Red Bayesiana

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1 Datasets

1.1 Dataset de entrenamiento

T	H	N	C
0	0	0	0
0	1	1	0
1	1	0	1
1	1	1	1
1	0	0	1
0	1	1	0
1	0	1	0
1	0	1	0
1	0	1	0
0	0	0	0

Table 1: Training dataset.

1.2 Dataset de prueba

T	H	N	C
0	1	0	?
0	1	1	?
0	0	1	?

Table 2: Test dataset.

2 Variables aleatorias

Las variables aleatorias a utilizar son:

- T = Temperatura
- H = Humedad
- N = Nubosidad
- C = Clase

3 Representación

Se modela la red bayesiana que tiene como estructura:

$$B =< G, P >$$

Donde G es un grafo y P son parámetros. Se presenta el siguiente grafo:

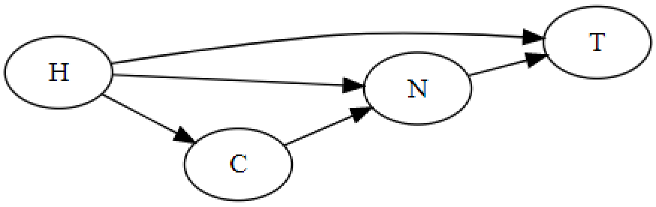


Figure 1: Grafo Acíclico Dirigido o DAG .

Donde G está compuesto de variables y aristas:

$$G =< V, A >$$

$$V = [T, H, N, C]$$

$$A = [H- > T, H- > N, H- > C, N- > T, C- > N]$$

Y P se compone de:

$$P = [P^T, P^H, P^N, P^C]$$

- Probabilidad de T que depende de las variables H y N (Distribución condicional)

$$P^T = P(T|H, N)$$

- Probabilidad de H que no depende de ninguna variable (Distribución marginal)

$$P^H = P(H)$$

- Probabilidad de N que depende de las variables H y C (Distribución condicional)

$$P^N = P(N|H, C)$$

- Probabilidad de C que depende de la variable H (Distribución condicional)

$$P^C = P(C|H)$$

3.1 Cálculo de probabilidades

Para el cálculo de todas las probabilidades se utilizara el hiperparámetro α donde $\alpha_1 = \alpha_2 = 1$.

3.1.1 Cálculo P(T|H,N)

Existen 4 distribuciones:

1. P(T|H=0,N=0)

   (a) P(T=0|H=0,N=0)

   $$\frac{P(T = 0 \wedge H = 0 \wedge N = 0) + \alpha_1}{P(H = 0 \wedge N = 0) + \alpha} = \frac{0.2 + 1}{0.3 + 2} = \frac{1.2}{2.3} = 0.5217$$

   (b) P(T=1|H=0,N=0)

   $$\frac{P(T = 1 \wedge H = 0 \wedge N = 0) + \alpha_2}{P(H = 0 \wedge N = 0) + \alpha} = \frac{0.1 + 1}{0.3 + 2} = \frac{1.1}{2.3} = 0.4783$$

2. P(T|H=0,N=1)

   (a) P(T=0|H=0,N=1)

   $$\frac{P(T = 0 \wedge H = 0 \wedge N = 1) + \alpha_1}{P(H = 0 \wedge N = 1) + \alpha} = \frac{0 + 1}{0.3 + 2} = \frac{1}{2.3} = 0.4348$$

(b) P(T=1|H=0,N=1)

$$\frac{P(T=1 \wedge H=0 \wedge N=1)+\alpha_2}{P(H=0 \wedge N=1)+\alpha} = \frac{0.3+1}{0.3+2} = \frac{1.3}{2.3} = 0.5652$$

3. P(T|H=1,N=0)

(a) P(T=0|H=1,N=0)

$$\frac{P(T=0 \wedge H=1 \wedge N=0)+\alpha_1}{P(H=1 \wedge N=0)+\alpha} = \frac{0+1}{0.1+2} = \frac{1}{2.1} = 0.4762$$

(b) P(T=1|H=1,N=0)

$$\frac{P(T=1 \wedge H=1 \wedge N=0)+\alpha_2}{P(H=1 \wedge N=0)+\alpha} = \frac{0.1+1}{0.1+2} = \frac{1.1}{2.1} = 0.5238$$

4. P(T|H=1,N=1)

(a) P(T=0|H=1,N=1)

$$\frac{P(T=0 \wedge H=1 \wedge N=1)+\alpha_1}{P(H=1 \wedge N=1)+\alpha} = \frac{0.2+1}{0.3+2} = \frac{1.2}{2.3} = 0.5217$$

(b) P(T=1|H=1,N=1)

$$\frac{P(T=1 \wedge H=1 \wedge N=1)+\alpha_2}{P(H=1 \wedge N=1)+\alpha} = \frac{0.1+1}{0.3+2} = \frac{1.1}{2.3} = 0.4783$$

3.1.2 Cálculo P(H)

Existe 1 sola distribución

1. P(H)

(a) P(H=0)

$$\frac{6+\alpha_1}{10+\alpha} = \frac{6+1}{10+2} = \frac{7}{12} = 0.583333$$

(b) P(H=1)

$$\frac{4+\alpha_2}{10+\alpha} = \frac{4+1}{10+2} = \frac{5}{12} = 0.416666$$

3.1.3 Cálculo P(N|H,C)

Existen 4 distribuciones:

1. P(N|H=0,C=0)

(a) P(N=0|H=0,C=0)

$$\frac{P(N=0 \wedge H=0 \wedge C=0)+\alpha_1}{P(H=0 \wedge C=0)+\alpha} = \frac{0.2+1}{0.5+2} = \frac{1.2}{2.5} = 0.48$$

(b) P(N=1|H=0,C=0)

$$\frac{P(N=1 \wedge H=0 \wedge C=0)+\alpha_2}{P(H=0 \wedge C=0)+\alpha} = \frac{0.3+1}{0.5+2} = \frac{1.3}{2.5} = 0.52$$

2. P(N|H=0,C=1)

(a) P(N=0|H=0,C=1)

$$\frac{P(N=0 \wedge H=0 \wedge C=1)+\alpha_1}{P(H=0 \wedge C=1)+\alpha} = \frac{0.1+1}{0.1+2} = \frac{1.1}{2.1} = 0.5238$$

(b) P(N=1|H=0,C=1)

$$\frac{P(N=1 \wedge H=0 \wedge C=1)+\alpha_2}{P(H=0 \wedge C=1)+\alpha} = \frac{0+1}{0.1+2} = \frac{1}{2.1} = 0.4762$$

3. P(N|H=1,C=0)

(a) P(N=0|H=1,C=0)

$$\frac{P(N=0 \wedge H=1 \wedge C=0)+\alpha_1}{P(H=1 \wedge C=0)+\alpha} = \frac{0+1}{0.2+2} = \frac{1}{2.2} = 0.45$$

(b) P(N=1|H=1,C=0)

$$\frac{P(N=1 \wedge H=1 \wedge C=0)+\alpha_2}{P(H=1 \wedge C=0)+\alpha} = \frac{0.2+1}{0.2+2} = \frac{1.2}{2.2} = 0.54$$

4. P(N|H=1,C=1)

(a) P(N=0|H=1,C=1)

$$\frac{P(N=0 \wedge H=1 \wedge C=1)+\alpha_1}{P(H=1 \wedge C=1)+\alpha} = \frac{0.1+1}{0.2+2} = \frac{1.1}{2.2} = 0.5$$

(b) P(N=1|H=1,C=1)

$$\frac{P(N=1 \wedge H=1 \wedge C=1)+\alpha_2}{P(H=1 \wedge C=1)+\alpha} = \frac{0.1+1}{0.2+2} = \frac{1.1}{2.2} = 0.5$$

3.1.4 Cálculo P(C|H)

Existen 2 distribuciones:

1. P(C|H=0)

(a) P(C=0|H=0)

$$\frac{P(C=0 \wedge H=0)+\alpha_1}{P(H=0)+\alpha} = \frac{0.5+1}{0.6+2} = \frac{1.5}{2.6} = 0.5769$$

(b) P(C=1|H=0)

$$\frac{P(C=1 \wedge H=0)+\alpha_2}{P(H=0)+\alpha} = \frac{0.1+1}{0.6+2} = \frac{1.1}{2.6} = 0.4231$$

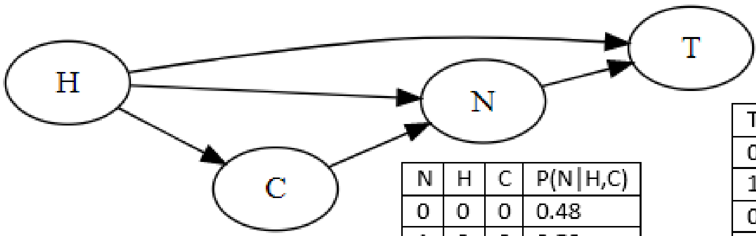
2. P(C|H=1)

(a) P(C=0|H=1)

$$\frac{P(C=0 \wedge H=1)+\alpha_1}{P(H=1)+\alpha} = \frac{0.2+1}{0.4+2} = \frac{1.2}{2.4} = 0.5$$

(b) P(C=1|H=1)

$$\frac{P(C=1 \wedge H=1)+\alpha_2}{P(H=1)+\alpha} = \frac{0.2+1}{0.4+2} = \frac{1.2}{2.4} = 0.5$$



H	P(H)
0	0.583333
1	0.416666

C	H	P(C|H)
0	0	0.5769
1	0	0.4231
0	1	0.5
1	1	0.5

N	H	C	P(N|H,C)
0	0	0	0.48
1	0	0	0.52
0	0	1	0.5238
1	0	1	0.4762
0	1	0	0.45
1	1	0	0.54
0	1	1	0.5
1	1	1	0.5

T	H	N	P(T|H,N)
0	0	0	0.5217
1	0	0	0.4783
0	0	1	0.4348
1	0	1	0.5652
0	1	0	0.4762
1	1	0	0.5238
0	1	1	0.5217
1	1	1	0.4783

Figure 2: Red bayesiana con tablas de probabilidad.

4 Inferencia

Una vez obtenido el grafo con las tablas de probabilidad de cada variable aleatoria (Figure 2), se procede a inferir resultados utilizando el dataset de prueba (Table 2). Para ello, se empleará la estimación de Maximo a Posteriori (MAP). Se infiere el valor de la variable "C" de los 3 datos del dataset de prueba:

1. Cuando T = 0, H = 1, N = 0 y C = ?

$$clase.max = max_c[P(C=0,T=0,H=1,N=0), P(C=1,T=0,H=1,N=0)]$$

   (a) P(C=0,T=0,H=1,N=0)

$$P(C=0,T=0,H=1,N=0) =$$

$$P(C=0|H=1)*P(T=0|H=1,N=0)*P(H=1)*P(N=0|H=1,C=0) =$$

$$(0.5)*(0.4762)*(0.416666)*(0.45) = 0.04464$$

   (b) P(C=1,T=0,H=1,N=0)

$$P(C=1,T=0,H=1,N=0) =$$

$$P(C=1|H=1)*P(T=0|H=1,N=0)*P(H=1)*P(N=0|H=1,C=1) =$$

$$(0.5)*(0.4762)*(0.416666)*(0.5) = 0.04960$$

   Para el test 1, la clase es C=1.

2. Cuando T = 0, H = 1, N = 1 y C = ?

$$clase.max = max_c[P(C=0,T=0,H=1,N=1), P(C=1,T=0,H=1,N=1)]$$

   (a) P(C=0,T=0,H=1,N=1)

$$P(C=0,T=0,H=1,N=1) =$$

$$P(C=0|H=1)*P(T=0|H=1,N=1)*P(H=1)*P(N=1|H=1,C=0) =$$

$$(0.5)*(0.5217)*(0.416666)*(0.54) = 0.05869$$

(b) P(C=1,T=0,H=1,N=0)

$$P(C = 1, T = 0, H = 1, N = 1) =$$

$$P(C = 1|H = 1) * P(T = 0|H = 1, N = 1) * P(H = 1) * P(N = 1|H = 1, C = 1) =$$
$$(0.5) * (0.5217) * (0.416666) * (0.5) = 0.05434$$

Para el test 2, la clase es C=0.

3. Cuando T = 0, H = 0, N = 1 y C = ?

$$clase.max = max_c[P(C = 0, T = 0, H = 0, N = 1), P(C = 1, T = 0, H = 0, N = 1)]$$

(a) P(C=0,T=0,H=0,N=1)

$$P(C = 0, T = 0, H = 0, N = 1) =$$

$$P(C = 0|H = 0) * P(T = 0|H = 0, N = 1) * P(H = 0) * P(N = 1|H = 0, C = 0) =$$
$$(0.5769) * (0.4348) * (0.583333) * (0.52) = 0.07608$$

(b) P(C=1,T=0,H=0,N=0)

$$P(C = 1, T = 0, H = 0, N = 1) =$$

$$P(C = 1|H = 0) * P(T = 0|H = 0, N = 1) * P(H = 0) * P(N = 1|H = 0, C = 1) =$$
$$(0.4231) * (0.4348) * (0.583333 * (0.4762) = 0.05110$$

Para el test 3, la clase es C=0.