Exercise 1

Exercise 1

For the following claims, decide whether they are true or false. Give reasons for your decision:

(a) The depth-first search always expands at least as many nodes as the A search with admissible heuristics.*

False. Depth-first search (with a lot of luck) can reach the target with exactly d expansions (d = depth of the most favorable solution).

(b) $h(n) = 0$ *is an admissible heuristic for the 8-puzzle.*

True, since in the 8-puzzle the cost is positive. Heuristics must never overestimate.

Exercise 1

(c) The A algorithm is not applicable in the field of robotics, since perceptions, states and actions are continuous.*

False. A* can not be used in continuous domains, but discretization is possible.

(d) Breadth-first search is complete even if step costs of 0 are allowed.

True. Question is a little misleading, since costs do not influence the decision which node to expand next in breadth-first search.

(e) On a chessboard, a rook can move in a straight line vertically or horizontally any number of squares, but it cannot jump over other pieces. Manhattan distance (MD) is an admissible heuristic for the problem of moving the rook from square A to square B with the least number of moves.

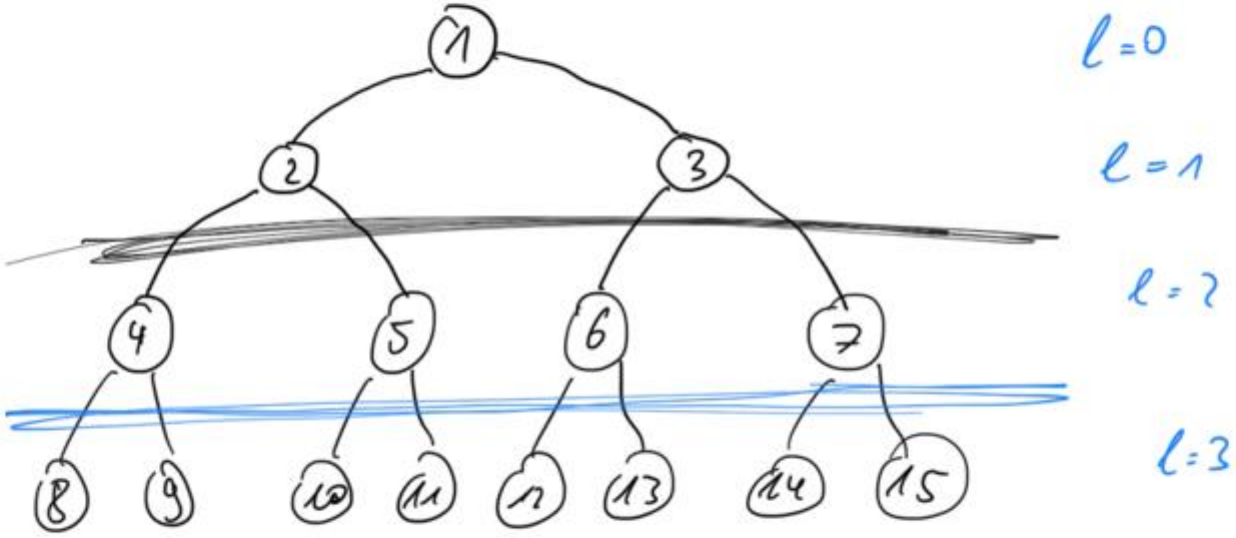
False, since MD overestimates

Exercise 2

Exercise 2

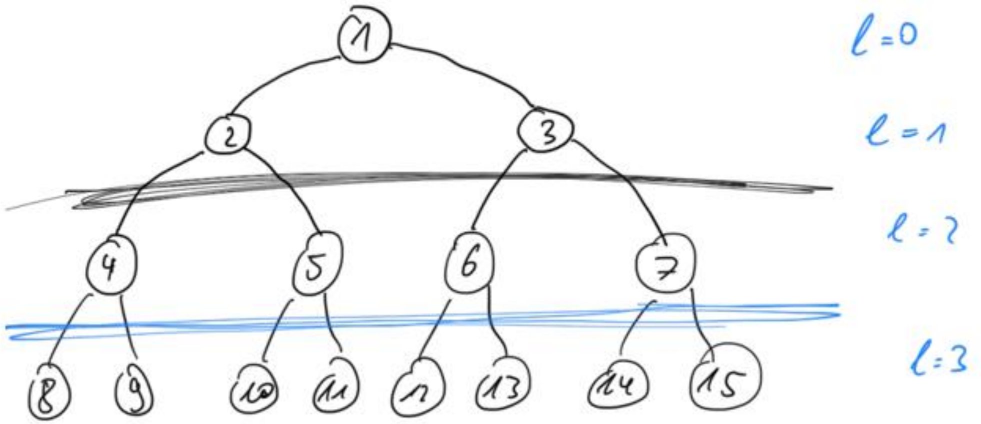
Imagine a state space in which the starting state is the number 1 and each state K has two successors: Number 2K and 2K + 1.

(a) Draw the state space for states 1 through 15



Exercise 2

(b) Assuming the target state is 11, in what order is node expansion performed using the following search algorithms?



Breadth-first search:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

Exercise 2

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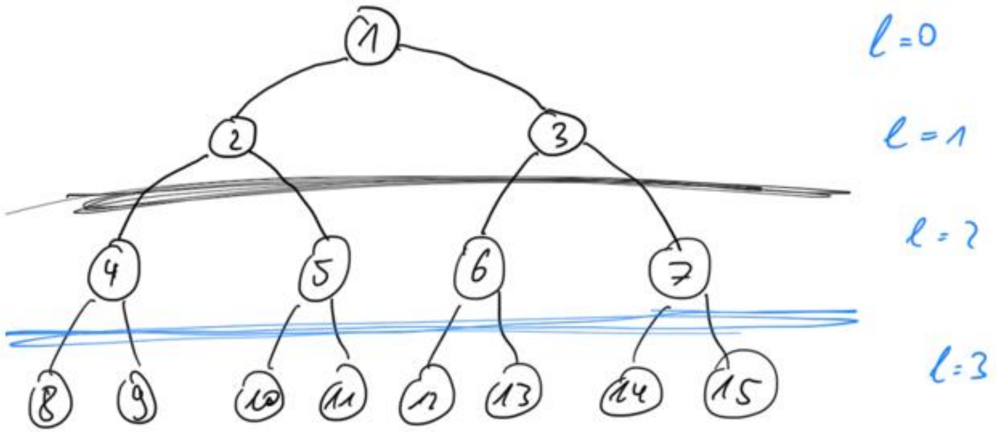

Limited depth-first search with $l = 2$:

1, 2, 4, 5, 3, 6, 7 (goal not found)

Exercise 2

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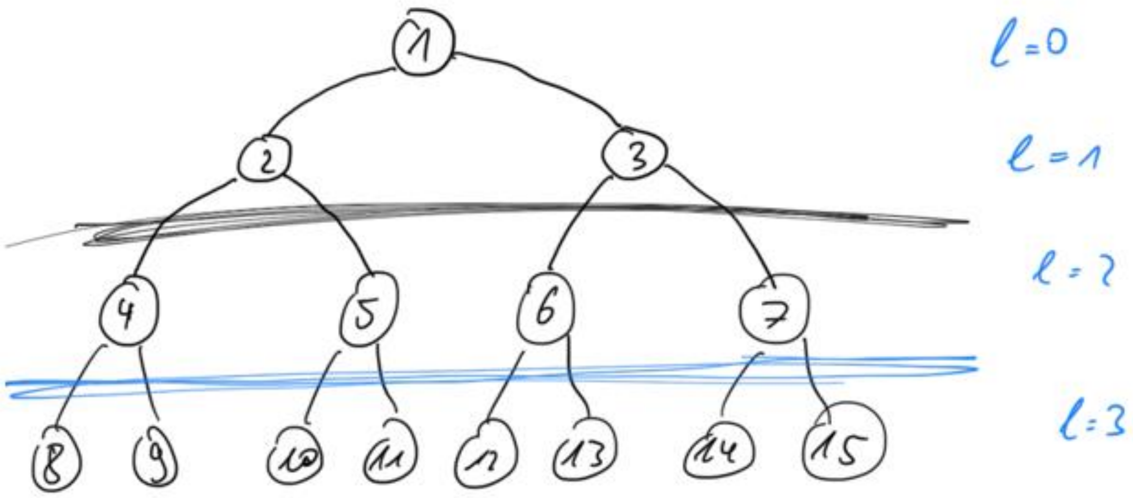
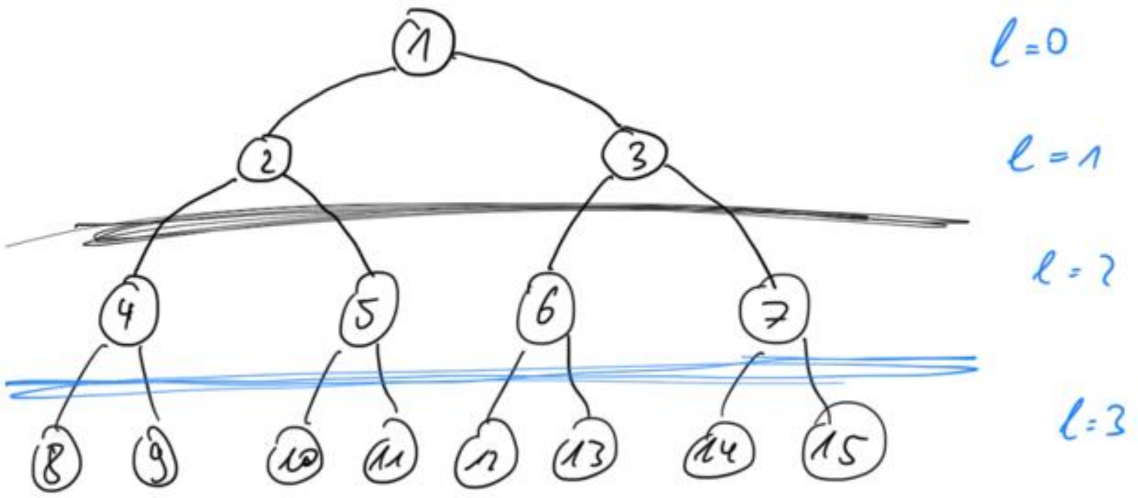


Iterativ depth-first search:

l=0 | 1

Exercise 2

(b) Assuming the target state is 11, in what order is node expansion performed using the following search algorithms?

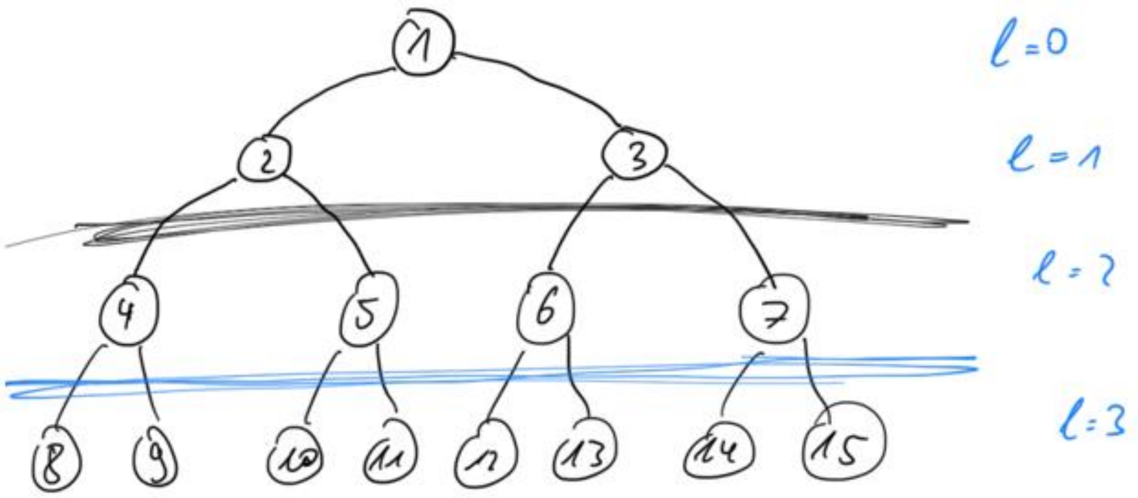


Iterativ depth-first search:

$l=0$	1
$l=1$	1 2 3

Exercise 2

(b) Assuming the target state is 11, in what order is node expansion performed using the following search algorithms?

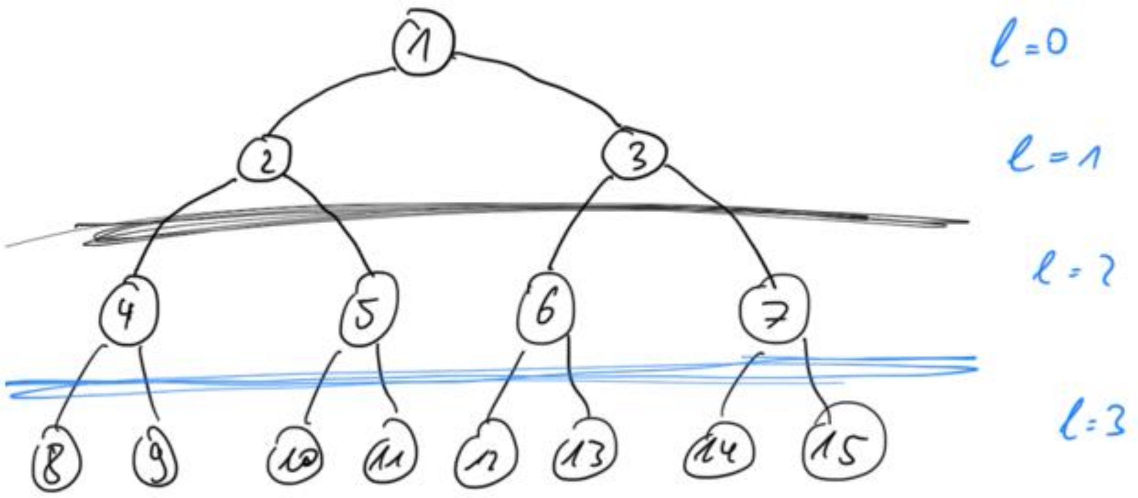


Iterativ depth-first search:

l=0	1
l=1	1 2 3
l=2	1 2 4 5 3 6 7

Exercise 2

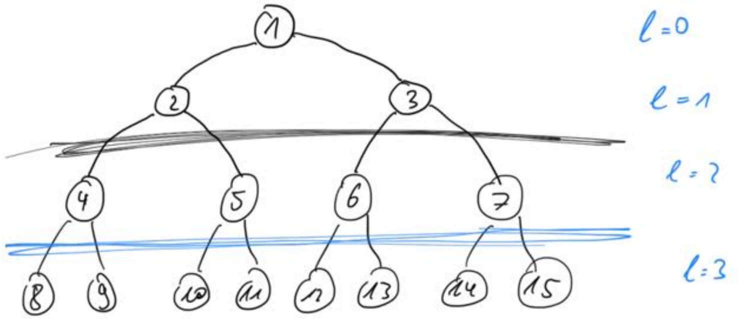
(b) Assuming the target state is 11, in what order is node expansion performed using the following search algorithms?



Iterativ depth-first search:

l=0	1
l=1	1 2 3
l=2	1 2 4 5 3 6 7
l=3	1 2 4 8 9 5 10 11

Exercise 2



(c) Would a bidirectional search help here? What is the branching factor in each direction?

Yes, bidirectional search would help, because the successor is unique.

Branching factor:

forward: 2 (two children)

backward: 1 (one parent)

Exercise 3

Reinforcement Learning (RL) vs. Planning

- RL: select the next action for a given state:

$$\pi(s_t) = a_t$$

- Planning: select multiple actions which yield a high reward <u>together</u>

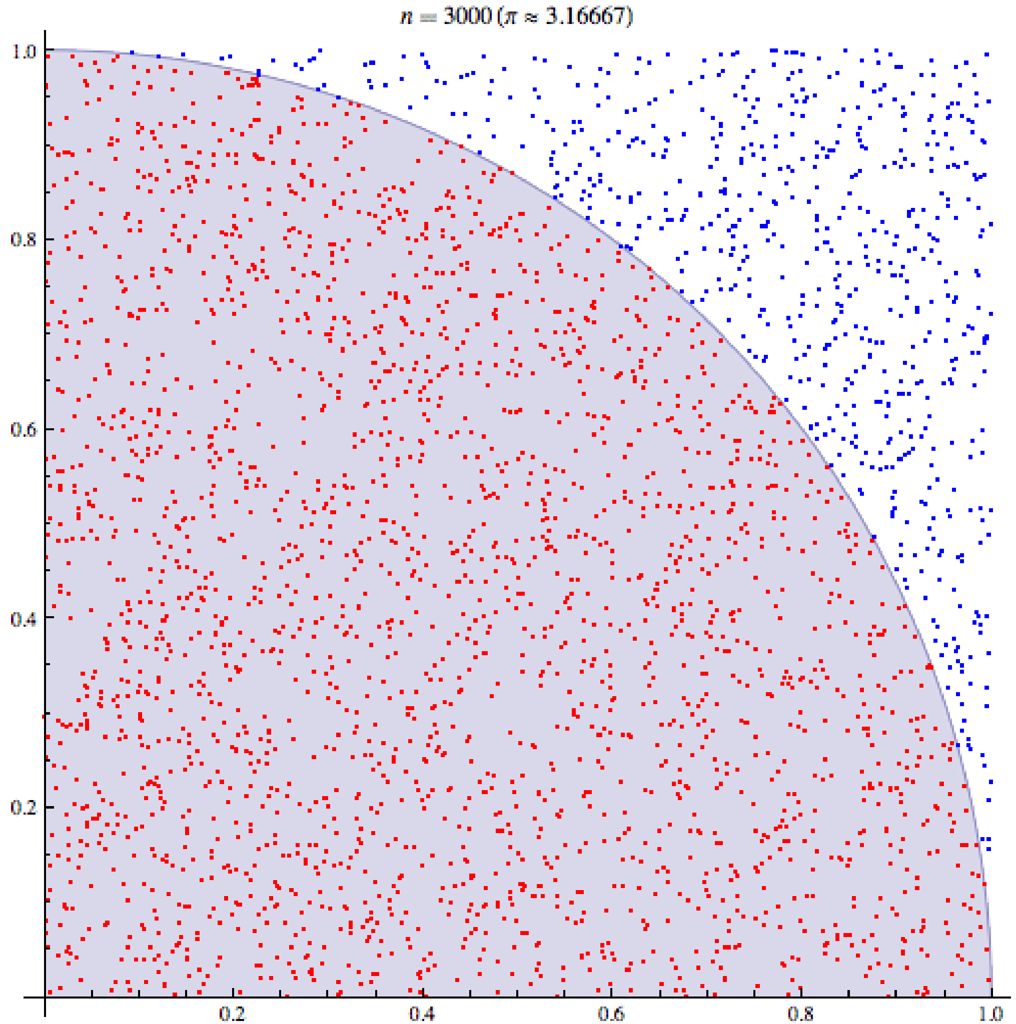
- One famous example: Monte Carlo Tree Search (MCTS)

Randomized Algorithms

- **Monte Carlo algorithm**: randomized algorithm whose output may be incorrect with a certain (small) probability

- **Las Vegas algorithm**: algorithm which always outputs the correct solution, but makes use of random choices in the calculation

- Example for Monte Carlo algorithm: Determine π using counting and random sampling



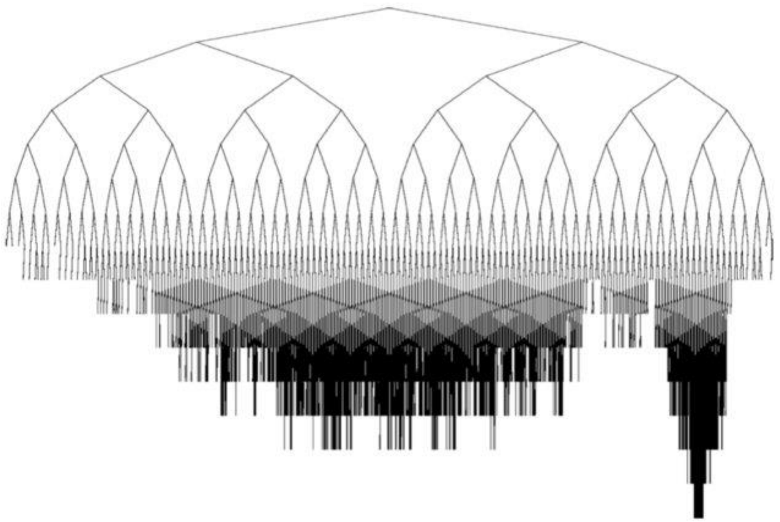
Monte Carlo Tree Search (MCTS)

What is the goal?

Search efficiently in a (very deep) tree with high branching factor

Idea:

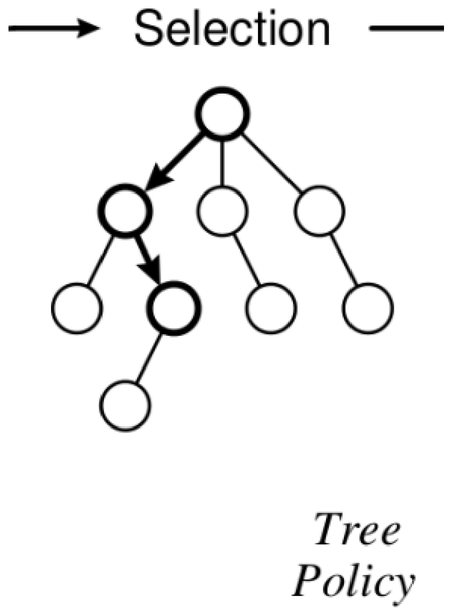
- Use simulations to determine the value of a node
- Use Upper Confidence Bound to decide whether to explore in breadth or to exploit in depth



Monte Carlo Tree Search (MCTS)

MCTS consists of four main steps (Browne et al., 2012):

1. Selection: Start at the root and select the best action until reaching a node that has not been fully explored yet



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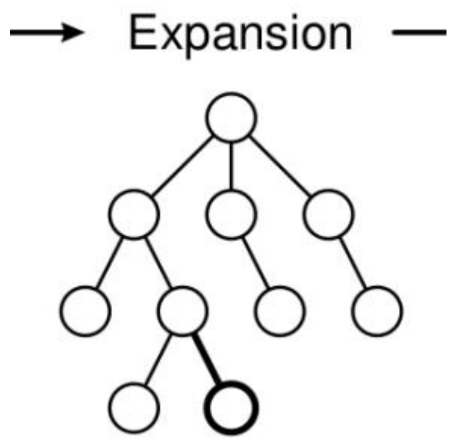

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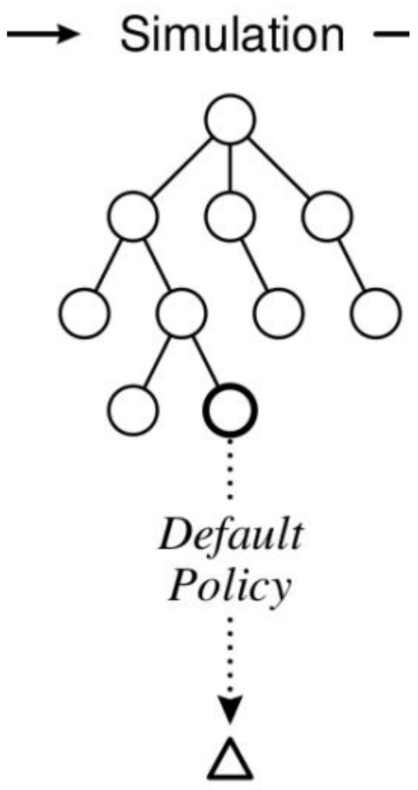
3. Simulation: From the newly added child randomly select actions until the episode terminates and → receiving a reward



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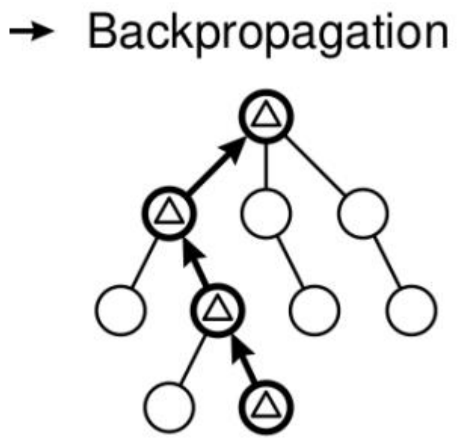
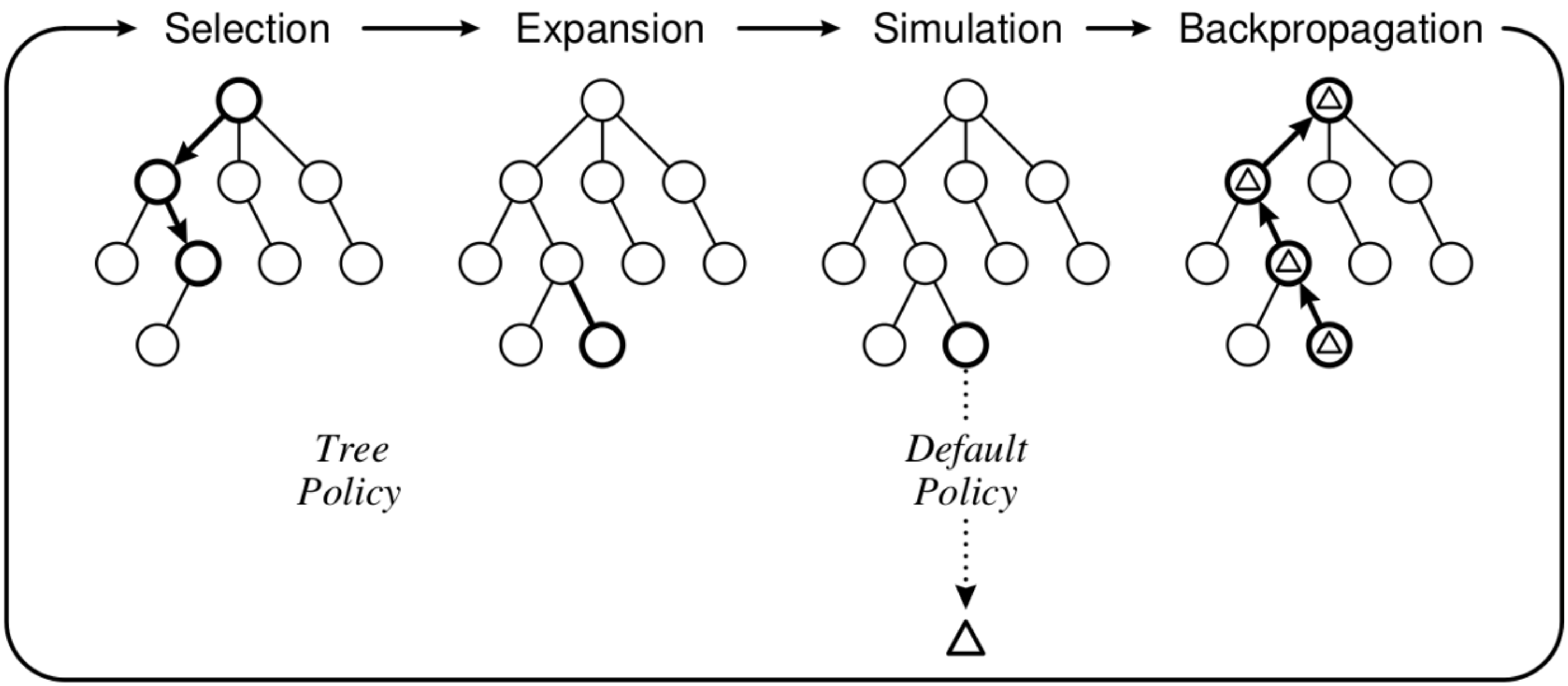
2. Expansion: Choose an action, and expand the tree by adding a child node.
3. Simulation: From the newly added child randomly select actions until the episode terminates and → receiving a reward
4. Backpropagation: Starting at the new child node, propagate the reward to the root by adjusting the visit count N(v) and the simulation reward Q(v) of the nodes along the path.



Monte Carlo Tree Search (MCTS)



Monte Carlo Tree Search (MCTS)

Tree Policy (optimal trade-off between exploration and exploitation):

Choose the child that maximizes the Upper Confidence Bound for Trees (UCT):

$$\frac{1}{n_i}\sum_{t=1}^{n_i} r_t + c\sqrt{\frac{\ln N}{n_i}}$$

N = number of times the parent node has been visited
n_i = number of times the child i has been visited
r_t = reward from the t-th visited of the child
c = exploration hyperparameter

Monte Carlo Tree Search (MCTS)

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$$\boxed{\frac{1}{n_i}\sum_{t=1}^{n_i} r_t} + c\sqrt{\frac{\ln N}{n_i}}$$

Exploitation (mean reward)

N = number of times the parent node has been visited
n_i = number of times the child i has been visited
r_t = reward from the t-th visited of the child
c = exploration hyperparameter

Monte Carlo Tree Search (MCTS)

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$$\frac{1}{n_i}\sum_{t=1}^{n_i} r_t + \boxed{c\sqrt{\frac{\ln N}{n_i}}}$$

Exploration (lower the more often node i was visited)

N = number of times the parent node has been visited
n_i = number of times the child i has been visited
r_t = reward from the t-th visited of the child
c = exploration hyperparameter

Monte Carlo Tree Search (MCTS)

Exploration vs Exploitation dilemma:

- Exploitation: choose the action that, according to the information so far, is the best one

- Exploration: choose other actions to see if there might be better actions to take

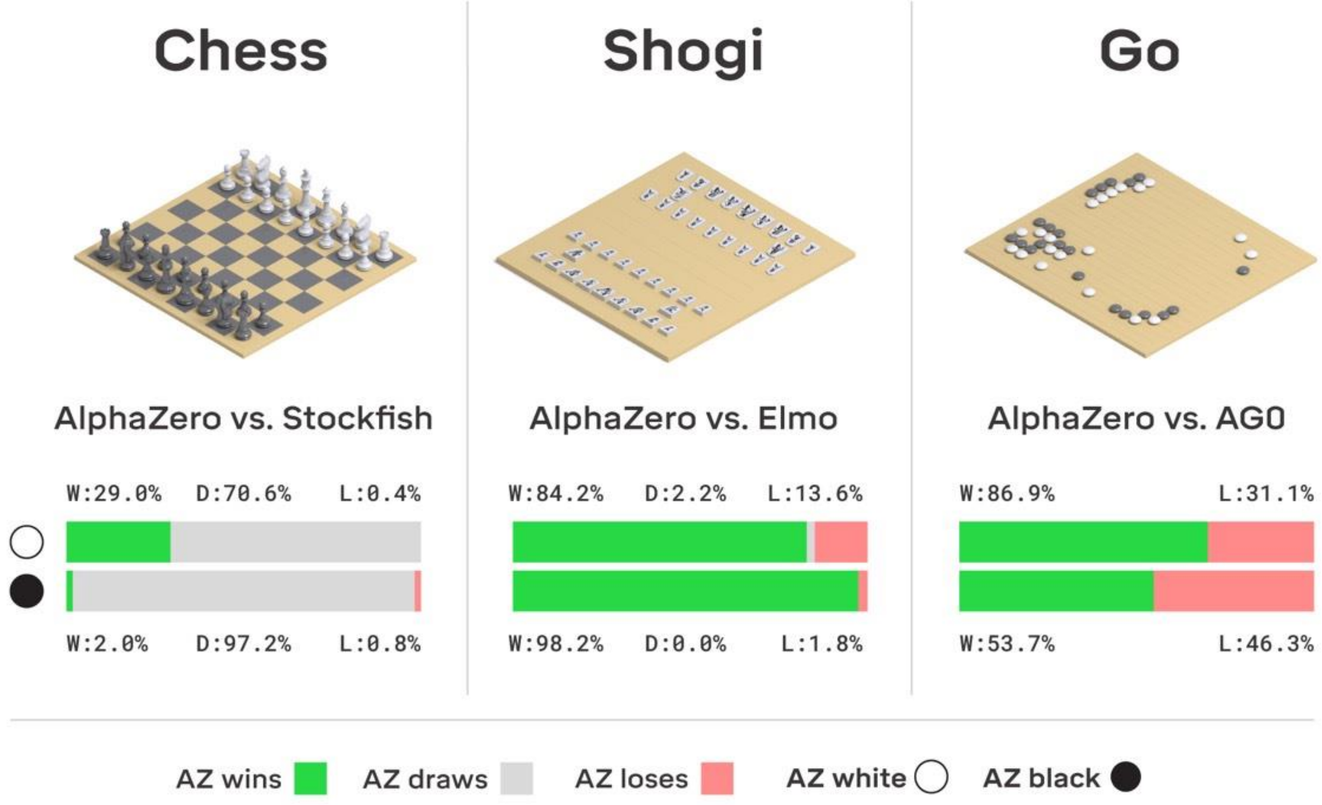
Monte Carlo Tree Search (MCTS)

Default Policy:

- In the simplest case: random
- More advanced (e.g. AlphaGo and AlphaZero):
  - Use as policy network instead of a random default policy
  - Don‘t use rollouts at all but use a value network to evaluate states

AlphaZero

- AlphaZero (DeepMind) uses a combination of CNNs, RL and MCTS
- MCTS particularly advantageous for domains with a high branching factor (number of possible actions)
- Example: Branching factor of Go: ~250



Exercise 3

- Use Monte Carlo Tree Search to find the largest entry in the following matrix.
- One action is to select one of the four (sub)areas.
- Thus, to select an entry you need 3 actions.
- You have a budget of exactly 11 rollouts.
- Choose $c = 5$ as your exploration hyperparameter.

30	33	39	38	32	34	55	53
41	50	49	54	42	45	62	66
32	41	44	49	40	40	55	58
45	45	46	59	49	52	64	65
37	37	43	44	38	42	53	54
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0

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$0, 3$

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40	45	43	57	45	58	56	65
50	48	58	59	59	66	70	77

$0, 3, 2 \rightarrow 46$

$<>$

$n = 0$

$\Sigma\, r = 0$

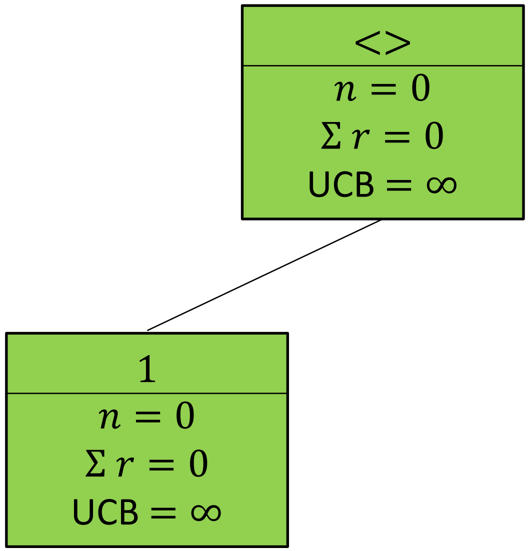
$\mathrm{UCB} = \infty$

$<>$

$n = 0$

$\Sigma\, r = 0$

$\text{UCB} = \infty$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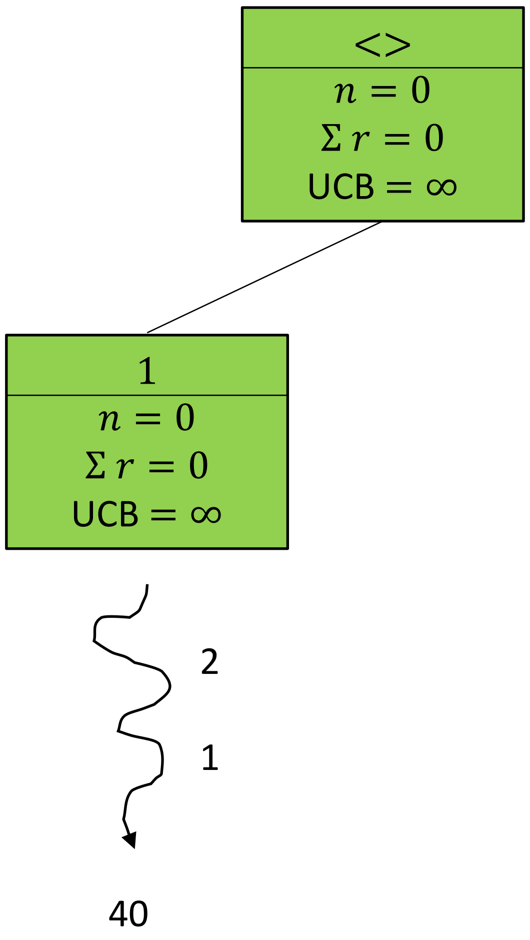

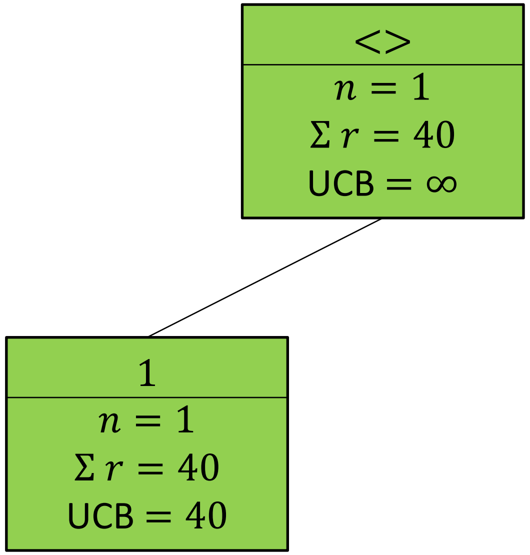
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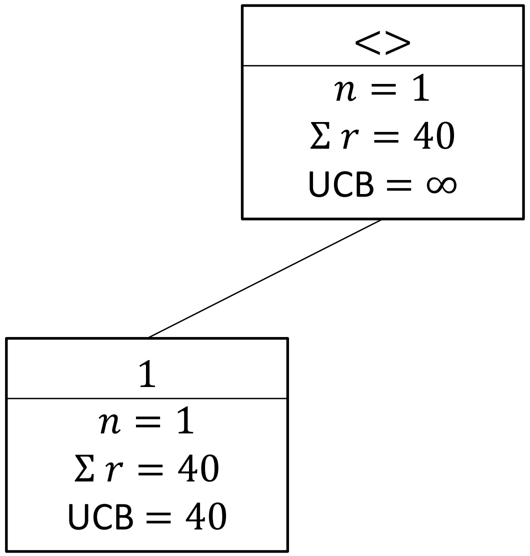
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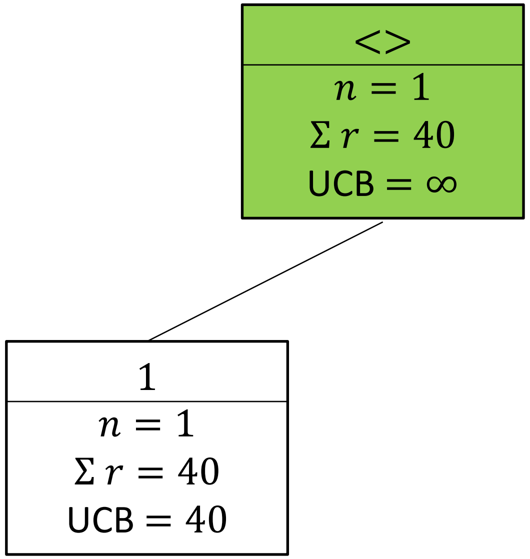

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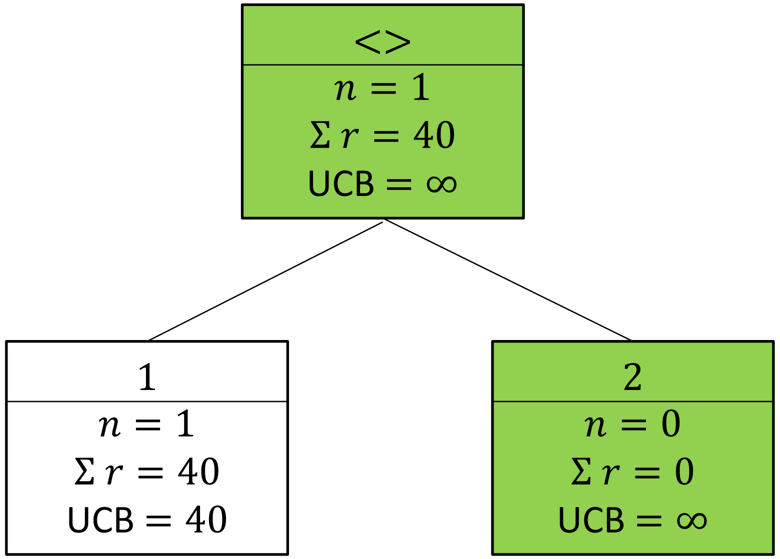
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UCB $= 40$
2
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$\Sigma\, r = 0$
UCB $= \infty$

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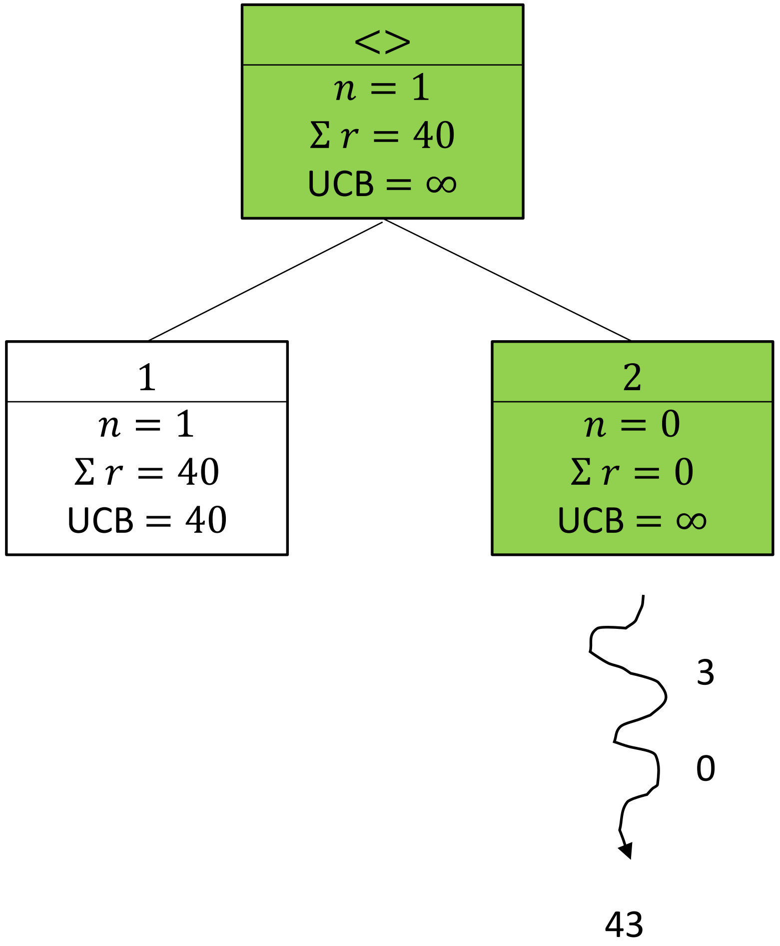

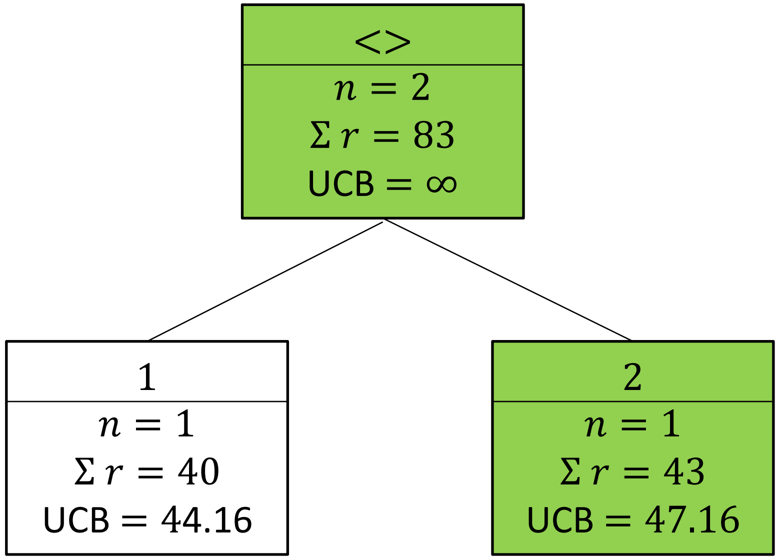
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UCB = ∞
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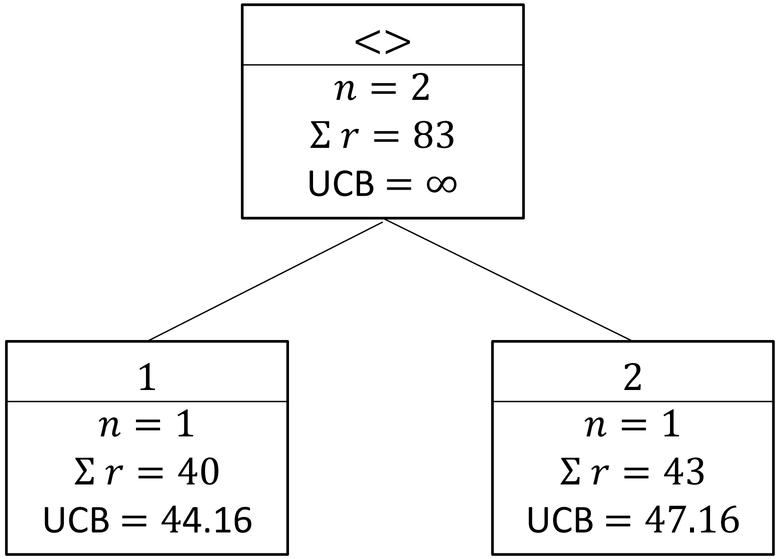
n = 1
Σ r = 40
UCB = 44.16
2
n = 1
Σ r = 43
UCB = 47.16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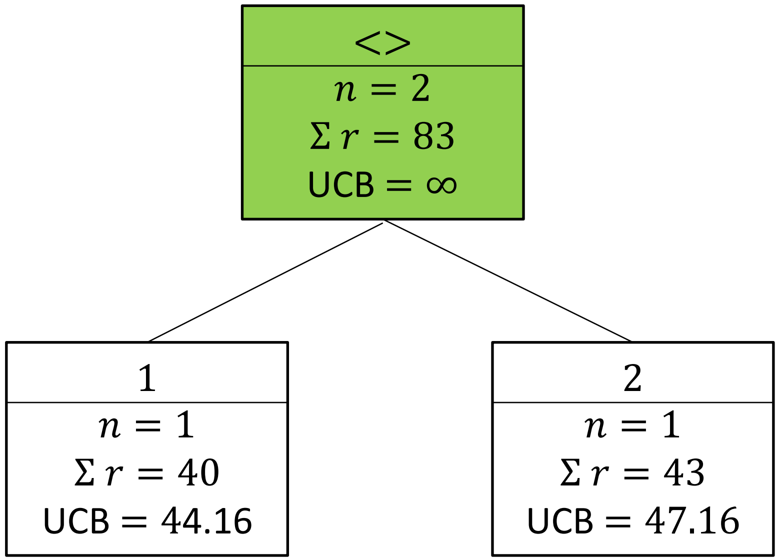
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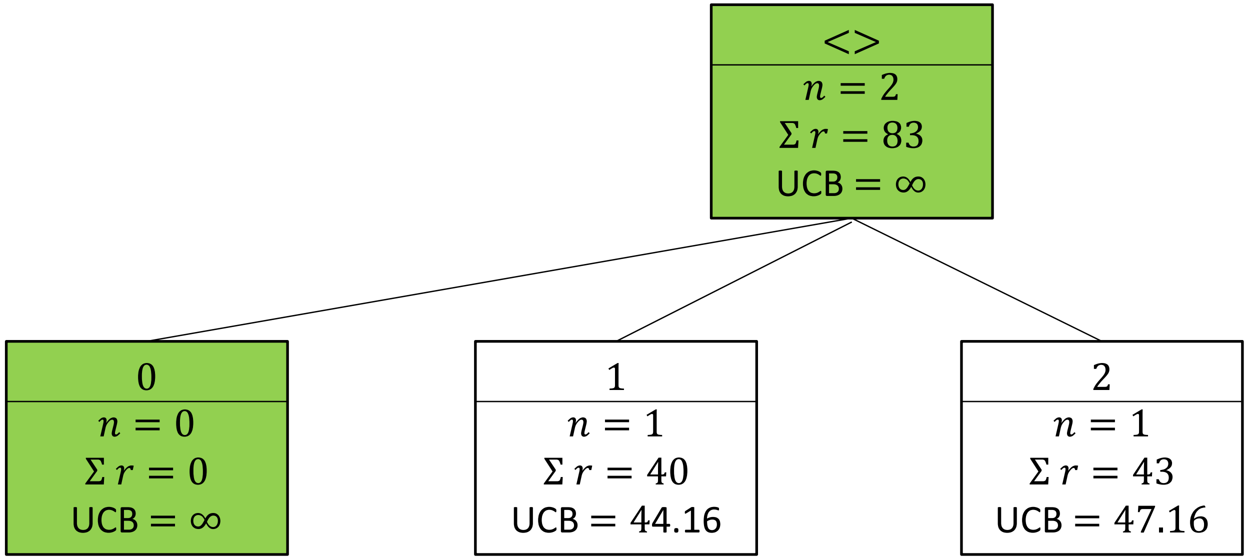
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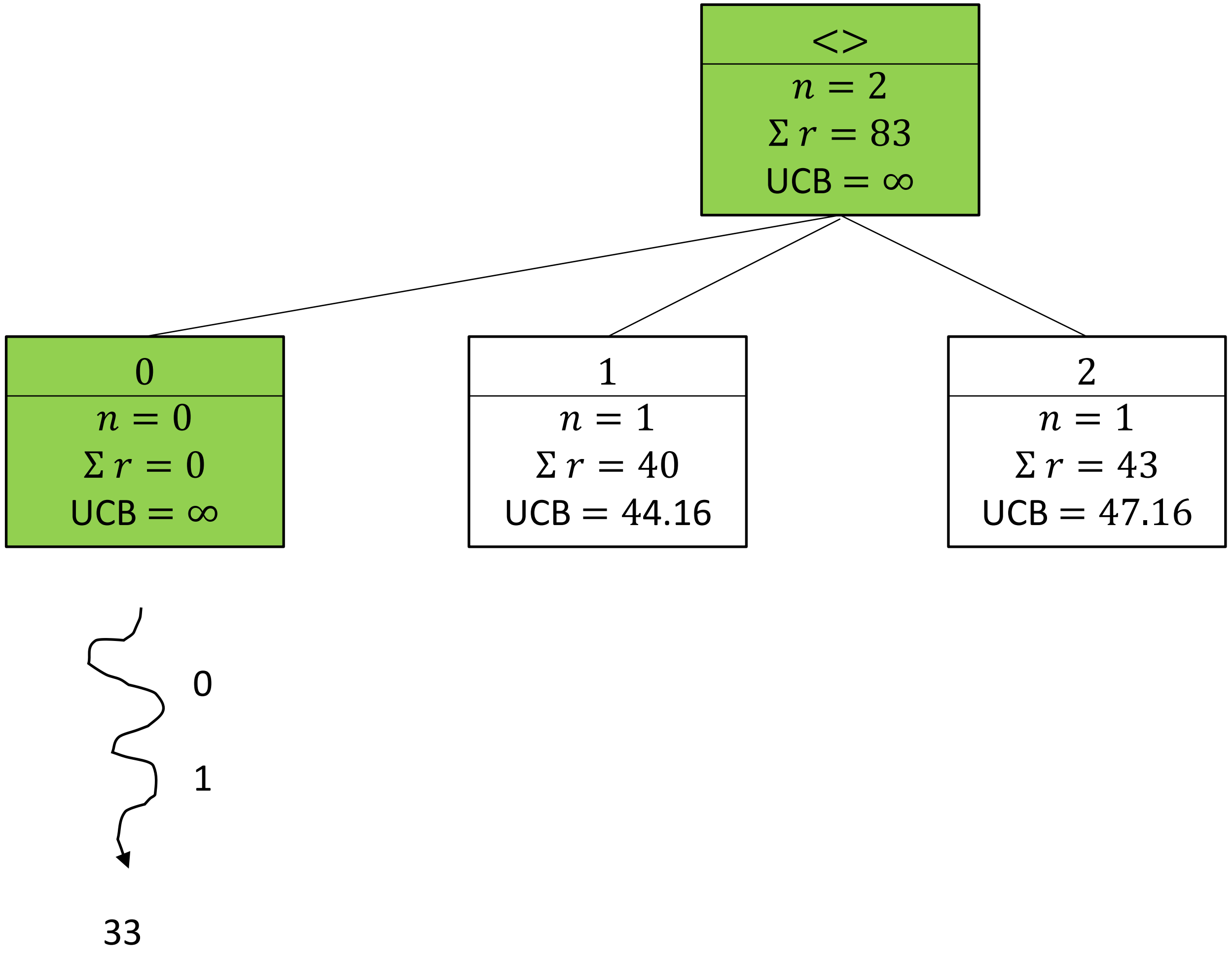

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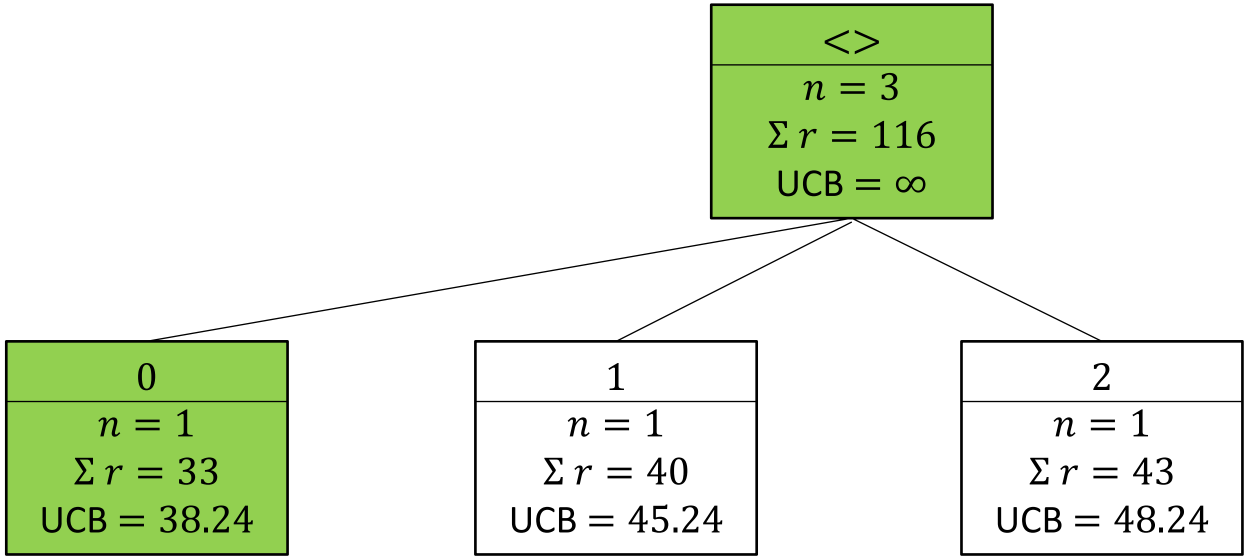
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2
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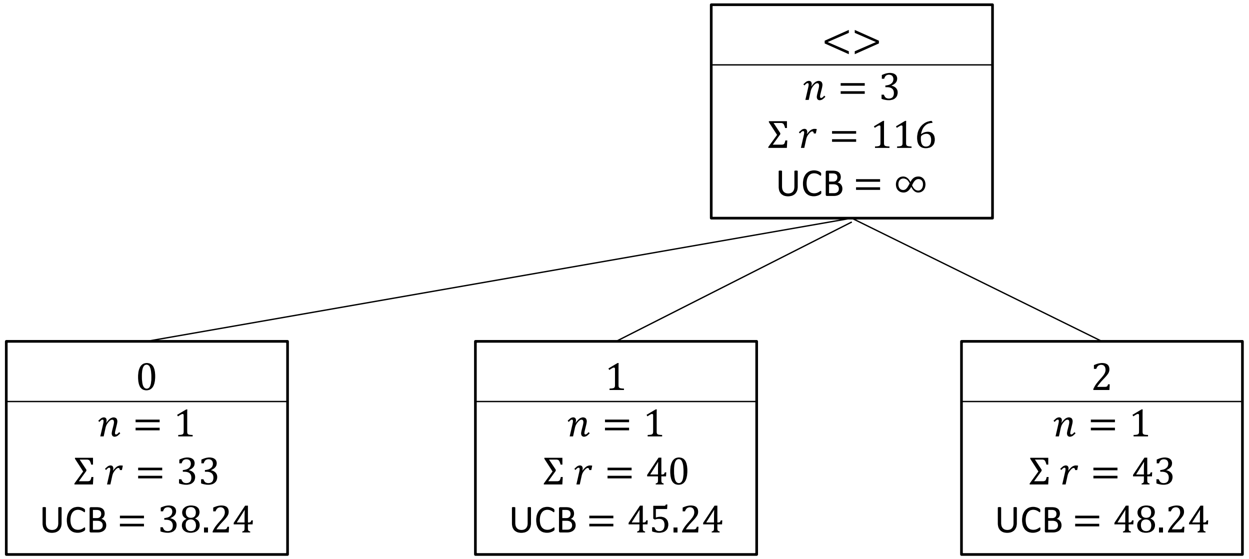
$\text{UCB} = 48.24$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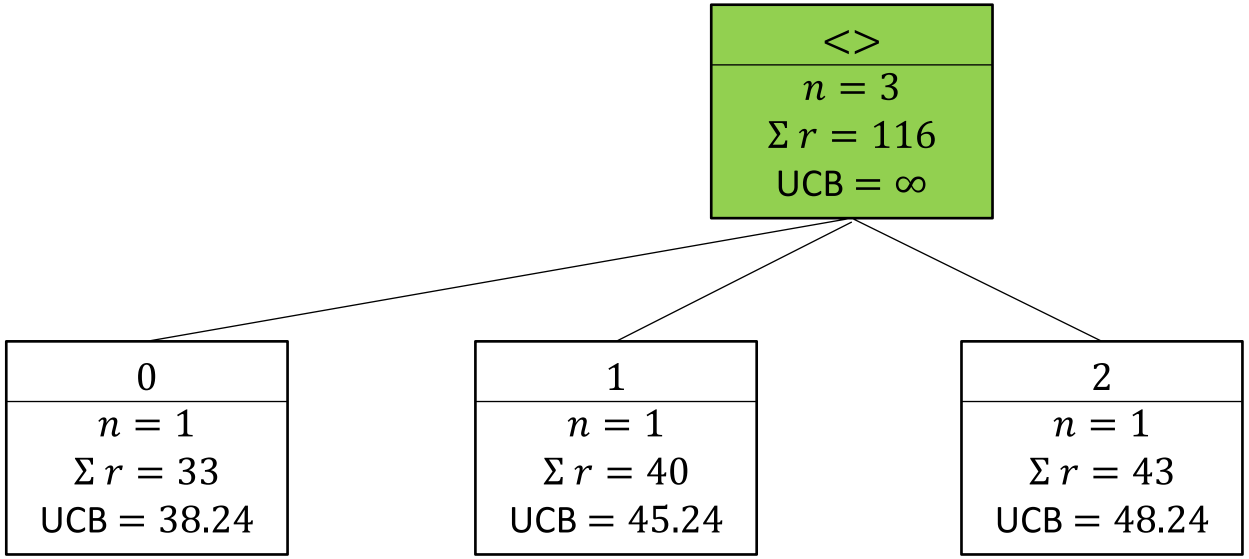
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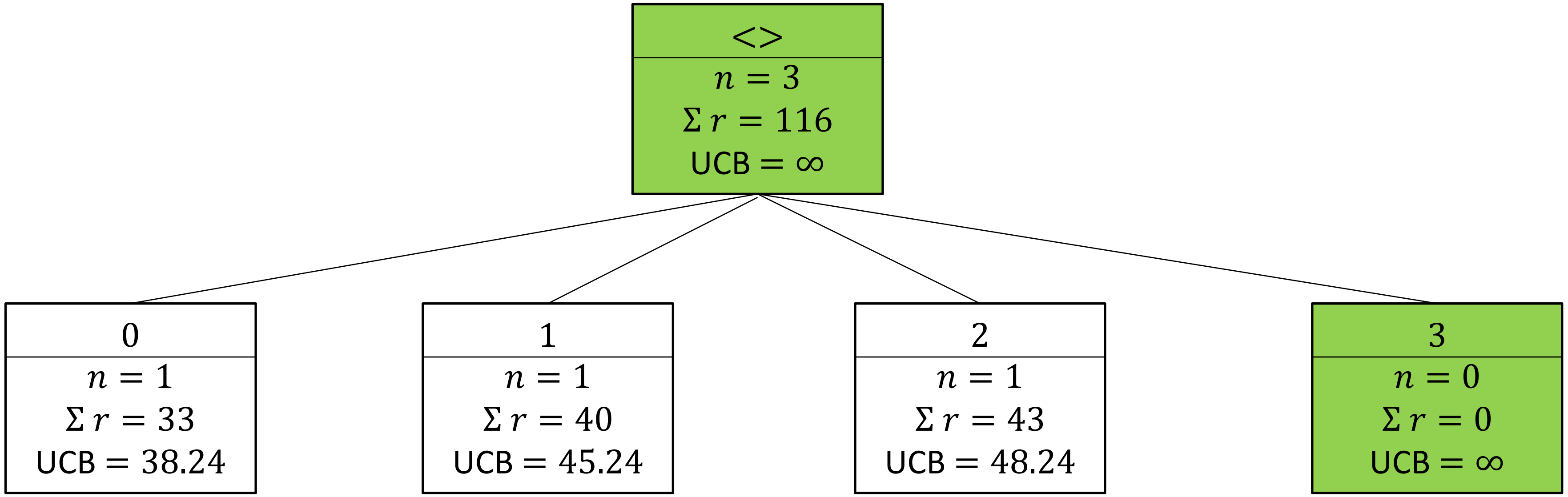
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$\Sigma\, r = 33$
UCB = 38.24
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$n = 1$
$\Sigma\, r = 40$
UCB = 45.24
2
$n = 1$
$\Sigma\, r = 43$
UCB = 48.24
3
$n = 0$
$\Sigma\, r = 0$
UCB = ∞

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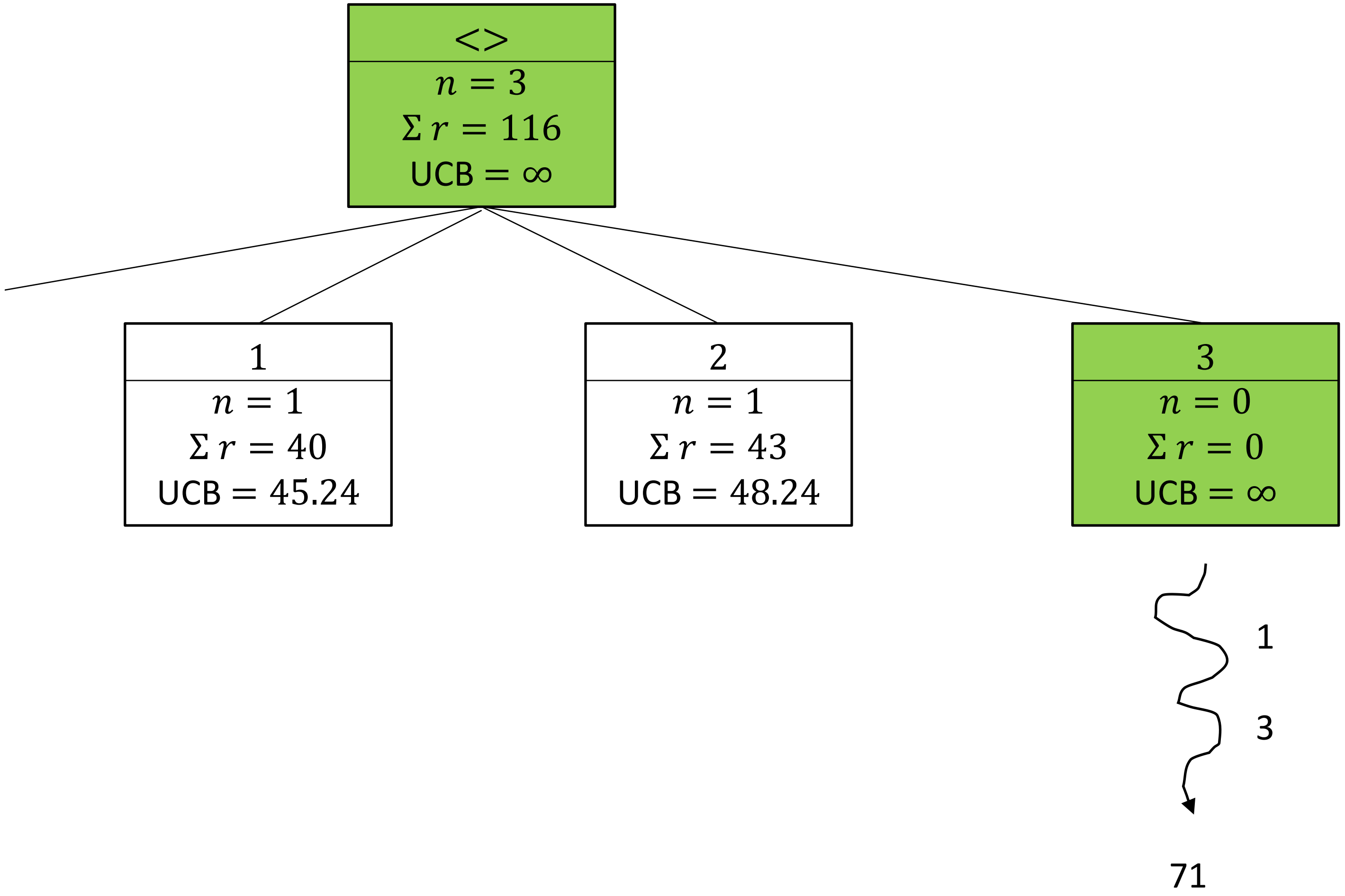
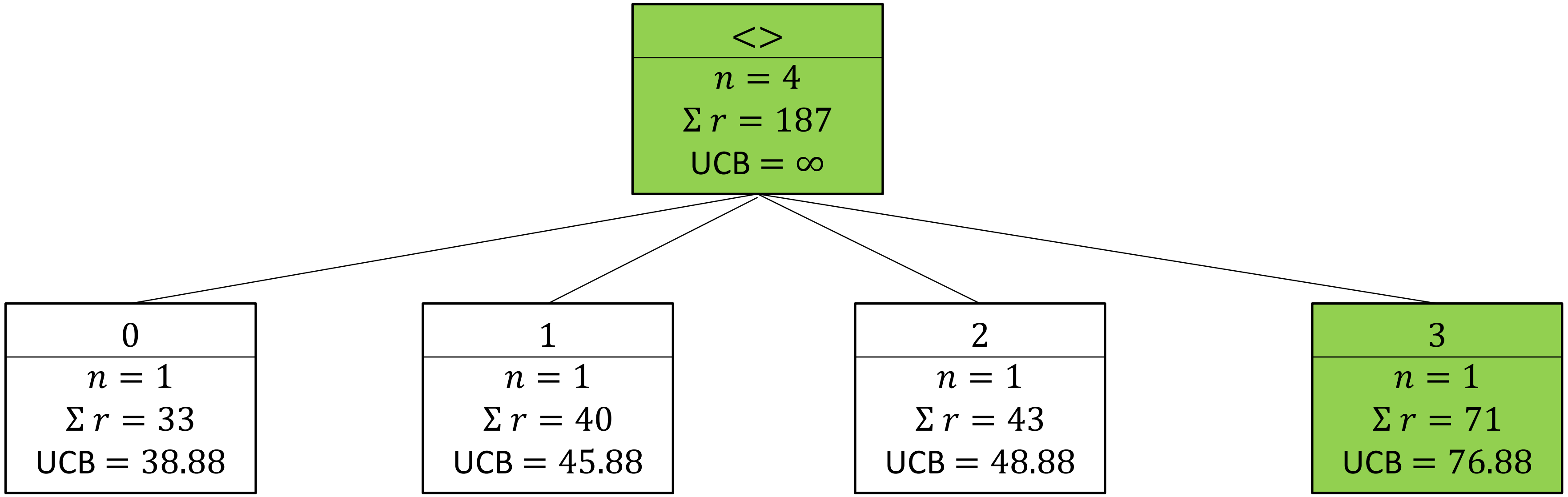
50	48	58	59	59	66	70	77

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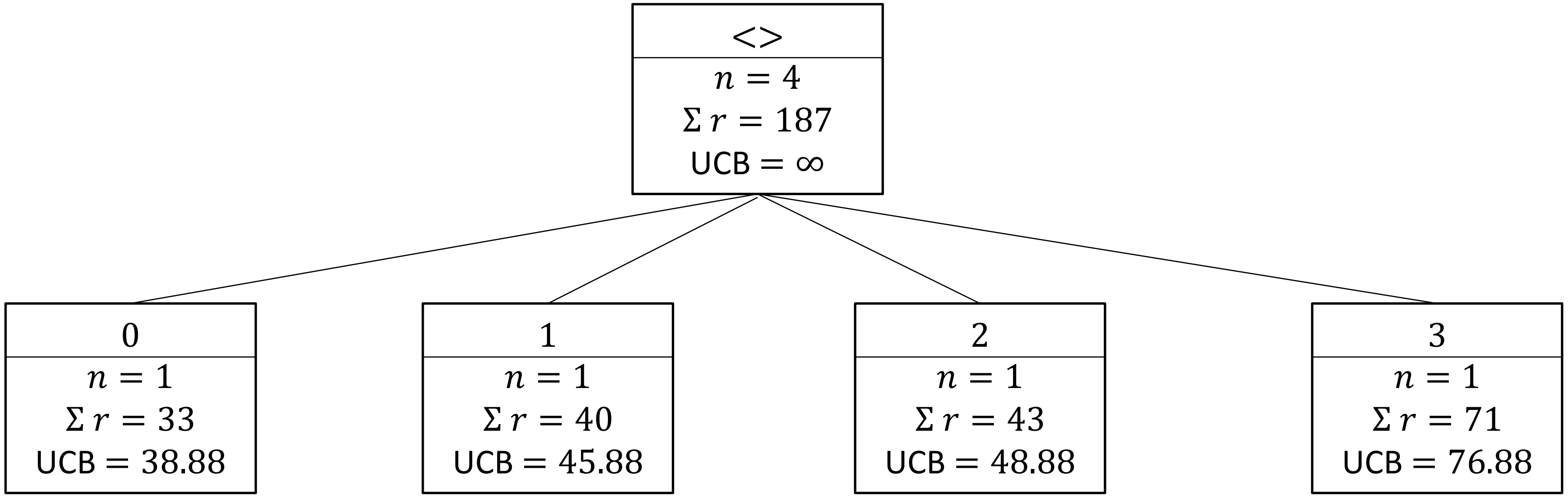
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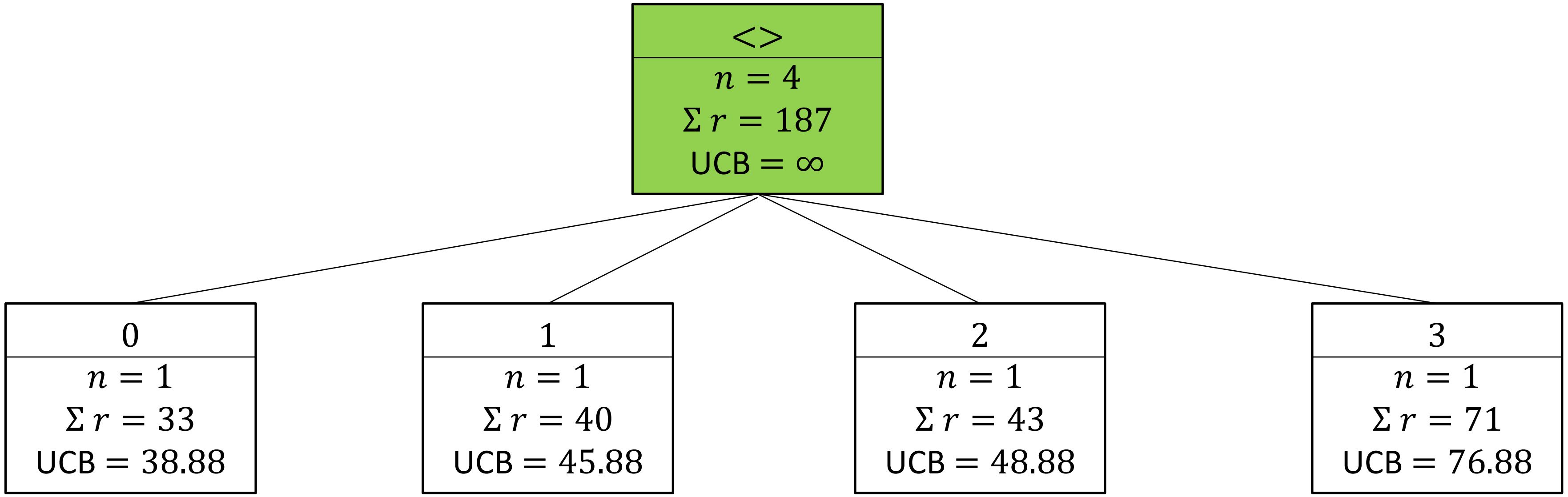
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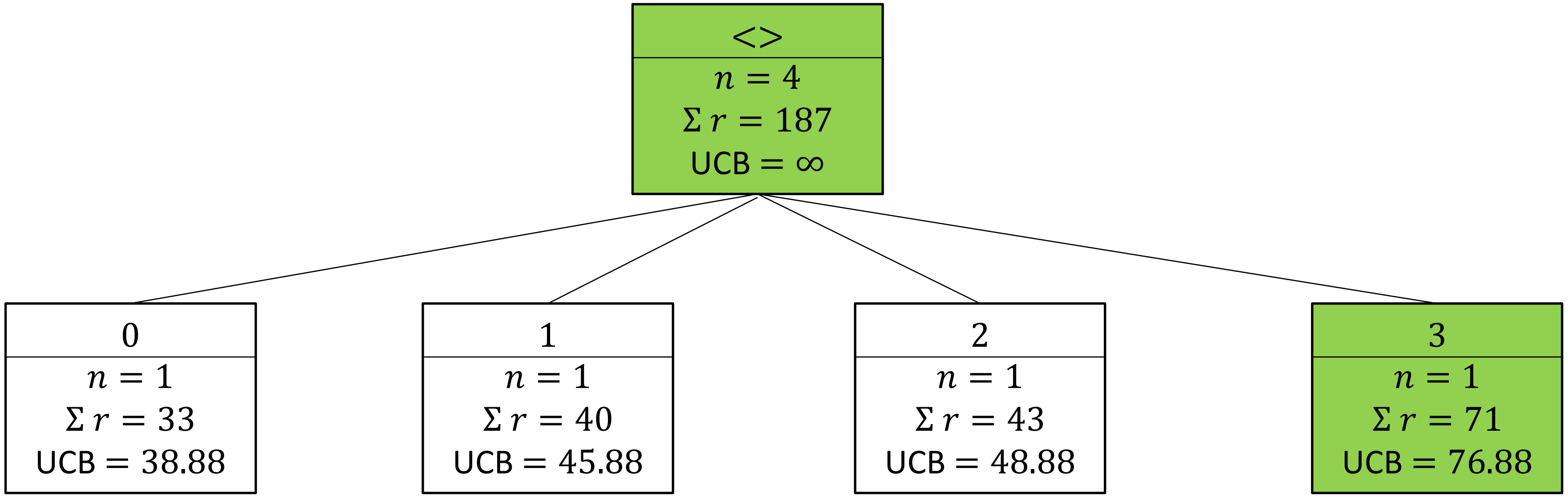
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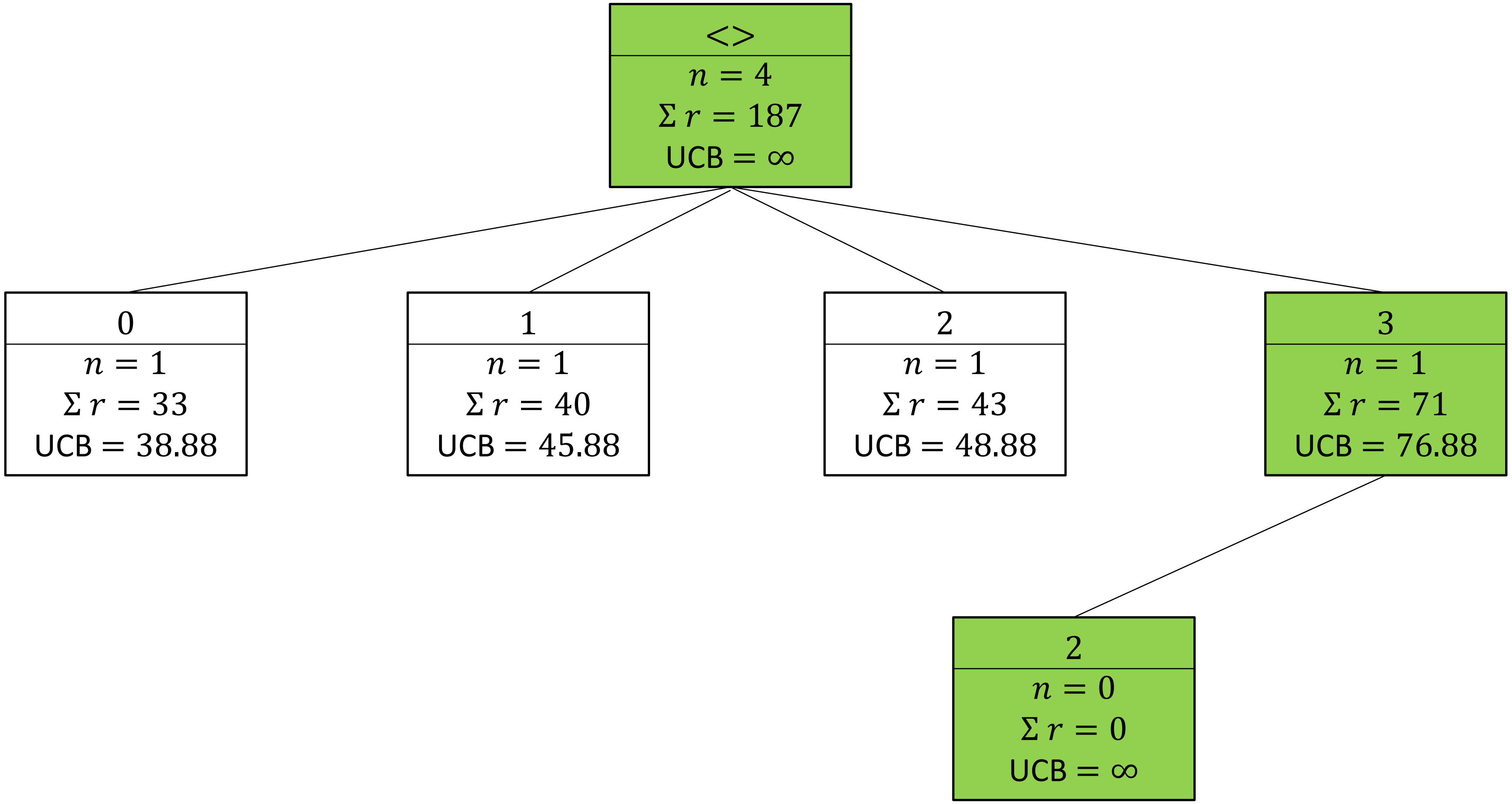
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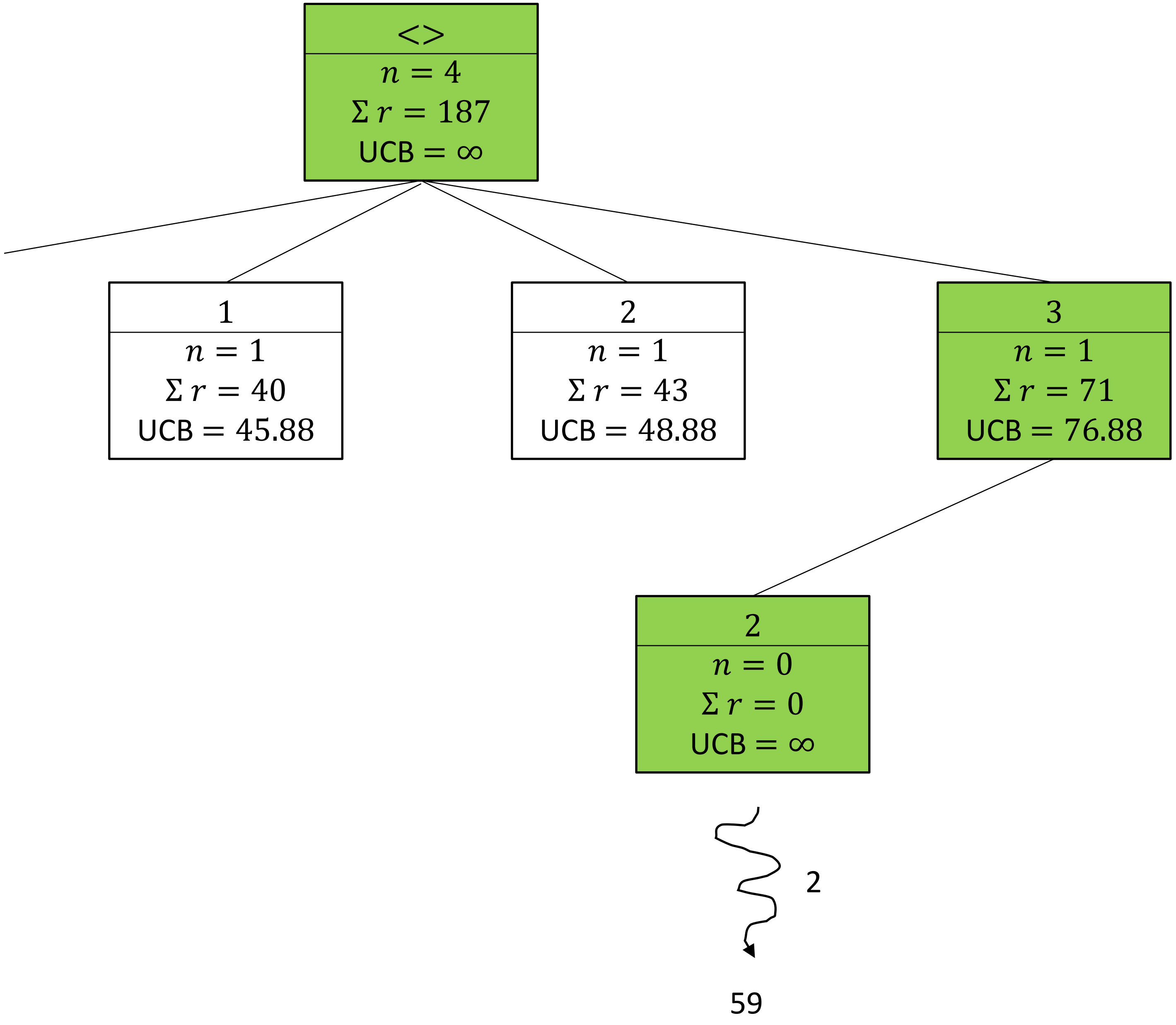

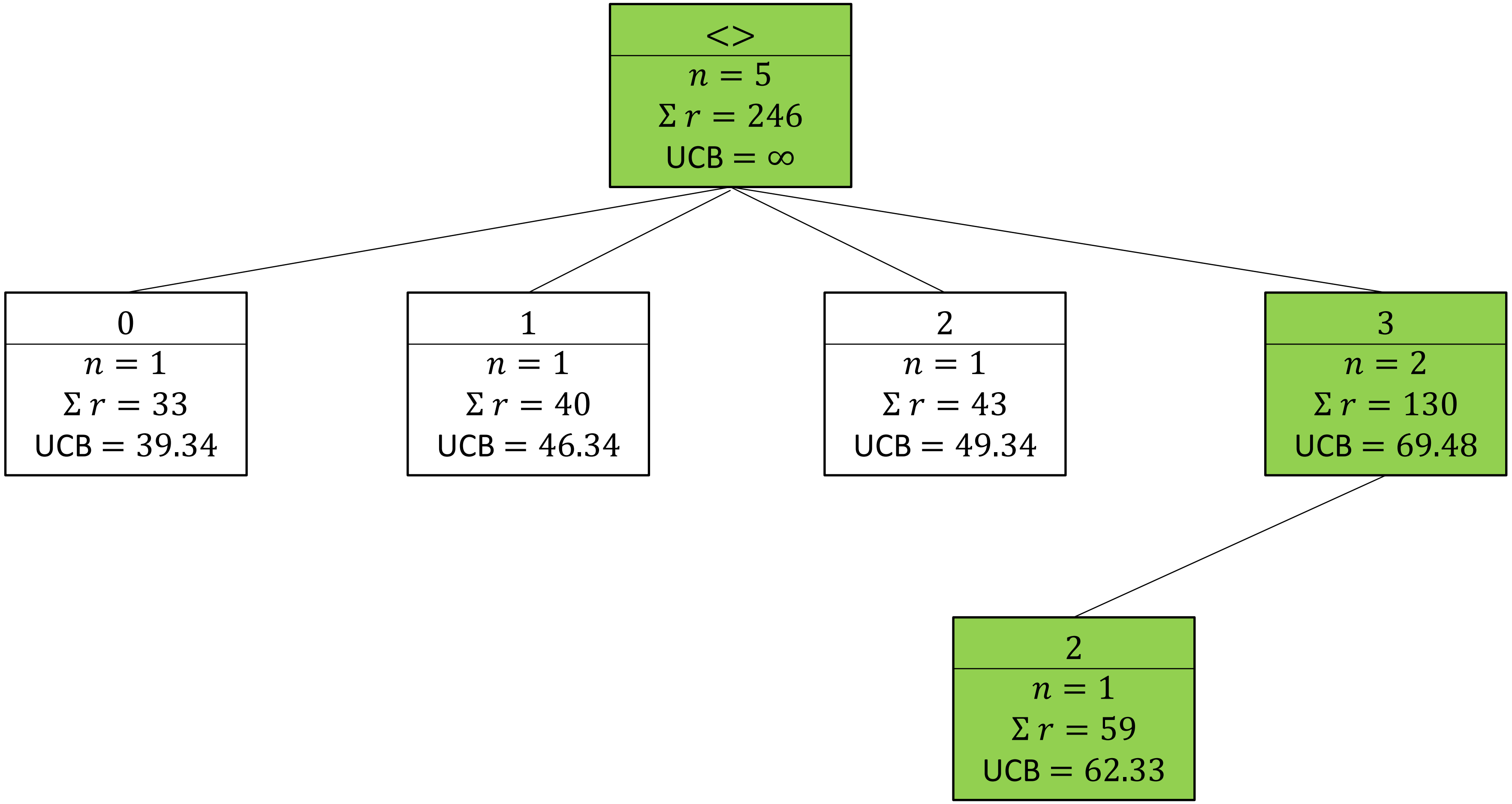
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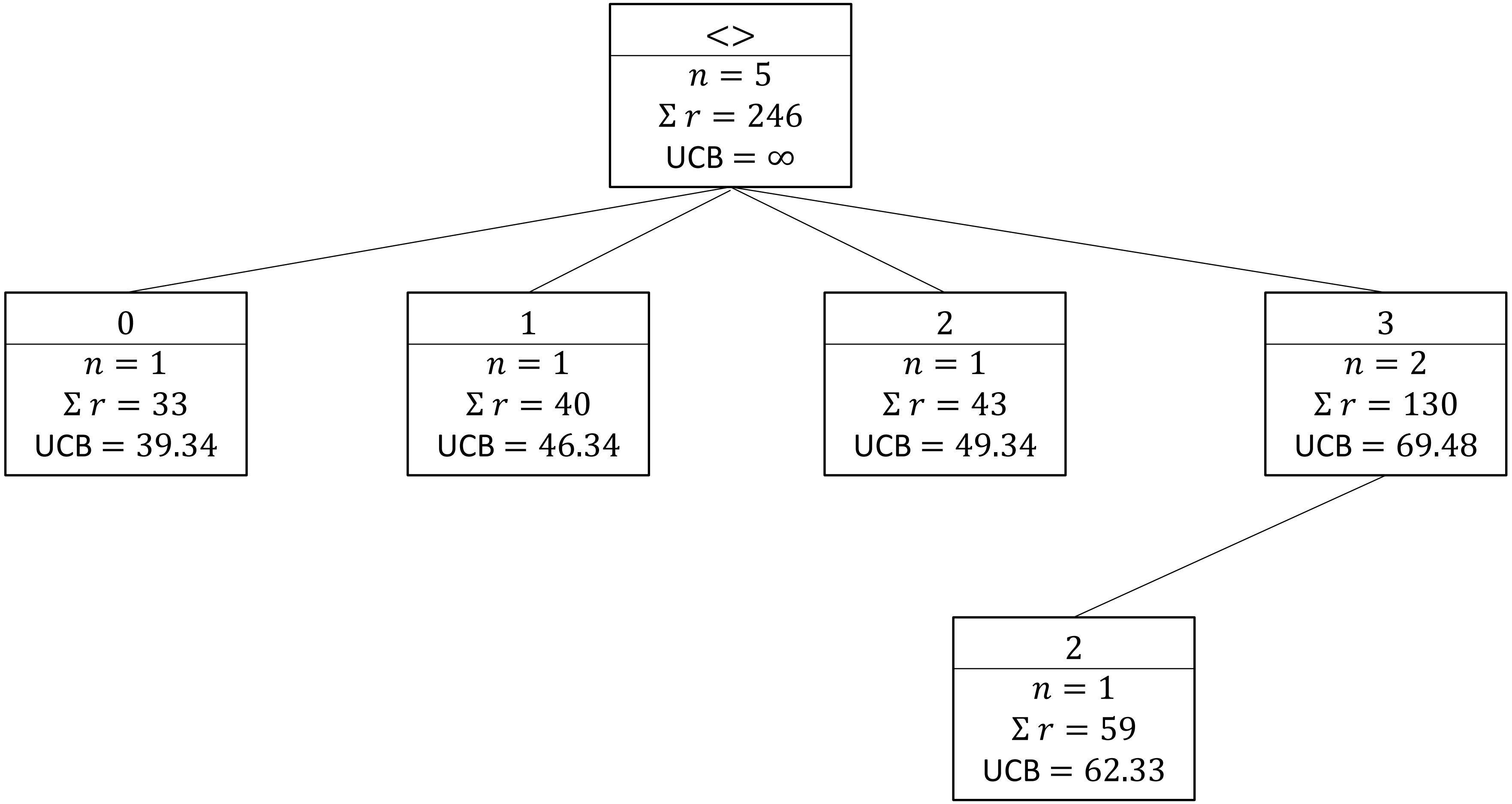
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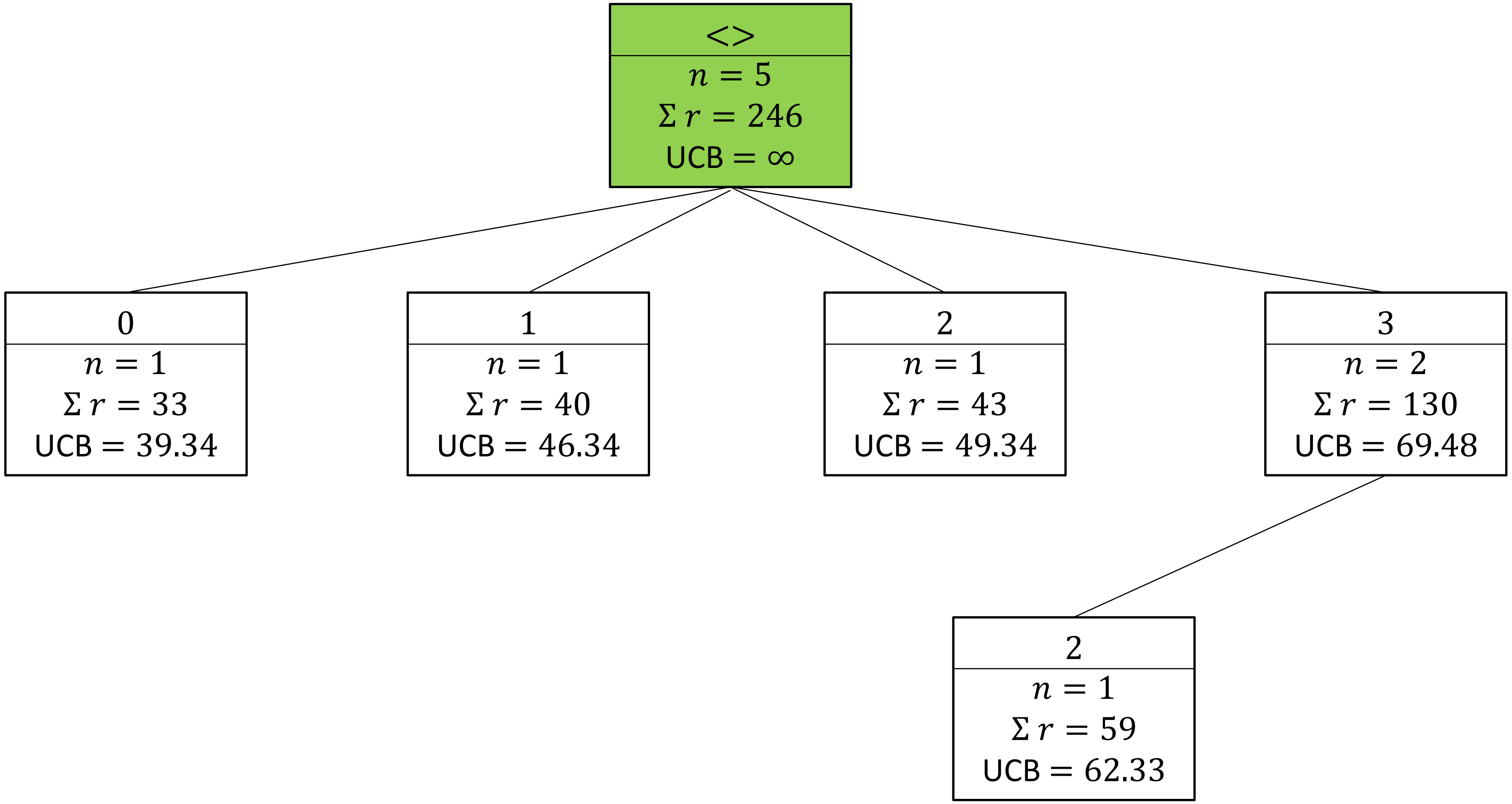
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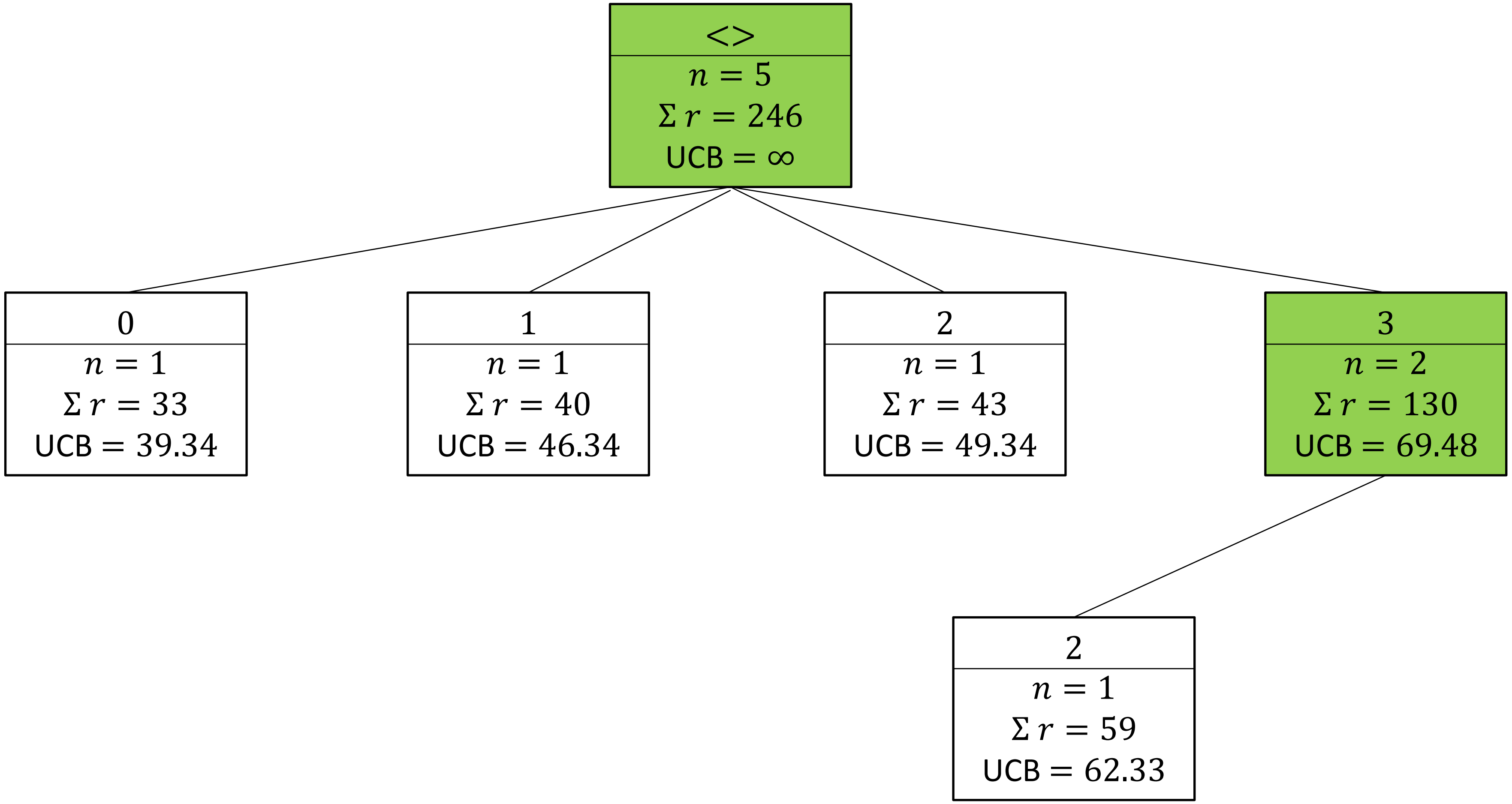
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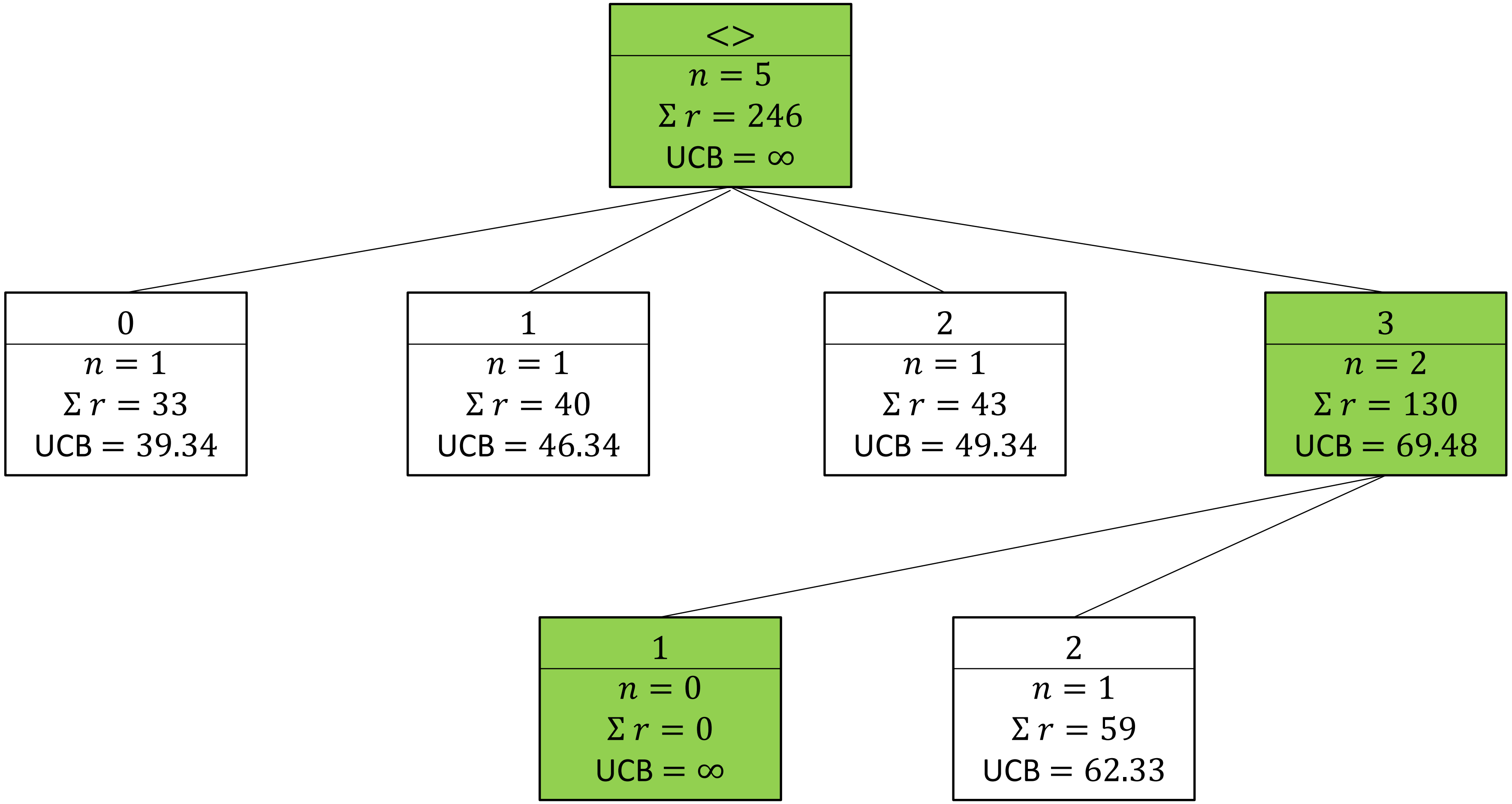
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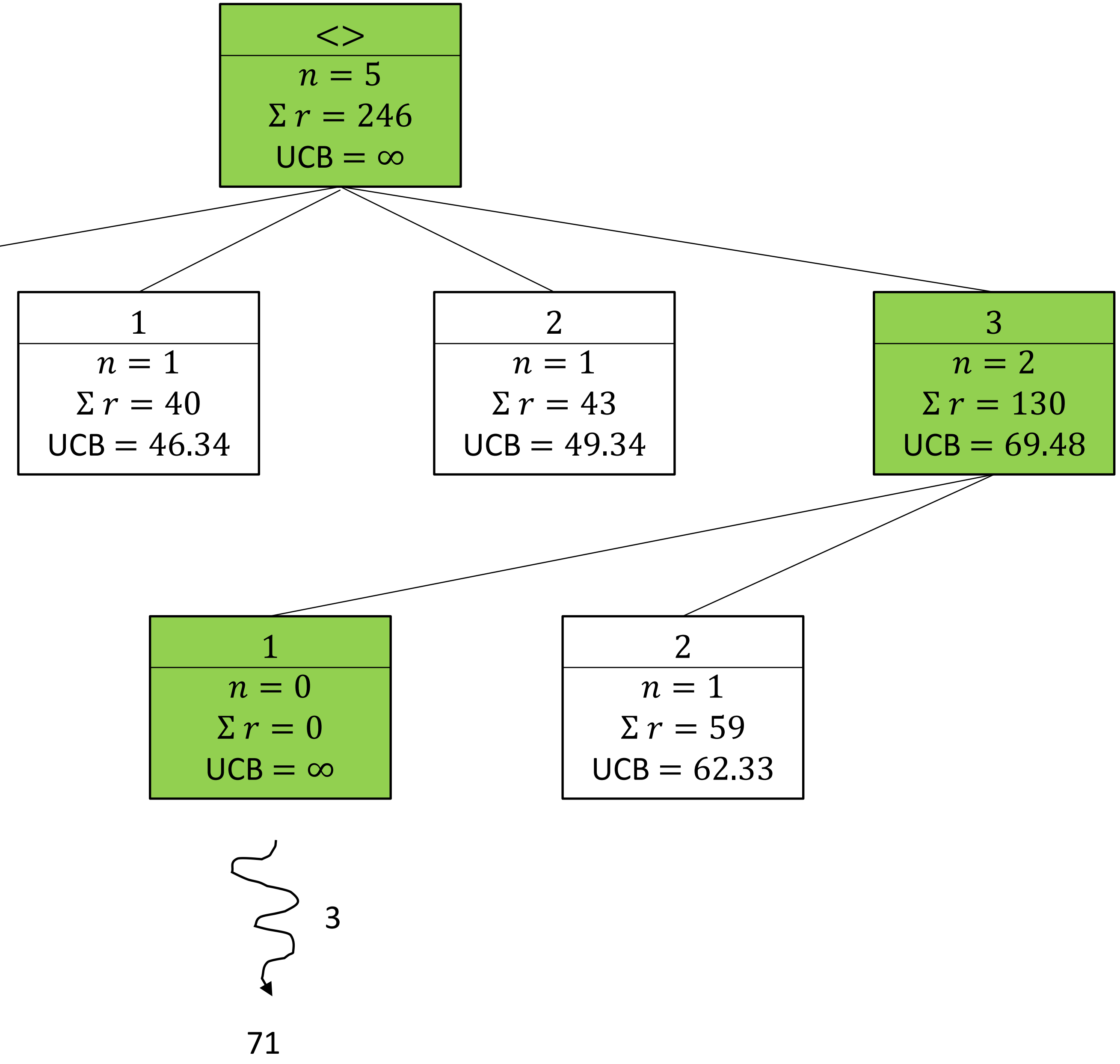

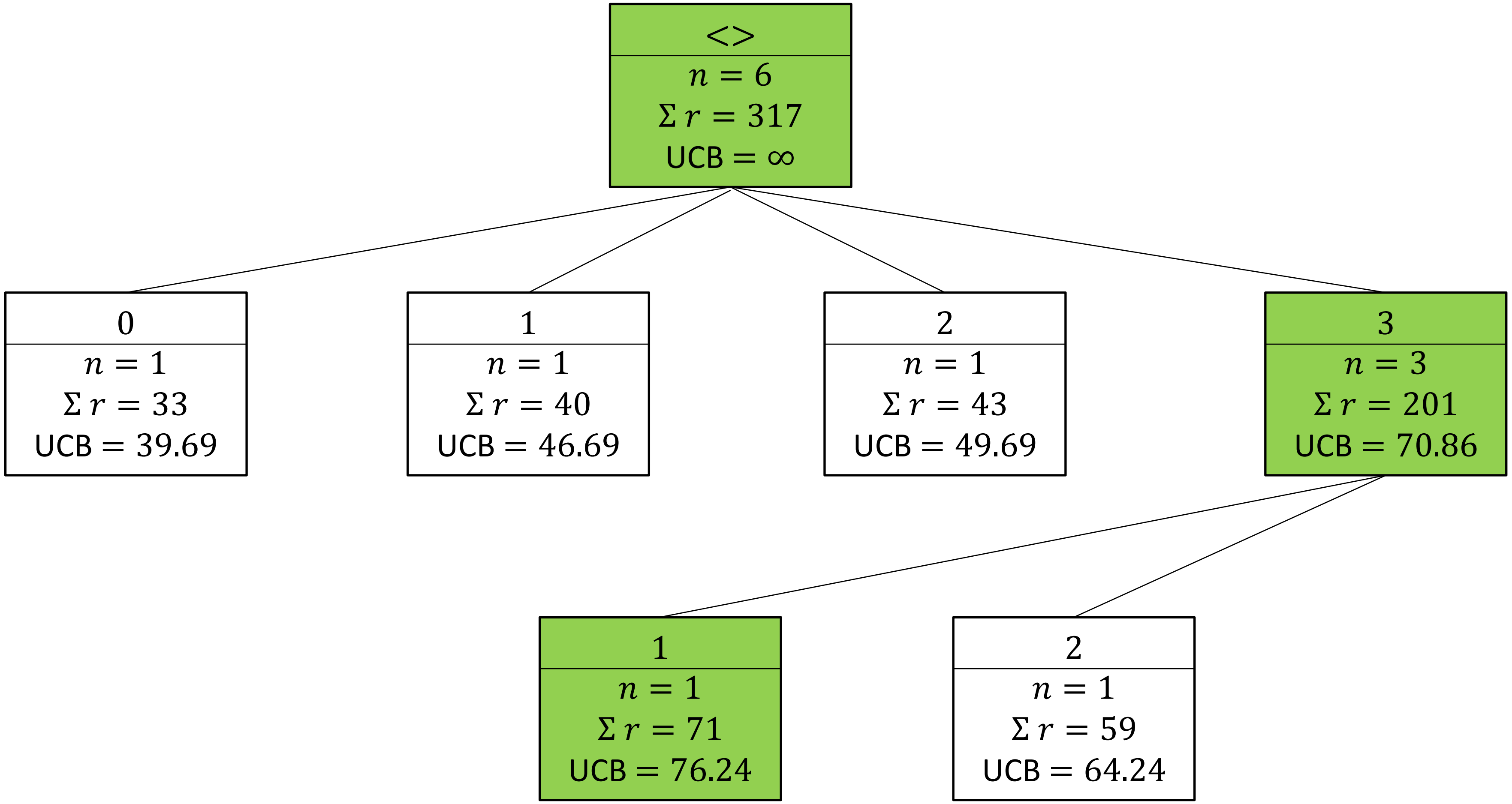
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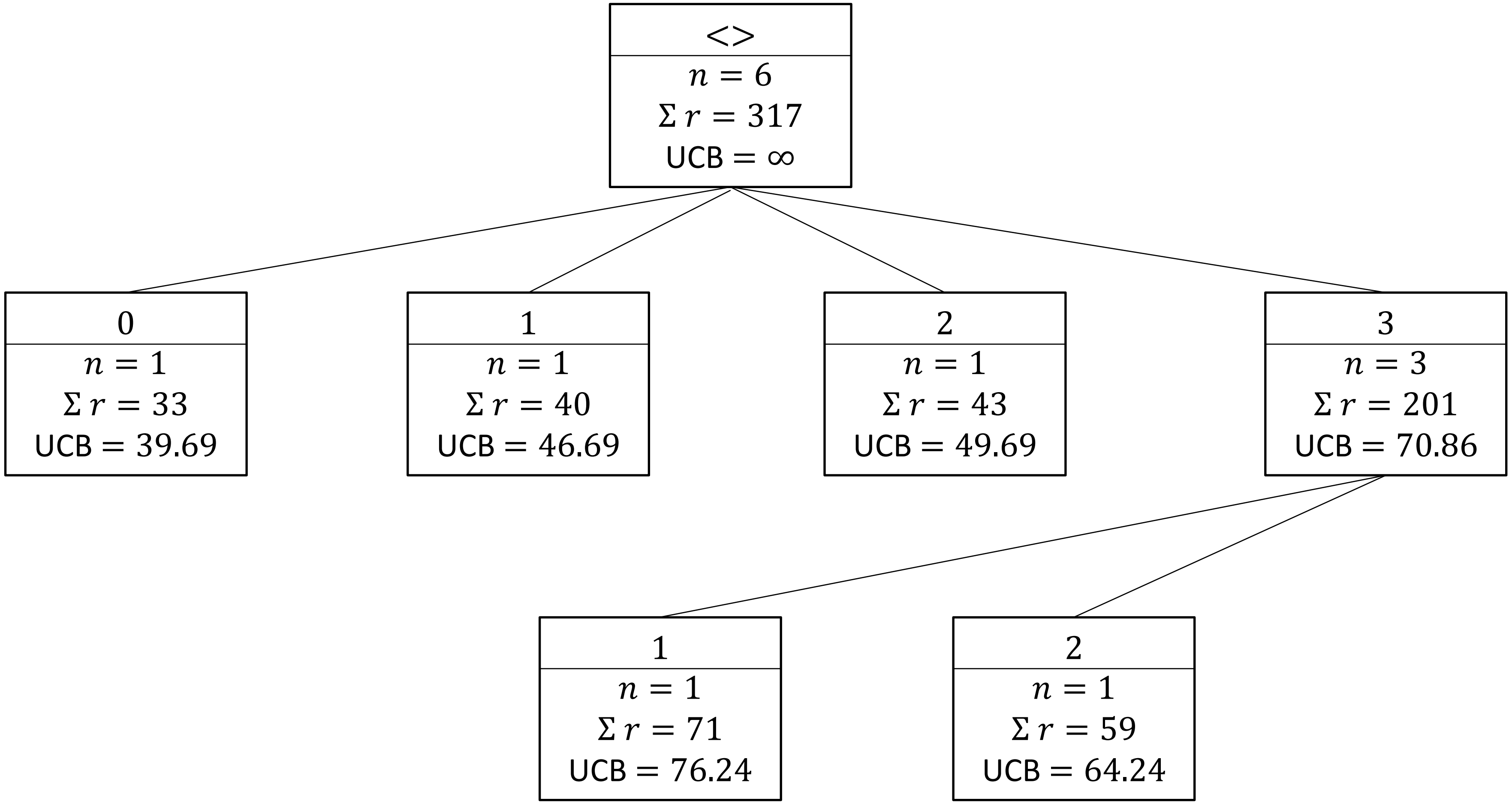
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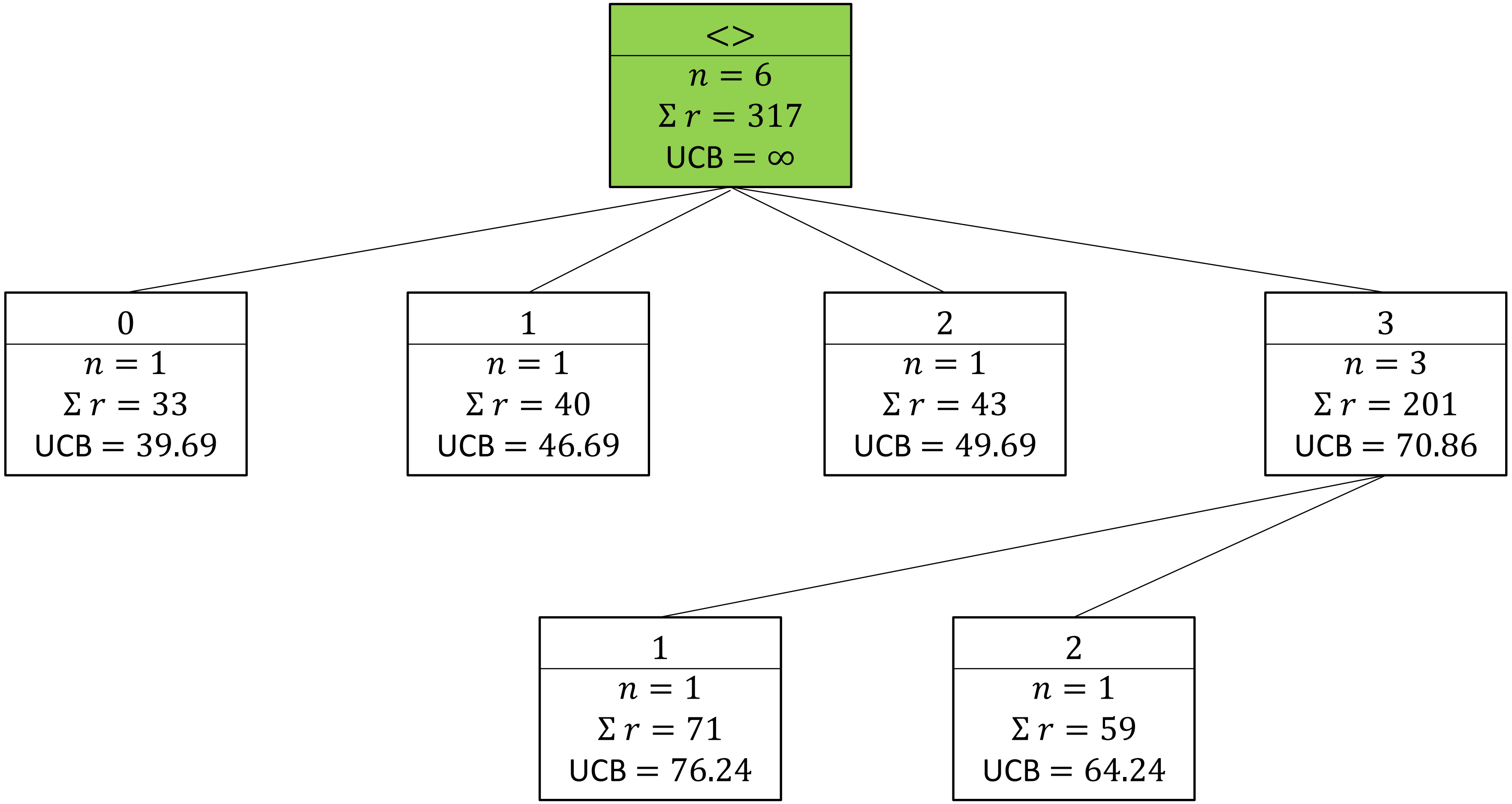
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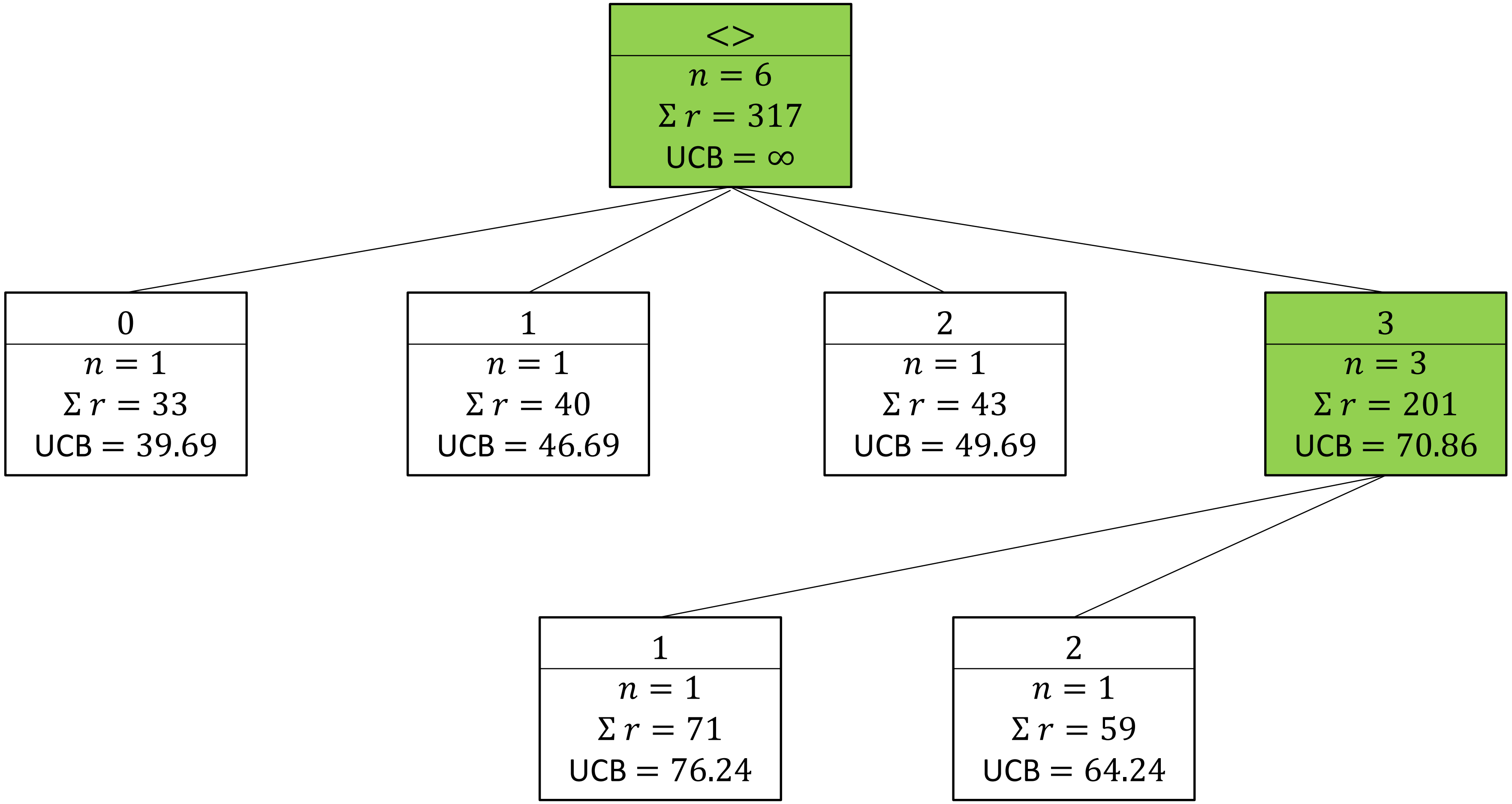
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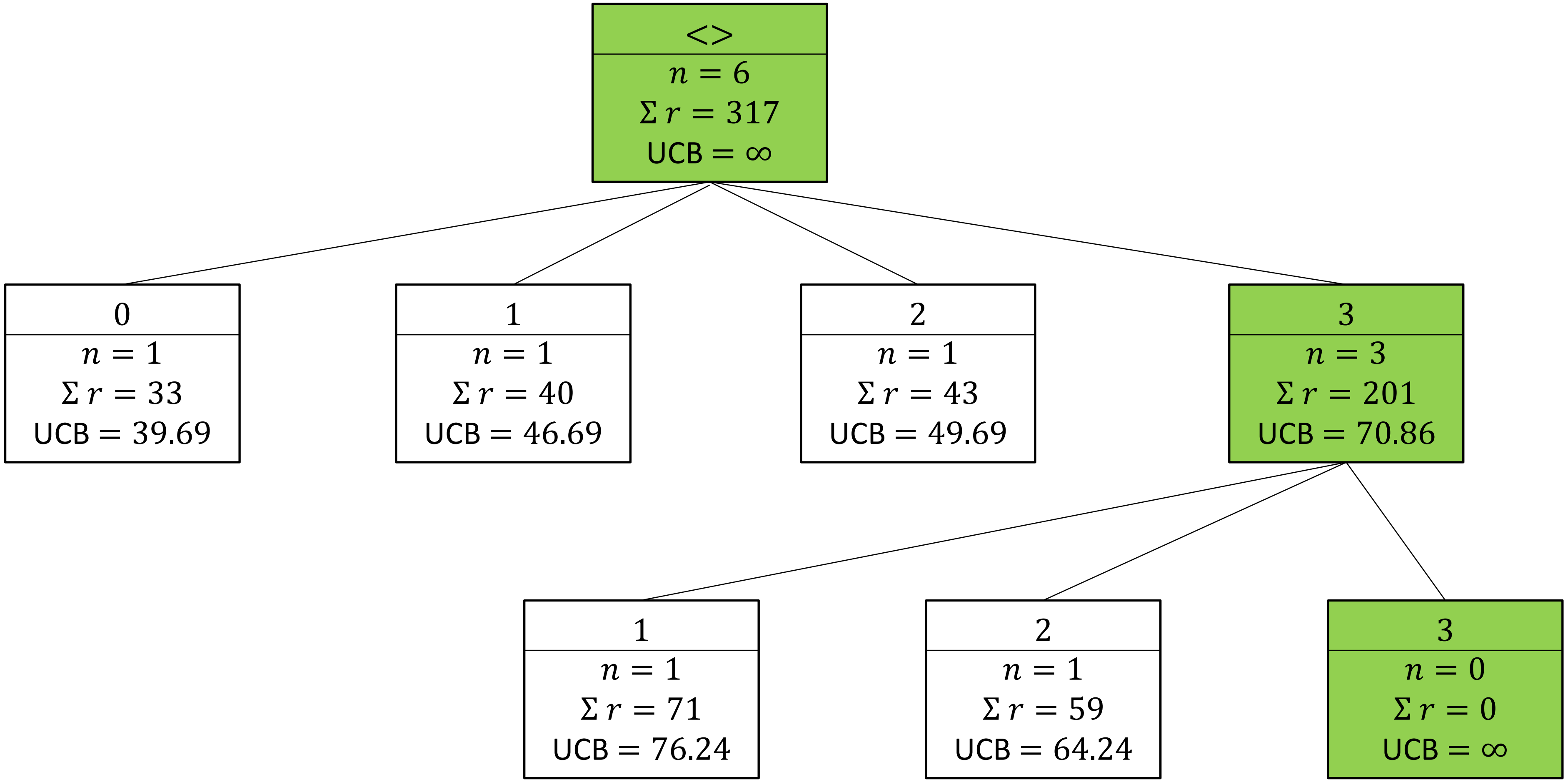
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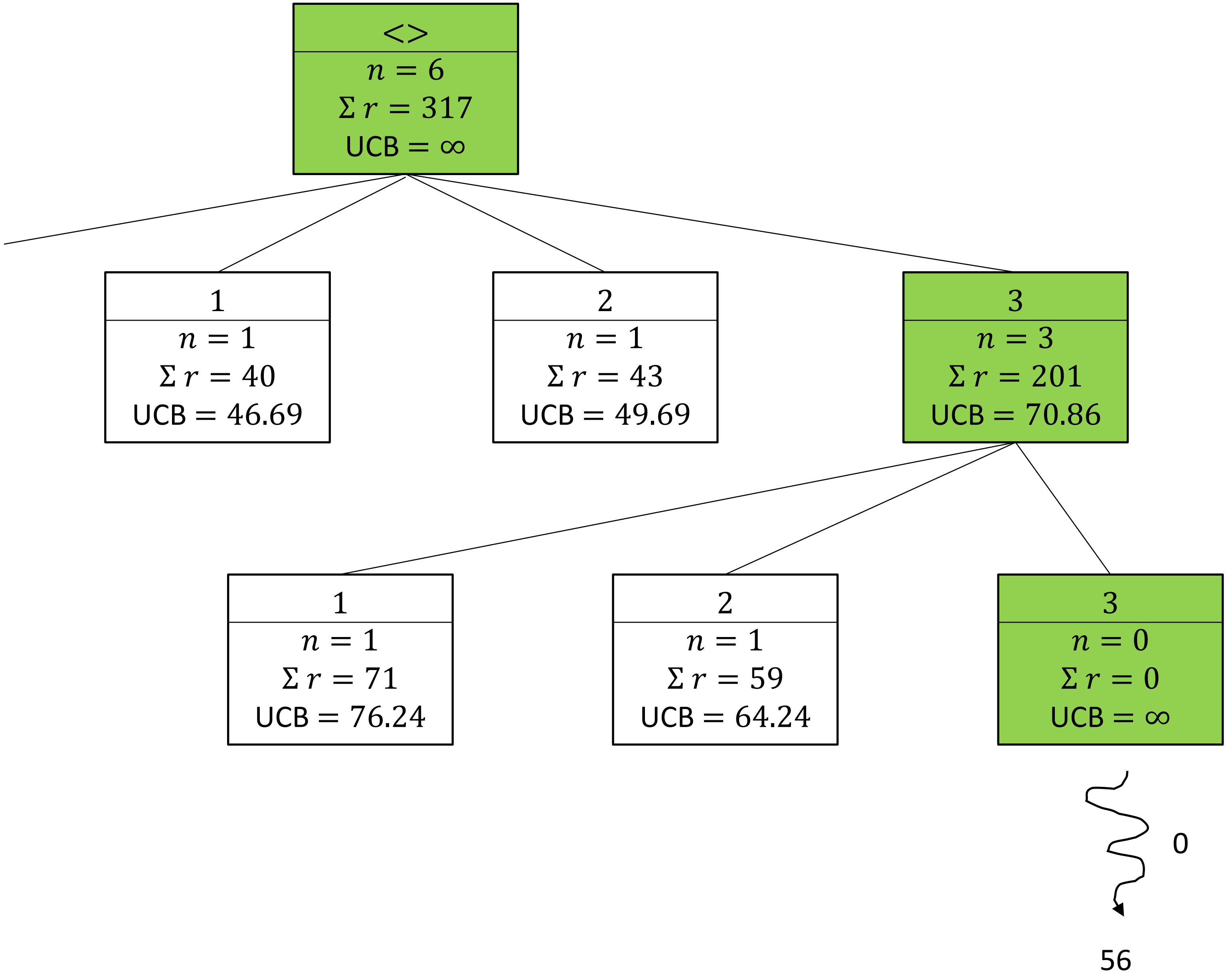

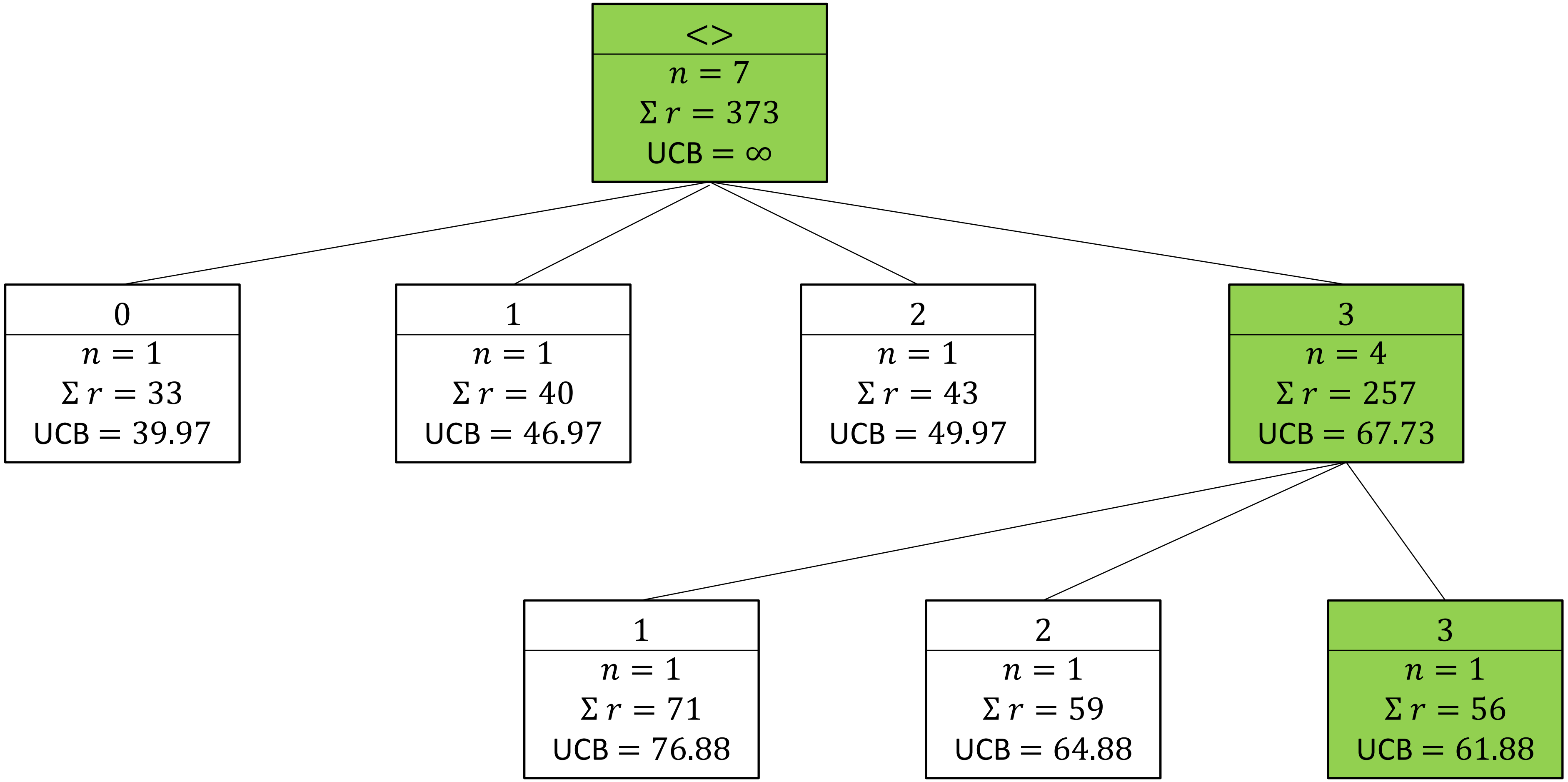
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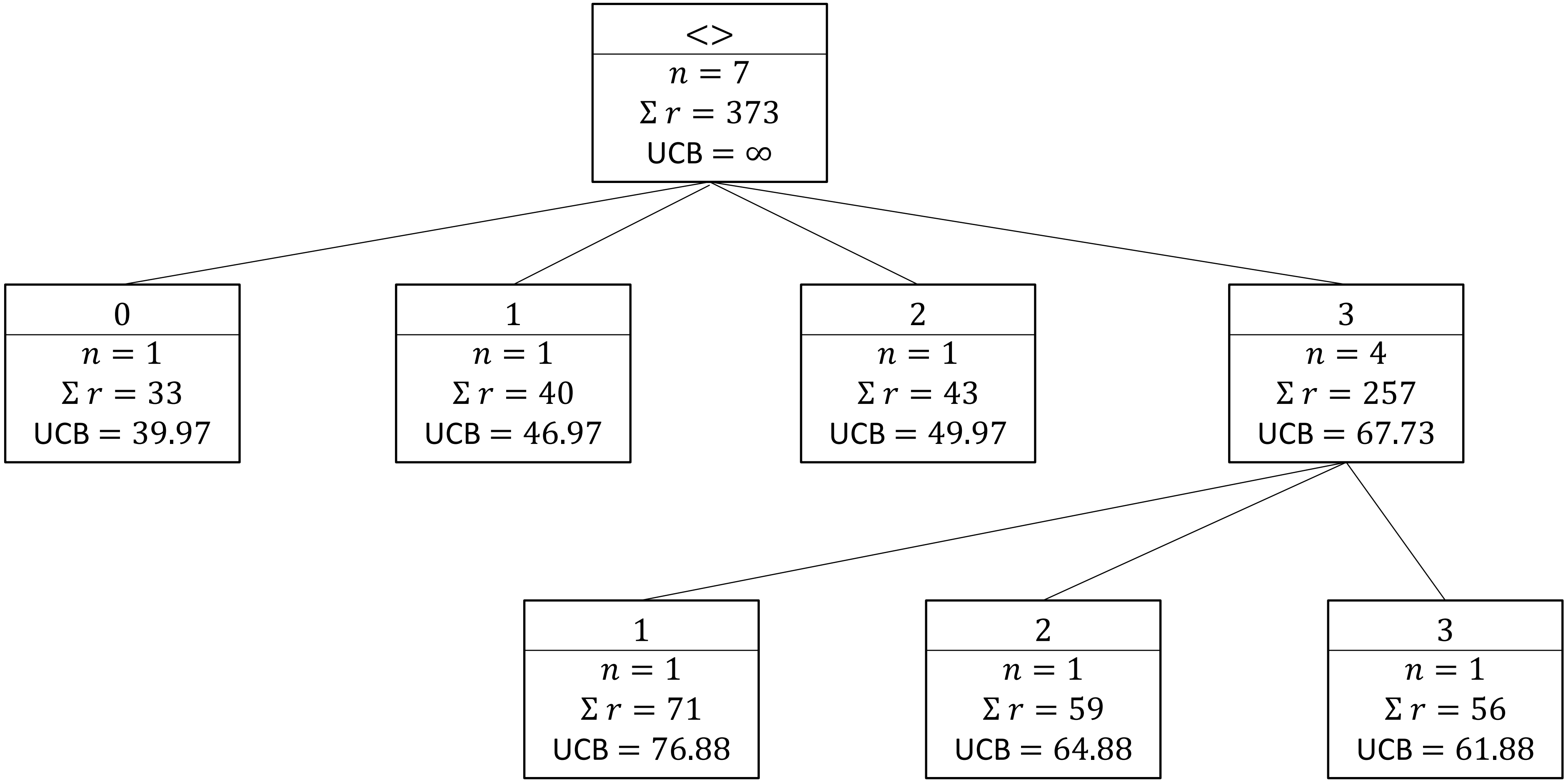
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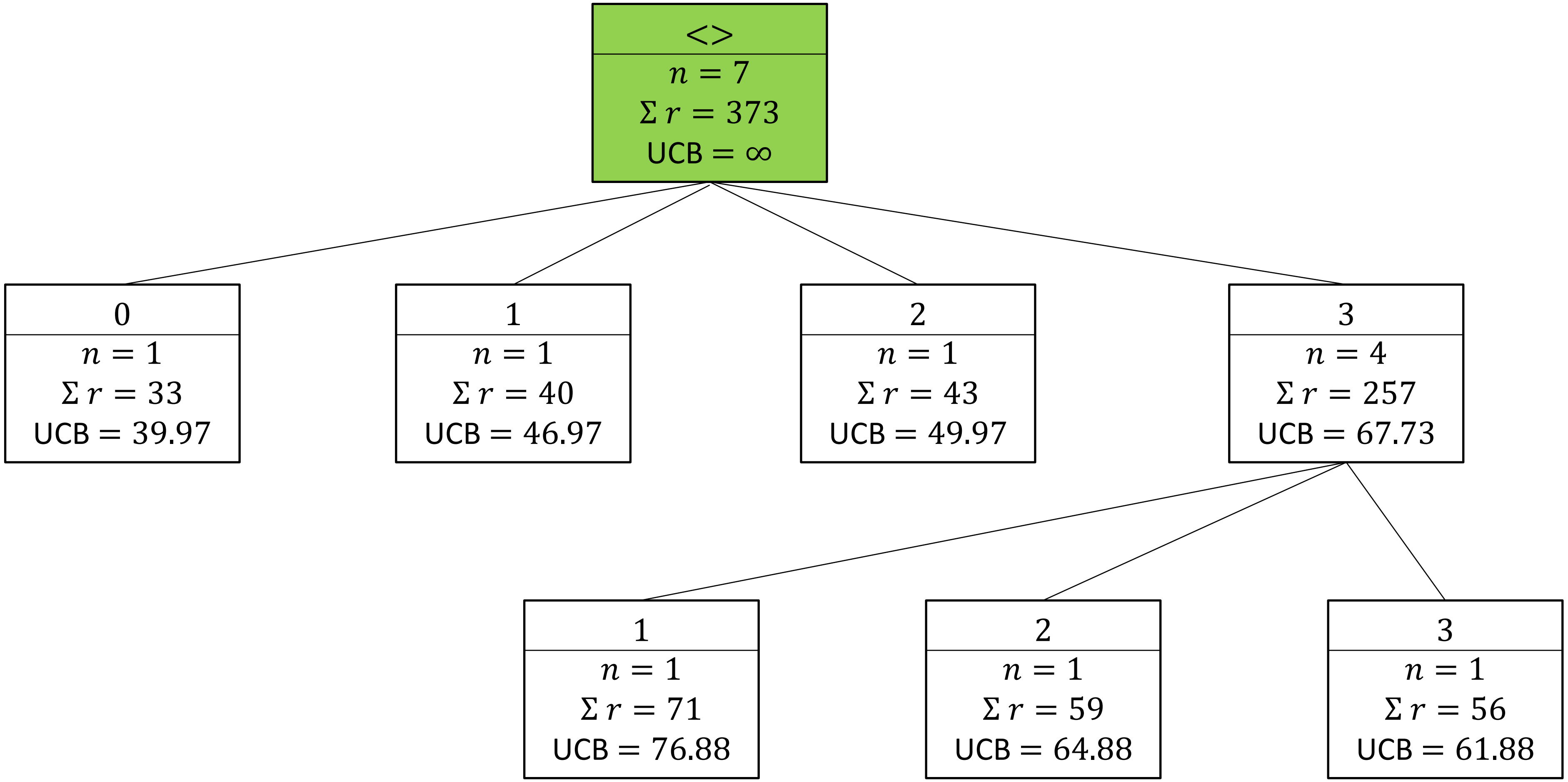
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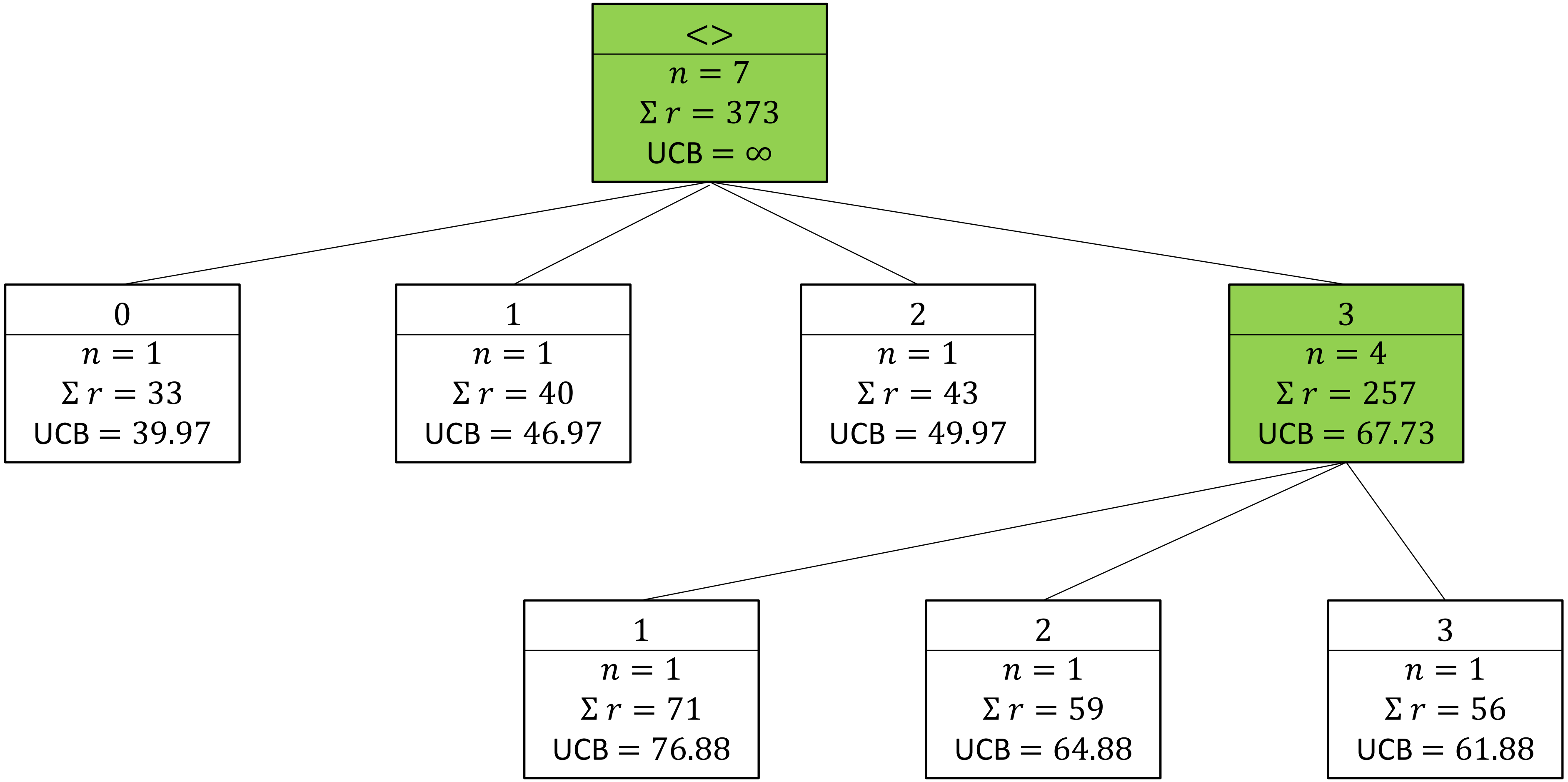
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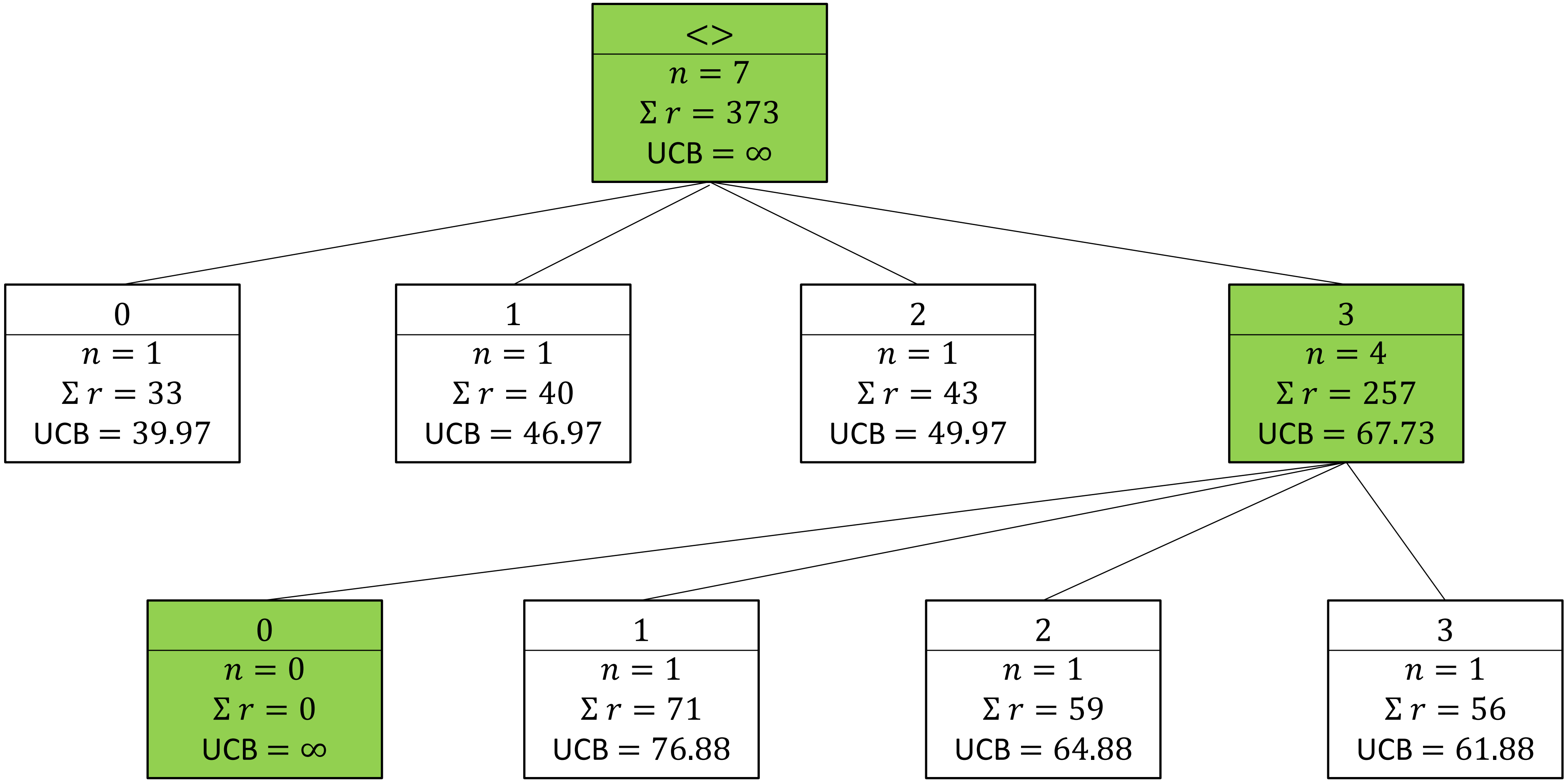
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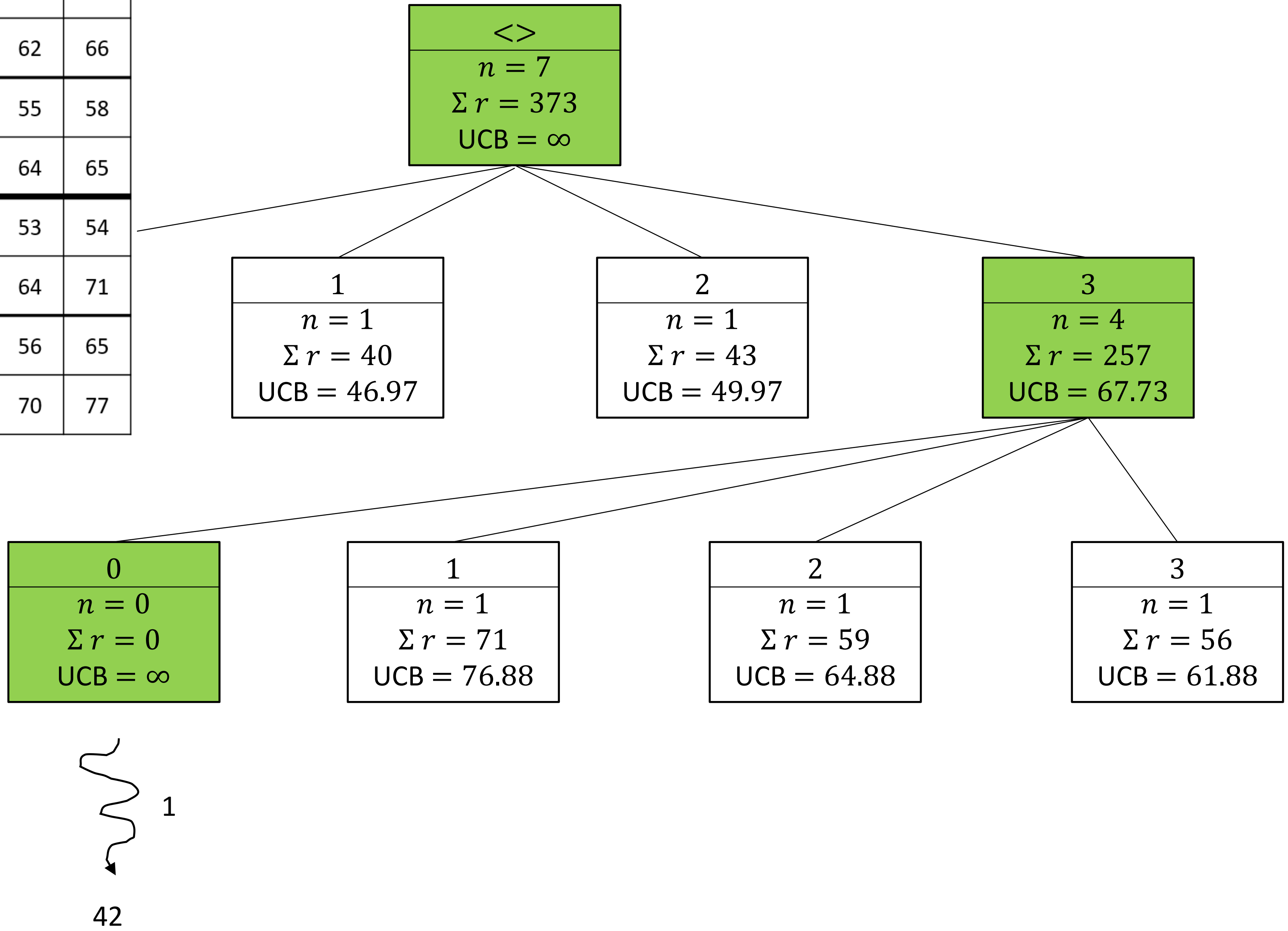

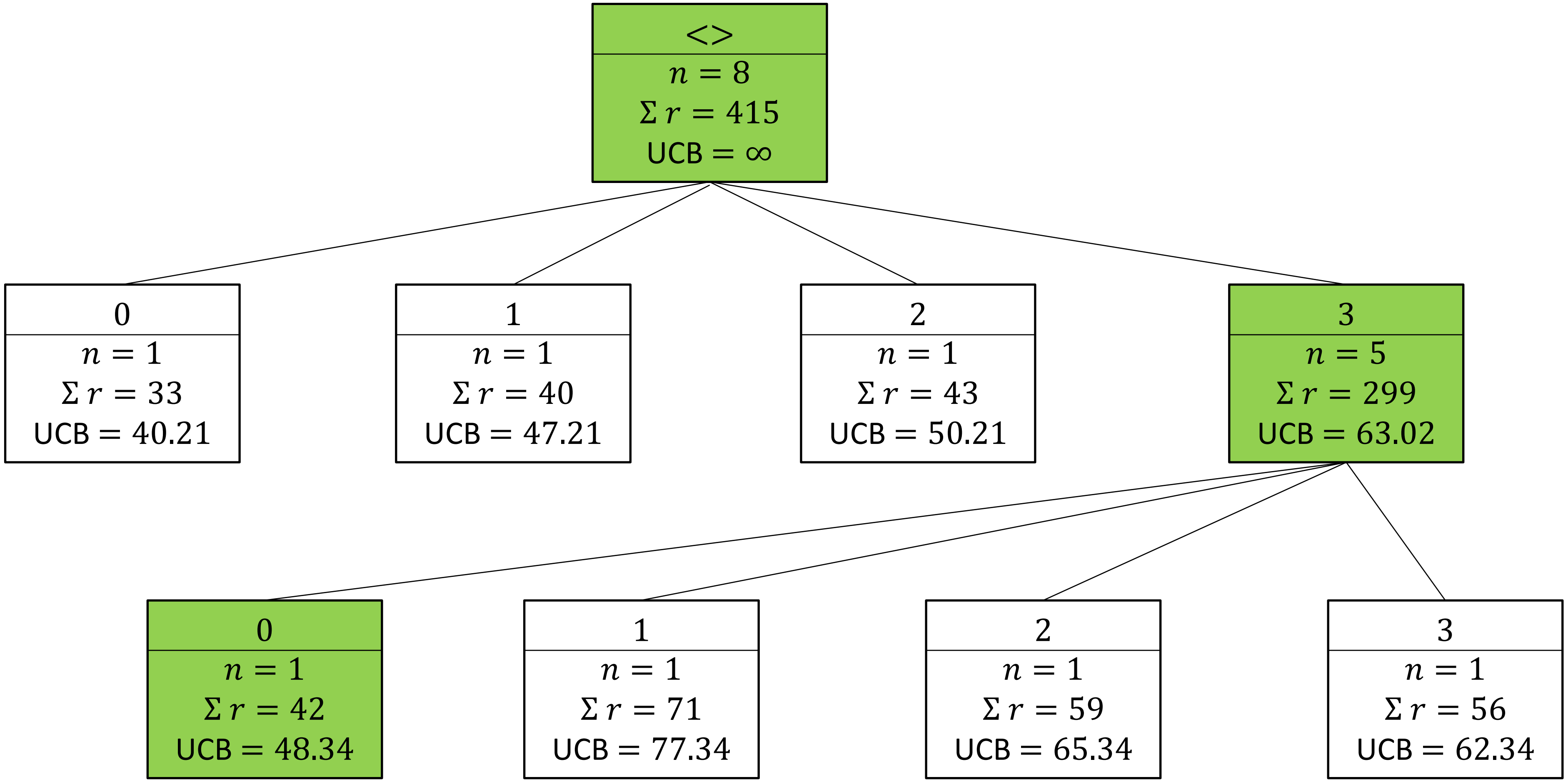
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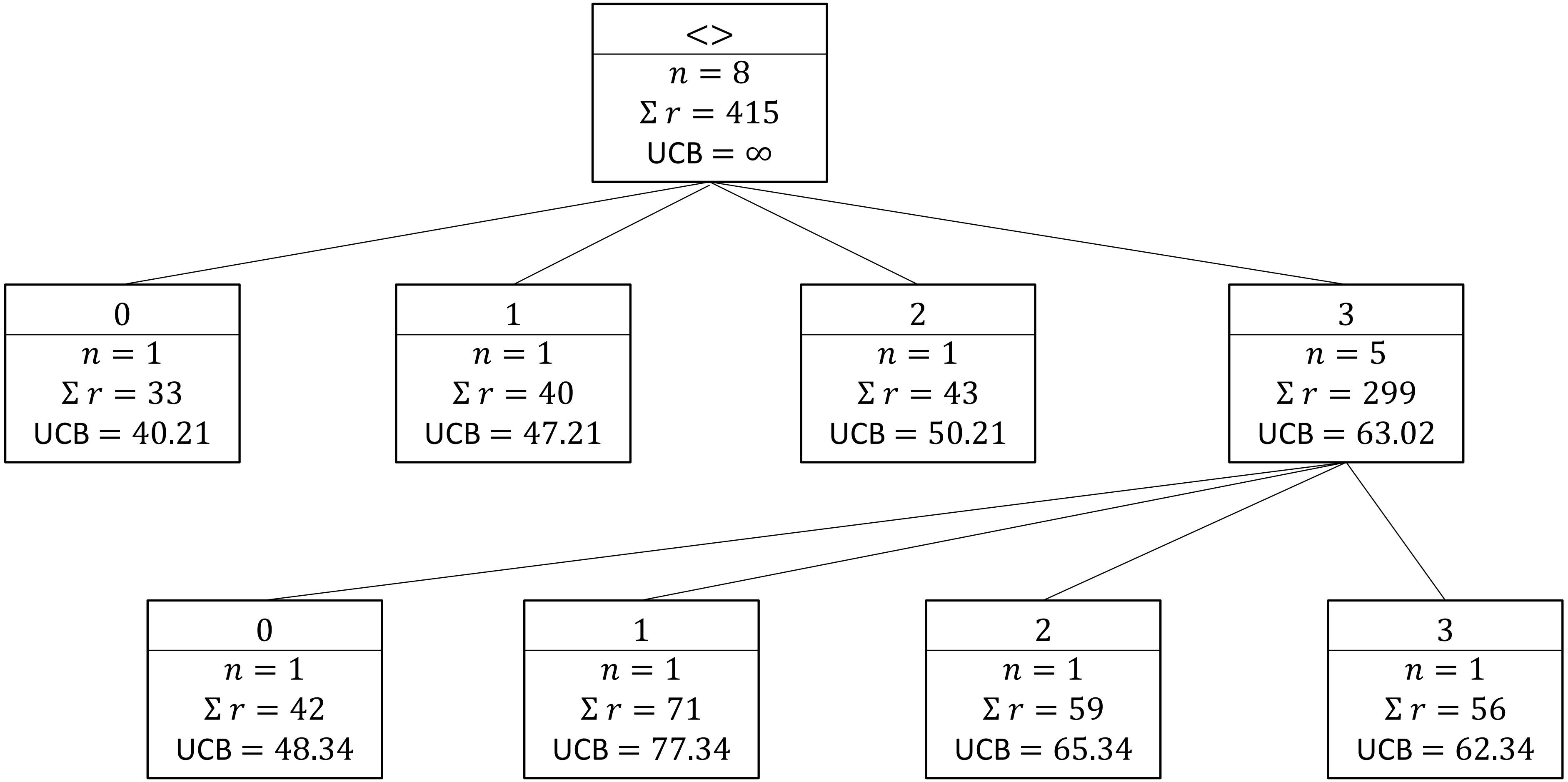
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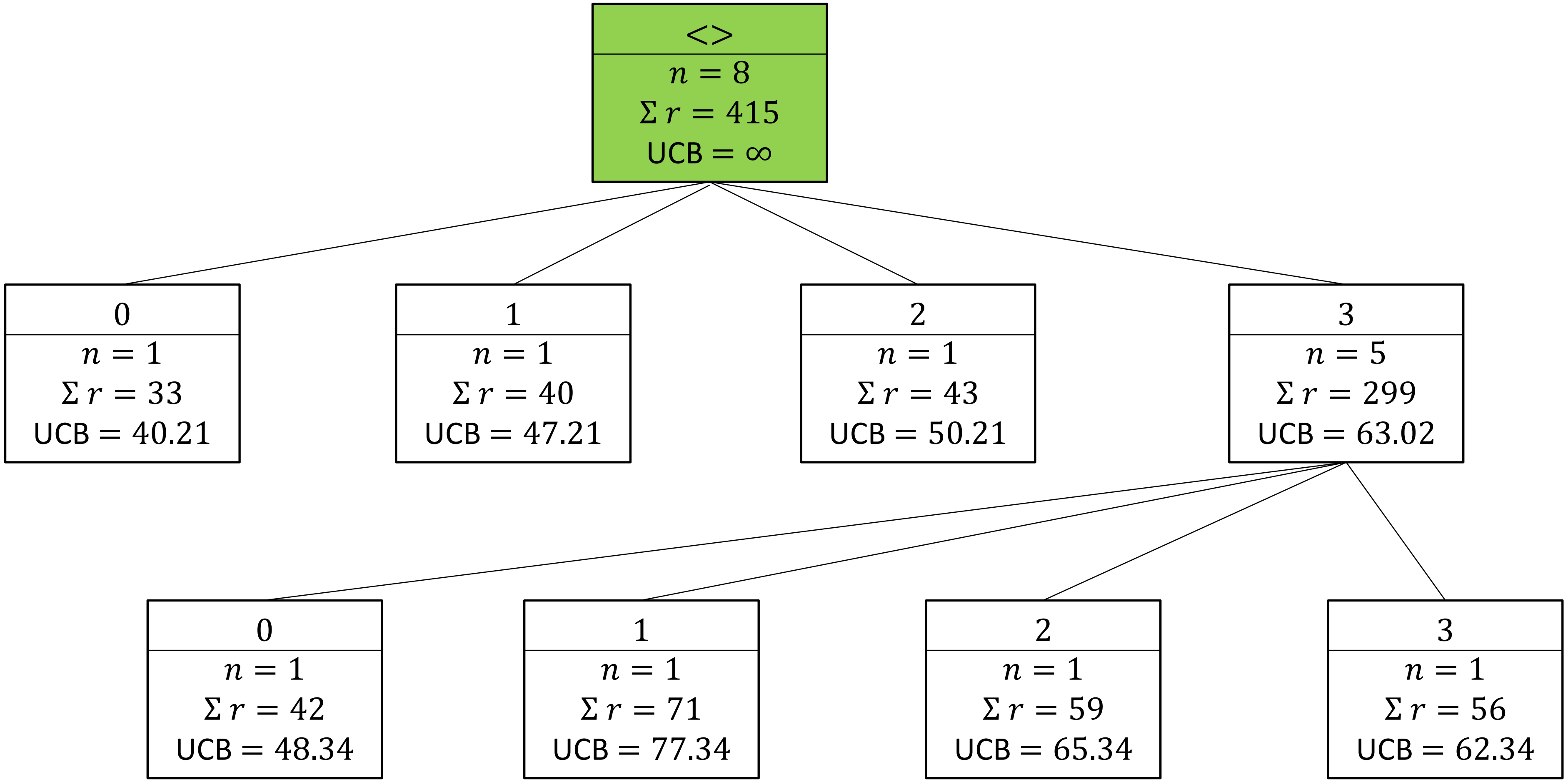
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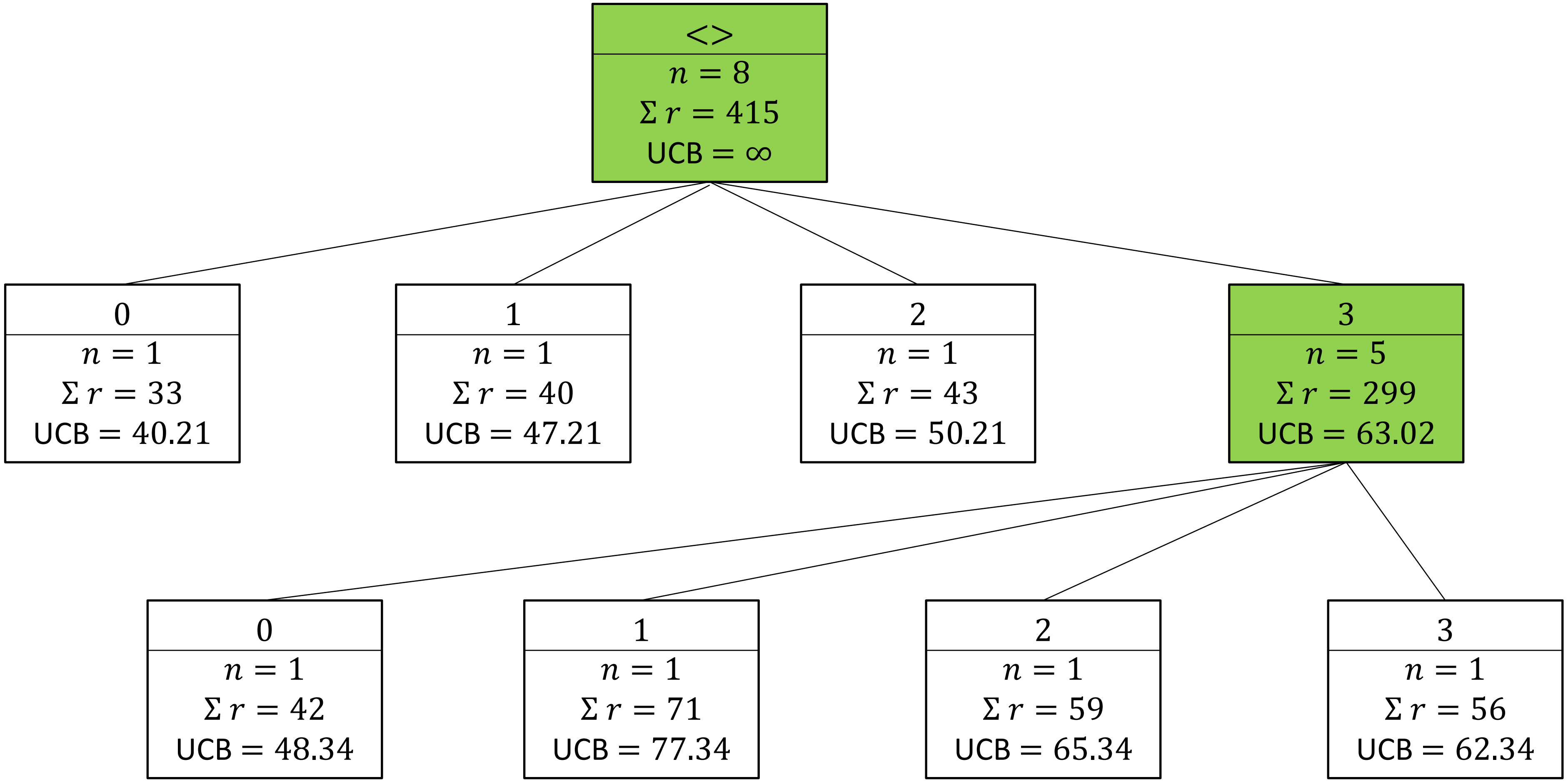
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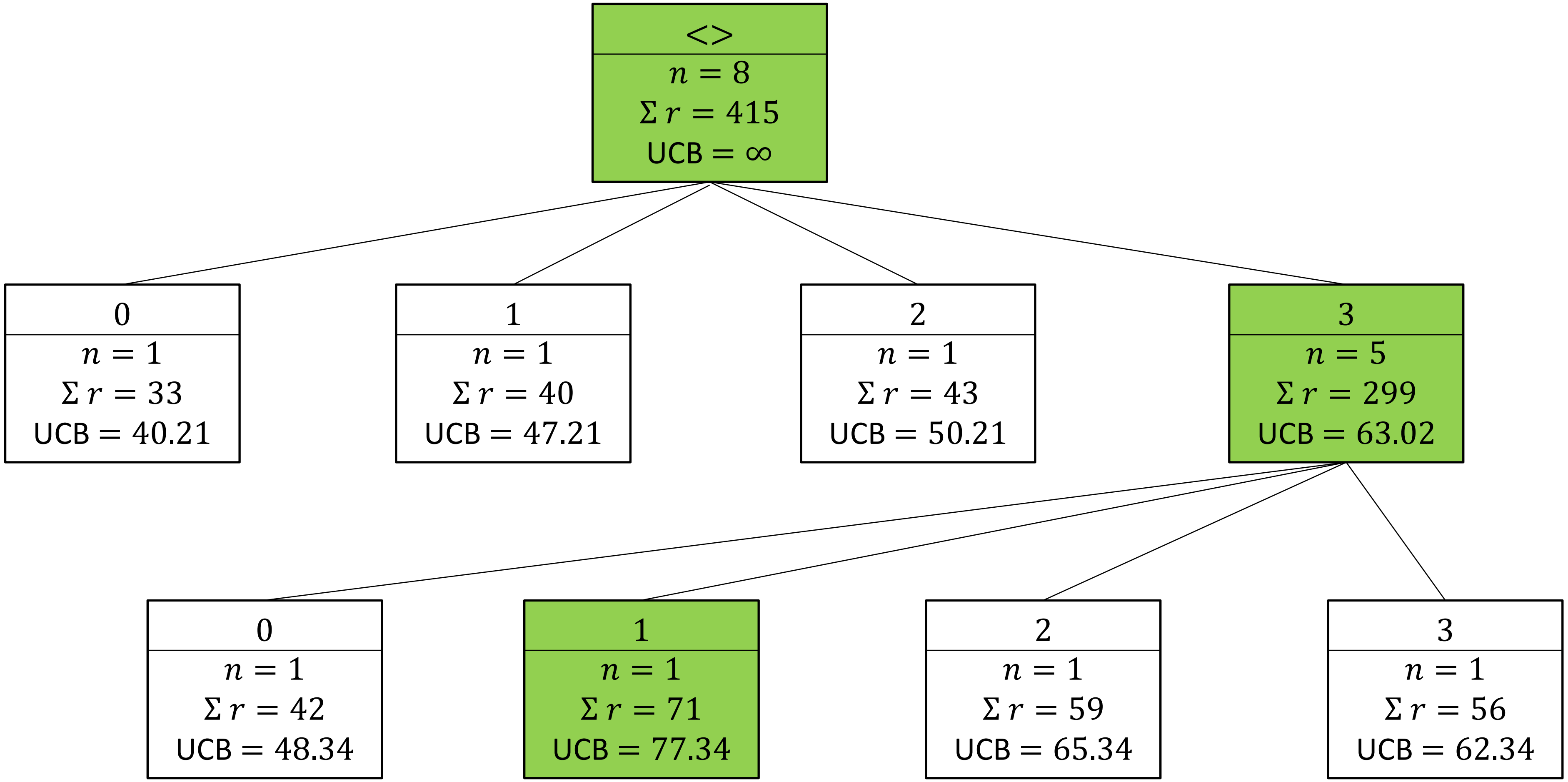
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Σ r = 415
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0
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1
n = 1
Σ r = 40
UCB = 47.21
2
n = 1
Σ r = 43
UCB = 50.21
3
n = 5
Σ r = 299
UCB = 63.02
0
n = 1
Σ r = 42
UCB = 48.34
1
n = 1
Σ r = 71
UCB = 77.34
2
n = 1
Σ r = 59
UCB = 65.34
3
n = 1
Σ r = 56
UCB = 62.34

30	33	39	38	32	34	55	53
41	50	49	54	42	45	62	66
32	41	44	49	40	40	55	58
45	45	46	59	49	52	64	65
37	37	43	44	38	42	53	54
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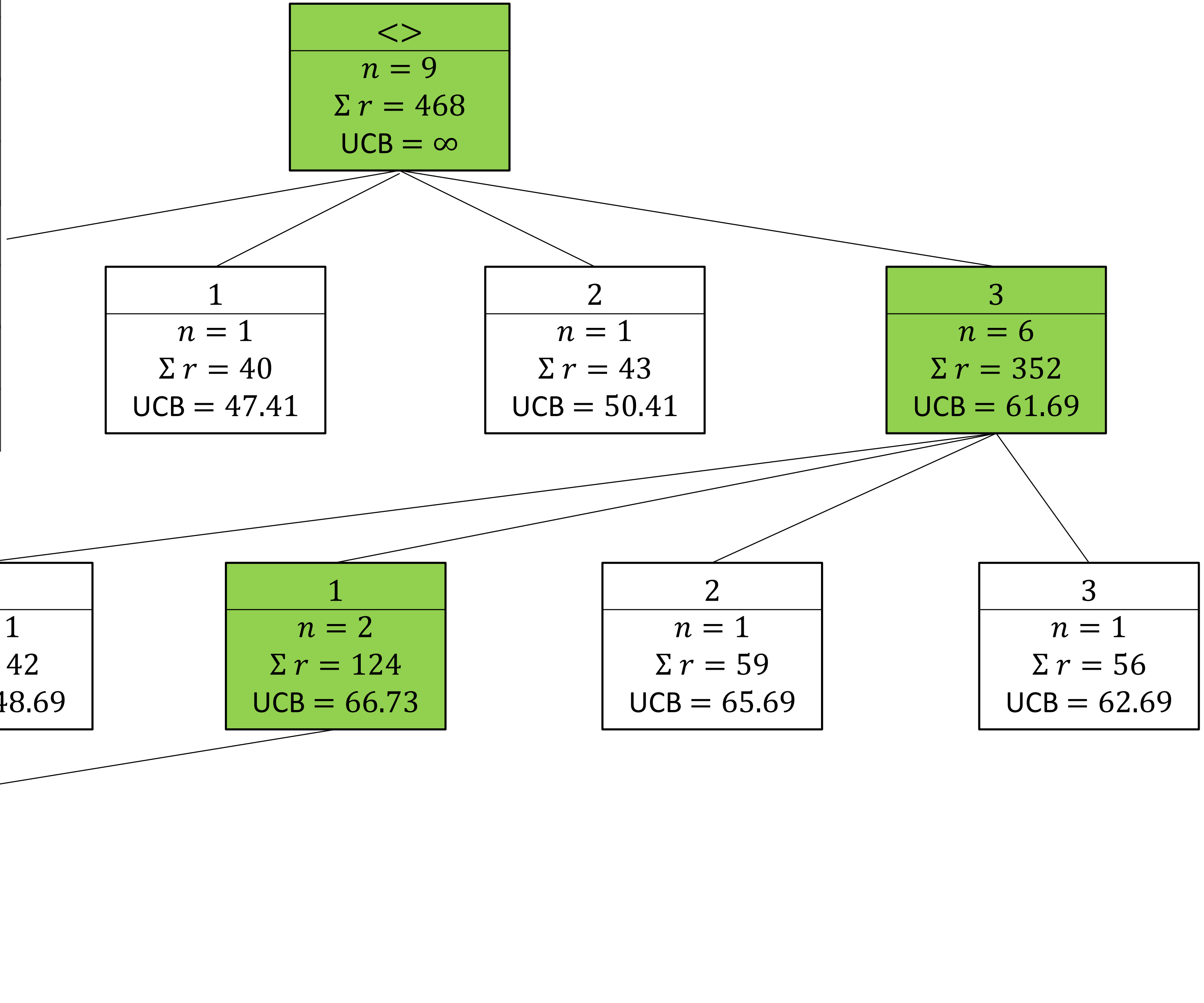

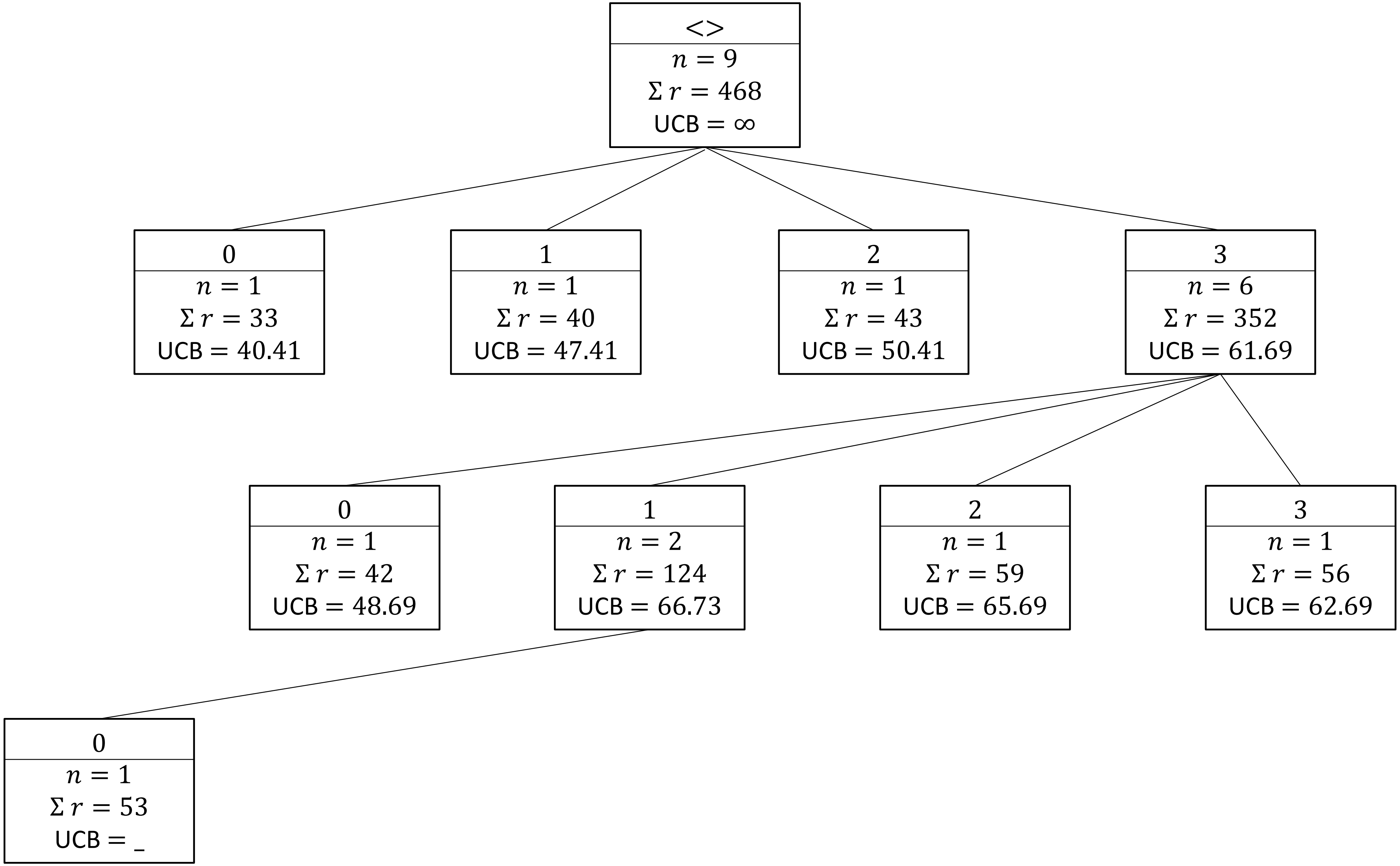
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UCB = ∞
0
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UCB = 40.41
1
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UCB = 47.41
2
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$\Sigma r = 43$
UCB = 50.41
3
$n = 6$
$\Sigma r = 352$
UCB = 61.69
0
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$\Sigma r = 42$
UCB = 48.69
1
$n = 2$
$\Sigma r = 124$
UCB = 66.73
2
$n = 1$
$\Sigma r = 59$
UCB = 65.69
3
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$\Sigma r = 56$
UCB = 62.69
0
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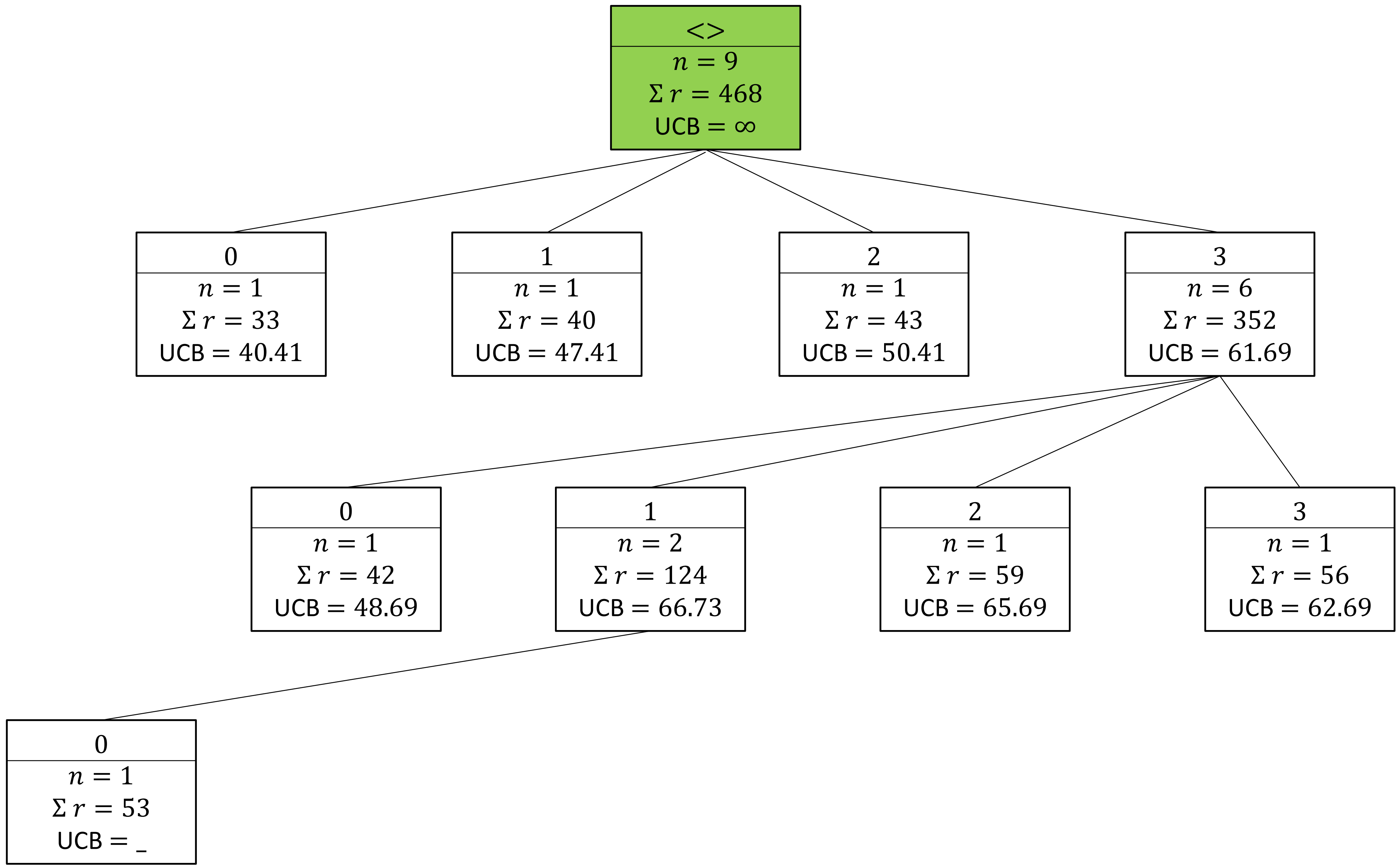
$\Sigma r = 53$
UCB = _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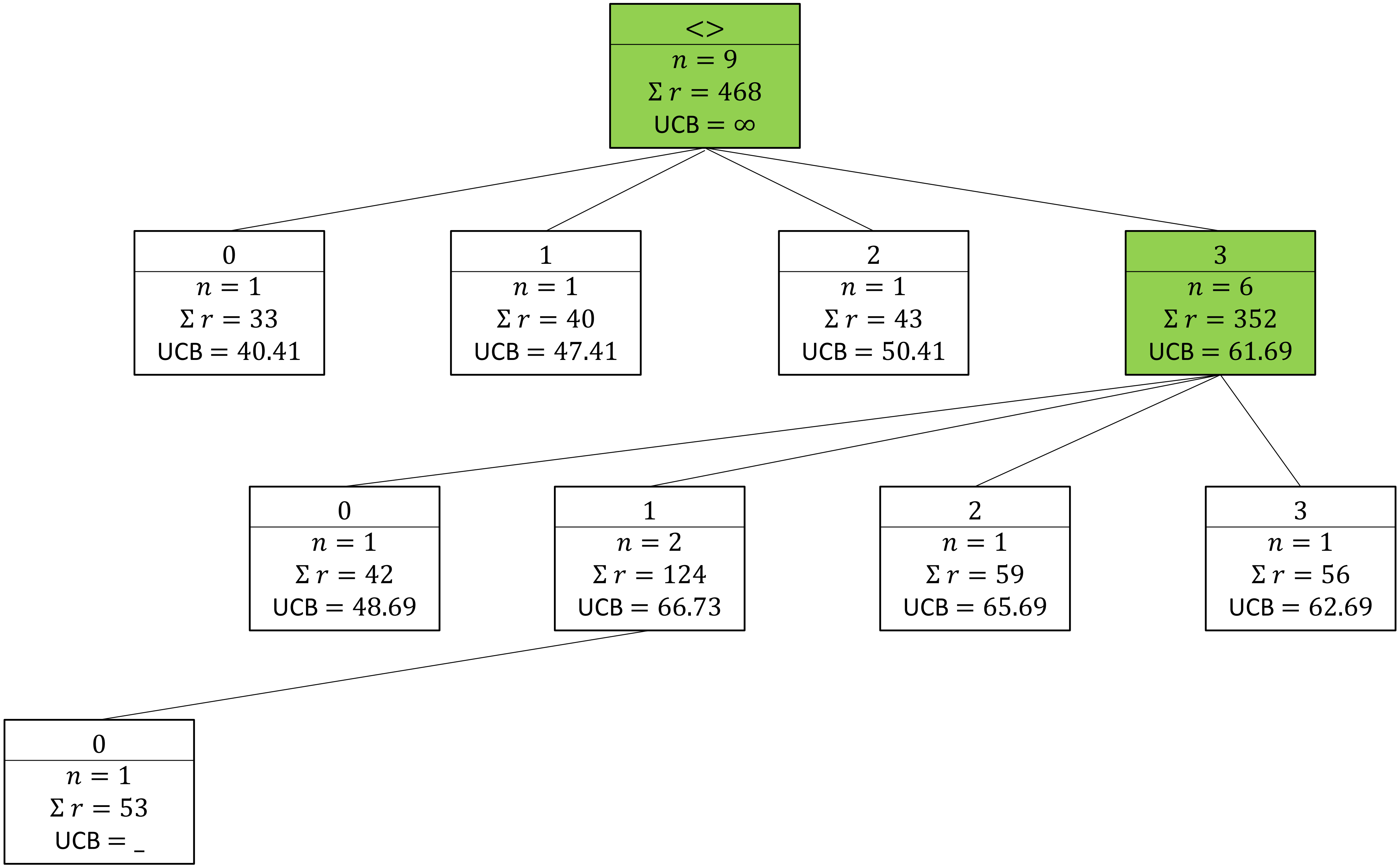
$\Sigma\, r = 40$
UCB = 47.41
2
$n = 1$
$\Sigma\, r = 43$
UCB = 50.41
3
$n = 6$
$\Sigma\, r = 352$
UCB = 61.69
0
$n = 1$
$\Sigma\, r = 42$
UCB = 48.69
1
$n = 2$
$\Sigma\, r = 124$
UCB = 66.73
2
$n = 1$
$\Sigma\, r = 59$
UCB = 65.69
3
$n = 1$
$\Sigma\, r = 56$
UCB = 62.69
0
$n = 1$
$\Sigma\, r = 53$
UCB = _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UCB = 47.41
2
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UCB = 50.41
3
$n = 6$
$\Sigma\, r = 352$
UCB = 61.69
0
$n = 1$
$\Sigma\, r = 42$
UCB = 48.69
1
$n = 2$
$\Sigma\, r = 124$
UCB = 66.73
2
$n = 1$
$\Sigma\, r = 59$
UCB = 65.69
3
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$\Sigma\, r = 56$
UCB = 62.69
0
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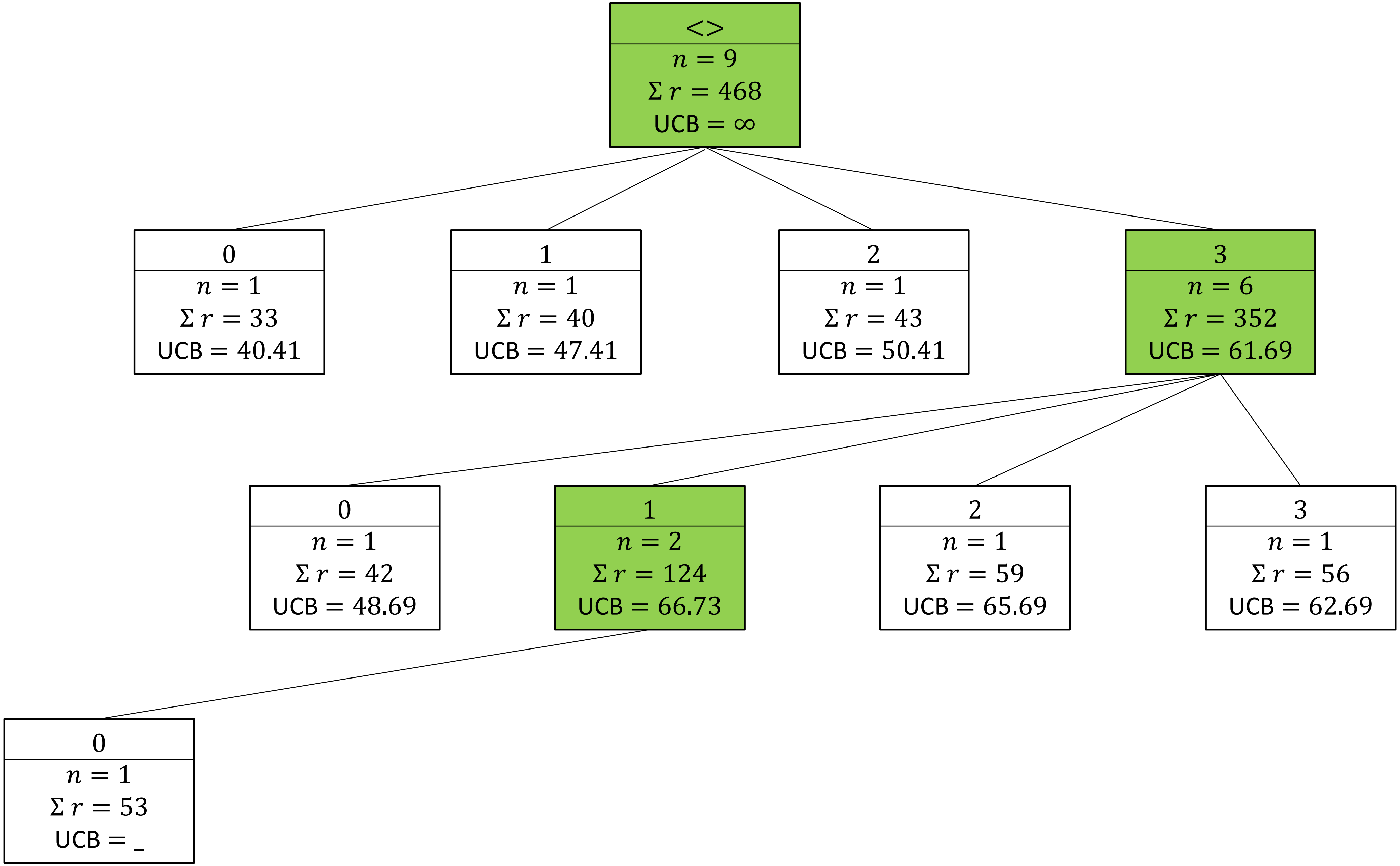
UCB = _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$\Sigma\, r = 40$
UCB = 47.41
2
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$\Sigma\, r = 43$
UCB = 50.41
3
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$\Sigma\, r = 352$
UCB = 61.69
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$\Sigma\, r = 42$
UCB = 48.69
1
$n = 2$
$\Sigma\, r = 124$
UCB = 66.73
2
$n = 1$
$\Sigma\, r = 59$
UCB = 65.69
3
$n = 1$
$\Sigma\, r = 56$
UCB = 62.69
0
$n = 1$
$\Sigma\, r = 53$
UCB = _

30	33	39	38	32	34	55	53
41	50	49	54	42	45	62	66
32	41	44	49	40	40	55	58
45	45	46	59	49	52	64	65
37	37	43	44	38	42	53	54
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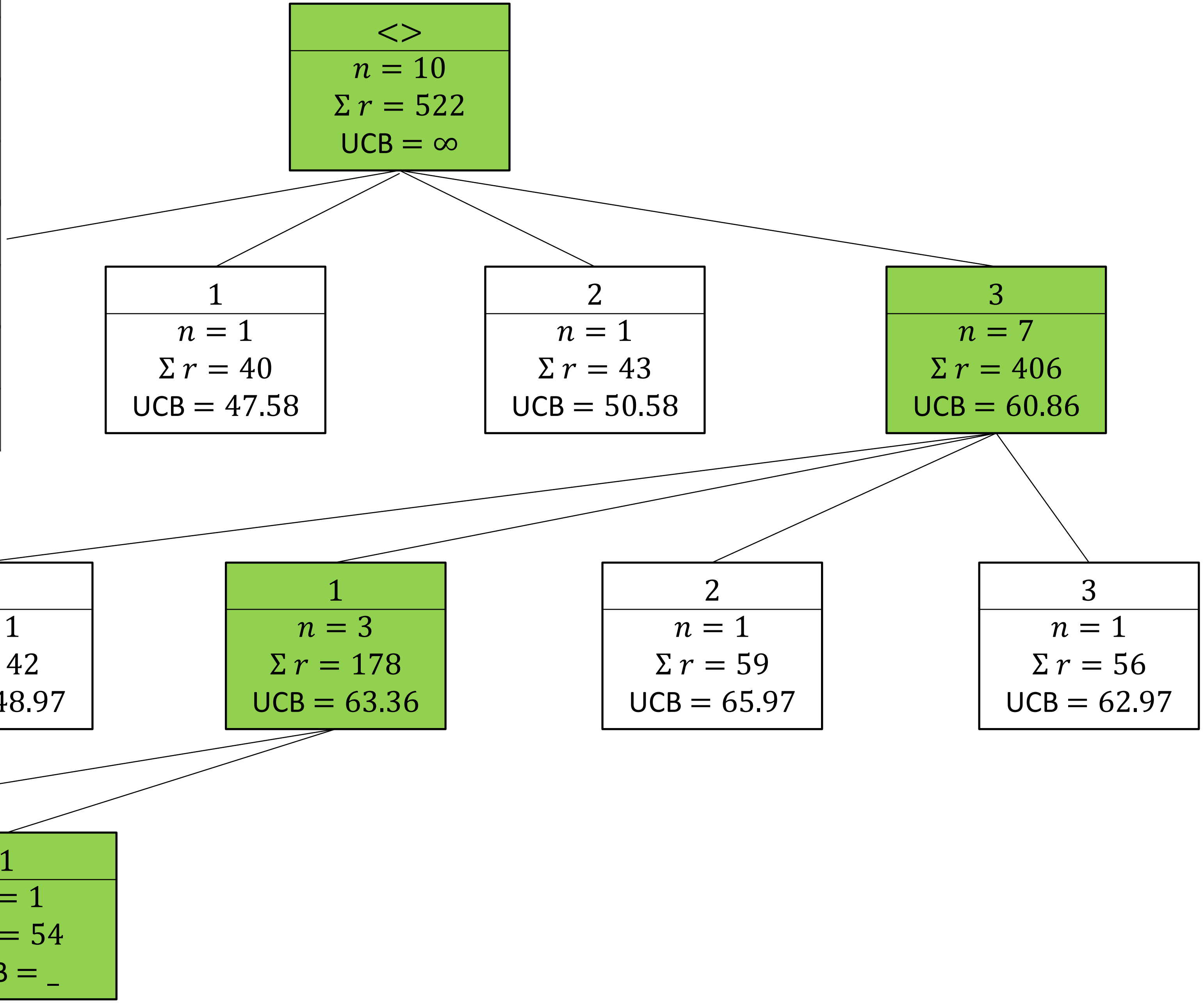
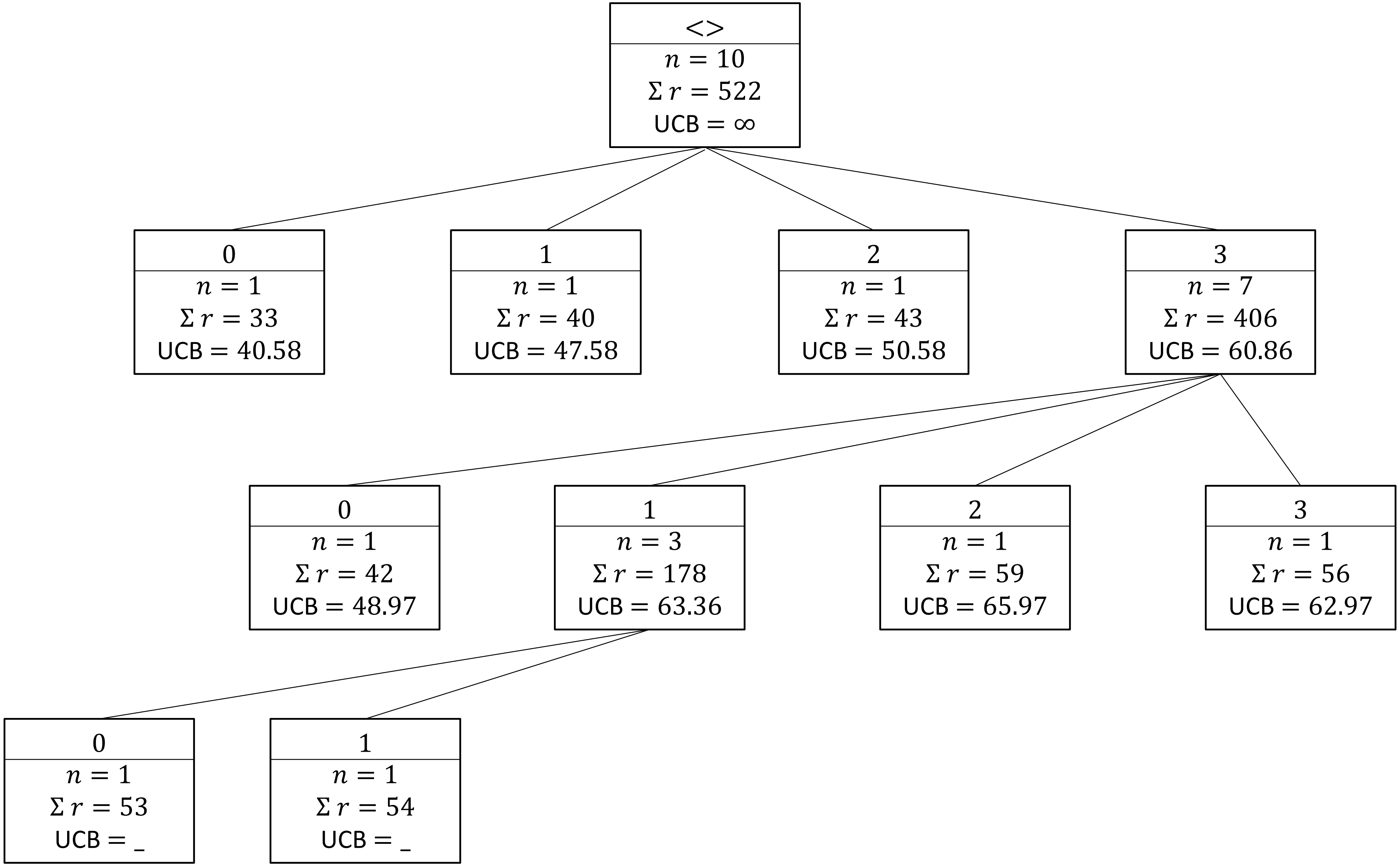
50	48	58	59	59	66	70	77

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1
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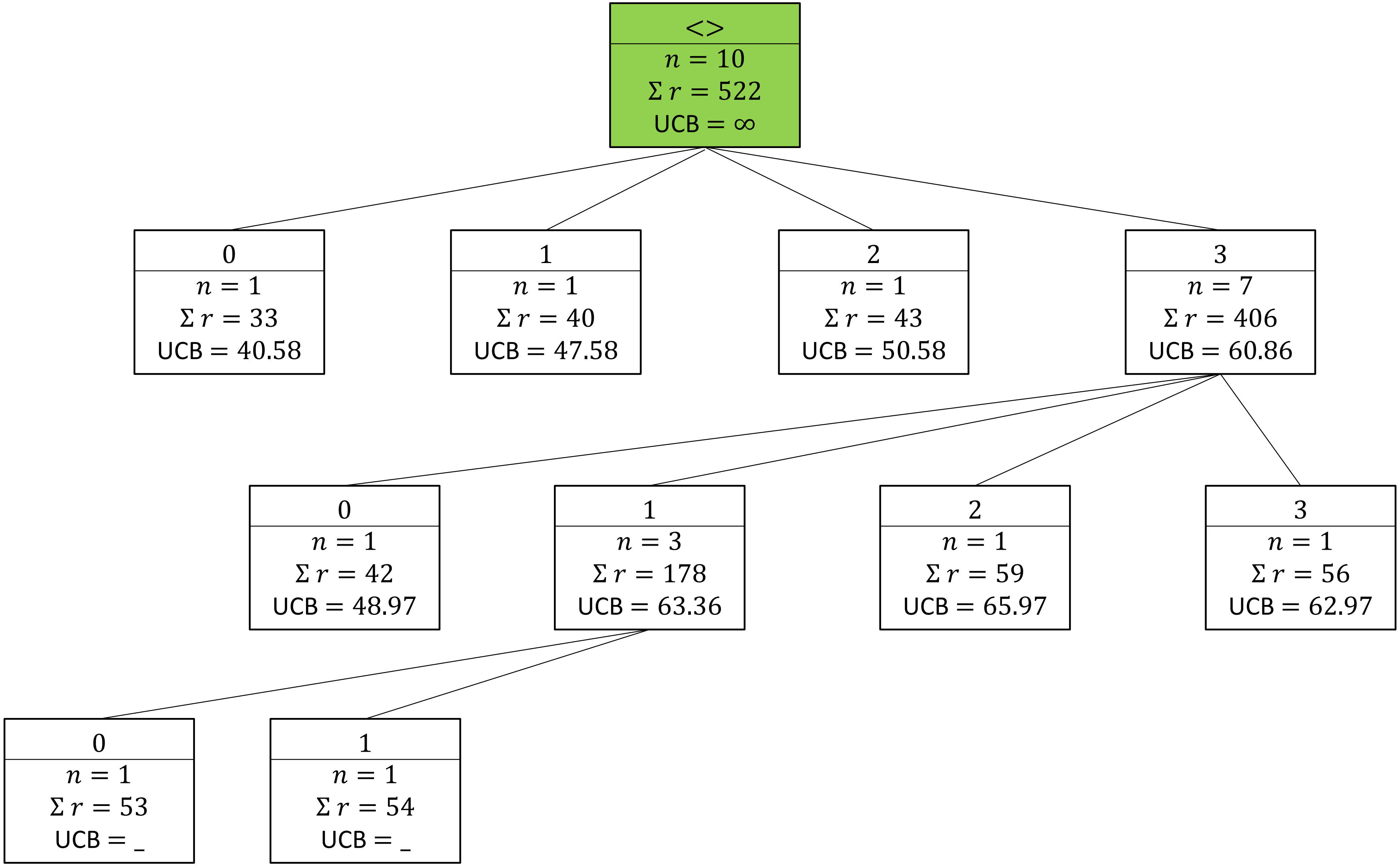
$\Sigma\, r = 40$
UCB = 47.58
2
$n = 1$
$\Sigma\, r = 43$
UCB = 50.58
3
$n = 7$
$\Sigma\, r = 406$
UCB = 60.86
0
$n = 1$
$\Sigma\, r = 42$
UCB = 48.97
1
$n = 3$
$\Sigma\, r = 178$
UCB = 63.36
2
$n = 1$
$\Sigma\, r = 59$
UCB = 65.97
3
$n = 1$
$\Sigma\, r = 56$
UCB = 62.97
0
$n = 1$
$\Sigma\, r = 53$
UCB = _
1
$n = 1$
$\Sigma\, r = 54$
UCB = _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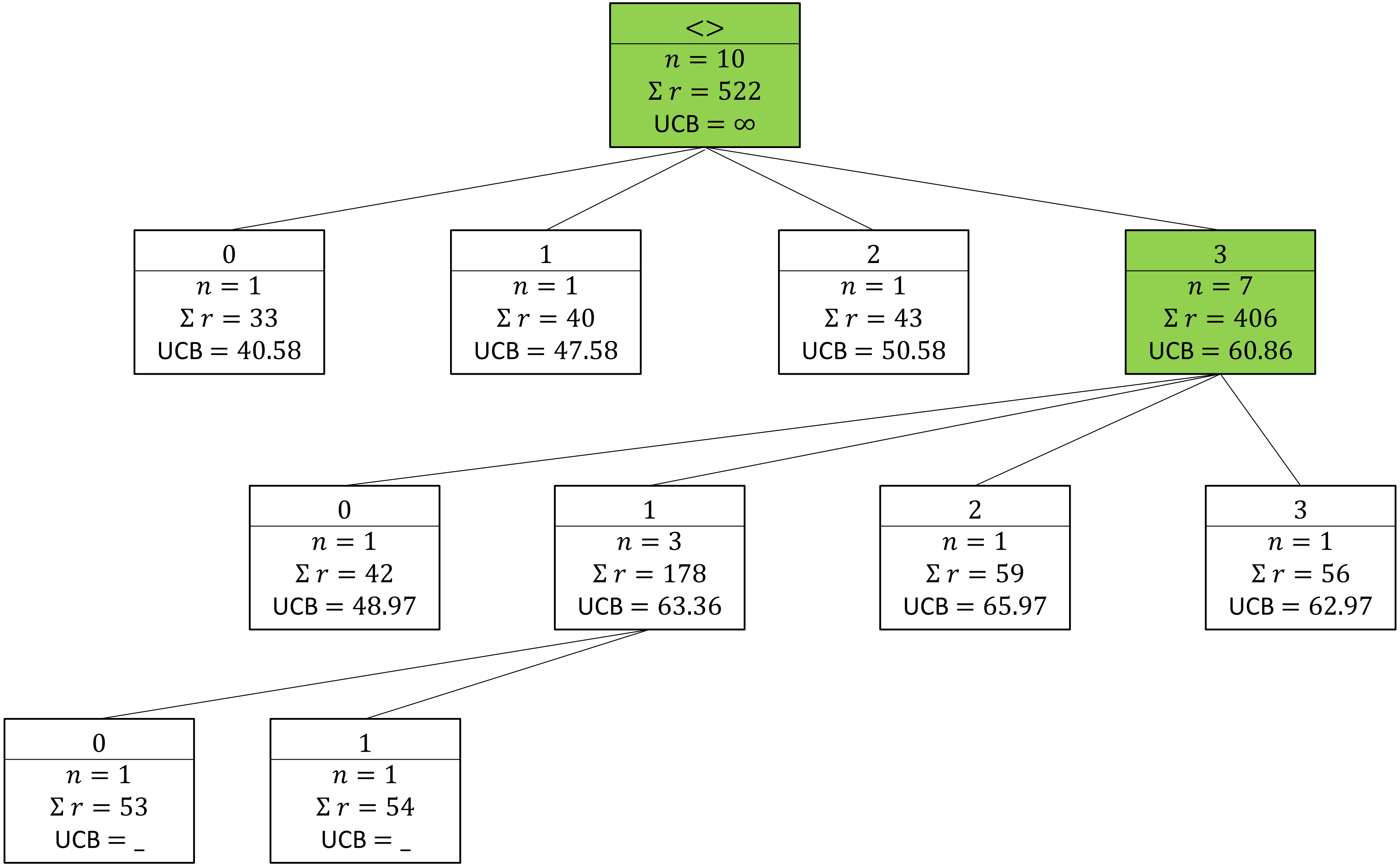
n = 10
Σ r = 522
UCB = ∞
0
n = 1
Σ r = 33
UCB = 40.58
1
n = 1
Σ r = 40
UCB = 47.58
2
n = 1
Σ r = 43
UCB = 50.58
3
n = 7
Σ r = 406
UCB = 60.86
0
n = 1
Σ r = 42
UCB = 48.97
1
n = 3
Σ r = 178
UCB = 63.36
2
n = 1
Σ r = 59
UCB = 65.97
3
n = 1
Σ r = 56
UCB = 62.97
0
n = 1
Σ r = 53
UCB = _
1
n = 1
Σ r = 54
UCB = _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$n = 10$
$\Sigma\, r = 522$
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0
$n = 1$
$\Sigma\, r = 33$
UCB = 40.58
1
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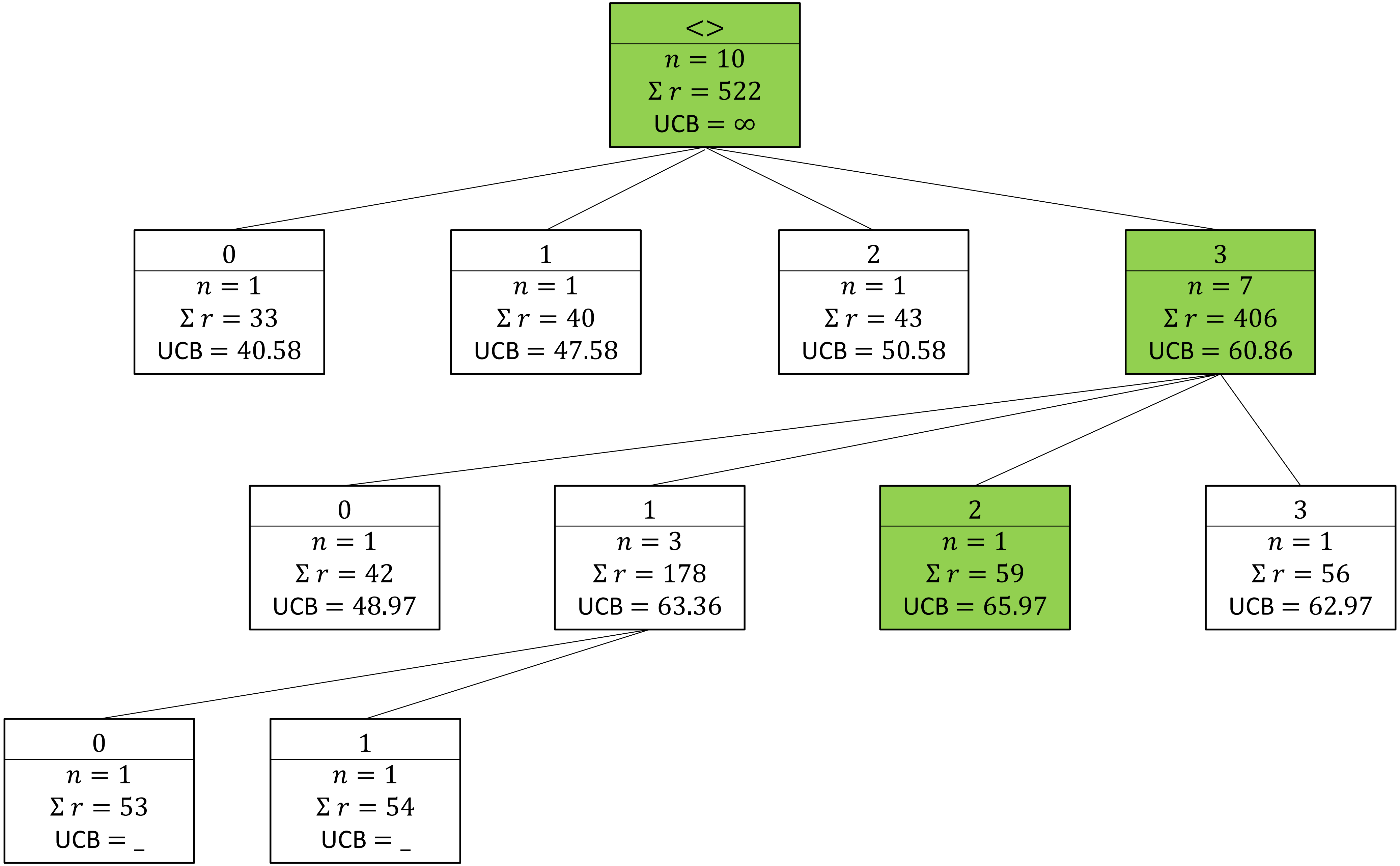
$\Sigma\, r = 40$
UCB = 47.58
2
$n = 1$
$\Sigma\, r = 43$
UCB = 50.58
3
$n = 7$
$\Sigma\, r = 406$
UCB = 60.86
0
$n = 1$
$\Sigma\, r = 42$
UCB = 48.97
1
$n = 3$
$\Sigma\, r = 178$
UCB = 63.36
2
$n = 1$
$\Sigma\, r = 59$
UCB = 65.97
3
$n = 1$
$\Sigma\, r = 56$
UCB = 62.97
0
$n = 1$
$\Sigma\, r = 53$
UCB = _
1
$n = 1$
$\Sigma\, r = 54$
UCB = _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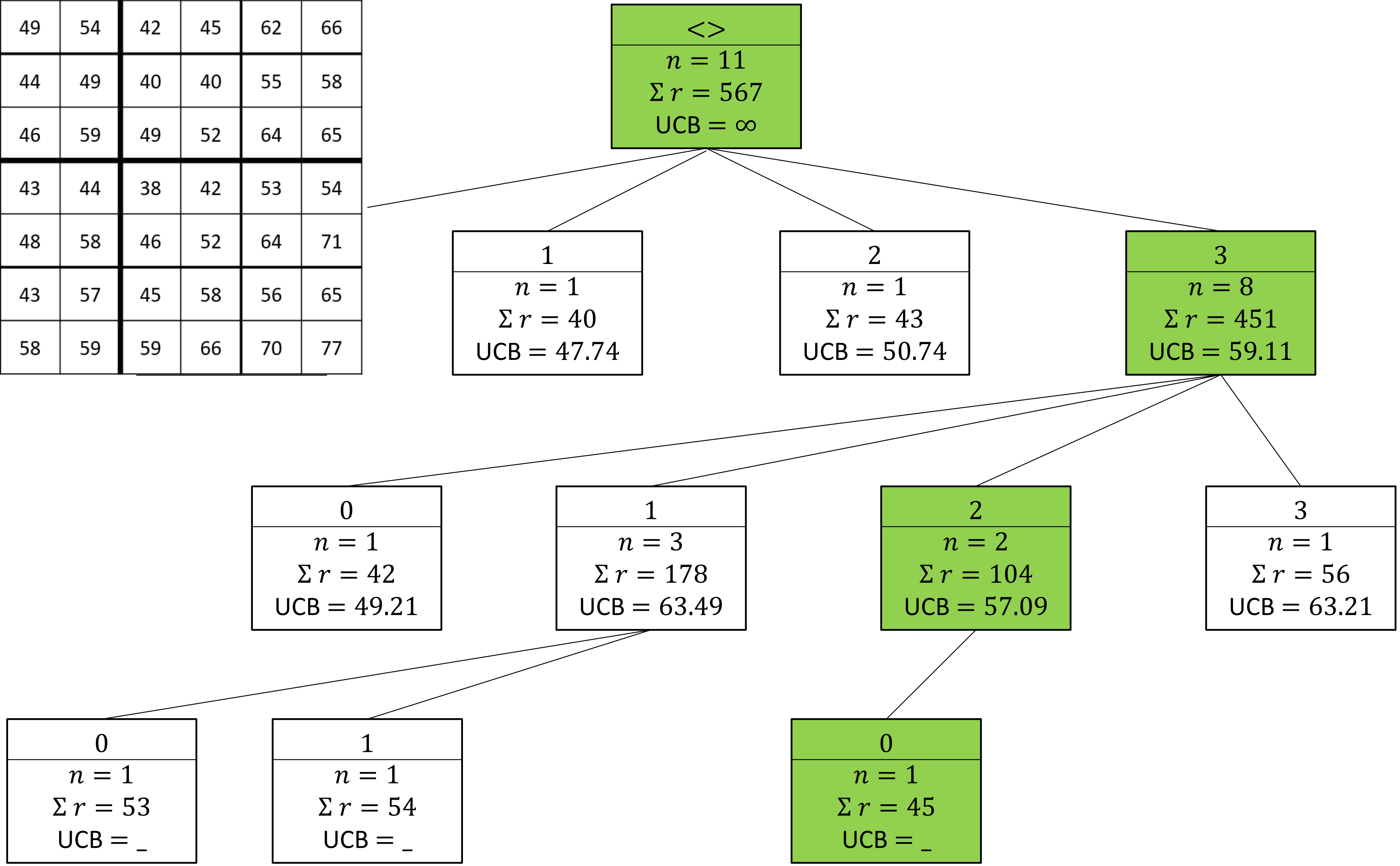
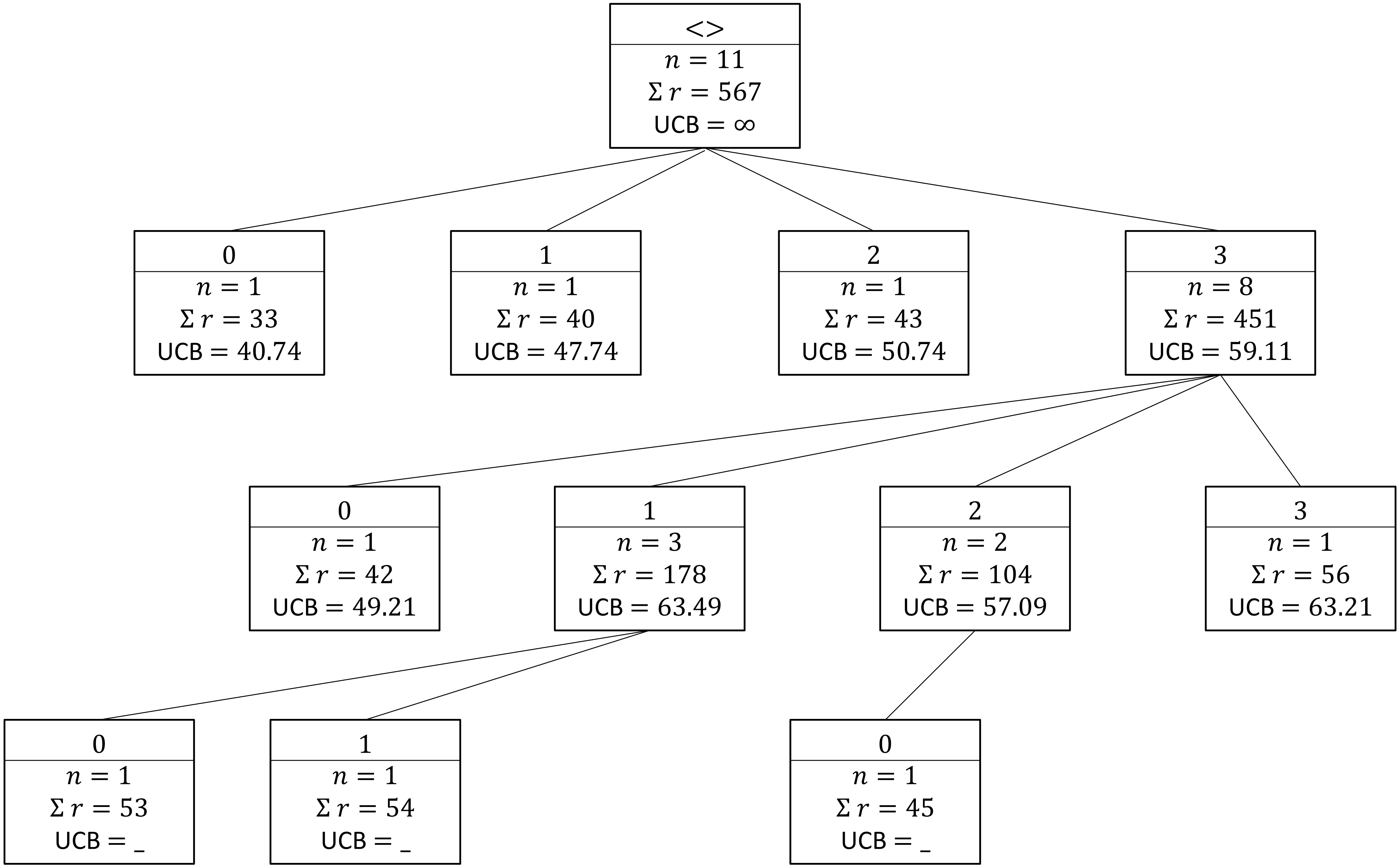
$\Sigma\, r = 522$
UCB = ∞
0
$n = 1$
$\Sigma\, r = 33$
UCB = 40.58
1
$n = 1$
$\Sigma\, r = 40$
UCB = 47.58
2
$n = 1$
$\Sigma\, r = 43$
UCB = 50.58
3
$n = 7$
$\Sigma\, r = 406$
UCB = 60.86
0
$n = 1$
$\Sigma\, r = 42$
UCB = 48.97
1
$n = 3$
$\Sigma\, r = 178$
UCB = 63.36
2
$n = 1$
$\Sigma\, r = 59$
UCB = 65.97
3
$n = 1$
$\Sigma\, r = 56$
UCB = 62.97
0
$n = 1$
$\Sigma\, r = 53$
UCB = _
1
$n = 1$
$\Sigma\, r = 54$
UCB = _

30	33	39	38	32	34	55	53
41	50	49	54	42	45	62	66
32	41	44	49	40	40	55	58
45	45	46	59	49	52	64	65
37	37	43	44	38	42	53	54
48	51	48	58	46	52	64	71
40	45	43	57	45	58	56	65
50	48	58	59	59	66	70	77




<>
$n = 11$
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0
$n = 1$
$\Sigma\, r = 33$
UCB = 40.74
1
$n = 1$
$\Sigma\, r = 40$
UCB = 47.74
2
$n = 1$
$\Sigma\, r = 43$
UCB = 50.74
3
$n = 8$
$\Sigma\, r = 451$
UCB = 59.11
0
$n = 1$
$\Sigma\, r = 42$
UCB = 49.21
1
$n = 3$
$\Sigma\, r = 178$
UCB = 63.49
2
$n = 2$
$\Sigma\, r = 104$
UCB = 57.09
3
$n = 1$
$\Sigma\, r = 56$
UCB = 63.21
0
$n = 1$
$\Sigma\, r = 53$
UCB = _
1
$n = 1$
$\Sigma\, r = 54$
UCB = _
0
$n = 1$
$\Sigma\, r = 45$
UCB = _

Influence of the hyperparameter c

If we had chosen a larger exploration hyperparameter. Would we have found the maximum element sooner or later?

$$\frac{1}{n_i}\sum_{t=1}^{n_i} r_t + c\sqrt{\frac{\ln N}{n_i}}$$

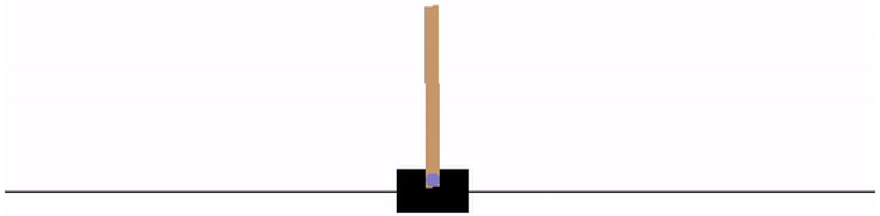
- Trick question!
- Since MCTS is a Monte Carlo algorithm (i.e. a randomized algorithm) we can‘t tell

Exercise 4

MCTS for Planning in MDPs

- OpenAI Gym (Benchmarks for RL research)
- CartPole is a famous domain
- Discrete action space: 2 actions (move the cart left or right)
- We use a fixed seed

  → no randomness in the environment (but still randomness in MCTS)



MCTS for Planning in MDPs

Solve the OpenAI Gym domain CartPole (for one fixed seed) with MCTS.

See PyCharm

Exercise 5

Exercise 5

The state space now changes compared to exercise 3:

***state** = (row, column)*

***next_states** = one move vertically or horizontally*

	0	1	2	3	4	5	6	7
0	30	33	39	38	32	34	55	53
1	41	50	49	54	42	45	62	66
2	32	41	44	49	40	40	55	58
3	45	45	46	59	49	52	64	65
4	37	37	43	44	38	42	53	54
5	48	51	48	58	46	52	64	71
6	40	45	43	57	45	58	56	65
7	50	48	58	59	59	66	70	77

Exercise 5

a) Starting from cell (2,5), always greedily choose the best neighbor.

State	Value
(2,5)	40

	0	1	2	3	4	5	6	7
0	30	33	39	38	32	34	55	53
1	41	50	49	54	42	45	62	66
2	32	41	44	49	40	40	55	58
3	45	45	46	59	49	52	64	65
4	37	37	43	44	38	42	53	54
5	48	51	48	58	46	52	64	71
6	40	45	43	57	45	58	56	65
7	50	48	58	59	59	66	70	77

Exercise 5

a) Starting from cell (2,5), always greedily choose the best neighbor.

State	Value
(2,5)	40
(2,6)	55

	0	1	2	3	4	5	6	7
0	30	33	39	38	32	34	55	53
1	41	50	49	54	42	45	62	66
2	32	41	44	49	40	40	55	58
3	45	45	46	59	49	52	64	65
4	37	37	43	44	38	42	53	54
5	48	51	48	58	46	52	64	71
6	40	45	43	57	45	58	56	65
7	50	48	58	59	59	66	70	77

Exercise 5

a) Starting from cell (2,5), always greedily choose the best neighbor.

State	Value
(2,5)	40
(2,6)	55
(3,6)	64

	0	1	2	3	4	5	6	7
0	30	33	39	38	32	34	55	53
1	41	50	49	54	42	45	62	66
2	32	41	44	49	40	40	55	58
3	45	45	46	59	49	52	64	65
4	37	37	43	44	38	42	53	54
5	48	51	48	58	46	52	64	71
6	40	45	43	57	45	58	56	65
7	50	48	58	59	59	66	70	77

Exercise 5

a) Starting from cell (2,5), always greedily choose the best neighbor.

State	Value
(2,5)	40
(2,6)	55
(3,6)	64
(3,7)	65
(3,6)	64
(3,7)	65
(3,6)	64

	0	1	2	3	4	5	6	7
0	30	33	39	38	32	34	55	53
1	41	50	49	54	42	45	62	66
2	32	41	44	49	40	40	55	58
3	45	45	46	59	49	52	64	65
4	37	37	43	44	38	42	53	54
5	48	51	48	58	46	52	64	71
6	40	45	43	57	45	58	56	65
7	50	48	58	59	59	66	70	77

Exercise 5

b) What happened and how does Tabu Search help prevent this?

Local optimum cycle.

Tabu Search breaks such cycles.
Here: $listSize = 1$ is enough

State	Value
(2,5)	40
(2,6)	55
(3,6)	64
(3,7)	65
(3,6)	64
(3,7)	65
(3,6)	64

Exercise 5

c) At how many places of the Simulated Annealing algorithm is randomness used?

Answer: 2

- Random selection of the next candidate state
- Random choice whether the the candidate state is accepted if $\Delta E < 0$

Simulated Annealing - Example

if $\Delta E = E_{new} - E_{old} > 0$:
    accept new state
else:
    accept new state with prob $\sim e^{\Delta E/T}$

Iteration	State	Value	T
1	(3,3)	59	10
2			
3			
4			
5			
6			
7			

E_{old}	E_{new}	$e^{\Delta E/T}$

	0	1	2	3	4	5	6	7
0	30	33	39	38	32	34	55	53
1	41	50	49	54	42	45	62	66
2	32	41	44	49	40	40	55	58
3	45	45	46	59	49	52	64	65
4	37	37	43	44	38	42	53	54
5	48	51	48	58	46	52	64	71
6	40	45	43	57	45	58	56	65
7	50	48	58	59	59	66	70	77

```
if ΔE = E_new − E_old > 0:
      accept new state
else:
      accept new state with prob ~e^{ΔE/T}
```

Iteration	State	Value	T
1	(3,3)	59	10
2			
3			
4			
5			
6			
7			

E_{old}	E_{new}	$e^{\Delta E/T}$

	0	1	2	3	4	5	6	7
0	30	33	39	38	32	34	55	53
1	41	50	49	54	42	45	62	66
2	32	41	44	49	40	40	55	58
3	45	45	46	59	49	52	64	65
4	37	37	43	44	38	42	53	54
5	48	51	48	58	46	52	64	71
6	40	45	43	57	45	58	56	65
7	50	48	58	59	59	66	70	77

```
if ΔE = E_new − E_old > 0:
     accept new state
else:
     accept new state with prob ~e^(ΔE/T)
```

Iteration	State	Value	T
1	(3,3)	59	10
2			
3			
4			
5			
6			
7			

E_{old}	E_{new}	$e^{\Delta E/T}$
59	44	$e^{-15/10} = 0.22$

	0	1	2	3	4	5	6	7
0	30	33	39	38	32	34	55	53
1	41	50	49	54	42	45	62	66
2	32	41	44	49	40	40	55	58
3	45	45	46	59	49	52	64	65
4	37	37	43	44	38	42	53	54
5	48	51	48	58	46	52	64	71
6	40	45	43	57	45	58	56	65
7	50	48	58	59	59	66	70	77

```
if ΔE = E_new − E_old > 0:
     accept new state
else:
     accept new state with prob ~e^{ΔE/T}
```

Iteration	State	Value	T
1	(3,3)	59	10
2	(3,3)	59	9
3			
4			
5			
6			
7			

E_{old}	E_{new}	$e^{\Delta E/T}$

	0	1	2	3	4	5	6	7
0	30	33	39	38	32	34	55	53
1	41	50	49	54	42	45	62	66
2	32	41	44	49	40	40	55	58
3	45	45	46	59	49	52	64	65
4	37	37	43	44	38	42	53	54
5	48	51	48	58	46	52	64	71
6	40	45	43	57	45	58	56	65
7	50	48	58	59	59	66	70	77

```
if ΔE = E_new − E_old > 0:
    accept new state
else:
    accept new state with prob ~e^{ΔE/T}
```

Iteration	State	Value	T
1	(3,3)	59	10
2	(3,3)	59	9
3			
4			
5			
6			
7			

E_{old}	E_{new}	$e^{\Delta E/T}$
59	49	$e^{-10/9} = 0.32$

	0	1	2	3	4	5	6	7
0	30	33	39	38	32	34	55	53
1	41	50	49	54	42	45	62	66
2	32	41	44	49	40	40	55	58
3	45	45	46	59	49	52	64	65
4	37	37	43	44	38	42	53	54
5	48	51	48	58	46	52	64	71
6	40	45	43	57	45	58	56	65
7	50	48	58	59	59	66	70	77

```
if ΔE = E_new − E_old > 0:
    accept new state
else:
    accept new state with prob ~e^{ΔE/T}
```

Iteration	State	Value	T
1	(3,3)	59	10
2	(3,3)	59	9
3	(3,4)	49	8
4			
5			
6			
7			

E_{old}	E_{new}	$e^{\Delta E/T}$

	0	1	2	3	4	5	6	7
0	30	33	39	38	32	34	55	53
1	41	50	49	54	42	45	62	66
2	32	41	44	49	40	40	55	58
3	45	45	46	59	49	52	64	65
4	37	37	43	44	38	42	53	54
5	48	51	48	58	46	52	64	71
6	40	45	43	57	45	58	56	65
7	50	48	58	59	59	66	70	77

if $\Delta E = E_{new} - E_{old} > 0$:
    accept new state
else:
    accept new state with prob $\sim e^{\Delta E/T}$

Iteration	State	Value	T
1	(3,3)	59	10
2	(3,3)	59	9
3	(3,4)	49	8
4			
5			
6			
7			

E_{old}	E_{new}	$e^{\Delta E/T}$

	0	1	2	3	4	5	6	7
0	30	33	39	38	32	34	55	53
1	41	50	49	54	42	45	62	66
2	32	41	44	49	40	40	55	58
3	45	45	46	59	49	52	64	65
4	37	37	43	44	38	42	53	54
5	48	51	48	58	46	52	64	71
6	40	45	43	57	45	58	56	65
7	50	48	58	59	59	66	70	77

```
if ΔE = E_new − E_old > 0:
    accept new state
else:
    accept new state with prob ~e^(ΔE/T)
```

$$\text{if } \Delta E = E_{new} - E_{old} > 0:$$

Iteration	State	Value	T
1	(3,3)	59	10
2	(3,3)	59	9
3	(3,4)	49	8
4	(3,5)	52	7
5			
6			
7			

E_{old}	E_{new}	$e^{\Delta E/T}$

	0	1	2	3	4	5	6	7
0	30	33	39	38	32	34	55	53
1	41	50	49	54	42	45	62	66
2	32	41	44	49	40	40	55	58
3	45	45	46	59	49	52	64	65
4	37	37	43	44	38	42	53	54
5	48	51	48	58	46	52	64	71
6	40	45	43	57	45	58	56	65
7	50	48	58	59	59	66	70	77

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    accept new state
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```

Iteration	State	Value	T
1	(3,3)	59	10
2	(3,3)	59	9
3	(3,4)	49	8
4	(3,5)	52	7
5			
6			
7			

E_{old}	E_{new}	$e^{\Delta E/T}$
52	40	$e^{-12/7} = 0.18$

	0	1	2	3	4	5	6	7
0	30	33	39	38	32	34	55	53
1	41	50	49	54	42	45	62	66
2	32	41	44	49	40	40	55	58
3	45	45	46	59	49	52	64	65
4	37	37	43	44	38	42	53	54
5	48	51	48	58	46	52	64	71
6	40	45	43	57	45	58	56	65
7	50	48	58	59	59	66	70	77

```
if ΔE = E_new − E_old > 0:
      accept new state
else:
      accept new state with prob ~e^{ΔE/T}
```

Iteration	State	Value	T
1	(3,3)	59	10
2	(3,3)	59	9
3	(3,4)	49	8
4	(3,5)	52	7
5	(3,5)	52	6
6			
7			

E_{old}	E_{new}	$e^{\Delta E/T}$

	0	1	2	3	4	5	6	7
0	30	33	39	38	32	34	55	53
1	41	50	49	54	42	45	62	66
2	32	41	44	49	40	40	55	58
3	45	45	46	59	49	52	64	65
4	37	37	43	44	38	42	53	54
5	48	51	48	58	46	52	64	71
6	40	45	43	57	45	58	56	65
7	50	48	58	59	59	66	70	77

Iteration	State	Value	T
1	(3,3)	59	10
2	(3,3)	59	9
3	(3,4)	49	8
4	(3,5)	52	7
5	(3,5)	52	6
6			
7			

```
if ΔE = E_new − E_old > 0:
     accept new state
else:
     accept new state with prob ~e^(ΔE/T)
```

if $\Delta E = E_{new} - E_{old} > 0$:
accept new state
else:
accept new state with prob $\sim e^{\Delta E/T}$

E_{old}	E_{new}	$e^{\Delta E/T}$

	0	1	2	3	4	5	6	7
0	30	33	39	38	32	34	55	53
1	41	50	49	54	42	45	62	66
2	32	41	44	49	40	40	55	58
3	45	45	46	59	49	52	64	65
4	37	37	43	44	38	42	53	54
5	48	51	48	58	46	52	64	71
6	40	45	43	57	45	58	56	65
7	50	48	58	59	59	66	70	77

if $\Delta E = E_{new} - E_{old} > 0$:
    accept new state
else:
    accept new state with prob $\sim e^{\Delta E/T}$

Iteration	State	Value	T
1	(3,3)	59	10
2	(3,3)	59	9
3	(3,4)	49	8
4	(3,5)	52	7
5	(3,5)	52	6
6	(3,6)	64	5
7			

E_{old}	E_{new}	$e^{\Delta E/T}$

	0	1	2	3	4	5	6	7
0	30	33	39	38	32	34	55	53
1	41	50	49	54	42	45	62	66
2	32	41	44	49	40	40	55	58
3	45	45	46	59	49	52	64	65
4	37	37	43	44	38	42	53	54
5	48	51	48	58	46	52	64	71
6	40	45	43	57	45	58	56	65
7	50	48	58	59	59	66	70	77

```
if ΔE = E_new − E_old > 0:
    accept new state
else:
    accept new state with prob ~e^(ΔE/T)
```

Iteration	State	Value	T
1	(3,3)	59	10
2	(3,3)	59	9
3	(3,4)	49	8
4	(3,5)	52	7
5	(3,5)	52	6
6	(3,6)	64	5
7			

E_{old}	E_{new}	$e^{\Delta E/T}$
64	55	$e^{-9/5} = 0.16$

	0	1	2	3	4	5	6	7
0	30	33	39	38	32	34	55	53
1	41	50	49	54	42	45	62	66
2	32	41	44	49	40	40	55	58
3	45	45	46	59	49	52	64	65
4	37	37	43	44	38	42	53	54
5	48	51	48	58	46	52	64	71
6	40	45	43	57	45	58	56	65
7	50	48	58	59	59	66	70	77

if $\Delta E = E_{new} - E_{old} > 0$:
    accept new state
else:
    accept new state with prob $\sim e^{\Delta E/T}$

Iteration	State	Value	T
1	(3,3)	59	10
2	(3,3)	59	9
3	(3,4)	49	8
4	(3,5)	52	7
5	(3,5)	52	6
6	(3,6)	64	5
7	(2,6)	55	4

E_{old}	E_{new}	$e^{\Delta E/T}$
64	55	$e^{-9/5} = 0.16$

	0	1	2	3	4	5	6	7
0	30	33	39	38	32	34	55	53
1	41	50	49	54	42	45	62	66
2	32	41	44	49	40	40	55	58
3	45	45	46	59	49	52	64	65
4	37	37	43	44	38	42	53	54
5	48	51	48	58	46	52	64	71
6	40	45	43	57	45	58	56	65
7	50	48	58	59	59	66	70	77

Exercise 6

Exercise 6

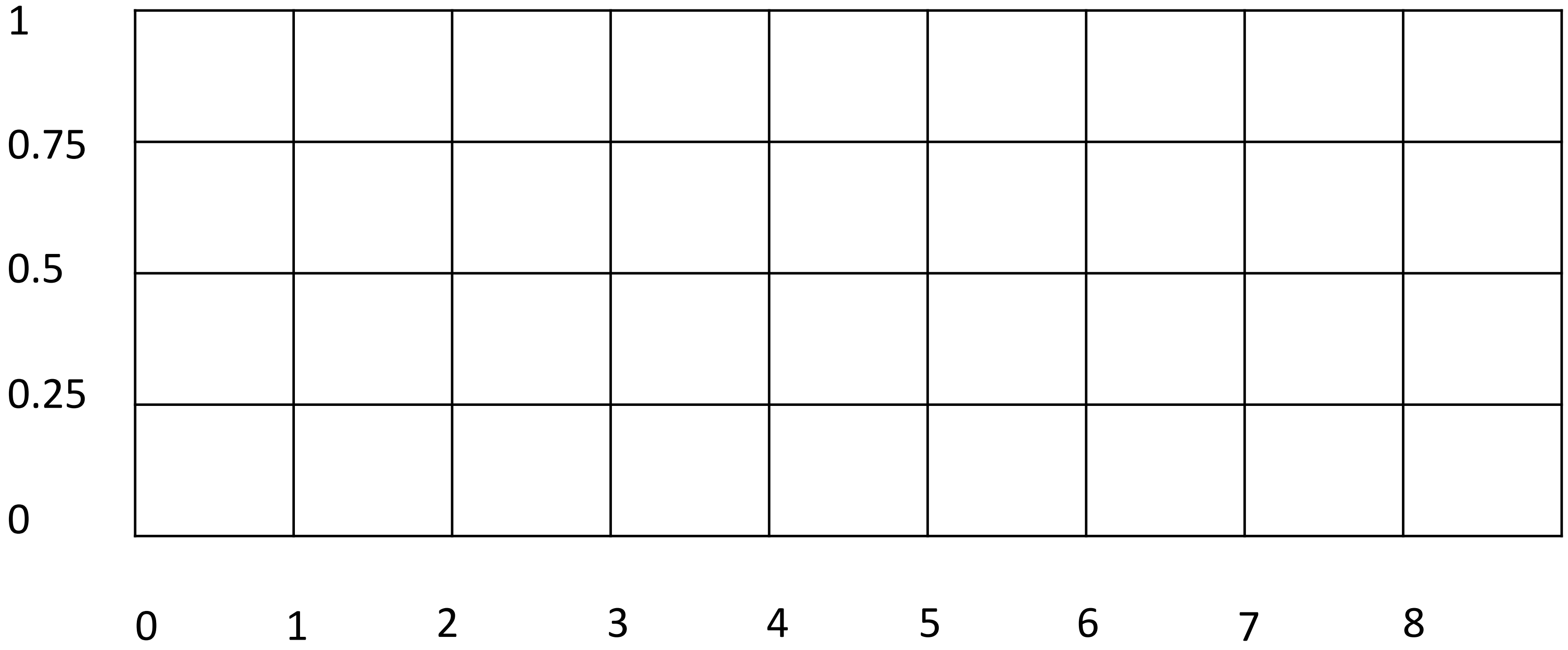
The "fuzzyfication" of an input variable x is to be performed with the help of the fuzzy sets Z (zero), S (small), M (medium) and L (large). The membership functions μ(x) are tabulated as follows:

	0	1	2	3	4	5	6	7	8
Z	1	0	0	0	0	0	0	0	0
S	0	1	1	0.5	0	0	0	0	0
M	0	0	0	0.5	1	0.5	0	0	0
L	0	0	0	0	0	0.5	1	1	1

Draw the graphs of the membership functions and determine the membership degree for the input value $x = 3.5$. Assume that the sum of the memberships to the fuzzy sets for each input value is exactly 1.

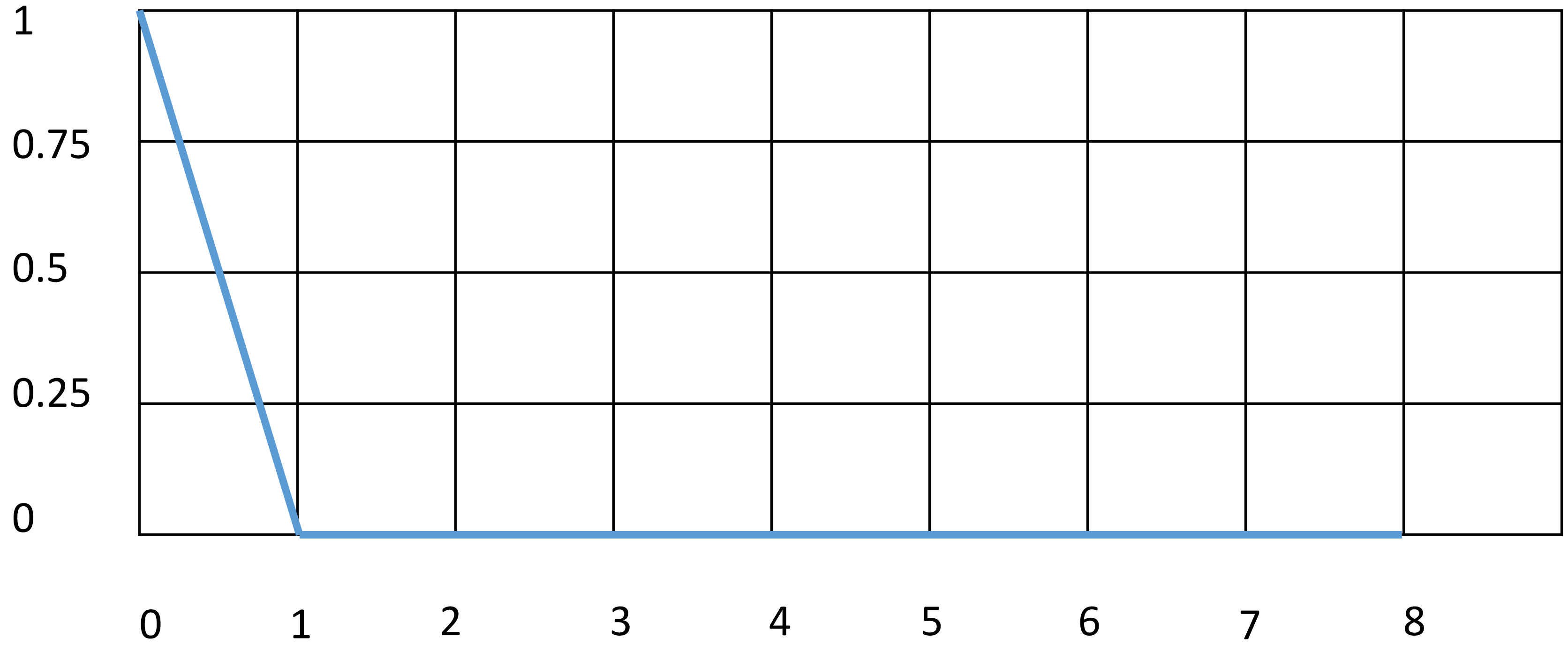
Exercise 6

	0	1	2	3	4	5	6	7	8
Z	1	0	0	0	0	0	0	0	0
S	0	1	1	0.5	0	0	0	0	0
M	0	0	0	0.5	1	0.5	0	0	0
L	0	0	0	0	0	0.5	1	1	1



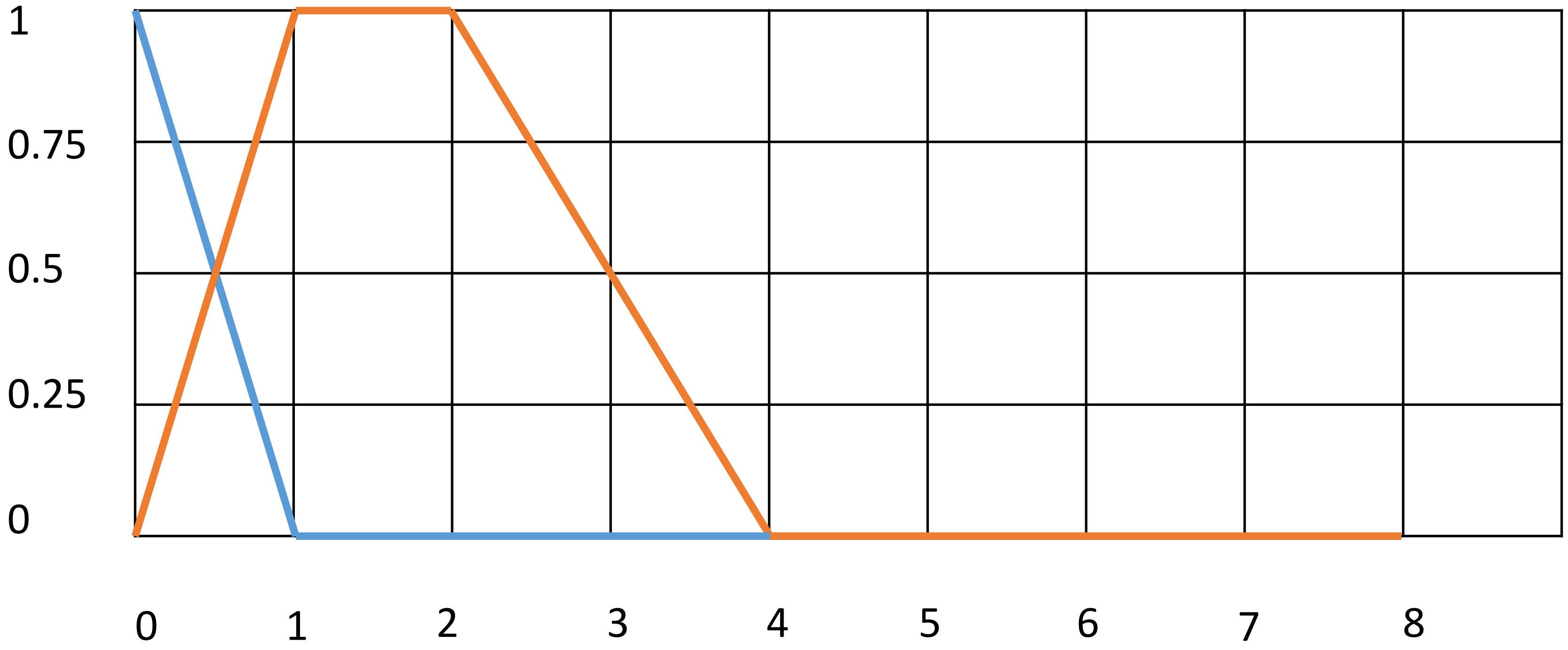
Exercise 6

	0	1	2	3	4	5	6	7	8
Z	1	0	0	0	0	0	0	0	0
S	0	1	1	0.5	0	0	0	0	0
M	0	0	0	0.5	1	0.5	0	0	0
L	0	0	0	0	0	0.5	1	1	1



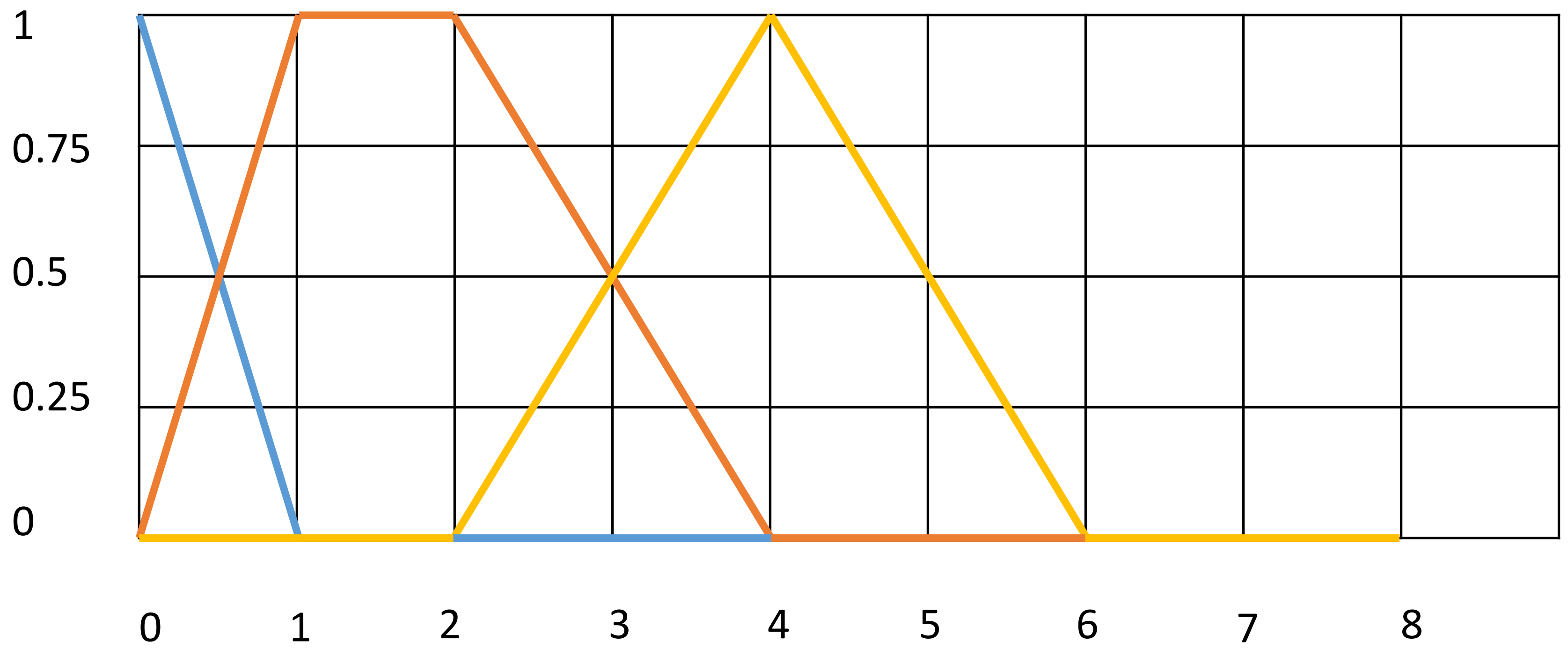
Exercise 6

	0	1	2	3	4	5	6	7	8
Z	1	0	0	0	0	0	0	0	0
S	0	1	1	0.5	0	0	0	0	0
M	0	0	0	0.5	1	0.5	0	0	0
L	0	0	0	0	0	0.5	1	1	1



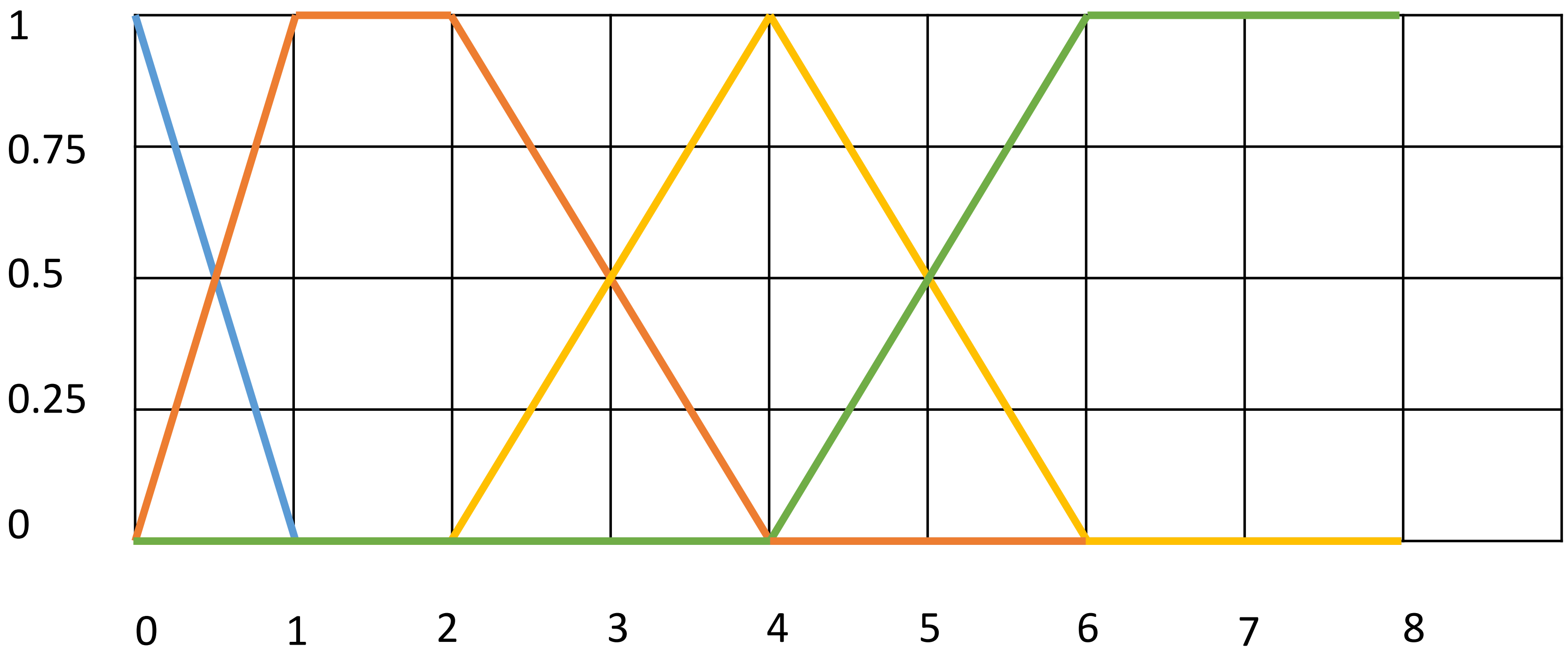
Exercise 6

	0	1	2	3	4	5	6	7	8
Z	1	0	0	0	0	0	0	0	0
S	0	1	1	0.5	0	0	0	0	0
M	0	0	0	0.5	1	0.5	0	0	0
L	0	0	0	0	0	0.5	1	1	1



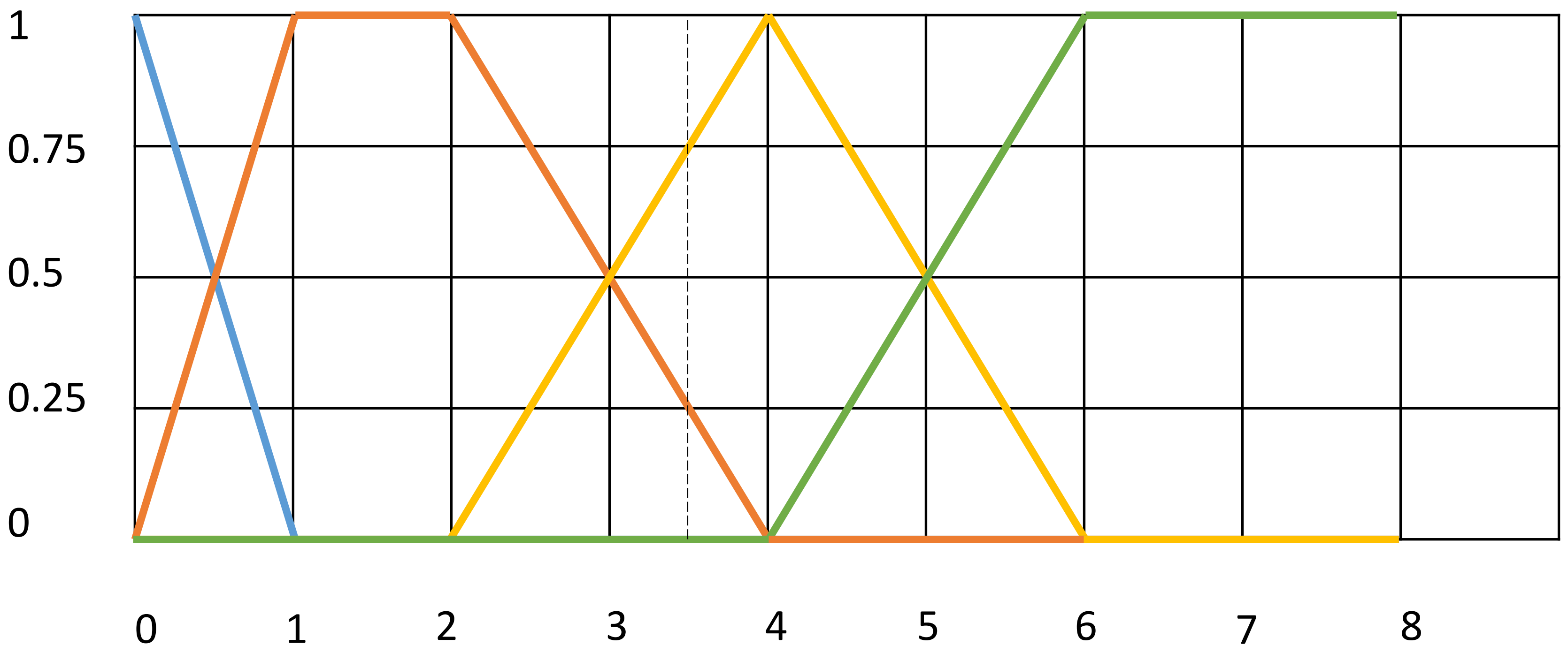
Exercise 6

	0	1	2	3	4	5	6	7	8
Z	1	0	0	0	0	0	0	0	0
S	0	1	1	0.5	0	0	0	0	0
M	0	0	0	0.5	1	0.5	0	0	0
L	0	0	0	0	0	0.5	1	1	1



Exercise 6

	0	1	2	3	4	5	6	7	8
Z	1	0	0	0	0	0	0	0	0
S	0	1	1	0.5	0	0	0	0	0
M	0	0	0	0.5	1	0.5	0	0	0
L	0	0	0	0	0	0.5	1	1	1



Z(3.5)=0

S(3.5)=0.25

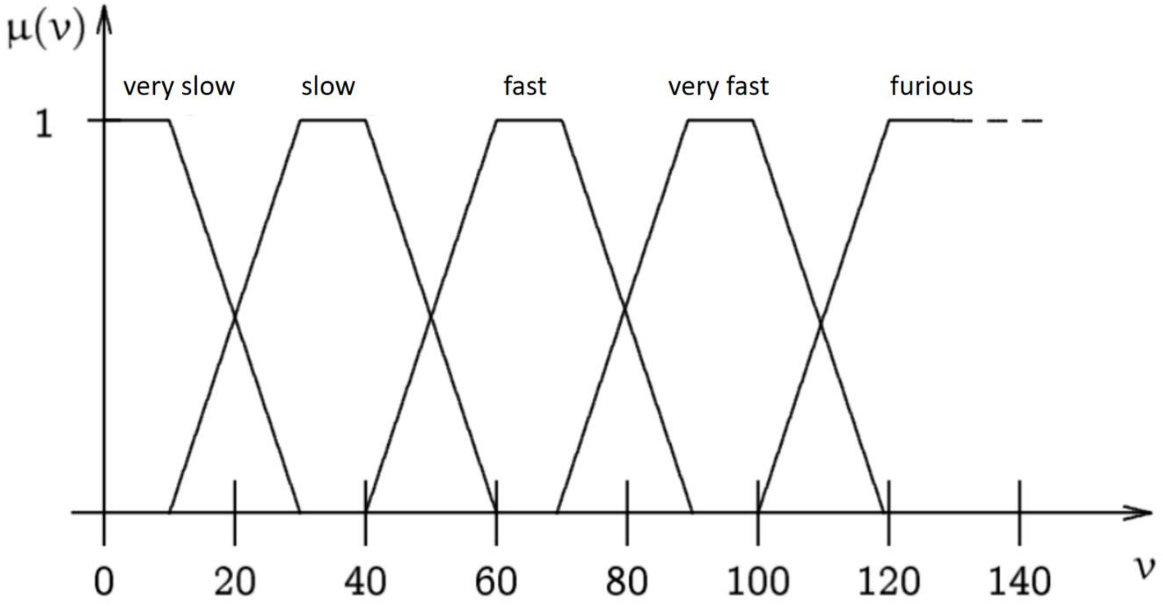
M(3.5)=0.75

L(3.5)=0

Exercise 7

Exercise 7

The speed v of a vehicle in a local area was "fuzzyfied" using the linguistic values very slow, slow, fast, very fast, furious. The following membership functions are used:



Exercise 7

Specify the resulting fuzzy sets when the following operators are executed:

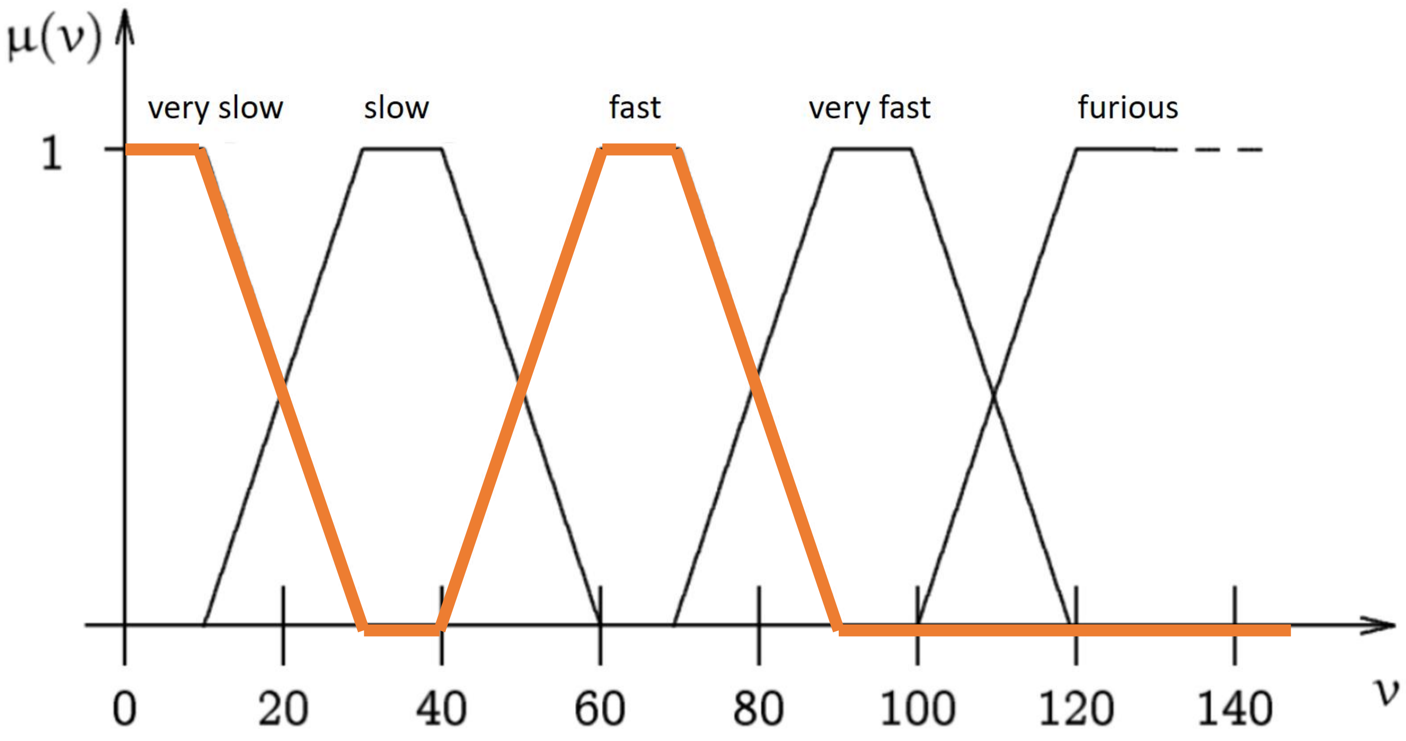
a) *very slow OR fast*

b) *slow AND fast*

c) *NOT fast*

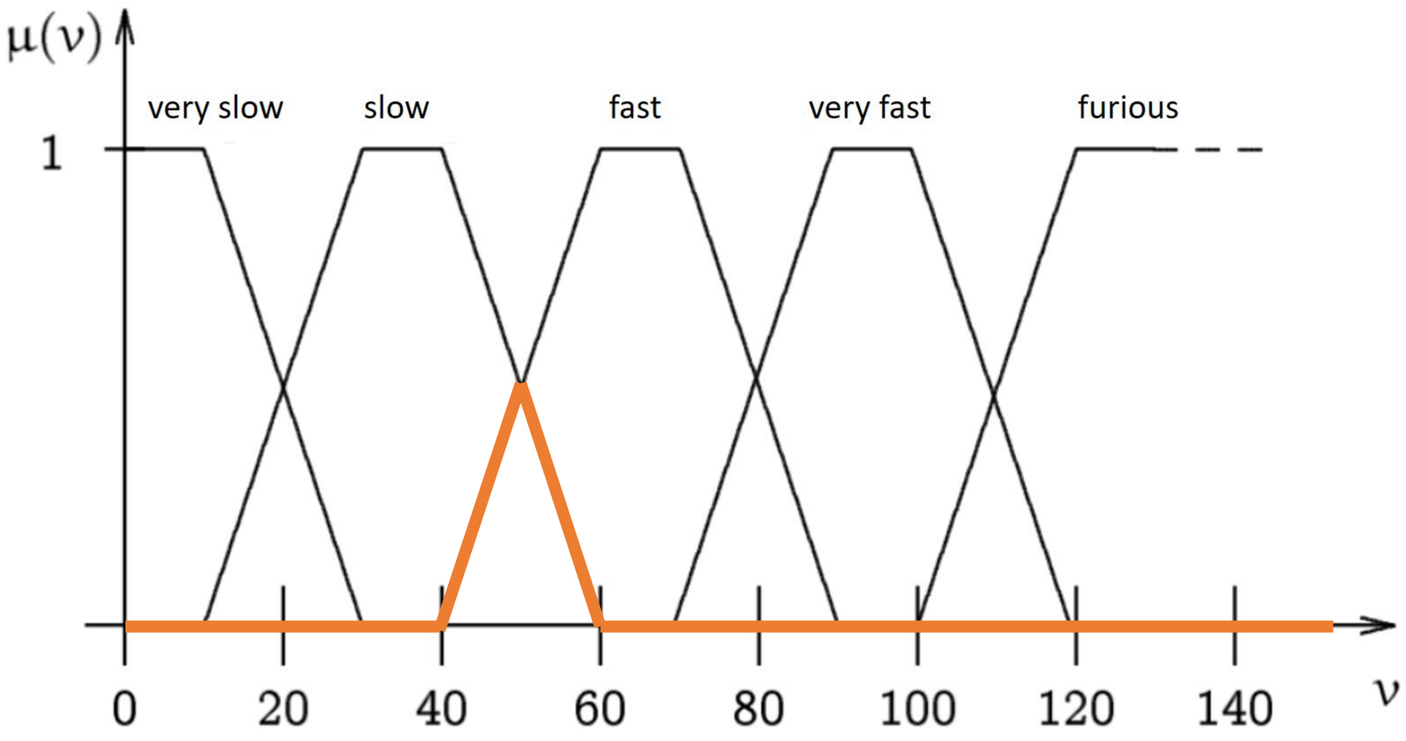
Exercise 7

very slow OR fast:



Exercise 7

slow AND fast:



Exercise 7

NOT fast: