Egenskaber for logaritmer

Sætning 1.1 (Regneregler for log.)**.** *For titalslogaritmen* $\log_{10} = \log$ *gælder der for* $a, b > 0$*, at*

1. $\log(a \cdot b) = \log(a) + \log(b)$.
2. $\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$.
3. $\log(a^x) = x\log(a)$.

Bevis. Ad *i*):

$$\begin{aligned}\log(a \cdot b) &= \log(10^{\log(a)} \cdot 10^{\log(b)}) \\ &= \log(10^{\log(a)+\log(b)}) \\ &= \log(a) + \log(b).\end{aligned}$$

Ad *ii*):

$$\begin{aligned}\log\left(\frac{a}{b}\right) &= \log\left(\frac{10^{\log(a)}}{10^{\log(b)}}\right) \\ &= \log(10^{\log(a)-\log(b)}) \\ &= \log(a) - \log(b).\end{aligned}$$

Ad *iii*):

$$\begin{aligned}\log(a^x) &= \log((10^{\log(a)})^x) \\ &= \log(10^{x\log(a)}) \\ &= x\log(a).\end{aligned}$$

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Sætning 1.2 (Regneregler for ln)**.** *For den naturlige logaritme* $\ln$ *gælder der for* $a, b > 0$*, at*

i) $\ln(a \cdot b) = \ln(a) + \ln(b)$,

ii) $\ln(\frac{a}{b}) = \ln(a) - \ln(b)$,

iii) $\ln(a^x) = x\ln(a)$.

Opgave 1

Bestem følgende ved brug af logaritmeregneregler

1) $\log(\sqrt{10})$
2) $\log(\sqrt[3]{100})$
3) $\log(\sqrt[n]{1000})$
4) $\log(2) + \log(50)$
5) $\log(200) - \log(20)$
6) $\log(2 \cdot 10^5)$

Opgave 2

Bestem følgende:

1) $\ln(e)$
2) $\ln(e^3)$
3) $\ln(\sqrt{e})$
4) $\ln(\sqrt[5]{e^4})$

Opgave 3

i) Bevis, at $\ln(ab) = \ln(a) + \ln(b)$.

ii) Bevis, at $\ln(\frac{a}{b}) = \ln(a) - \ln(b)$.

iii) Bevis, at $\ln(a^x) = x\ln(a)$.