Topics in Behavioral Decisons in Finance

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4.3 Ebert and Strack, 2015, AER

- dynamic perspective on gambling/ CPT decisions
- CPT agent never stops gambling/ investing
- "skewness preference in the small"
- $\Rightarrow$ simple, small, lottery-like risk with negative expectation.
- If I lose a bit more, I stop. investment: never exercise attractive options key assumption: probability weighting domininate loss aversion

4.3 Ebert and Strack, 2015, AER

- Formally, binary risk: $L = (g, p, L, 1-p)$

$$CPT = \begin{cases} [1 - w^+(p)]V(L) + w^+(p)V(g), & r \leq L \\ w^-(1-p)V(L) + w^+(p)V(g), & L < r \leq g \\ w^-(1-p)V(L) + [1 - w^-(1-p)]V(g), & g < r \end{cases} \tag{1}$$

- $w^-, w^+ : [0,1] \to [0,1]$, non-decreasing
- $w^+(0) = w^-(0) = 0$
- $w^+(1) = w^-(1) = 1$
- continuous, increasing V: $\mathbb{R} \to \mathbb{R}$ with $V(r) = 0$

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Assumption 1

- The value function U has finite left and right derivatives, $\partial_- V(x)$ and $\partial_+ V(x)$ at every wealth level x.
- Further, $\lambda = \sup_{x\in\mathbb{R}} \frac{\partial_- V(x)}{\partial_+ V(x)} < \infty$ exists.
- $\Rightarrow$??? too extreme, no infinite loss aversion(KT,1979 ??)

Assumption 2

There exists at least one $p \in (0, 1)$ such that:

(i) $w^+(p) > \frac{\lambda}{1-p+\lambda p} = b_\lambda(p)$

(ii) $w^-(1-p) < \frac{1-p}{1-p+\lambda p} = b_{1/\lambda}(1-p)$

(satisfied by any dominan function of w)

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- $b_\theta(p) = \frac{\theta p}{1-p+\theta p}$, $(\theta = 1)$
- $b_\theta(0) = 0, b_\theta(1) = 1$
- $\theta > 1 \Rightarrow$ concave, above 45°
- $\theta < 1 \Rightarrow$ convex, below 45°
- $\Rightarrow$ A2 satisfied: $\omega^+(5\%) > b_\lambda(5\%)$; $\quad \omega^-(95\%) > b_{\frac{1}{\lambda}}(95\%)$
- At least one probability sufficiently overweighted by $\omega^+ : \omega^+(p) > b_\lambda(p) \geq p$
- and the complimentary probability $1-p$ is ?? by $\omega^- : \omega^-(1-p) < b_{\frac{1}{\lambda}}(1-p) \leq 1-p$.

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- Under Assumptions 1 and 2, for every wealth level there exists an attractive zero-mean binary lottery that is arbitrarily small.

Corollary

- Under Assumptions 1 and 2, for every wealth level there exists an attractive, arbitrarily small binary lottery with negative mean.

- generally, risk aversion is defined as every fair risk

- Here: skewness preference in the small thus implies that, at every wealth level, a CPT agent is not risk averse

- $\Rightarrow$ sufficiently small risks are attractive to CPT

- Azevedo and Gottlieb (2012) $\Rightarrow$ skewness preference in the large, $p \approx 7.2$ for CPT weight $\Rightarrow$ not so small!

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Dynamic Consequences

- what does skewness preference in the small do?
- consider Markov diffusion $X = (X_t)_{t\in\mathbb{R}_+}$; $dX_t = \mu(X_t)dt + \sigma(X_t)dW_t$,
- $(W_t)_{t\in\mathbb{R}_+}$: a Brownian motion
- $\mu : \mathbb{R} \to \mathbb{R}$
- $\sigma : \mathbb{R} \to (0, \infty)$ Lipschitz continuous
- Investment or gambling strategies : integrable stopping times τ, adapted filtration $(\mathcal{F}_t)_{t\in\mathbb{R}_+}$ (all avaliable information)
- CPT utility of strategy τ given $\mathcal{F}_t$, $CPT(X_\tau, \mathcal{F}_t)$ (time inconsistency)
- $\Rightarrow \tau$ may be changed later

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- A naive investor does not anticipate that
- $\Rightarrow$ stops at t if $CPT(X_\tau, \mathcal{F}_t) \leq CPT(X_t, \mathcal{F}_t) = U(X_t)$
- $U(X_t) \geq \sup_{\tau \geq t} CPT(X_\tau, \mathcal{F}_t)$

Theorem 2

- Under Assumptions 1 and 2, the naive CPT agent never stops. (preference point can be dynamic and change)
- Information : simple two-threshold strategy: stops if utility drops, continue utility raised.
- a lot always exceeds stopping.