Topics in Behavioral Decisons in Finance

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5. Alternative Theories of Choice under Risk

5.1 Reference-Dependent Risk Attitudes

Kszegi, Rabin (2007, AER)

- choice of reference point in CPT exogenous.
- has huge impact for economic choices
- $\Rightarrow$ literature on disposition effect
- new model:
  - combines gain/loss utility with standard consumption utility
  - Rabin(2000):(50/50, $+$550,-500) $\Rightarrow$ $(+100,000,000,000,000,-4,000)$ $\Rightarrow$ reject
  - endogenously determines reference point
  - allows for stochastic reference points.

5.1 Reference-Dependent Risk Attitudes

1. surprise, low probability decisions, exogenous expectations; say pay \$55 to insure \$100 risk, 50%
   (a) status quo:0. $\rightarrow$ diminishing sensitivity do not insure
   (b) expect to pay 55: lose 45 gain 55 . loss aversion $\rightarrow$ insure.
2. anticipated risks
   (a) macdimahing personal equilibrium (UPE)
   behaviour where the stochastic outcome generated by utility-maximizing choices conditional on expectations equals expectations. i.e. follows planned behavior
   $\Rightarrow$ selects preferred personnal equilibrium (PPE)
   (b) choice-acclimating personal equilibrium(CPE)
   committed decision long before outcomes $\Rightarrow$ reference point influenced by choice. maximize expected utility given that if determines reference lottery and outcome lottery.
3. 1&2 : linear consumption utility$\Rightarrow$ small gambles.

   now large gambles, consumption utility non-linear $\Rightarrow$ reference point not very important.

Model

- $w \in \mathbb{R}$ wealth, $r \in \mathbb{R}$ reference:
  $U(w|r) = m(w) + \mu(m(w) - m(r))^2$
- reference point: belief of outcomes, g probility measure
  $U(w|g) = \int u(w|r)\, dg(r)$
  $\Rightarrow$ mixed feelings, $50 : 500/100; 50$:gain to 0, loss to 100.
- w has measure F
  $U(F|g) = \int\int u(w|r) dg(r) dF(w)$, no probability weighting for simplicily.

Assumptions on μ

- A0: $\mu(x)$ continuous , twice differentiable for $x \neq 0, \mu(0) = 0$.
- A1: $\mu(x)$ strictly increasing.
- A2: if $y > x \geq 0$, then $\mu(y) + \mu(-y) < \mu(x) + \mu(-x)$.
- A3: $\mu^{"}(x) \leq 0$ for $x > 0$ and $\mu^{"}(x) \geq 0$ for $x < 0$.
- A4: $\frac{\mu^{'}_{-}(0)}{\mu^{'}_{+}(0)} \equiv \lambda > 1$,
  where $\mu^{'}_{+}(0) \equiv \lim_{x \to 0} \mu^{'}(|x|)$ and $\mu^{'}_{-}(0) \equiv \lim_{x \to 0} \mu^{'}(-|x|)$.
- A2,A4: loss aversion
- A3: diminishing sensitivity
- A3': $\forall x \neq 0, \mu^{"}(x) = 0 \Rightarrow$ no diminishing sensitivity.
- general assumption, reference point $\neq$ expectations;rational expectations, generally people have some idea how they behave and their own environment.

1. Risk aversion in surprise situations

- m is linear: $m(w) = w$
- reference point fixed

Proposition (1)

Suppose $m(\cdot)$ is linear and $\mu(\cdot)$ satisfies A3'(no diminishing sensitivity). For any lotteries F, G, H, and constant w, if $U(w + F|g) \geq U(w|w)$, the $U(H + F|G) \geq U(H|G)$.

$\Rightarrow$

- If willing to accept F relative to riskless r, positive values of F are gains, negative ones are lossed.
- If F is added to lottery H relative to lottery G, positive outcomes of F eliminate losses from H relative to G, losses from F merely eliminate gains from H.
- $\Rightarrow$ more willing to accept F.
- If $H = w, G = F$ $\Rightarrow$ less risk averse in eliminating risk that is expected.

1. Risk aversion in surprise situations

Proposition (2)

Suppose $m(\cdot)$ is linear. For any lottery F with positive expected value:

(i) *There exist $A, \varepsilon > 0$ such that if G has $\Pr_G[r \in (k-A, k+A)] < \varepsilon$ for all constants k, then $U(H+F|G) > U(H|G) > U(H-F|G)$ for any lottery H.*

(ii) *For any continuously distributed lottery G, there is a $\bar{t} > 0$ s.t. for any $t \in (0, \bar{t}]$ and any lottery H, $U(H + t \cdot F|G) > U(H|G) > U(H - t \cdot F|G)$.*

Identifies attitudes towards F in which risk neutral.

(i) If sufficiently widely distributed reference lottery, accept F, and reject -F.

(ii) If fixed continuously distributed reference lottery, accept sufficiently small multiple of F, reject the same multiple of -F.

Prop. 1&2 do not imply no risk aversion, just lower risk aversion!

2. UPE and PPE Risk Attitudes

- now correctly anticipates choice set.
- connot commit to choice untill shortly before outcome
  $L = \{D_1, 1-q; D_2, q\}, D_1, D_2 \in \triangle(\mathbb{R})$
- for now $q = 0 \Rightarrow$ choice set is certain
- beliefs are set, reference point exogenous.

Definition (UPE)

A selection $F_1 \in D_1, F_2 \in D_2$ is an unacclimating personal equilibrium (UPE) if for each $l \in 1, 2$ and any $F_l' \in D_l$, $U\left(F_l \mid (1-q)F_1 + qF_2\right) \geq U\left(F_l' \mid (1-q)F_1 + \ qF_2\right)$.

(Koszegi, 2005 proves existence.)

- If the person expects to choose F_1 and F_2 from choice sets D_1 and D_2, then she expects the distribution of outcomes $(1-q)F_1 + qF_2$.
- Def. 1: If this is the expectation. she should be willing to choose F_1 and F_2.

2. UPE and PPE Risk Attitudes

Example:
wealth w, 50/50 chance $0, -100$ or pay -55, when is the lottery a UPE?

$$\begin{aligned} &\left[\frac{1}{2}(w \ -100) + \frac{1}{2}w\right] \\ &\qquad + \left[\frac{1}{4}\mu(100) + \frac{1}{4}\mu(-100)\right] \\ \geq &[w - 55] \\ &+ \left[\frac{1}{2}\mu(45) + \frac{1}{2}\mu(-55)\right] \end{aligned}$$

- UPE generally not unique.
- expectation: plan what to do at the time.
- idea: select best plan she will follow through on.

2. UPE and PPE Risk Attitudes

Definition (PPE)

A selection $F_1 \in D_1, F_2 \in D_2$ is a preferred personal equilibrium(PPE) if it is a UPE, and
$U\left((1-q)F_1 + qF_2 \mid (1-q)\mathrm{F}_1 + qF_2\right) \geq U\left((1-q)\mathrm{F}'_1 + q\ \mathrm{F}'_2 \mid (1-q)\mathrm{F}'_1 + q\ \mathrm{F}'_2\right)$ for all *UPE* selections $F'_l \in D_1, F'_2 \in D_2$.

$\Rightarrow$ choice optimal given expectations!

2. UPE and PPE Risk Attitudes

Proposition (3)

Suppose $m(\cdot)$ is linear. For any $w \in \mathbb{R}$ and mean-zero lottery $F \neq 0$ with bounded support, there exist $\bar{k}, \bar{t} > 0$ such that for any positive $t < \bar{t}, k < \bar{k}$, the unique PPE with the choice set $\{w, w + t(F + k)\}$ is to choose w.

- select riskless w over a sufficiently small, better-than-fair but unattractive bet.
- loss aversion makes gamble unattractive.
- key point: CPT: costs $\neq$ loss in status quo.
  here: expected costs, such as insurance premium is a cost, not a loss.
- $\Rightarrow$ explain insurance for likely events

2. UPE and PPE Risk Attitudes

Proposition (4)

Suppose $m(\cdot)$ is linear, $\mu(\cdot)$ satisfies A3'.
If $w + F$ is a PPE in the choice set $\{w, w + F\}$, then for any lottery H,
$U(w + F \mid H) > U(w \mid H)$.

$\Rightarrow$ If choose between risk and insurance, at least a risk aversion as ????
$\Rightarrow$ in experiments , people generally are in surprise settings / don't know what comes
$\Rightarrow$ underestimate risk aversion
Additionally (see papers) expecting risk decreases risk aversion.

3. CPE Risk Attitudes

- long commited choices

Definition (CPE)

For any choice set $D, F \in D$ is *a* choice-acclimating personal equilibrium (CPE) if $U(F \mid F) \geq U\left(F' \mid F'\right)$ for all $F' \in D$.

$\Rightarrow$ selecting F determines it as reference point.

Example:

lottery 50/50 with (0,-100) is CPE if:

$$\begin{aligned}
&\left[\frac{1}{2}(w-100)+\frac{1}{2} w\right] \\
&+\left[\frac{1}{4} \mu(100)+\frac{1}{4} \mu(-100)\right] \\
\geq &\, [w-55]+[0].
\end{aligned}$$

- UPE: premium 55 can be gain or loss;
- CPE: premium 55 is neither gain nor loss.

3. CPE Risk Attitudes

Some implications:

- unlike as UPE,PPE, for CPE pepple may want to choose stochastialy dominated options
- idea: give up unlikely gain to avoid losses

5.2 Salience Theory

Bordalo, Gennaioli, Shleifer (2012,QJE)

- risk preferences not stable
- Allais (1953) paradoxes:irrelevant choice implies risk lottery behavior.
- idea: salience to prominent outcomes
- building blocks:
  - ordering
  - diminishing sensitivity
  - salience weighting ($\neq$ probility weighting)
- say consumer choice: speed, price, design

5.2 Salience Theory

Allais (1953) paradoxes

$$L_1(z) = \begin{cases} \$2500 & \text{with prob.} & 0.33 \\ \$0 & & 0.01 \\ \$z & & 0.66 \end{cases}$$

$$L_2(z) = \begin{cases} \$2400 & \text{with prob.} & 0.34 \\ \$z & & 0.66 \end{cases}$$

In experiments: $z = 2400$

$$L_1(2400) = \begin{cases} 2500 \text{ with prob.} & 0.33 \\ 0 & 0.01 \prec L_2(2400) = 2400 \\ 2400 & 0.66 \end{cases}$$

$\Rightarrow$ risk averse

5.2 Salience Theory

In experiments: $z = 0$

$$L_1(0) = \begin{cases} 2500 \text{ with prob.} & 0.33 \\ 0 & 0.67 \end{cases} \succ L_2(0) = \begin{cases} 2400 \text{ with prob.} & 0.34 \\ 0 & 0.66 \end{cases}$$

$\Rightarrow$ risk loving

- CPT: in last gamble $w(p = 0.01)$ overweighted
- Salience theory: $L_1(2400), L_2(2400) \rightarrow$ 2500 only slightly higher than 2400; 0 a lot lower than 2400.
- Salience theory: $L_1(0), L_2(0) \rightarrow$ outcome zero is standard; 2500 stands out more.

Model

- $s \in S$: state
- π_s: probability, s.t. $\sum_{s \in S} \pi_s = 1$
- $\{L_1, L_2\}$: choice set
- x_s^i: payoffs
- value function as before, reference dependent V without decision weights, only local thinking:

$$V\left(L_i\right) = \sum_{s \in S} \pi_s v\left(x_s^i\right).$$

- With salience distortion:
- two steps: salience ranking, then decision weight π_s^i
- Formally: $\mathrm{x}_s = \left(x_s^i\right)_{i=1,2}$, payoffs in state s
- x_s^{-i}: payoff of lottery L_j, $j \neq i$
- $x_s^{\min}, x_s^{\max}$: largest / smallest payoffs in x_s

Model

Definition (1)

The salience of state s for lottery $L_i, \mathrm{i} = 1, 2$, is a continuous and bounded function $\sigma\left(x_s^i, x_s^{-i}\right)$ that satisfies three conditions:

- 1. Ordering. If for states $s, \tilde{s} \in S$ we have that $\left[x_s^{\min}, x_s^{\max}\right] \in \left[x_{\tilde{s}}^{\min}, x_{\tilde{s}}^{\max}\right]$, then

$$\sigma\left(x_s^i, x_s^{-i}\right) < \sigma\left(x_{\tilde{s}}^i, x_{\tilde{s}}^{-i}\right).$$

- 2. Diminishing sensitivity. If $x_s^j > 0$ for $j = 1, 2$, then for any $\epsilon > 0$,

$$\sigma\left(x_s^i + \epsilon, x_s^{-i} + \epsilon\right) < \sigma\left(x_s^i, x_s^{-i}\right).$$

- 3. Reflection. For any two states $s, \tilde{s} \in S$ s.t. $x_s^j, x_{\tilde{s}}^j > 0, (j = 1, 2)$, we have $\sigma\left(x_s^i, x_s^{-i}\right) < \sigma\left(x_{\tilde{s}}^i, x_{\tilde{s}}^{-i}\right)$
  if and only if $\sigma\left(-x_s^i, -x_s^{-i}\right) < \sigma\left(-x_{\tilde{s}}^i, -x_{\tilde{s}}^{-i}\right)$.

Model

Example:

$$\sigma\left(x_s^i, x_s^{-i}\right) = \frac{\left|x_s^i - x_s^{-i}\right|}{\left|x_s^i\right| + \left|x_s^{-i}\right| + \theta}, \theta > 0$$

- ordering: salience rises if distance of x_s^i and x_s^{-i} rises
- diminishing sensitivity: as average payoff gets farther from zero, salience reduces $\left|x_s^1\right| + \left|x_s^2\right|$
- reflection: salience is shaped by the magnitude, not sign
- (example: symmetric additional dropped for $N > 2$)
- Results mostly driven by ordering and diminishing sensitivity.

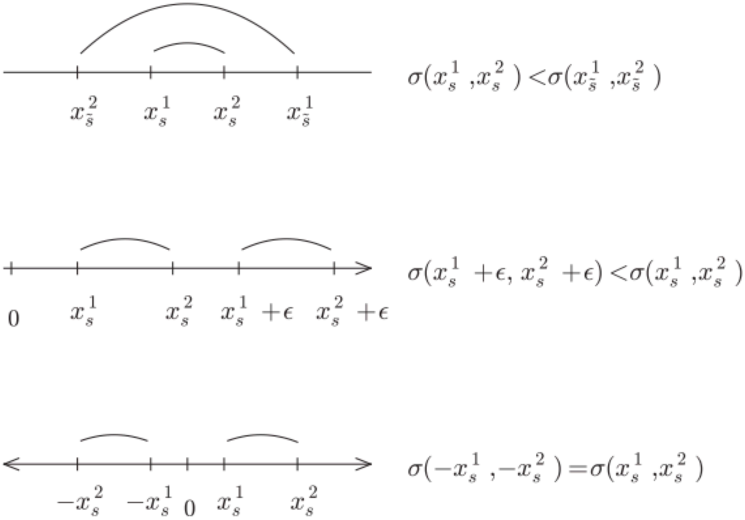

FIGURE I

Properties of a Salience Function, Equation (5)

Model

Definition (2)

Given states $s, \tilde{s} \in S$, we say that for lottery L_i s s is more salient than $\tilde{s}$ if $\sigma\left(x_s^i, x_s^{-i}\right) > \sigma\left(x_{\tilde{s}}^i, x_{\tilde{s}}^{-i}\right)$. Let $k_s^i \in \{1, \ldots, |S|\}$ be the salience ranking of state s for L_i, with lower k_s^i indicating higher salience. All states with the same salience obtain the same ranking(no jumps). Then the local thinker transforms the odds $\frac{\pi_{\tilde{s}}}{\pi_s}$ of $\tilde{s}$ relative to s into the odds $\frac{\pi_s^i}{\pi_s^i}$, given by:

$$\frac{\pi_{\tilde{s}}^i}{\pi_s^i} = \delta^{k_{\tilde{s}}^i - k_s^i} \cdot \frac{\pi_{\tilde{s}}}{\pi_s}, \quad \text{where } \delta \in (0, 1]$$

By normalizing $\sum_s \pi_s^i = 1$ and defining $\omega_s^i = \frac{\delta^{k_s^i}}{\left(\sum_r \delta^{k_r^i} \cdot \pi_r\right)}$

the decision weight is: $\pi_s^i = \pi_s \cdot \omega_s^i$

$\Rightarrow$ local thinker overweights most salient states.

Model

- $\delta = 1 \Rightarrow$ standard model, $\omega_s^i = 1$
- $\delta < 1 \Rightarrow$ local thinker
- state s is overweighted, if $\left(\omega_s^i > 1\right.$, or $\left.\delta^{k_s^i} > \sum_r \delta_r^{k_r^i} \cdot \pi_r\right)$
- $\delta \rightarrow 0$: decision based on most salient state
- δ is independent of objective state probabilities!

Some remarks:

- weighting depends on salience, not low-probability
- low-probility can be most overweighted, but also underweighted
- choice of state space / alternative lottery unclear
- some cases: no alternative $\Rightarrow$ take zero? Open question

Risk attitudes:

- suppose linear value function
- $L_0 = (x, 1)$: sure prospect
- $L_1 = (x + g, \pi_g; x - l, 1 - \pi_g)$, with $g\pi_g = (1 - \pi_g)\, l$: mean preserving spread, $x, g, x - l > 0$
- $s_g = (x + g, x), s_l = (x - l, x)$, :two states
- $V^{LT}(L) = \sum_{s \in S} \pi_s^i v\left(x_s^i\right) = \sum_{s \in S} \pi_s \omega_s^i v\left(x_s^i\right)$

Risk attitudes:

- $\delta < 1 \Rightarrow$ prefer L_1 if s_g more salient, $\sigma(x+g,x) > \sigma(x-l,x)$
- Using $g\pi_g = (1-\pi_g)\, l$, $\sigma\left(x + \frac{1-\pi_g}{\pi_g} \cdot l, x\right) > \sigma(x-l,x)$
- holds, if $\pi_g \simeq 0$ (gain unlikely) because g is high $\Rightarrow$ risk taking
- diminishing sensitivity: if $g = l, x - l < g + x$ implies that the loss is salient,$\pi_g = \frac{1}{2} \Rightarrow$ risk averse
- $\Rightarrow \pi_g^* < \frac{1}{2}$, below risk seeking, above averting

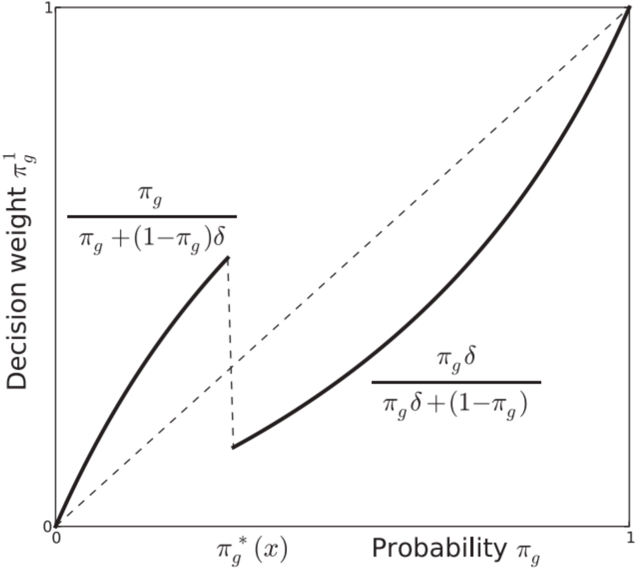

FIGURE II

Context-Dependent Probability Weighting Function

Risk attitudes:

Definition (3)

A salience function is convex if, for any state with positive payoffs (y, z) and any $x, \epsilon > 0$, the difference $\sigma(y + x,\, z + x) - \sigma(y + x + \epsilon, z + x + \epsilon)$ is a decreasing function of the payoff level x.
(concave if increasing in x).

Lemma (1)

If the salience function is convex, then $r = v^{LT}(L_0) - v^{LT}(L_1)$ weakly decreases with x.
(concave if increases with x).

$\Rightarrow$ if diminishing sensitivity weakens with x, a higher payoff level raises the relative attractiveness of L_1.
(π_g^* increases, raises risk seeking)

Some critical remarks:

- Kontek (2016, EL): certainty equivatent not necessarily defined
- monotonicity for $N > 2$ violated
- mixed experimental evidence if estimated from indifference curves
- also strong empirical support
- $\Rightarrow$ room for research