QUESTION Nº 2

GROUP 20

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$$\det\left(\begin{pmatrix} 4 & 8 & -1 & -2 \\ -2 & -8 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right) = 0 \quad (1)$$

$$\Rightarrow \det\begin{pmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -8-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{pmatrix} = 0$$

$$\underline{\underline{(4-\lambda)}}\underbrace{\begin{bmatrix} -8-\lambda & -2 & -4 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{bmatrix}}_{A'} - 8\underbrace{\begin{bmatrix} -2 & -2 & -4 \\ 0 & 5-\lambda & -10 \\ -1 & -14 & -13-\lambda \end{bmatrix}}_{A''} - 8$$

$$-1\underbrace{\begin{bmatrix} -2 & -8-\lambda & -4 \\ 0 & 10 & -10 \\ -1 & -13 & -13-\lambda \end{bmatrix}}_{A'''} + 2\underbrace{\begin{bmatrix} \cancel{4-\lambda} & \cancel{8} & \cancel{-1} \\ -2 & -8-\lambda & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{bmatrix}}_{A^{IV}}$$

$$\det A' \begin{bmatrix} -8-\lambda & -2 & -4 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{bmatrix}$$

$$(-8-\lambda)\begin{bmatrix} 5-\lambda & -10 \\ -14 & -13-\lambda \end{bmatrix} + 2\begin{bmatrix} 10 & -10 \\ -13 & -13-\lambda \end{bmatrix} - 4\begin{bmatrix} 10 & 5-\lambda \\ -13 & -14 \end{bmatrix}$$

$$(-8-\lambda)\Big[(5-\lambda)(-18-\lambda)-(-14)(-10)\Big]+(2)\Big[10(-17-\lambda)-(-13)(-10)\Big]-(4)\Big[(10)(-14)-(-13)(5-\lambda)\Big]$$

$$\Rightarrow(-8-\lambda)\Big[(-65-5\lambda+13\lambda+\lambda^2)-(140)\Big]+(2)\Big[(-130-10\lambda)-(130)\Big]-(4)\Big[(-140)-(-65+13\lambda)\Big]$$

$$\Rightarrow(-8-\lambda)\Big[-65+8\lambda+\lambda^2-140\Big]+2\Big[-130-10\lambda-130\Big]-4\Big[-140+65-13\lambda\Big]$$

$$\Rightarrow(-9-\lambda)\Big[-205+8\lambda+\lambda^2\Big]+2\Big[-260-10\lambda\Big]-4\Big[-75-13\lambda\Big]$$

$$\left(1845-72\lambda-\cancel{9}\lambda^2+205\lambda-8\lambda^2-\lambda^3\right)+\left(-520-20\lambda\right)+\left(300+52\lambda\right)$$

$$\left(1845+133\lambda-17\lambda^2-\lambda^3\right)+\left(-520-20\lambda\right)+\left(300+52\lambda\right)$$

$$1845+133\lambda-17\lambda^2-\lambda^3-520-20\lambda+300+52\lambda$$

$$\boxed{1625+165\lambda-17\lambda^2-\lambda^3=\det A'}$$

* $\det A''$

$$\begin{bmatrix}-2 & -2 & -4\\ 0 & 5-\lambda & -10\\ -1 & -14 & -13-\lambda\end{bmatrix}$$

$$(-2)\begin{bmatrix}5-\lambda & -10\\ -14 & -13-\lambda\end{bmatrix}(+2)\begin{bmatrix}0 & -10\\ -1 & -13-\lambda\end{bmatrix}(-4)\begin{bmatrix}0 & 5-\lambda\\ -1 & -14\end{bmatrix}$$

$$(-2)\Big[(5-\lambda)(-13-\lambda)-(-14)(-10)\Big](+2)\Big[-10\Big](-4)\Big[\cancel{+5+\lambda}\ 5-\lambda\Big]$$

$$(-2)\Big[-65-5\lambda+13\lambda+\lambda^2-140\Big]-20-20+4\lambda$$

$(-2)\left[-205+8\lambda+\lambda^2\right]-40+4\lambda$

$410-16\lambda-2\lambda^2-40+4\lambda$

$\boxed{370-12\lambda-2\lambda^2 = \det A''}$

$\det A'' \begin{bmatrix} -2 & -9-\lambda & -4 \\ 0 & 10 & -10 \\ -1 & -13 & -13-\lambda \end{bmatrix}$

$(-2)\begin{bmatrix} 10 & -10 \\ -13 & -13-\lambda \end{bmatrix} + (+9+\lambda)\begin{bmatrix} 0 & -10 \\ -1 & -13-\lambda \end{bmatrix} - 4\begin{bmatrix} 0 & 10 \\ -1 & -13 \end{bmatrix}$

$(-2)\left[(-130-10\lambda)-(+130)\right]+(9+\lambda)\left[-10\right]-4\left[10\right]$

$(-2)\left[-130-10\lambda-130\right]+(9+\lambda)\left[-10\right]-4\left[10\right]$

$(-2)\left[-260-10\lambda\right]-90-10\lambda-40$

$520+20\lambda-90-10\lambda-40$

$\boxed{390+10\lambda = \det A'''}$

$\det A^{IV} \begin{bmatrix} -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{bmatrix}$

$(-2)\begin{bmatrix} 10 & 5-\lambda \\ -13 & -14 \end{bmatrix} + (9+\lambda)\begin{bmatrix} 0 & 5-\lambda \\ -1 & -14 \end{bmatrix} - (+2)\begin{bmatrix} 0 & 10 \\ -1 & -13 \end{bmatrix}$

(-4-)

$(-2)[(-140) - (65+13\lambda)] + (9+\lambda)[5-\lambda] - (2)[10]$

$(-2)[-140+65-13\lambda] + (9+\lambda)[5-\lambda] - 20.$

$(-2)[-75-13\lambda] + 45 - 9\lambda + 5\lambda - \lambda^2 - 20$

$150 + 26\lambda + 45 - 9\lambda + 5\lambda - \lambda^2 - 20$

$\det A^{IV} = 175 + 22\lambda - \lambda^2$

So, $(4-\lambda) \times \det A$

$\Rightarrow (4-\lambda)(1625 + 165\lambda - 17\lambda^2 - \lambda^3)$

$\Rightarrow (6500 + 660\lambda - 68\lambda^2 - 4\lambda^3 - 1625\lambda - 165\lambda^2 + 17\lambda^3 + \lambda^4)$

$\Rightarrow (6500 - 965\lambda - 233\lambda^2 + 13\lambda^3 + \lambda^4)$

$\Rightarrow \boxed{6500 - 965\lambda - 233\lambda^2 + 13\lambda^3 + \lambda^4}$ (1)

So, $-8 \times \det A''$

$-8(370 - 12\lambda - 2\lambda^2)$

$-2960 + 96\lambda + 16\lambda^2$

$\Rightarrow \boxed{-2960 + 96\lambda + 16\lambda^2}$ (2)

So$_3$, $-1 \det A'''$

$(-1)(390+10\lambda)$

$$\boxed{-390-10\lambda}$$

So$_4$, $2 \det A^{IV}$

$2(175+22\lambda-\lambda^2)$

$$\boxed{350+44\lambda-2\lambda^2}$$

Generally, the whole determinant:

$6500-965\lambda-233\lambda^2+13\lambda^3+\lambda^4-2960+96\lambda+26\lambda^2-390-10\lambda$

$+350+44\lambda-2\lambda^2=0$

$$\boxed{3500-835\lambda-209\lambda^2+13\lambda^3+\lambda^4=0}$$

~~So let's solve the equation.~~

~~div of 3500 = {±1, ±2, ±5}~~

~~So, ±1 = root~~

~~±2 = root~~

~~Let's try 5 = $3500-835(5)-209(5)^2+13(5)^3+5^4 \stackrel{?}{=} 0$~~

~~= $3500-4175-5225+1625+625 \stackrel{?}{=} 0$~~

~~=~~

Final equation $3500 - 835\lambda - 219\lambda^2 + 13\lambda^3 + \lambda^4$

Let's test divisors of 3500

~~Tests~~: Divisors of 3500 : $\pm 1, \pm 2, \pm 4, \pm 5, \pm 7, \pm 10, \pm 14, \pm 20,$
$\pm 25, \pm 28, \pm 35, \pm 50, \pm 70, \pm 100, \pm 125, \pm 140, \pm 175,$
$\pm 250, \pm 350, \pm 500, \pm 700, \pm 875, \pm 1750, \pm 3500$

Let's test some of them:

*) Test $\lambda = 5$

$= 5^4 + 13(5^3) + - 219(5^2) - 835(5) + 3500$

$= 625 + 1625 + - 5475 - 4175 + 3500$

$= 5750 - 9650$

$= -3900 \neq 0$

*) Test $\lambda = -5$

$= -5^4 + 13(-5^3) - 219(-5^2) - 835(-5) + 3500$

$= 625 + 4175 + 3500 - 1625 - 5475$

$= 1200 \neq 0$

*) Test $\lambda = 10$

$= 10^4 + 13(10^3) - 219(10^2) - 835(10) + 3500$

$= -3750 \neq 0$

*) Test $\lambda = -20$

$= -20^4 + 13(-20^3) - 219(-20^2) - 835(-20) + 3500$

$= -11400 \neq 0$

*) Test $\lambda = 35$

$= 35^4 + 13(35^3) - 219(35^2) - 835(35) + 3500$

$= 1764000 \neq 0$

*) Test $\lambda = 14$

$= 14^4 + 13(14^3) - 219(14^2) - 835(14) + 3500$

$= 22974 \neq 0$

*) Test $\lambda = -14$

$= -14^4 + 13(14^3) - 219(14^2) - 835(14) + 3500$

$= -24990 \neq 0$

*) Test $\lambda = 7$

$= 7^4 + 13(7^3) - 219(7^2) - 835(7) + 3500$

$= -6216 \neq 0$

*) Test $\lambda = -2$

$= -2^4 + 13(-2^3) - 219(-2^2) - 835(-2) + 3500$

$= 4206 \neq 0$

So, There are no roots for this equation implies that there is no Eigen value nor Eigen vector for the provided matrix.

All of us:

1) Christian

2) Carine

3) Eva

4) Thierry

Have contributed to the work.

— Group 80 —