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An Approach to Generate Different CPG Patterns

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***Abstract* - In this paper a new approach is proposed to generate different patterns for legged robots locomotion control, which is performed by Cellular Neural Networks (CNN) playing the role of artificial Central Pattern Generator (CPG). An analog system is introduced by replacing the output with a sigmoid function to enable some academic research feasible in CNN state equations. A limit cycle is firstly proved existent applying Poincaré-Bendixson theorem and numerical calculation in the new system. Critical values of biases are also figured out when the limit cycle disappears and a local bifurcation occurs. And then, a conclusion is derived to show that suitable patterns can be achieved by modifying the values of biases in CNN system, which is a foundation to generate different patterns in CPG control strategy. Simulation results are given to illustrate the suitability of the proposed approach.**

Index Terms - Locomotion control, Intelligence control, Central pattern generator, Cellular neural networks.

I. INTRODUCTION

It has been a challenge in the past few years to develop new approaches and architectures for locomotion control in legged robots. Biology provides a wealth inspiration: rhythms and paces of animals are controlled by CPG in their bodies. Drawn inspiration from it, many research works have been published over the years on generating patterns for robots locomotion control through CPG models; see [1]~[5], for instance. Recently, much attention has been paid to the CNN based CPG [3]~[6], since the CNN is easy both to conceive and to implement. In [7], a limit cycle has been studied in a single two-layer CNN cell with proper choice of parameters, and then autowaves has been observed in [8], which are carriers of CPG patterns. Several robot models are built in [4], [5], such as the Hexapod Robots and the Lamprey Robot, which are driven by CPG patterns generated by CNN. There are also many approaches so far to achieve different patterns. In [3] a new method based on biological models was proposed to gain different patterns by loading different templates when necessary. This course can also be pursued through modifying the CNN structures [4]. However, both of above methods are difficult to implement by hardware, and there is little theoretical analysis of CNN state equations so far to show when the CNN will display complicate dynamic behaviors, and how to allow CNN to generate different patterns. In [9], an analog CNN system was built to analyze the bifurcation phenomenon. But there are some mistakes in the analysis process, besides the system is hard to achieve by hardware.

In this paper, we will consider the analog system as a bridge to analyze dynamic behaviors of CNN state equations. The system is built by using a sigmoid function as the output instead of the piecewise linear saturated one, which allows us to do further theoretical analysis of more complicate dynamic behaviors signified in CNN state equations. Existence of limit cycles is proved, which is a foundation of patterns generating. In the new system, we can obtain easily the critical value of bias when local bifurcation phenomenon occurs. Most important of all, it can be easily explain why different patterns will be displayed while the value of bias is changed, and a new method to achieve varied patterns is given consequently. Simulations show that conclusions achieved in the analog system are valid in the universal one.

II. TWO-LAYER CNN FOR CPG MODEL

CNN was proposed by L. O. Chua in 1988 and was applied in many fields during the past several years. Recently a two-layer CNN is applied into modeling CPG to generate patterns for legged robot locomotion controlling. The two layer CNN system described by the following state equations:

$$\begin{cases} \dot{x}_{1,ij} = -x_{1,ij} + (1+\mu)y_{1,ij} - s_1 y_{2,ij} + i_1 \\ \dot{x}_{2,ij} = -x_{2,ij} + s_2 y_{1,ij} + (1+\mu)y_{2,ij} + i_2 \end{cases} \tag{1}$$

where

$$y_{k,ij} = 0.5(|x_{k,ij}+1| - (|x_{k,ij}-1|) \tag{2}$$

k=1, 2, i=1, 2…M, j=1, 2…N , the positive integers M and N are dimensions of CNN, x, y and u respectively denote the state, output, and control, i_1 and i_2 are biases of the network.

Dynamic behavior of equation (1) with $i_1 = i_2 = 0$ was well studied in [10] and it was proven that the nonlinear system oscillates with a limit cycle centered to the origin by a suitable choice of the parameters μ , s_1 and s_2. Stability of the CNN state equations has also been investigated in [7] when both i_1 and i_2 are not equal to zero. Simulations show that, the slow-fast ratio of state variables in system (1) will change if biases vary. What's more, if the values of biases surpass a certain intervals, the limit cycle will disappear. However, it is difficult to give the theoretic explanation for these phenomenons since the output function (2) isn't differentiable globally and traditional theories of differential equations aren't valid any more here. In this paper, an analog system is introduced by replacing (2) with the following relation

$$y_{k,ij} = \tanh(2x_{k,ij}) . \tag{3}$$

Equation (3) is a globally differentiable function and it may enable us to do more research works on the dynamic behaviors of CNN state equations with this output function.

III. QUALITATIVE ANALYSIS OF NEW SYSTEM

In this section, we will investigate the variations of dynamic behaviors of the new system when the value of bias varies. Before that, we will study the stability of the analog system. By setting the parameters

$$\mu = 0.7\ ,s_1 = s_2 = 1\ ,i_1 = -i_2 = -0.17\ , \tag{4}$$

system (1) becomes

$$\begin{cases} \dot{x}_1 = -x_1 + 1.7\tanh(2x_1) - \tanh(2x_2) - 0.17 \\ \dot{x}_2 = -x_2 + \tanh(2x_1) + 1.7\tanh(2x_2) + 0.17 \end{cases} \tag{5}$$

Proposition 1: In area D_0={(x_1, x_2)| $-2.87 \le x_1 \le 2.53$, $-2.53 \le x_2 \le 2.87$}, system (5) has only one equilibrium point, P (6.84×10^{-3}, -7.74×10^{-2}), which is an unstable focus.

Proof: Defining $y_1 = -x_1 + 1.7\tanh(2x_1) - 0.17$, $y_2 = x_2 - 1.7\tanh(2x_2) - 0.17$ and solving the following equations

$$\begin{cases} -x_1 + 1.7\tanh(2x_1) - \tanh(2x_2) - 0.17 = 0 \\ -x_2 + \tanh(2x_1) + 1.7\tanh(2x_2) + 0.17 = 0 \end{cases} \tag{6}$$

we obtain

$$x_2 = f(y_1) = 0.25[\ln(y_1 + 1) - \ln(1 - y_1)] \tag{7}$$

$$x_1 = g(y_2) = 0.25[\ln(y_2 + 1) - \ln(1 - y_2)] \tag{8}$$

By (7) and (8), we can line out in Fig1 the isoclines of system (5) with the help of Matlab software. Solid line shows $\dot{x}_1 = 0$, while the dash line denotes $\dot{x}_2 = 0$. In the region of D_0, as the figure shows, there is only one intersection point between isoclines and we can figure out through Newton method [11] the approximate coordinate of point P is (6.84×10^{-3}, -7.74×10^{-2}), which is a solution of equation (6).

It is easy to show that the Jacobin matrix of the system (5) is

$$J_x = \begin{pmatrix} -1 + \dfrac{3.4}{\cosh^2(2x_1)} & \dfrac{2}{\cosh^2(2x_2)} \\ -\dfrac{2}{\cosh^2(2x_1)} & -1 + \dfrac{3.4}{\cosh^2(2x_2)} \end{pmatrix}$$

and its eignequation at point P is

$$1.024\lambda^2 - 4.834\lambda + 9.702 = 0 \tag{9}$$

In according with binomial theorem, equation (9) has two imaginary roots, which are latent roots of system (5) and the real parts are both bigger than zero. So we can draw a conclusion that point P is an unstable focus. ■

In Fig1, the isoclinals $\dot{x}_1 = 0$ divide D_0 into two parts: on its left side, $\dot{x}_1 > 0$ and on the right side, $\dot{x}_1 < 0$. Similarly, it can be known that above $\dot{x}_2 = 0$, $\dot{x}_2 < 0$, and $\dot{x}_2 > 0$ under $\dot{x}_2 = 0$. Before proving the existence of periodic solutions in equation (5), we give another proposition as follows, which can be easily seen by using Poincaré-Bendixson theorem [12].

Proposition 2: System (5) exists at least one limit cycle in the region of D_0, if all trajectories go from outer to inner on the boundary of D_0 and there are no other equilibrium points except unstable focuses in D_0.

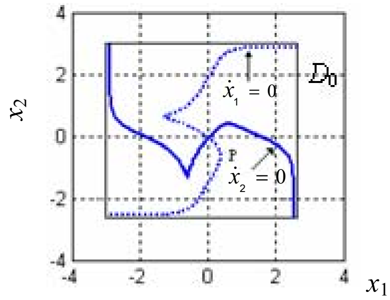


Fig1: Distribution of isoclines of system (5) in region of D_0.

Theorem 1: System (5) exists stable periodic solutions.

Proof: It's easy to check that $x_1 = -2.87$ and 2.53 are discontinuous points of relation (7), and x_2=2.87 and –2.53 are discontinuous points of relation (8). Define U={(x_1, x_2)| $-2.87+\varepsilon \le x_1 \le 2.53-\varepsilon$, $-2.53+\varepsilon \le x_2 \le 2.87-\varepsilon$ }, where ε is an arbitrarily small positive number. As Fig2 shows, points A, B, C and D are intersections between isoclinic lines and the boundary of U. Isoclinic lines of system (5) divide U into four parts: D_1-D_4, and in each part the signs of $\dot{x}_1$ and $\dot{x}_2$ are indicated in Table I. Curves with arrows in Fig2 denote directions of trajectories on the boundary of U, and they are all towards inside. According to Proposition 1 and 2, we reach a conclusion that system (5) exist stable cycles. ■

TABLE I
SINGS IN INTERVALS.

	D_1	D_2	D_3	D_4
Derivative of x_1	+	-	-	+
Derivative of x_2	-	-	+	+

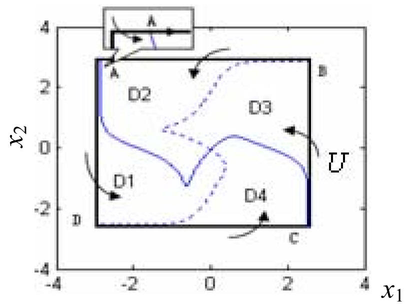
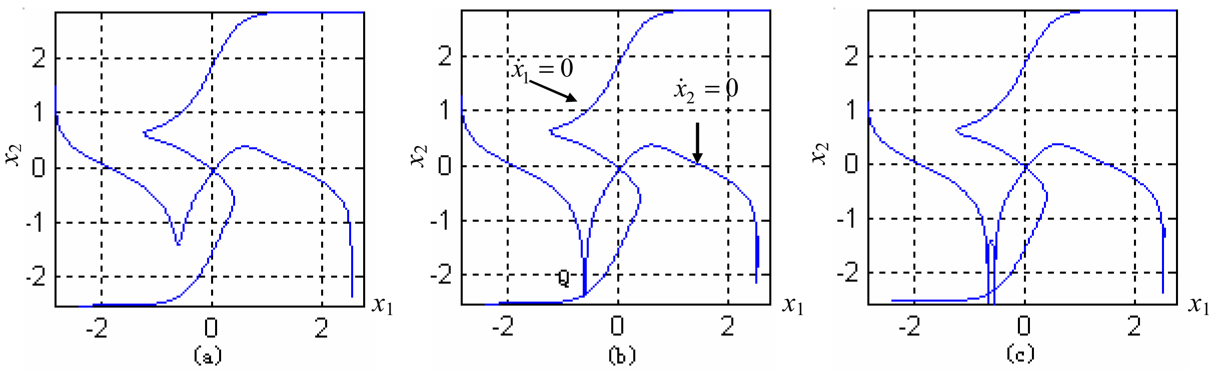


Fig2: Direction field on the boundaries of U in plane x_1-x_2.

In Theorem 1, a result has been derived from system (5) with given i_1 and i_2. It is known that a real system, such as the robotic system, will be inevitably disturbed by the variations of the environment, which will lead to the variations of parameters of the system (5), see [6]. In the following, we will study the dynamic behavior of system (5) when biases i_1 and i_2 change, i.e.,

$$\begin{cases} \dot{x}_1 = -x_1 + 1.7\tanh(2x_1) - \tanh(2x_2) + i_1 \\ \dot{x}_2 = -x_2 + \tanh(2x_1) + 1.7\tanh(2x_2) + i_2 \end{cases} . \tag{10}$$

Fix i_2, when the bias i_1 changes, other equilibrium may arise. Fig3 show the distribution of isoclines of system (10) when i_1 varies from -0.175 to -0.19, with a fixed



(a) i_1= − 0.175, (b) i_1= − 0.1824, (c) i_1= − 0.19

Fig3: Isoclines in plane x_1-x_2 for system (10) with the fixed parameter i_2= 0.17.

parameter i_2=0.17. Fig3 (b) reveals a critical state. The most thing that we concern about is whether the system will maintain a periodic solution or occur a bifurcation.

Theorem 2: Fixed i_2, system (10) will occur bifurcation phenomenon and limit cycles will vanish, if $i_1<I_c$, where I_c is a critical value. Specially, when i_2= − 0.17, I_c= − 0.1824.

Proof: To analyze the critical state in Fig 3(b), we suppose the new equilibrium point is $Q(\alpha, \beta)$. The derivative of relation (7) with respect to x_1 is

$$\frac{dx_2}{dx_1} = \frac{1}{4}\left(\frac{1}{1+y_1} + \frac{1}{1-y_1}\right)\left(-1 + \frac{3.4}{\cosh^2(2x_1)}\right).$$

Let dx_2/dx_1=0, we get $-1+3.4/(\cosh^2(2x_1))=0$, which leads to the solutions $x_1 = \pm 0.6109$. That's to say, relation (7) has two extreme points in the region of D_0 and Q is one of them. Since Q is on the left side of x_2 axis, we get $\alpha = x_1$= − 0.6109. Fig 3(b) also shows that point Q is in a region in which $x_2<-1$, so that $y_2 \approx 1$ and $\dot{y}_2 \approx 0$. To study the dynamic performance around Q of system (10), we rewrite it as follows,

$$\begin{cases} \dot{x}_1 = -x_1 + 1.7\tanh(2x_1) + 1 + i_1 \\ \dot{x}_2 = -x_2 + \tanh(2x_1) - 1.53 \end{cases}. \tag{11}$$

It is easy to obtain the Jacobin matrix of (11) and its eigenequation is

$$(\lambda+1)(\lambda - 3.4/\cosh^2(2\alpha)+1) = 0,$$

which means

$$\lambda_1 = -1 \ , \ \lambda_2 = 3.4/(\cosh^2(2\alpha)+1) = 0 .$$

What's more, the first equation of system (11) is independent of x_2 and the second equation is foreign to i_1, we can apply the Center Manifold Theorem [12] to study behaviors of the system around point Q by studying the following equation

$$\dot{x}_1 = f(x_1, i_1) = -x_1 + 1.7\tanh(2x_1) + 1 + i_1 .$$

Since

$$\frac{\partial^2 f}{\partial x_1^2}\Big|_Q = 13.6\sinh(2\alpha)\cosh^{-3}(2\alpha) = -3.3604 \neq 0,$$

$$\frac{\partial f}{\partial i_1}\Big|_Q = 1 \neq 0,$$

and eigenvalues of system (11) are −1 and 0, there must occur a local bifurcation at point Q [13].

Let $\dot{x}_1 = \dot{x}_2 = 0$ in system (10). We get

$$\begin{cases} \dot{x}_1 = -x_1 + 1.7\tanh(2x_1) - \tanh(2x_2) + i_1 = 0 \\ \dot{x}_2 = -x_2 + \tanh(2x_1) + 1.7\tanh(2x_2) + i_2 = 0 \end{cases}. \tag{12}$$

Applying x_1= − 0.6109 in the second equation of system (12), it can be figured out that x_2= − 2.3699 through Newton- Raphson method [11]. Then applying x_1= − 0.6109 and x_2= − 2.3699 in the first equation, we obtain i_1= − 0.1824, which is the critical value I_c when a bifurcation phenomenon occurs. ■

Remark I: If there is only one equilibrium point in area U, Poincaré-Bendixson theorem will be always valid, which means a limit cycle will always exist. Theorem 2 tells us the limit cycle will disappear if biases change over the critical value I_c.

Remark II: In fact, a way to calculate the critical values of the bias was implied in the proof process of Theorem 2. If parameters of system (5) are set as μ = 0.5, $s_1 = s_2$ = 1, i_2 = 0.3, we will achieve I_c= − 0.3483 in the same way mentioned before.

Remark III: An analogous conclusion, which is about a bifurcation in CNN state equations, was proved in [9]. But there are some mistakes in the proof process. Firstly the conclusion reached in that article that there exists a bifurcation around the new equilibrium is based on the assumption that a bifurcation is really existent. It's obviously improper. Secondly, the method to calculate the critical values of biases is imprecise. It's incorrect to apply x_1=−0.6109 to system (11), which is an approximate system, since CNN state equations are sensitive to the parameters.

We will do further research to find a method to generate different patterns through CNN. It's easy to verify that the solutions of (10) rely on variations of the biases continuously, since the derivative of the right side of (10) with respect to x_1 and x_2 respectively are bounded, and respect to i_1 and i_2 are both equal to 1 in the region of U. If biases change, the variance ratio of state variables, $\dot{x}_1$ and $\dot{x}_2$, will be varied, so that system (10) will display different slow-fast patterns. Consequently, we give another remark as follows.

Remark IV: Change the values of biases, system (5) will show different slow-fast ratio, if periodic solutions exist.

The above remark provides a method to change the patterns achieved from CNN. If values of the biases are varied, the CPG patterns will be modified. In the next section, we will denote that conclusions reached before are valid through some numerical simulations.

IV. Simulation Results

In application different CPG patterns can be generated through changing connection voltages between CNN cells. In this section, this process will be demonstrated by some simulations.

(1) Conclusions reached in section III are valid in the traditional single CNN cell state equations. Fig4 (a)~(c) shows the locus of the state variables by setting parameters except biases with (4), and bias are: (a) $i_1 = -0.17$, $i_2 = 0.17$; (b) $i_1 = -0.3$, $i_2 = 0.3$, (c) $i_1 = -0.31$, $i_2 = 0.3$ respectively. Solid lines denote x_1 oscillating for 100 seconds and dash lines display the state variable x_2. Fig4 (a) and (b) show different periodic solutions of the system, if we set the biased with different values. Fig4(c) indicates that the behavior of system (5) changes to a stable fixed point, for a local bifurcation exists.

(2) The changing of patterns generated by CNN has been analyzed above when the biases are varied, which enable us to achieve different CPG patterns. We build the same CNN structure mentioned in [4], and set the parameters except biases with relation (4). Fig5 shows locus of the CNN state variable x_1 while the values of biases varied from (a) $i_1 = -0.3$, $i_2 = 0.3$ to (b) $i_1 = -0.2$, $i_2 = 0.2$. Time of oscillation is 50 seconds. The heavier lines mark behaviors of cell 1, and then 4, 7 and 10 from left side to the right. Patterns are changed while the biases are modified.

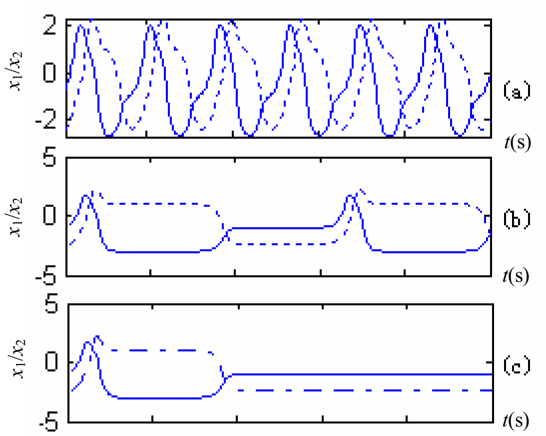


Fig4: State parameters x_1 and x_2 with different value of bias in traditional CNN in plane t－x_1/x_2.

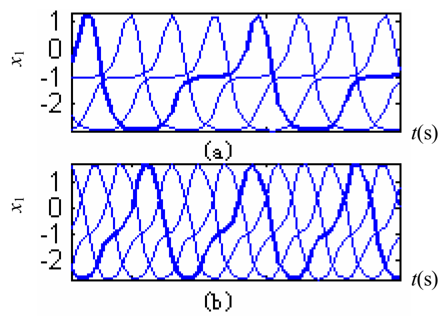


Fig5: Different period with different values of biases in CNN state-equations in plane t－x_1.

V. Conclusions

In this paper an approach has been proposed to achieve different CPG patterns through CNN structure. Academic studies in CNN state equations become feasible by using a sigmoid function as output instead of the piecewise linear saturated one. Theoretical analysis shows that the CPG patterns can be changed by modifying the biases of CNN. What's more, this strategy can be implemented by hardware since the value of bias is expressed by potentials between cells in CNN circuit. In addition, the analog system could be a bridge to study more complicated dynamic behaviors that CNN system displays, and the analysis of how the bias affects qualities of the system provided a new way to error analysis for robot system. Simulations show that conclusions achieved in the analog system are valid in the universal. However, whether the two systems are topological equivalent or isomorphic in academic, will be left for our future research.

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References

[1] M.G. Felipe, F. Yang, Z. Yang, "Building artificial CPGs with asymmetric Hopfield networks," *Proceedings of the IEEE-INNS -ENNS International Joint Conference*, vol.4, pp290–295, July 2000.
[2] H. Kimura, Y. Fukuoka, "Adaptive dynamic walking of a quadruped robot on irregular terrain by using neural system model," *Proceedings 2000 IEEE/RSJ International Conference* on Vol. 2, 31 Oct.-5 Nov. 2000 pp. 979 – 984.
[3] P. Arena, L. Fortuna, M. Frasca, C. Marchese, "Multi-template approach to artificial locomotion control," *IEEE Circuits and System,* Vol. 3, pp. 37 – 40, May 2001.
[4] P. Arena, L. Fortuna, "Analog cellular locomotion control of hexapod robots," *IEEE Control Systems Magazine*, vol. 22, 2002.
[5] P. Arena, L. Fortuna, M. Frasca, L. Patane, M. Pavone, "Climbing Obstacles via Bio-Inspired CNN-CPG and Adaptive Attitude Control," *IEEE Circuits and Systems*, pp. 5214 - 5217, May 2005.
[6] Y. -J. Zhang, Q. -B. Zhu, "Research in locomotion control based on central pattern generation," *Journal of Nanjing Normal University (Engneerng and Technology)*, Vol. 5, Issue 3. 2005.
[7] P. Arena, R. Caponetto, L. Fortuna, G. Manganaro, "Cellular neural networks to explore complexity," *Soft Computing, springer-verlag* vol.1, Number 3, pp.120-136, September 1997.
[8] M. Gomez-Gesteira, M. deCastro, V. Perez-Villar, L.O. Chua, "Experimental Chua's circuit arrays as an autowave simulator," *Circuits and Systems I: Fundamental Theory and Applications,* vol. 46, Iss. 4, pp.495 – 499, Apr.1999.
[9] P. Arena, L. Fortuna, M. Frasca, L. Patane, "CNN based central pattern generators with sensory feedback," *Proceedings of the 2002 7th IEEE International Workshop* on 22-24 July 2002, pp.275 – 282.
[10] F. Zou, J. -A. Nossek, "Bifurcation and chaos in cellular neural networks," *IEEE Circuits and Systems I*, vol. 40, pp. 166–173, 1993.
[11] C. Lin, "Method of numerical calculation," *Science Press*, 1999.
[12] J. Zhang, B. Feng, "Geometric theory and bifurcation of ordinary differential equation," *Beijing University Press*, 2000.
[13] Y. Kuznetsov, "Elements of applied bifurcation theory," *Springer*, New York, 1998.