CENTER FOR
ECONOMIC
BEHAVIOR &
INEQUALITY

Consumption-Saving

NumEcon

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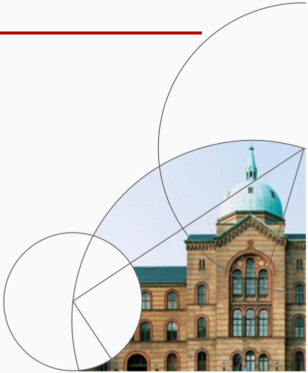
2020



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Introduction

Introduction

- **Why are consumption-saving models important?**
  1. Important topic in itself (70 percent of GDP)
  2. Central aspect of many other decisions
     a) Labor supply and retirement choices
     b) Portfolio choices
     c) Housing and location choices
- **Dynamic programming** essential for recent advances
  1. Idiosyncratic and aggregate uncertainty
  2. Ex ante and ex post heterogeneity
  3. Internal and external optimization frictions (bounded rationality, adjustment costs etc.)
- **Main paper:** Carroll (2020, QE), *Theoretical foundations of buffer stock saving*
- **Additional references:** See end of slides

Plan

PIH

Permanent Income Hypothesis (PIH)

- **Household problem**

$$
\begin{aligned}
V_0(M_0, P_0) &= \max_{\{C_t\}_{t=0}^{T}} \sum_{t=0}^{T} \beta^t \frac{C_t^{1-\rho}}{1-\rho}, \;\; \beta < 1, \; \rho \geq 1 \\
& \text{s.t.} \\
A_t &= M_t - C_t \\
B_{t+1} &= R \cdot A_t, \;\; R > 0 \\
M_{t+1} &= B_{t+1} + P_{t+1} \\
P_{t+1} &= G \cdot P_t, \;\; G > 0 \\
A_T &\geq 0
\end{aligned}
$$

- **Well-defined analytical solution for $T \to \infty$ if**
  1. Return impatience (RI): $(\beta R)^{1/\rho}/R < 1$
  2. Finite human wealth (FHW): $G/R < 1$
- **What do you think is missing?**

The Intertemporal Budget Constraint (IBC)

- **Substitution** implies

$$
\begin{aligned}
A_T &= M_T - C_T = (RA_{T-1} + P_T) - C_T \\
&= R(M_{T-1} - C_{T-1}) + P_T - C_T \\
&= R^2 A_{T-2} + RP_{T-1} - RC_{T-1} + P_T - C_T \\
&= R^{T+1} A_{-1} + \sum_{t=0}^{T} R^{T-t}(P_t - C_t)
\end{aligned}
$$

- Use **terminal condition** (why equality?)

$$
\begin{aligned}
A_T = 0 \Leftrightarrow R^{-T} A_T = 0 \Leftrightarrow RA_{-1} + \sum_{t=0}^{T} R^{-t}(P_t - C_t) &= 0 \Leftrightarrow \\
B_0 + H_0 &= \sum_{t=0}^{T} R^{-t} C_t
\end{aligned}
$$

where $H_0 \equiv \sum_{t=0}^{T} (G/R)^t P_0 = \frac{1-(G/R)^{T+1}}{1-G/R} P_0$

Static problem → Lagrangian

$$\mathcal{L} = \sum_{t=0}^{T} \beta^t \frac{C_t^{1-\rho}}{1-\rho} + \lambda \left[\sum_{t=0}^{T} R^{-t} C_t - (B_0 + H_0) \right]$$

- **First order conditions**

$$\forall t:\ 0 = \beta^t C_t^{-\rho} - \lambda R^{-t}$$

- ***Short-run*** **Euler** equation**:** $\frac{C_{t+1}}{C_t} = (\beta R)^{1/\rho}$
- ***Long-run*** **Euler** equation**:** $\frac{C_t}{C_0} = (\beta R)^{t/\rho}$

Consumption function

- Insert **Euler** into **IBC**

$$\begin{aligned}\sum_{t=0}^{T} R^{-t}(\beta R)^{t/\rho} C_0 &= B_0 + H_0 \Leftrightarrow \\ C_0 \sum_{t=0}^{T}((\beta R)^{1/\rho}/R)^t &= B_0 + H_0\end{aligned}$$

- **Solve** for C_0

$$C_0 = \frac{1-(\beta R)^{1/\rho}/R}{1-((\beta R)^{1/\rho}/R)^{T+1}}(B_0 + H_0)$$

  - **MPC:** $\frac{\partial C_0}{\partial B_0} \approx 1 - [(\beta R)^{1/\rho}/R] \approx 1 - R^{-1} \approx r$, where $R = 1 + r$
  - **MPCP:** $\frac{\partial C_0}{\partial P_0} \approx 1 - [(\beta R)^{1/\rho}/R]\frac{\partial H_0}{\partial P_0} \approx \frac{1-1/R}{1-G/R} \approx 1$

Side-note: Value function

- **Analytical expression** for the value function

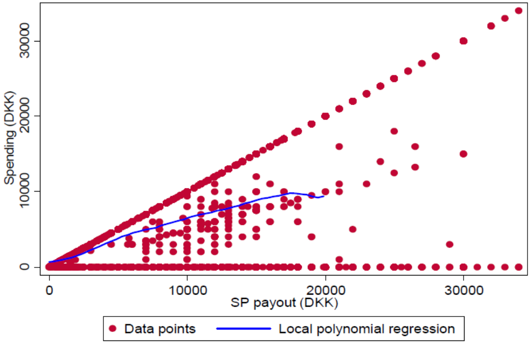
$$
\begin{aligned}
V_0(M_0, P_0) &= \sum_{t=0}^{T} \beta^t u((\beta R)^{t/\rho} C_0) \\
&= \sum_{t=0}^{T} \beta^t (\beta R)^{(1-\rho)t/\rho} \frac{C_0^{1-\rho}}{1-\rho} \\
&= \sum_{t=0}^{T} ((\beta R)^{1/\rho}/R)^t \frac{C_0^{1-\rho}}{1-\rho} \\
&= \frac{1-((\beta R)^{1/\rho}/R)^{T+1}}{1-(\beta R)^{1/\rho}/R} \frac{C_0^{1-\rho}}{1-\rho}
\end{aligned}
$$

Empirical evidence

- **Pro**
    1. Micro-founded consumption-saving
        - Theoretically appealing (humans are intentional)
        - Empirically appealing (testable implications on micro-data)
    2. Larger responses to permanent than to transitory shocks
    3. Consumption smoothing - save for retirement (future low income)
- **Con**
    1. Households seems to have a high MPC in the range 0.20-0.40
        - Survey studies
        - Tax rebates studies
        - Lottery studies
        - ARM payments studies
    2. Consumption responds to anticipated income changes
    3. Households with more volatile income have larger savings
    4. Consumption tracks income over the life-cycle
    5. (Households are only boundedly rational)

High MPC: Danish SP payout

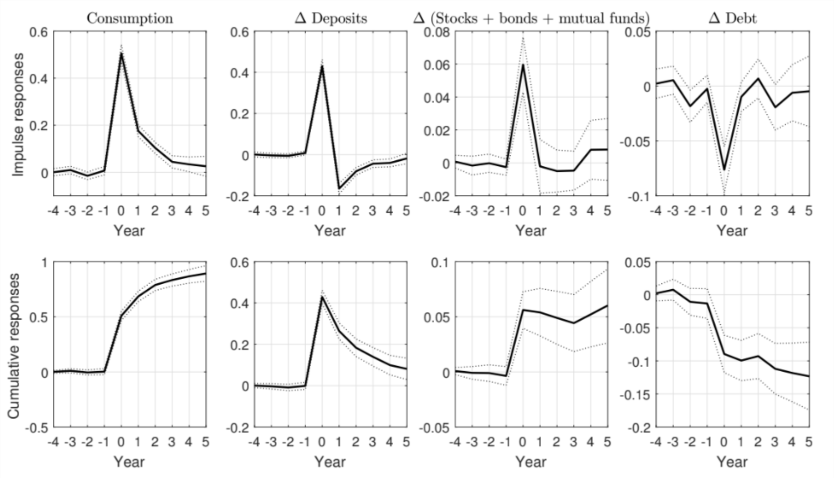
Figure 4: Spending and the size of the SP payout



NOTE: 5055 observations.

Source: Kreiner, Lassen og Leth-Petersen (AEJ:Pol, 2019)

High MPC: Norwegian lottery winners



Source: Fagereng, Holm, Natvik (WP, 2019)

Buffer-stock model

Buffer-stock model (Deaton-Carroll)

+ borrowing constraints

+ income uncertainty

$\Rightarrow$

$$
\begin{aligned}
V_0(M_0, P_0) &= \max_{\{C_t\}_{t=0}^{T}} \mathbb{E}_0 \sum_{t=0}^{T} \beta^t \frac{C_t^{1-\rho}}{1-\rho} \\
&\text{s.t.} \\
A_t &= M_t - C_t \\
M_{t+1} &= RA_t + Y_{t+1} \\
Y_{t+1} &= \xi_{t+1} P_{t+1} \\
\xi_{t+1} &= \begin{cases} \mu & \text{with prob. } \pi \\ (\epsilon_{t+1} - \pi\mu)/(1-\pi) & \text{else} \end{cases} \\
&\quad \epsilon_t \sim \exp \mathcal{N}(-0.5\sigma_\xi^2, \sigma_\xi^2) \\
P_{t+1} &= GP_t \psi_{t+1}, \;\; \psi_t \sim \exp \mathcal{N}(-0.5\sigma_\psi^2, \sigma_\psi^2) \\
A_t &\geq -\lambda P_t \\
A_T &\geq 0
\end{aligned}
$$

Note: Later analytical results hold only for $\mu = 0$ and $\pi > 0$

How to solve the model?

- **Borrowing constraints** $\rightarrow$ inequalities $\rightarrow$ high-dimensional **Kuhn-Tucker problem**
- **Uncertainty** $\rightarrow$ fully dynamic problem $\rightarrow$ no simple Lagrangian
- **No analytical solution with CRRA preferences**
  - Quadratic or CARA utility, which give some analytical results, have implausible properties

$$\text{CRRA: } u(c) = \frac{c^{1-\rho}}{1-\rho} \rightarrow \text{RRA} = \rho$$

$$\text{Qudratic: } u(c) = ac - \frac{b}{2}c^2 \rightarrow \text{ RRA} = \frac{b}{a-bc}c$$

$$\text{CARA: } u(c) = \frac{1}{\alpha}e^{-\alpha c} \rightarrow \text{ RRA} = \alpha c$$

where RRA = relative risk aversion = $\frac{-u''(c)}{u'(c)}c$

- **Solution:** Bellman equation $\rightarrow$ numerical dynamic programming

Bellman equation

$$
\begin{aligned}
V_t(M_t, P_t) &= \max_{C_t} \frac{C_t^{1-\rho}}{1-\rho} + \beta \mathbb{E}_t \left[V_{t+1}(M_{t+1}, P_{t+1}) \right] \\
&\text{s.t.} \\
A_t &= M_t - C_t \\
M_{t+1} &= RA_t + Y_{t+1} \\
Y_{t+1} &= \xi_{t+1} P_{t+1} \\
\xi_{t+1} &= \begin{cases} \mu & \text{with prob. } \pi \\ (\epsilon_{t+1} - \pi\mu)/(1-\pi) & \text{else} \end{cases} \\
P_{t+1} &= G P_t \psi_{t+1} \\
A_t &\geq -\lambda P_t \\
A_T &\geq 0
\end{aligned}
$$

Normalization I

- **Defining** $c_t \equiv C_t/P_t$, $m_t \equiv M_t/P_t$ etc. implies

$$
\begin{aligned}
A_t = M_t - C_t \quad &\Leftrightarrow \quad A_t/P_t = M_t/P_t - C_t/P_t \\
&\Leftrightarrow \quad a_t = m_t - c_t
\end{aligned}
$$

$$
\begin{aligned}
M_{t+1} = RA_t + Y_{t+1} \quad &\Leftrightarrow \quad M_{t+1}/P_{t+1} = RA_t/P_{t+1} + Y_{t+1}/P_{t+1} \\
&\Leftrightarrow \quad m_{t+1} = Ra_t P_t/P_{t+1} + \xi_{t+1} \\
&\Leftrightarrow \quad m_{t+1} = \frac{R}{G\psi_{t+1}} a_t + \xi_{t+1}
\end{aligned}
$$

The **adjustment factor** $\frac{1}{G\psi_{t+1}}$ is due to changes in permanent income

Normalization II

- **Defining** $v_t(m_t) = V_t(M_t, P_t)/P_t^{1-\rho}$ finally implies

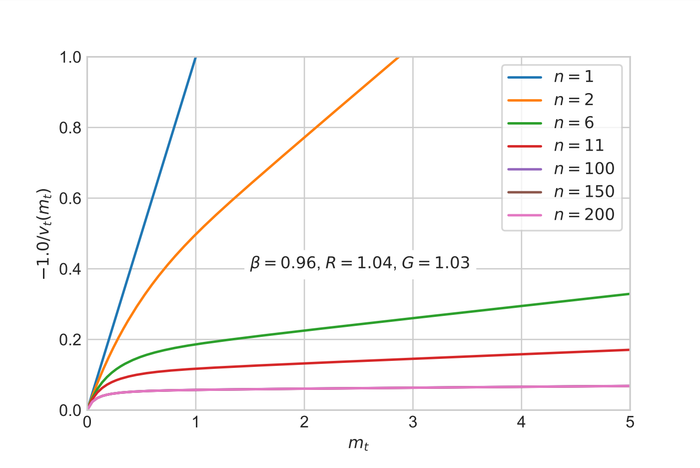
$$
\begin{aligned}
V_t(M_t, P_t) &= \max_{C_t} \frac{C_t^{1-\rho}}{1-\rho} + \beta \mathbb{E}_t \left[V_{t+1}(M_{t+1}, P_{t+1}) \right] \\
&= \max_{c_t} \frac{(c_t P_t)^{1-\rho}}{1-\rho} + \beta \mathbb{E}_t \left[V_{t+1}(M_{t+1}, P_{t+1}) \right] \Leftrightarrow \\
V_t(M_t, P_t)/P_t^{1-\rho} &= \max_{c_t} \frac{(c_t P_t)^{1-\rho}/P_t^{1-\rho}}{1-\rho} + \beta \mathbb{E}_t \left[V_{t+1}(M_{t+1}, P_{t+1})/P_t^{1-\rho} \right] \Leftrightarrow \\
v_t(m_t) &= \max_{c_t} \frac{c_t^{1-\rho}}{1-\rho} + \beta \mathbb{E}_t \left[V_{t+1}(M_{t+1}, P_{t+1})/P_{t+1}^{1-\rho} \cdot P_{t+1}^{1-\rho}/P_t^{1-\rho} \right] \\
&= \max_{c_t} \frac{c_t^{1-\rho}}{1-\rho} + \beta \mathbb{E}_t \left[(G\psi_{t+1})^{1-\rho} v_{t+1}(m_{t+1}) \right]
\end{aligned}
$$

Bellman equation in ratio form

$$
\begin{aligned}
v_t(m_t) &= \max_{c_t} \frac{c_t^{1-\rho}}{1-\rho} + \beta \mathbb{E}_t \left[(G\psi_{t+1})^{1-\rho} v_{t+1}(m_{t+1}) \right] \\
&\text{s.t.} \\
a_t &= m_t - c_t \\
m_{t+1} &= \frac{1}{G\psi_{t+1}} R a_t + \xi_{t+1} \\
\xi_{t+1} &= \begin{cases} \mu & \text{with prob. } \pi \\ (\epsilon_{t+1} - \pi\mu)/(1-\pi) & \text{else} \end{cases} \\
a_t &\geq -\lambda \\
a_T &\geq 0
\end{aligned}
$$

- **Benefit:** Dimensionality of state space reduced
  Can this always be done?
- Easy to solve by **VFI**

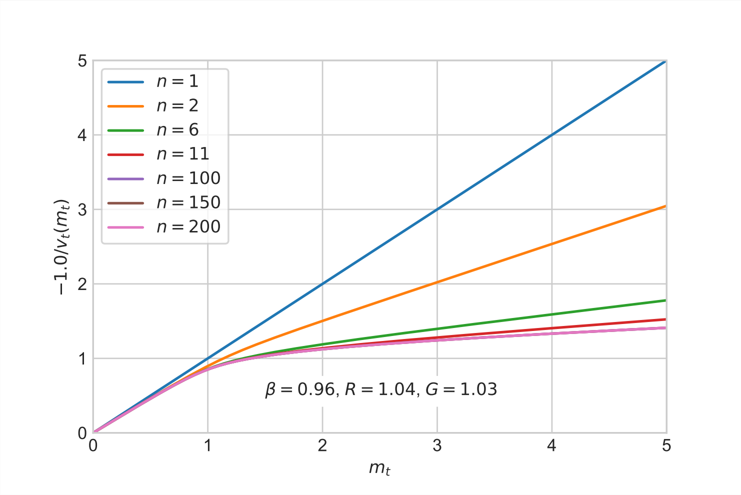
$T \to \infty$; Convergence of $-1.0/v_t(m_t) \to -1.0/v^\star(m_t)$



Other parameters: $\rho = 2$, $\pi = 0.005$, $\mu = 0.0$, $\sigma_\psi = \sigma_\xi = 0.10$

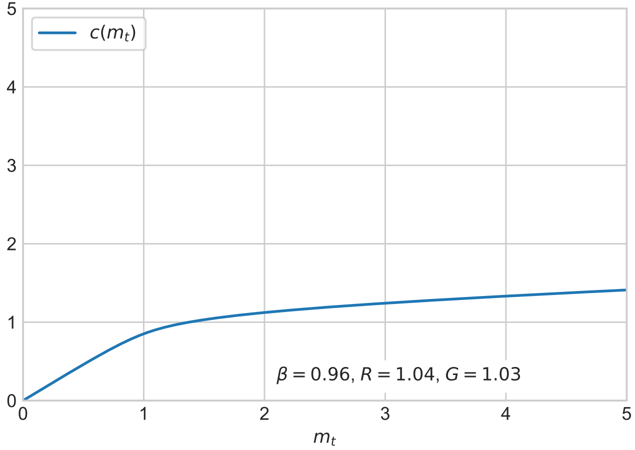
Note: $-1.0/v_t(m_t)$ is a numerically more stable object than $v_t(m_t)$

$T \to \infty$: Convergence of $c_t(m_t) \to c^\star(m_t)$

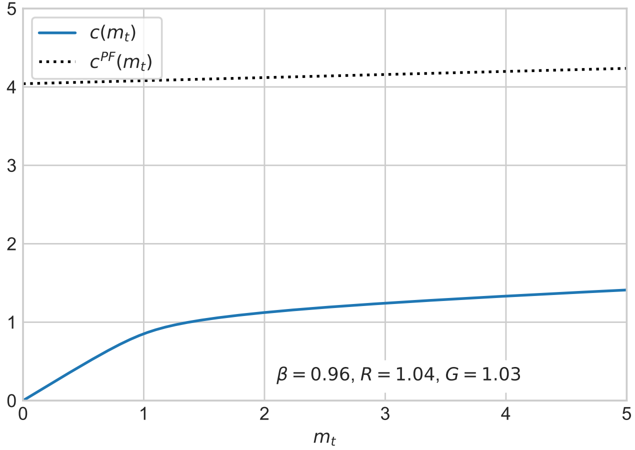


- What is the MPC?

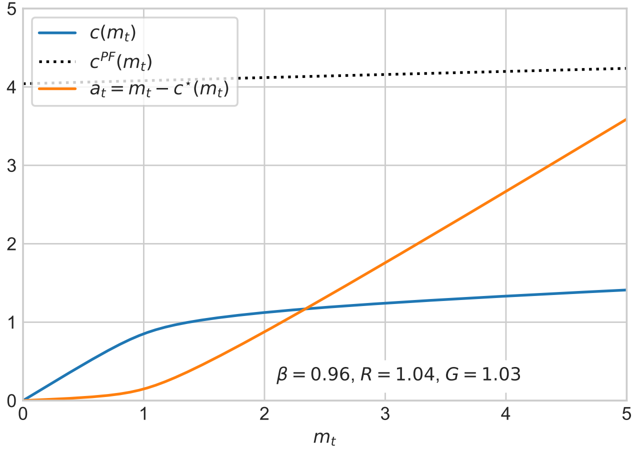
$T \to \infty$: The buffer-stock target



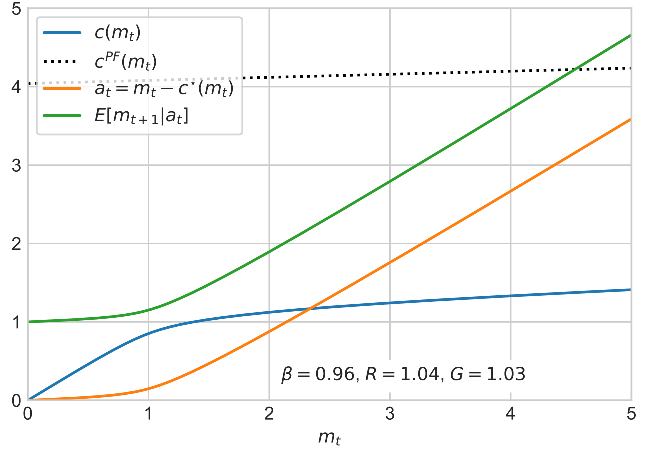
$T \to \infty$: The buffer-stock target



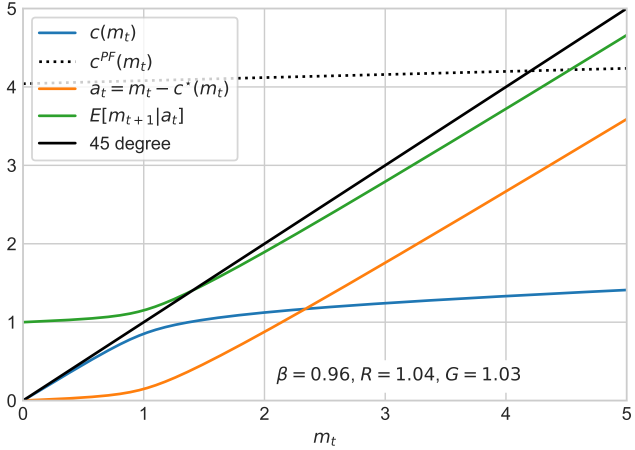
$T \to \infty$: The buffer-stock target



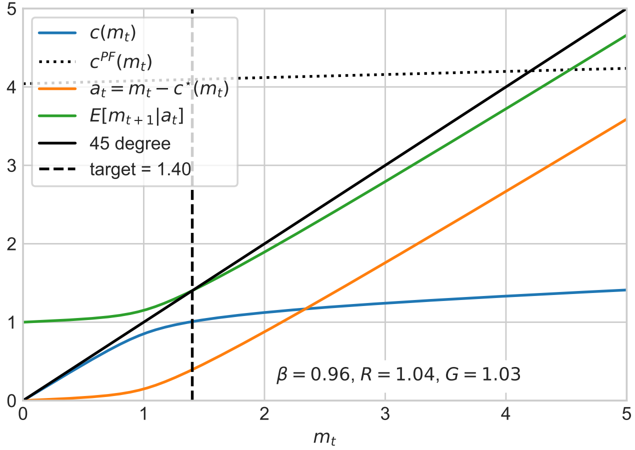
$T \to \infty$: The buffer-stock target



$T \to \infty$: The buffer-stock target



$T \to \infty$: The buffer-stock target



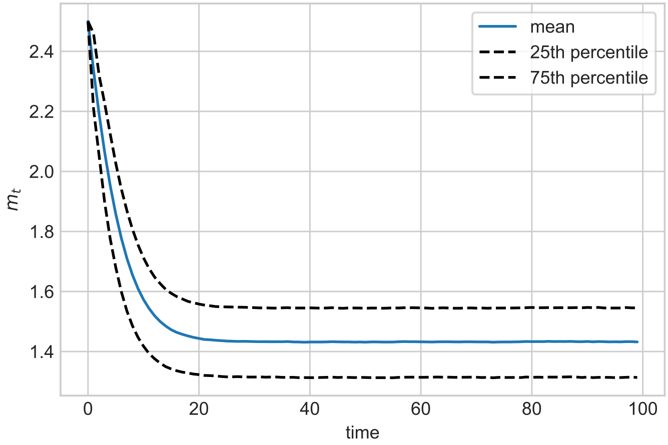
Simulation for $t \in \{0, 1, \ldots, T-1\}$

1. Choose m_0 and set $t = 0$
2. Calculate $c_t = c^\star(m_t)$
3. Calculate $a_t = m_t - c_t$
4. Draw (pseudo-)random numbers

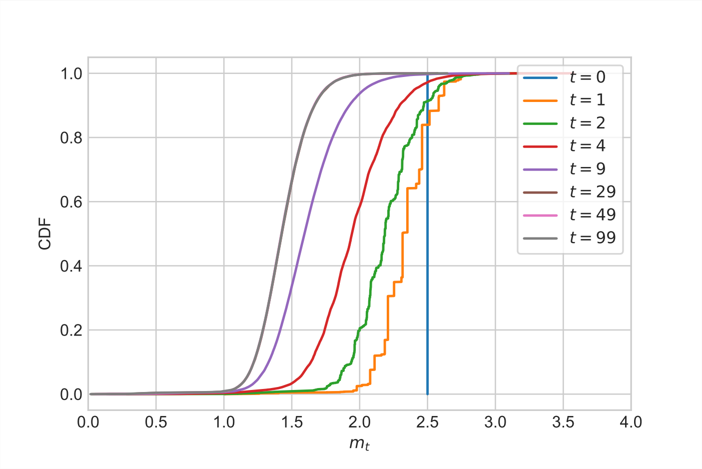
$$\begin{aligned}
\epsilon_{t+1} &\sim \exp \mathcal{N}(-0.5\sigma_\xi^2, \sigma_\xi^2) \\
\psi_{t+1} &\sim \exp \mathcal{N}(-0.5\sigma_\psi^2, \sigma_\psi^2) \\
\eta_{t+1} &\sim \mathcal{U}(0, 1)
\end{aligned}$$

5. Calculate $\xi_{t+1} = \begin{cases} \mu & \text{if } \eta_{t+1} < \pi \\ (\epsilon_{t+1} - \pi\mu)/(1-\pi) & \text{else} \end{cases}$
6. Calculate $m_{t+1} = \frac{R}{G\psi_{t+1}} a_t + \xi_{t+1}$
7. Set $t = t + 1$
8. Stop if $t \geq T$ else go to step 2

Simulation: Avg. cash-on-hand



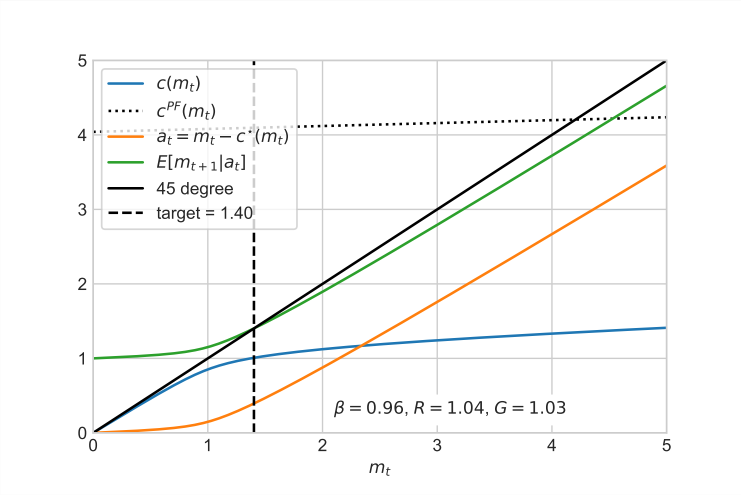
Simulation: Distribution of cash-on-hand



- **Proof of convergence:** Szeidl (2006)

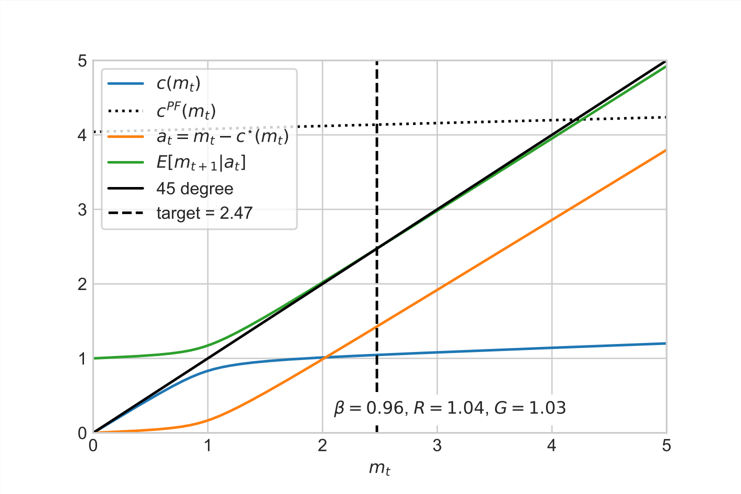
Some details

$\sigma_\psi = 0.10$



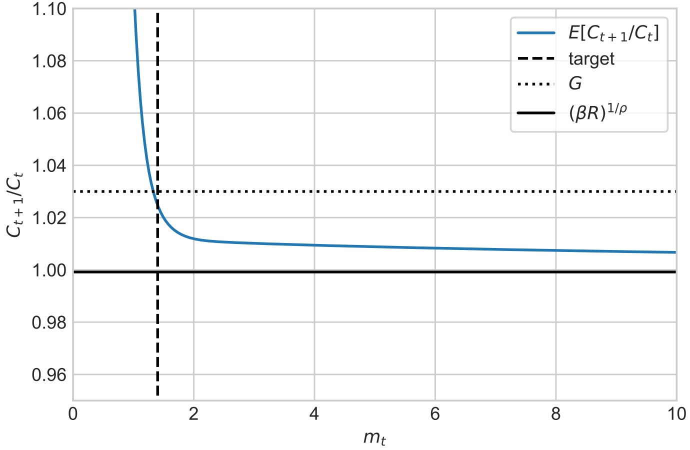
Target with standard risk: 1.40

$\sigma_\psi = 0.15$



Target with high risk: 2.47

Consumption growth I



Consumption growth II

- Remember **Euler-equation**

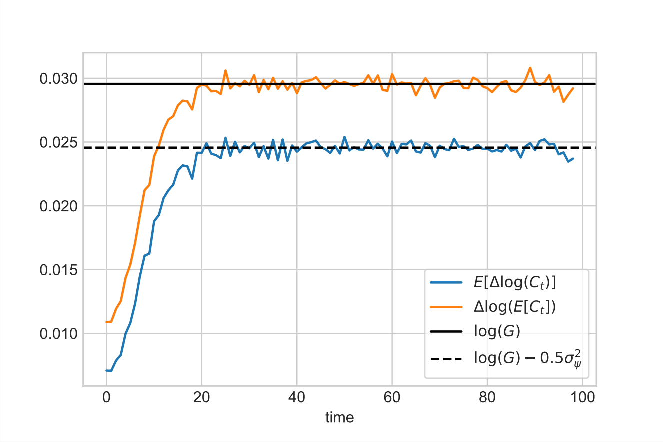
$$C_t^{-\rho} = \beta R \mathbb{E}_t \left[C_{t+1}^{-\rho} \right] \text{ if no uncertainty} \Rightarrow C_{t+1}/C_t = (\beta R)^{1/\rho}$$

- **Results**
  1. C_{t+1}/C_t is declining in m_t
  2. $\lim_{m_t \to \infty} C_{t+1}/C_t = (\beta R)^{1/\rho} = \text{RI}$
  3. $\lim_{m_t \to 0} C_{t+1}/C_t = \infty$
  4. $C_{t+1}/C_t < G$ at buffer-stock target
- **Intuition** for $C_{t+1}/C_t > (\beta R)^{1/\rho}$
  1. Uncertainty $\Rightarrow$ expected marginal utility $\uparrow$ [$C_{t+1}^{-\rho}$ is convex function]
  2. Consumer must be lowered today, $C_t \downarrow$
  3. Consumption growth will increase, $C_{t+1}/C_t \uparrow$

Further: *The above arguments are stronger for lower cash-on-hand relative to permanent income*

Consumption growth III

1. Growth of average consumption $= G$
2. Average consumption growth $= G - 0.5\sigma_\psi^2$

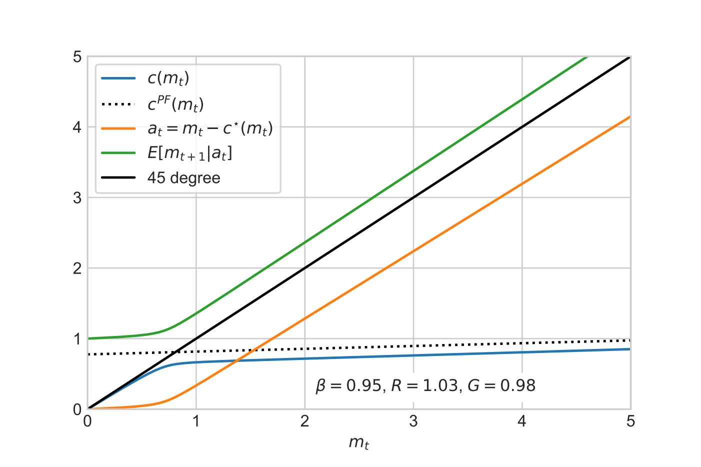


Always a buffer-stock target? I

1. **Utility impatience** (UI): $\beta < 1$
2. **Return impatience** (RI): $(\beta R)^{1/\rho}/R < 1$
3. **Weak return impatience** (WRI): $\pi^{1/\rho}(\beta R)^{1/\rho}/R < 1$
4. **Growth impatience** (GI): $(\beta R)^{1/\rho}\mathbb{E}_t[\psi_{t+1}^{-1}]/G < 1$
5. **Absolute impatience** (AI): $(\beta R)^{1/\rho} < 1$
6. **Finite value of autarky** (FVA): $\beta\mathbb{E}_t[(G\psi_{t+1})^{1-\rho}] < 1$

Always a buffer-stock target? II

- **GI ensures buffer-stock target**
- If not *GI* then inifinite accumulation is possible like:



Existence of solution

- **Existence of solution:** WRI + FVA
  - **Proof:** Use *Boyds weighted contraction mapping theorem*
  - **Standard assumptions:** FHW, RI, GI
- The **consumption function** is twice continuously differentiable**,** **increasing** and **concave**

The borrowing constraint

- Assume **perfect foresight** ($\sigma_\psi = \sigma_\epsilon = \pi = 0$), but **no borrowing**, $\lambda = 0$.
- **Solution:** RI + FHW is still *sufficient* (with $\lambda = \infty$ they are *necessary*)
- **Standard solutions:** RI + FHW
  1. **GI** $\Rightarrow$ *constraint will eventually be binding*

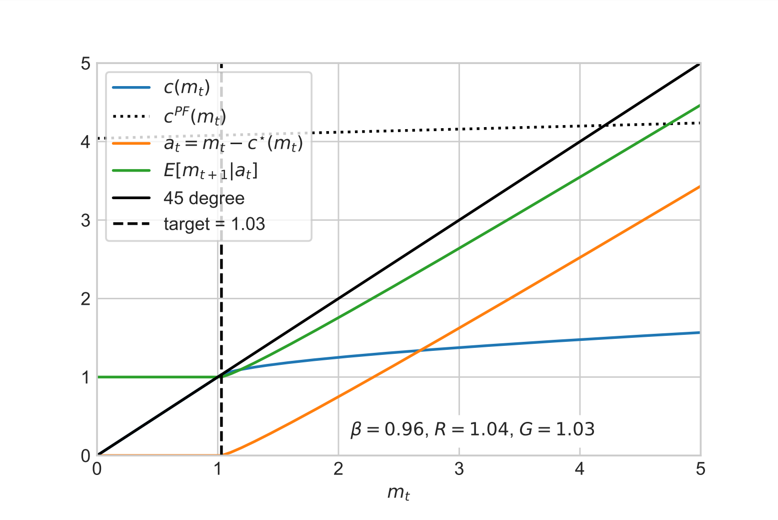
     $$c^\star(m_t) \text{ converge to } c^{PF}(m_t) \text{ from below as } m_t \to \infty$$

  2. **Not GI** $\Rightarrow$ *constraint is never reached*

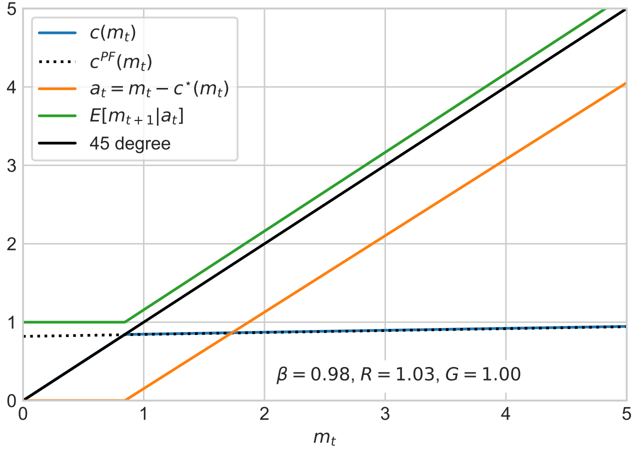
     $$c^\star(m_t) = c^{PF}(m_t) \text{ for } m_t \geq 1$$

- **Exotic solutions without FHW** exists (GI necessary)

Perfect foresight with $\lambda = 0$ and GI



Perfect foresight with $\lambda = 0$, but not GI



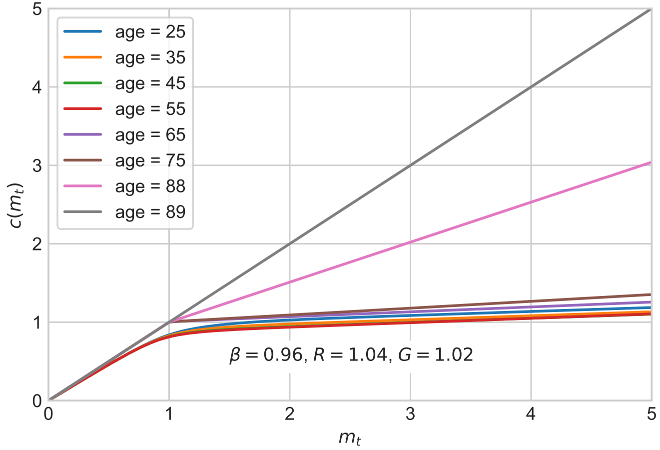
Life-cycle

Adding a life-cycle (normalized)

$$
\begin{aligned}
v_t(m_t, z_t) &= \max_{c_t} \frac{v(z_t)c_t^{1-\rho}}{1-\rho} + \beta\mathbb{E}_t\left[(GL_{t+1}\psi_{t+1})^{1-\rho} v_{t+1}(\bullet)\right] \\
&\text{s.t.} \\
a_t &= m_t - c_t \\
m_{t+1} &= \frac{1}{GL_t\psi_{t+1}} Ra_t + \xi_{t+1} \\
\xi_{t+1} &= \begin{cases} \mu & \text{with prob. } \pi \\ (\epsilon_{t+1} - \pi\mu)/(1-\pi) & \text{else} \end{cases} \\
a_t &\geq \lambda_t = \begin{cases} -\lambda & \text{if } t < T_R \\ 0 & \text{if } t \geq T_R \end{cases}
\end{aligned}
$$

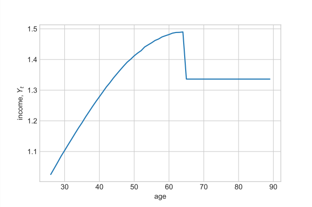
- **Demographics**: z_t (exogenous)
- **Income profile:** $P_{t+1} = GL_t P_t \psi_{t+1}$
- **No shocks in retirement:** $\psi_t = \xi_t = 1$ if $t > T_R$
- **Euler equation:** $C_t^{-\rho} = \beta R\mathbb{E}_t[\frac{v(z_{t+1})}{v(z_t)} C_{t+1}^{-\rho}]$

Consumption functions ($v(z_t) = 1$)

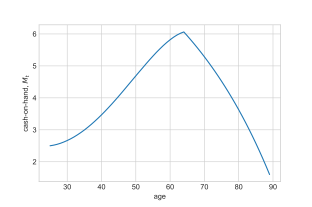


Simulation: LIfe-cycle profiles ($v(z_t) = 1$)

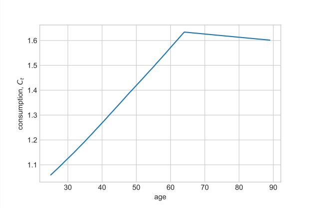
Income, Y_t (implied by G and L_t)



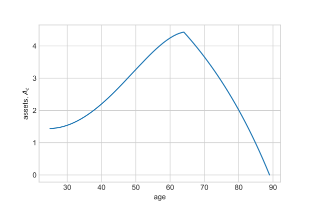
Cash-on-hand, M_t



Consumption, C_t



End-of-period assets, A_t



EGM

Euler-equation

- Assume for simplicity **no borrowing**: $\lambda = 0$
- All optimal **interior choices** must satisfy

$$
\begin{aligned}
C_t^{-\rho} &= \beta R \mathbb{E}_t \left[C_{t+1}^{-\rho} \right] \Leftrightarrow \\
c_t^{-\rho} &= \beta R \mathbb{E}_t \left[\left(G \psi_{t+1} c_{t+1} \right)^{-\rho} \right]
\end{aligned}
$$

- Else optimal choice is **constrained**

$$
\begin{aligned}
C_t^{-\rho} &\geq \beta R \mathbb{E}_t \left[C_{t+1}^{-\rho} \right] \Leftrightarrow \\
C_t &= M_t \Leftrightarrow \\
c_t &= m_t
\end{aligned}
$$

Endogenous grid method: Intuition

- **Obs.:** Given $C^\star_{t+1}(M_{t+1}, P_{t+1})$ and A_t and P_t we have

$$
\begin{aligned}
C_t^{-\rho} &= \beta R \mathbb{E}_t \left[(C^\star_{t+1}(M_{t+1}, P_{t+1}))^{-\rho} \right] \Leftrightarrow \\
C_t &= \mathbb{E}_t \left[\beta R \left(C^\star_{t+1}(M_{t+1}, P_{t+1}) \right)^{-\rho} \right]^{-\frac{1}{\rho}} \\
&= \mathbb{E}_t \left[\beta R \left(C^\star_{t+1}(RA_t + Y_{t+1}, P_{t+1}) \right)^{-\rho} \right]^{-\frac{1}{\rho}} \\
&= \mathbb{E}_t \left[\beta R \left(C^\star_{t+1}(RA_t + P_t \psi_{t+1} \xi_{t+1}, P_t \psi_{t+1}) \right)^{-\rho} \right]^{-\frac{1}{\rho}} \\
&\equiv F(A_t, P_t)
\end{aligned}
$$

- **Endogenous grid:** $A_t = M_t - C_t \Leftrightarrow M_t = C_t + A_t$
- **Conclusion:** $(M_t,\ P_t,\ C_t)$ is a solution to the Bellman equation because it satisfies the Euler equation
- **Perspectives:** Varying A_t (and P_t) we can map out the consumption function without using any numerical solver!
- **The borrowing constraint** is binding below the lowest M_t points we can generate

... in ratio form

- **Prerequisites**:

  1. Next-period **consumption function**: $c^{\star}_{t+1}(m_{t+1})$
  2. **Asset grid:** $\mathcal{G}_a = \{a_1, a_2, \ldots, a_{\#}\}$ with $a_1 = 10^{-6}$

- **Algorithm:** For each $a_i \in \mathcal{G}_a$

  1. Find consumption using Euler equation

$$c_i = \mathbb{E}_t \left[\beta R \left(G\psi_{t+1} c^{\star}_{t+1} \left(\frac{R}{G\psi_{t+1}} a_i + \xi_{t+1} \right) \right)^{-\rho} \right]^{-\frac{1}{\rho}}$$

  2. Find endogenous state

$$a_i = m_i - c_i \Leftrightarrow m_i = a_i + c_i$$

- The **consumption function**, $c_t(m_t)$, is given by

$$\{0, c_1, c_2, \ldots, c_{\#}\} \text{ for } \{\underline{a}_t, m_1, m_2, \ldots, m_{\#}\}$$

- *We can find all consumption functions in this way!*

Addendum: The natural borrowing constraint ($\lambda > 0$)

- The **optimal end-of-period asset choice satisfies** the backwards recursion

$$a_t \geq \underline{a}_t = \begin{cases} 0 & \text{if } t \geq T_R \\ -\min\left\{\Lambda_t, \lambda_t\right\} GL_t\underline{\psi} & \text{if } t < T_R \end{cases}$$

where

$$\mathit{\Lambda}_t \quad \equiv \quad \begin{cases} R^{-1}GL_t\underline{\psi}\,\underline{\xi} & \text{if } t = T_R - 1 \\ R^{-1}\left[\min\left\{\mathit{\Lambda}_{t+1}, \lambda_t\right\} + \underline{\xi}\right] GL_t\underline{\psi} & \text{if } t < T - 1 \end{cases}$$

and $\underline{\psi}$ and $\underline{\xi}$ are the minimum realizations of ψ_{t+1} and ξ_{t+1}

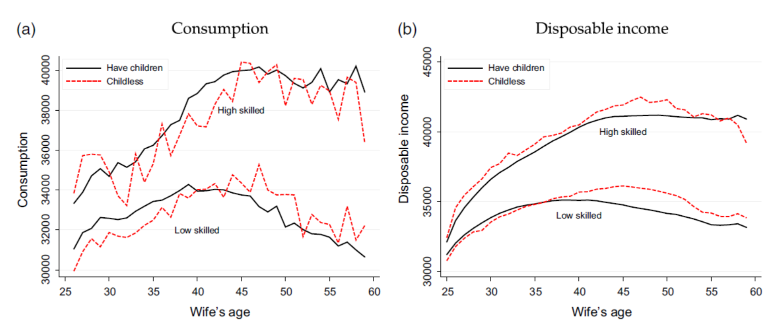
- **Proof:** Can be shown as a consequence of the household wanting to avoid $c_t = 0$ at *any cost* because $\lim_{c_t \to 0} u'(c_t) = \infty$.

Further perspectives

Three generations of models

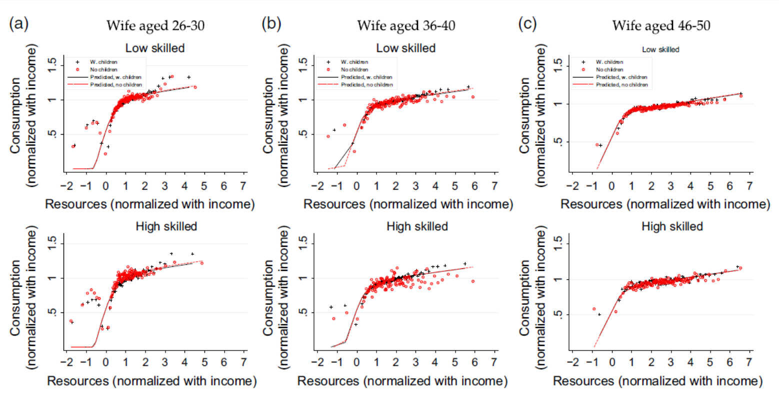
- **1st:** *Permanent income hypothesis* (Friedman, 1957)
  or *life-cycle model* (Modigliani and Brumberg, 1954)
- **2nd:** *Buffer-stock consumption model*
  (Deaton, 1991, 1992; Carroll 1992, 1997, 2012)Carroll (1992, 1997)
- **3nd:** *Multiple-asset buffer-stock consumption models*
  (e.g. Kaplan and Violante, 2014)

Denmark: Life-cycle profiles fit



Source: Jørgensen (2017)

Denmark: Consumption function fit



Source: Jørgensen (2017)

Level of wealth and MPC

- Consumption-saving models a few years ago **could not endogenously fit** both
  1. The level of wealth observed
  2. The high MPCs found in quasi experiments
- **Three solutions:**
  1. Exogenous **hands-too-mouth households** (Campbell and Mankiw, 1990)
  2. **Preference heterogeneity** (Carroll et al. 2017)
  3. **Wealthy hands-to-mouth** (Kaplan and Violante, 2014)
     Many households hold mostly illiquid assets with a high return
     $\rightarrow$ *consumption adjust in response to small income shock*

Kaplan-Violante model (two-asset model)

$$V_t(M_t, N_t, P_t) = \max\left\{v_t^{keep}(M_t, N_t, P_t), v_t^{adj.}(M_t + N_t - \lambda, P_t)\right\}$$

$$\begin{aligned}
v_t^{keep}(M_t, N_t, P_t) &= \max_{C_t} u(C_t, B_t) + \beta W_t(A_t, B_t, P_t) \text{ s.t.} \\
A_t &= M_t - C_t \\
B_t &= N_t \\
A_t &\geq -\omega P_t.
\end{aligned}$$

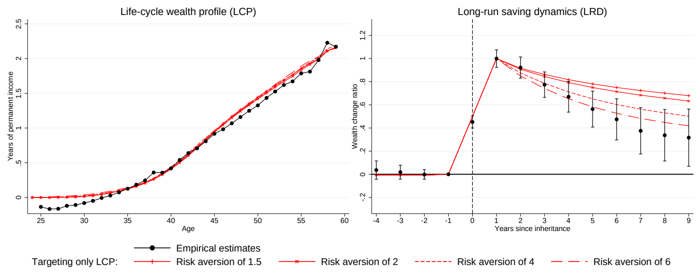
$$\begin{aligned}
\tilde{v}_t^{adj.}(X_t, P_t) &= \max_{B_t, C_t} u(C_t, B_t) + \beta W_t(A_t, B_t, P_t) \text{ s.t.} \\
M_t &= X_t - B_t \\
A_t &= M_t - C_t \\
A_t &\geq -\omega P_t.
\end{aligned}$$

$$W_t(A_t, B_t, P_t) = \mathbb{E}_t[V_t(RA_t + P_t\psi_{t+1}\xi_{t+1}, R_b B_t, P_t\psi_{t+1})]$$

Level of wealth and long-run dynamics I

- **Best test of a life-cycle consumption-saving model:**
  A *sudden*, *sizable* and *salient* shock to wealth
  $+$ long panel to observe how the extra wealth is spend
- **My own research (with Alessandro Martinello):**
  Compare individuals in the Danish register data who
  1. Receive a similar inheritance, but at different points in time
  2. From parents dying due to heart attacks or car crashes

Level of wealth and long-run dynamics II



- **Net worth:** Good fit for different levels of risk-aversion (ρ) when re-calibrating patience (β)
- **Also dynamics:** Good fit only if risk-aversion (ρ) is high

Frontier topics

- The dynamics of **durable consumption** (very volatile over the business cycle, involve non-convexities due to adjustment costs)
- The effects of **non-Gaussian** and **high frequency income uncertainty** (monthly Danish income since 2008 very interesting)
- **Housing** and a more detailed specification of the households' **balance sheets** (did expectations or credit availability drive the boom and bust in house prices?)
- Relevant **deviations from rationality** (learning, myopia, hyperbolic discounting, reference dependence, mental accounting)
- Fitting the **level and dynamics of inequality** – circumstances or behavior?
- All of this in **general equilibrium** with heterogeneous households

Summary

Summary

- **Dynamic programming** is needed to solve **empirically realistic consumption-saving models**
- The **buffer-stock consumption model**, and it's two asset cousin, can fit central stylized facts
    1. High MPC
    2. Responses to expected windfalls
    3. Households with more volatile income save more
    4. Consumption tracks income over the life-cycle
- Advances in micro-data, numerical methods and computational power are leading to **new discoveries**
- **EGM is a powerful solution method** (can be generalized, DCEGM, G2EGM, NEGM)
- Realistic consumption-saving behavior can be included in **general equilibrium models** $\rightarrow$ welfare analysis with full distributional effects

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