Exercise 1

Exercise 1

Recap: Local Beam Search

- Local Search
- Difference to Hill Climbing: maintain k instead of just one state
- Calculate for each mainainted state the neighboring states → out of this set choose the best k states → repeat
- Problem: low diversity of the k states (all k states are very similar to each other)
- Solution: stochastic beam search
- Beam Seach is often used in Natural Language Processing (NLP) (along with autoregressive models such as Transformer models)

Exercise 1

Give the name of the algorithm resulting from the following special cases:

a) Local Beam Search with $k = 1$

Hill Climbing

b) Local Beam Search with exactly one initial state and no restriction on the number of maintained states

Breadth-first search

Exercise 1

Give the name of the algorithm resulting from the following special cases:

c) Simulated Annealing with always $T = 0$

Simulated Annealing: accept next state if $\Delta E > 0$. Else accept the next state with probability $e^{\Delta E/T}$.

With $T = 0$ → never accept a state if $\Delta E \leq 0$. This is equal to **Hill Climbing**

d) Simulated Annealing with always $T = \infty$

With $T = \infty$ → Next state is always accepted. This is equal to a **Random Walk**

Exercise 2

Recap: Archetypal analysis

X = Observations

Z = Archetypes

$\propto$ = Assignment of observation i to archetype j

β = Assignment of archetype j to observation i

$$\min RSS = \min ||X - \alpha Z^T||_2$$

with:

$$Z = X^T \beta$$

$$\sum_{j=1}^{K} \propto_{ij} = 1 \qquad \forall\, i \in [1, n] \qquad \propto_{ij} \geq 0$$

$$\sum_{i=1}^{n} \beta_{ji} = 1 \qquad \forall\, j \in [1, K] \qquad \beta_{ji} \geq 0$$

Archetypal analysis

a) What is the constraint on minimizing the residual sum of squares with respect to the assignment of an observation to the archetypes?

- For each observation, the sum of the assignments to the archetypes is equal to 1
- Alternative: Observations are **convex combinations** of the archetypes

$$\forall\ i \in [1, n]: \sum_{j=1}^{K} \propto_{ij} = 1$$

Archetypal analysis

b) What is the constraint for minimizing the residual sum of squares in terms of assigning an archetype to observations?

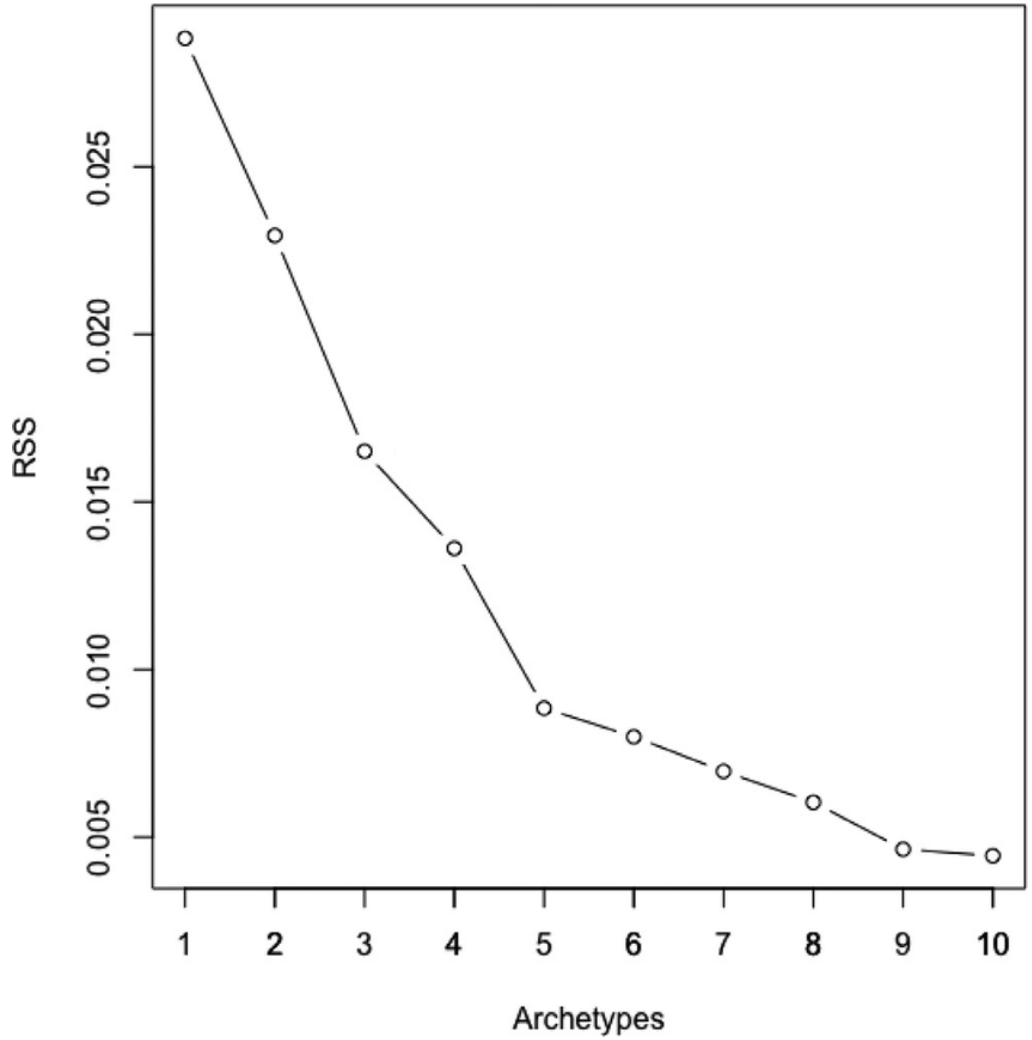
- For each archetype, the sum of the assignments to observations is equal to 1
- Alternative: Archetypes are **convex combinations** of the observations

$$\forall\, j \in [1, K]: \sum_{i=1}^{n} \beta_{ji} = 1$$

Archetypal analysis

c) Explain the structure of the Scree plot and how the elbow criterion works.

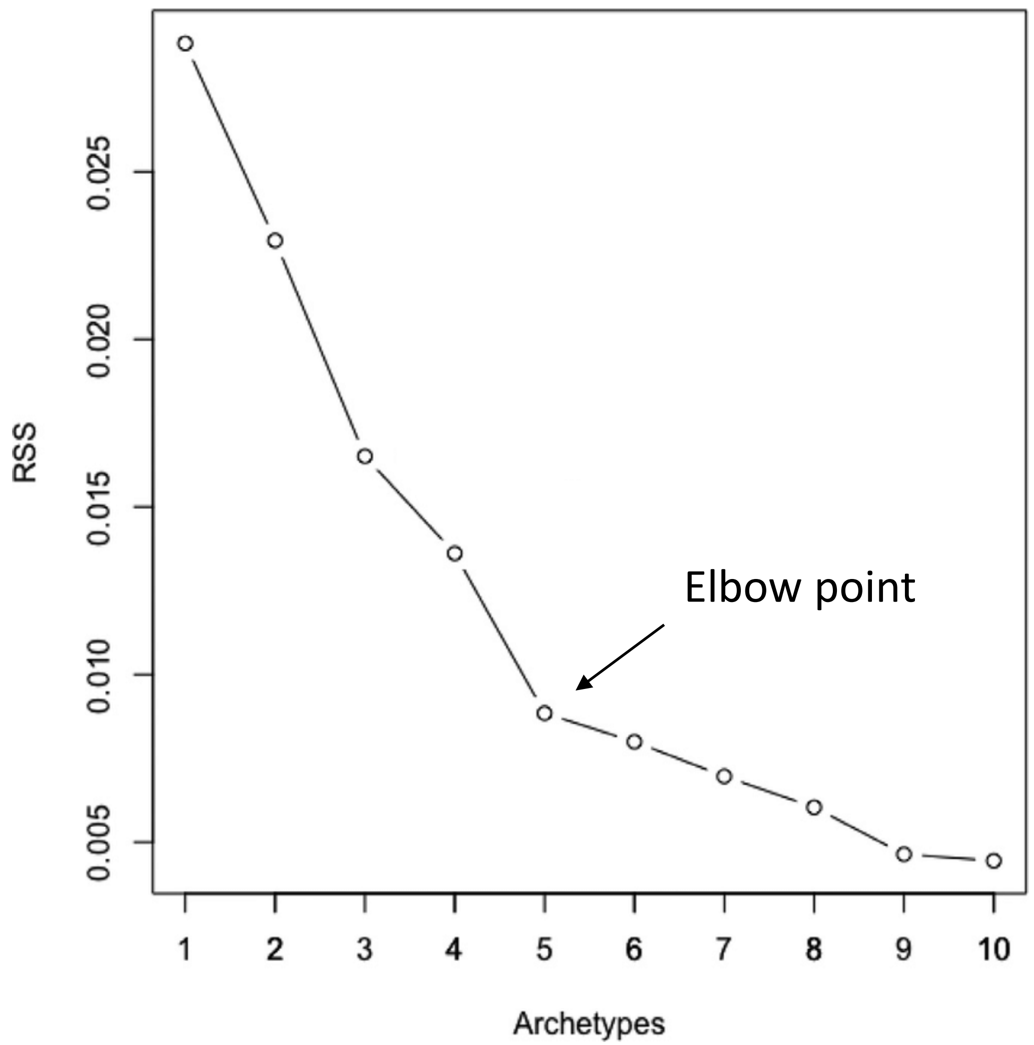
A Scree plot relates values of the RSS to the number of archetypes K



Archetypal analysis

c) Explain the structure of the Scree plot and how the elbow criterion works.

- Elbow criterion: method to determine appropriate value for K
- It is a tradeoff between "few K" and "small error“
- From a certain K the RSS does not improve significantly



Archetypal analysis

d) In Archetypal Analysis, the number K of archetypes must be known before execution. Give three ways how K can be determined.

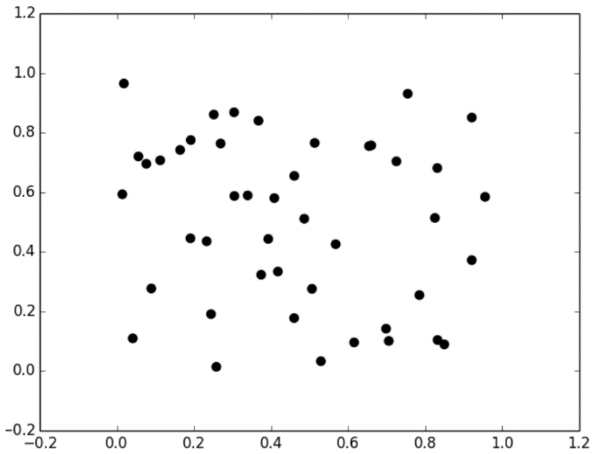
1. Elbow criterion

2. Determined by the user: E.g. the user wants 3 alternative routes

3. K is already known. E.g.: Striker, midfielder, defender

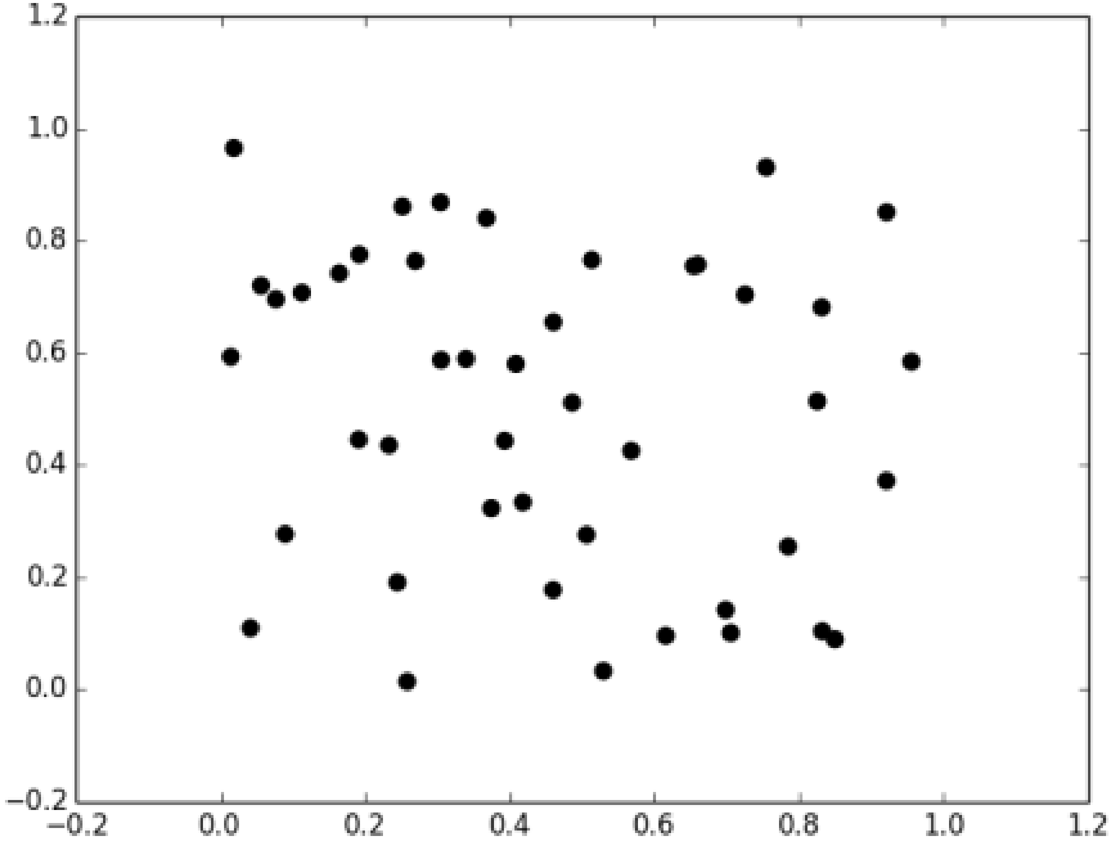
Archetypal analysis

e,f) Given is the following 2D point cloud:



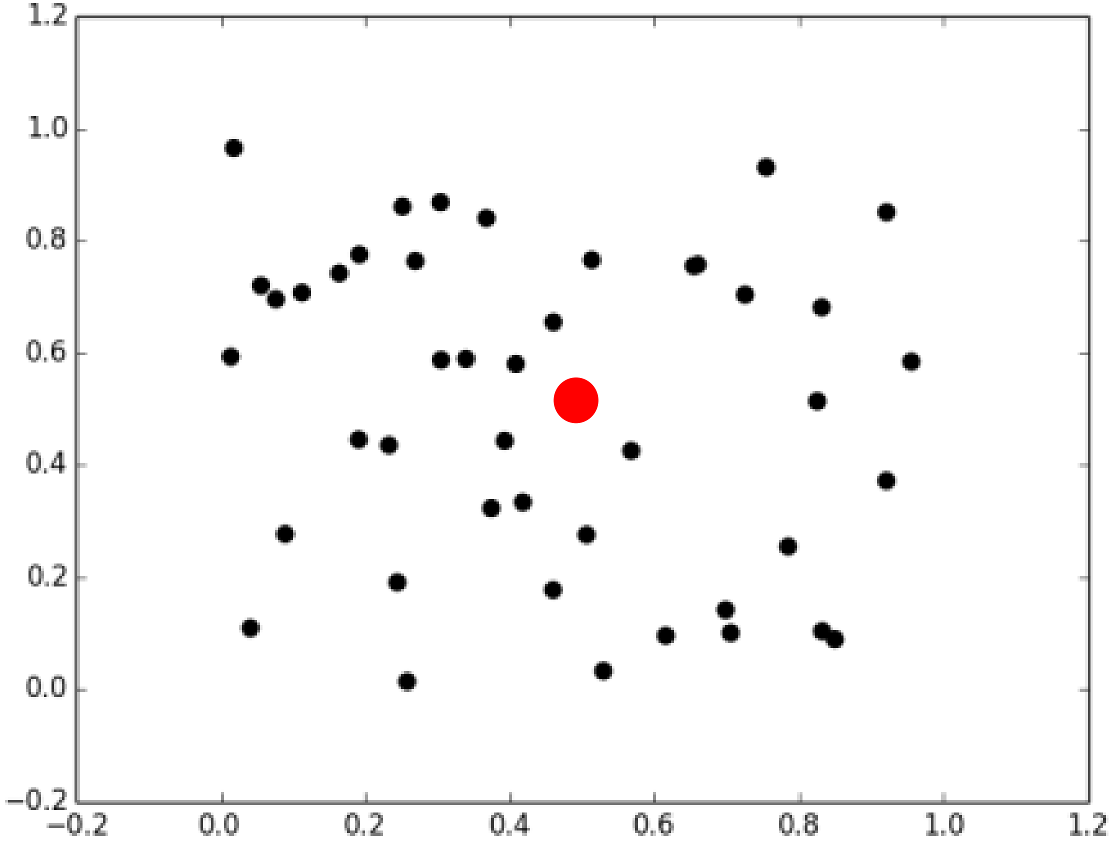
draw K archetypes each with $\mathrm{K} = 1, 3, 4, 5$ where you reasonably expect them to be (an estimate is enough). Then draw the convex hull. Which number of archetypes K seems to be best suited?

Archetypal analysis



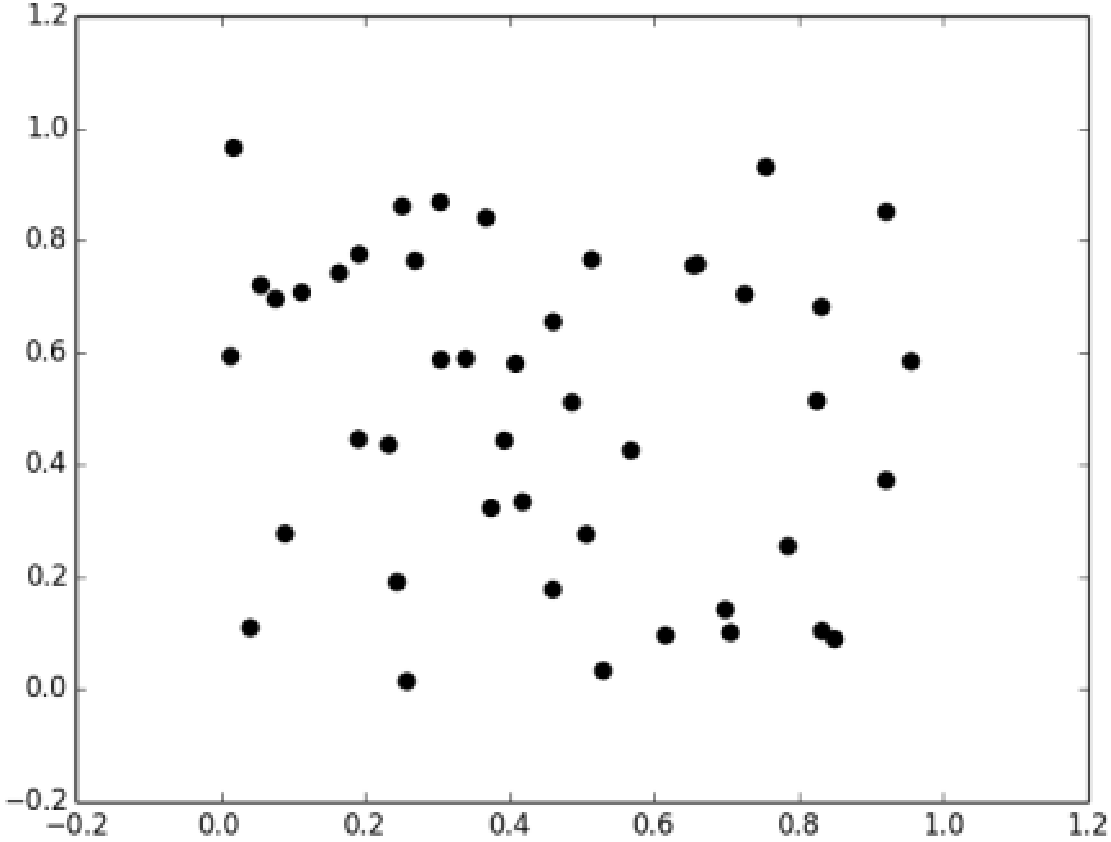
$K = 1$

Archetypal analysis



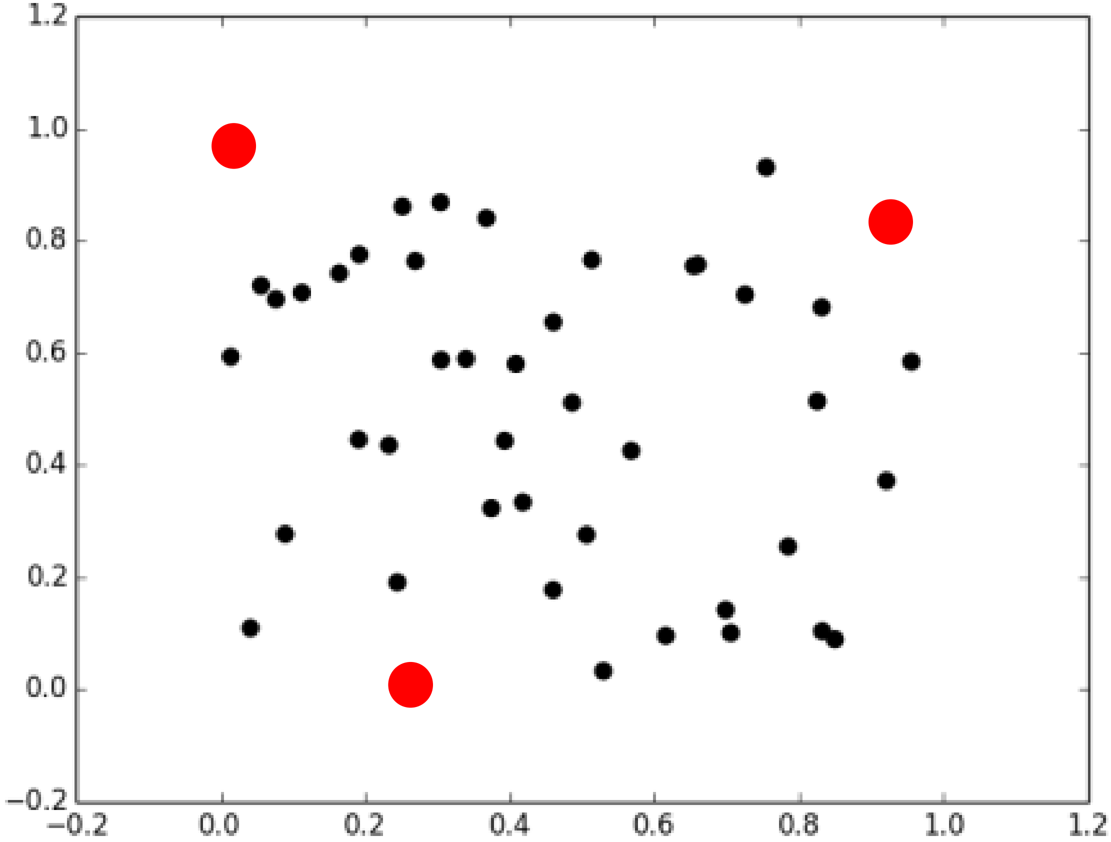
$K = 1$

Archetypal analysis



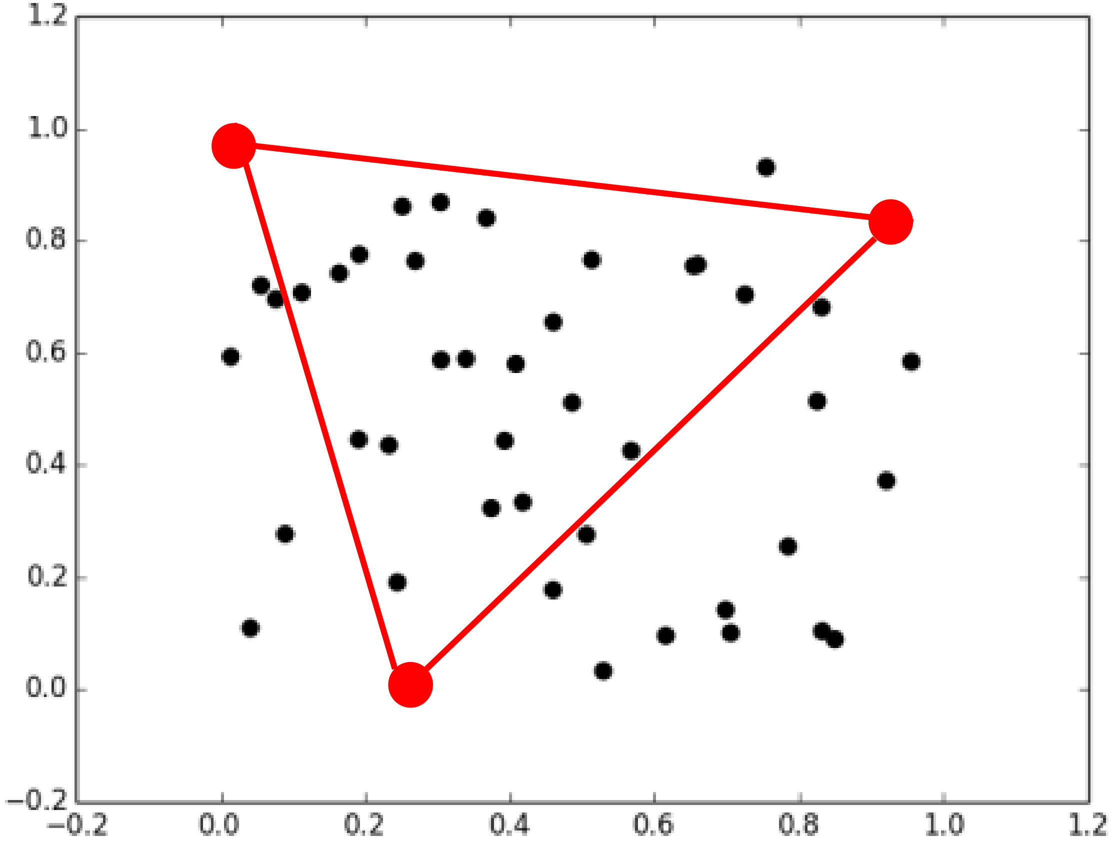
$K = 3$

Archetypal analysis



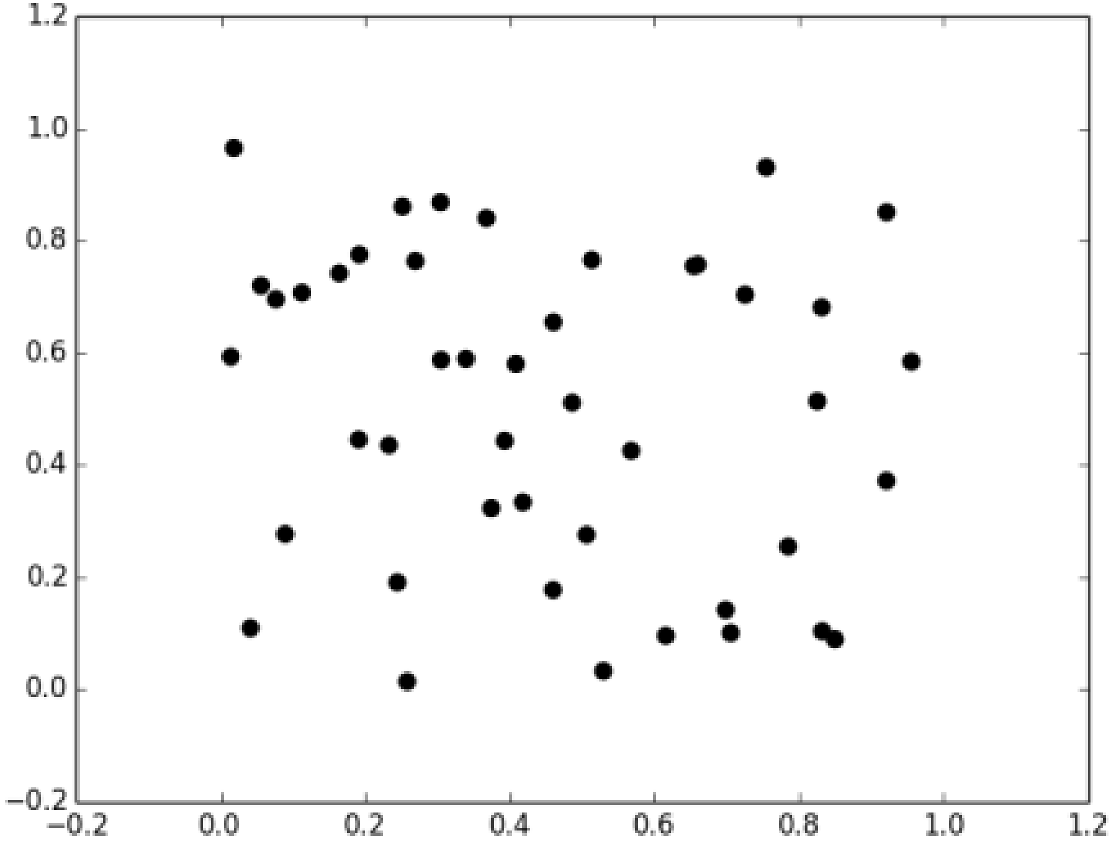
$K = 3$

Archetypal analysis



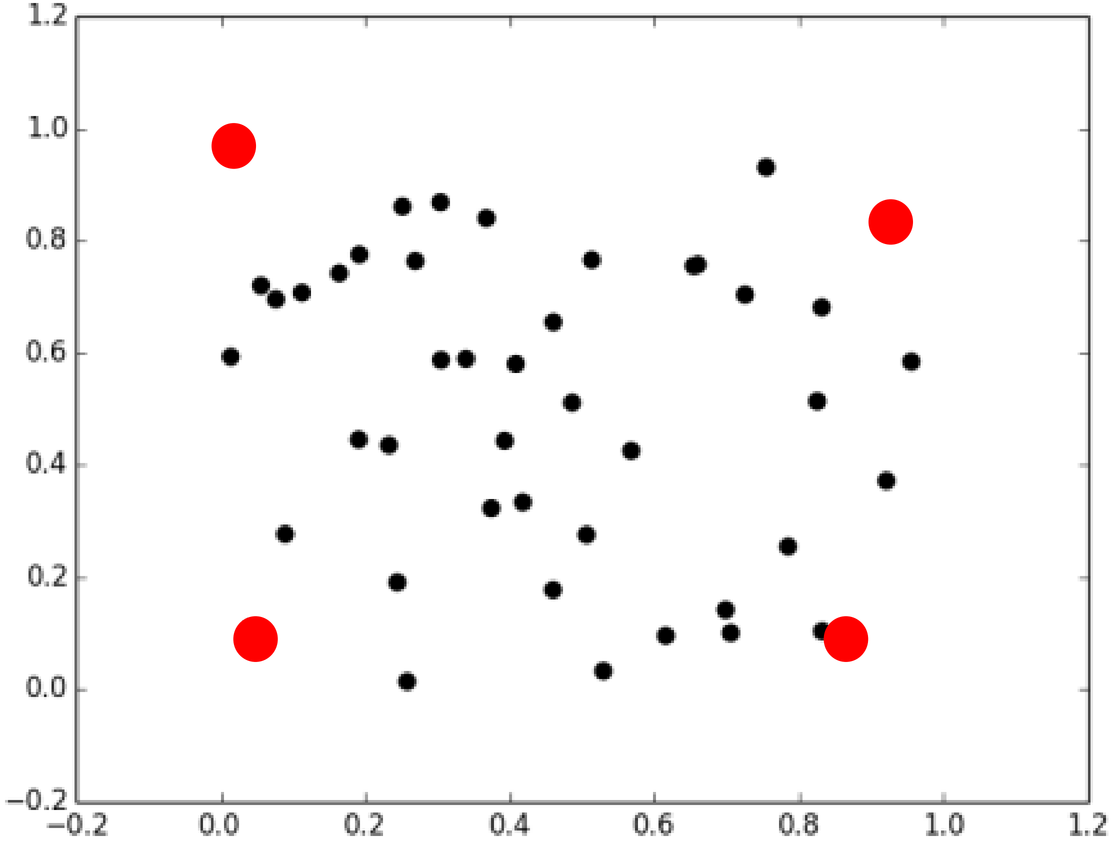
$K = 3$

Archetypal analysis



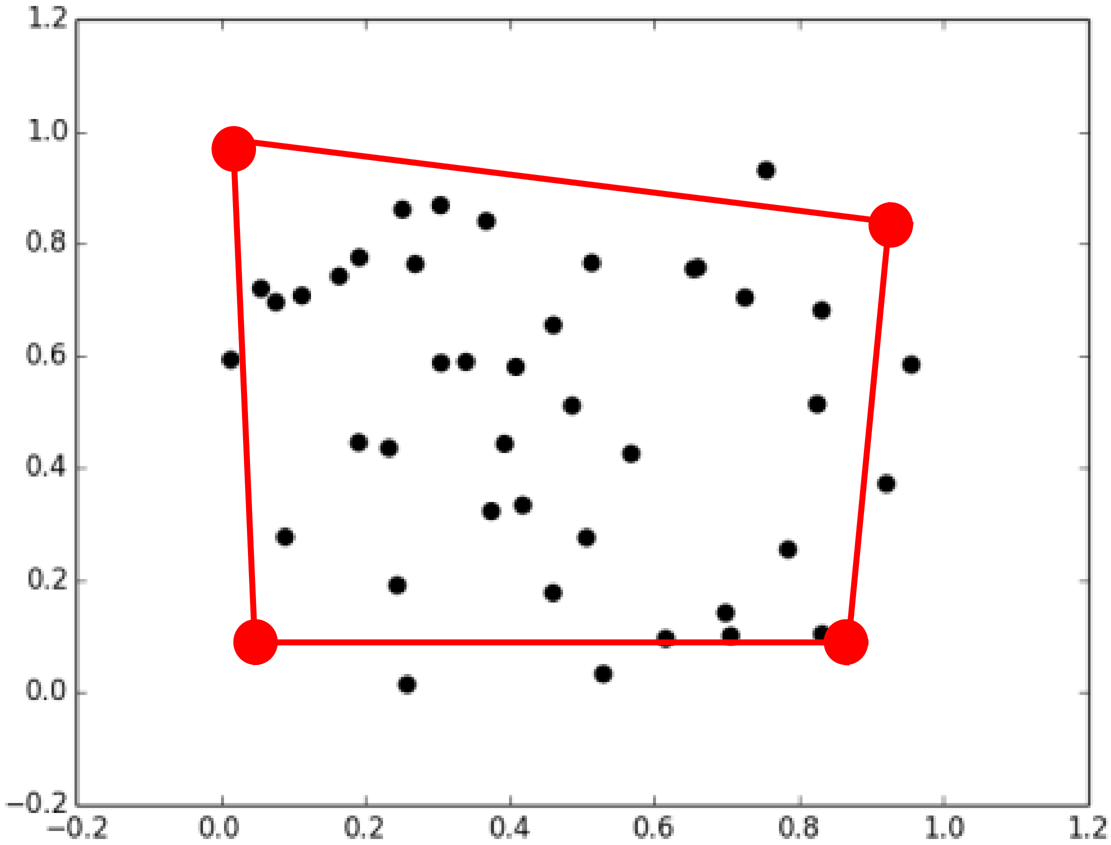
$K = 4$

Archetypal analysis



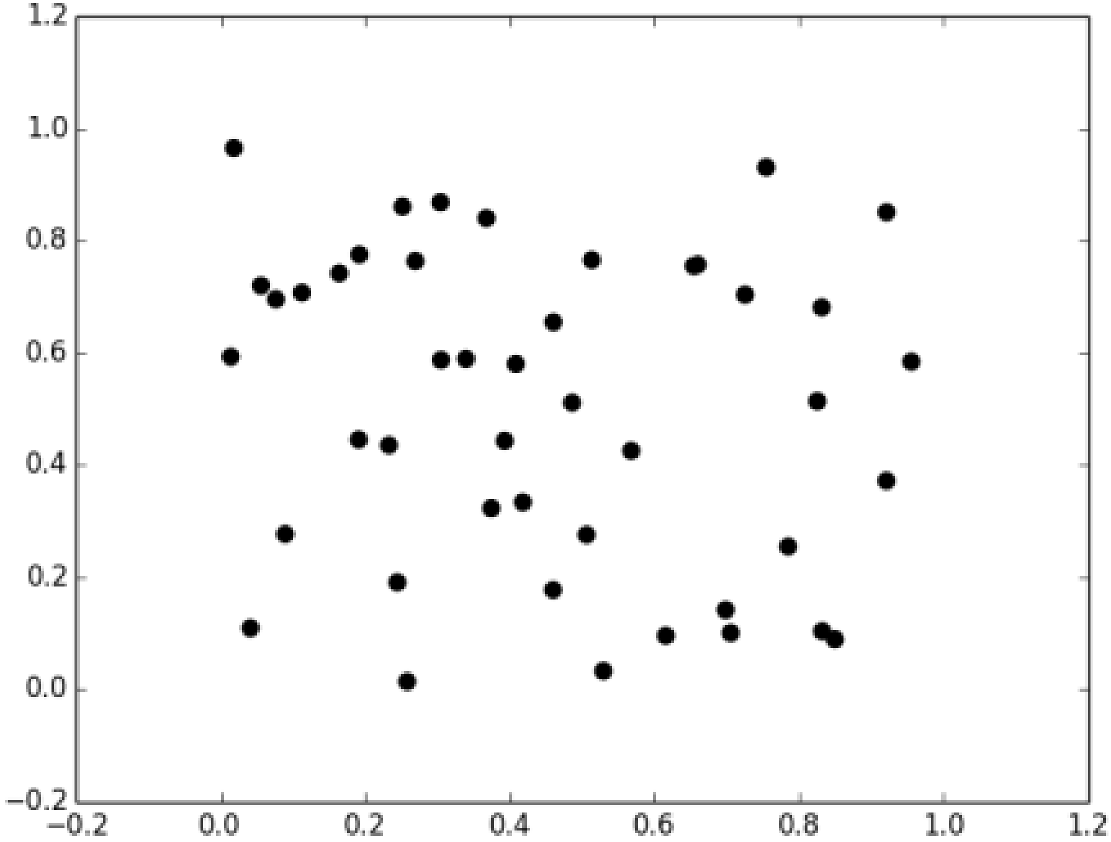
$K = 4$

Archetypal analysis



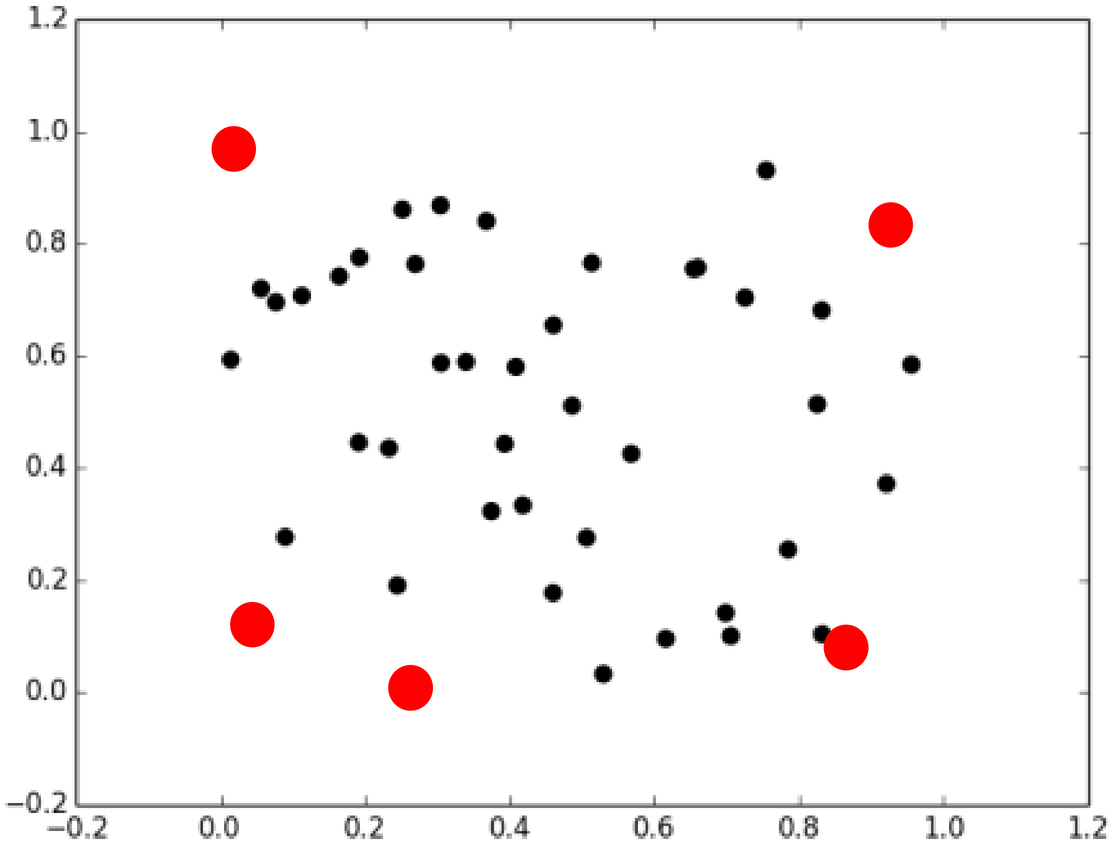
$K = 4$

Archetypal analysis



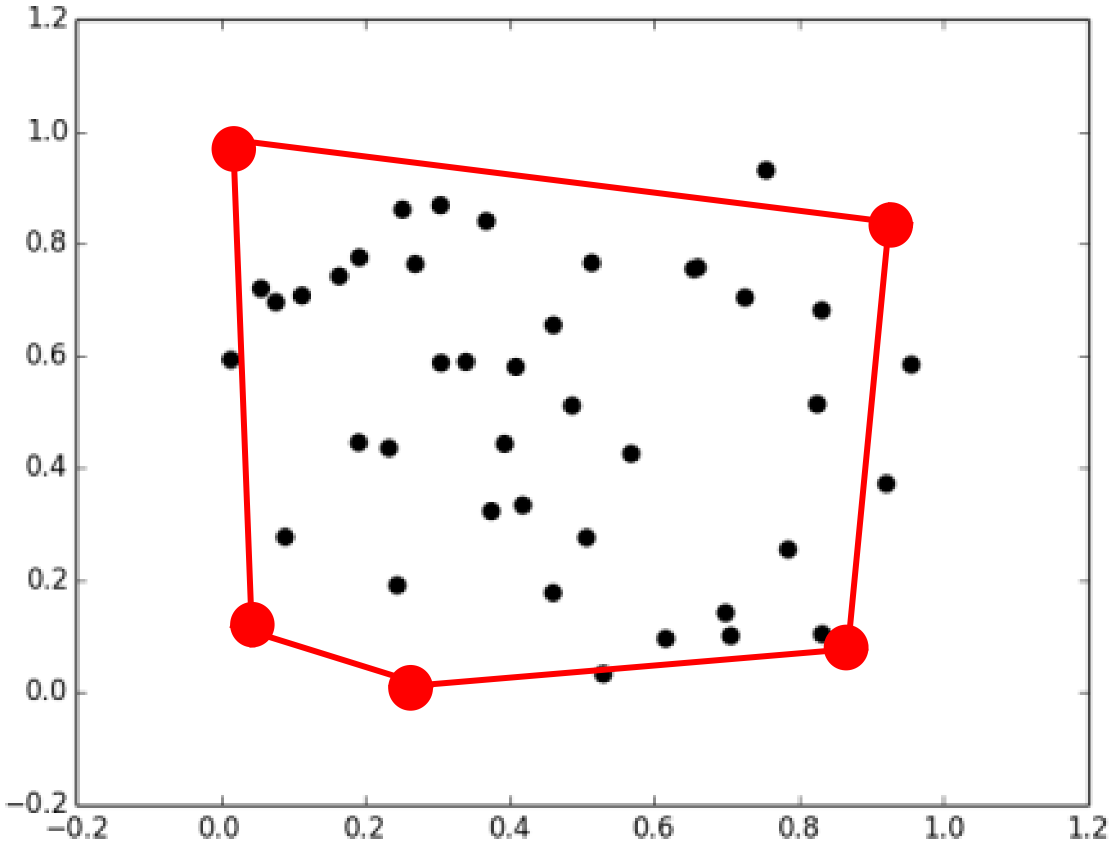
$K = 5$

Archetypal analysis



$K = 5$

Archetypal analysis



$K = 5$

Archetypal analysis

f) Which number of archetypes K seems to be best suited for the data set at hand and why?

$K = 4$, because of the elbow criterion

Exercise 3

Exercise 3

Which of the following statements are true?

a) Simulated Annealing maintains multiple solution candidates at the same time

No. Just one.

b) Evolutionary algorithms maintain multiple solution candidates at the same time

Yes.

Exercise 3

c) Simulated Annealing guarantees to find the global optimum at infinite runtime

Depends on the decay of T.

d) The temperature T in SA controls the tradeoff between exploration and exploitation

Yes.

Exercise 4

Recap: Game Theory

- Pure Strategy
  - A player selects deterministically
    one of the possible actions
- Mixed Strategy
  - A probability distribution over
    all possible actions
  - A mixed strategy is a probability distribution one uses to randomly choose among available actions in order to avoid being predictable.
  - E.g.: A = {0.4, 0.6}
- Dominant Strategy:
  - a strategy is dominant over another iff it is the best strategy regardless of the strategy of the opponent
- Nash equilibrium:
  - No player improves by individually changing its strategy

Exercise 4

Let the following game be given:

	B1	B2
A1	A:5, B:6	A:7, B:2
A2	A:4, B:5	A:9, B:1

Decide whether the following statements are true or false:

a) Player A has a dominant strategy

No! Why not?

Exercise 4

Let the following game be given:

	B1	B2
A1	A:5, B:6	A:7, B:2
A2	A:4, B:5	A:9, B:1

Decide whether the following statements are true or false:

b) Player B has a dominant strategy

Yes: Move 1; no matter if player A chooses move 1 or 2, it is always best for player B to choose move 1.

Exercise 4

Let the following game be given:

	B1	B2
A1	A:5, B:6	A:7, B:2
A2	A:4, B:5	A:9, B:1

Decide whether the following statements are true or false:

b) Player B has a dominant strategy

Yes: Move 1; no matter if player A chooses move 1 or 2, it is always best for player B to choose move 1: $6 > 2$ and $5 > 1$

Exercise 5

Exercise 5

Calculate the expected rewards for the following game with the mixed strategies:

$A = \{0.4, 0.6\}$

$B = \{0.9, 0.1\}$

	A1	A2
B1	A:5, B:3	A:4, B:6
B2	A:0, B:4	A:2, B:2

Answer: see whiteboard

Exercise 5

Calculate the Nash equilibrium in mixed strategies for the following game:

	B1	B2
A1	A:4, B:4	A:1, B:0
A2	A:2, B:2	A:6, B:9

Answer: see whiteboard

Exercise 6

Exercise 6

*Let the following game be given (*the values represent earnings in million €)*:*

	Mercedes, Ads	Mercedes, no Ads
BMW, Ads	2, 4	4, 2
BMW, no Ads	0.5, 9	3, 6

a) What is the Nash equilibrium (in pure strategies) of the above game?

The Nash equilibrium is: (Ads, Ads)

Exercise 6

*Let the following game be given (*the values represent earnings in million €)*:*

	Mercedes, Ads	Mercedes, no Ads
BMW, Ads	2, 4	4, 2
BMW, no Ads	0.5, 9	3, 6

b) If both companies could commit not to advertise by entering into a binding contract, should they do so? If so, why?

Yes, because more efficient for both companies ($3 > 2$ and $6 > 4$)

Exercise 6

c) Implement a program in Python which finds a Nash Eq assuming pure strategies.

See PyCharm

Exercise 6

d) Find a game with 3 players and 2 actions with no Nash Eq. Verify your solution using the program from c)

[a1,a2,a3]	r1	r2	r3
[0,0,0]			
[0,0,1]			
[0,1,0]			
[0,1,1]			
[1,0,0]			
[1,0,1]			
[1,1,0]			
[1,1,1]			

Exercise 6

d) Find a game with 3 players and 2 actions with no Nash Eq. Verify your solution using the program from c)

[a1,a2,a3]	r1	r2	r3
[0,0,0]	1	0	0
[0,0,1]	1	0	0
[0,1,0]	0	1	1
[0,1,1]	1	1	0
[1,0,0]	0	1	0
[1,0,1]	0	1	1
[1,1,0]	1	1	0
[1,1,1]	1	0	1

Exercise 7

Exercise 7

The four most important components of an Evolutionary Algorithm are:

- *Evaluation*
- *Selection*
- *Recombination*
- *Mutation*

Consider the Maximum Clique Problem. How would you implement these four components for this problem?

Exercise 7

Evaluation for Maximum Clique states?

```
def objective_maxClique(x):
        if x is a clique in G:
             return sum(x)
        else:
             return 0
```

Exercise 7

Selection for Maximum Clique states?

trivial.

Just choose the states with the highest objective function value.

Exercise 7

Recombination for Maximum Clique states?

C1 = [1, 0, 1, 0, 0, 0]

C2 = [0, 0, 1, 1, 0, 1]

Exercise 7

Recombination for Maximum Clique states?

C1 = [1, 0, 1, 0, 0, 0]

C2 = [0, 0, 1, 1, 0, 1]

C3 = [1, 0, 1, 1, 0, 1]

Exercise 7

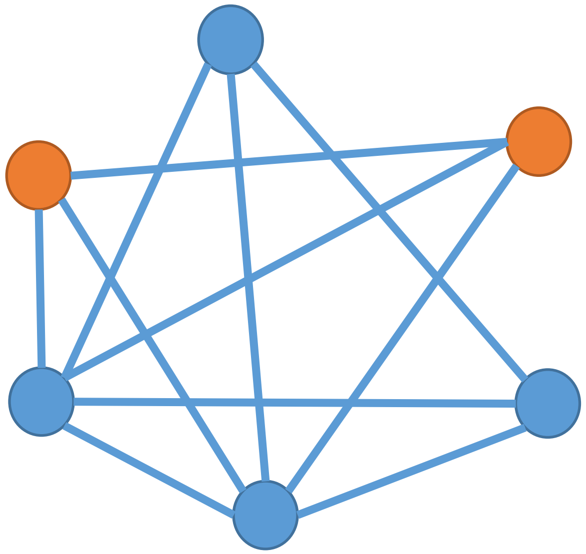
Recombination for Maximum Clique states?

C1 = [1, 0, 1, 0, 0, 0]

C2 = [0, 0, 1, 1, 0, 1]

C3 = [1, 0, 1, 1, 0, 1]

Value: 2



Exercise 7

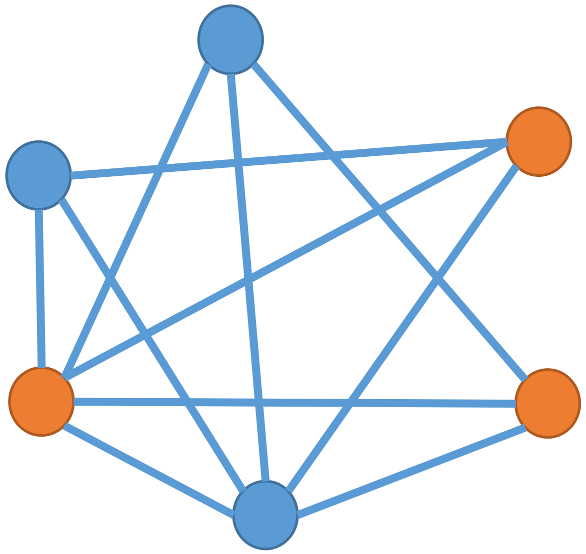
Recombination for Maximum Clique states?

C1 = [1, 0, 1, 0, 0, 0]

C2 = [0, 0, 1, 1, 0, 1]

C3 = [1, 0, 1, 1, 0, 1]

Value: 0



Exercise 7

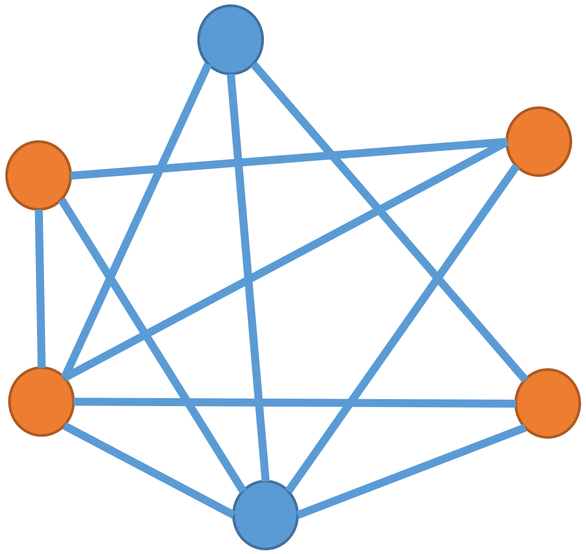
Recombination for Maximum Clique states?

C1 = [1, 0, 1, 0, 0, 0]

C2 = [0, 0, 1, 1, 0, 1]

C3 = [1, 0, 1, 1, 0, 1]

Value: 0



Exercise 7

Mutation for Maximum Clique states?

Randomly flip entries.

Example (1 mutation):

$$C = [1, 0, 0, 1, 0, 0]$$

$$C_{mutated} = [0, 0, 0, 1, 0, 0]$$

Exercise 7

Mutation for Maximum Clique states?

Randomly flip entries.

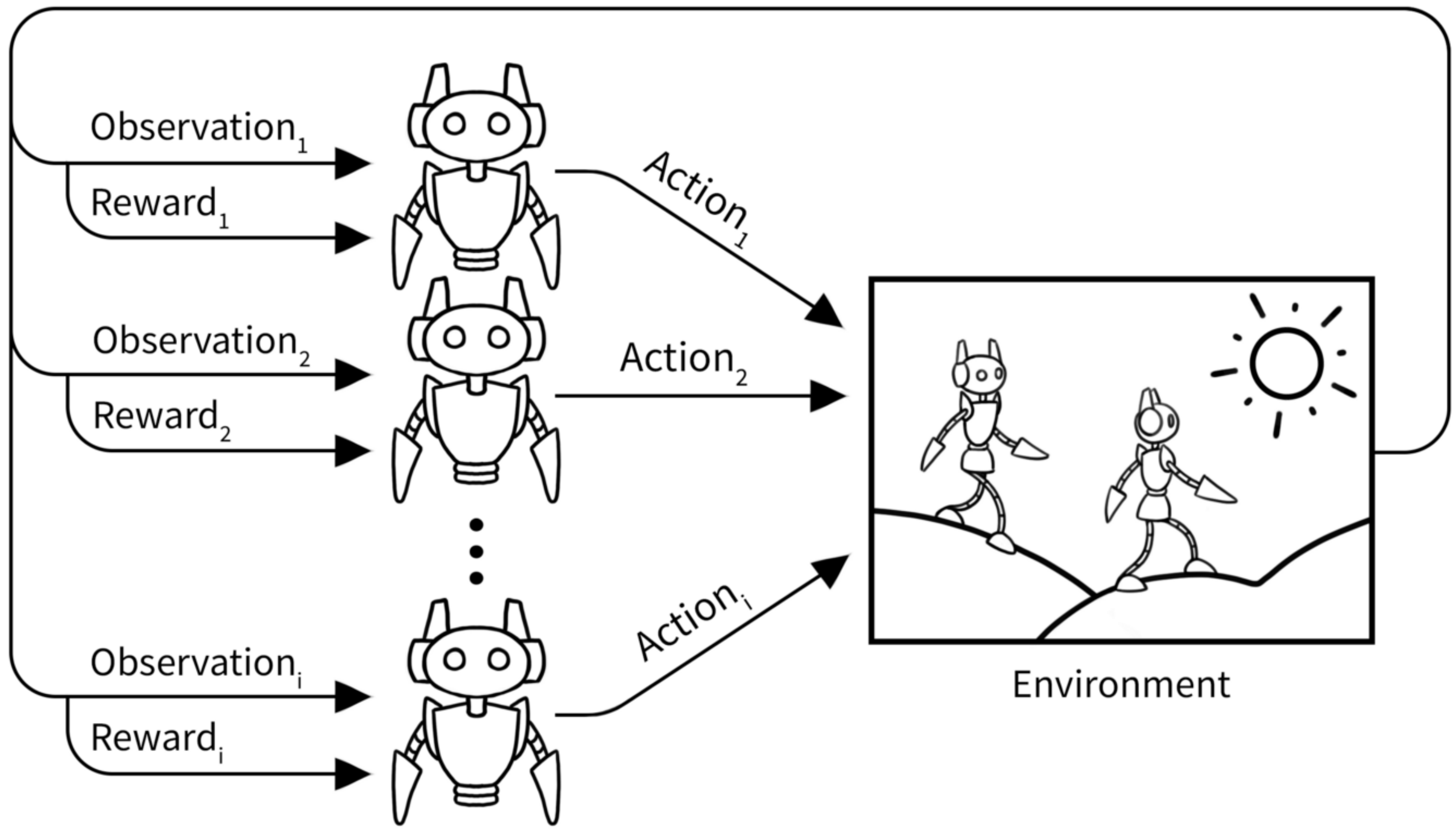
Example (2 mutations):

$$C = [1, 0, 0, 1, 0, 0]$$

$$C_{mutated} = [0, 1, 0, 1, 0, 0]$$

Outlook: Advanced Multi-Agent Systems

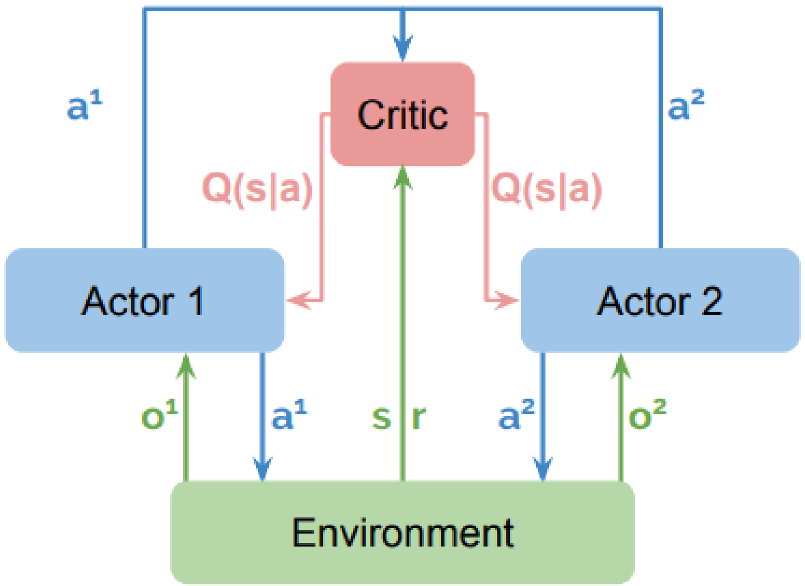
Multi-Agent Reinforcement Learning



MDP for Multi-Agent

- Let n be the number of agents
- S is the observation space
- $A_i, \ \ i = 1, 2, \dots, n$ is the action space for each agent
- The joint action space for all agents is defined by $A = A_1 \times A_1 \times \ \dots \times A_n$
- The state transition probability function is represented by $p : S \times A \times S \rightarrow [0, 1]$
- The reward function is specified as $r : S \times A \times S \rightarrow \mathbb{R}$
- The value function of each agent is dependent on the joint action and joint policy, which is characterized by $V_\pi : S \times A \rightarrow \mathbb{R}$

Some Axes of MARL



Centralized:

- One brain / algorithm deployed across many agents
- In Actor-Critic Architectures:
  Centralized Critic, but decentralized Actors

Decentralized:

- All agents learn individually
- Treat other agents as being part of the environment → Markov property becomes invalid since the environment is no longer stationary

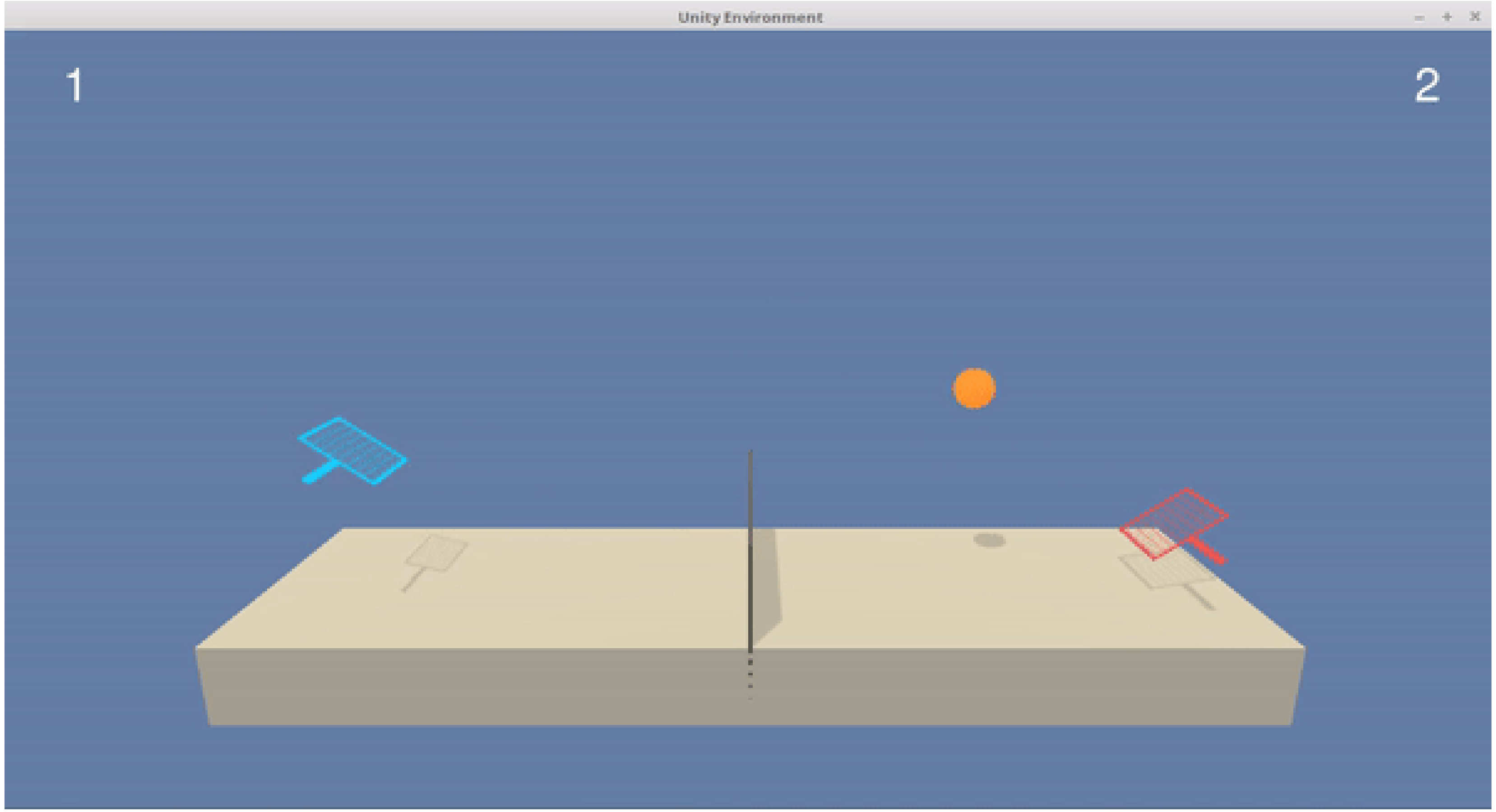
Cooperative:

- Agents work together to maximize the joint reward

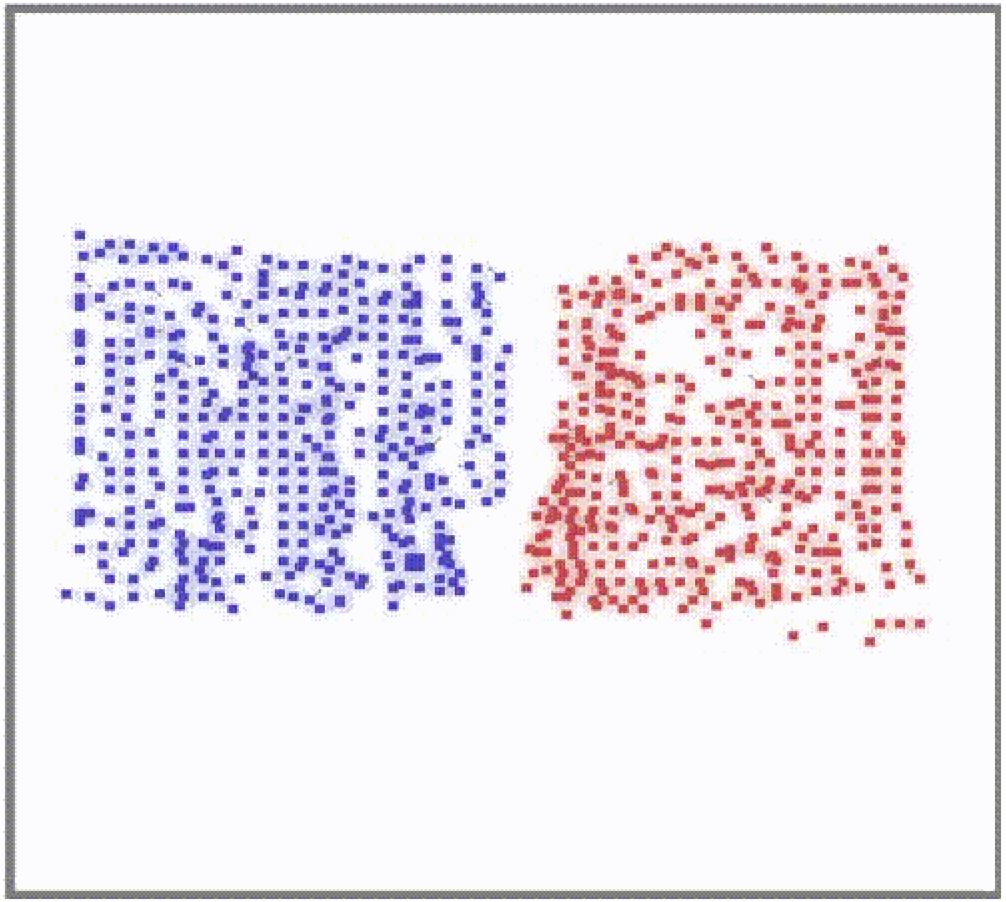
Competitive:

- Agents trying to maximizes solely their own (individual) reward

Pure Cooperative / Competitive MARL



Mixed-Cooperative-Competitive MARL



- Another example: Football
- Or any other game with 2 teams