

Exponential function via ODE

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The exponential function can be implemented by solving the following ODE:

$$\frac{dy}{dx} = y(x) \quad (1)$$

subject to the initial condition: $y(0) = 1$. In order to avoid numerical integrations over large intervals the argument have been reduced such that $0 \leq x < 1$ by using the following equations:

$$\exp(-x) = \frac{1}{\exp(x)}, \exp(x) = \left[\exp\left(\frac{x}{2}\right) \right]^2$$

This differential equation has been solved numerically by using the GNU Scientific Library. The used algorithm is the Runge-Kutta-Fehlberg (4,5) method. The numerical solution is within a relative error of 10^{-10} of the correct result. A comparison between the exponential function found in the library `<math.h>` and the one obtained by solving (1) can be seen in figure 1.

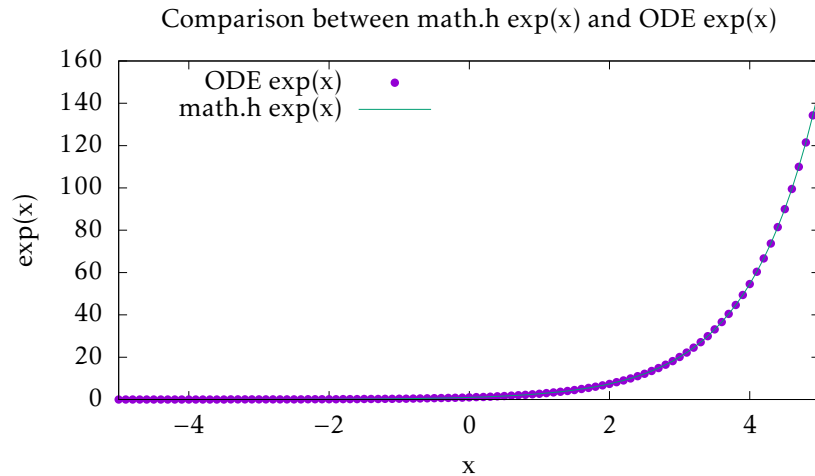


Figure 1: Comparison between the exponential function from the library `math.h` and the exponential function obtained by solving (1).