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Retailer's optimal ordering and payment strategy under two-level and flexible two-part trade credit policy

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ABSTRACT

Trade credit has been widely used as a marketing strategy to boost sales and a tool for short-term financing. The existing literature usually study flexible two-part trade credit contract and two-level trade credit policy separately. In this paper, an inventory model is formulated for a supply chain simultaneously adopting flexible trade credit contract and two-level trade credit policy. In particular, the supplier offers the retailer with a flexible two-part trade credit contract, which allows the retailer to pay any fraction of the purchasing cost within a short credit period at a discounted price and the rest at full price within the maturity of the contract. Meanwhile, the retailer allows his/her customer to defer payment for a fixed period of time. The inventory model aims at determining the retailer's optimal ordering and payment strategy. A solution procedure is proposed and closed-form solutions are derived. Numerical results indicate that offering flexible trade credit can reduce the retailer's total cost while offering two-level trade credit contract will increase the retailer's total cost and reduce the retailer's early payment fraction.

1. Introduction

Trade credit is a very common and reciprocal business practice between suppliers and buyers. Suppliers offer trade credit as a marketing strategy to boost sales, reduce on-hand inventory level and attract new customers. Buyers offered with trade credit can defer payment for goods already delivered, which results in reduced capital tied up in stocks. Moreover, buyers can even earn interest via accumulated sales revenue received during the credit period. Nowadays, trade credit is greater in volume than short-term bank credit in nearly all developed and developing countries (Zhou and Zhou, 2013). The volume of trade credit accounts for 15% of the total sales of large U.S. enterprises. About 30% of U.S. firms use trade credit as a conventional short-term financing tool (Lee & Rhee, 2010). In the long run, trade credit can help to improve the overall performance of a supply chain (Gupta & Wang, 2009).

One-part and two-part trade credit are the two most basic contract forms of trade credit (Zhou & Zhou, 2013). With one-part trade credit, the supplier allows a retailer to delay payment for a fixed period without any interest charges. However, with two-part trade credit, the retailer can make full payment either within a shorter credit period M_1 at discount rate β or within a longer credit period M_2 at no discount. Generally, a two-part trade credit takes the form of $(\beta/M_1, 0/M_2)(M_1 < M_2)$, which combines cash discount and permissible delay in payment. It is intuitive that retailers have no incentive to make early payment under one-part trade credit. They will always postpone payment up to the maturity of the credit period so as to earn interest through accumulated sales revenue received during the credit period. This will increase the supplier's default risk and slow down the supplier's cash inflow. On the contrary, a two-part trade credit contract entices retailers to pay for goods quickly by offering cash discount, which accelerates the supplier's cash inflow (Ho, Ouyang, & Su, 2008).

Generally, the conventional two-part trade credit contract allows no partial payment neither at M_1 nor M_2. That is, the retailer must make full payment to enjoy price discount or defer full payment up to the maturity of the trade credit contract. This assumption does not reflect the real business world. The retailer can ask to pay for any fraction of the full purchasing cost at a discount price when he/she has a larger bargain power over the supplier. Such a payment arrangement is referred to as a flexible two-part trade credit contract, which takes the form of $(\lambda/\beta/M_1, (1-\lambda)/0/M_2)$. $\lambda (0 \leqslant \lambda \leqslant 1)$ represents the fraction of the purchasing cost that the retailer pays at discount rate β by time M_1 and $(1-\lambda)$ represents the balanced fraction of the purchasing cost that the retailer pays at full price by time M_2. Flexible two-part trade credit contract is more beneficial to the retailer since the retailer will be under less capital pressure at time M_1 when he/she is capital constrained.

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Thereby, the retailer can control his/her financial costs more properly.

In a supply chain, when only the supplier offers trade credit to the retailer, it is a one-level trade credit. Under one-level trade credit, it is implicitly assumed that the customer has to pay for the items upon purchasing because no trade credit is offered to the customer. However, this is unrealistic in practice. As the market competition is fierce and the customer has a stronger bargain power in a buyer's market, the retailer is also willing to extend the benefit received from the supplier to the customer by offering trade credit, so as to stimulate sales and attract new customers. When both the supplier and the retailer offer trade credit to his/her adjacent downstream player, it is a two-level trade credit.

It is intuitive that trade credit terms will affect the retailer's inventory and payment decisions in a supply chain. These terms include the credit period length, cash discount rate, flexible/non-flexible payment option, one-level/two-level trade credit policy, etc. The retailer's operational and payment decisions include order size, ordering cycle time, early payment fraction, etc. When a two-level trade credit policy is adopted in a three-echelon supply chain, the retailer in the middle has to trade off between the credit period lengths offered by the supplier (i.e., M_1 and M_2), the credit period length he/she offers to the customer and his/her own ordering cycle time because the credit period lengths and the ordering cycle time will affect the retailer's opportunity cost and opportunity gains. Moreover, the early payment fraction λ will impact the retailer's financial costs because, to enjoy cash discount, the retail needs to loan when he/she is capital restricted to pay at time M_1. Therefore, it is crucial to examine these interplayed factors systematically to optimize the retailer's performance.

In this paper, we incorporate flexible trade credit contract with two-level trade credit policy. Specifically, the supplier offers a flexible two-part trade credit to the retailer while the retailer allows his/her customer to defer payment for a fixed period of time. The objective is to determine the retailer's optimal early payment fraction at time M_1 and the optimal ordering cycle time T, so as to minimize the retailer's annual total cost.

This paper enriches the existing literature by being the first research to simultaneously study flexible two-part trade credit and two-level trade credit contract in a three-echelon supply chain. In addition, a solution procedure is proposed and closed-form solutions are derived. An extensive numerical study is carried out to understand the impact of adopting flexible two-part trade credit and the impact of offering two-level trade credit policy. It is found out that the retailer's annual total cost can be reduced for being flexible in determining the early payment fraction at time M_1. Moreover, under two-level trade credit, the retailer's annual total cost is increased and the retailer is less motivated to pay early at time M_1. Key trade credit terms impact the retailer's optimal payment decisions.

The remaining of the paper is organized as follows. Relevant literature is reviewed in Section 2. The notations and the major assumptions are presented in Section 3. The mathematical model is built in Section 4. Four propositions are developed in Section 5, which solve the retailer's optimal early payment fraction and the optimal ordering cycle time. A numerical study is conducted in Section 6. Conclusions and possible future research are discussed in Section 7.

2. Literature review

Trade credit has been widely used to boost sales, reduce default risk and attract new customers. However, in the classical EOQ model, it is implicitly assumed that customers make full payment upon purchasing. Goyal (1985) extended the classical EOQ model by considering permissible delay in payment, which reflected the real business practice. The trade credit studied in Goyal (1985) allows customer to defer payment for a fixed period of time, which is, in essence, a one-part trade credit. Since the trade credit is only offered by the supplier to the retailer (or by the retailer to the customer), it is a one-level trade credit (Teng, 2002; Chung & Huang, 2003; Jaggi, Yadavalli, Verma, & Sharma, 2015; Wu, Zhao, & Xi, 2017). After that, many researchers further extended Goyal (1985) from different perspectives. Since item deterioration will affect the on-hand inventory level, the ordering frequency and ordering size, Aggarwal and Jaggi (1995) studied ordering policies for deteriorating items under permissible delay in payment; Jamal, Sarker, and Wang (1997) extended Aggarwal and Jaggi (1995) to allow shortage while Hwang and Shinn (1997) extended to consider price strategy; Chung and Huang (2007), Liang and Zhou (2011) and Liao et al. (2016) developed a two-warehouse inventory model for deteriorating items. In practice, imperfect production quality is very common which can be caused by factors such as weak process control, poor machine maintenance and unskilled operation. It is clear that imperfect production quality will affect the inventory decisions as well. Liao et al. (2018b) developed an inventory model incorporating imperfect production quality and permissible delay in payment in a two-warehouse setting. Khanna et al. (2016a) studied the retailer's optimal ordering decisions for imperfect quality items, where the vendor is offered with permissible delay in payment by the supplier. Khanna, Gautam, and Jaggi (2016b) studied a similar problem to Khanna et al. (2016a) but developed an integrated inventory model. Khanna, Gautam, and Jaggi (2017) examined the same problem by considering exponential declining demand rate while Jaggi, Gautam, and Khanna (2018) assumed price-dependent demand. With traditional one-part trade credit, full trade credit is granted to the total purchasing amount under no condition. Nonetheless, partial trade credit or quantity dependent trade credit are considered in some cases (Chung et al., 2005, 2013, 2015, 2016; Ouyang, Ho, & Su, 2009). Other factors have been considered in the literature exploring one-part trade credit. Cárdenas-Barrón, Shaikh, Tiwari, and Treviño-Garza (2018) formulated an inventory model under trade credit with nonlinear holding cost and nonlinear stock dependent demand. Tsao (2018) built an inventory model to consider default risk under one-part trade credit.

Nowadays, as it is always the buyer's market and the market competition is fierce, the retailer is also willing to offer his/her customer with trade credit at the same time that the retailer is offered with a trade credit by the supplier. In a supply chain, such a payment arrangement is a two-level trade credit (Teng & Luo, 2012; Ouyang, Yang, & Chan, 2013; Otrodi, Yaghin, & Torabi, 2019). Many researchers have studied two-level trade credit under permissible delay in payment. Huang (2003) was one of the first research exploring such a problem. Huang (2006) extended Huang (2003) to consider limited storage space and the difference between unit selling price and purchasing price. Teng and Goyal (2007) extended Huang (2003) by assuming that the credit period offered by the retailer was no shorter than that offered by the supplier. Huang (2007) formulated an EPQ model under two-level trade credit policy. Teng, Chang, Chern, and Chan (2007) extended Goyal (1985) to develop an inventory model under two-level trade credit. The credit period length offered by the supplier was assumed to be no shorter than that offered by the retailer. Pakhira, Maiti, and Maiti (2018) developed an inventory model under two-level trade credit, where customer demand was trade credit period and promotional cost dependent. Uncertain resources, such as changing bank interest and budget, were also considered. Srivastava, Chung, Liao, Lin, and Chuang (2019) corrected some mathematical shortcomings in Teng et al. (2007). Teng and Chang (2009) relaxed the assumption in Huang (2007) to consider that the credit period offered by the retailer was longer than that offered by the supplier. Ho (2011) developed a price and credit linked demand model. Zhou, Zhong, and Li (2012) formulated an uncooperative ordering model with inventory-dependent demand and limited shelf space under two-level trade credit policy. Liao et al. (2017) investigated the retailer's optimal replenishment decisions for deteriorating items under two-level trade credit policy in a two-warehouse setting. Shah et al. (2017a) formulated an inventory model under two-level trade credit for time varying deteriorating items having fixed lifetime. Demand was time and price sensitive.

Preservation technology was incorporated to reduce deterioration. Shah et al. (2017b) studied a similar problem but assumed credit-period dependent demand while Shah, Jani, and Chaudhari (2018) considered trade credit dependent quadratic demand. Khanna, Kishore, Sarkar, and Jaggi (2018) studied a two-level trade credit for imperfect quality items where the manufacturer offered full trade credit to the wholesaler while the wholesaler offered full/partial permissible delay in payment to his/her downstream old/new retailers.

Since the retailer has the incentive to postpone payment up to the maturity of the credit period under permissible delay in payment, it increases the supplier's default risk. To accelerate cash inflow and reduce default risk, the supplier will offer both cash discount and deferment in payment, which is a two-part trade credit (Chang & Teng, 2004; Su, Ouyang, Ho, & Chang, 2007; Chung, 2010; Zhong & Zhou, 2012). Huang and Chung (2003) extended Goyal (1985) to include cash discount. Chang and Teng (2004) studied a retailer's optimal ordering and replenishment decisions for deteriorating items under two-part trade credit policy. Chung, Liao, Lin, Chuang, and Srivastava (2019) studied a similar problem and derived the optimal solution of the annual total relevant cost in Chang and Teng (2004) without adopting the Taylor-series approximation method. Ouyang, Teng, Chuang, and Chuang (2005) discussed optimal inventory policy with non-instantaneous receipt under two-part trade credit. Teng (2006) used discount cash-flow approach to determine the retailer's optimal ordering quantity under two-part trade credit. Closed-form solution was derived and compared with the classical economic order quantity. Ho et al. (2008) developed an integrated supplier-buyer inventory model with price-sensitive market demand and two-part trade credit contract. An iterative algorithm was established to obtain the optimal pricing, ordering, shipping and payment policy, which maximized the joint expected total profit per unit time. Chung and Liao (2011) studied the same problem in Ho et al. (2008) but proposed a simplified solution algorithm. Kreng and Tan (2011) developed an inventory model for deteriorating items under order-size dependent two-part trade credit. The supplier offered cash discount to the retailer when the retailer's order size was larger than a predetermined quantity. Chung, Lin, and Srivastava (2012) proposed a solution procedure to solve the problem in Kreng and Tan (2011) without assuming small deteriorating rate. Liao et al. (2018a) studied a variant of two-part trade credit, which was referred to as a progressive payment scheme. With such a payment arrangement, two credit periods were offered by the supplier to the retailer. If the retailer could pay off full purchasing cost by the shorter credit period, no interest would be charged by the retailer; otherwise, the retailer could decide to pay off purchasing cost after the shorter or the longer credit period at the cost of different interest charges. The research reviewed above explore two-part trade credit contract under one-level trade credit policy. However, the research simultaneously exploring two-part, two-level trade credit is relatively few. Chung, Liao, Ting, Lin, and Srivastava (2018) explored the retailer's optimal inventory decisions under a two-level trade credit policy by incorporating cash discount and quantity discount.

With conventional two-part trade credit, cash discount is offered to full purchasing cost. That is, the retailer needs to pay off the purchasing cost within a shorter credit period to enjoy price discount; otherwise, the retail can choose to postpone payment up to the maturity of a longer credit period at full price. However, when the retailer has stronger bargain power or when the market competition is fierce, the retailer will ask to pay any fraction of the purchasing cost within the short credit period but still enjoys price discount. This is a flexible two-part trade credit policy (Zhou, Zhong, & Wang, 2011). The existing literature on flexible two-part trade credit contract is quite few. Zhou, Zhong, and Wahab (2013) considered a flexible two-part trade credit contract in a two-echelon supply chain. The retailer's optimal ordering policy and payment decision were derived. The results indicated that the retailer could obtain more benefits from the flexible payment policy. Yang, Zhuo, Zha, and Wan (2016) proposed a continuous newsvendor model for a two-period supply chain, where the retailer was enticed to pay early with a flexible trade credit contract.

This paper is the first one to incorporate flexible trade credit contract with two-level trade credit policy. Specifically, the upstream trade credit is a flexible two-part trade credit and the downstream trade credit is a one-part trade credit. In this way, the trade credit contract model developed in Zhou et al. (2013) becomes a special case of our model when the downstream trade credit period is set to zero.

3. Problem description and notations

In this paper, a three-tier supply chain, which consists of a supplier, a retailer and a direct customer of the retailer, is considered. To promote sales, the supplier offers a flexible two-part trade credit contract to the retailer, which takes the form of $(\lambda/\beta/M_1, (1-\lambda)/0/M_2)$ $(0 < M_1 < M_2)$. The retailer allows his/her customer to defer payment for a fixed period N, which is assumed to be shorter than M_1 (Huang, 2003, 2006; Chung & Huang, 2007; Liao, 2008). Only one product type is considered. Market demand is assumed to be deterministic and no backlog order is allowed. The retailer places order regularly to the supplier. Each order incurs an ordering cost. On-hand inventory incurs inventory holding cost. The objectives of the research are to determine the retailer's optimal ordering cycle time and the early payment fraction, so as to minimize the retailer's annual total cost.

Notations for the model formulation are defined as follows:

Variables:	
D	annual market demand
Q	ordering size in an ordering cycle time, which is equal to $T \cdot D$
A_r	ordering cost per order
c_r	purchasing cost per unit
p	selling price per unit
I_e	opportunity interest earned per dollar per year for the retailer
I_c	loan interest charged per dollar per year for the retailer
g_r	opportunity gain per unit per year for the retailer, which equals to $I_e \cdot p$
s_r	opportunity cost per unit per year for the retailer (exclusive of the holding cost), which equals to $I_c \cdot c_r$. It's natural to have $s_r \geqslant g_r$.
h_r	holding cost per unit per year
β	the discount rate for early payment at time M_1 under two-part trade credit
M_1	the short discount period to enjoy price discount under two-part trade credit
M_2	the maturity of the two-part trade credit offered by the supplier to the retailer
N	the credit period offered by the retailer to his/her customer
$TC_r(T, \lambda)$	retailer's annual total cost
Decision variables:	
T	the ordering cycle time of the retailer
λ	the retailer's payment fraction at time M_1

Therefore, the retailer's annual total ordering cost is A_r/T, annual inventory holding cost equals to $DTh_r/2$ and discount revenue is $\beta\lambda c_r D$. Under the current payment arrangement, it's possible for the retailer to have opportunity gain through the accumulated sales revenue received during the credit period. Also, it's possible for the retailer to incur opportunity cost when he/she finances to pay off purchasing cost or makes early payment to enjoy price discount.

It's clear that the retailer's annual total cost depends on the values of T, N, M_1 and M_2. Therefore, the retailer's annual total costs under different cases (as summarized in Table 1) are formulated in Section 4.

4. Model formulation

4.1. Case 1: $T \leqslant M_1$

In case 1, it is assumed that the retailer's ordering cycle time T is no longer than M_1.

1) Subcase 1.1: $T + N \leqslant M_1$

Table 1
Cases dependent on the values of T, N, M_1 and M_2.

Case 1: $T \leqslant M_1$		Case 2: $M_1 < T \leqslant M_2$		Case 3: $T > M_2$
Subcase 1.1: $T + N \leqslant M_1$		Subcase 2.1: $M_1 < T + N \leqslant M_2$	Subcase 2.1.1: $\lambda T + N \leqslant M_1$	Subcase 3.1: $\lambda T + N \leqslant M_1$
Subcase 1.2: $M_1 < T + N \leqslant M_2$	Subcase 1.2.1: $\lambda T + N \leqslant M_1$		Subcase 2.1.2: $M_1 < \lambda T + N \leqslant M_2$	
	Subcase 1.2.2: $M_1 < \lambda T + N \leqslant M_2$	Subcase 2.2: $T + N > M_2$	Subcase 2.2.1: $\lambda T + N \leqslant M_1$	Subcase 3.2: $M_1 < \lambda T + N \leqslant M_2$
Subcase 1.3: $T + N > M_2$	Subcase 1.3.1: $\lambda T + N \leqslant M_1$		Subcase 2.2.2: $M_1 < \lambda T + N \leqslant M_2$	
	Subcase 1.3.2: $M_1 < \lambda T + N \leqslant M_2$		Subcase 2.2.3: $\lambda T + N > M_2$	Subcase 3.3: $\lambda T + N > M_2$
	Subcase 1.3.3: $\lambda T + N > M_2$			

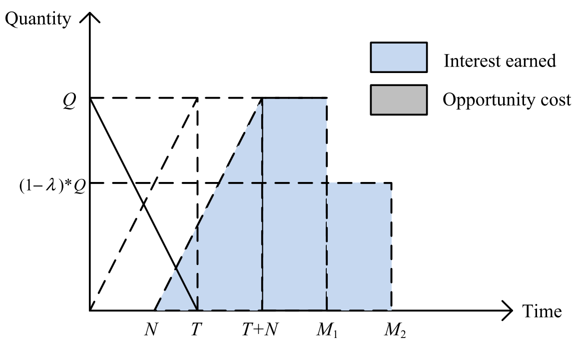


Fig. 1. Subcase 1.1.

Under such a circumstance (as shown in Fig. 1), the retailer collects full payment from the customer at time $T + N$, pays λ fraction of the purchasing cost at time M_1 and pays off the remaining purchasing cost at time M_2. The retailer incurs no opportunity cost in this case; instead, the retailer gains interest on average sales revenue received during the interval $[N, T + N]$, on full sales revenue received during the interval $[T + N, M_1]$ and on $(1 - \lambda)$ fraction of the full sales revenue received during the interval $[M_1, M_2]$. Denote the retailer's annual total cost as $TC_{r1}(T, \lambda)$, which equals,

$$TC_{r1}(T, \lambda) = A_r/T + h_r DT/2 - \beta\lambda c_r D - g_r D(M_1 - N - T/2) - (1 - \lambda)g_r D(M_2 - M_1) \quad (1)$$

2) Subcase 1.2: $M_1 < T + N \leqslant M_2$

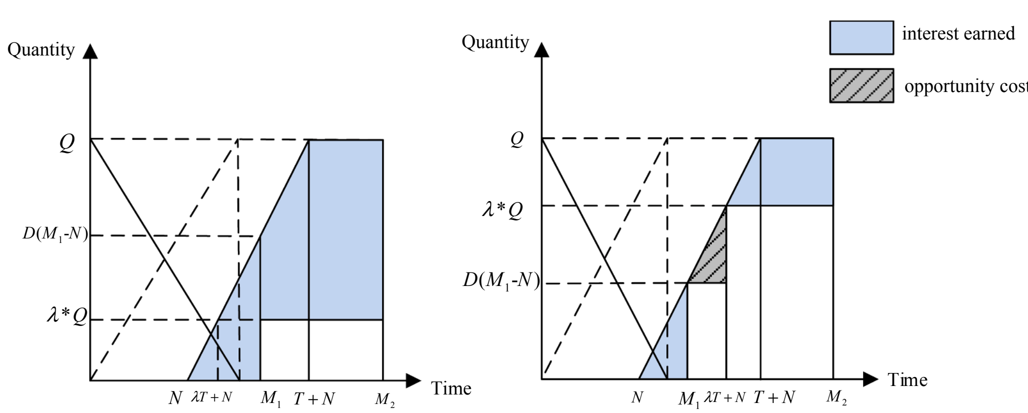
Two scenarios are considered for subcase 1.2 (as shown in Fig. 2).

a) Subcase 1.2.1: $\lambda T + N \leqslant M_1$

Under such a case, the retailer receives enough sales revenue to pay λ fraction of the purchasing cost at time M_1. Since the retailer collects full sales revenue from the customer at time $T + N$, the remaining purchasing cost can be paid off at time M_2 without loan. Therefore, the retailer incurs no opportunity cost. Instead, the retailer's opportunity gain obtained during the interval $[N, M_2]$ consists of three parts: (1) interest earned on average sales revenue received during the interval $[N, M_1]$; (2) interest earned on average sales revenue plus interest earned on "accumulated sales revenue at time M_1 minus λ fraction of the purchasing cost" during the interval $[M_1, T + N]$; (3) interest earned on $(1 - \lambda)$ fraction of the full sales revenue received during the interval $[T + N, M_2]$. It is found out that the retailer's annual total cost equals to $A_r/T + h_r DT/2 - \beta\lambda c_r D - g_r D(M_1 - N - T/2) - (1 - \lambda)g_r D(M_2 - M_1) = TC_{r1}(T, \lambda)$

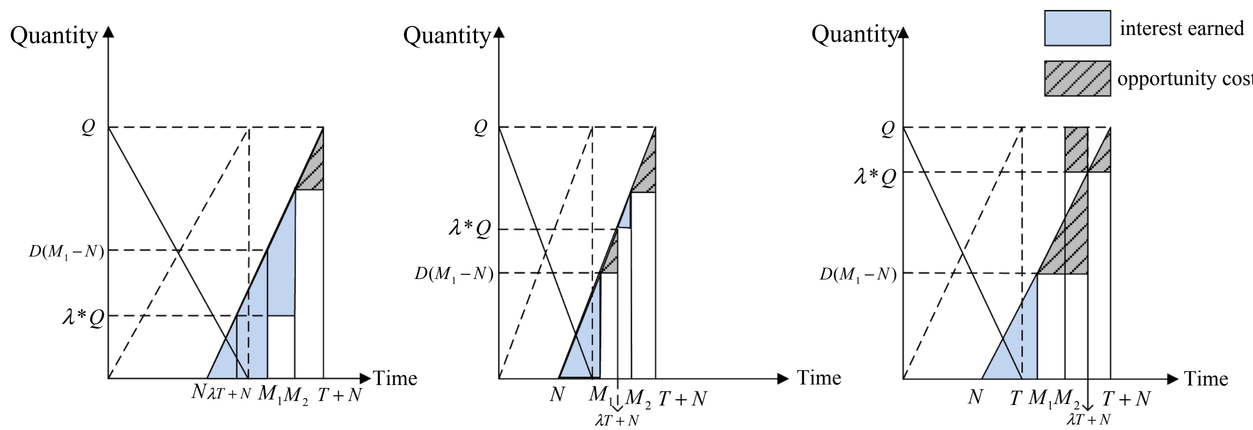
b) Subcase 1.2.2: $M_1 < \lambda T + N \leqslant M_2$

Under such a case, the retailer's accumulated sales revenue is not enough to pay λ fraction of the purchasing cost at time M_1 because the credit period M_1 is shorter than $\lambda T + N$. To enjoy price discount, the retailer needs to finance the items sold during the interval $[M_1, \lambda T + N]$ at an interest rate of I_c. Therefore, the retailer's opportunity gain consists of three parts: (1) interest earned on average sales revenue received during the interval $[N, M_1]$; (2) interest earned on average sales revenue received during the interval $[\lambda T + N, T + N]$; (3) interest earned on $(1 - \lambda)$ fraction of the full sales revenue received during the interval $[T + N, M_2]$. Thus, the retailer's annual opportunity cost is $(1 - \beta)s_r D(\lambda T + N - M_1)^2/(2T)$ and the annual opportunity gain is $g_r D(M_1 - N)^2/(2T) + g_r DT(1 - \lambda)^2/2 + g_r D(1 - \lambda)(M_2 - T - N)$. Denote the retailer's annual total cost under subcase 1.2.2. as $TC_{r2}(T, \lambda)$, which equals to,



a) subcase 1.2.1 $\lambda T + N \le M_1$ b) subcase 1.2.2 $M_1 < \lambda T + N \le M_2$

Fig. 2. Subcase 1.2.



a) Subcase 1.3.1 $\lambda T+N\le M_1$ b) Subcase 1.3.2 $M_1<\lambda T+N\le M_2$ c) Subcase 1.3.3 $\lambda T+N>M_2$

Fig. 3. Subcase 1.3.

$$TC_{r2}(T,\lambda)= A_r/T+h_rDT/2-\beta\lambda c_rD-g_rD(M_1-N)^2/(2T)-g_rDT(1-\lambda)^2/2 -g_rD(1-\lambda)(M_2-T-N) +(1-\beta)s_rD(\lambda T+N-M_1)^2/(2T) \tag{2}$$

3) Subcase 1.3: $T+N>M_2$

In subcase 1.3, $T+N>M_2$ is explored. In particular, three subcases are studied (as shown in Fig. 3).

a) Subcase 1.3.1: $\lambda T+N\leqslant M_1$

Under such a case, the retailer has enough accumulated sales revenue to pay λ fraction of the purchasing cost at time M_1. However, the retailer's accumulated sales revenue at time M_2 is not large enough to pay off the remaining purchasing cost because the credit period M_2 is shorter than $T+N$. Therefore, the retailer has to finance the items sold during the interval $[M_2, T+N]$ at an interest rate of I_c. Based on the above, the retailer's annual opportunity gain comprises of the following: (1) interest earned on average sales revenue received during the interval $[N, M_1]$; (2) interest earned on average sales revenue plus interest earned on "the accumulated sales revenue at M_1 minus λ faction of the purchasing cost" received during the interval $[M_1, M_2]$. Therefore, the retailer's annual opportunity gain is $g_rD(M_2-N)^2/(2T)-g_r\lambda D(M_2-M_1)$ and the annual opportunity cost is $s_rD(T+N-M_2)^2/(2T)$. Denote the retailer's annual total cost in this case as $TC_{r3}(T,\lambda)$, which equals

$$TC_{r3}(T,\lambda)= A_r/T+h_rDT/2-\beta\lambda c_rD-g_rD(M_2-N)^2/(2T) +g_r\lambda D(M_2-M_1)+s_rD(T+N-M_2)^2/(2T) \tag{3}$$

b) Subcase 1.3.2: $M_1<\lambda T+N\leqslant M_2$

Under such a case, the retailer's accumulated sales revenue is not enough to pay λ fraction of the purchasing cost at time M_1. Therefore, the retailer has to finance the items sold during the interval $[M_1, \lambda T+N]$ and pays off the debt at time $\lambda T+N$. However, the retailer is unable to pay off the remaining purchasing cost at time M_2 without financing the items sold during the interval $[M_2, T+N]$. So, the retailer's opportunity gain consists of the interest earned on average sales revenue received during the intervals $[N, M_1]$ and $[\lambda T+N, M_2]$. The retailer's opportunity cost comprises of the loan interest incurred during the intervals $[M_1, \lambda T+N]$ and $[M_2, T+N]$. Thus, the retailer's annual opportunity cost is $(1-\beta)s_rD(\lambda T+N-M_1)^2/(2T)+s_rD(T+N-M_2)^2/(2T)$ and the retailer's annual opportunity gain is $g_rD(M_1-N)^2/(2T)+g_rD(M_2-N-\lambda T)^2/(2T)$. Denote the retailer's annual total cost in subcase 1.3.2 as $TC_{r4}(T,\lambda)$, which equals

$$TC_{r4}(T,\lambda)= A_r/T+h_rDT/2-\beta\lambda c_rD-g_rD(M_1-N)^2/(2T) -g_rD(M_2-N-\lambda T)^2/(2T) +(1-\beta)s_rD(\lambda T+N-M_1)^2/(2T)+s_rD(T+N-M_2)^2/(2T) \tag{4}$$

c) Subcase 1.3.3: $\lambda T+N>M_2$

Similar to subcase 1.3.2, the retailer needs to loan at both time M_1 and M_2 since M_1 is shorter than $\lambda T+N$ and M_2 is shorter than $T+N$. Therefore, the retailer's opportunity gain consists of the interest earned on the average sales revenue received during the time interval $[N, M_1]$. The retailer's opportunity cost consists of two parts: (1) the loan interest during the interval $[M_1, \lambda T+N]$, which is incurred to pay for λ fraction of the purchasing cost at time M_1; (2) the loan interest during the intervals $[M_2, \lambda T+N]$ and $[\lambda T+N, T+N]$, which is incurred to pay for $(1-\lambda)$ fraction of the purchasing cost at time M_2. Thus, the retailer's annual opportunity gain is $g_rD(M_1-N)^2/(2T)$ and the annual opportunity cost is $(1-\beta)s_rD(\lambda T+N-M_1)^2/(2T)+(1-\lambda)s_r D(\lambda T+N-M_2)+(1-\lambda)^2s_rDT/2$. Denote the retailer's annual total cost in this case as $TC_{r5}(T,\lambda)$, which equals

$$TC_{r5}(T,\lambda)= A_r/T+h_rDT/2-\beta\lambda c_rD-g_rD(M_1-N)^2/(2T) +(1-\lambda)^2s_rDT/2 +(1-\beta)s_rD(\lambda T+N-M_1)^2/(2T) +(1-\lambda)s_rD(\lambda T+N-M_2) \tag{5}$$

4.2. Case 2: $M_1<T\leqslant M_2$

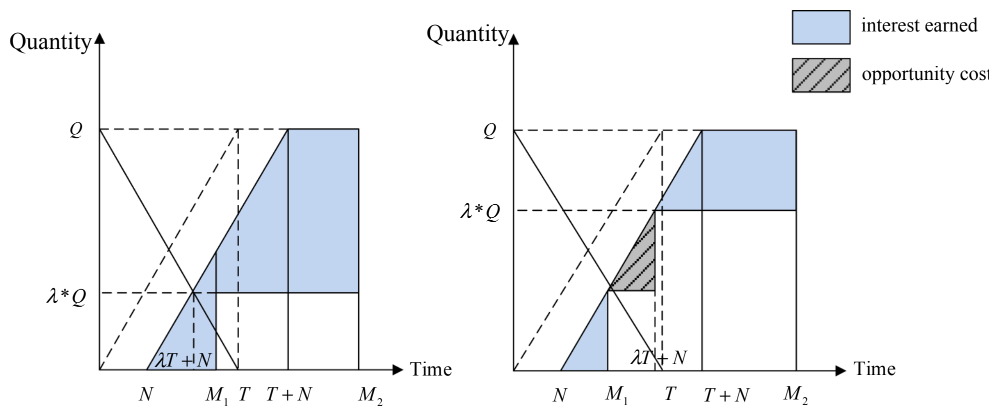
In case 2, it is assumed that the retailer's ordering cycle time T is longer than credit period M_1 but no longer than credit period M_2.

1) Subcase 2.1: $M_1<T+N\leqslant M_2$

In subcase 2.1, $M_1<T+N\leqslant M_2$ is explored. In particular, two subcases are discussed (as shown in Fig. 4).

a) Subcase 2.1.1: $\lambda T+N\leqslant M_1$

Analysis to the retailer's opportunity gain and opportunity cost in this case can refer to that for subcase 1.2.1. Moreover, the retailer's annual total cost in this case equals $TC_{r1}(T,\lambda)$.



a) Subcase 2.1.1 $\lambda T+N\leq M_1$ b) Subcase 2.1.2 $M_1<\lambda T+N\leq M_2$

Fig. 4. Subcase 2.1.

b) Subcase 2.1.2: $M_1 < \lambda T + N \leqslant M_2$

Analysis to the retailer's opportunity gain and opportunity cost in this case can refer to that for subcase 1.2.2. Moreover, the retailer's annual total cost in this case equals $TC_{r2}(T, \lambda)$.

2) subcase 2.2: $T + N > M_2$

In subcase 2.2, $T + N > M_2$ is studied. Specifically, three subcases are discussed (as shown in Fig. 5).

a) subcase 2.2.1: $\lambda T + N \leqslant M_1$

Analysis to the retailer's opportunity gain and opportunity cost in this case can refer to that for subcase 1.3.1. Moreover, the retailer's annual total cost in this case equals $TC_{r3}(T, \lambda)$.

b) subcase 2.2.2: $M_1 < \lambda T + N \leqslant M_2$

Analysis to the retailer's opportunity gain and opportunity cost in this case can refer to that for subcase 1.3.2. Moreover, the retailer's annual total cost in this case equals $TC_{r4}(T, \lambda)$.

c) subcase 2.2.3: $\lambda T + N > M_2$

Analysis to the retailer's opportunity gain and opportunity cost in this case can refer to that for subcase 1.3.3. Moreover, the retailer's annual total cost in this case equals $TC_{r5}(T, \lambda)$.

4.3. *Case 3:* $T > M_2$

In case 3, it is assumed that the retailer's ordering cycle time T is longer than credit period M_2. Specifically, three subcases (as shown in Fig. 6) are explored.

1) Subcase 3.1 $\lambda T + N \leqslant M_1$

Analysis to the retailer's opportunity gain and opportunity cost in this case can refer to that for subcase 1.3.1. Moreover, the retailer's annual total cost in this case equals to $TC_{r3}(T, \lambda)$.

2) Subcase 3.2: $M_1 < \lambda T + N \leqslant M_2$

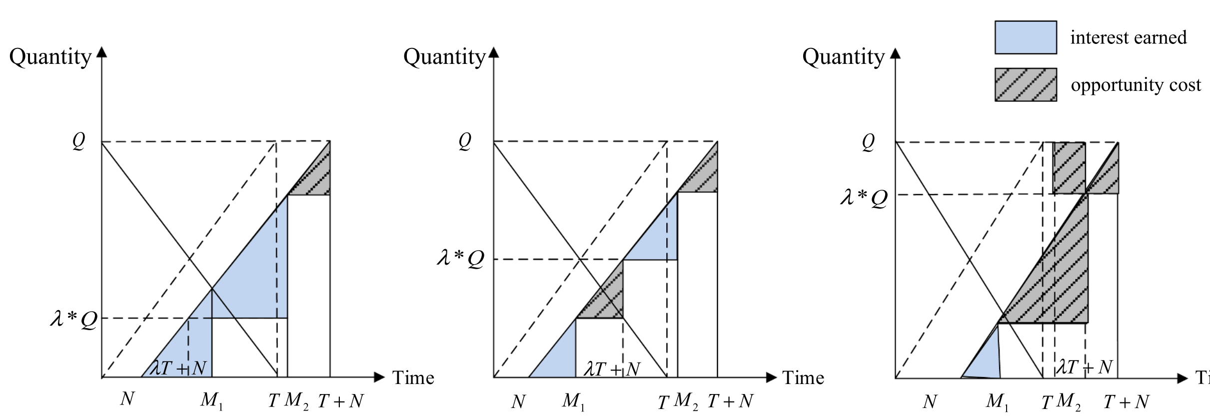
Analysis to the retailer's opportunity gain and opportunity cost in this case can refer to that for subcase 1.3.2. Moreover, the retailer's annual total cost in this case equals to $TC_{r4}(T, \lambda)$.

3) Subcase 3.3: $\lambda T + N > M_2$

Analysis to the retailer's opportunity gain and opportunity cost in this case can refer to that for subcase 1.3.3. Moreover, the retailer's annual total cost in this case equals to $TC_{r5}(T, \lambda)$.

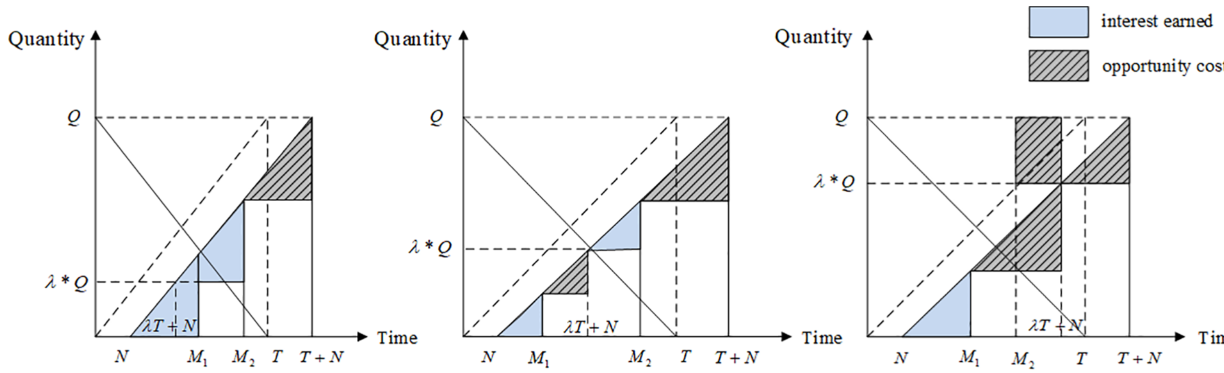
5. Theoretical results and optimal solutions

After examining these five functions $TC_{ri}(T, \lambda)$, $i = 1, 2, 3, 4, 5$, it is found that they can be integrated into two individually piecewise



a) Subcase 2.2.1 $\lambda T+N\leq M_1$ b) Subcase 2.2.2 $M_1<\lambda T+N\leq M_2$ c) Subcase 2.2.3 $\lambda T+N>M_2$

Fig. 5. Subcase 2.2.



a) subcase 3.1 $\lambda T+N \le M_1$ b) subcase 3.2 $M_1 < \lambda T+N \le M_2$ c) subcase 3.3 $\lambda T+N > M_2$

Fig. 6. Case 3.

continuous functions of T and λ depending on the values of $(M_1 - N)/(M_2 - N)$ and λ. When $(M_1 - N)/(M_2 - N) > \lambda$, $(M_1 - N) < (M_2 - N) < (M_1 - N)/\lambda < (M_2 - N)/\lambda$, hence, $TC_{ri}(T, \lambda), i = 1, 3, 4, 5$ is continuous with respect to $T \geqslant 0$ and $0 \leqslant \lambda < (M_1 - N)/(M_2 - N)$. When $(M_1 - N)/(M_2 - N) \leqslant \lambda$, $(M_1 - N) < (M_1 - N)/\lambda < (M_2 - N) < (M_2 - N)/\lambda$, hence, $TC_{ri}(T, \lambda), i = 1, 2, 4, 5$ is continuous with respect to $T \geqslant 0$ and $\lambda \geqslant (M_1 - N)/(M_2 - N)$. To sum up, the retailer's annual total cost can be expressed as follows:

1) If $(M_1 - N)/\lambda > (M_2 - N)$

$$TC_r(T, \lambda) = \begin{cases} TC_{r1}(T, \lambda) & T \leqslant (M_1 - N) \text{ and } (M_1 - N) < T \leqslant (M_2 - N) \\ TC_{r3}(T, \lambda) & (M_2 - N) < T \leqslant (M_1 - N)/\lambda \\ TC_{r4}(T, \lambda) & (M_1 - N)/\lambda < T \leqslant (M_2 - N)/\lambda \\ TC_{r5}(T, \lambda) & T > (M_2 - N)/\lambda \end{cases} \tag{6}$$

To simplify the notation, given any value of λ, use $TC_r(T)$ and $TC_{ri}(T)$ to denote $TC_r(T, \lambda)$ and $TC_{ri}(T, \lambda)(i = 1, 2..., 5)$, respectively. Therefore, it can be derived that $TC_{r1}(M_2 - N) = TC_{r3}(M_2 - N)$, $TC_{r3}[(M_1 - N)/\lambda] = TC_{r4}[(M_1 - N)/\lambda]$ and $TC_{r4}[(M_2 - N)/\lambda] = TC_{r5}[(M_2 - N)/\lambda]$. Hence, $TC_r(T)$ is continuous with respect to T over $(0, \infty)$.

2) if $(M_1 - N)/\lambda \leqslant (M_2 - N)$

$$TC_r(T, \lambda) = \begin{cases} TC_{r1}(T, \lambda) & T \leqslant (M_1 - N) \text{ and } (M_1 - N) < T \leqslant (M_1 - N)/\lambda \\ TC_{r2}(T, \lambda) & (M_1 - N)/\lambda < T \leqslant (M_2 - N) \\ TC_{r4}(T, \lambda) & (M_2 - N) < T \leqslant (M_2 - N)/\lambda \\ TC_{r5}(T, \lambda) & T > (M_2 - N)/\lambda \end{cases} \tag{7}$$

Similarly, it can be derived that $TC_{r1}[(M_1 - N)/\lambda] = TC_{r2}[(M_1 - N)/\lambda]$, $TC_{r2}(M_2 - N) = TC_{r4}(M_2 - N)$ and $TC_{r4}[(M_2 - N)/\lambda] = TC_{r5}[(M_2 - N)/\lambda]$. Hence, $TC_r(T)$ is continuous with respect to T over $(0, \infty)$.

As $TC_r(T, \lambda)$ is continuous with respect to T and λ, the following lemma can be obtained.

Lemma 1. *For any given λ, $TC_{ri}(T, \lambda)(i = 1, 2..., 5)$ is convex over $(0, +\infty)$. Therefore, there exists a unique optimal $T_{ri}(i = 1, 2..., 5)$ satisfying the first-order condition of $dTC_{ri}(T, \lambda)/dT = 0(i = 1, 2..., 5)$, where*

$$T_{r1} = \sqrt{\frac{2A_r}{(h_r + g_r)D}}$$

$$T_{r2} = \sqrt{\frac{2A_r + [(1-\beta)s_r - g_r]D(M_1 - N)^2}{(h_r + g_r)D + [(1-\beta)s_r - g_r]D\lambda^2}}$$

$$T_{r3} = \sqrt{\frac{2A_r + (s_r - g_r)D(M_2 - N)^2}{(h_r + s_r)D}}$$

$$T_{r4} = \sqrt{\frac{2A_r + [(1-\beta)s_r - g_r]D(M_1 - N)^2 + (s_r - g_r)D(M_2 - N)^2}{(h_r + s_r)D + [(1-\beta)s_r - g_r]D\lambda^2}}$$

$$T_{r5} = \sqrt{\frac{2A_r + [(1-\beta)s_r - g_r]D(M_1 - N)^2}{(h_r + s_r)D - \beta s_r D\lambda^2}}$$

Proof. Please refer to Appendix A for details.

5.1. Determination of the optimal t for a given λ

Scenario 1: $(M_1 - N) > \lambda(M_2 - N)$

Denote $\Delta_1 = TC'_{r1}(M_2 - N)$, $\Delta_2 = TC'_{r3}[(M_1 - N)/\lambda]$ and $\Delta_3 = TC'_{r4}[(M_2 - N)/\lambda]$, where,

$$\Delta_1 = -A_r/(M_2 - N)^2 + (h_r + g_r)D/2 \tag{8}$$

$$\Delta_2 = -\lambda^2 A_r/(M_1 - N)^2 + h_r D/2 + s_r D[1 - \lambda^2(M_2 - N)^2/(M_1 - N)^2]/2 + g_r D\lambda^2(M_2 - N)^2/2(M_1 - N)^2 \tag{9}$$

$$\Delta_3 = -\lambda^2 A_r/(M_2 - N)^2 + h_r D/2 + s_r D(1 - \lambda^2)/2 + (1-\beta)\lambda^2 s_r D[1 - (M_1 - N)^2/(M_2 - N)^2]/2 + g_r D\lambda^2(M_1 - N)^2/2(M_2 - N)^2 \tag{10}$$

It can be further derived that $\Delta_1 - \Delta_2 < 0$ and $\Delta_2 - \Delta_3 < 0$. Therefore, $\Delta_1 < \Delta_2 < \Delta_3$. As $TC_r'(T)$ is monotonously increasing with respect to T, Lemma 2 can be attained.

Lemma 2. *Based on the values of Δ_1, Δ_2 and Δ_3, the retailer's optimal ordering cycle time T_r^* can be determined as follows:*

(i) *If $\Delta_1 \geqslant 0$, then $TC_r(T_r^*) = TC_{r1}(T_{r1})$. Hence, $T_r^* = T_{r1}$ and $T_{r1} \leqslant M_2 - N$.*
(ii) *If $\Delta_1 < 0$ and $\Delta_2 \geqslant 0$, then $TC_r(T_r^*) = TC_{r3}(T_{r3})$. Hence, $T_r^* = T_{r3}$ and $M_2 - N < T_{r3} \leqslant (M_1 - N)/\lambda$.*
(iii) *If $\Delta_2 < 0$ and $\Delta_3 \geqslant 0$, then $TC_r(T^*) = TC_{r4}(T_{r4})$. Hence, $T_r^* = T_{r4}$ and $(M_1 - N)/\lambda < T_{r4} \leqslant (M_2 - N)/\lambda$.*
(iv) *If $\Delta_3 < 0$, then $TC_r(T^*) = TC_{r5}(T_{r5})$. Hence, $T_r^* = T_{r5}$ and*

$T_{r5} > (M_2 - N)/\lambda$.

Define $f_1(\lambda) = \Delta_1$, $f_2(\lambda) = \Delta_2$ and $f_3(\lambda) = \Delta_3$. Since $\Delta_1 < \Delta_2 < \Delta_3$, it can be derived that $f_1(\lambda) < f_2(\lambda) < f_3(\lambda)$ for all the $\lambda \in [0, (M_1 - N)/(M_2 - N)]$. Denote $M_t \equiv T_{r1}$. Hence, the following two situations are discussed based on the values of M_t and $M_2 - N$.

(1) If $M_t \leqslant M_2 - N$

It can be derived under this condition that $0 \leqslant f_1(\lambda) < f_2(\lambda) < f_3(\lambda)$.

(2) If $M_t > M_2 - N$

Hence, it can be derived under this condition that $f_1(\lambda) < 0$. Moreover, both $f_2(\lambda)$ and $f_3(\lambda)$ are strictly decreasing with respect to λ. Furthermore, it is derived that $\lim_{\lambda\to 0} f_2(\lambda) = \lim_{\lambda\to 0} f_3(\lambda) = (h_r + s_r)D/2 > 0$ and $\lim_{\lambda\to(M_1-N)/(M_2-N)} f_2(\lambda) = -A_r/(M_2 - N)^2 + (h_r + g_r)D/2 < 0$.

According to the intermediate value theorem, there must exist a λ, which makes $f_2(\lambda) = 0$. Denote the zero point of $f_2(\lambda) = 0$ as λ_1, which equals to

$$\lambda_1 = (M_1 - N)\sqrt{(h_r + s_r)D/[(s_r - g_r)D(M_2 - N)^2 + 2A_r]} \tag{11}$$

If $0 \leqslant \lambda \leqslant \lambda_1$, then $f_1(\lambda) < 0 \leqslant f_2(\lambda)$. Set

$$R = -A_r/(M_2 - N)^2 - [(1-\beta)s_r - g_r]D(M_1 - N)^2/2(M_2 - N)^2 - \beta s_r D/2 + (h_r + s_r)D(M_2 - N)^2/2(M_1 - N)^2$$

Since $\lim_{\lambda\to(M_1-N)/(M_2-N)} f_3(\lambda) = (M_1 - N)^2 R/(M_2 - N)^2$, then we have the following:

(2.1) If $R \geqslant 0$

Hence, it can be obtained that $f_3(\lambda) \geqslant 0$ for $\lambda \in [0, (M_1 - N)/(M_2 - N)]$.

(2.2) If $R < 0$

Hence, there must exist a unique λ_2 satisfying $f_3(\lambda) = 0$, where

$$\lambda_2 = (M_2 - N)\sqrt{\frac{(h_r + s_r)D}{[(1-\beta)s_r - g_r]D(M_1 - N)^2 + \beta s_r D(M_2 - N)^2 + 2A_r}} \tag{12}$$

Therefore, there will be: a) if $\lambda_2 < \lambda < (M_1 - N)/(M_2 - N)$, then $f_3(\lambda) < 0$; b) if $0 \leqslant \lambda < \lambda_2$, then $f_3(\lambda) \geqslant 0$.

Based on the analysis above, Proposition 1 is obtained.

Proposition 1. *When $(M_1 - N) > \lambda(M_2 - N)$, the retailer's optimal ordering cycle time T_r^*, given λ, can be determined as follows:*

(i) *When $M_t \leqslant M_2 - N$ and $0 \leqslant \lambda < (M_1 - N)/(M_2 - N)$, $\Delta_3 > \Delta_2 > \Delta_1 \geqslant 0$. Hence, $T_r^* = T_{r1}$.*
(ii) *When $M_t > M_2 - N$ and $0 \leqslant \lambda \leqslant \lambda_1$, $\Delta_1 < 0$, $\Delta_3 > \Delta_2 \geqslant 0$. Hence, $T_r^* = T_{r3}$.*
(iii) *When $M_t > M_2 - N$, ① if $R \geqslant 0$ and $\lambda_1 < \lambda < (M_1 - N)/(M_2 - N)$ or ② if $R < 0$ and $\lambda_1 < \lambda \leqslant \lambda_2$, then $\Delta_3 \geqslant 0$ and $\Delta_1 < \Delta_2 < 0$. Hence, $T_r^* = T_{r4}$.*
(iv) *When $M_t > M_2 - N$, $R < 0$ and $\lambda_2 < \lambda \leqslant (M_1 - N)/(M_2 - N)$, then $\Delta_1 < \Delta_2 < \Delta_3 < 0$. Hence, $T_r^* = T_{r5}$.*

Scenario 2: $\lambda(M_2 - N) > (M_1 - N)$

Denote $\Delta_4 = TC'_{r1}[(M_1 - N)/\lambda]$ and $\Delta_5 = TC'_{r2}(M_2 - N)$, where

$$\Delta_4 = -\lambda^2 A_r/(M_1 - N)^2 + h_r D/2 + g_r D/2 \tag{13}$$

$$\Delta_5 = -A_r/(M_2 - N)^2 + h_r D/2 + g_r D(M_1 - N)^2/2(M_2 - N)^2 + g_r D(1 - \lambda^2)/2 + (1-\beta)s_r D[\lambda^2 - (M_1 - N)^2/(M_2 - N)^2]/2 \tag{14}$$

From Eqs. (13) and (14), it is found out that $\Delta_4 - \Delta_5 < 0$ and $\Delta_5 - \Delta_3 \leqslant 0$. Hence, we have $\Delta_4 < \Delta_5 \leqslant \Delta_3$. Based on the analysis above, Lemma 3 can be obtained.

Lemma 3.

(i) *If $\Delta_4 \geqslant 0$, then $TC_r(T_r^*) = TC_{r1}(T_{r1})$. Hence, $T_r^* = T_{r1}$ and $T_{r1} \leqslant (M_1 - N)/\lambda$.*
(ii) *If $\Delta_4 < 0$ and $\Delta_3 > \Delta_5 \geqslant 0$, then $TC_r(T_r^*) = TC_{r2}(T_{r2})$. Hence, $T_r^* = T_{r2}$ and $(M_1 - N)/\lambda < T_{r2} \leqslant M_2 - N$.*
(iii) *If $\Delta_4 < \Delta_5 < 0$ and $\Delta_3 \geqslant 0$, then $TC_r(T_r^*) = TC_{r4}(T_{r4})$. Hence, $T_r^* = T_{r4}$ and $(M_2 - N) < T_{r4} \leqslant (M_2 - N)/\lambda$.*
(iv) *If $\Delta_3 < 0$, then $TC_r(T_r^*) = TC_{r5}(T_{r5})$. Hence, $T_r^* = T_{r5}$ and $T_{r5} > (M_2 - N)/\lambda$.*

Denote $f_4(\lambda) = \Delta_4$ and $f_5(\lambda) = \Delta_5$. As $\Delta_4 < \Delta_5 < \Delta_3$, it can be derived that $f_4(\lambda) < f_5(\lambda) < f_3(\lambda)$ for $\lambda \in [(M_1 - N)/(M_2 - N), 1]$. Moreover, $f_4(\lambda)$ is found to be strictly decreasing with respect to λ. Hence, we have $\lim_{\lambda\to(M_1-N)/(M_2-N)} f_4(\lambda) = -A_r/(M_2 - N)^2 + (h_r + g_r)D/2$ and $\lim_{\lambda\to 1} f_4(\lambda) = -A_r/(M_1 - N)^2 + (h_r + g_r)D/2$.

Therefore, the following three situations are discussed based on the values of M_t, $M_1 - N$ and $M_2 - N$.

(1) If $M_2 - N > M_1 - N \geqslant M_t \equiv T_{r1}$

It is derived that $\lim_{\lambda\to(M_1-N)/(M_2-N)} f_4(\lambda) \geqslant 0$ and $\lim_{\lambda\to 1} f_4(\lambda) \geqslant 0$. Hence, $0 \leqslant f_4(\lambda) < f_5(\lambda) < f_3(\lambda)$.

(2) If $M_2 - N \geqslant M_t > M_1 - N$

It is derived that $\lim_{\lambda\to(M_1-N)/(M_2-N)} f_4(\lambda) \geqslant 0$ and $\lim_{\lambda\to 1} f_4(\lambda) < 0$. Since $f_4(\lambda)$ is strictly decreasing with respect to λ, there must exist a unique $\lambda_3 \in ((M_1 - N)/(M_2 - N), 1]$ satisfying $f_4(\lambda) = 0$, where

$$\lambda_3 = (M_1 - N)/M_t \tag{15}$$

(2.1) If $(M_1 - N)/(M_2 - N) \leqslant \lambda \leqslant \lambda_3$

Hence, we have $0 \leqslant f_4(\lambda) < f_5(\lambda) < f_3(\lambda)$.

(2.2) If $\lambda_3 < \lambda \leqslant 1$

Hence, we have $f_4(\lambda) < 0$. Since $f_5(\lambda)$ is found to be strictly increasing with respect to λ, it can be derived that $\lim_{\lambda\to(M_1-N)/(M_2-N)} f_5(\lambda) = -A_r/(M_2 - N)^2 + (h_r + g_r)D/2 \geqslant 0$. Thus, $f_4(\lambda) < 0 \leqslant f_5(\lambda) < f_3(\lambda)$.

(3) If $M_t > M_2 - N > M_1 - N$

It can be obtained that $f_4(\lambda) < 0$ and $\lim_{\lambda\to(M_1-N)/(M_2-N)} f_5(\lambda) \leqslant 0$. Moreover, $f_5(\lambda)$ is found to be strictly increasing in λ. Set

$$\lim_{\lambda\to 1} f_5(\lambda) = -A_r/(M_2 - N)^2 - [(1-\beta)s_r - g_r]D(M_1 - N)^2/2(M_2 - N)^2 - \beta s_r D/2 + (h_r + s_r)D/2 = 0 \tag{16}$$

With Eq. (16), define M_r satisfy Eq. (17),

$$M_2 - N = M_r(M_1 - N) \equiv \sqrt{[2A_r + [(1-\beta)s_r - g_r]D(M_1 - N)^2]/[(h_r + g_r)D + [(1-\beta)s_r - g_r]D]} \quad (17)$$

It can be derived from Eq. (17) that $M_1 - N < M_r(M_1 - N) < M_t$.

(3.1) If $M_2 - N > M_r(M_1 - N)$

It can be derived under this condition that $\lim_{\lambda\to 1} f_5(\lambda) > 0$. Thus, there must exist a unique $\lambda_4 \in ((M_1 - N)/(M_2 - N), 1]$ satisfying $f_5(\lambda) = 0$, where

$$\lambda_4 = \sqrt{\frac{[(1-\beta)s_r - g_r]D(M_1 - N)^2 - (h_r + s_r)D(M_2 - N)^2 + 2A_r}{[(1-\beta)s_r - g_r]D(M_2 - N)^2}} \quad (18)$$

(3.1.1) If $\lambda_4 \leqslant \lambda < 1$

Then we have $f_4(\lambda) < 0 \leqslant f_5(\lambda) < f_3(\lambda)$.

(3.1.2) If $(M_1 - N)/(M_2 - N) \leqslant \lambda < \lambda_4$

Then we have $f_4(\lambda) < f_5(\lambda) \leqslant 0 < f_3(\lambda)$.

(3.2) If $M_2 - N \leqslant M_r(M_1 - N)$

Thus, we have $\lim_{\lambda\to 1} f_5(\lambda) \leqslant 0$. Hence, $f_5(\lambda) \leqslant 0$ for $\lambda \in ((M_1 - N)/(M_2 - N), 1]$, which leads to $f_4(\lambda) < f_5(\lambda) \leqslant 0$ for $\lambda \in ((M_1 - N)/(M_2 - N), 1]$. Since

$$\lim_{\lambda\to(M_1-N)/(M_2-N)} f_3(\lambda) = (M_1 - N)^2 R/(M_2 - N)^2$$

$$\lim_{\lambda\to 1} f_3(\lambda) = -A_r/(M_2 - N)^2 - [(1-\beta)s_r - g_r]D(M_1 - N)^2/2(M_2 - N)^2 - \beta s_r D/2 + (h_r + s_r)D/2$$

it can be derived that $\lim_{\lambda\to 1} f_3(\lambda) \leqslant 0$ under $M_2 - N \leqslant M_r(M_1 - N)$. Based on the value of R, we have:

(3.2.1) If $R \leqslant 0$

So we have $\lim_{\lambda\to(M_1-N)/(M_2-N)} f_3(\lambda) \leqslant 0$. Hence, $f_4(\lambda) < f_5(\lambda) < f_3(\lambda) < 0$ for $\lambda \in ((M_1 - N)/(M_2 - N), 1]$.

(3.2.2) If $R > 0$

It is derived that $\lim_{\lambda\to(M_1-N)/(M_2-N)} f_3(\lambda) > 0$. Hence, there must exist a unique zero point λ_2 satisfying $f_3(\lambda) = 0$. Therefore, if $(M_1 - N)/(M_2 - N) \leqslant \lambda < \lambda_2$, then we have $f_4(\lambda) < f_5(\lambda) \leqslant 0 \leqslant f_3(\lambda)$; if $\lambda_2 < \lambda \leqslant 1$, then we have $f_4(\lambda) < f_5(\lambda) < f_3(\lambda) < 0$.

Based on the analysis for Scenario 2 above, Proposition 2 is obtained.

Proposition 2. *When $\lambda(M_2 - N) > (M_1 - N)$, the retailer's optimal ordering cycle time T_r^*, given λ, can be determined as follows:*

(i) *When $M_2 - N > M_1 - N \geqslant M_t$, $\Delta_3 > \Delta_5 > \Delta_4 \geqslant 0$. Hence, $T_r^* = T_{r1}$.*
(ii) *When $M_2 - N \geqslant M_t > M_1 - N$: ① if $(M_1 - N)/(M_2 - N) < \lambda \leqslant \lambda_3$, then $\Delta_3 > \Delta_5 > \Delta_4 \geqslant 0$. Hence, $T_r^* = T_{r1}$. ② if $\lambda_3 < \lambda \leqslant 1$, then $\Delta_3 > \Delta_5 > 0$, $\Delta_4 \leqslant 0$. Hence, $T_r^* = T_{r2}$.*
(iii) *When $M_t > M_2 - N > M_r(M_1 - N)$:①if $(M_1 - N)/(M_2 - N) < \lambda < \lambda_4$, then $\Delta_3 > 0$, $\Delta_4 < \Delta_5 \leqslant 0$. Hence, $T_r^* = T_{r4}$; ②if $\lambda_4 < \lambda \leqslant 1$, then $\Delta_4 \leqslant 0$, $\Delta_3 > \Delta_5 > 0$. Hence, $T_r^* = T_{r2}$.*
(iv) *When $M_t > M_r(M_1 - N) > M_2 - N$: ①if $R < 0$ and $(M_1 - N)/(M_2 - N) < \lambda \leqslant 1$, then $\Delta_4 < \Delta_5 < \Delta_3 < 0$. Hence, $T_r^* = T_{r5}$; ② if $R \geqslant 0$ and $(M_1 - N)/(M_2 - N) < \lambda \leqslant \lambda_2$, then $\Delta_3 > 0$ and $\Delta_4 < \Delta_5 \leqslant 0$. Hence, $T_r^* = T_{r4}$; ③if $R \geqslant 0$ and $\lambda_2 < \lambda \leqslant 1$, then $\Delta_4 < \Delta_5 < \Delta_3 < 0$. Hence, $T_r^* = T_{r5}$.*

With Proposition 1 and 2, the retailer's optimal ordering cycle time is summarized in Proposition 3.

Proposition 3. *Given λ, the retailer's optimal ordering cycle time T_r^* equals to,*

(1) *When $M_2 - N > M_1 - N \geqslant M_t$, $T_r^* = T_{r1} = M_t$.*
(2) *When $M_2 - N \geqslant M_t > M_1 - N$: ① if $0 \leqslant \lambda \leqslant \lambda_3$, then $T_r^* = T_{r1}$. ② if $\lambda_3 < \lambda \leqslant 1$, then $T_r^* = T_{r2}(\lambda)$.*
(3) *When $M_t > M_2 - N > M_r(M_1 - N) > M_1 - N$: ① if $0 \leqslant \lambda \leqslant \lambda_1$, then $T_r^* = T_{r3}$. ②if $\lambda_1 < \lambda < \lambda_4$, then $T_r^* = T_{r4}(\lambda)$. ③ if $\lambda_1 \leqslant \lambda \leqslant 1$, then $T_r^* = T_{r2}(\lambda)$.*
(4) *When $M_t > M_r(M_1 - N) \geqslant M_2 - N > M_1 - N$: ① if $0 \leqslant \lambda \leqslant \lambda_1$, then $T_r^* = T_{r3}$. ② if $\lambda_1 < \lambda \leqslant \lambda_2$, then $T_r^* = T_{r4}(\lambda)$. ③ if $\lambda_2 < \lambda \leqslant 1$, then $T_r^* = T_{r5}(\lambda)$.*

5.2. Determination of the optimal λ with T_r^*

Based on Proposition 3, the retailer's optimal payment fraction λ at time M_1 is discussed below. In accordance with the four scenarios in Proposition 3, the following four scenarios are explored.

Scenario 1: $M_2 - N > M_1 - N \geqslant M_t$

Replace T_r^* with T_{r1}, the retailer's annual total cost can be expressed as,

$$TC_r = TC_{r1}(\lambda) = \sqrt{2A_rD(h_r + g_r)} - \lambda\beta c_r D - g_r D(M_2 - N) + \lambda g_r D(M_2 - M_1) \quad (19)$$

Take the first-order derivative of $TC_{r1}(\lambda)$ with respect to λ, we obtain

$$\frac{dTC_{r1}}{d\lambda} = [g_r(M_2 - M_1) - \beta c_r]D \quad (20)$$

Define $W_1 = g_r(M_2 - M_1) - \beta c_r$. So, we have the following lemma.

Lemma 4. *If $M_2 - N > M_1 - N \geqslant M_t$, the retailer's optimal payment fraction λ^* at time M_1 can be determined as follows:*

(i) *If $W_1 > 0$, then $TC_{r1}(\lambda)$ increases in λ. Hence, $\lambda^* = 0$.*
(ii) *If $W_1 \leqslant 0$, then $TC_{r1}(\lambda)$ decreases in λ. Hence, $\lambda^* = 1$.*

Scenario 2: $M_2 - N \geqslant M_t > M_1 - N$

Under the condition of $M_2 - N \geqslant M_t > M_1 - N$, the retailer's annual total cost is,

$$TC_r(\lambda) = \begin{cases} TC_{r1}(\lambda), & 0 \leqslant \lambda \leqslant \lambda_3 \\ TC_{r2}(\lambda), & \lambda_3 < \lambda \leqslant 1 \end{cases} \quad (21)$$

$TC_{r2}(\lambda)$ is obtained by replacing T_r^* with T_{r2}, which equals to

$$TC_{r2}(\lambda) = -\lambda\beta c_r D - (1-\beta)\lambda s_r D(M_1 - N) - (1-\lambda)g_r D(M_2 - N) + \sqrt{2A_r + [(1-\beta)s_r - g_r]D(M_1 - N)^2} \times \sqrt{(h_r + s_r)D + [(1-\beta)s_r - g_r]D\lambda^2} \quad (22)$$

With Eq. (22), it can be derived that $dTC_{r2}{}^2(\lambda)/d\lambda^2 > 0$. Since $TC_{r1}(\lambda_3) = TC_{r2}(\lambda_3)$ and $TC'_{r1}(\lambda_3) = TC'_{r2}(\lambda_3) = W_1 \times D$, $TC_r(\lambda)$ is continuous with respect to λ. Moreover, the first-order derivative of $TC_r(\lambda)$ with respect to λ is increasing in λ. Define λ_2^* as the zero point of $TC'_{r2}(\lambda_3) = 0$, which equals to,

$$\lambda_2^* = \sqrt{\frac{D(h_r + g_r)\{[(1-\beta)s_r - g_r](M_1 - N) - W_1\}^2}{[(1-\beta)s_r - g_r]\{2[(1-\beta)s_r - g_r](A_r + D(M_1 - N)W_1) - DW_1^2\}}} \quad (23)$$

Therefore, we have the following lemma.

Lemma 5. *If $M_2 - N \geqslant M_t > M_1 - N$, the retailer's optimal payment fraction λ^* at time M_1 is determined as follows:*

(i) *If $W_1 > 0$, then $\lambda^* = 0$.*
(ii) *If $W_1 \leqslant 0$, then $\lambda^* = \lambda_2^*$.*
(iii) *If $TC'_{c2}(1) < 0$, then $\lambda^* = 1$.*

Scenario 3: $M_t > M_2 - N > M_r(M_1 - N) > M_1 - N$

Under the condition of $M_t > M_2 - N > M_r(M_1 - N) > M_1 - N$, the retailer's annual total cost is,

$$TC_r(\lambda) = \begin{cases} TC_{r3}(\lambda), & 0 \leqslant \lambda \leqslant \lambda_1 \\ TC_{r4}(\lambda), & \lambda_1 < \lambda \leqslant \lambda_4 \\ TC_{r2}(\lambda), & \lambda_4 < \lambda \leqslant 1 \end{cases} \tag{24}$$

Replace T_r^* with T_{r3} in $TC_{r3}(\lambda)$, $TC_{r3}(\lambda)$ can be expressed as follows:

$$\begin{aligned} TC_{r3}(\lambda) = & \sqrt{[2A_r + (s_r - g_r)D(M_2 - N)^2](h_r + s_r)D} \\ & - \lambda\beta c_r D \\ & - s_r D(M_2 - N) \\ & + \lambda g_r D(M_2 - M_1) \end{aligned} \tag{25}$$

Replace T_r^* with T_{r4} in $TC_{r4}(\lambda)$, $TC_{r4}(\lambda)$ can be expressed as follows:

$$\begin{aligned} TC_{r4}(\lambda) = & \sqrt{2A_r + [(1-\beta)s_r - g_r)]D(M_1 - N)^2 + (s_r - g_r)D(M_2 - N)^2} \\ & \times \sqrt{(h_r + s_r)D + [(1-\beta)s_r - g_r)]D\lambda^2} - \lambda\beta c_r D \\ & - (1-\beta)\lambda s_r D(M_1 - N) \\ & - s_r D(M_2 - N) \\ & + \lambda g_r D(M_2 - N) \end{aligned} \tag{26}$$

With Eqs. (25) and (26), it is derived that $TC_{r3}(\lambda_1) = TC_{r4}(\lambda_1)$ and $TC_{r4}(\lambda_4) = TC_{r2}(\lambda_4)$. This implies that $TC_r(\lambda)$ is continuous with respect to λ. Moreover, we have $dTC_{r4}{}^2(\lambda)/d\lambda^2 > 0$. Denote $W_2 = W_1 + [(1-\beta)s_r - g_r][\lambda_4(M_2 - N) - (M_1 - N)]$. Hence, $TC'_{r3}(\lambda_1) = TC'_{r4}(\lambda_1) = W_1 \cdot D$ and $TC'_{r4}(\lambda_4) = TC'_{r2}(\lambda_4) = W_2 \cdot D$. This indicates that $TC'_{r4}(\lambda)$ is continuous and monotonously increasing with respect to λ. Define λ_4^* as the zero point of $TC'_{r4}(\lambda_4) = 0$, which equals to

$$\lambda_4^* = \sqrt{\frac{D(h_r + s_r)\{[(1-\beta)s_r - g_r](M_1 - N) - W_1\}^2}{[(1-\beta)s_r - g_r]\{[(1-\beta)s_r - g_r][2A_r + (s_r - g_r)D(M_2 - N)^2 + 2D(M_1 - N)W_1] - DW_1^2\}}} \tag{27}$$

Based on the above analysis, the retailer's optimal payment fraction under this scenario is obtained.

Lemma 6. *If $M_t > M_2 - N > M_r(M_1 - N) > M_1 - N$, the retailer's optimal payment fraction λ^* at time M_1 is determined as follows:*

(i) *If $W_1 \geqslant 0$, then $\lambda^* = 0$.*
(ii) *If $W_1 < 0$ and $W_2 > 0$, then $\lambda^* = \lambda_4^*$.*
(iii) *If $W_2 \leqslant 0$ and $TC'_{r2}(1) \geqslant 0$, then $\lambda^* = \lambda_2^*$.*
(iv) *If $TC'_{r2}(1) < 0$, then $\lambda^* = 1$.*

Scenario 4: $M_t > M_r(M_1 - N) \geqslant M_2 - N > M_1 - N$

Under the condition of $M_t > M_r(M_1 - N) \geqslant M_2 - N > M_1 - N$, the retailer's annual total cost is,

$$TC_r(\lambda) = \begin{cases} TC_{r3}(\lambda), & 0 \leqslant \lambda \leqslant \lambda_1; \\ TC_{r4}(\lambda), & \lambda_1 < \lambda \leqslant \lambda_2; \\ TC_{r5}(\lambda), & \lambda_2 < \lambda \leqslant 1. \end{cases} \tag{28}$$

Replace T_r^* with T_5 in $TC_{r5}(\lambda)$, $TC_{r5}(\lambda)$ can be expressed as follows:

$$\begin{aligned} TC_{r5}(\lambda) = & \sqrt{2A_r + [(1-\beta)s_r - g_r]D(M_1 - N)^2} \\ & \times \sqrt{(h_r + s_r)D - \beta s_r D\lambda^2} \\ & - \lambda\beta c_r D \\ & - (1-\beta)\lambda s_r D(M_1 - N) \\ & - (1-\lambda)s_r D(M_2 - N) \end{aligned} \tag{29}$$

Since $TC_{r3}(\lambda_1) = TC_{r4}(\lambda_1)$ and $TC_{r4}(\lambda_2) = TC_{r5}(\lambda_2)$, $TC_r(\lambda)$ is continuous with respect to λ. Define $W_3 = (1-\beta)s_r(M_2 - M_1) - \beta c_r$. It can be derived that $d^2TC_{r5}(\lambda)/d\lambda^2 < 0$. Therefore, $TC'_{r3}(\lambda_1) = TC'_{r4}(\lambda_1) = W_1 \cdot D$ and $TC'_{r4}(\lambda_2) = TC'_{r5}(\lambda_2) = W_3 \cdot D$. This indicates that $TC'_r(\lambda)$ is continuous with respect to λ over [0, 1]. Thus, the following lemma is attained.

Lemma 7. *If $M_t > M_r(M_1 - N) \geqslant M_2 - N > M_1 - N$, the retailer's optimal payment fraction λ^* at time M_1 is determined as follows:*

(i) *When $TC'_{r3}(\lambda_1) > 0$: ①if $TC_r(\lambda = 0) \leqslant TC_r(\lambda = 1)$, then $\lambda^* = 0$. ②if $TC_r(\lambda = 0) > TC_r(\lambda = 1)$, then $\lambda^* = 1$.*
(ii) *When $TC'_{r3}(\lambda_1) \leqslant 0$ and $TC'_{r4}(\lambda_2) > 0$: ①if $TC_r(\lambda = \lambda_4^*) \leqslant TC_r(\lambda = 1)$, then $\lambda^* = \lambda_4^*$. ②if $TC_r(\lambda = \lambda_4^*) > TC_r(\lambda = 1)$, then $\lambda^* = 1$.*
(iii) *When $TC'_{r4}(\lambda_2) \leqslant 0$, $\lambda^* = 1$.*

Combine the results in Lemma 4 to Lemma 7, the retailer's optimal payment fraction at time M_1 is summarized in Proposition 4.

Proposition 4. *Given the values of M_1, M_2 and N, the retailer's optimal payment fraction λ^* at time M_1 can be determined as follows:*

(i) *If $M_2 - N > M_1 - N \geqslant M_t$, then λ^* is determined by Lemma 4.*
(ii) *If $M_2 - N \geqslant M_t > M_1 - N$, then λ^* is determined by Lemma 5.*
(iii) *If $M_t > M_2 - N > M_r(M_1 - N) > M_1 - N$, then λ^* is determined by Lemma 6.*
(iv) *If $M_t > M_r(M_1 - N) \geqslant M_2 - N > M_1 - N$, then λ^* is determined by Lemma 7.*

In Section 5, an inventory model is built to analyze the retailer's optimal payment and ordering strategy. The retailer's payment fraction λ at time M_1 and ordering cycle time T are the two decision variables of this model. The objective is to find the optimal λ^* and T^*, which can minimize the retailer's annual total cost. Since the retailer's annual total cost is a continuous piecewise function with respect to T and λ, the two decision variables are derived by fixing one of them as given at a time. The optimal ordering cycle time T^* is derived first given payment fraction λ. Then, the optimal payment fraction λ^* is obtained by replacing T with the derived T^*. The optimal λ^* and T^* are presented in Proposition 3 and 4. Clearly, when discount rate β is set to be 0, our proposed model becomes a "one-part" trade credit contract model under two-level trade credit policy. Moreover, when λ is 0 or 1, the proposed model turns into the conventional nonflexible two-part trade credit contract model under two-level trade credit policy. In addition, when $N = 0$, the formulated model degrades into a one-level, flexible two-part trade credit model.

6. Numerical examples and sensitivity analysis

6.1. Impact of flexible payment strategy

Under conventional two-part trade credit contract, the retailer must make full payment both at time M_1 and time M_2. When λ is set to 1 in our proposed model, this corresponds to the retailer's decision to pay off at time M_1 to enjoy price discount. When λ is set to 0 in our proposed model, this corresponds to the retailer's decision to pay off at time M_2 to enjoy deferred payment. To analyze the impact of flexible trade credit on the retailer's annual total cost, a cross comparison of the retailer's annual total cost under flexible and nonflexible trade credit is made.

Table 2
Retailer's annual total cost under flexible and nonflexible payment strategy.

Varied parameters		Flexible payment strategy		Nonflexible payment strategy			
		λ(%)	TC_r($)	λ(%)	$TC_{\lambda=0}$($)	λ(%)	$TC_{\lambda=1}$($)
β	0.80%	0.00	6318.81	0	6318.81	100	6427.59
	0.90%	21.77	6317.10	0	6318.81	100	6387.18
	1.00%	33.83	6305.97	0	6318.81	100	6346.77
	1.10%	46.05	6289.98	0	6318.81	100	6306.35
	1.20%	100.00	6265.94	0	6318.81	100	6365.94
	1.30%	100.00	6225.53	0	6318.81	100	6225.53
M_1	15	0.00	6318.81	0	6318.81	100	6461.57
	17	0.00	6318.81	0	6318.81	100	6415.29
	19	26.64	6312.06	0	6318.81	100	6369.49
	21	41.04	6298.30	0	6318.81	100	6324.17
	23	55.55	6278.13	0	6318.81	100	6279.33
	25	100.00	6234.98	0	6318.81	100	6234.98
	27	100.00	6191.11	0	6318.81	100	6191.11
M_2	33	100.00	6346.77	0	6464.39	100	6346.77
	35	100.00	6346.77	0	6422.18	100	6346.77
	37	49.86	6345.52	0	6380.46	100	6346.77
	39	39.15	6319.97	0	6339.24	100	6346.77
	41	28.53	6291.17	0	6298.51	100	6346.77
	43	0.00	6258.27	0	6258.27	100	6346.77
	45	0.00	6218.52	0	6218.52	100	6346.77
h_r	8	30.93	5704.03	0	5715.77	100	5747.44
	10	33.83	6305.98	0	6318.81	100	6346.77
	12	36.50	6860.29	0	6874.14	100	6898.58
	14	38.98	7376.76	0	7391.55	100	7412.67
	16	41.32	7862.21	0	7877.89	100	7895.84
A_r	400	37.76	5573.63	0	5587.91	100	5610.65
	450	35.63	5949.85	0	5963.36	100	5988.93
	500	33.83	6305.98	0	6318.81	100	6346.77
	550	32.27	6644.91	0	6657.17	100	6687.14
	600	30.91	6968.92	0	6980.69	100	7012.39
I_c	0.2	41.32	6293.25	0	6308.01	100	6309.39
	0.22	33.83	6305.98	0	6318.81	100	6346.77
	0.24	30.21	6317.57	0	6329.40	100	6383.72
	0.26	28.11	6328.62	0	6339.78	100	6420.25
	0.28	26.74	6339.33	0	6349.96	100	6456.37
I_e	0	50.73	6331.42	0	6467.06	100	6363.50
	0.02	49.38	6325.94	0	6430.29	100	6359.32
	0.04	47.23	6320.31	0	6393.32	100	6355.14
	0.06	43.28	6314.13	0	6356.17	100	6350.95
	0.08	33.83	6305.97	0	6318.81	100	6346.77
	0.1	0.00	6281.26	0	6281.26	100	6342.58
N	5	43.24	6201.14	0	6218.52	100	6234.98
	8	37.60	6263.67	0	6278.33	100	6301.69
	11	31.94	6327.31	0	6339.24	100	6369.49
	14	26.26	6392.09	0	6401.26	100	6438.37
	17	20.56	6457.98	0	6464.39	100	6508.34

For the flexible trade credit, $\lambda \in [0, 1]$; however, for the nonflexible trade credit, λ equals to either 1 or 0. The detailed solving process of the corresponding ordering cycle time T^* and the retailer's annual total cost $TC(T^*)$ under nonflexible trade credit can refer to Appendix B.

Suppose $\beta = 1\%$, $M_1 = 20$ days, $M_2 = 40$ days, $N = 10$ days, $D = 4000$ units/year, $p = \$20$/unit, $C_r = \$10$/unit, $I_c = 0.22/\$$/unit/year, $I_e = 0.08/\$$/unit/year, $A_r = \$500$/order and $h_r = \$10$/unit/year. As is shown in Table 2, the annual total cost of the retailer under flexible payment strategy is always smaller than that under the nonflexible payment strategy. The annual total cost under flexible payment strategy TC_r and the annual total cost under the nonflexible payment strategy $TC_{\lambda=1}$ decrease in β and M_1; however, the annual total cost under the nonflexible payment strategy $TC_{\lambda=0}$ remains constant because the retailer chooses not to pay early. Therefore, no matter how β and M_1 change, they will not affect $TC_{\lambda=0}$. Furthermore, both TC_r and $TC_{\lambda=0}$ decrease in M_2 while $TC_{\lambda=1}$ remains constant. When $\lambda = 1$, the retailer has paid off all the purchasing cost at time M_1. Hence, the credit period M_2 has no impact on $TC_{\lambda=1}$. In addition, it is intuitive that TC_r, $TC_{\lambda=0}$ and $TC_{\lambda=1}$ all increase in h_r, A_r, I_c and N but decrease in I_e.

6.2. Impact of various key parameters on λ, T and TC

To better understand the impact of trade credit terms on the retailer's optimal payment and ordering decisions, a sensitivity analysis is explored. Figs. 7–11 examine the impact of β, M_1, M_2, I_c and I_e on the retailer's optimal payment fraction λ, optimal ordering cycle time T and annual total cost TC under both one-level and two-level trade credit policies. Fig. 12 study the impact of credit period N on the retailer's optimal decision under two-level trade credit.

It is seen from Fig. 7 that when β is small, the retailer will not pay in advance at time M_1(i.e., $\lambda = 0$). With a small discount rate, it is more profitable for the retailer to earn interest via received accumulated sales revenue than to pay in advance. When β increases to 0.86%, the retailer will pay a portion of the purchasing cost in advance at time M_1 and λ increases in β. It is clear that a larger discount rate will incentivize the retailer to pay early. When β keeps increasing to 1.18% and higher, the retailer will always make full payment at time M_1. This is because when the discount rate is large enough, the benefit of making early full payment will always exceed the sum of the interest earned and the possible financing costs incurred. However, the value of β is constrained by $(1-\beta)s_r > g_r$ and it cannot be increased forever. In contrast, the value of T changes slightly with β. The value of T roughly stabilizes at 52 (days). A stable T reflects that there is little change in the ordering size. It is intuitive that a longer T leads to an increase in the ordering size, whereby, incurring a higher loan cost. However, a shorter T represents a higher ordering frequency, which results in increased ordering cost. When β is small, the ordering cycle time is relatively long and constant. When β increases to 0.86%, the value of T drops a little as the retailer has the motivation to order more frequently. Although the retailer's ordering cost might be increased, the retailer will benefit more from paying early. When β keeps increasing to 1.18% and higher, T increases a little. This reflects the tradeoff of the ordering

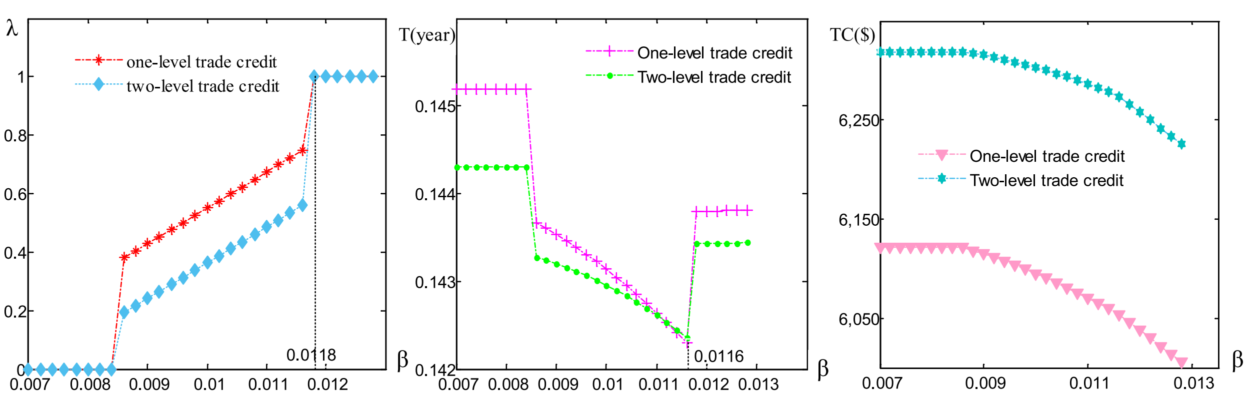


Fig. 7. Retailer's optimal decision under varied discount rate β.

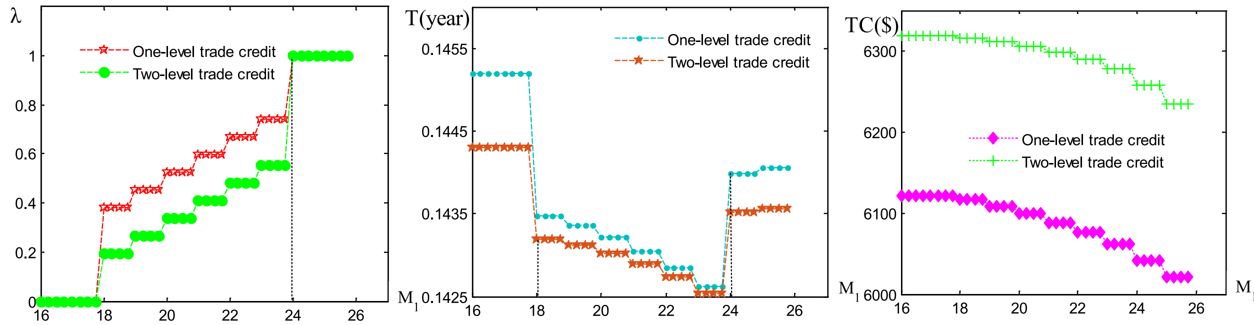


Fig. 8. Retailer's optimal decision under varied discount period M_1.

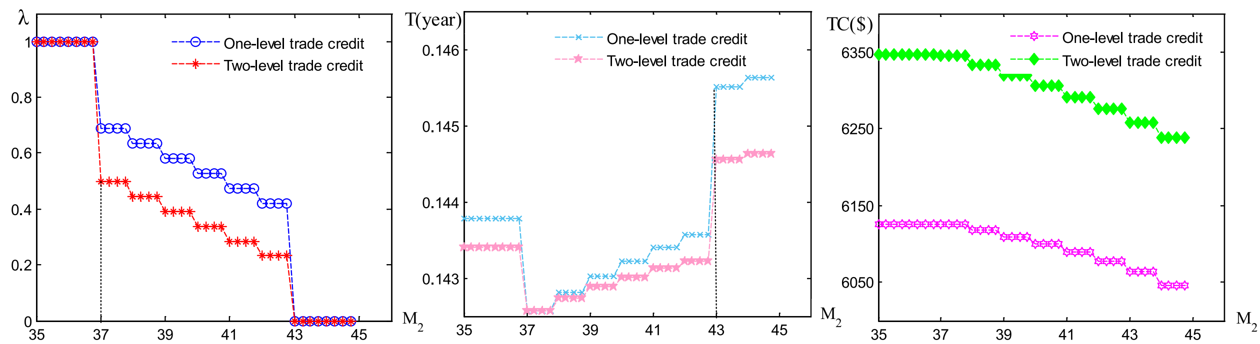


Fig. 9. Retailer's optimal decision under varied M_2.

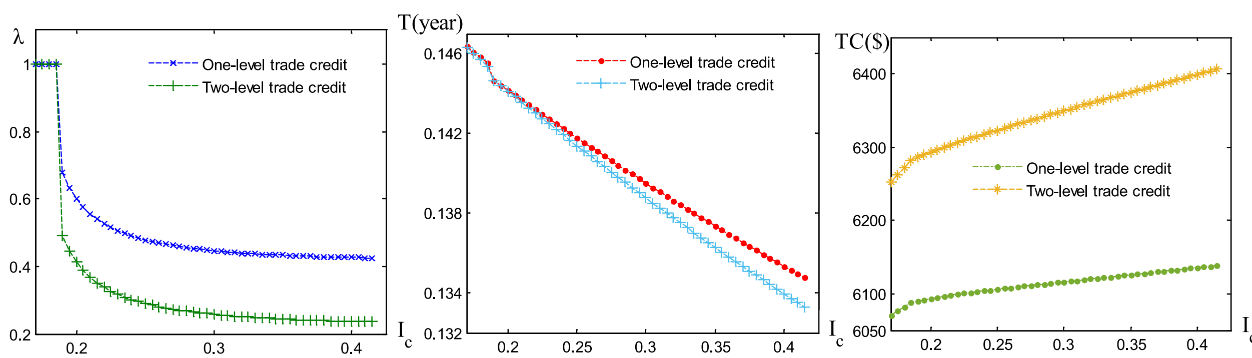


Fig. 10. Retailer's optimal decision under varied I_c.

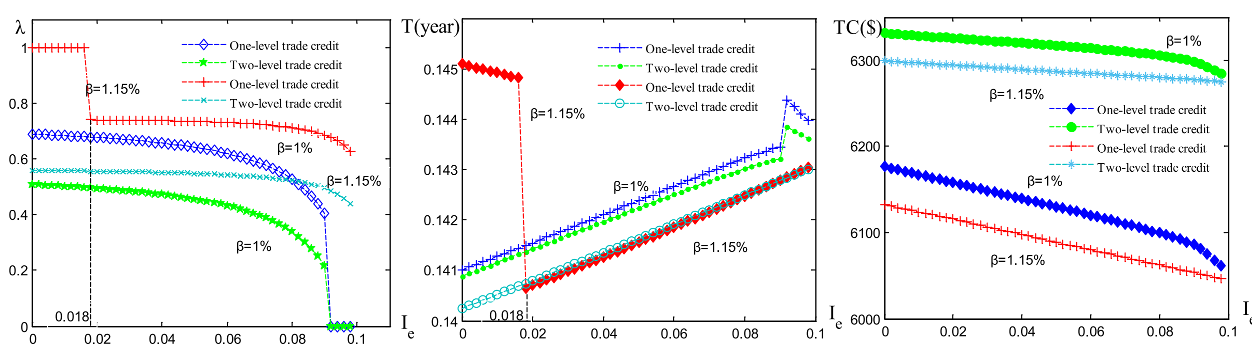


Fig. 11. Retailer's optimal decision under varied I_e.

cost and financing cost. The changing pattern of TC is in line with the common sense. That is, a larger discount rate, a lower annual total cost. The sensitivity analysis above clearly indicates that λ and TC are very sensitive to β even when β slightly changes. Therefore, the supplier can use a larger discount rate to induce the retailer to make early payment at time M_1 so as to accelerate his/her cash inflow.

Fig. 8 shows that the impact of M_1 on λ, T and TC is similar to that of β. There are also two inflection points of the retailer's payment fraction λ with the increase in M_1. When $M_1 < 18$, the discount benefit is smaller than both loan opportunity cost at M_1 and interest income earned during the time interval $[M_1, M_2]$. Hence, the retailer's payment fraction is 0. However, when $M_1 \in [18, 24]$, the discount benefit surpasses interest income during the interval $[M_1, M_2]$ but is still lower than loan opportunity cost at M_1. Therefore, the retailer will pay only a portion of

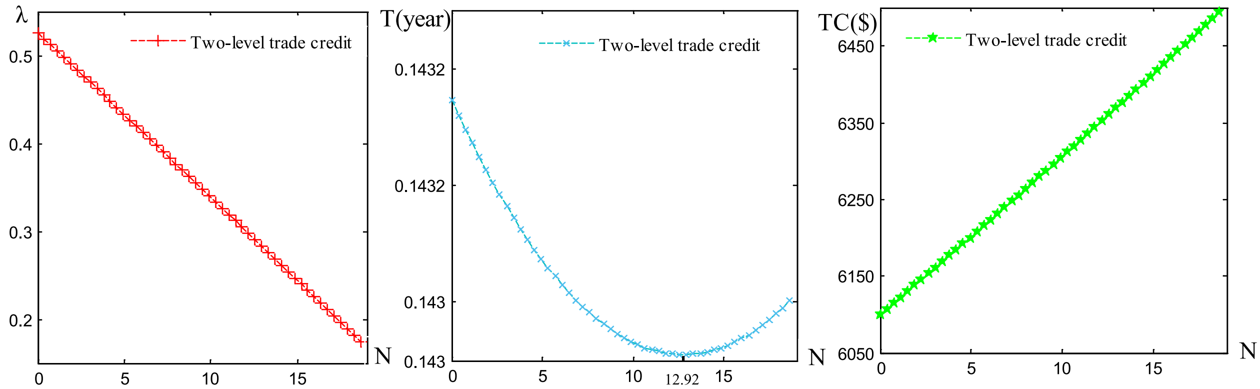


Fig. 12. Retailer's optimal decision under varied N.

the purchasing cost (i.e., $\lambda \in (0, 1)$). When $M_1 > 18$, the discount benefit exceeds both loan opportunity cost at M_1 and interest income during the time interval $[M_1, M_2]$. Thus, the retailer will pay off purchasing cost at M_1. Similarly, there is little change in the ordering cycle time T when M_1 changes. The changing pattern of TC is in line with the common sense as a prolonged credit period M_1 will increase the retailer's accumulated interest income. The sensitivity analysis on M_1 illustrates that the supplier can motivate the retailer to make full payment at time M_1 by increasing the discount period M_1; however, an increase in M_1 prolongs the time the supplier takes to retrieve full payment from the retailer.

Fig. 9 presents that the impact of M_2 on λ, T and TC is quite opposite to that of β and M_1. With a short credit period M_2(i.e., $M_2 < 37$), both the financing cost and interest income would be relatively low during the time interval $[M_1, M_2]$ while the discount benefit of paying early at time M_1 would be relatively high. Therefore, the retailer will pay off purchasing cost at time M_1. When $M_2 \in [37, 43]$, the interest income increases in M_2, this discourages the retailer from making full payment at M_1 via loan. To trade off, the retailer will pay a portion of the total purchasing cost. When $M_2 > 43$, no payment will be made at time M_1. Similarly, T changes very slightly within a narrow range, which equals to 0.002 year (i.e., less than a day). The changing pattern of the total cost is also quite intuitive since a prolonged credit period M_2 will increase the retailer's interest income. The sensitivity analysis illustrates that the retailer benefits more from a longer M_2. However, a longer credit period M_2 will defer the retailer from paying early at time M_1, which affects the supplier's cash inflow at time M_1. Therefore, the supplier has the motivation to reduce M_2 to accelerate capital circulation.

The impact of loan interest rate I_c on the retailer's optimal decisions is shown in Fig. 10. It's intuitive that a higher loan interest rate will increase the retailer's loan cost. Therefore, when I_c is small, the retailer will make full payment at M_1 to enjoy price discount. When I_c is increased to 0.19 and larger, the retailer's loan cost increases. Hence, the retailer will pay only a portion of the total purchasing cost. However, the retailer's payment fraction λ converges on 0.2, which implies that when loan cost is relatively high, the retailer has the motivation to use the discount benefit received at time M_1 to offset part of the financing cost. As to T, it drops very little (i.e., less than a day) in I_c, but the total cost TC increases in I_c.

As the retailer's optimal decisions on λ, T and TC are sensitive to β, the impact of I_e is examined under different discount rates. It is quite straightforward to see from Fig. 11 that, for both one-level and two-level trade credits, given I_e, λ increases in β. However, given β, λ decreases in I_e because the retailer is less motivated to pay early. On the contrary, it will be more beneficial for the retailer to accumulate sales revenue to earn interest. The variation of ordering cycle time T is small as well, which is less than a day. Obviously, a larger interest rate I_e leads to a lower total cost.

The sensitivity analysis above indicates that the retailer's optimal λ and T are determined by trading off the interest income, the financing cost and the discount benefit. These costs are interacted. It's seen from Figs. 7–11 that the retailer's optimal payment fraction λ at M_1 under one-level trade credit is always higher than its two-level trade credit counterpart. Moreover, the total cost TC under one-part trade credit policy is always lower than its two-level trade credit counterpart. This is because the retailer's payment return time is prolonged (by offering trade credit to his/her customer) under two-level trade credit policy. This results in less accumulated sales revenue at time M_1 and higher financing cost if the retailer loans to pay off the purchasing cost early.

To clearly understand the impact of the downstream trade credit (offered to the end customers), the retailer's optimal decisions under varied credit period N are shown in Fig. 12. It's obvious that a shorter credit period N will accelerate the collection of the sales revenue by the retailer. The payment fraction λ decreases in N(not linearly) because the retailer's payment return time is protracted. As to the ordering cycle time T, it reduces in N first, which reflects the retailer's intention to order more frequently to get more discount benefit. When N is increased to 12.92 days and longer, T increases. It indicates that the retailer has the intention to order less frequently since the increased ordering cost will exceed the discount benefit attained. It's easy to understand that TC increases in N because longer credit period slows down the retailer's cash inflow, leading to an increase in his/her total cost. A protracted credit period N reflects the retailer's intention to promote sales. When customer demand increases due to the offering of a downstream trade credit, the increased sales revenue is expected to cover the extra ordering, financing and opportunity costs incurred. Since constant customer demand is assumed in this sensitivity analysis, it leads to the result that TC increases in N.

6.3. Managerial insights

An extensive numerical study is carried out in Sections 6.1 and 6.2 to understand the impact of adopting flexible two-part trade credit, offering two-level trade credit and some key parameters on the retailer's optimal decisions and annual total cost. Some managerial insights are drawn as follows:

(1) The retailer's annual total cost can be reduced for being flexible in determining the early payment fraction at time M_1. Therefore, flexible payment strategy is always preferred by the retailer especially when the retailer is capital restricted.
(2) Adopting two-level trade credit contract will increase the retailer's annual total cost because offering downstream trade credit to the customer prolongs the retailer's payment return time, resulting in a larger opportunity cost. Moreover, the retailer is less motivated to pay early at time M_1 under two-level trade credit (i.e., smaller λ) when compared with that under one-level trade credit. Therefore,

adopting two-level trade credit will slow down the supplier's cash inflow.

(3) Key trade credit terms impact the retailer's optimal payment decisions. A larger discount rate β and a longer credit period M_1 will motivate the retailer to pay early to enjoy price discount; however, a longer credit period M_2 and N will entice the retailer to postpone payment up to the maturity of the two-part trade credit contract. Ordering cycle time T is insensitive to key trade credit terms with constant customer demand.

7. Conclusions and future research

In this paper, a two-level trade credit policy incorporated with flexible two-part trade credit contract is explored. Specifically, the retailer in the middle tier of a supply chain is offered with a flexible two-part trade credit by the upper stream supplier and allows the downstream customer to defer payment for a fixed period of time, which is a one-part trade credit. An inventory model is formulated with the objective to minimize the retailer's annual total cost. Fourteen subcases, depending on the values of T, N, M_1 and M_2, are discussed. Closed-form solutions of the retailer's optimal payment fraction and ordering cycle time are obtained in two steps. First, given payment fraction λ, the retailer's optimal ordering cycle time T^* is discussed (as shown in Proposition 3). Then, with the obtained T^*, the retailer's optimal payment fraction λ^* is developed (as shown in Proposition 4). An extensive numerical analysis is conducted to study the impact of adopting flexible two-part trade credit, the impact of offering two-level trade credit and the impact of various key parameters on the retailer's optimal decisions and annual total cost. The numerical results are analyzed and some managerial insights are drawn.

This research can be extended to consider credit period N-dependent customer demand and to consider flexible two-part trade credit offered by the retailer to his/her customer. In addition, an integrated model can also be explored to optimize the joint total cost of the supply chain or on how to design trade credit policy to boost the customer's demand in future study.

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Appendix A

For Eqs. (1)–(5), given the value of λ, the first-order derivative of $TC_{ri}(T,\lambda)(i=1,2...,5)$ with respect to T can be derived as follows:

$$dTC_{r1}(T,\lambda)/dT=(h_r+g_r)D/2-A_r/T^2 \tag{A.1}$$

$$\begin{aligned}dTC_{r2}(T,\lambda)/dT= & -A_r/T^2+[h_r+(1-\lambda^2)g_r]D/2+g_rD(M_1-N)^2/(2T^2)\\ & +s_r(1-\beta)D[\lambda^2T^2-(M_1-N)^2]/(2T^2)\end{aligned} \tag{A.2}$$

$$dTC_{r3}(T,\lambda)/dT=-A_r/T^2+(h_r+s_r)D/2+(g_r-s_r)D(M_2-N)^2/(2T^2) \tag{A.3}$$

$$\begin{aligned}dTC_{r4}(T,\lambda)/dT= & -A_r/T^2+(h_r+s_r)D/2+g_rD(M_1-N)^2/(2T^2)+s_rD(1-\beta)\lambda^2/2\\ & -s_rD(1-\beta)(M_1-N)^2/(2T^2)-s_rD(M_2-N)^2/(2T^2)\\ & +g_rD(M_2-\lambda T-N)(M_2+\lambda T-N)/(2T^2)\end{aligned} \tag{A.4}$$

$$\begin{aligned}dTC_{r5}(T,\lambda)/dT= & -A_r/T^2+h_rD/2+g_rD(M_1-N)^2/(2T^2)+s_rD(1-\beta)\lambda^2/2\\ & -s_rD(1-\beta)(M_1-N)^2/(2T^2)+s_rD(1-\lambda)-s_rD(1-\lambda)^2/2\end{aligned} \tag{A.5}$$

Based on (A.1)–(A.5), the second-order derivative of $TC_{ri}(T,\lambda)(i=1,2...,5)$ with respect to T can be further derived below:

$$dTC_{r1}{}^2(T,\lambda)/dT^2=2A_r/T^3>0 \tag{A.6}$$

$$dTC_{r2}{}^2(T,\lambda)/dT^2=2A_r/T^3+[(1-\beta)s_r-g_r]D(M_1-N)^2/T^3>0 \tag{A.7}$$

$$dTC_{r3}{}^2(T,\lambda)/dT^2=2A_r/T^3+(s_r-g_r)D(M_2-N)^2/T^3>0 \tag{A.8}$$

$$\begin{aligned}dTC_{r4}{}^2(T,\lambda)/dT^2= & \ 2A_r/T^3+[(1-\beta)s_r-g_r]D(M_1-N)^2/T^3\\ & +(s_r-g_r)D(M_2-N)^2/T^3>0\end{aligned} \tag{A.9}$$

$$dTC_{r5}{}^2(T,\lambda)/dT^2=2A_r/T^3+[(1-\beta)s_r-g_r]D(M_1-N)^2/2>0 \tag{A.10}$$

As the second-order derivative of $TC_{ri}(T,\lambda)(i=1,2...,5)$ with respect to T is positive, it implies that $TC_{ri}(T,\lambda)(i=1,2...,5)$ is convex over $(0,+\infty)$ given λ.

Appendix B

For the nonflexible trade credit, we consider $\lambda=0$ and $\lambda=1$, respectively.

(1) When $\lambda=0$, the annual total cost function $TC(T)$ can be expressed as,

$$TC(T)=\begin{cases}TC_1(T) & T+N\leqslant M_2\\ TC_2(T) & T+N>M_2\end{cases} \tag{B.1}$$

where

$$TC_1(T)=A_r/T+h_rDT/2-g_rD(M_2-N-T/2) \tag{B.2}$$

$$TC_2(T)=A_r/T+h_rDT/2-g_rD(M_2-N)^2/(2T)+s_rD(T+N-M_2)^2/(2T) \tag{B.3}$$

Take the first-order derivative of $TC(T)$ with respect to T,

$$TC'(T) = \begin{cases} TC'_1(T) & T + N \leqslant M_2 \\ TC'_2(T) & T + N > M_2 \end{cases} \tag{B.4}$$

$$TC_1'(T) = -A_r/T^2 + (h_r + g_r)D/2 \tag{B.5}$$

$$TC_2'(T) = -A_r/T^2 + (g_r - s_r)D(M_2 - N)^2/(2T^2) + (h_r + s_r)D/2 \tag{B.6}$$

Obviously $TC_1'(M_2 - N) = TC_2'(M_2 - N)$, $TC'(T)$ is continuous with respect to T and monotonously increase with T. So, let $TC'(T) = 0$, the optimal ordering cycle time can be expressed as follows:

$$T^* = \begin{cases} \sqrt{\frac{2A_r}{(h_r + g_r)D}} & TC'_1(M_2 - N) > 0 \\ \sqrt{\frac{2A_r + D(s_r - g_r)(M_2 - N)^2}{(h_r + s_r)D}} & TC'_1(M_2 - N) \leqslant 0 \end{cases} \tag{B.7}$$

(2) When $\lambda = 1$, the annual total cost function $TC(T)$ can be expressed as,

$$TC(T) = \begin{cases} TC_1(T) & T + N \leqslant M_1 \\ TC_2(T) & T + N > M_1 \end{cases} \tag{B.8}$$

$$TC_1(T) = A_r/T + h_r DT/2 - \beta c_r D - g_r D(M_1 - N - T/2) \tag{B.9}$$

$$TC_2(T) = A_r/T + h_r DT/2 - \beta c_r D - g_r D(M_1 - N)^2/(2T) + (1 - \beta)s_r D(T + N - M_1)^2/(2T) \tag{B.10}$$

Similarly, take the first-order derivative of $TC(T)$ with respect to T:

$$TC'(T) = \begin{cases} TC'_1(T) & T + N \leqslant M_1 \\ TC'_2(T) & T + N > M_1 \end{cases} \tag{B.11}$$

$$TC_1'(T) = -A_r/T^2 + (h_r + g_r)D/2 \tag{B.12}$$

$$TC_2'(T) = -A_r/T^2 + (g_r - (1 - \beta)s_r)D(M_1 - N)^2/(2T^2) + (h_r + (1 - \beta)s_r)D/2 \tag{B.13}$$

$TC'(T)$ is also continuous with respect to T and monotonously increase with T. So, let $TC'(T) = 0$, the optimal ordering cycle time can be expressed as follows:

$$T^* = \begin{cases} \sqrt{\frac{2A_r}{(h_r + g_r)D}} & TC'_1(M_1 - N) > 0 \\ \sqrt{\frac{2A_r + D((1-\beta)s_r - g_r)(M_1 - N)^2}{(h_r + (1-\beta)s_r)D}} & TC'_1(M_1 - N) \leqslant 0 \end{cases} \tag{B.14}$$

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