Simulating the Maximum Throughput of a Given Channel

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1 Introduction

This report demonstrates a designed digital communication system optimized to have minimum bit error rate while maximizing the transmitted message bits per second. The system is designed in reference to a given channel with an unknown constant frequency band limitation and unknown variable transmission delay. The simulation of the designed system is conducted in the MATLAB software. The simulation code is attached to the report as a separate file titled: ”Digital_Communications_Simulation_Christian_Kamps.m”. Provided MATLAB files that must be included are: ”root_raised_cosine.m” and ”channel.p” which are also attached to this report.

Section 2 contains a list of the system requirements. Section 3 contains the design and mathematical expression of the digital communication system. Section 4 analyzes the given channel characteristics. Section 5 compares and analyzes possible pulse shapes. Section 6 compares and analyzes possible modulation schemes. Section 7 contains the analysis for maximizing the throughput of the designed system. Section 8 contains several plots of the final implemented design. Section 9 contains the conclusion.

2 System Requirements

The designed digital communication system must adhere to certain design requirements presented in the lab manual. All system requirements are given below.

- The communication system must give a bit error rate (BER) that is less than 10^{-3} at an SNR (E_b/N_0) of 9 dB, where E_b the transmitted energy per bit and N_0 is the single-sided noise power spectral density.
- The system should maximize the throughput (transmitted message bits per second) while still meeting the BER requirement.
- The duration of the pulse cannot exceed $16T$ seconds, where T is the symbol period.
- The message word must be 1200 bits long.

3 Digital Communication System

This section of the report describes the designed digital communication system. A block diagram illustrating the various signal transformations in the system is given below in Figure 1.

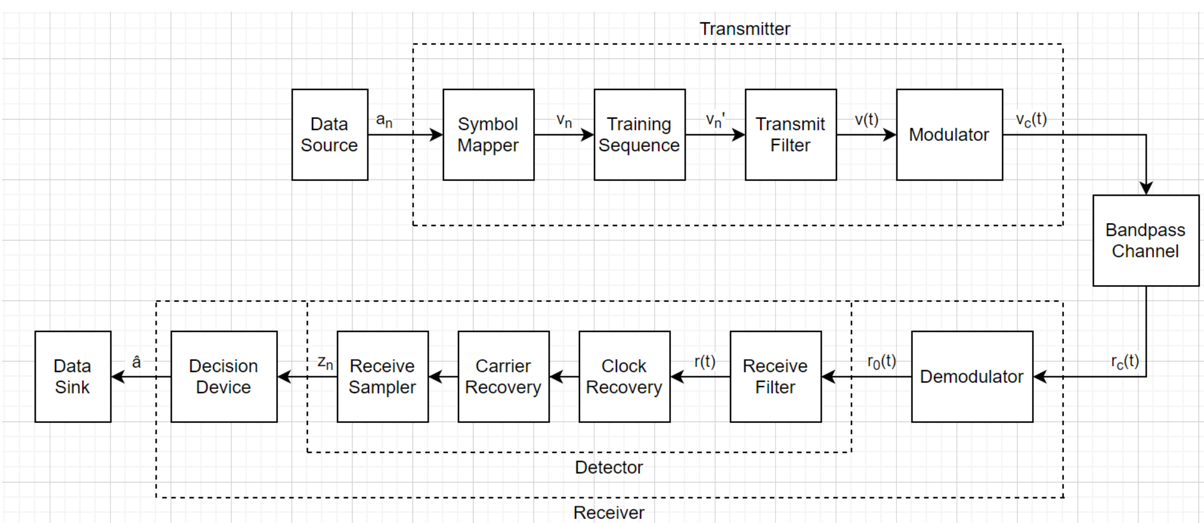

Fig. 1. Block diagram of the designed digital communication system

The digital communication system has three key components: the transmitter, the channel, and the receiver. The purpose of the transmitter is to transform a sequence of bits (transmitted message) into a signal, such that it can be transmitted over a channel. The purpose of the channel is to simulate a theoretically imperfect transmission of data such that the received signal is distorted. The purpose of the receiver is to transform the distorted received signal, such that it can be expressed as a sequence of bits (received message). The purpose of the digital communication system is for the received message to accurately represent the transmitted message such that there are as few errors as possible between the received and transmitted messages.

3.1 Data Source

The data source generates a message **a** composed of a random sequence of bits of bit length N_a, where a_n is the n^{th} message bit.

$$\mathbf{a} = [a_0, a_1, a_2, ..., a_{N_a-1}] \tag{1}$$

$$a_n \in \{0, 1\} \tag{2}$$

3.2 Symbol Mapper

The symbol mapper converts consecutive bits from the message sequence to real or complex valued amplitudes. The conversion process is dependent on the modulation scheme. The converted message sequence produces a sequence of symbols, where v_n is the n^{th} symbol. Each modulation scheme has a unique constellation diagram and symbol length (the number of input bits per symbol). The characteristics, advantages, and disadvantages of several modulation schemes are discussed in Section 6 of this report. The relation between the message sequence **a** and the generated symbol sequence **v** is given below in Equation 3 where SM refers to the symbol mapping transformation.

$$\mathbf{v} = SM[\mathbf{a}] \tag{3}$$

3.3 Training Sequence

A sequence of predetermined symbols, called the training sequence $\mathbf{v}_{train}$, is added to the beginning of of the transmitted symbols **v**. The purpose of the training sequence is discussed in Section 4.1. The new sequence of transmitted symbols **v**′ is given in Equation 4 below.

$$\mathbf{v}' = \mathbf{v}_{train} + \mathbf{v} \tag{4}$$

3.4 Transmit Filter

The transmit filter is a pulse shaping filter that generates an analogue pulse train determined by the output of the transmitted symbols v'_n and the pulse shape $h_T(t)$. The resultant signal $v(t)$ is known as the baseband signal and is given by Equation 5 below, where T is the symbol period or pulse duration and N_v is the number of total transmitted symbols.

$$v(t) = \sum_{n=0}^{N_v-1} v'_n \, h_T(t - nT) \tag{5}$$

3.5 Modulator

A modulator is used to up-convert the baseband signal to a higher frequency band by modulating the amplitude using a carrier wave. The frequency of the carrier signal f_c determines the center frequency of the modulated signal. The modulator carrier wave for real symbol amplitude is given below in Equation 6.

$$c_{v_{real}}(t) = \cos(2\pi f_c t) \tag{6}$$

The modulator carrier wave for complex symbol amplitudes is given below in Equation 7.

$$c_{v_{complex}}(t) = \cos(2\pi f_c t) + j\sin(2\pi f_c t) = e^{j2\pi f_c t} \tag{7}$$

The amplitude modulated signal $v_c(t)$ is given by Equation 8 below.

$$v_c(t) = \Re\left\{v(t)\sqrt{2}c_v(t)\right\} \tag{8}$$

The modulated signal $v_c(t)$ is the bandpass transmitted signal that is sent through the channel.

3.6 Channel

The channel is a theoretical representation of the distortion that occurs as the transmitted bandpass signal $v_c(t)$ propagates across the channel. To simulate the effect of noise distortion on the transmitted bandpass signal $v_c(t)$ during transmission across the channel, a noise signal $w_c(t)$ is added to the transmitted bandpass signal $v_c(t)$. In this report, the noise signal $w_c(t)$ is modeled as a Gaussian random process with zero mean and a double-sided noise power spectral density (PSD) of $N_0/2$. Furthermore, the given channel has unknown transmission delay. The transmission delay τ_c delays the transmitted bandpass signal $v_c(t)$. Therefore, the received signal is a summation of the bandpass transmitted signal $v_c(t)$ time delayed by an unknown amount τ_c and the noise signal $w_c(t)$. The received signal $r_c(t)$ for this system is defined below in Equation 9.

$$r_c(t) = v_c(t - \tau_c) + w_c(t) \tag{9}$$

3.7 Demodulator

A demodulator is used to down-convert the received baseband signal $r_c(t)$ to the original baseband by modulating the amplitude using a carrier wave. The demodulator carrier wave for real symbol amplitude is given below in Equation 10.

$$c_{r_{real}}(t) = \cos(2\pi f_c t) \tag{10}$$

The demodulator carrier wave for complex symbol amplitudes is given below in Equation 11.

$$c_{r_{complex}}(t) = \cos(2\pi f_c t) - j\sin(2\pi f_c t) = e^{-j2\pi f_c t} \tag{11}$$

The demodulated signal $r_o(t)$ for complex symbol amplitudes is given by Equations 12 through 15 below.

$$\begin{aligned} r_o(t) &= r_c(t)\sqrt{2}c_r(t) && (12)\\ &= r_c(t)\sqrt{2}e^{-j2\pi f_c t} && (13)\\ &= [v_c(t-\tau_c) + w_c(t)]\sqrt{2}e^{-j2\pi f_c t} && (14)\\ &= v_c(t-\tau_c)\sqrt{2}e^{-j2\pi f_c t} + w_c(t)\sqrt{2}e^{-j2\pi f_c t} && (15) \end{aligned}$$

The demodulated noise $w_o(t)$ is defined to simplify Equation 15 above and is given below in Equation 16.

$$w_o(t) = w_c(t)\sqrt{2}e^{-j2\pi f_c t} \tag{16}$$

Equation 14 can be substituted into Equation 13 to produce a simplified expression for the demodulated signal $r_o(t)$ given in Equations 17 through 21 below.

$$\begin{aligned} r_o(t) &= v_c(t-\tau_c)\sqrt{2}e^{-j2\pi f_c t} + w_o(t) && (17)\\ &= \Re\left\{\alpha_c v(t-\tau_c)\sqrt{2}e^{j2\pi f_c(t-\tau_c)}\right\}\sqrt{2}e^{-j2\pi f_c t} + w_o(t) && (18)\\ &= \frac{1}{2}[v(t-\tau_c)\sqrt{2}e^{j2\pi f_c(t-\tau_c)} + v^*(t-\tau_c)\sqrt{2}e^{-j2\pi f_c(t-\tau_c)}]\sqrt{2}e^{-j2\pi f_c t} + w_o(t) && (19)\\ &= v(t-\tau_c)e^{-j2\pi f_c\tau_c} + v^*(t-\tau_c)e^{-j2\pi(2f_c)t}e^{j2\pi f_c\tau_c} + w_o(t) && (20)\\ & && (21) \end{aligned}$$

The second term in $r_o(t)$ is centered at a high frequency ($2f_c$). The detector contains a matched filter which will filter out this frequency, so the term can be removed. The final term for $r_o(t)$ is given below in Equation 22 and 23 below.

$$\begin{aligned} r_o(t) &= v(t-\tau_c)e^{-j2\pi f_c\tau_c} + w_o(t) && (22)\\ &= \sum_{n=0}^{N_v-1} v_n\, h_T(t-\tau_c-nT)e^{-j2\pi f_c\tau_c} + w_o(t) && (23) \end{aligned}$$

3.8 Detector

The purpose of the detector is to detect the transmitted symbols in the demodulated signal. The detector consists of a filter and a sampler. The detector also uses carrier and clock recovery techniques to remove the effects of propagation delay from the signal.

3.8.1 Receive Filter

The detector acts as a lowpass filter that removes noise and high-frequency signal components that are added to the transmitted filter during demodulation. The detector used in this design is a matched filter. A matched filter uses impulse response $h_R(t)$ given below in Equation 24.

$$h_R(t) = h_T(LT - t) \tag{24}$$

The output of the matched filter $r(t)$ is given below by Equation 25 through 29.

$$\begin{aligned} r(t) &= r_o(t-\tau)h_R(\tau)\,d\tau && (25)\\ &= r_o(t) * h_R(t) && (26)\\ &= \int_{-\infty}^{\infty} r_o(t-\tau)h_R(\tau)\,d\tau && (27)\\ &= \int_{-\infty}^{\infty}\left[\sum_{n=0}^{N_v-1} v_n\, h_T(t-\tau-\tau_c-nT)e^{-j2\pi f_c\tau_c} + w_o(t-\tau)\right]h_R(t)\,d\tau && (28)\\ &= \sum_{n=0}^{N_v-1} v_n e^{-j2\pi f_c\tau_c}\int_{-\infty}^{\infty} h_T(t-\tau-\tau_c-nT)h_R(\tau)\,d\tau + \int_{-\infty}^{\infty} w_o(t-\tau)h_R(\tau)\,d\tau && (29) \end{aligned}$$

Filtered additive Gaussian noise $w(t)$ is given below in Equation 30.

$$w(t) = \int_{-\infty}^{\infty} w_o(t-\tau)h_R(\tau)\, d\tau \tag{30}$$

The combined impulse response $h_{TR}(t)$ of the transmit filter $h_T(t)$ and receive filter $h_R(t)$ is given below in Equation 31.

$$h_{TR}(t) = h_T(t) * h_R(t) = \int_{-\infty}^{\infty} h_T(t-\tau)h_R(\tau)\, d\tau \tag{31}$$

Therefore, the matched filter $r(t)$ can be defined as Equation 32 given below.

$$r(t) = \sum_{n=0}^{N_v-1} v_n e^{-j2\pi f_c \tau_c} h_{TR}(t-\tau_c-nT) + w(t) \tag{32}$$

3.8.2 Clock Recovery

Clock recovery refers to the technique in which the propagation delay τ_c is derived in order to correctly sample the matched filter $r(t)$. This technique is explained in detail in Section 4.1.2.

3.8.3 Carrier Recovery

Carrier recovery refers to the technique in which the phase error ϕ_c is derived in order to remove its effects on the sampled output r_n. This technique is explained in detail in Section 4.1.1.

3.8.4 Receive Sampler

The matched filter is sampled at n intervals of the symbol period T to get the received samples r_n. The first sample is taken at time LT where L is the truncation factor. All samples are time shifted by the propagation delay τ_c which must be derived in order to correctly sample the signal. The received samples are given by Equation 33 and 34 below.

$$\begin{aligned} r_n &= r(nT+LT+\tau_c) &(33)\\ &= \sum_{m=0}^{N_v-1} v_m e^{-j2\pi f_c\tau_c} h_{TR}(nT+LT+\tau_c-\tau_c-mT) + w(nT+LT+\tau_c) &(34) \end{aligned}$$

The sampled noise w_n is given below in Equation 35.

$$w_n = w(nT+LT+\tau_c) \tag{35}$$

The phase error ϕ_c is given below in Equation 36.

$$\phi_c = -2\pi f_c \tau_c \tag{36}$$

The received samples r_n is then expressed as given in Equation 37 below.

$$r_n = \sum_{m=0}^{N_v-1} v_m e^{j\phi_c} h_{TR}([n-m]T+LT) + w_n \tag{37}$$

Ideally, each sample of r_n should only depend on one transmitted symbol v_n. If transmitted symbols v_n have overlapping non-zero values at the sampling intervals, then the samples r_n will depend on more than one transmitted symbol v_n. When samples r_n depend on more than one transmitted symbol v_n, the sampled value is distorted and the probability of bit errors increases. This is known as intersymbol interference (ISI).

To avoid ISI, specific pulse shapes $h_{TR}(t)$ are utilized such that only one transmitted symbol v_n exists at sampling intervals nT. The condition to avoid ISI is given below in Equation 38.

$$h_{TR}(nT+LT) = \delta_n = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{if } n \neq 0 \end{cases} \tag{38}$$

Given an ideal pulse shape $h_{TR}(t)$ is utilized to avoid ISI, the equation for received samples r_n is given in Equation 39 and 40 below.

$$\begin{aligned} r_n &= \sum_{m=0}^{N_v-1} v_m \delta_{n-m} e^{j\phi_c} + w_n &(39)\\ &= v_n e^{j\phi_c} + w_n &(40) \end{aligned}$$

The received samples r_n without propagation delay is given below in Equation 41.

$$r_n = v_n + w_n \tag{41}$$

From Equation 41 and Equation 40 above, the propagation delay alters the magnitude of the transmitted symbols v_n and then rotates those symbols in the signal constellation. The component added by propagation delay is defined below in Equation 42.

$$q = e^{j\phi_c} \tag{42}$$

If v_n is scaled and shifted by q and then fed into the decision device, there is an extremely high probability of bit error. Therefore, the system must derive an expression for $e^{j\phi_c}$ in order to remove its component from the sampled signal. The sampled signal that has this component from delay propagation removed z_n is given below in Equation 43.

$$z_n = \frac{r_n}{q} = \frac{r_n}{e^{j\phi_c}} = v_n + \frac{w_n}{e^{j\phi_c}} \tag{43}$$

3.9 Decision Device

A decision device determines the value of each message bit based on the value of each received sample. The criteria the decision device uses to determine the message bit value is dependent on the modulation scheme. The relation between the sampled received sequence **z** and the bit output of the decision device **â** is given below in Equation 44 where DD refers to the decision device transformation.

$$\hat{\mathbf{a}} = DD[\mathbf{z}] \tag{44}$$

4 Channel Characteristics

The given channel has unknown constant frequency band limitation and unknown variable transmission delay. Both of these unknown characteristics affect the system's design. This section of the report analyzes the unknown characteristics for appropriate design solutions.

4.1 Propagation Delay

The given channel introduces propagation delay which affects the designed system in two distinct ways. Firstly, the received samples r_n are delayed by an unknown amount τ_c. Secondly, the received samples r_n have phase error which distorts the transmitted symbol components v_n by a factor of $e^{j\phi_c}$. Equation 45 below describes these two relationships.

$$r_n = r(nT + LT + \tau_c) = v_n e^{j\phi_c} + w_n \tag{45}$$

This section describes the two techniques used to derive the unknown propagation delay τ_c and the phase error $e^{j\phi_c}$ to obtain the correct received samples r_n .

4.1.1 Carrier Recovery

Carrier recovery is a technique in which the phase error ϕ_c is derived in order to remove its effects on the sampled output r_n. Specifically, the term $e^{j\phi_c}$ must be removed from the received samples r_n such that r_n is only dependent on the transmitted symbol v_n and the noise component w_n. For simplicity, the unwanted term $e^{j\phi_c}$ will be denoted the propagation delay component q. The propagation delay component q is given in Equation 46 below.

$$q = e^{j\phi_c} \tag{46}$$

The received samples r_n is simplified and given as Equation 47 below.

$$r_n = v_n e^{j\phi_c} + w_n = v_n q + w_n \tag{47}$$

Ignoring the noise component w_n, an expression for the propagation delay component q can be derived from Equation 47 above, given in Equation 48 below.

$$q = \frac{r_n}{v_n} \tag{48}$$

Therefore, if the receiver knows the transmitted symbol v_n, then the expression for q can be derived. To ensure the receiver always knows a transmitted symbol, a specific sequence of N_T symbols is appended before the transmitted message $v(t)$. This sequence is known as a "training" sequence. Furthermore, a single transmitted symbol of v_n should not be used because additive Gaussian noise is also a component of the received sample. The noise component will derive imprecise estimate of q if one symbol of v_n is used. This problem can be alleviated by taking the average value of v_n and r_n over multiple samples (for multiple values of n). A precise estimate for q based off taking multiple samples is given below in Equation 49, where N_T is the training sequence length.

$$q = \frac{1}{N_T} \sum_{n=0}^{N_T-1} \frac{r_n}{v_n} \tag{49}$$

4.1.2 Clock Recovery

Clock recovery refers to the technique in determining the correct sampling times to sample the filtered received signal $r(t)$ to produce the received sampled sequence r_n. The received sampled sequence r_n for a system with propagation delay τ_c is given in Equation 50 below.

$$r_n = r(nT + LT + \tau_c) \tag{50}$$

Given Equation 50 above, the system must know the delay τ_c or else the received samples will be sampled at incorrect timings. To find the delay τ_c, the receiver over-samples the filtered received signal $r(t)$ to produce an over-sampled version of the received samples $\widetilde{r}_n$ that is given below in Equation 51.

$$\widetilde{r}_n = r(nT/\eta) \tag{51}$$

The receiver parses through the over-sampled received samples $\widetilde{r}_n$ by intervals of η (η is the oversampling ratio) in an attempt to find the training sequence. The receiver analyzes the first N_T samples to look for the training sequence. If the training sequence cannot be found, the receiver begins the parse again except starting at the second sample of the over-sampled received samples $\widetilde{r}_n$. The receiver repeats the process of starting one sample higher than before until it finds the the training sequence.

Due to additive noise, it is unlikely that the training sequence will have the same values in the receiver. Therefore, the receiver parses for the sequence that closest resembles the training sequence. The correlation coefficient μ_k that defines the degree of similarity between a sequence and the training sequence is given below in Equation 52.

$$\mu_k = \left| \frac{1}{N_T} \sum_{n=0}^{N_T-1} \widetilde{r}_{(L+n)\eta+k} v_n^* \right| \tag{52}$$

For this design, the correlation coefficient is calculated for all values of k, and the maximum value of μ_k is chosen to represent the sequence that closest matches the training sequence. The k value that corresponds to the largest correlation coefficient is the propagation delay given in T_s seconds.

4.1.3 BER due to Training Sequence Length

Increasing the training sequence length N_T ensures the phase error ϕ_c and propagation delay τ_c are both estimated correctly. In order to choose an appropriate training sequence length for the system, the BER is measured for a simple 4-QAM system with Gaussian distributed noise. Figure 2 below shows the BER for various training sequence lengths N_T.

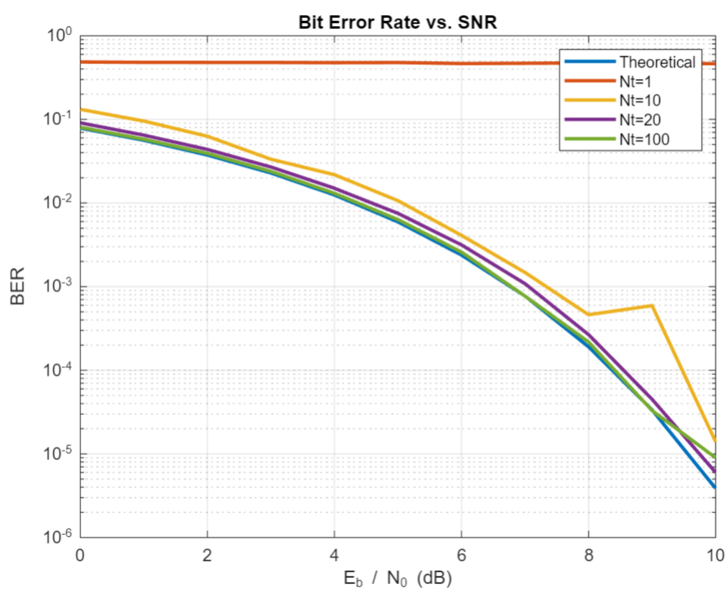

Fig. 2. BER for various training sequence lengths

Figure 2 above illustrates that increasing the training sequence length N_T increases the probability that the system correctly estimates the propagation delay and phase error. Therefore, the ideal training sequence length is as high as possible. For this design, a training sequence length of $N_T = 100$ is chosen because it results in a BER close to the theoretical.

$$N_{Tideal} = 100 \tag{53}$$

4.2 Band-Limitation

The frequency response of the given channel is limited to an unknown but constant range of frequencies. To determine the range of frequencies that the channel filters, normally distributed random values are transmitted through the channel to develop a received message that is band limited. By plotting the received message's frequency response, the band-limitation can be visualised. A plot of the received message's double-sided PSD is given below in Figure 3.

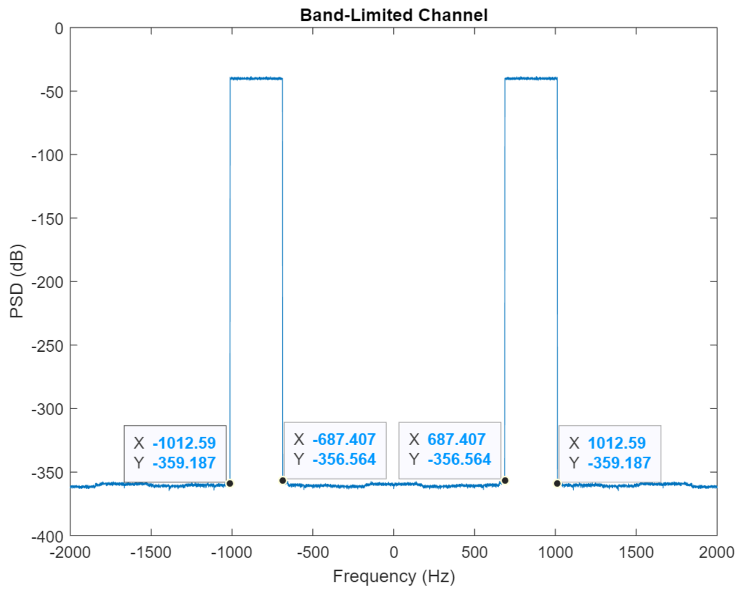

Fig. 3. PSD of the received message demonstrating the band limitation of the channel

The band-limitation frequency of the channel is defined below in Equation 54 to 58.

$$f_L \approx 687\ Hz \tag{54}$$

$$f_H \approx 1013\ Hz \tag{55}$$

$$f_0 = \frac{f_L + f_H}{2} = 850\ Hz \tag{56}$$

$$f_c = f_0 - f_L = 163\ Hz \tag{57}$$

$$BW = 2f_c = 326\ Hz \tag{58}$$

The passband of the channel is given from $f_L \approx 687\ Hz$ to $f_H \approx 1013\ Hz$, indicating a bandwidth of $BW = 326\ Hz$ centered at frequency $f_0 = 850\ Hz$.

Given the bandwidth BW of the channel, the pulse shape can be designed to minimize intersymbol interference (ISI) by containing the majority of the transmitted signals frequency response within the passband of the channel.

4.3 Carrier Frequency

Figure 4 below contains the PSD of a transmitted signal $v_c(t)$ for various carrier frequencies f_c.

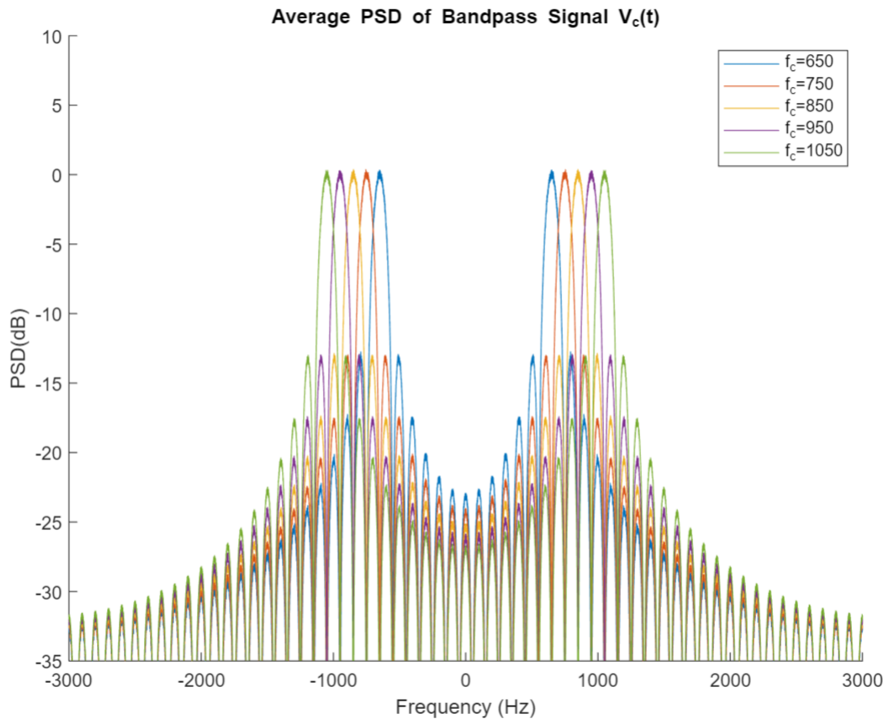

Fig. 4. PSD of a transmitted signal $v_c(t)$ for various carrier frequencies f_c

Figure 4 above illustrates how the the carrier frequency controls up-conversion of the baseband signal to a higher frequency in modulation. The carrier frequency dictates the center frequency of the transmitted signal $v_c(t)$ as it passes through the channel. Normally, the majority of the frequency components of a signal are centered around the center frequency, so the system should be designed such that the modulator up-converts the baseband signal so that a majority of the signal is not filtered through the given channel. Therefore, given the center frequency f_0 of the channel, the modulator and de-modulator of the system can be properly designed by setting the carrier frequency f_c to equal to center frequency of the channel filter f_0. The ideal carrier wave frequency is given below in Equation 59.

$$f_{c_{ideal}} = f_0 = 850\ Hz \tag{59}$$

5 Pulse Shapes

Two pulse shapes are analyzed in this report. Their parameters and their contribution to ISI are compared in order to choose the ideal pulse shape for the designed system.

5.1 Intersymbol Interference due to Puslse Shapes

Pulse signals $h_{TR}(t)$ that are limited to the symbol period (T or less seconds) ensures that only one symbol is transmitted per interval. However, time-limited pulse shapes require theoretically infinite bandwidth to transmit so they are not ideal or practical. Alternatively, pulse shapes that are longer than a symbol period (longer than T seconds) can be used such that they use finite bandwidth to transmit the signal. Longer pulse shapes often have infinite duration. These pulse shapes are not practical and cannot be implemented. In practice, infinite duration pulse shapes are truncated to a specific amount of symbol periods.

Truncating pulse shapes cause two issues. Firstly, truncated pulses produce ISI at the output of the matched filter that increase the probability of bit error. Secondly, truncated pulses are no longer perfectly band-limited, and therefore produces additional frequency components. The additional frequency components appear as ”sidelobes” in the PSD of the transmitted signal and are known as spectral regrowth. The power of the spectral regrowth is proportional to their interference with adjacent channels and their contribution to the signal in the time domain. For example, if the power of the spectral regrowth is low then removing part of the spectral regrowth will minimally affect signal in the time domain. Truncated pulses are truncated to L symbol periods, where L represents the pulse length. Increasing the pulse length decreases the pulse truncation.

5.2 Rectangular Filter

The rectangular pulse is a unit step with normalized unit energy. The rectangular pulse is given in the time domain by Equation 60 below and in the frequency domain by Equation 61 below.

$$h(t) = \begin{cases} \frac{1}{\sqrt{T}} & 0 \geq t \geq T \\ 0 & otherwise \end{cases} \tag{60}$$

$$H(f) = \text{sinc}(f) \tag{61}$$

The rectangular pulse satisfies the ISI conditions for the time domain such that only one pulse exists for a symbol period. However, in the frequency domain the pulse has infinite duration meaning that a majority of the frequency components will be filtered by the given channel. This system's channel filters all signal above 1013 Hz and below 687 Hz, so a large majority of the frequency components will be filtered. This will distort the received signal causing a large amount of ISI and BER.

5.3 Root-Raised Cosine Filter

The root-raised cosine (RRC) in the time domain is given below in Equation 62, where β is the roll-off factor that controls the bandwidth, and T is the symbol period.

$$h(t) = \begin{cases} \frac{1}{T}\left[1+\beta\left(\frac{4}{\pi}-1\right)\right] & t=0 \\ \frac{\beta}{T\sqrt{2}}\left[\left(1+\frac{2}{\pi}\right)\sin\left(\frac{\pi}{4\beta}\right)+\left(1-\frac{2}{\pi}\right)\cos\left(\frac{\pi}{4\beta}\right)\right] & t=\pm\frac{T}{4\beta} \\ \frac{1}{T}\frac{\sin\left(\pi\frac{t}{T}(1-\beta)\right)+4\beta\frac{t}{T}\cos\left(\pi\frac{t}{T_s}(1+\beta)\right)}{\pi\frac{t}{T}\left[1-\left(4\beta\frac{t}{T}\right)^2\right]} & otherwise \end{cases} \tag{62}$$

The RRC signal in the frequency domain is given below in Equation 63.

$$H(F) = \begin{cases} T & \text{if}|f| \leq \frac{1-\beta}{2T} \\ \frac{T}{2}\left[1+\cos\left(\frac{\pi T}{\beta}\left[|f|-\frac{1-\beta}{2T}\right]\right)\right] & \text{if}\frac{1-\beta}{2T} < |f| \leq \frac{1+\beta}{2T} \\ 0 & \text{if}|f| > \frac{1+\beta}{2T} \end{cases} \tag{63}$$

The RRC signal satisfies the ISI conditions in the time domain such that the signal equals zero at every symbol period except the initial. Therefore, the RRC signal avoids ISI by having only one transmitted symbol per transmitted pulse. In the frequency domain, the RRC signal is finite or psuedo-finite (in the case of truncation) such that the majority of the frequency components exist as one interval. This property allows the signal to be sent through the given channel filter with minimal distortion as removing low power frequency components contributes to a small amount of distortion.

To implement the RRC signal as the transmit filter $h_T(t)$ and as the receive filter $h_R(t) = h_T(-t)$, we truncate the signal amplitude in the frequency domain such that the transmit filter multiplied by the received filter in the frequency domain (convolution) produces the RRC signal $h_{TR}(t)$. This relation is given in Equation 64 below.

$$H_T(t) = \sqrt{H_{TR}(f)} \tag{64}$$

5.3.1 Truncation

The RRC signal has infinite duration in the time domain so it must be truncated in practice. The RRC signal must also be delayed by $LT/2$ to make the signal causal. When truncated, the RRC signal produces spectral regrowth. The more the signal is truncated (the lower the pulse length L), the more spectral regrowth appears. High spectral regrowth is characterized as higher individual side lobe bandwidth and power. The less the signal is truncated (the larger the pulse length L), the less spectral regrowth appears. Low spectral regrowth is characterized as lower individual side lobe bandwidth and power.

Figure 5 below shows the PSD of the RRC signal for various pulse lengths L in the designed system.

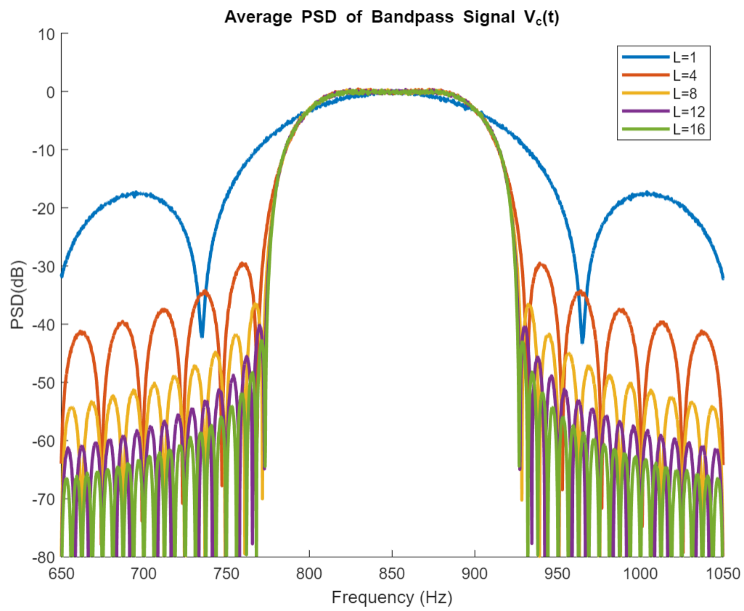

Fig. 5. PSD of the RRC signal for various pulse lengths

Figure 5 above illustrates that as the signal becomes more truncated, the power of the side lobes increase. This means that a truncated RRC signal is more susceptible to ISI from a band-limited channel as the side lobes carry a larger portion of power. This is further illustrated by a BER plot for the same pulse legnth values given in Figure 6 below.

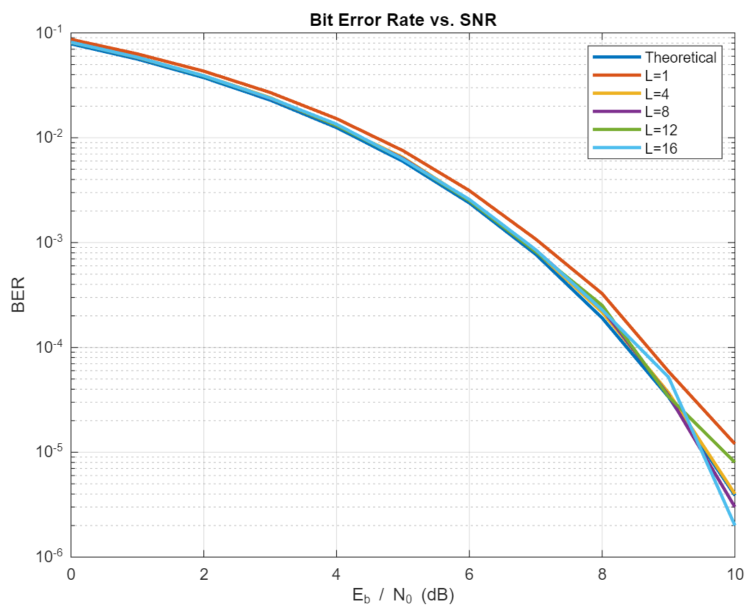

Fig. 6. BER of the RRC signal for various pulse lengths

Figure 6 above illustrates that as the signal becomes less truncated, the BER decreases. Therefore, if the RRC signal is chosen as the pulse shape, the RRC signal should have minimal truncation to produce minimal ISI and BER. The longest symbol length is given in the requirements as $16T$, so the maximum pulse length for the RRC signal is $L = 16$. The ideal pulse length for the RRC signal is given in Equation 65 below.

$$L_{ideal} = 16 \tag{65}$$

5.3.2 *Roll-off Factor*

The roll-off factor β changes the null to null bandwidth of the main lobe of the PSD. The roll-off factor β is also inversely proportional to the the power of the side lobes of the PSD.

Figure 7 below shows the PSD of the RRC signal for various roll-off factors β in the designed system.

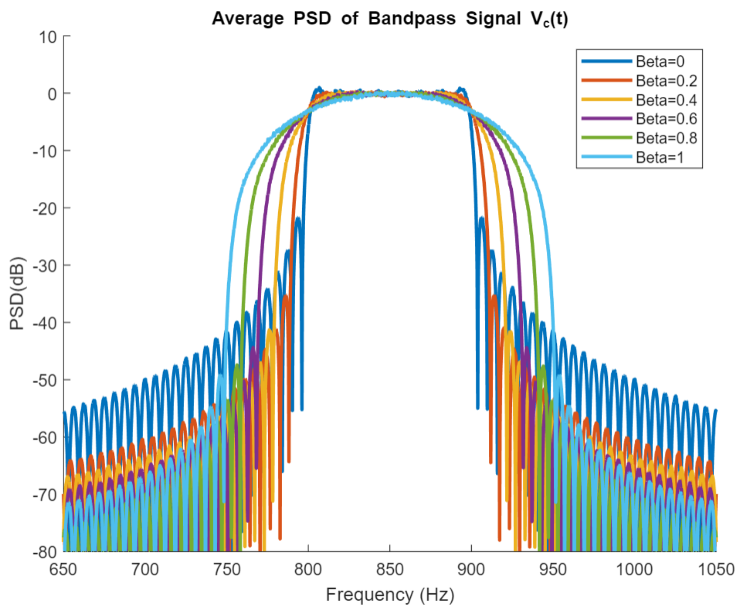

Fig. 7. PSD of the RRC signal for various roll-off factors

Figure 7 above illustrates that as the roll-off factor increases, the bandwidth of the main lobe increases and the power of the sidelobes decrease. This scenario is ideal for band-limited channels as it ensures that the main frequency components are not filtered, while the low power frequency components are filtered. This situation enables the system to pass a majority of the signal through the band-limited channel such that minimal distortion will occur. This is further illustrated by a BER plot for the same roll-off factor values given in Figure 8 below.

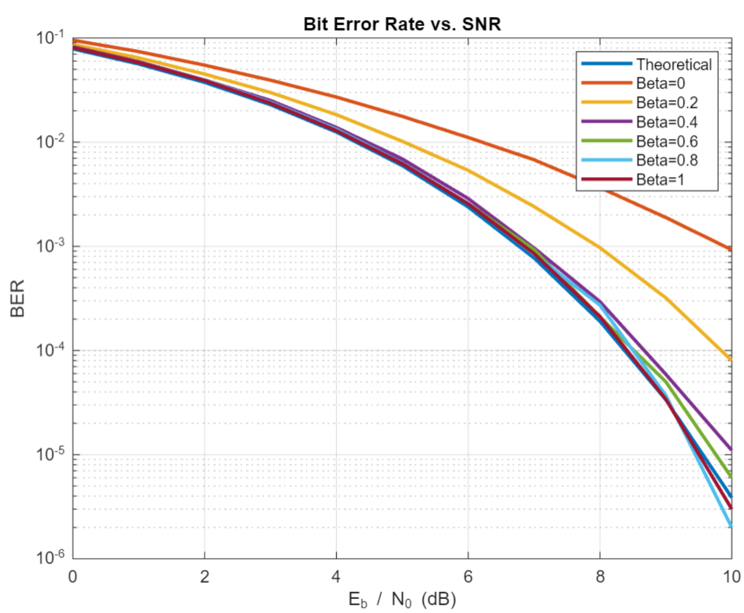

Fig. 8. BER of the RRC signal for various roll-off factor values

Figure 8 above illustrates that as the roll-off factor β increases, the BER decreases. Therefore, if the RRC signal is chosen as the pulse shape, the RRC signal should have maxmimum roll-off factor β to produce minimal ISI and BER. The ideal roll-off factor β for the RRC signal is given in Equation 66 below.

$$\beta_{ideal} = 1 \tag{66}$$

5.4 Comparing the Pulse Shapes

The PSD of the transmitted signal $v_c(t)$ is given below in Figure 9 for both pulse shapes.

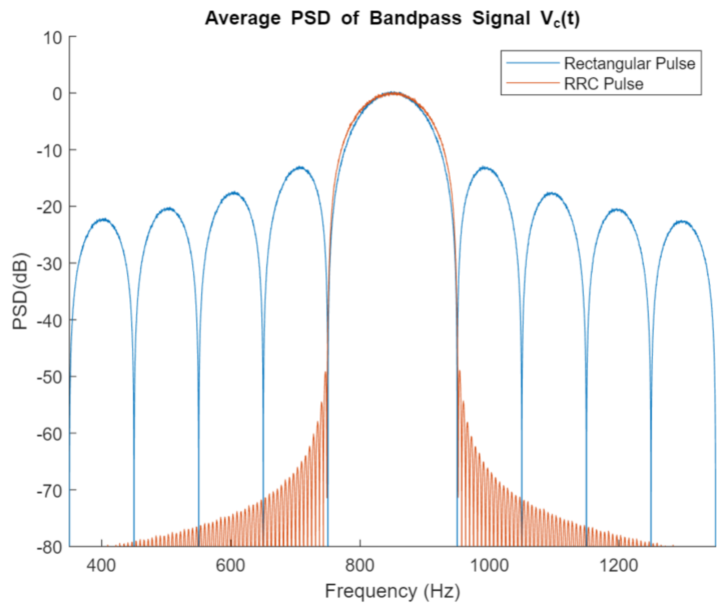

Fig. 9. PSD of the Rectangular and RRC signal

Figure 9 above illustrates that the RRC signal is the better choice for band-limited channels because a majority of the frequency components will exist within the band-limited channel. When the signal passes through the band-limited channel a majority of the rectangular pulse's frequency components will be filtered causing large distortion in comparison to the RRC pulse. This is further illustrated by Figure 10 below, where the BER of the rectangular pulse is compared against the RRC pulse with ideal parameters for the designed system.

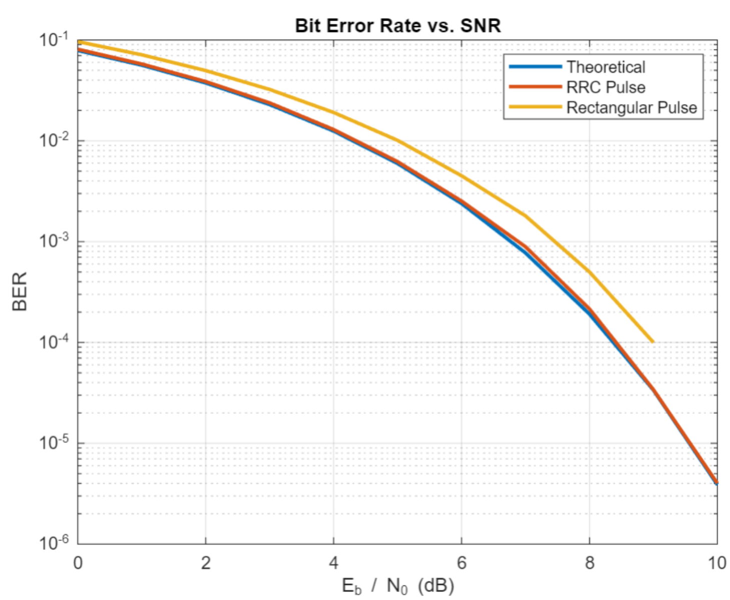

Fig. 10. BER of the Rectangular and RRC signal with ideal parameters

Figure 10 above illustrates the the RRC pulse with ideal parameters will produce much less bit errors than the rectangular pulse due the frequency component composition in conjunction with the band-limitation of the given channel. Furthermore, the RRC pulse produces almost idealistic BER so other pulse shapes do not need to be considered in this theoretical scenario.

6 MODULATION SCHEMES

Modulation schemes refer to the process in which sequences of bits are mapped to real and complex values to represent symbols. The theoretical BER of grey-mapped 4-QAM/BPSK, 16-QAM, and 16-QAM with grey mapping is given below in Figure 11.

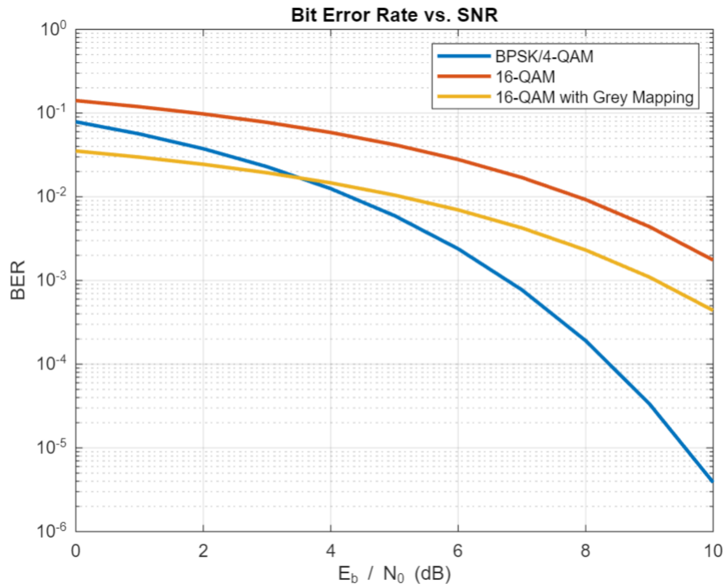

Fig. 11. BER of different modulation schemes

The theoretical BER given above in Figure 11 are on average the minimum BER for a system whose only error component is Gaussian noise. Therefore, the only theoretical BER that satisfies the requirement of a BER of under 10^{-3} at 9 dB is 4-QAM/BPSK modulation. 4-QAM sends 2 bits per symbol, while BPSK sends 1 bits per symbol. The throughput must be maximizes for this design, so 4-QAM is chosen as the modulation scheme because it sends more bits per symbol than BPSK.

6.1 4-QAM

4-QAM refers to the quadrature amplitude modulation scheme that transmits 2 bits per symbol. Figure 12 below illustrates the decision regions on the constellation diagram for each of the 2 bit combinations.

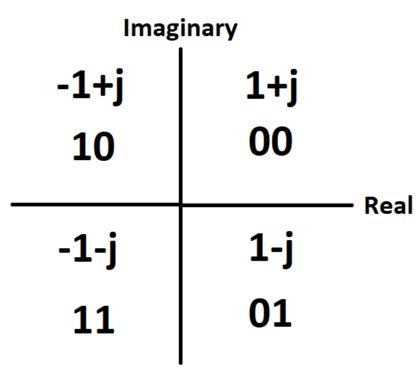

Fig. 12. Decision regions of grey-mapped 4-QAM

The decision regions are mapped with grey mapping to minimize symbol error such that adjacent decision regions only differ by at most 1 bit. The grey mapping technique lowers the total BER by minimizing bit error when a symbol error occurs. Each of the bit combinations are mapped to the complex value of their decision region. The symbol mapping transformation is given in Table 1 below.

Bits	Symbol
00	1+j
01	1-j
10	-1+j
11	-1-j

TABLE 1

The decision device transforms received samples r_n based on the decision regions. The decision regions for 4-QAM exist on the real and imaginary axis. The decision device transformation is given in Table 2 below, where ”first bit” refers to the right bit and ”second bit” refers to the left bit in the sequence.

Symbol Component	>0	<0
Real Component	second bit = 0	second bit = 1
Imaginary Component	first bit = 0	first bit = 1

TABLE 2

7 Maximum Throughput

The ideal system parameters for maximizing the throughput, given the requirements stated in Section 2, are given below.

- RRC pulse shape
- 4-QAM
- $L = 16$
- $\beta = 1$
- $f_c = 850$ Hz
- $N_T = 100$
- $N_a = 1200$
- $\eta = 64$

The message length N_a is set by the system requirements. The oversampling ratio η defines the data resolution of each symbol period and is set at 64 as a balance between resolution and simulation speed.

The system must be designed to maximize throughput. The throughput of the system is given in Equation 67 below, where M is the constellation size.

$$throughput = \frac{\log_2(M)}{T} = \frac{\log_2(4)}{T} = \frac{2}{T}\ bits\ per\ second \tag{67}$$

Given the design parameters, the symbol period T must be minimized such that the BER at an SNR of 9 dB is under 10^{-3}. Figure 13 below shows the BER of the designed system for 100 symbol period values T between $0.0001s$ and $0.01s$

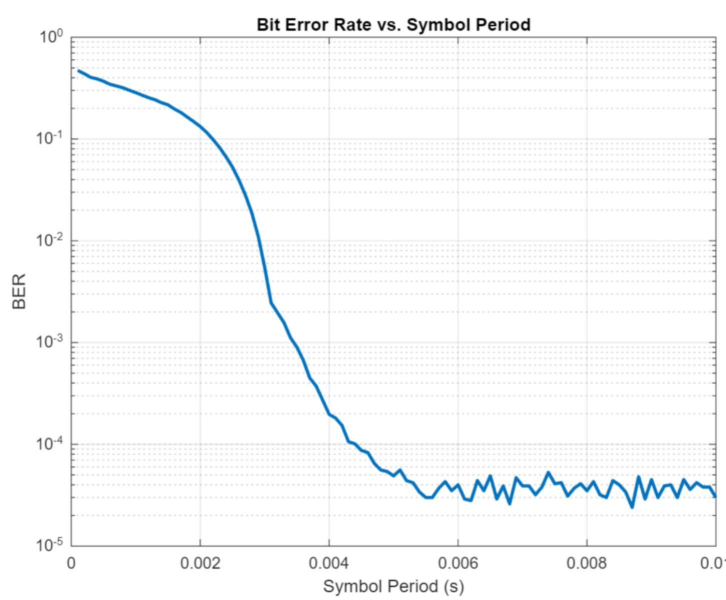

Fig. 13. BER of of different symbol periods

Figure 14 below shows the minimum symbol period T that gives an BER below 10^{-3} at an SNR of 9 dB.

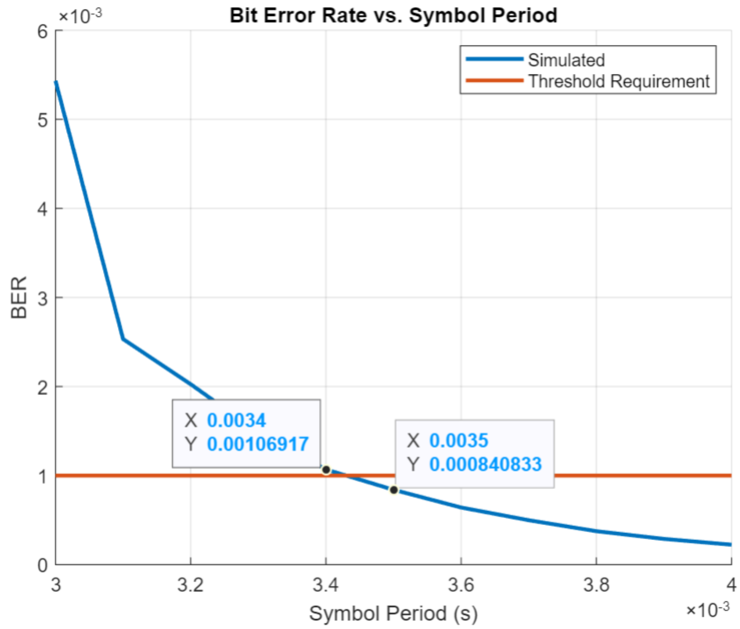

Fig. 14. Minimum symbol period for required BER

Figure 14 above shows that the minimum symbol period for the required BER is $T = 0.0035s$. Equation 68 below gives the minimum symbol period T_{min} that maximizes the throughput.

$$T_{min} = 0.0035s \tag{68}$$

Given the minimum symbol period T_{min}, the maximum throughput is derived in Equation 69 below.

$$Maximum\ Throughput = \frac{2}{T_{min}} \approx 517\ bits\ per\ second \tag{69}$$

8 Final Design

Given the ideal parameters to maximize throughput listed in Section 7, the BER for the designed system is shown in Figure 15 below.

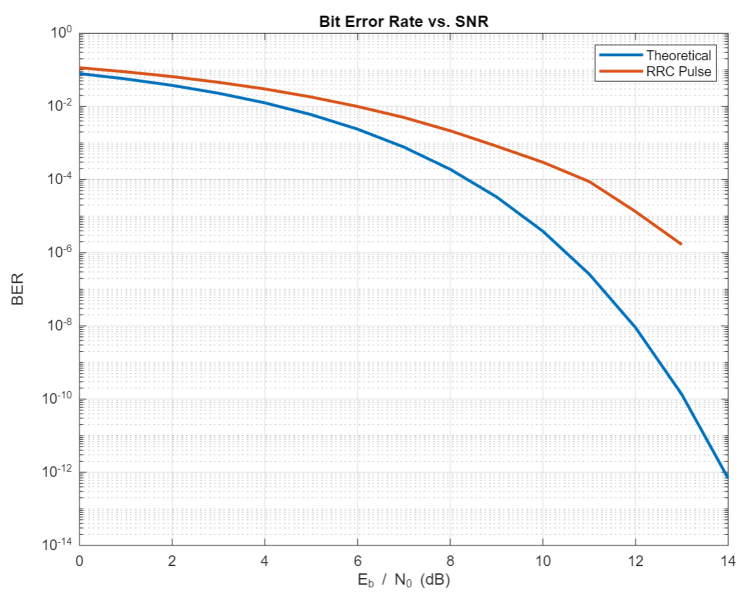

Fig. 15. BER for the designed system

Figure 15 above shows that the system meets the required BER.

The PSD of the transmitted signal $v_c(t)$ for the designed system is given below in Figure 16.

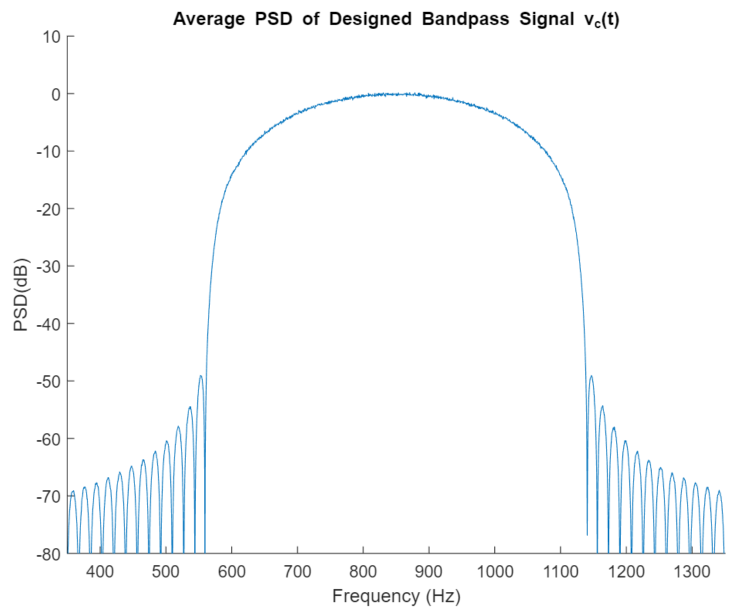

Fig. 16. PSD of the bandpass transmitted signal $v_c(t)$ for the designed system

Figure 16 above illustrates that a majority of the main frequency components will not be filtered by the channel.

9 Conclusion

This report demonstrated a designed digital communication that was optimized to have maximum throughput given a distorted channel. The designed solution was implemented and simulated in the MATLAB software. Optimizations were chosen in reference to minimizing the BER for the given band-limited channel with unknown propagation delay. Design decisions include choosing the ideal carrier frequency f_c, the pulse shape $h_{TR}(t)$, and the modulation scheme. Certain possible design choices were not analyzed such as 8-QAM, and sinc pulse shapes. Several techniques were utilized to minimize the BER such as band-limitation, consideration of ISI, clock recovery techniques and carrier recovery techniques. Once the BER was optimized to be as low as possible, the symbol period was decreased to the minimum amount to maximize throughput. The maximum throughput for this design was about 517 bits per second.