3	3	2	1	2	3
2	2	1	0	1	2
1	3	2	1	2	3

Node	f = g + h	expanded
•••	•••	
B5	2+4=6	✓
A4	2+4=6	✓
A1	3+3=6	✓
C1	3+1=4	✓
D1	4+2=6	✓
A5	3+5=8	✓
C5	3+3=6	✓
E1	5+3=8	
D5	4+4=8	
C4	4+2=6	

	a	b	\mathbf{c}	d	e
5					
4					
3		s			
2			G		
1					

	a	b	\mathbf{c}	d	e
5	5	4	3	4	5
4	4	3	2	3	4
3	3	2	1	2	3
2	2	1	0	1	2
1	3	2	1	2	3

Node	f = g + h	expanded
•••	•••	
E1	5+3=8	
D5	4+4=8	
C4	4+2=6	

	\mathbf{a}	b	С	d	e
5					
4					
3		S			
2			G		
1					

	a	b	\mathbf{c}	d	e
5	5	4	3	4	5
4	4	3	2	3	4
3	3	2	1	2	3
2	2	1	0	1	2
1	3	2	1	2	3

Node	f = g + h	expanded
	•••	•••
E1	5+3=8	
D5	4+4=8	
C4	4+2=6	\checkmark
D4	5+3=8	
C3	5+1=6	

	\mathbf{a}	b	\mathbf{c}	d	e
5					
4					
3		s			
2			G		
1					

	\mathbf{a}	b	\mathbf{c}	d	e
5	5	4	3	4	5
4	4	3	2	3	4
3	3	2	1	2	3
2	2	1	0	1	2
1	3	2	1	2	3

Node	f = g + h	expanded
***	***	•••
E1	5+3=8	
D5	4+4=8	
C4	4+2=6	✓
D4	5+3=8	
C3	5+1=6	✓
D3	6+2=8	
C2	6+0=6	

	а	b	C	d	е
5	8	6	6	8	
4	6	4	6	8	
3	4	S	6	8	
2	4	2	G		
1	6	4	4	6	8

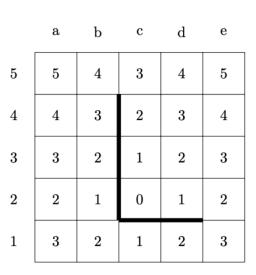
	a	b	\mathbf{c}	d	e
5	5	4	3	4	5
4	4	3	2	3	4
3	3	2	1	2	3
2	2	1	0	1	2
1	3	2	1	2	3

Node	f = g + h	expanded
	•••	•••
E1	5+3=8	
D5	4+4=8	
C4	4+2=6	✓
D4	5+3=8	
C3	5+1=6	✓
D3	6+2=8	
C2	6+0=6	

(b) is the heuristic admissible?

Yes, since it never overestimates

Can someone explain why a heuristic shall not overestimate?



- (c) How many nodes must A^* expand when $h^*(n)$ is used as a heuristic, where $h^*(n)$ is the actual cost of an optimal path from cell n to goal G?
 - B3 (start), then follow shortest path
 - So 6, if target is not expanded: B3, B4, B5, C5, C4, C3

Exercise 4

The 8-puzzle consists of 9 fields (3×3), on which 8 movable tiles are arranged; thus, one field remains free. Tiles that lie next to the free field can be moved into it. The goal is to arrange the tiles so that the following target state is reached:

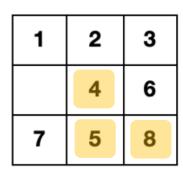
1	2	3
4	5	6
7	8	

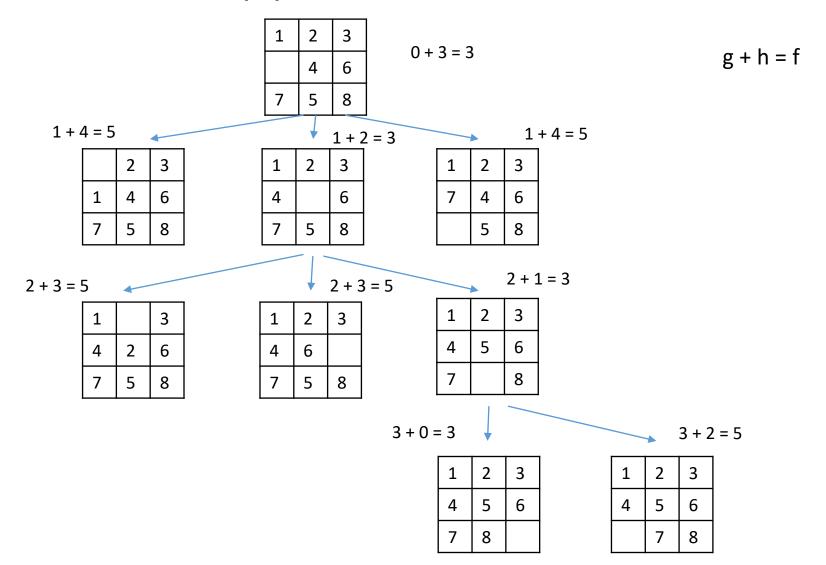
Run the A* algorithm to solve the 8-puzzle. Use the following start state for this:

1	2	3
	4	6
7	5	8

(a) Draw the search tree with the states visited by the algorithm as nodes. Also enter the corresponding f, g and h values for each node. The heuristic to be used is the number of misplaced tiles ("misplaced tiles" heuristic). So for the initial state, the heuristic would estimate a cost of 3 (tiles 4, 5 and 8 are misplaced). Moving a tile has an actual cost of 1. We assume that the algorithm remembers states that have already been visited and does not visit them again.

1	2	3
4	5	6
7	8	





(b) What is the maximum cost that the heuristic from (a) can estimate in the 8-puzzle?

All 8 tiles wrong \rightarrow 8

Exercise 4: Application of A*

(c) Another heuristic that can be used in the 8-puzzle is the summed Manhattan-Distance of all misplaced tiles to each tile's destination.

(i) Is there a state for which this heuristic estimates a lower value than the "Misplaced Tiles" heuristic?

- No
- A misplaced tile automatically has Manhattan distance of at least 1
- > Summed MD can not be lower then the number of misplaced tiles

Exercise 4: Application of A*

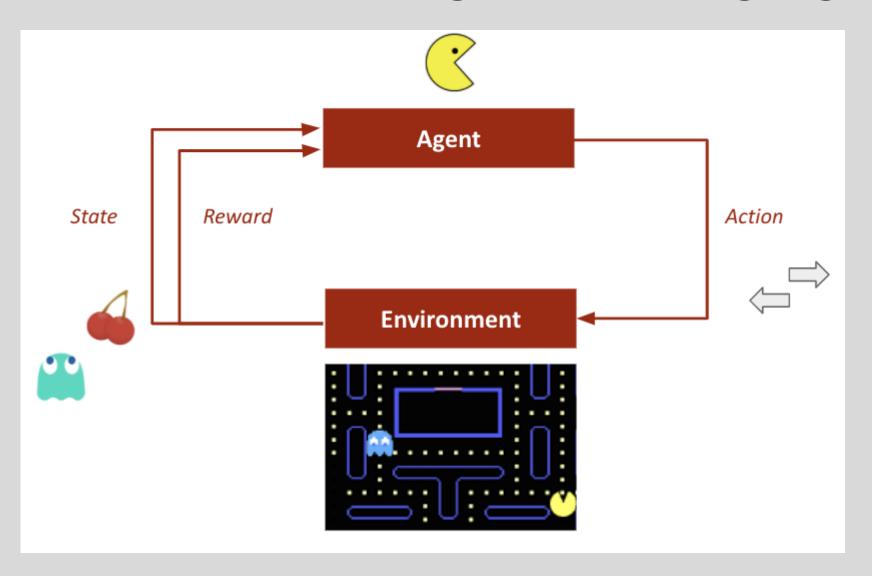
(c) Another heuristic that can be used in the 8-puzzle is the summed Manhattan-Distance of all misplaced tiles to each tile's destination.

(ii) Give an example of a state where this heuristic estimates a higher cost than the "Misplaced Tiles" heuristic

→ At least one misplaced tile must have MD > 1

3	2	1
6	5	4
	8	7

Intermezzo



• Markov Decision Process (MDP): $< S, A, T, R, \gamma >$

- *S* = *Set of all possible states*
- A = Set of all possible actions the agent can perform
- $T: S \times A \rightarrow S$ state transition function (if deterministic \rightarrow MDP, if probabilistic \rightarrow POMDP)
- $R: S \times A \rightarrow \mathbb{R}$ reward function
- $\gamma \in [0,1]$ = discount factor (prefer instantaneous rewards over delayed rewards)

- Markov Decision Process (MDP): $< S, A, T, R, \gamma >$
- Policy: mapping of states to actions

$$\pi(s_t) = a_t$$

• Learn a policy which maximizes expected cumulative reward

$$\pi^* = \underset{\pi}{\operatorname{argmax}} \left(\sum_{t} \gamma^t r(s_t) \mid a_t \sim \pi(s_t) \right)$$

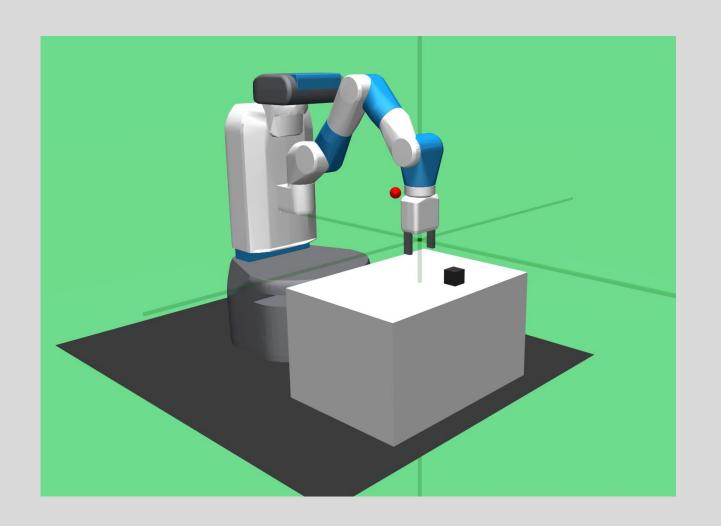
Goal reaching domain:

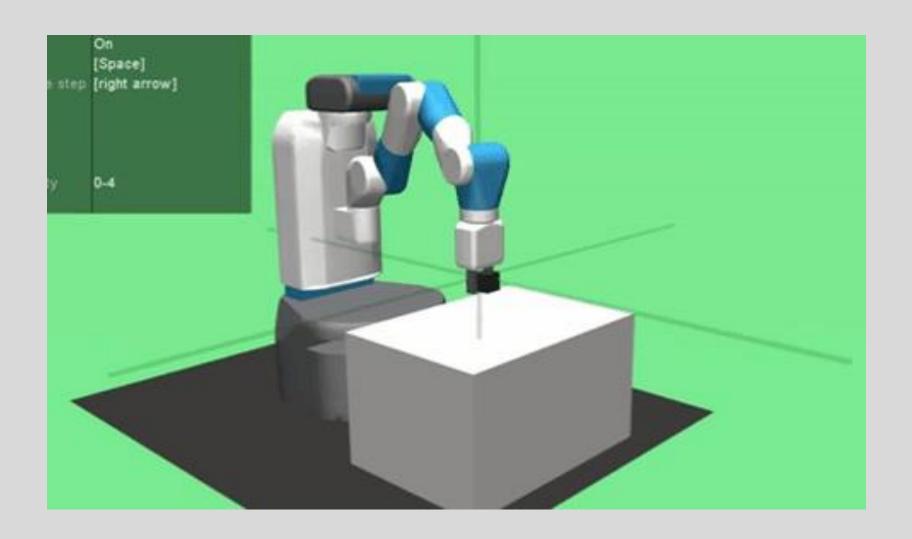
• Agent is given a goal g_d (desired goal) at the beginning of every episode which it shall achieve

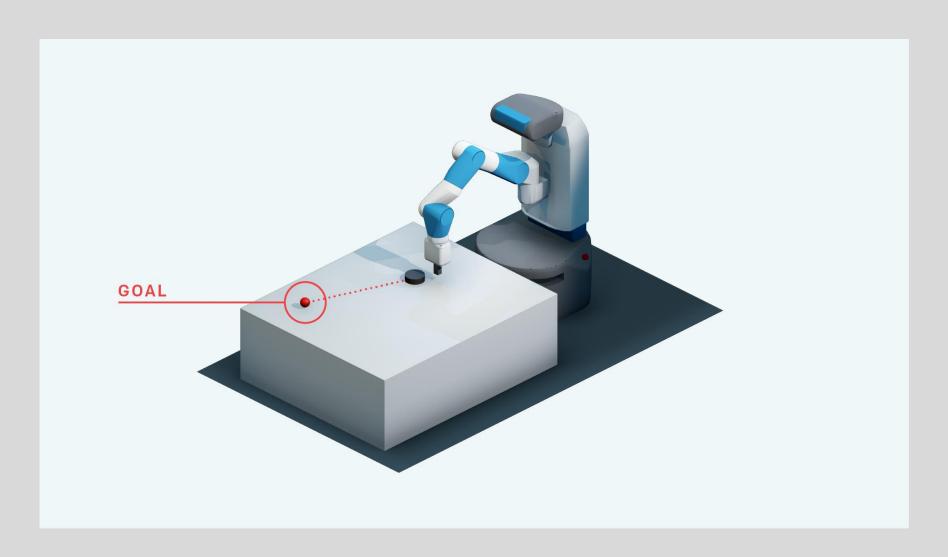
$$\pi(s_t, g_d) = a_t$$

Problem: sparse rewards. Example:

$$r(s,g)=\begin{cases} 1 & \text{if } s==g \\ 0 & \text{else} \end{cases}$$

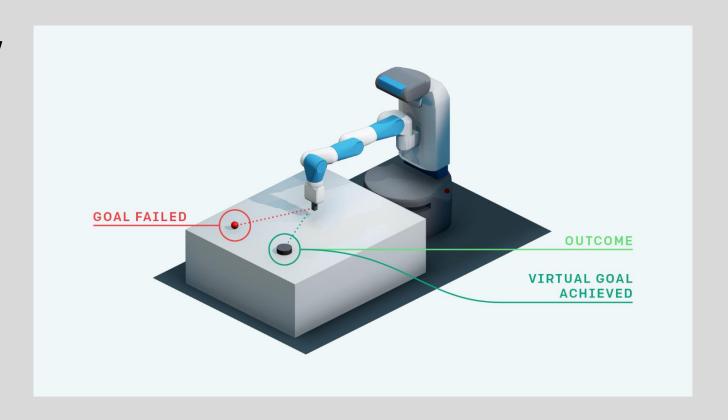






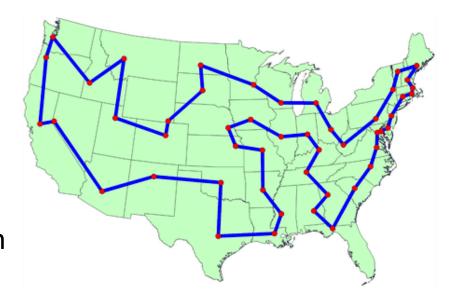
 Sparse rewards → very few training signals → very slow training in RL

- Solution: Hindsight Experience Replay (HER)
 - Even if you fail to reach the actual goal, you still learned how you reach the actually achieved goal



Traveling Salesman Problem:

- Visit every city
- Finish the tour in the same city you started from
- Minimize the length of the tour

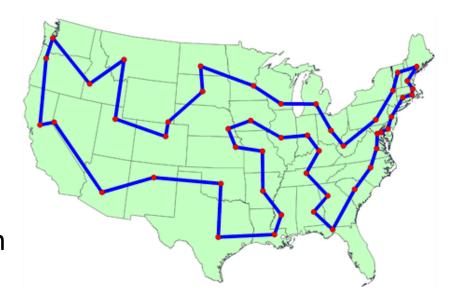


How can a problem solution (a state) be represented for these problems?

For a problem with N cities we need an array x of length N. x[i] = the city to be visited in step i

Traveling Salesman Problem:

- Visit every city
- Finish the tour in the same city you started from
- Minimize the length of the tour



How can a neighboring state be constructed given another state?

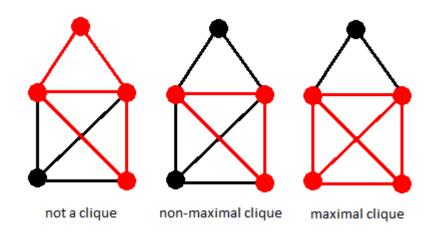
Given state *x*

set of neighbor states:

 $N(x) = \{x \text{ with } x[i] \text{ and } x[j] \text{ swapped } | \forall i \in [1, N] \forall j \in [1, N]: i \neq j\}$

Maximum Clique:

- Given an undirected graph
- Determine the largest set of nodes
 with all nodes being pairwise connected by an edge

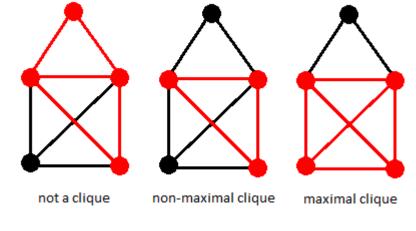


How can a problem solution (a state) be represented for these problems?

For a problem with N vertices we need a binary array x of length N. $x[i] = 1 \Leftrightarrow \text{node } i$ is part of the clique

Maximum Clique:

- Given an undirected graph
- Determine the largest set of nodes
 with all nodes being pairwise connected by an edge



How can a neighboring state be constructed given another state?

Given state *x*

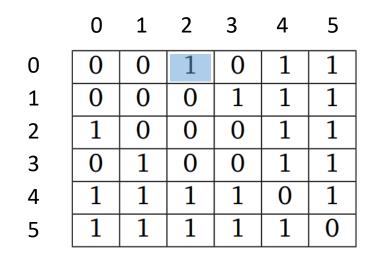
set of neighbor states:

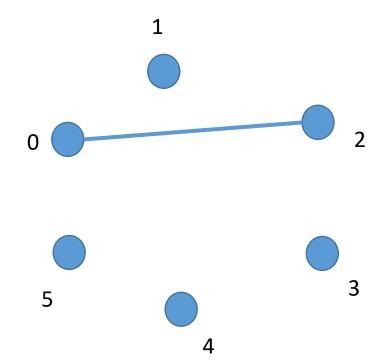
 $N(x) = \{x \text{ with flipped } x[i] \ \forall \ i \in [1, N] \}$

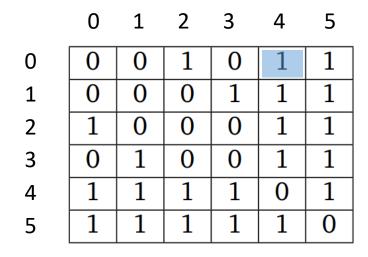
Consider the following (undirected) graph represented by an adjacency matrix:

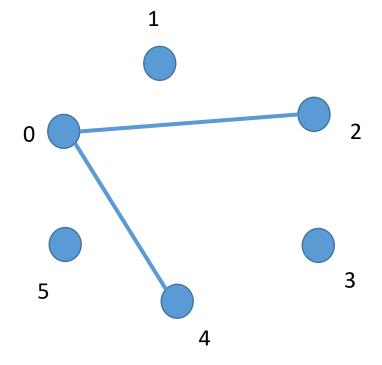
0	0	1	0	1	1
0	0	0	1	1	1
1	0	0	0	1	1
0	1	0	0	1	1
1	1	1	1	0	1
1	1	1	1	1	0

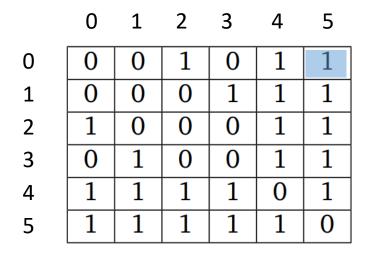
a) Draw the graph

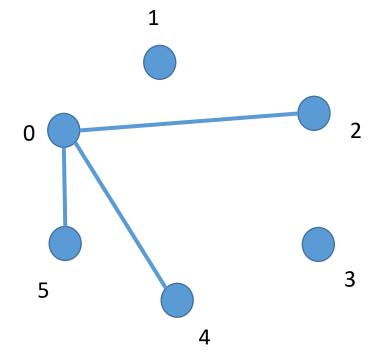


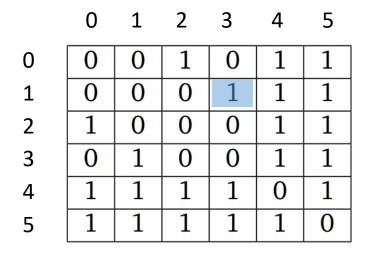


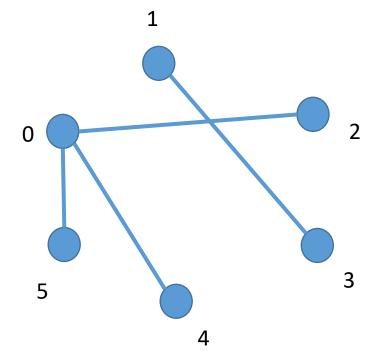


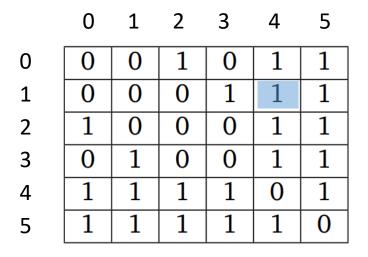


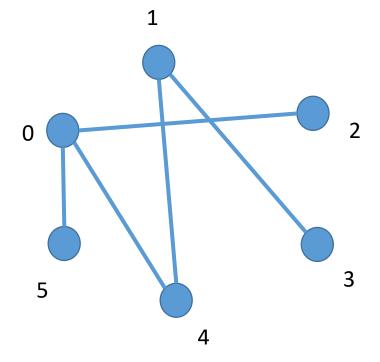


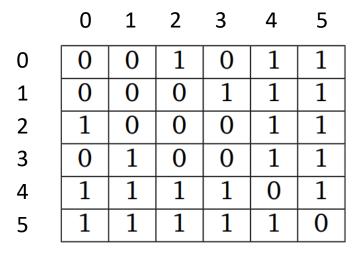


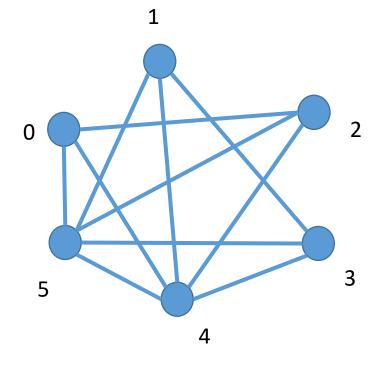




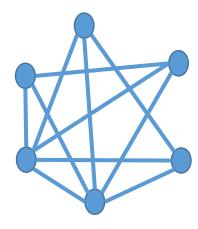


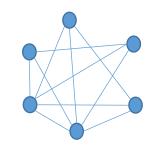




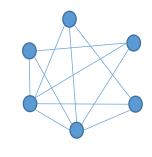


b) Write the objective function for Maximum Clique and the graph G in Python.



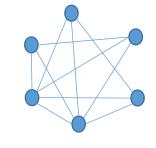


Iteration	S _{current}	Value of Scurrent	Tabu list	S _{best}	Value of s_{best}
1	[0,0,0,0,0,0]	0	[[0,0,0,0,0]]	[0,0,0,0,0,0]	0
2					



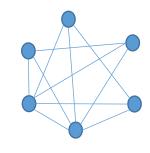
Iteration	Scurrent	Value of $s_{current}$	Tabu list	S _{best}	Value of s_{best}
1	[0,0,0,0,0,0]	0	[[0,0,0,0,0]]	[0,0,0,0,0,0]	0
2					

Neighbors of s _{current}	Value
[1,0,0,0,0,0]	1
[0,1,0,0,0,0]	1
[0,0,1,0,0,0]	1
[0,0,0,1,0,0]	1
[0,0,0,0,1,0]	1
[0,0,0,0,0,1]	1



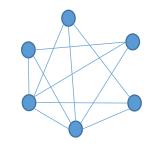
Iteration	Scurrent	Value of $S_{current}$	Tabu list	S _{best}	Value of s_{best}
1	[0,0,0,0,0,0]	0	[[0,0,0,0,0]]	[0,0,0,0,0,0]	0
2	[1,0,0,0,0,0]	1	[[0,0,0,0,0,0], [1,0,0,0,0,0]]	[1,0,0,0,0,0]	1

Neighbors of s _{current}	Value
[1,0,0,0,0,0]	1
[0,1,0,0,0,0]	1
[0,0,1,0,0,0]	1
[0,0,0,1,0,0]	1
[0,0,0,0,1,0]	1
[0,0,0,0,0,1]	1



Iteration	S _{current}	Value of Scurrent	Tabu list	s_{best}	Value of s_{best}
1	[0,0,0,0,0,0]	0	[[0,0,0,0,0]]	[0,0,0,0,0,0]	0
2	[1,0,0,0,0,0]	1	[[0,0,0,0,0,0], [1,0,0,0,0,0]]	[1,0,0,0,0,0]	1
3					

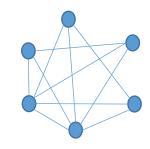




Iteration	Scurrent	Value of	Tabu list	S _{best}	Value of s_{best}
		Scurrent			or spest
1	[0,0,0,0,0,0]	0	[[0,0,0,0,0]]	[0,0,0,0,0,0]	0
2	[1,0,0,0,0,0]	1	[[0,0,0,0,0,0], [1,0,0,0,0,0]]	[1,0,0,0,0,0]	1
3					

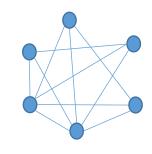
Neighbors of $s_{current}$	Value
[0,0,0,0,0,0]	tabu
[1,1,0,0,0,0]	0
[1,0,1,0,0,0]	2
[1,0,0,1,0,0]	0
[1,0,0,0,1,0]	2
[1,0,0,0,0,1]	2 66





Iteration	Scurrent	Value of	Tabu list	S _{best}	Value
		Scurrent			of s _{best}
1	[0,0,0,0,0,0]	0	[[0,0,0,0,0]]	[0,0,0,0,0,0]	0
2	[1,0,0,0,0,0]	1	[[0,0,0,0,0,0], [1,0,0,0,0,0]]	[1,0,0,0,0,0]	1
3	[1,0,1,0,0,0]	2	[[0,0,0,0,0,0], [1,0,0,0,0,0], [1,0,1,0,0,0]]	[1,0,1,0,0,0]	2

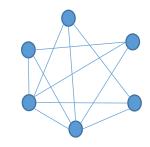
Neighbors of $s_{current}$	Value
[0,0,0,0,0,0]	tabu
[1,1,0,0,0,0]	0
[1,0,1,0,0,0]	2
[1,0,0,1,0,0]	0
[1,0,0,0,1,0]	2
[1,0,0,0,0,1]	2 67



Iteration	S _{current}	Value of Scurrent	Tabu list	S _{best}	Value of s_{best}
1	[0,0,0,0,0,0]	0	[[0,0,0,0,0]]	[0,0,0,0,0,0]	0
2	[1,0,0,0,0,0]	1	[[0,0,0,0,0,0], [1,0,0,0,0,0]]	[1,0,0,0,0,0]	1
3	[1,0,1,0,0,0]	2	[[0,0,0,0,0,0], [1,0,0,0,0,0], [1,0,1,0,0,0]]	[1,0,1,0,0,0]	2
4					

c) Maximize the oracle 0, 0]. Use a tabu list s

Neighbors of $s_{current}$	Value
[0,0,1,0,0,0]	1
[1,1,1,0,0,0]	0
[1,0,0,0,0,0]	tabu
[1,0,1,1,0,0]	0
[1,0,1,0,1,0]	3
[1,0,1,0,0,1]	3

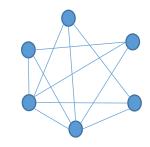


starting state [0, 0, 0, 0, ncluding) iteration 8.

Iteration	S _{current}	Value of $S_{current}$	[1,0,1,0,0,1]	3		S _{best}	Value of s_{best}
1	[0,0,0,0,0,0]	0	[[0,0,0,0,0]]		[0,0,0,0,0,0]	0	
2	[1,0,0,0,0,0]	1	[[0,0,0,0,0,0], [1,0,0,0,0,0]]		[1,0,0,0,0,0]	1	
3	[1,0,1,0,0,0]	2	[[0,0,0,0,0,0], [1,0,0,0,0], [1,0,1,0,0,0]]		[1,0,1,0,0,0]	2	
4							

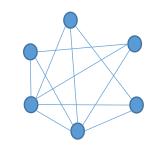
c) Maximize the oracle 0, 0]. Use a tabu list s

Neighbors of $s_{current}$	Value
[0,0,1,0,0,0]	1
[1,1,1,0,0,0]	0
[1,0,0,0,0,0]	tabu
[1,0,1,1,0,0]	0
[1,0,1,0,1,0]	3
[1,0,1,0,0,1]	3



starting state [0, 0, 0, 0, ncluding) iteration 8.

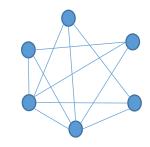
Iteration	S _{current}	Value of Scurrent	[1,0,1,0,0,1]	3		s_{best}	Value of s_{best}
1	[0,0,0,0,0,0]	0	[[0,0,0,0,0]]		[0,0,0,0,0,0]	0	
2	[1,0,0,0,0,0]	1	[[0,0,0,0,0,0], [1,0,0,0,0,0]]		[1,0,0,0,0,0]	1	
3	[1,0,1,0,0,0]	2	[[0,0,0,0,0,0], [1,0,0,0,0], [1,0,1,0,0,0]]		[1,0,1,0,0,0]	2	
4	[1,0,1,0,1,0]	3	[[1,0,0,0,0,0], [1,0,1,0,0,0], [1,0,1,0,1,0]]		[1,0,1,0,1,0]	3	



Iteration	Scurrent	Value of Scurrent	Tabu list	S _{best}	Value of s_{best}
1	[0,0,0,0,0,0]	0	[[0,0,0,0,0]]	[0,0,0,0,0,0]	0
2	[1,0,0,0,0,0]	1	[[0,0,0,0,0,0], [1,0,0,0,0,0]]	[1,0,0,0,0,0]	1
3	[1,0,1,0,0,0]	2	[[0,0,0,0,0,0], [1,0,0,0,0,0], [1,0,1,0,0,0]]	[1,0,1,0,0,0]	2
4	[1,0,1,0,1,0]	3	[[1,0,0,0,0,0], [1,0,1,0,0,0], [1,0,1,0,1,0]]	[1,0,1,0,1,0]	3
5					

c) Maximize the oracle 0, 0]. Use a tabu list s

Neighbors of $s_{current}$	Value
[0,0,1,0,1,0]	2
[1,1,1,0,1,0]	0
[1,0,0,0,1,0]	2
[1,0,1,1,1,0]	0
[1,0,1,0,0,0]	tabu
[1,0,1,0,1,1]	4

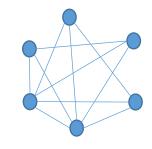


starting state [0, 0, 0, 0, ncluding) iteration 8.

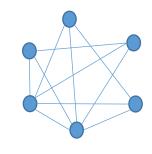
Iteration	S _{current}	Value of Scurrent	2 / / / / 2	4		s_{best}	Value of s_{best}
1	[0,0,0,0,0,0]	0	[[0,0,0,0,0,0]]		[0,0,0,0,0,0]	0	
2	[1,0,0,0,0,0]	1	[[0,0,0,0,0,0], [1,0,0,0,0,0]]		[1,0,0,0,0,0]	1	
3	[1,0,1,0,0,0]	2	[[0,0,0,0,0,0], [1,0,0,0,0], [1,0,1,0,0,0]]		[1,0,1,0,0,0]	2	
4	[1,0,1,0,1,0]	3	[[1,0,0,0,0,0], [1,0,1,0,0,0], [1,0,1,0,1,0]]		[1,0,1,0,1,0]	3	
5							

c) Maximize the oracle 0, 0]. Use a tabu list s

Neighbors of $s_{current}$	Value
[0,0,1,0,1,0]	2
[1,1,1,0,1,0]	0
[1,0,0,0,1,0]	2
[1,0,1,1,1,0]	0
[1,0,1,0,0,0]	tabu
[1,0,1,0,1,1]	4



Iteration	S _{current}	Value of Scurrent	[1,0,1,0,1,1]	4		s_{best}	Value of s_{best}
1	[0,0,0,0,0,0]	0	[[0,0,0,0,0,0]]		[0,0,0,0,0,0]	0	
2	[1,0,0,0,0,0]	1	[[0,0,0,0,0,0], [1,0,0,0,0,0]]		[1,0,0,0,0,0]	1	
3	[1,0,1,0,0,0]	2	[[0,0,0,0,0,0], [1,0,0,0,0], [1,0,1,0,0,0]]		[1,0,1,0,0,0]	2	
4	[1,0,1,0,1,0]	3	[[1,0,0,0,0,0], [1,0,1,0,0,0], [1,0,1,0,1,0]]		[1,0,1,0,1,0]	3	
5	[1,0,1,0,1,1]	4	[[1,0,1,0,0,0], [1	[[1,0,1,0,0,0], [1,0,1,0,1,0], [1,0,1,0,1,1]]		[1,0,1,0,1,1]	4

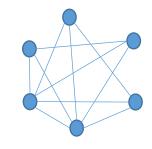


c) Maximize the oracle function from (b) using Tabu Search. Use the starting state [0, 0, 0, 0, 0, 0, 0]. Use a tabu list size of 3. Complete the following table until (including) iteration 8.

Iteration	Scurrent	Value of $S_{current}$	Tabu list	S _{best}	Value of s_{best}
1	[0,0,0,0,0,0]	0	[[0,0,0,0,0]]	[0,0,0,0,0,0]	0
2	[1,0,0,0,0,0]	1	[[0,0,0,0,0,0], [1,0,0,0,0,0]]	[1,0,0,0,0,0]	1
3	[1,0,1,0,0,0]	2	[[0,0,0,0,0,0], [1,0,0,0,0,0], [1,0,1,0,0,0]]	[1,0,1,0,0,0]	2
4	[1,0,1,0,1,0]	3	[[1,0,0,0,0,0], [1,0,1,0,0,0], [1,0,1,0,1,0]]	[1,0,1,0,1,0]	3
5	[1,0,1,0,1,1]	4	[[1,0,1,0,0,0], [1,0,1,0,1,0], [1,0,1,0,1,1]]	[1,0,1,0,1,1]	4
6					

c) Maximize the oracle 0, 0]. Use a tabu list s

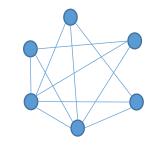
Neighbors of $s_{current}$	Value
[0,0,1,0,1,1]	3
[1,1,1,0,1,1]	0
[1,0,0,0,1,1]	3
[1,0,1,1,1,1]	0
[1,0,1,0,0,1]	3
[1,0,1,0,1,0]	tabu



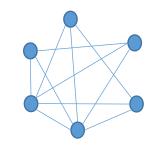
Iteration	Scurrent	Value of $S_{current}$	[1,0,1,0,1,0]	tabu		S _{best}	Value of s_{best}
1	[0,0,0,0,0,0]	0	[[0,0,0,0,0,0]]		[0,0,0,0,0,0]	0	
2	[1,0,0,0,0,0]	1	[[0,0,0,0,0,0], [1,0,0,0,0,0]] [[1,0,0,0,0,0]	1	
3	[1,0,1,0,0,0]	2	[[0,0,0,0,0,0], [1,0,0,0,0], [1,0,1,0,0,0]]		[1,0,1,0,0,0]	2	
4	[1,0,1,0,1,0]	3	[[1,0,0,0,0,0], [1,0,1,0,0,0], [1,0,1,0,1,0]]		[1,0,1,0,1,0]	3	
5	[1,0,1,0,1,1]	4	[[1,0,1,0,0,0], [1,	0,1,0,1,0], [1,0,1,0,1,1]]		[1,0,1,0,1,1]	4
6							

c) Maximize the oracle 0, 0]. Use a tabu list s

Neighbors of $s_{current}$	Value
[0,0,1,0,1,1]	3
[1,1,1,0,1,1]	0
[1,0,0,0,1,1]	3
[1,0,1,1,1,1]	0
[1,0,1,0,0,1]	3
[1,0,1,0,1,0]	tabu



Iteration	S _{current}	Value of Scurrent	[1,0,1,0,1,0]	tabu		S _{best}	Value of s_{best}
1	[0,0,0,0,0,0]	0	[[0,0,0,0,0,0]]		[0,0,0,0,0,0]	0	
2	[1,0,0,0,0,0]	1	[[0,0,0,0,0,0], [1,0,0,0,0,0]]		[1,0,0,0,0,0]	1	
3	[1,0,1,0,0,0]	2	[[0,0,0,0,0,0], [1,0,0,0,0], [1,0,1,0,0,0]]		[1,0,1,0,0,0]	2	
4	[1,0,1,0,1,0]	3	[[1,0,0,0,0,0], [1,0,1,0,0,0], [1,0,1,0,1,0]]		[1,0,1,0,1,0]	3	
5	[1,0,1,0,1,1]	4	[[1,0,1,0,0,0], [1,	0,1,0,1,0], [1,0,1,0,1,1]]		[1,0,1,0,1,1]	4
6	[0,0,1,0,1,1]	3	[[1,0,1,0,1,0], [1,	0,1,0,1,1], [0,0,1,0,1,1]]		[1,0,1,0,1,1]	4

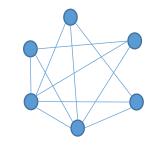


c) Maximize the oracle function from (b) using Tabu Search. Use the starting state [0, 0, 0, 0, 0, 0, 0]. Use a tabu list size of 3. Complete the following table until (including) iteration 8.

Iteration	S _{current}	Value of $S_{current}$	Tabu list	S _{best}	Value of s_{best}
1	[0,0,0,0,0,0]	0	[[0,0,0,0,0]]	[0,0,0,0,0,0]	0
2	[1,0,0,0,0,0]	1	[[0,0,0,0,0,0], [1,0,0,0,0,0]]	[1,0,0,0,0,0]	1
3	[1,0,1,0,0,0]	2	[[0,0,0,0,0,0], [1,0,0,0,0], [1,0,1,0,0,0]]	[1,0,1,0,0,0]	2
4	[1,0,1,0,1,0]	3	[[1,0,0,0,0,0], [1,0,1,0,0,0], [1,0,1,0,1,0]]	[1,0,1,0,1,0]	3
5	[1,0,1,0,1,1]	4	[[1,0,1,0,0,0], [1,0,1,0,1,0], [1,0,1,0,1,1]]	[1,0,1,0,1,1]	4
6	[0,0,1,0,1,1]	3	[[1,0,1,0,1,0], [1,0,1,0,1,1], [0,0,1,0,1,1]]	[1,0,1,0,1,1]	4
7					

c) Maximize the oracle 0, 0]. Use a tabu list s

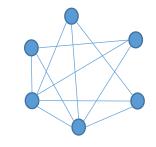
Neighbors of s _{current}	Value
[1,0,1,0,1,1]	tabu
[0,1,1,0,1,1]	0
[0,0,0,0,1,1]	2
[0,0,1,1,1,1]	0
[0,0,1,0,0,1]	2
[0,0,1,0,1,0]	2



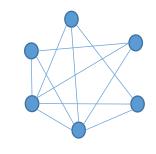
Iteration	S _{current}	Value of $S_{current}$	[0,0,1,0,1,0]	2		S _{best}	Value of s_{best}
1	[0,0,0,0,0,0]	0	[[0,0,0,0,0,0]]		[0,0,0,0,0,0]	0	
2	[1,0,0,0,0,0]	1	[[0,0,0,0,0,0], [1,0,0,0,0,0]] [1		[1,0,0,0,0,0]	1	
3	[1,0,1,0,0,0]	2	[[0,0,0,0,0,0], [1,0,0,0,0], [1,0,1,0,0,0]] [[1,0,1,0,0,0]	2	
4	[1,0,1,0,1,0]	3	[[1,0,0,0,0,0], [1,0,1,0,0,0], [1,0,1,0,1,0]]		[1,0,1,0,1,0]	3	
5	[1,0,1,0,1,1]	4	[[1,0,1,0,0,0], [1,	0,1,0,1,0], [1,0,1,0,1,1]]		[1,0,1,0,1,1]	4
6	[0,0,1,0,1,1]	3	[[1,0,1,0,1,0], [1,	0,1,0,1,1], [0,0,1,0,1,1]]		[1,0,1,0,1,1]	4
7							

c) Maximize the oracle 0, 0]. Use a tabu list s

Neighbors of $s_{current}$	Value
[1,0,1,0,1,1]	tabu
[0,1,1,0,1,1]	0
[0,0,0,0,1,1]	2
[0,0,1,1,1,1]	0
[0,0,1,0,0,1]	2
[0,0,1,0,1,0]	2



Iteration	Scurrent	Value of $S_{current}$	[0,0,1,0,1,0]	2		s_{best}	Value of s_{best}
1	[0,0,0,0,0,0]	0	[[0,0,0,0,0,0]]		[0,0,0,0,0,0]	0	
2	[1,0,0,0,0,0]	1	[[0,0,0,0,0,0], [1,0,0,0,0,0]] [2		[1,0,0,0,0,0]	1	
3	[1,0,1,0,0,0]	2	[[0,0,0,0,0,0], [1,0,0,0,0], [1,0,1,0,0,0]] [[1,0,1,0,0,0]	2	
4	[1,0,1,0,1,0]	3	[[1,0,0,0,0,0], [1,0,1,0,0,0], [1,0,1,0,1,0]]		[1,0,1,0,1,0]	3	
5	[1,0,1,0,1,1]	4	[[1,0,1,0,0,0], [1,0,1,0], [1,0,1,0,1,1]] [[1,0,1,0,1,1]	4	
6	[0,0,1,0,1,1]	3	[[1,0,1,0,1,0], [1,	0,1,0,1,1], [0,0,1,0,1,1]]		[1,0,1,0,1,1]	4
7	[0,0,0,0,1,1]	2	[[1,0,1,0,1,1], [0,	0,1,0,1,1], [0,0,0,0,1,1]]		[1,0,1,0,1,1]	4

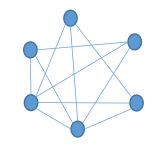


c) Maximize the oracle function from (b) using Tabu Search. Use the starting state [0, 0, 0, 0, 0, 0, 0]. Use a tabu list size of 3. Complete the following table until (including) iteration 8.

Iteration	Scurrent	Value of Scurrent	Tabu list	S _{best}	Value of s_{best}
1	[0,0,0,0,0,0]	0	[[0,0,0,0,0]]	[0,0,0,0,0,0]	0
2	[1,0,0,0,0,0]	1	[[0,0,0,0,0,0], [1,0,0,0,0,0]]	[1,0,0,0,0,0]	1
3	[1,0,1,0,0,0]	2	[[0,0,0,0,0,0], [1,0,0,0,0], [1,0,1,0,0,0]]	[1,0,1,0,0,0]	2
4	[1,0,1,0,1,0]	3	[[1,0,0,0,0,0], [1,0,1,0,0,0], [1,0,1,0,1,0]]	[1,0,1,0,1,0]	3
5	[1,0,1,0,1,1]	4	[[1,0,1,0,0,0], [1,0,1,0,1,0], [1,0,1,0,1,1]]	[1,0,1,0,1,1]	4
6	[0,0,1,0,1,1]	3	[[1,0,1,0,1,0], [1,0,1,0,1,1], [0,0,1,0,1,1]]	[1,0,1,0,1,1]	4
7	[0,0,0,0,1,1]	2	[[1,0,1,0,1,1], [0,0,1,0,1,1], [0,0,0,0,1,1]]	[1,0,1,0,1,1]	4
8					

c) Maximize the oracle0, 0]. Use a tabu list s

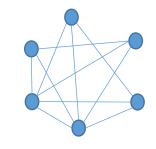
Neighbors of s _{current}	Value
[1,0,0,0,1,1]	3
[0,1,0,0,1,1]	3
[0,0,1,0,1,1]	tabu
[0,0,0,1,1,1]	3
[0,0,0,0,0,1]	1
[0,0,0,0,1,0]	1



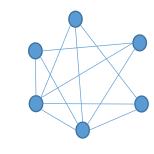
Iteration	Scurrent	Value of $s_{current}$	[0,0,0,0,1,0]	1	S _{best}	Value of s_{best}
1	[0,0,0,0,0,0]	0	[[0,0,0,0,0,0]]		[0,0,0,0,0,	_
2	[1,0,0,0,0,0]	1			[1,0,0,0,0,	0] 1
3	[1,0,1,0,0,0]	2	[[0,0,0,0,0,0], [1,	[[0,0,0,0,0,0], [1,0,0,0,0], [1,0,1,0,0,0]]		
4	[1,0,1,0,1,0]	3	[[1,0,0,0,0,0], [1,0,1,0,0,0], [1,0,1,0,1,0]]		[1,0,1,0,1,	0] 3
5	[1,0,1,0,1,1]	4	[[1,0,1,0,0,0], [1,0,1,0,1,0], [1,0,1,0,1,1]]		[1,0,1,0,1,	1] 4
6	[0,0,1,0,1,1]	3	[[1,0,1,0,1,0], [1,	0,1,0,1,1], [0,0,1,0,1,1]]	[1,0,1,0,1,	1] 4
7	[0,0,0,0,1,1]	2	[[1,0,1,0,1,1], [0,	0,1,0,1,1], [0,0,0,0,1,1]]	[1,0,1,0,1,	1] 4
8						

c) Maximize the oracle 0, 0]. Use a tabu list s

Neighbors of $s_{current}$	Value
[1,0,0,0,1,1]	3
[0,1,0,0,1,1]	3
[0,0,1,0,1,1]	tabu
[0,0,0,1,1,1]	3
[0,0,0,0,0,1]	1
[0,0,0,0,1,0]	1



Iteration	S _{current}	Value of		[0,0,0,0,1,0]	1		S _{best}	Value
		Scurrent						of s _{best}
1	[0,0,0,0,0,0]	0	[[0,0,0,0,0]]		[0,0,0,0,0,0]	0		
2	[1,0,0,0,0,0]	1		[[0,0,0,0,0,0], [1,0,0,0,0,0]] [[1,0,0,0,0,0]	1	
3	[1,0,1,0,0,0]	2		[[0,0,0,0,0,0], [1,0,0,0,0], [1,0,1,0,0,0]]		[1,0,1,0,0,0]	2	
4	[1,0,1,0,1,0]	3	[[1,0,0,0,0,0], [1,0,1,0,0,0], [1,0,1,0,1,0]]		[1,0,1,0,1,0]	3		
5	[1,0,1,0,1,1]	4	[[1,0,1,0,0,0], [1,0,1,0,1,0], [1,0,1,0,1,1]]		[1,0,1,0,1,1]	4		
6	[0,0,1,0,1,1]	3		[[1,0,1,0,1,0], [1,	.0,1,0,1,1], [0,0,1,0,1,1]]		[1,0,1,0,1,1]	4
7	[0,0,0,0,1,1]	2	[[1,0,1,0,1,1], [0,0,1,0,1,1], [0,0,0,0,1,1]]		[1,0,1,0,1,1]	4		
8	[1,0,0,0,1,1]	3		[[0,0,1,0,1,1], [0,	.0,0,0,1,1], [1,0,0,0,1,1]]		[1,0,1,0,1,1]	4



c) Maximize the oracle function from (b) using Tabu Search. Use the starting state [0, 0, 0, 0, 0, 0, 0]. Use a tabu list size of 3. Complete the following table until (including) iteration 8.

Iteration	S _{current}	Value of Scurrent	Tabu list	S _{best}	Value of s _{best}
1	[0,0,0,0,0,0]	0	[[0,0,0,0,0]]	[0,0,0,0,0,0]	0
2	[1,0,0,0,0,0]	1	[[0,0,0,0,0,0], [1,0,0,0,0,0]]	[1,0,0,0,0,0]	1
3	[1,0,1,0,0,0]	2	[[0,0,0,0,0,0], [1,0,0,0,0], [1,0,1,0,0,0]]	[1,0,1,0,0,0]	2
4	[1,0,1,0,1,0]	3	[[1,0,0,0,0,0], [1,0,1,0,0,0], [1,0,1,0,1,0]]	[1,0,1,0,1,0]	3
5	[1,0,1,0,1,1]	4	[[1,0,1,0,0,0], [1,0,1,0,1,0], [1,0,1,0,1,1]]	[1,0,1,0,1,1]	4
6	[0,0,1,0,1,1]	3	[[1,0,1,0,1,0], [1,0,1,0,1,1], [0,0,1,0,1,1]]	[1,0,1,0,1,1]	4
7	[0,0,0,0,1,1]	2	[[1,0,1,0,1,1], [0,0,1,0,1,1], [0,0,0,0,1,1]]	[1,0,1,0,1,1]	4
8	[1,0,0,0,1,1]	3	[[0,0,1,0,1,1], [0,0,0,0,1,1], [1,0,0,0,1,1]]	[1,0,1,0,1,1]	4

Given N players, each with individual playing strength s_i . Example: s = [10, 5, 12, 39, 32, ...]

Your task is to divide the N players into two teams of as equal strength as possible (the problem is also known as number partitioning). The team strength is the sum of all player strengths.

Implement the Tabu Search algorithm in Python and use it to solve the above presented task.

Exercise 7: Tabu in Python

see PyCharm

Any two optimization algorithms are equivalent when their performance is averaged across all possible problems

Intuitive understanding:

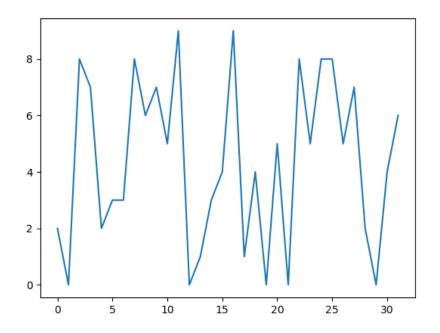
An (uninformed) optimization algorithm is a sequence of oracle evaluations

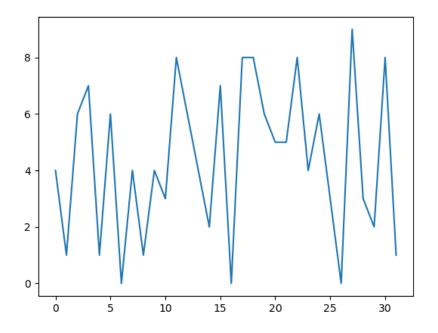
f(x) # function to be optimized

$$f(x_1), f(x_2), f(x_3), ...$$

The states $x_1, x_2, x_3, ...$ are determined by the optimization algorithm

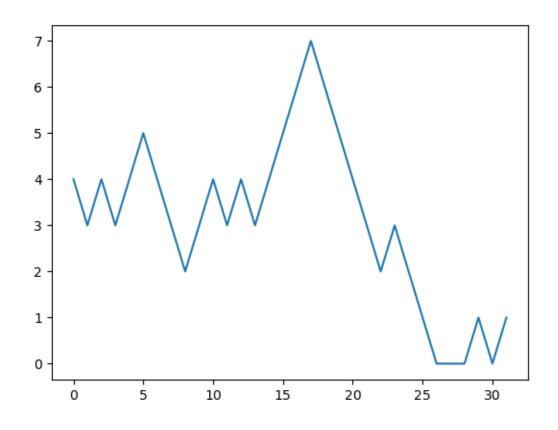
BUT: there is no winning in random solution landscapes \rightarrow a random selection of $x_1, x_2, x_3, ...$ is on average as good as the most clever optimization algorithm





- Consequences

 we must make assumptions about the solution landscape
- By far the most often used assumption: smoothness
- There is now a correlation between adjacent states
- Local Search Methods (like Tabu Search) now perform better then random search



Exercise 1 (a): Important Concepts

• Agent: Entity that perceives and acts

• Environment: World in which the agent perceives and acts

• Agent function: (policy in RL). Function that maps perceptions to actions

• Rationality: Choose the action that maximizes the expected performance metric based on knowledge and the previous perception

sequence

• Autonomy: An agent's behavior depends on its own experience and not

on fixed programming

Exercise 1 (a): Important Concepts

• *Model based agent*: Agent whose actions are derived directly from an internal model of the current state of the environment

• Goal oriented agent: Agent that selects actions that it believes will help reach a given goal

• Benefit oriented Agent that selects actions that it believes will maximize expected utility

• Reflex agent: Agent whose actions are only conditioned on the current perception

Exercise 1 (b): Important Concepts

Are reflexive actions - like pulling one's hand back from a hot stove top - rational, intelligent, or both?

- Sub-question to answer this question: what is the performance metric of the agent → Negative burning degree (maximize)
- Rational, because the action maximizes the performance metric
- Not intelligent, because the agent does not think about the action and its consequences

Exercise 1 (c): Important Concepts

Justify why it makes sense to perform the formulation of the problem <u>after</u> the formulation of the goal?

Solution:

- Goal formulation = what aspects of the world are we interested in?
- Problem formulation = how do interesting aspects need to be manipulated (we must know what the interesting aspects are)

Exercise 1 (d): Important Concepts

- State space: Graph where nodes represent states and edges represent actions
- State: Situation in which an agent finds itself
- Search tree: Tree of states (root = starting state)
- Goal: state we want to reach
- Action: transition between states
- Successor function: Given a state the successor function returns a set of state-action pairs
- Branching factor: maximum number of actions for a state in the search tree