

# Bayesian Inference Calculations

## 1 Problem Context

Before selecting the final case, a voting process was conducted. Carine, Eve Nice, Thierry, and Christian all chose the medical case. As a result, the topic "Medical practicalities using Bayes' Theorem" received unanimous approval from all group members. Group 20 we are going to chose a real-world problem where Bayesian probability is applicable is in medicine, defining calculating Bayesian inference to update the probability of a hypothesis (having a disease) given new evidence (a positive test result).

## 2 Assumed Probabilities

- $P(H)$  (Prior probability of having the disease): 0.01 (1
- $P(\text{not } H)$  (Probability of not having the disease): 0.99
- $P(E/H)$  (Likelihood of positive test given disease, sensitivity): 0.95
- $P(E/\text{not } H)$  (False positive rate, 1 - specificity): 0.05

## 3 Calculations

### 3.1 Step 1: Total Probability of Evidence ( $P(E)$ )

Calculating total probability of evidence using the law of total probability:  $P(E) = [P(E/H) * P(H)] + [P(E/\text{not } H) * P(\text{not } H)]$   $P(E) = (0.95 * 0.01) + (0.05 * 0.99)$   $P(E) = 0.0095 + 0.0495 = 0.059$

### 3.2 Step 2: Posterior Probability ( $P(H/E)$ )

Calculating posterior probability using Bayes' theorem:  $P(H/E) = (P(E/H) * P(H)) / P(E)$   $P(H/E) = (0.95 * 0.01) / 0.059$   $P(H/E) = 0.0095 / 0.059$  approx 0.1610 (16.10

## 4 Interpretation

Calculating posterior probability of having the disease given a positive test is approximately 16.1

## 5 Table of Probabilities

Probability	Value
$P(H)$ (Prior)	0.01
$P(\text{not } H)$	0.99
$P(E/H)$ (Sensitivity)	0.95
$P(E/\text{not } H)$ (False Positive)	0.05
$P(E)$ (Total)	0.059
$P(H/E)$ (Posterior)	0.1610

Table 1: Probabilities Used in Bayesian Inference