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Part 1: Key Probability Distributions - Poisson Distribution

Relevant Question

What is the probability of receiving exactly 3 customer calls in a 10-minute interval at a call center, given an average rate of 2 calls per 10 minutes?

Real-World Example

In a call center, the number of incoming customer calls during a fixed time interval (e.g., 10 minutes) often follows a Poisson distribution if the calls occur independently and at a constant average rate. This example is useful for staffing decisions, predicting busy periods, and optimizing resources.

Poisson Distribution Formulas

The Poisson distribution is defined by the following key formula:

- **Probability Mass Function (PMF)**:
  [P(X = k) = (e^(-λ) * λ^k) / k!]
  1. (P(X = k)): Probability of exactly (k) occurrences.
  2. (λ): Average rate of occurrence (e.g., 2 calls per 10 minutes).
  3. (k): Number of occurrences (e.g., 0, 1, 2, 3, ...).
  4. (e): Base of the natural logarithm (approximately 2.71828).
  5. (k!): Factorial of (k) (e.g., (0! = 1), (1! = 1), (2! = 2), (3! = 6)).
- **Mean (Expected Value)**:
  [E(k) = λ]
  1. The average number of occurrences over the interval, equal to (λ) (e.g., 2 for this example).
- **Variance**:
  [Var(k) = λ]
  1. The variance is also equal to (λ), indicating the spread of the distribution (e.g., 2).
- **Standard Deviation**:
  [SD(k) = sqrt(λ)]
  1. The standard deviation is the square root of (λ) (e.g., (sqrt(2) \approx 1.414)).

Analysis of the Poisson Distribution

- **Characteristics**: The Poisson distribution models the number of times an event occurs in a fixed interval of time or space, given a known average rate ((λ)). It is discrete, with probabilities summing to 1 over all possible (k) values. The mean and variance are both equal to (λ) (here, 2).
- **Behavior in This Example**: With (λ = 2), the distribution peaks around (k = 2), reflecting the average number of calls. The probability decreases for (k) values far from (λ) (e.g., (k = 0) or (k = 9)), showing the rarity of extreme events.
- **Differences from Other Distributions**:
    1. **Binomial Distribution**: Unlike the binomial (which models a fixed number of trials with two outcomes), Poisson applies to events over a continuous interval with no upper limit on occurrences.
    2. **Normal Distribution**: Poisson is discrete and skewed for small (λ), while the normal distribution is continuous and symmetric, approximating Poisson only for large (λ).
- **Real-World Relevance**: In the call center example, Poisson helps predict call volumes, aiding in staffing and queue management, especially when calls are random and independent.

Expected Output

- **Probability**: For (k = 3), (P(X = 3) = (e^-2 * 2^3) / 3! approx (0.1353 * 8) / 6 approx 0.1804) (manual approximation aligns with code output).
- **Graph**: A bar plot will show probabilities peaking near (k = 2), with values like:
    1. (P(X = 0) approx 0.1353)
    2. (P(X = 1) approx 0.2707)
    3. (P(X = 2) approx 0.2707)
    4. (P(X = 3) approx 0.1804)
    5. Decreasing thereafter.

Note

Python codes are in a separate file named: grp20_poisson.ipynb