Topics in Behavioral Decisons in Finance

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4. Applications and Limitations of CPT

Three sample applications to highlight:

i) how to apply CPT
ii) interesting applications on ???
iii) key limitations of CPT

4.1. Azevedo and Gottlieb (2012,JET)

- What happens in strategic and market environments?
- Consider monopolistic insurer, CPT customer
  - $\Rightarrow$ Monopolist can charge any arbitrarily large price ploicy features unbounded gains or losses with probability approaching zero.
  - $\Rightarrow$ Counterintuitive: insurance raises risk; no solution to firm's problem
- For any insurance policy, there is always another policy that makes both the firm and consumers simultaneously better off.
- If consumer's wealth is bounded, the insurer can extract all wealth with probability approaching one. (Bertrand competition, no equilibrium exists)

4.1. Azevedo and Gottlieb (2012,JET)

Additionally,

i) individuals pay arbitrarily large amounts for lotteries with finite expected value
ii) individuals refuse all actuarially fair gambles

Case

(i) implies no equilibrium if supply is endogenous.
(ii) rules out simultaneous risk-seeking and aversion.

(why we use CPT in the first place)

Model

- Gamble: $\mathcal{L} = (g, p; L, 1-p), g \geq 0, -L \leq 0$
- CPT: $V(\mathcal{L}) = w^+(p)v(g) - \lambda w^-(1-p)v(L)$.
  $w^+, w^- : [0,1] \to [0,1], \quad v : \mathbb{R}_+ \to \mathbb{R}$
- Assumption: v is continuous, strictly increasing $, v(0) = 0$
  w^+, w^- are continuous, strictly increasing,
  $$w^+(0) = w^-(0) = 0;\ w^+(1) = w^-(1) = 1$$
- Risk-neutral firm designs $\mathcal{L}$, perfect information, monopolist.
- profit $: \pi(\mathcal{L}) = (1-p)L - pg$
  $\Rightarrow$ maximise expected profit s.t. $V(\mathcal{L}) \geq 0$ (participate)

4.1. Azevedo and Gottlieb (2012,JET)

Proposition

suppose either :

(1) $\lim_{p\to 0} w^+(p)v(\frac{1}{p}) = \infty$ *,or*

(2) $\lim_{p\to 0} w^-(p)v(\frac{1}{p}) = 0$

Then, for any K_1, K_2*, there exists a lottery* $\mathcal{L}$ *s.t.* $\pi(\mathcal{L}) \geq K_1$ *and* $V(\mathcal{L}) > v(K_2)$
In particular, $\prod(V) = \infty$.

Proof:

- Consider the lottery$(\frac{1}{p}, p; \frac{1+\pi}{1-p}, 1-p)$.
- Assum(1) $\Rightarrow \mathbb{E}[\cdot] = (1-p)\frac{1+\pi}{1-p} - \frac{1}{p}p = \pi$
  and $\lim_{p\to 0} w^+(p)v(\frac{1}{p}) - \lambda w^-(1-p)v(\frac{1+\pi}{1-p}) \geq \lim_{p\to 0} w^+(p)v(\frac{1}{p}) - \lambda v(1+\pi)$
  $\Rightarrow$ arbitrarily high utility, profit $\pi > 0$.

- Assum(2). Consider:$(\frac{u}{p}, p; \frac{\pi+u}{1-p}, 1-p), u, \pi \geq 0.$
  $\mathbb{E}[\cdot] = (1-p)\frac{\pi+u}{1-p} - \frac{u}{p}p = \pi$
- CPT:$w^{+}(p)v(\frac{u}{p}) - \lambda w^{-}(1-p)v(\frac{u+\pi}{1-p})$
- the limit:
  $\lim_{p\to 1} w^{+}(p)v(\frac{u}{p}) - \lambda w^{-}(1-p)v(\frac{u+\pi}{1-p})$
  $= v(u) - \lambda \lim_{p\to 0} w^{-}(p)v(\frac{\pi+u}{p}) = v(u)$
- $\Rightarrow$ any profit $\pi \geq 0$,any utility $v(u) \in [0, v(\infty))$,p close to 1.

4.1. Azevedo and Gottlieb (2012,JET)

Interpretation:

(1) Cond.1: lottery, large sum $\frac{1}{p}$ with low p,and 0 else

- Expected value is one, gain $\frac{1}{p}$ grows unboundedly as $p \to 0$.
- Customer pays arbitrary amounts for expected value of 1.
- $\Rightarrow$ firm infinite expected profits, gives arbitrarily large certainty equivalents.

(2) Cond.2: large loss $\frac{1}{p}$ utility

- Expected payment $= -1$,unbounded loss $\frac{1}{p}$
- $\Rightarrow$ arbitrarily small risk premium.
- Customer accepts negative expected payoff for arbitrarily small prices, firm again has infinite profits,customer infinite utility.
- Catastrophe bonds below fair price.

4.1. Azevedo and Gottlieb (2012,JET)

- Under both 1 and 2:
  firm has infinite profits for any lottery, there always is a better one for both firm and customers as $p \to 0$.
- Additionally, $v(x) = x^{\alpha}, 0 < \alpha \leq 1$.
- $\Rightarrow$ homogeneous of degree α.
- For any $\mathcal{L} = (g, p; L, 1-p)$ and $c\mathcal{L} = (cg, p; cL, 1-p)$,
  $V(c\mathcal{L}) = c^{\alpha} V(\mathcal{L}), c > 0$.
- $\Rightarrow v(c\mathcal{L}) \geq V(\mathcal{L})$.
- If individual accepts $\mathcal{L}$, he must also accept $c\mathcal{L}$.
- $\Rightarrow$ firm profit $\pi \to c\pi$.
- Hence, if seller can obtain a positive profit, it can obtain any positive profit.

4.1. Azevedo and Gottlieb (2012,JET)

- The choice of ω empirically does not matter.
- all estimated forms of ω satisfy our conditions.
- adding a reference point $r \neq 0$ does not change anything.
- Competition *àla* Bertrand $\Rightarrow$ no equilibrium.
- Heterogeneity does not matter for result.
- Wealth constraints $B > 0$ bounds profits, rest remains
- one solution: global $+$ local utility (next lecture)

4.2. Barberis (2012,MS)

- Why do people gamble?
- Why can CPT shed light? Loss acersion?

  Even probabilityies $\Rightarrow$ unappealing $\neq$ lottery
- General point: time inconsistency.
- shows:
  - CPT agent gambles for many parameters, no skewness and zero or negative expected value.
  - CPT implies time inconsistency.
  - 50/50 bets.
  - Inityal plan: play if winning, quit on lossing.
    isolated bet unattractive,
    c???? bets good: low loss, high ???,
    probability weighting: expected value: $p(5 * winning)$
    $\frac{1}{5}^5 = \frac{1}{32} \Rightarrow$ low, overweighted
    after a winning bets:$\frac{1}{2} \Rightarrow$ underweighted
    $\Rightarrow$ preference change over time.

4.2. Barberis (2012,MS)

- predicts heterogeneity in gambling behavior:
  1. naive: plans, deviate from plan.
  2. sophisticated:knows time incosistency, cannot commit
     $\Rightarrow$ predicts losing
  3. sophisticated with commitment
     $\Rightarrow$ predicts behaviors, sticks with (eg: only takes limited cash).

Model:

- same as before:

$$v(x) = \begin{cases} x^{\alpha} & \text{for} \quad x \geq 0, \\ -\lambda(-x)^{\alpha} & \text{for} \quad x < 0, \end{cases} \tag{1}$$

  and

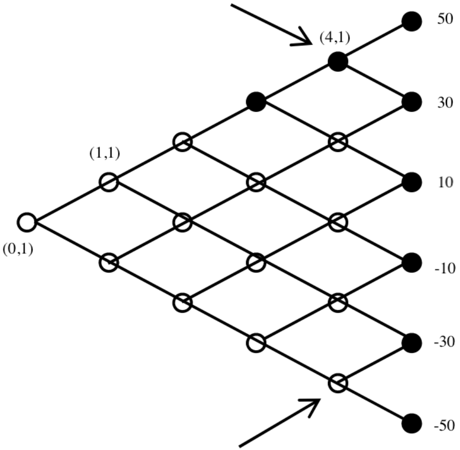
$$\omega(p) = \frac{p^{\delta}}{(p^{\delta}+(1-p)^{\delta})^{\frac{1}{\delta}}}$$

- $\alpha \in (0,1], \lambda > 1, \delta \in (0,1)$
- $T+1$ dates, $t = 0, 1, ..., T$.
- at $t = 0$, $50:50$ bet: gain /lose h.
  at any time bet, if declines once, game is over.

4.2. Barberis (2012,MS)

Take $T = 5$ for example

- Binomial Tree Representation



(black: no gamble; white: gamble)

Binomial Tree Representation

- each node (t, j).
- $t \in 0, \ldots, T$
- $j \in 1, \ldots, t+1$, how far down:$j = 1$ top node.
- Assume: $r = 0$,CPT of whole evening
- time inconsistency:
  $(4, 1)$: initial probability $\frac{1}{2}$: gain 50 or down 30 $\Rightarrow$ gamble
- if actually at $(4, 1)$:
  $v(40) \geq v(50)\omega(\frac{1}{2}) + v(30)(1 - \omega(\frac{1}{2})) \Rightarrow$ usually exit.
  $\Rightarrow v(40) - v(30) \geq (v(50) - v(30))\omega(\frac{1}{2})$; holds for all $\alpha, \delta \in (0, 1)$.
- similar: initial plan: stop at (4,5), not keep gambling

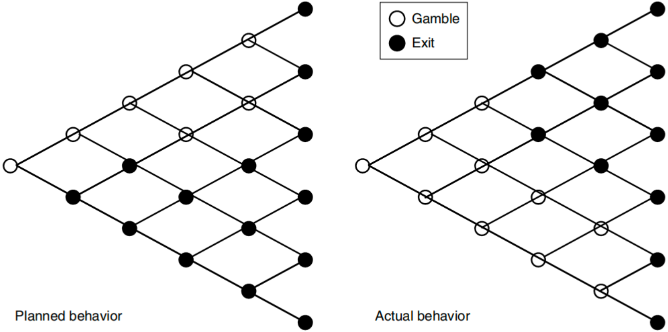
4.2. Barberis (2012,MS)

- general description for fixed strategies $s \in S(0,1)$
- $\tilde{g_s}$: the accumulated winnings or losses
- $\tilde{g_s} \backsim (30, \frac{7}{32}; 10, \frac{9}{32}; -10, \frac{10}{32}; -30, \frac{5}{32}; -50, \frac{1}{32})$
- $\Rightarrow \max_{s \in S(0,1)} V(\tilde{g_s})$
- generally no analytically solution.

4.2. Barberis (2012,MS)

Numerical analysis
naive agent

- many parameters enter, that is $V(\tilde{g}_s) > 0$
- deviations for naive agent: $(\alpha, \delta, \lambda) = (0.95, 0.5, 1.5)$



4.2. Barberis (2012,MS)

- mostly: plan a "loss exit" plan
- some: plan a "gain exit" plan
- probility weighting most important

sophisticated without commitment

- aware of time incosistency
- uses backward induction
- node $(t, j), t \in [0, T-1]$
  $V(\tilde{g_{t,j}}) > v(h(t+2-2j))$;
  $v(h(t+2-2j))$: leave immediately;
  $V(\tilde{g_{t,j}})$: the value of continuing to gamble.
  $\Rightarrow$ rarly enters casino; only if α, λ small,δ large

4.2. Barberis (2012,MS)

sophisticated with commitment

- same as naive agent, but goes through
- no time inconsistency
- want to gamble in the region of losses
- take a small amount of cash

4.2. Barberis (2012,MS)

Proposition

- An EU maximiser stops all Brownian motions with ??? and small variance at every wealth level where his utility function is of expanetial growth.
- $\Rightarrow$ ever ???? stops immediately.

Some Implications

- ??? ,infinite horizon casino $dX_t = \mu dt + \sigma dW_t, \mu(x) \equiv \mu < 0, \sigma(x) = \sigma > 0$

  $\Rightarrow$ gamble utility ??? , appears almost ???.
- American option: $\max(X_t - \mu, 0)$ if ex. at time t.
  $dX_t = X_t(\mu dt + \sigma dW_t)$
- $\Rightarrow$ never exerise real option, even if $X_0 > \mu$
- no disposition ????

Constraints

- naive
- probability weighting dominant.