Loss Sharing in Central Clearinghouses: Winners and Losers

Christian Kubitza
University of Bonn
christian.kubitza@uni-bonn.de

Loriana Pelizzon
Leibniz Institute SAFE,
Goethe University Frankfurt,
and Ca'Foscari University of Venice
pelizzon@safe-frankfurt.de

Mila Getmansky Sherman
Isenberg School of Management
University of Massachusetts Amherst
msherman@isenberg.umass.edu

This version: September 20, 2021

Previous versions have been circulated under the title "Pitfalls of Central Clearing in the Presence of Systematic Risk". We are grateful for helpful comments and suggestions by Bruno Biais, Agostino Capponi, Fousseni Chabi-Yo, Rama Cont, Marco D'Errico, Darrell Duffie, Paul Glasserman, Matthias Graulich, Wenqian Huang, Argyris Kahros, Nikunj Kapadia, Olga Lewandowska, Bing Liang, Fangzhou Lu, Albert Menkveld, Stephen O'Connor, Rafael Repullo, Emil Siriwardane, Pietro Stecconi, Guillaume Vuillemey, Jessie Jiaxu Wang, Joshua White, Haoxiang Zhu, and participants at the 2020 CEBRA meeting, 2020 AFA meeting, 2019 conference on regulation and operation of modern financial markets, 2019 SIAM conference on financial mathematics & engineering, 2018 SAFE conference, 2018 ECB workshop on money markets, and seminars at University of Oxford, the European Securities and Markets Authority (ESMA), UMASS Amherst, Villanova University, and Goethe-University Frankfurt. Any errors are our own. Christian Kubitza gratefully acknowledges financial support from the International Center for Insurance Regulation at Goethe-University Frankfurt and from the research cluster ECONtribute, funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy – EXC 2126/1–390838866, and travel grants from the German Insurance Science Association (DVfVW). Loriana Pelizzon gratefully acknowledges research support from the Leibniz Institute for Financial Research SAFE. Mila Getmansky Sherman gratefully acknowledges financial support from National Science Foundation Award 1940223 and 1733942. Corresponding author: `msherman@isenberg.umass.edu`.

Loss Sharing in Central Clearinghouses: Winners and Losers

Abstract

Central clearing counterparties (CCPs) were created to reduce default losses in derivatives markets. We show that not all market participants benefit, and some are worse off. Loss sharing rules and their interaction with market network structure affect who wins or loses. We develop a simple model which shows that currently implemented rules largely benefit market participants with flat portfolios but not participants with directional portfolios or those located in the periphery of the network, consistent with their reluctance to voluntarily clear in practice. We then investigate how alternative loss sharing rules can offset cross-sectional differences in clearing benefits.

JEL classification: G18, G23, G28, G12.

Keywords: Central Clearing, Counterparty Risk, Loss Sharing, OTC markets, Derivatives.

1. Introduction

Whether we choose to bolster the tools for CCP resilience, CCP recovery or CCP resolution, we will need to be aware of potential trade-offs in the way losses are allocated, and remember that there may be no ideal approach.[1]

Default losses arise when counterparties do not fulfill their obligations (e.g., when they default). The risk of default losses (i.e., counterparty risk) is one of the most important risks in over-the-counter (OTC) derivatives markets and has been identified as a major contributor to the amplification of the 2007-08 financial crisis. To mitigate this risk in derivatives markets, regulators worldwide have been pushing for the central clearing of OTC derivative transactions through central clearing counterparties, so-called *CCPs* (G20 (2009)).[2] The main tasks of CCPs are to reduce the total amount of default losses and allocate remaining losses to non-defaulting clearing members (so-called *loss sharing*).[3] Therefore, it is important to understand how central clearing affects the final distribution of losses across market participants, i.e., who benefits from central clearing and how does not.

In this paper, we take a first step to investigate the distributional aspects of loss sharing. Our main analysis explores differences in the effect of central clearing on default losses across market participants (i.e., winners vs. losers). Compared to an uncleared market, we show that central clearing often reduces expected default losses only for a subset of entities (the "winners"), but not for *all* market participants. We then explore how the distribution of winners and losers changes with loss sharing rules, market's network structure, and correlation of derivative prices.

A key result of our analysis is that market participants in the periphery of the market and with directional portfolios can face larger expected default losses when they centrally clear their transactions compared to an uncleared market (i.e., they lose from central clearing). The intuition

[1]Benoît Cœuré's remarks at the Federal Reserve Bank of Chicago 2015 Symposium on Central Clearing, Chicago (20 April). Available at `https://www.ecb.europa.eu/press/key/date/2015/html/sp150411.en.html`.

[2]OTC derivatives markets are very large, with a worldwide outstanding notional amount of $542 trillion in 2017 (Bank for International Settlements (BIS)). Before the 2007-08 financial crisis, the derivatives market architecture was dominated by bilateral trades (Financial Stability Board (FSB) (2017)). The G20 initiative in 2009 was followed by the Dodd-Frank Wall Street Reform and Consumer Protection Act (DFA) in 2010 and the European Market Infrastructure Regulation (EMIR) in 2012, with the mandatory central clearing of certain OTC derivatives as a key element. More recently, a central clearing mandate has also been suggested for other asset classes, such as US treasuries (Duffie (2020), Fleming and Keane (2021)).

[3]Loss sharing is required by post-crisis regulation, such as the European Market Infrastructure Regulation (EMIR).

is that central clearing brings little netting benefits to such entities but exposes them to potentially large contributions to loss sharing. This result provides a possible explanation for the reluctance of market participants, and particularly peripheral entities with directional portfolios, to use central clearing in practice.[4] Anecdotal evidence suggests that current CCPs' loss allocation rules in particular disincentivize a large set of market participants from central clearing (Novick et al. (2018)), which motivates our analysis of loss sharing.[5]

Despite the increasing importance of central clearing in derivatives markets, research on loss sharing in clearinghouses is still scarce. We extend the literature by building a tractable model, that clearly shows which market participants benefit or lose from using central clearing. Who wins or loses is a function of the market's network structure, loss sharing rules, and systematic risk. We derive comparative statics that provide guidance on how these characteristics interact with the distributional effects of loss sharing across different types of market participants. The analysis provides important insights for policymakers and facilitates future research by highlighting the relevant economic trade-offs that arise with loss sharing.

Our main contribution is to investigate the effect of the interaction between (1) market network structure and (2) loss sharing rules on market participants' default losses. We consider three different network structures, namely a flat-complete, heterogeneous-complete, and a core-periphery structure. The latter is a dominant structure we observe in the OTC market in practice (Getmansky et al. (2016)), while current policy efforts (such as centralized trading) push toward a more complete market network. The set of networks allows us to examine heterogeneity in the effect of clearing across different levels of network completeness and across market participants that differ in portfolio directionality and number of counterparties.

[4] Central clearing is optional for single-name CDS, foreign exchange forwards, and commodity and equity derivatives, which largely remain uncleared (Abad et al. (2016), Office of the Comptroller of the Currency (2016), Financial Stability Board (FSB) (2017)). For example, the Financial Stability Board (FSB) (2017) reports that only 28% of outstanding CDS notionals were cleared in December 2016. The share of notional cleared was only 1% in the foreign exchange market (Wooldridge (2017)) and substantially below 20% in commodity and equity derivatives market in 2016 (Financial Stability Board (FSB) (2017)). CCP membership requirements do not generally prohibit end-users from becoming clearing members. However, Bank for International Settlements (BIS) (2018) reports that only very few financial institutions other than banks and broker-dealers, such as insurers, investment or pension funds, or non-financial companies, are clearing members. For example, the European insurance company *Allianz* reports interest rate swap positions with more than 2 billion EUR notional outstanding at end-2020 (Source: `https://www.allianz.com/en/investor_relations/results-reports/annual-reports.html`), while it is not a clearing member of any of the authorized European central clearinghouses for interest rate swaps (LCH, Eurex, Nasdaq, KDPW, CME Clearing Europe) according to their membership lists as of April 2021.

[5] It is desirable to maximize clearing participation, e.g., to enhance transparency and netting benefits (e.g., see Duffie and Zhu (2011)).

Loss sharing rules might be proportional to net portfolio risk, to gross portfolio risk, or a combination of the two. We start by considering loss sharing proportional to net portfolio risk, which closely resembles current market practice. The analysis highlights that market participants face different levels of expected default losses depending on the market network structure and their position within the network. For example, market participants with exactly offsetting positions have zero net portfolio risk after multilateral netting. Thus, such market participants do not contribute to loss sharing although they benefit from a reduction in counterparty risk compared to an uncleared market. Instead, losses are fully borne by other market participants. We then examine an alternative loss sharing rule, namely loss sharing proportional to gross risk.[6] A main take-away from this analysis is that loss sharing proportional to gross risk can make central clearing incentive compatible *for all market participants*, in the core and periphery of the market, while this is not the case when loss sharing is proportional to net risk. Therefore, the trade-off between net and gross-oriented loss sharing rules is important for the distribution of default losses and clearing incentives across market participants.

The case that CCPs face losses that they need to allocate to clearing members is not a purely theoretical consideration but does occur in practice. For example, the default of a single trader at the Swedish clearinghouse *Nasdaq Clearing AB* caused EUR 107 million to be shared among surviving clearing members in September 2018 (Faruqui et al. (2018)). More generally, derivatives markets price in significant risk of CCP failure (Boissel et al. (2017)).

Understanding the interaction between loss sharing and market network structure is imperative for several reasons. First, it matters from a financial stability point of view since large default losses allocated to systemically important entities can lead to severe systemic consequences. Second, loss allocation affects market participants' incentives to use central clearing. Large participation in central clearing is relevant to achieve more diversification at CCPs as well as increased transparency (e.g., Acharya and Bisin (2014)), mitigation of adverse selection (e.g., Vuillemey (2019)) and counterparty risk (e.g., Bernstein et al. (2019)). Third, due to their effect on expected default losses, loss sharing rules might affect derivative prices, with a potential feedback on hedging costs for

[6] Rules based on gross risk are not uncommon. For example, the Basel III leverage ratio is based on derivatives' gross notional amount (see `https://www.bis.org/publ/bcbs270.htm`). Cont (2015) stresses that portfolios with a small ratio of net-to-gross notional value can result in substantial liquidity risk for CCPs, providing an additional rationale for using (partially) gross-based rules in central clearing.

the real economy. Finally, as clearing regulation is still being refined, market participants try to exert influence on loss sharing rules (e.g., ABN AMRO Clearing Bank N.V. et al. (2020)). Therefore, assessing the financial implications of loss sharing rules is important to understand market participants' incentives.

Our results are as follows. First, at the aggregate level we show that the total amount of default losses largely depends on the structure of the network. In case of a complete network, i.e., when everybody trades with everybody, netting benefits are very large, which results in lower expected total default losses that need to be allocated by the CCP compared to a core-periphery network, which is incomplete by definition.

Second, we zoom in on the distribution of losses across market participants. If loss sharing is proportional to net portfolio risk, central clearing induces a wedge between market participants in a core-periphery market. Specifically, participants in the core with a flat portfolio benefit substantially more from central clearing, relative to not clearing, than those in the periphery or with a directional portfolio. In fact, in simulations of our calibrated model central clearing *increases* default losses borne by peripheral entities, such that they do not benefit at all. The intuition is that due to off-setting positions across counterparties, central clearing substantially reduces net portfolio risk for entities with a flat portfolio. This leads to a small loss sharing contribution. In contrast, peripheral entities have limited netting opportunities. Thus, peripheral entities contribute far more to loss sharing than market participants with flat portfolios, compared to the expected default loss without central clearing. This leads to an uneven distribution of relative clearing benefits across entities. The result is consistent with the reluctance of end-users (which are typically in the periphery with directional portfolios) relative to dealers (which are typically in the core with flat portfolios) to centrally clear their trades in practice (Bank for International Settlements (BIS) (2018)), and with the claim of end-users that they are "penalized" by loss sharing rules (Novick et al. (2018)).

Third, we investigate an alternative loss sharing rule. While losses in our baseline results are shared proportionally to *net* portfolio risk among clearing members, we now examine the effect of sharing losses proportionally to *gross* risk, i.e., gross notional. We show that this loss sharing rule completely removes heterogeneity across market participants. Specifically, every market participant benefits from loss sharing to the same extent, relative to an uncleared market. The intuition is that, compared to an allocation proportional to net risk, allocating losses proportionally to gross risk forces entities in the core to bear losses that otherwise would be allocated to peripheral entities. Thereby, peripheral entities' total contribution is reduced, which makes central clearing more attractive for them. As a result, all entities have the same benefit of central clearing compared to an uncleared market, both in the core and in the periphery. While entities with a flat portfolio are worse off with the gross-based rule compared to the net-based rule, central clearing with the gross-based rule is still beneficial for them in calibrated examples. Importantly, changes in loss allocation rules do <u>not</u> affect the total benefit achieved by the CCP netting across defaulting and surviving clearing members. Instead, it affects the *distribution* of remaining losses (i.e., after netting) among surviving clearing members.

We also investigate the effects of systematic risk, i.e., correlation among the prices of different derivatives contracts. Correlation impairs netting benefits. As a result, we show that central clearing becomes less favorable for market participants in the core with directional portfolios, relative to not clearing. The extreme case is still the one for peripheral traders, for which expected default losses are smaller without central clearing. Nonetheless, also in the case with systematic risk, a loss sharing rule based on gross notional reduces the wedge between core and peripheral entities, as well as between entities with flat and directional portfolios. Finally, we show that our baseline results also apply to tail risks, i.e., default losses in times of market turmoil, and when defaults are correlated across market participants.

Since our analysis concentrates on the risk of default losses, our model does not incorporate other advantages and disadvantages of central clearing, such as the effects on capital requirements, margin costs, transparency, and market liquidity. Nonetheless, we discuss implications from our model about such related equilibrium trade-offs. For example, we argue based on our results that a structural change of the OTC market from a core-periphery structure to a more complete network (e.g., all-to-all trades via centralized trading) would reduce the wedge in the effect of clearing across all entities. Hence, centralized trading could incentivize more entities to use central clearing, even with loss sharing rules based on net risk.

2. Literature Review

We contribute to a growing literature on central clearing and its role in derivatives markets. We are complementary to previous studies and, from a market participant's perspective, provide theoretical evidence for the reluctance of peripheral traders to centrally clear.

Previous studies have examined loss sharing and its interaction with CCP collateral and fee policies (Capponi et al. (2017), Capponi and Cheng (2018), Huang (2018)) as well as its impact on clearing members' propensity to engage in risk-shifting (Biais et al. (2016), Capponi et al. (2019)). These studies typically assume homogeneous market participants. Instead, we focus on heterogeneity across market participants. We are, to the best of our knowledge, the first to investigate distributional effects of loss sharing on default losses across market participants. Thereby, we focus on the role of loss sharing rules, network structure, systematic risk, and tail risk. We show that the loss sharing rule prevailing in practice, based on net portfolio risk, has a highly skewed impact on clearing members and benefits mostly those with flat portfolios. We show that the benefits are homogeneous when a loss sharing rule proportional to gross risk is implemented. Furthermore, we shed light on how market network structure, systematic risk in derivatives prices, and tail risk affects the impact of central clearing.

Duffie and Zhu (2011) and Lewandowska (2015) study the impact of multilateral versus bilateral netting on counterparty risk exposure. Their main result is that a sufficiently large number of clearing members guarantees that central clearing reduces counterparty risk. Cont and Kokholm (2014) follow this rationale and study the effect of correlation of derivative prices *across* derivative classes on the benefit of multilateral netting. They conclude that multilateral netting is likely to reduce counterparty risk exposure compared to bilateral netting in practice. Ghamami and Glasserman (2017) study the capital and collateral costs of central clearing and find that there is no cost incentive for single market participants to centrally clear derivatives, which is driven primarily by margin costs in their model. Their result is contrasted by the Financial Stability Board (FSB) (2018)'s assessment that central clearing reforms create an overall incentive to clear.

Our framework extends the model of Duffie and Zhu (2011). While Duffie and Zhu (2011) focus on the case that all counterparties - and thus the CCP - default, we examine a more general case that the CCP suffers losses due the default of any number of clearing members and then allocates these losses to surviving clearing members. Thereby, we expand the understanding of central clearing in several dimensions, such as its relation to loss sharing rules, heterogeneity of clearing members, and tail risk, and provide new insights for financial regulation. Duffie and Zhu (2011) suggest that netting opportunities are the main driver for benefits of central clearing. Complementing their approach, we bring two other important dimensions into the picture: loss sharing and market network structure. These elements become particularly relevant when we allow for correlation across entities' positions, extending Duffie and Zhu (2011)'s model, in which case portfolio directionality is an important driver for central clearing benefits. We show that a loss sharing rule based on net exposure is not beneficial for directional and peripheral market participants. For this reason, we explore a simple alternative loss sharing rule based on gross exposure. This rules combines the benefit of netting at the CCP level and loss sharing in a way that central clearing can be beneficial for everyone.

Tail risk is also studied by Huang et al. (2019) and Menkveld (2017), who take a CCP's perspective and identify extreme price movements as well as concentrated portfolios as important risks to CCP stability. Complementing their analysis, we take a market participant's perspective and compare central clearing to an uncleared market.

Empirical evidence on the impact of central clearing on derivative markets has been growing only recently, fueled by the increasing availability of granular data. Recent examples are Loon and Zhong (2014), Duffie et al. (2015), Du et al. (2016), and Bellia et al. (2019) for single-name CDS, Menkveld et al. (2015) for equity, Mancini et al. (2016) for interbank repo, and Cenedese et al. (2020) and Dalla Fontana et al. (2019) for IRS markets. Boissel et al. (2017) estimate that prices in the European repo market implied a substantial risk of CCP failure during the 2011 European sovereign debt crisis. Bellia et al. (2019) provide empirical evidence that dealers typically clear contracts with risky counterparties that result in small CCP margins being paid, i.e., contracts with large netting benefits. This result suggests that counterparty risk and netting considerations are indeed highly relevant for decisions to centrally clear. This result is consistent with the historical evidence documented by Vuillemey (2019), who shows that a spike in counterparty risk during the global coffee crisis in 1880-81 motivated a group of well-established coffee traders to create a CCP specifically to mitigate counterparty risk.

3. A model for central clearing and loss sharing

This section describes our model. The key ingredients are: a network of market participants that are subject to default risk, a model for derivative prices, and the loss sharing mechanism of the CCP.

Default losses result from replacement costs, which are changes in contract values during the settlement period, i.e., the time between the most recent exchange of collateral (i.e., variation margin) and liquidation (i.e., settlement) after a counterparty's default.[7] Without loss of generality, we consider a one-period model. At time $t = 0$, derivative contracts are written (or, equivalently, all contracts are marked to market by the exchange of variation margin) and, subsequently, counterparties might default. At time $t = 1$, contracts are settled.

[Place Figure 1 about here]

To capture the effect of loss sharing on market participants' net worth, we compare a central clearing architecture with an uncleared market for a given set of derivative trades. Derivative trades are sorted into $K \in \mathbb{N}$ derivative classes. This classification can result for different reasons, for example from grouping derivatives according to the type of underlying, such as interest rate, credit, commodities, or equities. One could also distinguish between derivatives that are sufficiently standardized for central clearing and those that are not. This interpretation is particularly relevant since we assume that a CCP clears (only) all derivative trades within one specific derivative class K.

There are $\gamma \in \mathbb{N}$ market participants (or, equivalently, *entities*), indexed $j = 1, ..., \gamma$, that trade in all derivative classes K. Each market participant can default with an exogenous probability $\pi \in (0, 1)$. A defaulted market participant does not honor any obligations arising from derivative contracts to other market participants (or the CCP). However, liabilities from surviving market participants (or the CCP) toward a defaulting market participant are being paid (with the exact allocation specified below).

We denote by v_{ij}^k the position of entity i with j in class k. v_{ij}^k reflects the volume and direction of trade. By symmetry, $v_{ij}^k = -v_{ji}^k$. The absolute size $|v_{ij}^k|$ determines the contract volume and thus

[7] The length of the settlement period depends on the liquidity of contracts and typically ranges from 2 to 5 days (Arnsdorf (2012)).

reflects the notional. Since we will be mainly interested in heterogeneity in portfolio directionality but not heterogeneity in size, throughout the paper we assume $v_{ij}^k \in \{-1, 0, 1\}$ for all i, j, k.

3.1. Market setting: derivative prices

We assume that, during the settlement period, entity i's net portfolio profit with j in derivative class k is given by $X_{ij}^k = v_{ij}^k r_{ij}^k$ (see also Figure 1).

r_{ij}^k is the return (at market value, scaled by contract size v_{ij}^k) of the net value of contracts traded between entities i and j in derivative class k during the settlement period. We initially assume that all contract returns are normally distributed with zero mean, $\mathbb{E}[r_{ij}^k] = 0$.[8] Symmetry substantially reduces the dimension of our model and improves its tractability.[9] We consider a single-factor model for contract returns, such that

$$r_{ij}^k = \beta_{ij}^k M + \sigma_{ij}^k \varepsilon_{ij}^k, \tag{1}$$

where $\varepsilon_{ij}^k \sim \mathcal{N}(0, 1)$ is idiosyncratic risk. It is $\varepsilon_{ij}^k = \varepsilon_{ji}^k$ (due to symmetry of trades), ε_{ij}^k and ε_{hl}^m are independent for different derivative classes $k \neq m$ and different entity pairs $(h, l) \notin \{(i, j), (j, i)\}$, and ε_{ij}^k is independent from M for all i, j, k.[10] The systematic risk factor $M \sim \mathcal{N}(0, \sigma_M^2)$ serves as a latent variable that reflects macroeconomic conditions (e.g., the S&P 500 stock market index), and β_{ij}^k is the systematic risk exposure of the contract portfolio traded between i and j in class k.

It will be useful to reparametrize r_{ij}^k in terms of the total volatility, $\sigma_{X,ij}^k = \sqrt{\mathrm{var}(r_{ij}^k)}$, and correlation with M, $\rho_{X,M,ij}^k = \mathrm{cor}\left(r_{ij}^k, M\right)$, such that $\beta_{ij}^k = \rho_{X,M,ij}^k \sigma_{X,ij}^k / \sigma_M$ and $\sigma_{ij}^k = \sigma_{X,ij}^k \sqrt{1 - (\rho_{X,M,ij}^k)^2}$.

Throughout the paper, we assume a positive correlation between returns r_{ij}^k and the systematic risk factor M, $\beta_{ij}^k > 0$. This comes without loss of generality, since the final profit and loss, X_{ij}^k, ultimately depends on the long and short position of entities, reflected by the sign of v_{ij}^k. For

[8] Due to the small time horizon of the settlement period, the risk-free rate and risk premium in derivative prices are negligible. Thus, we assume that they are equal to zero, i.e., $\mathbb{E}[r_{ij}^k] = 0$. Expected returns will, however, be non-zero when we condition on a specific realization of the systematic risk factor.

[9] The assumption of normally distributed prices might not be justified for individual contracts, since these often exhibit heavily skewed and fat-tailed market values. However, due to diversification arising from aggregating across underlying names as well as long and short positions across derivatives traded in the same derivative class with the same counterparty, it is reasonable that exposures are substantially less skewed or fat-tailed, particularly for large dealers. The assumption of normality allows us to work with closed-form analytical solutions or approximations for the most part of the paper.

[10] Due to symmetry, the gain of i is the loss of j, such that $r_{ij}^k = r_{ji}^k$, and $v_{ij}^k = -v_{ji}^k$.

example, if $v_{ij}^k > 0$, then entity i is long in the systematic risk factor, $\text{cor}(X_{ij}^k, M) > 0$. Since symmetry implies that $v_{ji}^k = -v_{ij}^k$, market participant j is then short in the systematic risk factor, $\text{cor}(X_{ji}^k, M) < 0$.

We empirically calibrate contract returns in the model based on 5-day returns of index CDS between January 2006 and December 2009 to capture the elevated stress during crises.[11] Index CDS are already subject to a clearing obligation in the US and EU. The systematic risk factor M is proxied by the S&P 500. The detailed calibration procedure is documented in the Online Appendix.

For simplicity and tractability, we assume that all contracts exhibit the same distributional properties and skip entity-specific indices where possible: $\beta \equiv \beta_{ij}^k$ and $\sigma \equiv \sigma_{ij}^k$ for all $i \neq j$ and $k = 1, ..., K$, which implies that $\rho_{X,M} \equiv \rho_{X,M,ij}^k = \beta \frac{\sigma_M}{\sqrt{\beta^2 \sigma_M^2 + \sigma^2}}$. Thus, there is a monotonic relationship between $\rho_{X,M}$ and β.

3.2. Market setting: network of trades

We consider three stylized network structures which are commonly found in practice. For simplicity, we assume that networks are the same within each asset class k.[12]

3.2.1. Flat and complete network

In a flat-complete network, each entity trades with each other entity and each entity's portfolio within each derivative class is flat across counterparties, that is

$$v_{ij}^k \neq 0 \quad \forall i, j \in \{1, ..., \gamma\}, i \neq j, k = 1, ..., K \tag{2}$$

$$\text{and} \sum_{\substack{j=1 \\ j \neq i}}^{\gamma} v_{ij}^k \approx 0 \quad \forall i = 1, ..., \gamma, k = 1, ..., K, \tag{3}$$

where the last condition holds with equality if γ is uneven. If Equation (3) holds with equality for all entities i and derivative prices are perfectly correlated (i.e., if there was no idiosyncratic risk),

[11] A settlement period of 5 days is a common assumption in practice. For example, initial margins for OTC foreign exchange and IRS trades is based on a 5-day settlement period at CME (see their CPMI-IOSCO Quantitative Disclosures for 2019Q3).

[12] It is straightforward to modify this assumption, which would not qualitatively change our results but substantially complicate the analysis and require additional and more detailed assumptions.

each entity's portfolio is risk-less.

3.2.2. Heterogeneous and complete network

In a heterogeneous-complete network, each entity trades with each other entity. However, contrary to the flat-complete network, not every entity's portfolio is flat across counterparties. Instead, this network structure includes entities with different portfolio directionalities. Equation (4) illustrates differences in portfolio directionality in a network with five entities, where a cell (i, j) is the derivative position of entity i (in row i) with counterparty j (in column j):

$$(v_{ij}^k)_{i,j\in\{1,\ldots,\gamma\}} = \begin{pmatrix} & 1 & 1 & 1 & 1 & \text{(fully directional)} \\ -1 & & 1 & 1 & 1 & \\ -1 & -1 & & 1 & 1 & \text{(flat)} \\ -1 & -1 & -1 & & 1 & \\ -1 & -1 & -1 & -1 & & \text{(fully directional)} \end{pmatrix} \quad \text{for all } k = 1, .., K. \tag{4}$$

In this network, portfolios may be flat, partially or fully directional across counterparties. In particular, each heterogeneous-complete network includes one entity $i = \frac{\gamma+1}{2}$ with a flat portfolio, $\frac{\gamma-1}{2}$ entities with directional portfolios across counterparties with a net portfolio value that is positively correlated with the systematic risk factor, $\sum_{j=1,j\neq i}^{\gamma} v_{ij}^k > 0$, and $\frac{\gamma-1}{2}$ entities with directional portfolios across counterparties with a net portfolio value that is negatively correlated with the systematic risk factor, $\sum_{j=1,j\neq i}^{\gamma} v_{ij}^k < 0$. This network exists only if $\gamma \geq 3$ is uneven:

$$\sum_{j=1,j\neq i}^{\gamma} v_{ij}^k \in (0, \gamma-1] \quad \forall i = \frac{\gamma+1}{2} + 1, ..., \gamma, \quad \text{(core, directional)} \tag{5}$$

$$\sum_{j=1,j\neq i}^{\gamma} v_{ij}^k = 0 \quad \text{if } i = \frac{\gamma+1}{2}, \quad \text{(core, flat)} \tag{6}$$

$$\text{and} \quad \sum_{j=1,j\neq i}^{\gamma} v_{ij}^k \in [-(\gamma-1), 0) \quad \forall i = 1, ..., \frac{\gamma+1}{2} - 1 \quad \text{(core, directional)}. \tag{7}$$

While this network is clearly a simplification, it is transparent and allows us to shed light on the trade-off between directionality in portfolios.[13]

[13] We specify an exogenous market network of positions in order to focus on the effect of central clearing and loss sharing on default losses. Our assumptions about the network structure are realistic, e.g., in the CDS market

3.2.3. Core-periphery network

Finally, we consider a core-periphery network, which can be found in many OTC markets in practice (Getmansky et al. (2016), Di Maggio et al. (2017), Li and Schürhoff (2019)). In this case, central intermediaries ("dealers") in the core of the network trade (1) with each other and (2) with entities in the periphery. Peripheral entities only trade with a small number of dealers. We assume that there are $\frac{\gamma-1}{2}$ entities in the core of the network, and $\frac{\gamma+1}{2}$ entities in the periphery. The network exists and includes more than one core entity if $\frac{\gamma-1}{2}$ is uneven, i.e., $\gamma = 2k+1 \geq 5$ with uneven $k \in \mathbb{N}$, $k \geq 3$. Each peripheral entity trades exactly with one core entity, while each core entities trades with one or two periphery entities and with all other core entities. The following illustrates the network with $\gamma = 11$ entities:

$$(v_{ij}^k)_{i,j\in\{1,\ldots,\gamma\}} = \begin{pmatrix}
 & & & & & 1 & & & & & \\
 & & & & 1 & & & & & & \\
 & & & 1 & & & & & & & \\
 & & -1 & & 1 & 1 & 1 & 1 & & & \\
 & -1 & & -1 & & 1 & 1 & 1 & & & \\
-1 & & & -1 & -1 & & 1 & 1 & & & 1 \\
 & & & -1 & -1 & -1 & & 1 & & 1 & \\
 & & & -1 & -1 & -1 & -1 & & 1 & & \\
 & & & & & & & -1 & & & \\
 & & & & & & -1 & & & & \\
 & & & & & -1 & & & & &
\end{pmatrix}. \tag{8}$$

The core in Equation (8) is marked in gray: rows (and columns) 4-8 correspond to entities in the core. On the contrary, rows (columns) 1-3 and 9-11 correspond to entities in the periphery. In this example, no two entities in the periphery trade with each other, each entity in the periphery trades with one entity in the core, and all entities in the core trade with each other. Peripheral entities

(Getmansky et al. (2016)). While it may be unknown whether a specific entity will be long or short in the future, the market structure is likely to be stable over time. Moreover, business models and strategies of many entities naturally lead to the direction of trade sides. For example, insurers take pay-float positions to hedge the negative duration mismatch on their balance sheet. Another example are asset managers (e.g., hedge funds), that have been replacing dealers as largest net sellers of CDS protection since the 2008 financial crisis (Siriwardane (2019)).

are thus purely directional, while core entities differ in directionality,

$$\sum_{j=1,j\neq i}^{\gamma} v_{ij}^k = 1 \quad \forall i = \frac{3\gamma-1}{4}+1,...,\gamma, \quad \text{(periphery)} \tag{9}$$

$$\sum_{j=1,j\neq i}^{\gamma} v_{ij}^k \in (0,\frac{\gamma}{2}) \quad \forall i = \frac{\gamma+1}{2}+1,...,\frac{3\gamma-1}{4}, \quad \text{(core, directional)} \tag{10}$$

$$\sum_{j=1,j\neq i}^{\gamma} v_{ij}^k = 0 \quad \text{if } i = \frac{\gamma+1}{2}, \quad \text{(core, flat)} \tag{11}$$

$$\sum_{j=1,j\neq i}^{\gamma} v_{ij}^k \in [\frac{\gamma}{2},0) \quad \forall i = \frac{\gamma+1}{4}+1,...,\frac{\gamma+1}{2}-1, \quad \text{(core, directional)} \tag{12}$$

$$\text{and} \sum_{j=1,j\neq i}^{\gamma} v_{ij}^k = -1 \quad \forall i = 1,...,\frac{\gamma+1}{4} \quad \text{(periphery).} \tag{13}$$

3.3. Default losses and loss sharing

Our model for entity defaults is inspired by Merton (1974)'s credit risk model, but additionally allows for correlation of defaults. We model each entity j's value of assets A_j. If A_j is below an exogenous debt threshold, j defaults. In the default model, the random value of entity j's value of assets at the settlement period begin is given by

$$A_j = \exp\left(\mu_{A_j} - \frac{\sigma_{A_j}^2}{2} + \sigma_{A_j} W_j\right), \tag{14}$$

where $(W_1,...,W_\gamma)$ are jointly standard normally distributed and correlated according to the correlation matrix $(\rho_{A_j,A_h})_{j,h\in\{1,...,\gamma\}}$.[14] μ_{A_j} and σ_{A_j} are the drift and volatility of the asset value process, respectively.[15]

If asset values are correlated, $\rho_{A_j,A_h} > 0$ for $j \neq h$, defaults are correlated across market participants. Correlation of defaults can result from interconnectedness between (financial) institutions, e.g., interbank liabilities, such that the financial distress of one entity spills over to other entities. A prime example has been Lehman Brothers' default during the 2007-08 financial crisis, which am-

[14]In an earlier working paper version, we also allowed for correlation between asset values and systematic risk factor, which introduces a wedge between entities with a negative vs. positive correlation between their derivatives portfolio and the systematic risk factor. Here, we focus on correlation across derivatives prices but assume independence between derivative prices and defaults, which substantially improves the tractability of our model by allowing for closed-form solutions.

[15]The model is described in the Online Appendix in detail.

plified losses of other financial institutions. Considering correlation of defaults is important since it can potentially impair the effectiveness of loss sharing. We define by D_j a binary random variable that equals one if market participant j defaults, which happens when A_j breaches an exogenous debt threshold. We calibrate the debt threshold and distributional parameters of $(A_j)_j$ for a given default probability $\pi \in (0,1)$, i.e., $\mathbb{P}(D_j = 1) = \pi$, and asset correlation $\rho_{A_j,A_h} \equiv \rho_{A,A}$ for all j,h.

Market participants exchange collateral (i.e., initial margin) with each other and with the CCP.[16] First, we consider a non-centrally cleared market. We assume that all entity pairs have bilateral (close-out) netting agreements with each other. Netting agreements aggregate outstanding positions into one single claim (Bergman et al. (2004)) and are common market practice (e.g., Mengle (2010)). Bilateral netting offsets gains and losses of different derivative trades across different derivative classes (e.g., IRS and CDS) with a single counterparty.

If all derivative classes are bilaterally traded, then the expected loss due to the default of entity i's counterparties, i.e. entity i's expected *default loss*, is

$$\mathbb{E}\left[DL_i^K\right] = \mathbb{E}\left[\sum_{j=1, j\neq i}^{\gamma} D_j \max\left(\sum_{k=1}^{K} X_{ij}^k - C_{ij}^K, 0\right)\right], \tag{15}$$

where C_{ij}^K is the total collateral provided by j to i. Note that a counterparty j's default results in a default loss for i only if it coincides with an adverse price movement in excess of the collateral C_{ij}^K posted by j to i.

We reparametrize the collateral as a Value-at-Risk of the uncleared portfolio between i and j, such that $C_{ij}^K = VaR_{\alpha_{uc}}\left(\sum_{k=1}^K X_{ij}^k\right)$ with $\alpha_{uc} \in (0,1)$ being the confidence level.[17] The larger α_{uc}, the more protected is i against a default of j. It is then straightforward to show that i's expected default loss corresponds to (see Proposition A.1 in the Appendix)

$$\mathbb{E}\left[DL_i^K\right] = \pi\xi(\alpha_{uc}) \sum_{j=1, j\neq i}^{\gamma} 1_{\{\sum_{k=1}^K |v_{ij}^k|>0\}} \sqrt{\sigma_M^2 \beta^2 \left(\sum_{k=1}^K v_{ij}^k\right)^2 + \sigma^2 \sum_{k=1}^K \left(v_{ij}^k\right)^2}, \tag{16}$$

where $\pi \in (0,1)$ is the probability of each entity's default and $\xi(\alpha) = (1-\alpha)\Phi^{-1}(1-\alpha) + \varphi\left(\Phi^{-1}(\alpha)\right)$.

Second, we introduce central clearing and loss sharing. Following Duffie and Zhu (2011), we

[16] In Biais et al. (2016)'s model, the exchange of collateral is efficient due to moral hazard frictions.

[17] Using a Value-at-Risk approach is common industry practice (e.g., ISDA (2013)) and partly mandated by the regulator (Bank for International Settlements (BIS) (2019)).

examine the case that one derivative class is centrally cleared while others remain uncleared. This is a natural starting point given that central clearing is mandated and utilized in only a small number of derivative classes in practice. If derivative class K is centrally cleared by a CCP, all entities $i = 1, ..., \gamma$ become clearing members at the CCP while the CCP is the single counterparty to all positions in this derivative class. Thus, there is netting across counterparties, which is called *multilateral netting*. For example, in Figure 2, A can reduce its total exposure from \$100 to \$40 with multilateral netting, as the exposure of \$100 to B is offset with a loss of \$60 to C.

[Place Figure 2 about here]

It is important to note that peripheral entities are not prohibited from becoming clearing members in practice.[18] To avoid becoming a clearing member, entities might either abstain from central clearing (if it is optional) or choose to use client clearing. For example, Fiedor et al. (2017) find that most non-G16 banks, insurance companies, and pension funds choose to use client clearing instead of becoming a direct clearing member in the European interest rate swap market, in which central clearing is mandatory.[19] These entities, which are typically in the periphery of the network and have directional portfolios, choose to avoid becoming clearing members and participating in loss sharing. We do not explicitly model client clearing, and focus on the incentives to participate in loss sharing relative to an uncleared market.[20]

If a clearing member's default results in a loss for the CCP, this loss is offset by contributions from the surviving clearing members. The CCP suffers losses only in case at least one clearing member j defaults and the net liability of j toward the CCP exceeds the collateral C_j^{CCP} posted

[18] Instead, regulation forces CCPs to provide non-discriminatory access to clearing and to use membership requirements only to control the risk for the CCP (e.g., see European Regulation No 648/2012 (EMIR) Article 37). For example, the membership criteria of LCH include minimum levels of capital and experienced staff, but they do not restrict access for particular types of financial institutions (`https://www.lch.com/membership/ltd-membership`).

[19] In other OTC derivatives markets, in which central clearing is not mandatory, client clearing is less common. For example, less than 5% of initial margin for OTC foreign exchange derivatives and less than 10% of initial margin for OTC CDS at the London-based clearinghouse LCH attributes to client clearing activities (Source: LCH LTD and SA CPMI-IOSCO Quantitative Disclosures 2020Q4).

[20] Client clearing implementation varies across CCPs and jurisdictions (e.g., see, Braithwaite (2016)). Often, clients are shielded from CCP loss sharing and dealers guarantee their clients' obligations to the CCP. In this case, using client clearing is similar to trading an uncleared contract in our model.

by j to the CCP.[21] The aggregate default loss at the CCP level is then given by

$$DL^{CCP} = \sum_{j=1}^{\gamma} D_j \max\left(\sum_{\substack{g=1,\\ g\neq j}}^{\gamma} X_{gj}^{K} - C_j^{CCP}, 0 \right). \tag{17}$$

The CCP's task is to allocate DL^{CCP} to surviving (i.e., non-defaulting) clearing members. We consider two different loss sharing rules, namely allocating losses (1) proportionally to net risk and (2) proportionally to gross risk. Since a clearing member i's portfolio net risk is proportional to the (initial) margin C_i^{CCP} in our model, the first loss sharing rule is equivalent to allocating losses proportionally to initial margin. The second loss sharing rule is equivalent to allocating losses proportionally to a clearing member i's gross notional cleared, which is $\sum_{j=1,j\neq i}^{\gamma} |v_{ij}^{K}|$.

Loss sharing proportional to net risk is very close to current market practice and, e.g., suggested by Duffie (2015).[22] Under this rule, a clearing member contributes more if the net risk in the cleared derivatives portfolio is larger. The natural alternative is loss sharing proportional to gross risk, i.e., gross notional. In this case, clearing members do not benefit from netting: two members with the same gross notional are allocated the same loss even if one member's portfolio is directional and the other one's is perfectly hedged.[23]

A clearing member i's expected loss in the centrally cleared derivative class K due to counterparty defaults is given by its expected loss sharing contribution (LSC), which in the case of

[21] The mechanism in our model is most closely related to cash calls. In practice, default losses are absorbed not only by the defaulter's collateral and surviving members but also by a share of the CCP's capital, its *skin-in-the-game* (SITG), and a pre-funded default fund contribution of the defaulting clearing member. Pre-funded default fund contributions are 4% of initial margin for cleared OTC IRS at LCH and 7% for cleared CDS at ICE Clear Credit in 2021 (Source: CPMI-IOSCO Quantitative Disclosures 2021Q1), which are the largest CCPs for USD- and Euro-denominated IRS and CDS, respectively. Since pre-funded default fund contributions are replenished regularly, they are similar to cash calls in our model. CCP's SITG is significantly smaller and typically below 20% of pre-funded default fund contributions (ESRB (2021)). Therefore, we do neither consider SITG nor pre-funded default funds explicitly in the model. We do not expect that including them would change our results qualitatively, but it would substantially complicate the analysis. For a detailed discussion of the use of a CCP's pre-funded resources to cover losses, we refer to Duffie (2015), Armakolla and Laurent (2017), and Elliott (2013).

[22] For example, the default rules of LCH, one of the largest CCPs worldwide, specify that contributions to loss sharing are proportional to pre-funded contributions to the CCP's default fund, which are proportional to the CCP's exposure to each clearing member and replenished regularly (typically each month), e.g., see *LCH Default Rules for Listed Rates* available at `https://www.lch.com/resources/rules-and-regulations`. Since a CCP's exposure is based on changes in a clearing member's *net* portfolio value, it is proportional to net portfolio risk and, thus, the initial margin in our model with Normally distributed prices.

[23] Other loss sharing mechanisms may lie in-between these two extremes. Since gross exposures seize to exist once they have been novated and netted by a CCP, in practice loss sharing proportionally to gross risk would need to be computed by aggregating the gross *flow* of cleared transactions instead of the existing (net) *stock* of outstanding exposures.

allocating losses proportionally to net risk is[24]

$$\mathbb{E}[LSC_i^{\propto\text{net}}] = \mathbb{E}\left[\frac{(1-D_i)C_i^{CCP}}{\sum_{g=1}^{\gamma}(1-D_g)C_g^{CCP}}DL^{CCP} \mid \sum_{g=1}^{\gamma}(1-D_g)>0\right], \tag{18}$$

and in the case of allocating losses proportionally to gross risk is

$$\mathbb{E}[LSC_i^{\propto\text{gross}}] = \mathbb{E}\left[\frac{(1-D_i)\sum_{j=1,j\neq i}^{\gamma}|v_{ij}^K|}{\sum_{g=1}^{\gamma}(1-D_g)\sum_{j=1,j\neq g}^{\gamma}|v_{gj}^K|}DL^{CCP} \mid \sum_{g=1}^{\gamma}(1-D_g)>0\right]. \tag{19}$$

The loss sharing contributions can be rewritten as (see Propositions A.2 and A.3 in the Appendix)

$$\mathbb{E}[LSC_i^{\propto\text{net}}] = \mathbb{E}\left[\left(\frac{\sum_{g=1}^{\gamma}\bar{\sigma}_g^{CCP}}{\sum_{g=1}^{\gamma}(1-D_g)\bar{\sigma}_g^{CCP}}-1\right)\xi(\alpha_{CCP})(1-D_i)\bar{\sigma}_i^{CCP} \mid \sum_{g=1}^{\gamma}(1-D_g)>0\right], \tag{20}$$

and

$$\mathbb{E}[LSC_i^{\propto\text{gross}}] = \mathbb{E}\left[\frac{(1-D_i)v_{i*}^K}{\sum_{g=1}^{\gamma}(1-D_g)v_{g*}^K}\xi(\alpha_{CCP})\sum_{j=1}^{\gamma}D_j\bar{\sigma}_j^{CCP} \mid \sum_{g=1}^{\gamma}(1-D_g)>0\right], \tag{21}$$

where $\bar{\sigma}_g^{CCP} = \sqrt{\sigma_M^2\beta^2\left(\sum_{j=1,j\neq g}^{\gamma}v_{jg}^K\right)^2+\sigma^2\sum_{j=1,j\neq g}^{\gamma}(v_{jg}^K)^2}$ is the volatility in entity g's centrally cleared portfolio and $v_{i*}^K=\sum_{j=1,j\neq i}^{\gamma}|v_{ij}^K|$ is entity i's total gross notional cleared. If one derivative class is centrally cleared and $K-1$ remaining classes are uncleared, entity i's overall expected default loss is $\mathbb{E}\left[DL_i^{K-1}+LSC_i^{CCP}\right]$.

Throughout the analysis, we assume that the clearing and uncleared margins are both based on a 99% confidence level, which is consistent with common market practice.[25] We assume that, if not specified differently, entities default with probability $\pi=0.05$ and assets in the default model have correlation $\rho_{A,A}=0.1$.[26]

We examine the effect of loss sharing on an entity's expected default loss relative to an uncleared

[24]We condition on at least one entity surviving since it is extremely unlikely that all entities default at the same time, and in practice it seems likely that a government would bail out a CCP in the case that all clearing members default, given the systemic importance of most CCPs.

[25]For example, CME sets initial margins at the 99% VaR for futures and options, and at the 99.7% VaR for interest rate swaps (see CME's CPMI-IOSCO Quantitative Disclosures 2019Q3). The effect of margin differences in our model is straightforward to assess since $\xi(\alpha)$ is decreasing with α: the larger α_{CCP} relative to α_{uc}, the smaller is $\mathbb{E}[LSC_i^{CCP}+DL_i^{K-1}]$ relative to $\mathbb{E}[DL_i^K]$.

[26]The results are also robust toward other levels of correlation of defaults. The detailed calibration is reported in the Online Appendix.

market, which we define by[27]

$$\Delta E_i = \frac{\mathbb{E}[{DL_i}^{K-1} + LSC_i^{CCP}] - \mathbb{E}[{DL_i}^{K}]}{\mathbb{E}[{DL_i}^{K}]}. \tag{22}$$

If $\Delta E_i < 0$, loss sharing reduces entity i's expected default losses compared to an uncleared market. For notational convenience, we sometimes call loss sharing *more favorable* if ΔE_i is smaller, which means that loss sharing results in a larger reduction (or, equivalently, smaller increase) of expected default losses relative to an uncleared market.

In the following, we investigate the distribution of ΔE_i across market participants i and its interaction with network structure, loss sharing rules, and systematic risk. To motivate our focus on ΔE_i, suppose that market participants decide whether to use central clearing based on its effect on expected default losses, i.e., counterparty risk. Counterparty risk is highlighted as a key determinant for central clearing participation, e.g., by Bellia et al. (2019), Financial Stability Board (FSB) (2018), and Vuillemey (2019). Furthermore, suppose that regulators maximize a policy objective $\mathcal{O} = \mathcal{O}(\omega)$, $\omega \in \Omega$, where Ω is the space of potential policies (e.g., regulation of loss sharing rules or trading platforms). Each policy ω is associated with a set $\mathcal{S}(\omega) \subseteq \mathbb{N}$ of market participants that choose to use central clearing under this policy. Then, the optimal policy solves

$$\max_{\omega \in \Omega} \ \mathcal{O}(\omega) \tag{23}$$

$$\text{subject to } \Delta E_i(\omega) \leq 0 \quad \text{for all } i \in \mathcal{S}(\omega). \tag{24}$$

Hence, $\Delta E_i(\omega) \leq 0$ is the incentive compatibility constraint conditional on a policy ω that incentivizes market participant i to use central clearing. Indeed, policymakers have been undertaking substantial effort to incentivize market participants to use central clearing (e.g., see Financial Stability Board (FSB) (2018)), which suggests that $\frac{d\mathcal{O}}{d|S|} > 0$. To achieve policies involving large central clearing participation, it is of paramount importance to understand the effect of central clearing on default losses, ΔE_i, and its heterogeneity across market participants. The following analysis serves as a first step by highlighting the key economic trade-offs.

[27] While we derive closed-form analytic expressions for uncleared default losses, the expected contributions to loss sharing are not available in closed-form (but only conditional on default events $(D_j)_{j=1,\ldots,\gamma}$. Therefore, we evaluate Equations (20) and (21) by Monte-Carlo simulations with 200,000 realizations of default vectors $(D_j)_{j=1,\ldots,\gamma}$.

4. Network structure and total default losses

Before examining market participants' default losses, we compare the total expected default loss in the centrally cleared derivative class K, $\mathbb{E}[DL^{CCP}]$, across different network structures. It is important to note (1) that different network structures imply different levels of total losses due to differences in (aggregate) netting opportunities and (2) that these differences depend on the level of systematic risk. Since networks also differ in total trading volume, we focus on differences in network structure by comparing the total expected default loss per dollar gross notional cleared.

In the absence of systematic risk (i.e., if $\rho_{X,M} = 0$), each entity's portfolio risk in the flat-complete and in the heterogeneous-complete networks is the same: trades have only idiosyncratic risk and each entity has the same number and volume of trades. As a result, if $\rho_{X,M} = 0$, the total expected default loss per gross notional cleared is the same in both networks.

In contrast, peripheral entities only trade with one counterparty by assumption, which reduces the ability to multilaterally net across gains and losses across counterparties. Therefore, a peripheral entity's (cleared) portfolio's risk per gross notional cleared is larger than that for a core entity. As a result, the total expected default loss per gross notional cleared is larger in the core-periphery network than in complete networks. Figure 3 (a) illustrates this result.

[Place Figure 3 about here]

In the presence of systematic risk, the directionality of entities' portfolios becomes relevant. From above, the volatility of an entity i's cleared (class-K) portfolio is

$$\bar{\sigma}_i^{CCP} = \sqrt{\sum_{j=1,j\neq i}^{\gamma} (v_{ji}^K)^2 \left(\sigma_M^2 \beta^2 \left(\frac{\sum_{j=1,j\neq i}^{\gamma} v_{ji}^K}{\sum_{j=1,j\neq i}^{\gamma} |v_{ji}^K|} \right)^2 \sum_{j=1,j\neq i}^{\gamma} |v_{ji}^K| + \sigma^2 \right)}. \tag{25}$$

Therefore, holding the total portfolio size (i.e., gross notional) $\sum_{j=1,j\neq i}^{\gamma} |v_{ji}^K|$ fixed, larger directionality reflected in a larger net-to-gross ratio $\left(\frac{\sum_{j=1,j\neq i}^{\gamma} v_{ji}^K}{\sum_{j=1,j\neq i}^{\gamma} |v_{ji}^K|} \right)^2$ results in larger portfolio risk $\bar{\sigma}_i^{CCP}$. In other words, entities with a more directional portfolio have higher portfolio risk. This implies that the total class-K expected default loss is larger in a heterogeneous-complete network compared a flat-complete network. Figure 3 (b) illustrates this result.

A core-periphery network additionally includes peripheral entities with a purely directional portfolio and only one counterparty. Perfectly directional entities in the core and peripheral entities both have the largest possible net-to-gross ratio of one. Thus, the total class-K expected default loss in a heterogeneous-complete network is similar to that in a core-periphery network, with the important difference that the latter includes relatively more entities with a large net-to-gross-ratio. As a result, the total class-K expected default loss is larger in a core-periphery network than in a heterogeneous-complete network.

Overall, these results show that total default losses highly depend on network structure. Proposition 1 provides a formal proof of the results and shows that they hold independently of the number of market participants γ. The proofs of this and other propositions are provided in the Online Appendix.

Proposition 1 (Class-k total expected default loss)**.** *Fix $\gamma > 2$ and denote by GN the total gross notional centrally cleared,*

$$GN = \sum_{i=1}^{\gamma} \sum_{j=1, j \neq i}^{\gamma} |v_{ji}^{K}|. \tag{26}$$

The total expected default loss per dollar gross notional in the centrally cleared derivative class K is strictly larger in the core-periphery network than in the heterogeneous-complete network,

$$\frac{\mathbb{E}\left[DL_{core-periphery}^{CCP}\right]}{GN_{core-periphery}} > \frac{\mathbb{E}\left[DL_{heterogeneous-complete}^{CCP}\right]}{GN_{heterogeneous-complete}}. \tag{27}$$

In the absence of systematic risk (i.e., with $\rho_{X,M} = 0$), the class-K total expected default loss per dollar gross notional coincides in the heterogeneous-complete and flat network,

$$\frac{\mathbb{E}\left[DL_{heterogeneous-complete}^{CCP}\right]}{GN_{heterogeneous-complete}} = \frac{\mathbb{E}\left[DL_{flat}^{CCP}\right]}{GN_{flat}}, \tag{28}$$

but in the presence of systematic risk it is strictly smaller in the flat network,

$$\frac{\mathbb{E}\left[DL_{heterogeneous-complete}^{CCP}\right]}{GN_{heterogeneous-complete}} > \frac{\mathbb{E}\left[DL_{flat}^{CCP}\right]}{GN_{flat}}. \tag{29}$$

5. Effect of loss sharing on expected default losses

In this section, we examine the relative effect of loss sharing on expected default losses, ΔE, and how it depends on network structure, market participants' position in networks, and the directionality of market participants' portfolio. First, we consider the case without systematic risk (i.e., if $\rho_{X,M} = 0$) and, second, we examine the effect of systematic risk.

5.1. The case without systematic risk

We start by considering a loss allocation that is proportional to the net risk of the cleared portfolio, $\bar{\sigma}_i^{CCP}$, and, thus, in our model proportional to initial margins. With only idiosyncratic risk, volatility of the cleared portfolio is independent of portfolio directionality. However, in the core-periphery network, it differs between core and peripheral entities. Due to multilateral netting across counterparties, the larger the number of counterparties, the smaller is the volatility of the cleared portfolio relative to the expected uncleared default loss.[28] That is particularly relevant for comparing entities in the core to those in the periphery of the network: there are strong multilateral netting benefits for core entities, as they trade with many counterparties, and weak benefits for peripheral entities, as they trade with only one counterparty. As a consequence, when losses are allocated proportionally to net risk (and thus, portfolio volatility), peripheral entities bear more losses than core entities, relative to their expected default loss without central clearing. In other words, loss sharing is less beneficial (or even more harmful) for peripheral than for core entities. Figure 4 (a) illustrates this result.[29]

[Place Figure 4 about here]

In contrast to the core-periphery network, the effect of loss sharing does neither differ across entities within nor across the heterogeneous-complete and flat-complete networks. The reason is that - in the absence of systematic risk - differences in directionality do not affect portfolio risk

[28]This directly follows from the fact that the expected uncleared default losses depends linearly on the number of counterparties while the volatility of the cleared portfolio is proportional to its square-root (see Equation (25)).

[29]In the exemplary situation of Figure 4, all entities in the core-periphery network face an increase in expected default losses. The reason is that we assume that only one derivative class is being centrally cleared while others remain uncleared, following Duffie and Zhu (2011). Duffie and Zhu (2011) show for this case that central clearing can increase default losses if the number of counterparties is small relative to the number of derivative classes. This result also applies in our model. Central clearing becomes beneficial when the number of counterparties is sufficiently large.

and thus both the relative loss sharing contribution as well as the class-K total expected losses coincide. Nonetheless, as we have shown in the previous section, the class-K total expected loss is larger in the core-periphery network than in complete networks, due to differences in netting opportunities. As a result, loss sharing is generally more beneficial in complete networks than in the core-periphery network, as Figure 4 (a) shows. The following proposition summarizes these results.

Proposition 2 (Loss sharing proportional to net risk without systematic risk)**.** *Assume that derivative positions are idiosyncratic and consider a core-periphery network with $\gamma > 3$. If losses are allocated proportionally to net risk, it is*

$$\Delta E^{\propto net}_{periphery} > \Delta E^{\propto net}_{core} \tag{30}$$

and

$$\frac{d}{d\gamma}\left(\Delta E^{\propto net}_{periphery} - \Delta E^{\propto net}_{core}\right) > 0. \tag{31}$$

The change in expected default losses due to loss sharing is strictly larger in the core-periphery network than in the heterogeneous-complete and flat-complete networks for any pair of entities,

$$\min\left(\Delta E^{\propto net}_{periphery}, \Delta E^{\propto net}_{core}\right) > \max\left(\Delta E^{\propto net}_{\textit{flat-complete}}, \Delta E^{\propto net}_{\textit{heterog-complete}}\right). \tag{32}$$

Next, we examine the impact of altering the loss sharing mechanism. If losses are allocated proportionally to gross risk, the relative effect of loss sharing is the same across entities in the core-periphery network, as Figure 4 (b) illustrates. The reason is that gross notional removes the impact of multilateral netting and is proportional to the expected uncleared class-K default loss. Therefore, the loss sharing contribution of core entities with a large multilateral netting potential (i.e., with many counterparties) is the same as that of peripheral entities with no multilateral netting potential relative to their uncleared default loss, respectively. By removing the impact of multilateral netting, core entities' expected LSC increases compared to one that is proportional to net risk, while peripheral entities' expected LSC decreases. As a result, all entities benefit from loss sharing to the same extent - relative to their uncleared expected default loss.

Although differences within the core-periphery network vanish, differences across networks remain. Figure 4 (b) shows that loss sharing is relatively more beneficial in the flat-complete and heterogeneous-complete networks than in the core-periphery network. The reason is that, due to overall netting opportunities, the total expected default loss in the centrally cleared derivative class is smaller in the former than in the core-periphery network (see Section 4). Intuitively, loss allocation mechanisms only affect the allocation of losses *within* networks, but not the overall expected loss *across* networks. The following proposition summarizes these results.

Proposition 3 (Loss sharing proportional to gross risk without systematic risk)**.** *Assume that derivative positions are idiosyncratic and consider a core-periphery network with $\gamma > 3$. If losses are allocated proportionally to gross risk, it is*

$$\Delta E^{\propto gross}_{periphery} = \Delta E^{\propto gross}_{core} \tag{33}$$

independent of γ. The increase in expected default losses due to loss sharing is strictly larger in the core-periphery network than in the heterogeneous-complete and flat-complete networks for any pair of entities,

$$\min\left(\Delta E^{\propto gross}_{periphery}, \Delta E^{\propto gross}_{core}\right) > \max\left(\Delta E^{\propto gross}_{\text{flat-complete}}, \Delta E^{\propto gross}_{\text{heterog-complete}}\right). \tag{34}$$

5.2. The case with systematic risk

In this section, we assume that derivative prices are correlated. More specifically, each derivative's price is correlated with the systematic risk factor M. Without loss of generality, we assume a positive correlation $\rho_{X,M} > 0$. The important effect of systematic risk is on netting opportunities: the more correlated derivative prices are, the smaller are netting benefits. As a result, portfolio risk differs with portfolio directionality, that is, with the net-to-gross ratio

$$v_i^* = \frac{|\sum_{j=1,j\neq i}^{\gamma} v_{ji}^K|}{\sum_{j=1,j\neq i}^{\gamma} |v_{ji}^K|}. \tag{35}$$

The more directional a portfolio, the smaller are netting benefits and the larger is its overall correlation with the systematic risk factor and, thus, its total volatility.

In Section 4 we have highlighted that this effect leads to a larger total default loss in a heterogeneous-complete network, where some entities have directional portfolio, compared to a flat-complete network, where all entities have flat portfolios. In this section, we examine the implications on the entity-level, i.e., on an entity's expected loss sharing contribution. On the entity-level, the presence of systematic risk implies that entities with different portfolio directionality post different levels of margins, due to differences in portfolio risk. Entities with a more directional portfolio post more margin.

As a result, if loss sharing is proportional to net risk, it is relatively less beneficial for entities with a more directional portfolio. Figure 5 (a) illustrates this effect. It applies within the heterogeneous-complete and the core-periphery network, where cleared portfolios of core entities differ in directionality. Peripheral entities are similar to core entities with a purely directional portfolio, but additionally have lower multilateral netting since they only trade with one counterparty. Thus, as before, peripheral entities have much lower benefits from loss sharing than core entities. As a result, in Figure 5 (a) central clearing is incentive compatible only for entities in the core but not for peripheral entities, from the perspective of expected default losses (i.e., $\Delta E_i \leq 0$). Instead, in the flat complete network, all entities have a flat portfolio and, thus, have the same benefit of loss sharing.

[Place Figure 5 about here]

Due to the presence of systematic risk, loss allocation rules attain a second task, namely to smooth the effect of loss sharing across core entities with different portfolio directionality. As Figure 5 (b) illustrates, if losses are allocated proportionally to gross risk, the effect of loss sharing is the same within networks – independent of portfolio directionality and of whether entities are in the core or periphery. The intuition is similar to above. The allocation proportional to gross risk removes the impact of netting opportunities on the relative loss allocation. Thus, if two surviving entities have the same portfolio size (such as any two core entities in the core-periphery network), they bear the same amount of losses. As a result, all entities within a given network benefit from loss sharing to the same extent - relative to their uncleared expected default loss. In Figure 5 (b) central clearing is then incentive compatible for all market participants, both in the core and periphery (i.e., $\Delta E_i \leq 0$). Importantly, while core entities with a flat portfolio benefit less with a rule that is proportional to gross risk compared to one proportional to net risk, they still benefit from central clearing compared to an uncleared market, i.e., it is still incentive compatible for them. The following proposition summarizes these results.

Proposition 4 (Portfolio directionality and systematic risk). *Assume that* $|\rho_{X,M}| > 0$ *and* $\gamma \geq 2$. *Define by* $v_i^* = \frac{|\sum_{j=1,j\neq i}^{\gamma} v_{ji}^K|}{\sum_{j=1,j\neq i}^{\gamma} |v_{ji}^K|}$ *entity* i*'s net-to-gross ratio.*

If centrally cleared losses are allocated proportionally to net risk, the effect of central clearing on entity i*'s expected default loss depends on portfolio directionality and differs across core and periphery. Specifically, it is*

$$\Delta E^{\propto net}_{peripheral} \geq \Delta E^{\propto net}_{core\ (directional)} > \Delta E^{\propto net}_{core\ (flat)}, \tag{36}$$

where the first inequality binds only if $v_i^* = 0$ *and* $\gamma = 3$*, and is strict otherwise. Moreover,*

$$\frac{d}{dv_i^*} \Delta E^{\propto net}_{core\ (directional)} > 0. \tag{37}$$

If centrally cleared losses are allocated proportionally to gross risk, then

$$\Delta E^{\propto gross}_{peripheral} = \Delta E^{\propto net}_{core\ (gross)} = \Delta E^{\propto net}_{core\ (flat)}, \tag{38}$$

independent of portfolio directionality.

6. Tail risk and robustness

The primary purpose of central clearing is to enhance financial stability during crises times (see, e.g., G20 (2009), Financial Stability Board (FSB) (2017)). In these times, CCPs should ideally absorb losses arising from counterparty defaults and thereby decrease the spillover of risk in the overall financial system. Thus, it is of prevalent importance to contrast the effect of loss sharing during extreme and moderate times.

We examine loss sharing during different degrees of stress times by conditioning on particular realizations of the systematic risk factor M.[30] One can interpret this approach as an analysis of

[30]This rationale and approach is similar to the (marginal) expected shortfall of Acharya et al. (2017): while they examine the capital shortfall of financial institutions during crises, we study loss sharing risk during crises.

loss sharing during days on which the economy (e.g., the S&P 500) performs particularly poorly (or well). Proposition 5 derives an entity's expected default loss with and without loss sharing conditional on the systematic risk factor M. The Proposition shows that the effect of loss sharing now depends not only on the *directionality* of portfolios (which we measure by $\left(\sum_{k=1}^{K} v_{ij}^{k}\right)^2$) but also on the direction, namely on $\sum_{k=1}^{K} v_{ij}^{k}$. In other words, the effect of loss sharing is different for an entity with an overall positive correlation of cleared trades with the systematic risk factor ($\sum_{k=1}^{K} v_{ij}^{k} > 0$) compared to an overall negative correlation ($\sum_{k=1}^{K} v_{ij}^{k} < 0$) even if the absolute level of correlation is the same.

Proposition 5 (Expected losses conditional on M)**.** *Assume that K derivative classes are uncleared and that $v_{ij}^{k} \in \{-1, 0, 1\}$, and denote by $C_{ij}^{K} = VaR_{\alpha_{uc}}\left(\sum_{k=1}^{K} X_{ij}^{k}\right)$ the uncleared margin. Then, entity i's expected default loss conditional on M equals*

$$\mathbb{E}[DL_i^K \mid M] = \pi \sum_{j=1, j\neq i}^{\gamma-1} 1_{\{\sum_{k=1}^{K} |v_{ij}^{k}|>0\}} \cdot \Bigg[\left(M\beta \sum_{k=1}^{K} v_{ij}^{k} - C_{ij}^{K} \right) \Phi \left(\frac{M\beta \sum_{k=1}^{K} v_{ij}^{k} - C_{ij}^{K}}{\sigma\sqrt{K}} \right) + \sigma\sqrt{K}\varphi \left(-\frac{M\beta \sum_{k=1}^{K} v_{ij}^{k} - C_{ij}^{K}}{\sigma\sqrt{K}} \right) \Bigg]. \tag{39}$$

If class K is centrally cleared and losses are allocated proportionally to net risk, entity i's expected loss sharing contribution conditional on M is given by

$$\mathbb{E}[LSC_i^{\propto net} \mid M] = \mathbb{E}\left[\frac{(1-D_i)\bar{\sigma}_i^{CCP}}{\sum_{g=1}^{\gamma}(1-D_g)\bar{\sigma}_g^{CCP}} \sum_{j=1}^{\gamma} D_j \mathbb{E}[e_j^{CCP} \mid M] \mid \sum_{g=1}^{\gamma}(1-D_g) > 0 \right] \tag{40}$$

and if losses are allocated proportionally to gross risk, entity i's expected loss sharing contribution conditional on M is given by

$$\mathbb{E}[LSC_i^{\propto gross} \mid M] = \mathbb{E}\left[\frac{(1-D_i)\sum_{j=1,j\neq i}^{\gamma}|v_{ij}^{K}|}{\sum_{g=1}^{\gamma}(1-D_g)\sum_{j=1,j\neq g}^{\gamma}|v_{gj}^{K}|} \sum_{j=1}^{\gamma} D_j \mathbb{E}[e_j^{CCP} \mid M] \mid \sum_{g=1}^{\gamma}(1-D_g) > 0 \right], \tag{41}$$

where in both cases

$$\mathbb{E}[e_j^{CCP} \mid M] = \left(M\beta \sum_{h=1,h\neq j}^{\gamma} v_{hj}^K - C_j^{CCP} \right) \Phi \left(\frac{M\beta \sum_{h=1,h\neq j}^{\gamma} v_{hj}^K - C_j^{CCP}}{\sigma \sqrt{\sum_{h=1,h\neq j}^{\gamma} (v_{hj}^K)^2}} \right) + \sigma \sqrt{\sum_{h=1,h\neq j}^{\gamma} (v_{hj}^K)^2} \varphi \left(- \frac{M\beta \sum_{h=1,h\neq j}^{\gamma} v_{hj}^K - C_j^{CCP}}{\sigma \sqrt{\sum_{h=1,h\neq j}^{\gamma} (v_{hj}^K)^2}} \right), \tag{42}$$

$\bar{\sigma}_i^{CCP} = \sqrt{\sigma_M^2 \beta^2 \left(\sum_{j=1,j\neq i}^{\gamma} v_{ji}^K \right)^2 + \sigma^2 \sum_{j=1,j\neq i}^{\gamma} (v_{ji}^K)^2}$ *is net portfolio risk, and* $C_j^{CCP} = VaR_{\alpha_{CCP}} \left(\sum_{h=1,h\neq j}^{\gamma} X_{hj}^k \right)$ *is the margin posted to the CCP.*

The overall effect of loss sharing depends crucially on the severity of realizations $\bar{M}$ of M, which is captured by the cdf $q = \mathbb{P}(M \leq \bar{M})$. We parameterize these by $\bar{M} = \sigma_M \Phi^{-1}(q)$, where $\Phi^{-1}(\cdot)$ is the inverse cdf of the standard normal distribution. The larger $|q - 1/2|$, the larger is $|\bar{M}|$.

We compute a measure for tail risk that is independent of portfolio *direction* and only depends on entity's portfolio directionality. The reason is that risk can result from both the left and right tail of the distribution of M and, thus, entities whose portfolios have the same level of correlation but different *signs* of correlation with the systematic risk factor are exposed to the same level of tail risk. Following this rationale, we define tail risk by the sum of expected losses for extremely large positive and for extremely large negative realization of the systematic risk factor, which is

$$\mathbb{E}[E_i^{BN,K} \mid M = \sigma_M \Phi^{-1}(q)] + \mathbb{E}[E_i^{BN,K} \mid M = \sigma_M \Phi^{-1}(1-q)] \tag{43}$$

in the case that all derivative trades are uncleared, and analogously for the case that they are cleared. The effect of loss sharing on tail risk is then defined by

$$\Delta E_q^{\propto \text{net}} = \frac{\mathbb{E}[LSC_i^{\propto \text{gross}} + E_i^{BN,K-1} \mid M = \sigma_M \Phi^{-1}(q)] + \mathbb{E}[LSC_i^{\propto \text{gross}} + E_i^{BN,K-1} \mid M = \sigma_M \Phi^{-1}(1-q)]}{\mathbb{E}[E_i^{BN,K} \mid M = \sigma_M \Phi^{-1}(q)] + \mathbb{E}[E_i^{BN,K} \mid M = \sigma_M \Phi^{-1}(1-q)]} \tag{44}$$

if losses are allocated proportionally to net risk, and analogously $\Delta E_q^{\propto \text{gross}}$ if losses are allocated proportionally to gross risk. Since the expressions in Proposition 5 cannot be compared analytically, we rely on 200,000 simulated realizations of the default vector to evaluate them and compare the effect of loss sharing on tail risk across networks and across entities within networks.

Figure 6 (a) illustrates the effect of loss sharing on tail risk at the $q = 10\%$ level when losses are allocated proportionally to net risk. It is qualitatively very close to the effect on expected default losses. Loss sharing substantially increases tail risk for peripheral entities compared to its effect on core entities. The largest benefit of loss sharing is generated in a complete-flat network, while in other networks its benefit decreases with portfolio directionality.

[Place Figure 6 about here]

In contrast, if losses are allocated proportionally to gross risk, differences in the effect of loss sharing across entities within the same network vanish. This result for tail risk is analogous to the results for expected default losses. It highlights that an allocation of losses proportionally to gross risk does not only balance the effect of loss sharing across entities at the mean but also in the tail of the loss distribution.

We assess the sensitivity of our numerical results with respect to the correlation of defaults. First, correlation of defaults does not affect expected uncleared default losses, as Equation (16) shows. Second, a larger correlation of entity defaults increases the tails of the total default loss distribution and each entity's loss sharing contribution, but also reduces the likelihood that other entities default conditional on entity i surviving, while entity i does not have to contribute to losses when i defaults. Thus, the effect of correlation of defaults is ambiguous.

We increase the pairwise correlation of entities' assets from $\rho_{A,A} = 0.1$ to $\rho_{A,A} = 0.5$, but find that the effect on both ΔE and ΔE_q is negligible.[31] This result suggests that, with stronger correlation of defaults, the higher likelihood of large loss sharing contributions is largely offset by a lower likelihood that other entities default conditional on an entity i's survival.

7. Empirical predictions, equilibrium trade-offs, and policy implications

Since expected default losses are an important determinant for market participants' expected profits, they affect demand and supply in derivatives markets.[32] Therefore, and especially in light of post-crisis regulation that mandates central clearing for certain derivatives, it is important to

[31] Results are available on request.

[32] Indeed, several studies document that the risk of default losses is an important determinant for OTC market equilibria, such as Bellia et al. (2019), Bernstein et al. (2019), Vuillemey (2019), and Cenedese et al. (2020).

understand how and through which channels loss sharing affects the level and distribution of default losses.

First, we observe that, in practice, market participants are on average reluctant to centrally clear derivatives in the absence of a clearing mandate. E.g., only 28% of CDS trades and less than 1% of foreign exchange derivatives were voluntarily cleared, as of December 2016 (Wooldridge (2017)). From the perspective of loss sharing and its impact on expected default losses, we show that central clearing is indeed not necessarily beneficial for all market participants compared to an uncleared market. In contrast, loss sharing may expose market participants to additional risk. Therefore, the additional losses faced due to loss sharing can disincentivize market participants from central clearing.

Second, our results provide an explanation for the observation that clearing members are predominantly dealers and large banks in practice, while only very small number of end-users, such as investment funds and non-financial firms, participate in central clearing (Bank for International Settlements (BIS) (2018)). Indeed, large derivatives end-users such as Blackrock claim that loss sharing at CCPs "unfairly penalizes end-investors, who in general hold directional positions, vs. CMs [clearing members] or dealers, who generally manage to a flat market position" (Novick et al. (2018)). Consistent with this statement, our results show that, if loss sharing is proportional to net portfolio risk, entities with directional positions (such as end-users) cannot typically reduce expected default losses with central clearing. This result is amplified when entities with directional positions only trade with a limited number of counterparties, i.e., are in the "periphery" of the market. Indeed, loss sharing is largely proportional to net portfolio risk in practice (see Footnote 22). Our results thus provide a possible explanation for the tendency of end-users not to become clearing members in practice. Instead, the results suggest that dealers substantially benefit from loss sharing, which is consistent with the price discount they give for centrally cleared relative to uncleared transactions (Cenedese et al. (2020)).

Our results highlight that heterogeneity in the effect of central clearing across market participants can be reduced by (1) moving from a net-based to gross-based loss sharing rule or (2) moving from a core-periphery to a complete network structure. Loss sharing rules are currently not mandated by regulation. However, the desire to increase clearing participation can be a potential reason to mandate CCPs to also consider gross risk in loss sharing rules. Alternatively, when peripheral entities become more connected (the network becomes more complete), they have more netting benefits and, thereby, are incentivized to centrally clear even with a rule based on net risk. For instance, centralized trading platforms, such as a limited order book, intermediate trade between any pair of market participants and, thus, break up core-periphery networks. Indeed, centralized trading platforms become increasingly popular in OTC derivatives markets, partly due to regulation (e.g., mandating that certain derivatives are traded on swap execution facilities, see Riggs et al. (2020)).

Third, default losses are an important consideration for regulators. Default losses are a form of financial contagion that, if sufficiently large, may contribute to systemic risk. Our results indicate that loss sharing based on net risk is beneficial particularly for interconnected entities with flat portfolios, e.g., dealers. Since dealers are often systematically important in the financial system (e.g., Billio et al. (2012)), regulators might want to more-than-proportionally reduce dealers' exposure to default losses. Loss sharing based on net risk can be consistent with this objective.[33]

Fourth, we stress that derivatives positions are endogenous to loss sharing rules in practice. Given that loss sharing proportional to net risk favors less directional portfolios, a clearing obligation may incentivize end-users to hold less directional positions. From a welfare perspective, this implies an important trade-off: on the one hand, if portfolios are less directional, there is less total net risk for the CCP. This effect can mitigate financial contagion in the derivatives market and, thus, may reduce externalities from a simultaneous failure of financial institutions. On the other hand, end-users may forego hedging benefits when they reduce portfolio directionality.[34] Therefore, in markets with many end-users that have a large marginal utility from hedging, a clearing obligation may reduce welfare if the marginal expected loss sharing contribution for holding a more directional portfolio is too large. While it is ultimately an empirical question which effect dominates, the impact of loss sharing rules on hedging benefits can provide a rationale for strengthening the supervision of CCPs' loss allocation rules whenever there is a clearing obligation.

[33] We stress that central clearing may improve overall financial stability also in other dimensions, e.g., by increasing transparency (Acharya and Bisin (2014)) and facilitating fast auctioning of defaulting members' portfolios. Indeed, the cleared share of Lehman's derivative trades was hedged and closed out within three weeks of Lehman's failure, suggesting that central clearing may stabilize derivatives markets (see Sir Jon Cunliffe's speech from 5 June 2018, *Central clearing and resolution - learning some of the lessons of Lehmans*).

[34] In standard models, such as Biais et al. (2012, 2016), end-users buy derivatives to protect themselves against risks outside of the derivatives market. In this case, they forego hedging benefits when choosing a portfolio that is less directional than the one that provides full insurance.

Fifth, we stress that market structure is endogenous to loss sharing rules in practice. If loss sharing is proportional to net portfolio risk, more interconnected entities (in the core of the market) benefit more from loss sharing than less interconnected ones (in the periphery). Hence, entities have an incentive to increase their centrality in the market. On the one hand, increasing network centrality might amplify too-interconnected-to-fail externalities. On the other hand, when entities move closer to the market core using centralized trading platforms, inefficiencies from discriminatory pricing might decline (Hau et al. (2020)).

Finally, relative to loss sharing based on net portfolio risk, a rule based on gross notional would benefit end-users but harm interconnected entities with flat portfolios in the core of the market. The latter are typically large market dealers, that provide liquidity and intermediate trade (Getmansky et al. (2016)). Therefore, a rule based on gross notional would likely penalize liquidity provision by dealers, while at the same increasing loss sharing benefits for end-users and, thereby, increase CCP participation. On the one hand, impediment of liquidity provision may reduce trade gains in derivatives markets. On the other hand, central clearing has been found to facilitate trade, as it tackles counterparty risk and adverse selection inefficiencies (e.g., Bernstein et al. (2019), Vuillemey (2019)). Central clearing might also facilitates the adoption of more centralized trading technologies, reducing the importance of liquidity provision by dealers. Thus, the ultimate effect on market liquidity of loss sharing based on gross notional depends on the trade-off between a reduction in liquidity provision by dealers and the impact of a wider adoption of central clearing on overall market liquidity.

8. Conclusion

The recent global financial crisis 2007-08 exposed vulnerabilities in the derivatives market architecture, which was dominated by uncleared trades. The introduction of mandatory central clearing has clearly increased transparency in derivatives markets; however, was it successful in reducing counterparty risks in derivative markets as well?

To address this question, we present a theoretical analysis of the impact of central clearing on default losses in derivatives markets. The focus of the model is on loss sharing in central clearinghouses, namely the allocation of losses caused by the default of clearing members to surviving clearing members. We show that the effect of loss sharing on entities' expected default losses, relative to an uncleared market, can differ substantially across market participants and is highly sensitive toward (1) the derivatives market network structure, (2) loss sharing rules, (3) directionality in market participants' derivatives portfolios, and (4) correlation of derivatives prices. In many realistic situations, loss sharing actually *increases* a market participant's expected default losses relative to the absence of central clearing. Such market participants lose from central clearing.

In particular, our results show that market participants in the core of the market with flat portfolios, e.g., dealers, substantially benefit from loss sharing compared to an uncleared market - at the expense of (peripheral) entities with directional portfolios, e.g., end-users. Under the assumption that the decision to clear is affected by market participants' objective to minimize risk, as suggested by Bellia et al. (2019), Financial Stability Board (FSB) (2018), and Vuillemey (2019), our result is consistent with the reluctance of end-users to become clearing members in practice. The result emerges due to sharing of default losses among surviving clearing members proportionally to their *net* portfolio risk. While this is a current standard practice, we contrast this rule with an alternative loss sharing rule that allocates losses proportionally to *gross* risk. We show that the latter removes heterogeneity across market participants in the benefit of central clearing compared to an uncleared market. Hence, with loss sharing proportional to gross risk, central clearing is beneficial for all market participants. Furthermore, we show that changes in the market structure from a core-periphery toward a more complete network also smooth heterogeneity across market participants. Thus, promoting either loss sharing rules based on gross risk or centralized trading are potential policy tools to incentivize central clearing. Overall, our analysis shows that it is feasible that all market participants have the same relative benefit from loss sharing, i.e., are "winners", and are thus incentivized to use central clearing.

Figures

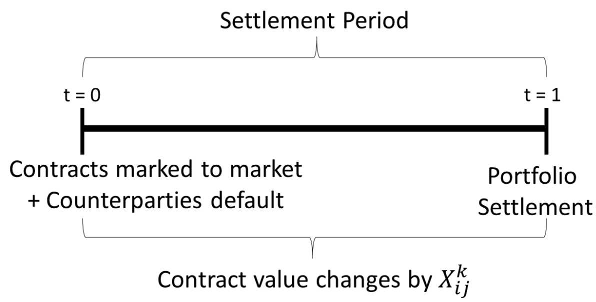
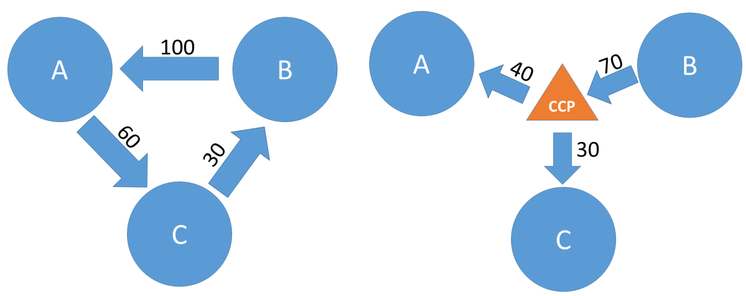
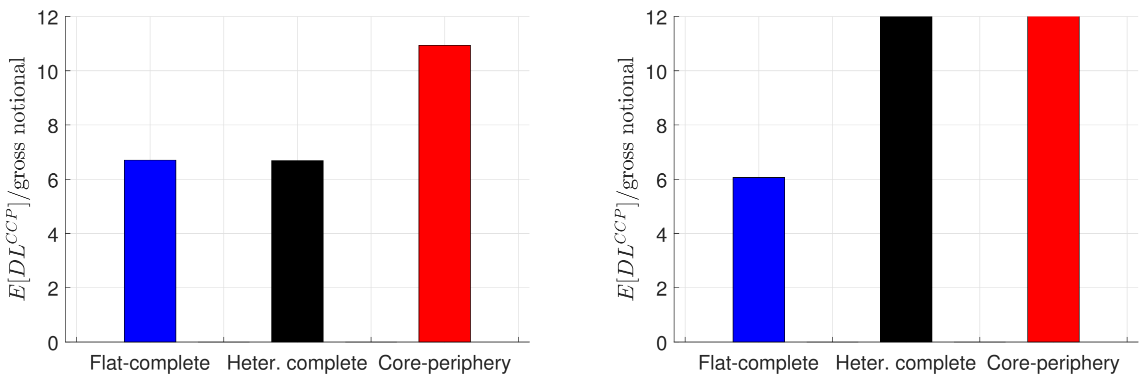


Fig. 1. Timeline of the model.
Losses due to counterparty default occur between time $t = 0$, the most recent date where contracts have been marked to market and counterparties might default, and time $t = 1$, at which time the portfolio is settled.



(a) Bilateral netting. (b) Multilateral netting.

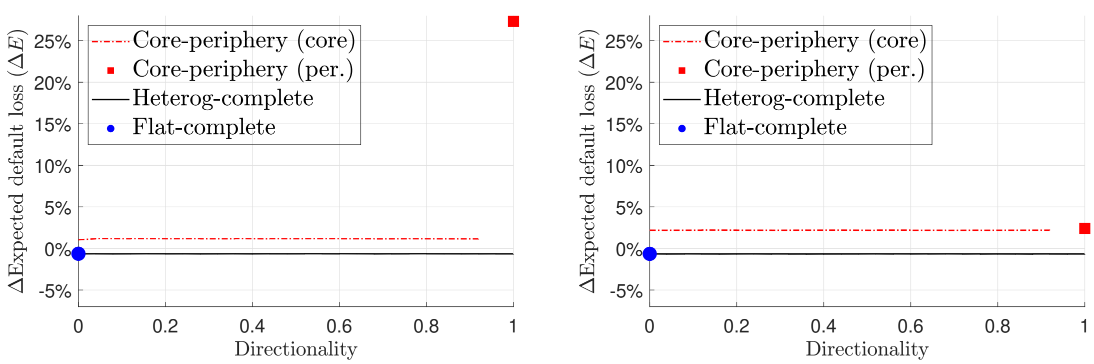
Fig. 2. Illustration of bilateral and multilateral netting.
(a) Bilateral netting and (b) multilateral netting across counterparties. Arrows illustrate the flow of profits and losses (e.g., B owes \$100 to A).



(a) No systematic risk.

(b) With systematic risk ($\rho_{X,M} = 0.43$).

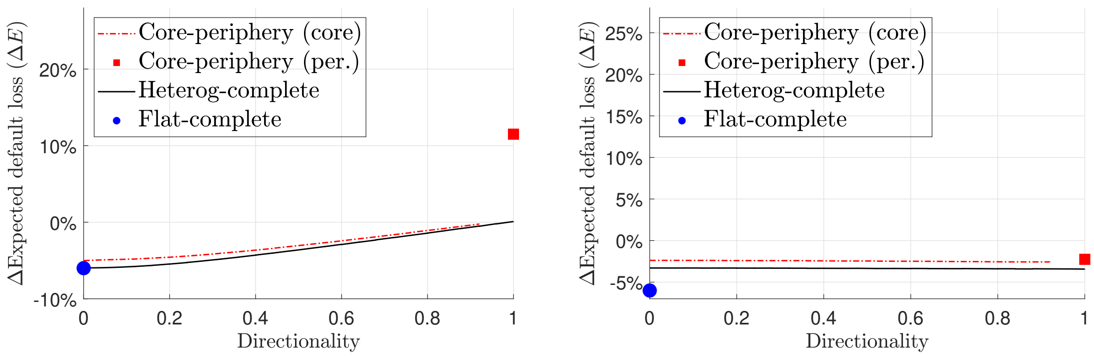
Fig. 3. Total expected default loss in the centrally cleared derivative class.
The figures depict $\mathbb{E}[DL^{CCP}]/GN$, the total expected default loss per gross notional cleared for different networks per dollar of total gross notional notional cleared across networks. The number of market participants is $\gamma = 51$.



(a) Proportional to net risk.

(b) Proportional to gross risk.

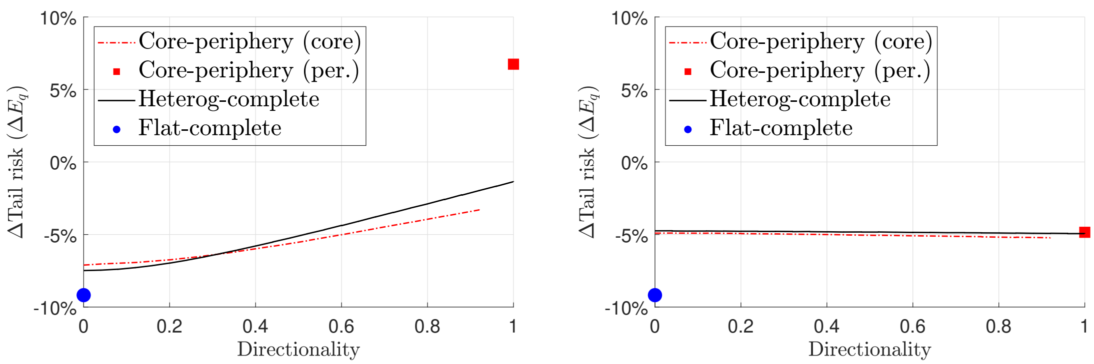
Fig. 4. Effect of loss sharing on expected default loss without systematic risk.
The figures depict ΔE, the effect of loss sharing on entities' expected default loss relative to an uncleared market. Within the heterogeneous-complete and core-periphery networks, entities differ in the net-to-gross ratio (i.e., portfolio directionality) $\frac{|\sum_{j=1,j\neq i}^{\gamma} v_{ji}^K|}{\sum_{j=1,j\neq i}^{\gamma} |v_{ji}^K|}$, which we show on the x-axis. The number of market participants is $\gamma = 51$.



(a) Proportional to net risk. (b) Proportional to gross risk.

Fig. 5. Effect of loss sharing on expected default loss with systematic risk.

The figures depict ΔE, the effect of loss sharing on entities' expected default loss relative to an uncleared market when the correlation between derivative prices and the systematic risk factor is $\rho_{X,M} = 0.43$. Within the heterogeneous-complete and core-periphery networks, entities differ in the net-to-gross ratio (i.e., portfolio directionality) $\frac{|\sum_{j=1,j\neq i}^{\gamma} v_{ji}^{K}|}{\sum_{j=1,j\neq i}^{\gamma} |v_{ji}^{K}|}$, which we show on the x-axis. The number of market participants is $\gamma = 51$.



(a) Proportional to net risk. (b) Proportional to gross risk.

Fig. 6. Effect of loss sharing on tail risk.

The figures depict ΔE_q, the effect of loss sharing on entities' tail risk of facing default losses relative to an uncleared market. The correlation between derivative prices and the systematic risk factor is assumed to be $\rho_{X,M} = 0.43$. Within the heterogeneous-complete and core-periphery networks, entities differ in the net-to-gross ratio (i.e., portfolio directionality) $\frac{|\sum_{j=1,j\neq i}^{\gamma} v_{ji}^{K}|}{\sum_{j=1,j\neq i}^{\gamma} |v_{ji}^{K}|}$, which we show on the x-axis. The number of market participants is $\gamma = 51$.

References

Abad, J., Aldasorol, I., Aymanns, C., D'Errico, M., Rousová, L. F., Hoffmann, P., Langfield, S., Neychev, M., Roukny, T., 2016. Shedding light on dark markets: First insights from the new EU-wide OTC derivatives dataset. European Systemic Risk Board Occasional Paper 11.

ABN AMRO Clearing Bank N.V., Allianz Global Investors, Barclays, BlackRock, Inc., Citigroup Inc., Commonwealth Bank of Australia, Credit Suisse AG, DeutscheBank AG, Franklin Templeton, Goldman Sachs Group, Inc., The Guardian Life Insurance Company of America, Ivy Investments, JPMorgan Chase & Co., Nordea Bank Abp, Societe Generale S.A., State Street Global Markets, T. Rowe Price Group, Inc., The Vanguard Group, TIAA, UBS AG, 2020. A path forward for CCP Resilience, Recovery, and Resolution. Industry Report .

Acharya, V., Bisin, A., 2014. Counterparty risk externality: Centralized versus over-the-counter markets. Journal of Economic Theory 149, 153–182.

Acharya, V., Pedersen, L., Philippon, T., Richardson, M., 2017. Measuring systemic risk. Review of Financial Studies 30, 2–47.

Armakolla, A., Laurent, J.-P., 2017. CCP resilience and clearing membership. *Working Paper*.

Arnsdorf, M., 2012. Quantification of central counterparty risk. Journal of Risk Management in Financial Institutions 5, 273–287.

Bank for International Settlements (BIS), 2018. Analysis of central clearing interdependencies.

Bank for International Settlements (BIS), 2019. Margin requirements for non-centrally cleared derivatives.

Bellia, M., Panzica, R., Pelizzon, L., Peltonen, T., 2019. The demand for central clearing: To clear or not to clear, that is the question. ESRB Working Paper Series 62.

Bergman, W. J., Bliss, R. R., Johnson, C. A., Kaufman, G. G., 2004. Netting, financial contracts, and banks: The economic implications. Federal Reserve Bank of Chicago Working Paper 2004-02.

Bernstein, A., Hughson, E., Weidenmier, M., 2019. Counterparty risk and the establishment of the new york stock exchange clearinghouse. Journal of Political Economy 127, 689–729.

Biais, B., Heider, F., Hoerova, M., 2012. Clearing, counterparty risk, and aggregate risk. IMF Economic Review 60, 193–222.

Biais, B., Heider, F., Hoerova, M., 2016. Risk-sharing or risk-taking? Counterparty risk, incentives and margins. Journal of Finance 71, 1669–1698.

Billio, M., Getmansky, M., Lo, A. W., Pelizzon, L., 2012. Econometric measures of connectedness and systemic risk in the finance and insurance sectors. Journal of Financial Economics 104, 535–559.

Boissel, C., Derrien, F., Ors, E., Thesmar, D., 2017. Systemic risk in clearing houses: Evidence from the European repo market. Journal of Financial Economics 125, 511–536.

Braithwaite, J., 2016. The dilemma of client clearing in the otc derivatives markets. European Business Organization Law Review 17.

Capponi, A., Cheng, W., 2018. Clearinghouse margin requirements. Operations Research forthcoming.

Capponi, A., Cheng, W. A., Sethuraman, J., 2017. Clearinghouse default waterfalls: Risk-sharing, incentives, and systemic risk. *Working Paper*.

Capponi, A., Wang, J. J., Zhang, H., 2019. Central counterparty and the design of collateral requirements. *Working Paper*.

Cenedese, G., Ranaldo, A., Vasios, M., 2020. OTC premia. Journal of Financial Economics 136, 86–105.

Cont, R., 2015. The end of the waterfall: default resources of central counterparties. Working Paper .

Cont, R., Kokholm, T., 2014. Central clearing of OTC derivatives: Bilateral vs multilateral netting. Statistics & Risk Modeling 31, 3–22.

Dalla Fontana, S., Holz auf der Heide, M., Pelizzon, L., Scheicher, M., 2019. The anatomy of the Euro area interest rate swap market. ECB Working Paper 2242.

Di Maggio, M., Kermani, A., Song, Z., 2017. The value of trading relations in turbulent times. Journal of Financial Economics 124, 266–284.

Du, W., Gadgil, S., Gordy, M. B., Vega, C., 2016. Counterparty risk and counterparty choice in the credit default swap market. Federal Reserve Board Working Paper 87.

Duffie, D., 2015. Resolution of failing central counterparties. In: Jackson, T., Scott, K., Taylor, J. E. (eds.), *Making Failure Feasible: How Bankruptcy Reform Can End 'Too Big To Fail*, Hoover Institution Press, chap. 4, pp. 87–109.

Duffie, D., 2020. Still the World's Safe Haven? Redesigning the U.S. Treasury Market After the COVID-19 Crisis. Hutchins Center Working Paper 62.

Duffie, D., Scheicher, M., Vuillemey, G., 2015. Central clearing and collateral demand. Journal of Financial Economics 116, 237–256.

Duffie, D., Zhu, H., 2011. Does a central clearing counterparty reduce counterparty risk? Review of Asset Pricing Studies 1, 74–95.

Elliott, D., 2013. Central counterparty loss-allocation rules. Bank of England Financial Stability Paper 20.

ESRB, 2021. ESRB risk dashboard. 11 March 2021.

Faruqui, U., Huang, W., Takáts, E., 2018. Clearing risks in otc derivatives markets: the CCP-bank nexus. BIS Quarterly Review .

Fiedor, P., Lapschies, S., Országhová, L., 2017. Networks of counterparties in the centrally cleared eu-wide interest rate derivatives market. ESRB Working Paper Series 54.

Financial Stability Board (FSB), 2017. Review of OTC derivatives market reforms: Effectiveness and broader effects of the reforms.

Financial Stability Board (FSB), 2018. Incentives to centrally clear over-the-counter (OTC) derivatives - A post-implementation evaluation of the effects of the G20 financial regulatory reforms.

Fleming, M., Keane, F., 2021. The Netting Efficiencies of Marketwide Central Clearing. Federal Reserve Bank ofNew York Staff Reports 964.

G20, 2009. Leader's statement - The Pittsburgh Summit - September 24-25 2009. Available at `https://www.fsb.org/source/g20/`.

Getmansky, M., Girardi, G., Lewis, C., 2016. Interconnectedness in the CDS Market. Financial Analysts Journal 72, 62–82.

Ghamami, S., Glasserman, P., 2017. Does OTC derivatives reform incentivize central clearing? Journal of Financial Intermediation 32, 76–87.

Hau, H., Hoffmann, P., Langfield, S., 2020. Discriminatory pricing of over-the-counter derivatives. Management Science forthcoming.

Huang, W., 2018. Central counterparty capitalization and misaligned incentives. *Working Paper*.

Huang, W., Menkveld, A. J., Yu, S., 2019. Central counterparty exposure in stressed markets. *Working Paper*.

ISDA, 2013. Standard initial margin model for non-cleared derivatives. *Available at* `https://www.isda.org/a/cgDDE/simm-for-non-cleared-20131210.pdf`.

Lewandowska, O., 2015. OTC clearing arrangements for bank systemic risk regulation: A simulation approach. Journal of Money, Credit and Banking 47, 1177–1203.

Li, D., Schürhoff, N., 2019. Dealer networks. Journal of Finance forthcoming.

Loon, Y. C., Zhong, Z. K., 2014. The impact of central clearing on counterparty risk, liquidity, and trading: Evidence from the credit default swap market. Journal of Financial Economics 112, 91–115.

Mancini, L., Ranaldo, A., Wrampelmeyer, J., 2016. The euro interbank repo market. Review of Financial Studies 29, 1747–1779.

Markit, 2015. Markit CDX High Yield & Markit CDX Investment Grade Index Rules.

Mengle, D., 2010. The importance of close-out netting. ISDA Research Notes 1/2010.

Menkveld, A. J., 2017. Crowded positions: An overlooked systemic risk for central clearing parties. Review of Asset Pricing Studies 7, 209–242.

Menkveld, A. J., Pagnotta, E. S., Zoican, M. A., 2015. Does central clearing affect price stability? Evidence from nordic equity markets. *Working Paper* .

Merton, R. C., 1974. On the pricing of corporate debt: The risk structure of interest rates. Journal of Finance 29, 449–470.

Novick, B., De Jesus, H., Fisher, S., Kiely, E., Osman, M., Hsu, V., 2018. An end-investor perspective on central clearing. Looking back to look forward. *Blackrock Viewpoint Sept 2018.*

Office of the Comptroller of the Currency, 2016. Quarterly report on bank trading and derivatives activities. 4th Quarter 2016.

Riggs, L., Onur, E., Reiffen, D., Zhu, H., 2020. Swap trading after Dodd-Frank: Evidence from index CDS. Journal of Financial Economics 137, 857–886.

Siriwardane, E. N., 2019. Limited investment capital and credit spreads. Journal of Finance 74, 2303–2347.

Vuillemey, G., 2019. The value of central clearing. Journal of Finance forthcoming.

Wooldridge, P., 2017. Central clearing makes further inroads. In: *BIS Quarterly Review June 2017 - International banking and financial market developments*.

Online Appendix

A. Additional Statements

Proposition A.1 (Bilateral counterparty risk)**.** *Assume that K derivative classes are bilaterally netted, $|v_{ij}^k| = 1$, π is the probability of a counterparty's default, and $C_{ij}^K = VaR_{\alpha_{uc}}\left(\sum_{k=1}^K X_{ij}^k\right)$. Then, entity i's expected loss sharing contribution is*

$$\mathbb{E}\left[DL_i^K\right] = \pi\xi(\alpha_{uc}) \sum_{j=1, j\neq 1}^{\gamma} 1_{\{\sum_{k=1}^K |v_{ij}^k|>0\}} \sqrt{\sigma_M^2\beta^2\left(\sum_{k=1}^K v_{ij}^k\right)^2 + K\sigma^2}, \tag{A.1}$$

where $\xi(\alpha) = (1-\alpha)\Phi^{-1}(1-\alpha) + \varphi\left(\Phi^{-1}(\alpha)\right)$.

Proposition A.2 (Expected loss sharing contribution $\propto$ net)**.** *Assume that derivative class K is centrally cleared, CCP default losses are allocated proportionally to net risk, π is the probability of a counterparty's default, $\beta \equiv \beta_{ij}^K$, $\sigma \equiv \sigma_{ij}^K$, and $C_i^{CCP} = VaR_{\alpha_{CCP}}\left(\sum_{j=1,j\neq i}^{\gamma} X_{ji}^k\right)$. Then, entity i's expected loss sharing contribution is*

$$\mathbb{E}[LSC_i{}^{\propto net}] = \mathbb{E}\left[\left(\frac{\sum_{g=1}^{\gamma}\bar{\sigma}_g^{CCP}}{\sum_{g=1}^{\gamma}(1-D_g)\bar{\sigma}_g^{CCP}} - 1\right)\xi(\alpha_{CCP})(1-D_i)\bar{\sigma}_i^{CCP} \mid \sum_{g=1}^{\gamma}(1-D_g) > 0\right], \tag{A.2}$$

where $\xi(\alpha) = (1-\alpha)\Phi^{-1}(1-\alpha) + \varphi\left(\Phi^{-1}(\alpha)\right)$ and $\bar{\sigma}_i^{CCP} = \sqrt{\sigma_M^2\beta^2\left(\sum_{j=1,j\neq i}^{\gamma} v_{ji}^K\right)^2 + \sigma^2\sum_{j=1,j\neq i}^{\gamma}(v_{ji}^K)^2}$.

Proposition A.3 (Expected loss sharing contribution $\propto$ gross)**.** *Assume that derivative class K is centrally cleared, CCP default losses are allocated proportionally to gross notional, π is the probability of a counterparty's default, $\beta \equiv \beta_{ij}^K$, $\sigma \equiv \sigma_{ij}^K$, and $C_i^{CCP} = VaR_{\alpha_{CCP}}\left(\sum_{j=1,j\neq i}^{\gamma} X_{ji}^k\right)$. Then, entity i's expected loss sharing contribution is*

$$\mathbb{E}[LSC_i{}^{\propto gross}] = \mathbb{E}\left[\frac{(1-D_i)v_{i*}^K}{\sum_{g=1}^{\gamma}(1-D_g)v_{g*}^K}\xi(\alpha_{CCP})\sum_{j=1}^{\gamma} D_j\bar{\sigma}_j^{CCP} \mid \sum_{g=1}^{\gamma}(1-D_g) > 0\right], \tag{A.3}$$

where $\xi(\alpha) = (1-\alpha)\Phi^{-1}(1-\alpha) + \varphi\left(\Phi^{-1}(\alpha)\right)$ and $v_{i*}^K = \sum_{j=1,j\neq i}^{\gamma}|v_{ij}^K|$.

B. Proofs of Statements

For the following proofs, we will make extensive use of the following property of the Normal distribution: for $Y \sim \mathcal{N}(\mu, \sigma^2)$ the truncated expectation is given by $\mathbb{E}[Y \mid Y > 0] = \mu + \sigma \frac{\varphi(-\mu/\sigma)}{\Phi(\mu/\sigma)}$, and thus $\mathbb{E}[\max(Y, 0)] = \mathbb{E}[Y \mid Y > 0]\Phi(\mu/\sigma) = \mu\Phi(\mu/\sigma) + \sigma\varphi(-\mu/\sigma)$.

B.1. Proof of Proposition 1.

Proof. Let $\gamma > 2$. The CCP's expected default loss is given by

$$\mathbb{E}\left[DL^{CCP}\right] = \mathbb{E}\left[\sum_{j=1}^{\gamma} D_j \max\left(\sum_{\substack{g=1,\\ g\neq j}}^{\gamma} X_{gj}^K - C_j^{CCP}, 0\right)\right]. \tag{B.1}$$

Define

$$\begin{aligned}
(\bar{\sigma}_j^{CCP})^2 =& \mathrm{var}\left(\sum_{g=1, g\neq j}^{\gamma} X_{gj}^K\right) = \sigma_M^2 \left(\sum_{g=1, g\neq j}^{\gamma} v_{gj}^K \beta_{gj}^K\right)^2 + \sum_{g=1, g\neq j}^{\gamma} (v_{gj}^K \sigma_{gj}^K)^2 \\
=& \sigma_M^2 \beta^2 \left(\sum_{g=1, g\neq j}^{\gamma} v_{gj}^K\right)^2 + \sigma^2 \sum_{g=1, j\neq j}^{\gamma} (v_{gj}^K)^2, \\
\xi(\alpha) =& (1-\alpha)\Phi^{-1}(1-\alpha) + \varphi\left(\Phi^{-1}(\alpha)\right).
\end{aligned}$$

Then, it is

$$\mathbb{E}\left[\max\left(\sum_{\substack{g=1,\\ g\neq j}}^{\gamma} X_{gj}^K - C_j^{CCP}, 0\right)\right] = \xi(\alpha_{CCP})\bar{\sigma}_j^{CCP}. \tag{B.2}$$

In a flat network, all entities trade with all other $(\gamma - 1)$ entities and cleared portfolios are not exposed to systematic risk, implying $\bar{\sigma}_j^{CCP} = \sqrt{\gamma - 1}\sigma$ for all $j = 1, ..., \gamma$.

In a heterogeneous-complete network, all entities trade with all other $(\gamma - 1)$ entities but differ in net-to-gross ratio $v_j^* = \frac{\left(\sum_{g=1, g\neq j}^{\gamma} v_{gj}^K\right)^2}{\sum_{g=1, g\neq j}^{\gamma} (v_{gj}^K)^2}$ such that $\bar{\sigma}_j^{CCP} = \sqrt{\gamma - 1}\sqrt{\sigma_M^2 \beta^2 v_j^* + \sigma^2}$ with $v_j^* \in \left\{\frac{0}{\gamma-1}, \frac{1}{\gamma-1}, ..., 1\right\}$.

In a core-periphery network, by definition of core and peripheral entities, it is

$\bar{\sigma}_j^{CCP} = \sqrt{\sum_{g=1,g\neq j}^{\gamma}(v_{gj}^K)^2}\sqrt{\sigma_M^2\beta^2 v_j^* + \sigma^2}$ with

$$v_j^* = \begin{cases} \frac{\left(\sum_{g=1,g\neq j}^{\gamma} v_{gj}^K\right)^2}{\frac{\gamma-1}{2}} \leq 1, & \text{if } j \text{ is core (directional)}, \\ 0, & \text{if } j \text{ is core (flat)}, \\ 1, & \text{if } j \text{ is periphery} \end{cases} \tag{B.3}$$

and

$$\sum_{g=1,g\neq j}^{\gamma}(v_{gj}^K)^2 = \begin{cases} \frac{\gamma-1}{2}, & \text{if } j \text{ is core (directional)}, \\ \frac{\gamma+1}{2}, & \text{if } j \text{ is core (flat)}, \\ 1, & \text{if } j \text{ is periphery}. \end{cases} \tag{B.4}$$

Given the definition of networks and using that there is 1 flat entity, $\frac{\gamma-1}{2} - 1$ core directional and $\frac{\gamma+1}{2}$ peripheral entities in a core-periphery network, it is then

$$\mathbb{E}\left[DL^{CCP}\right] = \mathbb{E}\left[\sum_{j=1}^{\gamma} D_j \xi(\alpha_{CCP})\bar{\sigma}_j^{CCP}\right] \tag{B.5}$$

$$= \xi(\alpha_{CCP})\sum_{j=1}^{\gamma}\bar{\sigma}_j^{CCP}\mathbb{E}\left[D_j\right] \tag{B.6}$$

$$= \pi\xi(\alpha_{CCP})\sum_{j=1}^{\gamma}\bar{\sigma}_j^{CCP} \tag{B.7}$$

$$= \begin{cases} \pi\gamma\xi(\alpha_{CCP})\sigma\sqrt{\gamma-1}, & \text{flat}, \\ \pi\xi(\alpha_{CCP})\sum_{j=1}^{\gamma}\sqrt{\gamma-1}\sqrt{\sigma_M^2\beta^2 v_j^* + \sigma^2}, & \text{heterog-complete}, \\ \pi\xi(\alpha_{CCP})\left(\sqrt{\frac{\gamma+1}{2}}\sigma + \sum_{j=2}^{(\gamma-1)/2}\sqrt{\frac{\gamma-1}{2}}\sqrt{\sigma_M^2\beta^2 v_i^* + \sigma^2} + \frac{\gamma+1}{2}\sqrt{\sigma_M^2\beta^2 + \sigma^2}\right) & \text{core-periphery}. \end{cases} \tag{B.8}$$

If there is no systematic risk, i.e., $\beta \equiv 0$, then $\mathbb{E}\left[DL_{\text{flat}}^{CCP}\right] = \mathbb{E}\left[DL_{heterog-complete}^{CCP}\right]$ and

$$\mathbb{E}\left[DL_{core-periphery}^{CCP}\right] = \pi\xi(\alpha_{CCP})\sigma\left(\sqrt{\frac{\gamma+1}{2}} + \frac{\gamma-3}{2}\sqrt{\frac{\gamma-1}{2}} + \frac{\gamma+1}{2}\right) \tag{B.9}$$

Total gross notional GN is $\gamma(\gamma-1)$ in the flat and heterogeneous-complete networks and

$\frac{\gamma+1}{2}+\frac{\gamma-3}{2}\frac{\gamma-1}{2}+\frac{\gamma-1}{2}$. Thus, the CCP's expected default loss per 1$ gross notional is

$$\frac{\mathbb{E}\left[DL^{CCP}\right]}{GN}=\begin{cases}\pi\xi(\alpha_{CCP})\frac{\sigma}{\sqrt{\gamma-1}}, & \text{flat,}\\ \pi\xi(\alpha_{CCP})\frac{\sum_{j=1}^{\gamma}\sqrt{\sigma_M^2\beta^2v_j^*+\sigma^2}}{\gamma\sqrt{\gamma-1}}, & \text{heterog-complete,}\\ \pi\xi(\alpha_{CCP})\frac{\sqrt{\frac{\gamma+1}{2}}\sigma+\sum_{j=2}^{(\gamma-1)/2}\sqrt{\frac{\gamma-1}{2}}\sqrt{\sigma_M^2\beta^2v_i^*+\sigma^2}+\frac{\gamma+1}{2}\sqrt{\sigma_M^2\beta^2+\sigma^2}}{\frac{\gamma+1}{2}+\frac{\gamma-3}{2}\frac{\gamma-1}{2}+\frac{\gamma+1}{2}}. & \text{core-periphery,}\end{cases}$$

Clearly, $\frac{\mathbb{E}\left[DL^{CCP}_{heterog-complete}\right]}{GN_{heterog-complete}}\geq\frac{\mathbb{E}\left[DL^{CCP}_{\text{flat}}\right]}{GN_{\text{flat}}}$ with equality if, and only if, $\beta=0$, i.e., in the absence of systematic risk.

The relative expected default loss in case of a core-periphery network can be rewritten as

$$\pi\xi(\alpha_{CCP})\left[\frac{\sigma}{\sqrt{\frac{\gamma+1}{2}}}\theta_1+\sum_{j=2}^{(\gamma-1)/2}\frac{\sqrt{\sigma_M^2\beta^2v_i^*+\sigma^2}}{\sqrt{\frac{\gamma-1}{2}\frac{\gamma-3}{2}}}\theta_2+\sqrt{\sigma_M^2\beta^2+\sigma^2}(1-\theta_1-\theta_2)\right] \tag{B.10}$$

with $\theta_1=\frac{\frac{\gamma+1}{2}}{\frac{\gamma+1}{2}+\frac{\gamma-3}{2}\frac{\gamma-1}{2}+\frac{\gamma+1}{2}}$ and $\theta_2=\frac{\frac{\gamma-3}{2}\frac{\gamma-1}{2}}{\frac{\gamma+1}{2}+\frac{\gamma-3}{2}\frac{\gamma-1}{2}+\frac{\gamma+1}{2}}$. It is $\frac{\sigma}{\sqrt{\frac{\gamma+1}{2}}}>\frac{\sigma}{\sqrt{\gamma-1}}$, $\sum_{j=2}^{(\gamma-1)/2}\frac{\sigma}{\sqrt{\frac{\gamma-1}{2}\frac{\gamma-3}{2}}}=\frac{\sigma}{\sqrt{\frac{\gamma-1}{2}}}>\frac{\sigma}{\sqrt{\gamma-1}}$, and $\sigma>\frac{\sigma}{\sqrt{\gamma-1}}$ for $\gamma>2$ (in the case $\gamma=3$, there exist no core directional entities and the intermediate term drops out while other inequalities remain to hold). Therefore, in the case without systematic risk ($\beta=0$), it is

$$\frac{\mathbb{E}\left[DL^{CCP}_{core-periphery}\right]}{GN_{core-periphery}}>\frac{\mathbb{E}\left[DL^{CCP}_{heterog-complete}\right]}{GN_{heterog-complete}}=\frac{\mathbb{E}\left[DL^{CCP}_{\text{flat}}\right]}{GN_{\text{flat}}}.$$

Moreover, in the case with systematic risk, it is $\frac{\sqrt{\sigma_M^2\beta^2v_i^*+\sigma^2}}{\sqrt{\frac{\gamma-1}{2}}}<\sqrt{\sigma_M^2\beta^2+\sigma^2}$ for all $\gamma\geq 2$ since $v_i^*\leq 1$. Then, for all $\gamma>3$ it is

$$\frac{\mathbb{E}\left[DL^{CCP}_{core-periphery}\right]}{GN_{core-periphery}}>\frac{\mathbb{E}\left[DL^{CCP}_{heterog-complete}\right]}{GN_{heterog-complete}}>\frac{\mathbb{E}\left[DL^{CCP}_{\text{flat}}\right]}{GN_{\text{flat}}}.$$

□

B.2. Proof of Proposition 2.

Proof. Let $\beta_{ij}^k\equiv 0$. Consider the core-periphery network. Then, given that $v_{ij}^k\in\{-1,0,1\}$ and that a directional entity in the core trades with $\frac{\gamma-1}{2}-1$ counterparties in the core and one entity in the periphery, a flat entity in the core trades with $\frac{\gamma-1}{2}-1$ counterparties in the core and two entities in the periphery, and an entity in the periphery trades with only one counterparty (both in all K asset classes),

$$\bar{\sigma}_g^{CCP} = \sigma\sqrt{\sum_{j=1,j\neq g}^{\gamma}(v_{jg}^K)^2} = \begin{cases} \sigma\sqrt{\frac{\gamma-1}{2}}, & \text{if } g \text{ is core (directional),} \\ \sigma\sqrt{\frac{\gamma+1}{2}}, & \text{if } g \text{ is core (flat),} \\ \sigma, & \text{if } g \text{ is periphery.} \end{cases} \tag{B.11}$$

In the case with loss sharing proportional to net risk, using Propositions A.2 it is

$$\mathbb{E}[LSC_i{}^{\propto\text{net}}] = \begin{cases} \sigma\sqrt{\frac{\gamma-1}{2}}\xi(\alpha_{CCP})H, & \text{if } i \text{ is core (directional),} \\ \sigma\sqrt{\frac{\gamma+1}{2}}\xi(\alpha_{CCP})H, & \text{if } i \text{ is core (flat),} \\ \sigma\xi(\alpha_{CCP})H, & \text{if } i \text{ is periphery,} \end{cases} \tag{B.12}$$

with $H = \mathbb{E}\left[\left(\frac{\sum_{g=1}^{\gamma}\bar{\sigma}_g^{CCP}}{\sum_{g=1}^{\gamma}(1-D_g)\bar{\sigma}_g^{CCP}} - 1\right)(1-D_i) \mid \sum_{g=1}^{\gamma}(1-D_g) > 0\right]$ (which is independent from i by assumption that D_i is identically distributed across i). Moreover, using Proposition A.1,

$$\mathbb{E}[DL_i^K] = \begin{cases} \pi\xi(\alpha_{uc})\frac{\gamma-1}{2}\sigma\sqrt{K}, & \text{if } i \text{ is core (directional),} \\ \pi\xi(\alpha_{uc})\frac{\gamma+1}{2}\sigma\sqrt{K}, & \text{if } i \text{ is core (flat),} \\ \pi\xi(\alpha_{uc})\sigma\sqrt{K}, & \text{if } i \text{ is periphery.} \end{cases} \tag{B.13}$$

Therefore, the effect of loss sharing proportional to net risk for entity i is

$$\Delta E_i^{\propto\text{net}} = \frac{\mathbb{E}[DL_i{}^{K-1} + LSC_i^{\propto\text{net}}]}{\mathbb{E}[DL_i{}^K]} - 1 \tag{B.14}$$

$$= \begin{cases} \frac{\pi\xi(\alpha_{uc})\frac{\gamma-1}{2}\sigma\sqrt{K-1}+\sigma\sqrt{\frac{\gamma-1}{2}}\xi(\alpha_{CCP})H}{\pi\xi(\alpha_{uc})\frac{\gamma-1}{2}\sigma\sqrt{K}} - 1 = \frac{\sqrt{K-1}}{\sqrt{K}} + \frac{\xi(\alpha_{CCP})H}{\pi\xi(\alpha_{uc})\sqrt{\frac{\gamma-1}{2}}\sqrt{K}} - 1, & \text{if } i \text{ is core (directional),} \\ \frac{\pi\xi(\alpha_{uc})\frac{\gamma+1}{2}\sigma\sqrt{K-1}+\sigma\sqrt{\frac{\gamma+1}{2}}\xi(\alpha_{CCP})H}{\pi\xi(\alpha_{uc})\frac{\gamma+1}{2}\sigma\sqrt{K}} - 1 = \frac{\sqrt{K-1}}{\sqrt{K}} + \frac{\xi(\alpha_{CCP})H}{\pi\xi(\alpha_{uc})\sqrt{\frac{\gamma+1}{2}}\sqrt{K}} - 1, & \text{if } i \text{ is core (flat),} \\ \frac{\pi\xi(\alpha_{uc})\sigma\sqrt{K-1}+\sigma\xi(\alpha_{CCP})H}{\pi\xi(\alpha_{uc})\sigma\sqrt{K}} - 1 = \frac{\sqrt{K-1}}{\sqrt{K}} + \frac{\xi(\alpha_{CCP})H}{\pi\xi(\alpha_{uc})\sqrt{K}} - 1, & \text{if } i \text{ is periphery,} \end{cases} \tag{B.15}$$

which implies that $\Delta E_i^{\propto\text{net}} = \Delta E_{\text{periphery}}^{\propto\text{net}}$ if, and only if, $\sqrt{\frac{\gamma-1}{2}} = 1 \Leftrightarrow \gamma = 3$ when i is a directional-core entity, and $\Delta E_i^{\propto\text{net}} = \Delta E_{\text{periphery}}^{\propto\text{net}}$ if, and only if, $\sqrt{\frac{\gamma+1}{2}} = 1 \Leftrightarrow \gamma = 1$ when i is a flat-core entity; and that

$$\frac{d}{d\gamma}\left(\Delta E^{\propto\text{net}}_{\text{periphery}} - \Delta E^{\propto\text{net}}_{i}\right) = \begin{cases} \frac{d}{d\gamma}\left(-\frac{\xi(\alpha_{CCP})H}{\pi\xi(\alpha_{uc})\sqrt{\frac{\gamma-1}{2}}\sqrt{K}}\right) > 0, & \text{if } i \text{ is core (directional)}, \\ \frac{d}{d\gamma}\left(-\frac{\xi(\alpha_{CCP})H}{\pi\xi(\alpha_{uc})\sqrt{\frac{\gamma+1}{2}}\sqrt{K}}\right) > 0, & \text{if } i \text{ is core (flat)}, \end{cases}$$

and, hence, that $\Delta E^{\propto\text{net}}_{\text{core}} < \Delta E^{\propto\text{net}}_{\text{periphery}}$ for all $\gamma > 3$. From above,

$$\min\left(\Delta E^{\propto\text{net}}_{\text{periphery}}, \Delta E^{\propto\text{net}}_{\text{core (flat)}}, \Delta E^{\propto\text{net}}_{\text{core (directional)}}\right) = \Delta E^{\propto\text{net}}_{\text{core (flat)}} \tag{B.16}$$

and the effect of loss sharing is the same across entities in the flat-complete and heterogeneous complete networks,

$$\Delta E^{\propto\text{net}}_{\text{flat-complete}} = \Delta E^{\propto\text{net}}_{\text{heterog-complete}}, \tag{B.17}$$

where the effect of loss sharing in the flat-complete network is

$$\begin{aligned} \Delta E^{\propto\text{net}}_{\text{flat-complete}} &= \frac{\pi\xi(\alpha_{uc})(\gamma-1)\sigma\sqrt{K-1} + \mathbb{E}\left[\frac{(1-D_i)}{\sum_{g=1}^{\gamma}(1-D_g)}\xi(\alpha_{CCP})\sum_{j=1}^{\gamma} D_j\sigma\sqrt{\gamma-1} \mid \sum_{g=1}^{\gamma}(1-D_g) > 0\right]}{\pi\xi(\alpha_{uc})(\gamma-1)\sigma\sqrt{K}} - 1 \\ &= \frac{\sqrt{K-1}}{\sqrt{K}} + \frac{\xi(\alpha_{CCP})\mathbb{E}\left[\frac{(1-D_i)\sum_{j=1}^{\gamma} D_j}{\sum_{g=1}^{\gamma}(1-D_g)} \mid \sum_{g=1}^{\gamma}(1-D_g) > 0\right]}{\pi\xi(\alpha_{uc})\sqrt{\gamma-1}\sqrt{K}} - 1. \end{aligned}$$

Therefore, the effect of loss sharing differs across networks if, and only if,

$$
\begin{aligned}
&\Delta E^{\propto \text{net}}_{\text{flat-complete}} < \Delta E^{\propto \text{net}}_{\text{core (flat)}} \\
\Leftrightarrow\ & \frac{\sqrt{K-1}}{\sqrt{K}} + \frac{\xi(\alpha_{CCP})\mathbb{E}\left[\frac{(1-D_i)\sum_{j=1}^{\gamma} D_j}{\sum_{g=1}^{\gamma}(1-D_g)} \mid \sum_{g=1}^{\gamma}(1-D_g) > 0\right]}{\pi\xi(\alpha_{uc})\sqrt{\gamma-1}\sqrt{K}} - 1 \\
&\quad < \frac{\sqrt{K-1}}{\sqrt{K}} + \frac{\xi(\alpha_{CCP})\mathbb{E}\left[\frac{(1-D_i)\bar{\sigma}_i^{CCP}}{\sum_{g=1}^{\gamma}(1-D_g)\bar{\sigma}_g^{CCP}} \sum_{j=1}^{\gamma} D_j \bar{\sigma}_j^{CCP} \mid \sum_{g=1}^{\gamma}(1-D_g) > 0\right]}{\pi\xi(\alpha_{uc})\sqrt{\frac{\gamma+1}{2}}\sqrt{K}} - 1 \\
\Leftrightarrow\ & \frac{\mathbb{E}\left[\frac{(1-D_i)\sum_{j=1}^{\gamma} D_j}{\sum_{g=1}^{\gamma}(1-D_g)} \mid \sum_{g=1}^{\gamma}(1-D_g) > 0\right]}{\sqrt{\gamma-1}} < \frac{\sqrt{\frac{\gamma+1}{2}}\mathbb{E}\left[\frac{(1-D_i)\sum_{j=1}^{\gamma} D_j\bar{\sigma}_j^{CCP}}{\sum_{g=1}^{\gamma}(1-D_g)\bar{\sigma}_g^{CCP}} \mid \sum_{g=1}^{\gamma}(1-D_g) > 0\right]}{\sqrt{\frac{\gamma+1}{2}}} \\
\Leftrightarrow\ & \frac{\mathbb{E}\left[\frac{(1-D_i)\sum_{j=1}^{\gamma} D_j}{\sum_{g=1}^{\gamma}(1-D_g)} \mid \sum_{g=1}^{\gamma}(1-D_g) > 0\right]}{\sqrt{\gamma-1}} < \mathbb{E}\left[\frac{(1-D_i)\sum_{j=1}^{\gamma} D_j\bar{\sigma}_j^{CCP}}{\sum_{g=1}^{\gamma}(1-D_g)\bar{\sigma}_g^{CCP}} \mid \sum_{g=1}^{\gamma}(1-D_g) > 0\right].
\end{aligned}
$$

Using that $\sigma \leq \bar{\sigma}_g^{CCP} \leq \sigma\sqrt{\frac{\gamma+1}{2}}$, the inequality holds if

$$
\begin{aligned}
& \frac{\mathbb{E}\left[\frac{(1-D_i)\sum_{j=1}^{\gamma} D_j}{\sum_{g=1}^{\gamma}(1-D_g)} \mid \sum_{g=1}^{\gamma}(1-D_g) > 0\right]}{\sqrt{\gamma-1}} < \mathbb{E}\left[\frac{(1-D_i)\sum_{j=1}^{\gamma} D_j\sigma}{\sum_{g=1}^{\gamma}(1-D_g)\sigma\sqrt{\frac{\gamma+1}{2}}} \mid \sum_{g=1}^{\gamma}(1-D_g) > 0\right] \\
\Leftrightarrow\ & \frac{\mathbb{E}\left[\frac{(1-D_i)\sum_{j=1}^{\gamma} D_j}{\sum_{g=1}^{\gamma}(1-D_g)} \mid \sum_{g=1}^{\gamma}(1-D_g) > 0\right]}{\sqrt{\gamma-1}} < \frac{\mathbb{E}\left[\frac{(1-D_i)\sum_{j=1}^{\gamma} D_j}{\sum_{g=1}^{\gamma}(1-D_g)} \mid \sum_{g=1}^{\gamma}(1-D_g) > 0\right]}{\sqrt{\frac{\gamma+1}{2}}} \\
\Leftrightarrow\ & \sqrt{\gamma-1} > \sqrt{\frac{\gamma+1}{2}} \\
\Leftrightarrow\ & \gamma - 1 > \frac{\gamma+1}{2} \\
\Leftrightarrow\ & \gamma > 3.
\end{aligned}
$$

$\square$

B.3. Proof of Proposition 3.

Proof. Let $\beta_{ij}^k \equiv 0$. In the case with loss sharing proportional to gross notional, using Propositions A.3 it is

$$\mathbb{E}[LSC_i{}^{\propto\text{gross}}] = \begin{cases} \frac{\gamma-1}{2}H, & \text{if } i \text{ is core (directional)},\\ \frac{\gamma+1}{2}H, & \text{if } i \text{ is core (flat)},\\ \sigma H, & \text{if } i \text{ is periphery}, \end{cases} \tag{B.18}$$

with $H = \xi(\alpha_{CCP})\mathbb{E}\left[\frac{(1-D_i)}{\sum_{g=1}^{\gamma}(1-D_g)v_{g*}^K}\sum_{j=1}^{\gamma} D_j\bar{\sigma}_j^{CCP} \mid \sum_{g=1}^{\gamma}(1-D_g)>0\right]$ (which is independent from i by assumption that D_i is identically distributed across i). $\mathbb{E}[DL_i^K]$ is as above. Therefore, it is

$$\Delta E_i^{\propto\text{gross}} = \frac{\mathbb{E}[DL_i{}^{K-1} + LSC_i^{\propto\text{gross}}]}{\mathbb{E}[DL_i{}^K]} - 1 \tag{B.19}$$

$$= \begin{cases} \frac{\pi\xi(\alpha_{uc})\frac{\gamma-1}{2}\sigma\sqrt{K-1}+\frac{\gamma-1}{2}H}{\pi\xi(\alpha_{uc})\frac{\gamma-1}{2}\sigma\sqrt{K}} - 1 = \frac{\sqrt{K-1}}{\sqrt{K}} + \frac{H}{\pi\xi(\alpha_{uc})\sigma\sqrt{K}} - 1, & \text{if } i \text{ is core (directional)},\\ \frac{\pi\xi(\alpha_{uc})\frac{\gamma+1}{2}\sigma\sqrt{K-1}+\frac{\gamma+1}{2}H}{\pi\xi(\alpha_{uc})\frac{\gamma+1}{2}\sigma\sqrt{K}} - 1 = \frac{\sqrt{K-1}}{\sqrt{K}} + \frac{H}{\pi\xi(\alpha_{uc})\sigma\sqrt{K}} - 1, & \text{if } i \text{ is core (flat)},\\ \frac{\pi\xi(\alpha_{uc})\sigma\sqrt{K-1}+H}{\pi\xi(\alpha_{uc})\sigma\sqrt{K}} - 1 = \frac{\sqrt{K-1}}{\sqrt{K}} + \frac{H}{\pi\xi(\alpha_{uc})\sigma\sqrt{K}} - 1, & \text{if } i \text{ is periphery}, \end{cases} \tag{B.20}$$

which implies that $\Delta E_{\text{periphery}}^{\propto\text{gross}} = \Delta E_{\text{core}}^{\propto\text{gross}}$ independent of portfolio directionality and independent of γ. From above, it is

$$\begin{aligned} \Delta E_{\text{flat-complete}}^{\propto\text{gross}} &= \frac{\sqrt{K-1}}{\sqrt{K}} + \frac{\xi(\alpha_{CCP})\mathbb{E}\left[\frac{(1-D_i)(\gamma-1)}{\sum_{g=1}^{\gamma}(1-D_g)(\gamma-1)}\sum_{j=1}^{\gamma} D_j\sigma\sqrt{\gamma-1} \mid \sum_{g=1}^{\gamma}(1-D_g)>0\right]}{\pi\xi(\alpha_{uc})(\gamma-1)\sigma\sqrt{K}} - 1\\ &= \frac{\sqrt{K-1}}{\sqrt{K}} + \frac{\xi(\alpha_{CCP})\mathbb{E}\left[\frac{(1-D_i)\sum_{j=1}^{\gamma} D_j}{\sum_{g=1}^{\gamma}(1-D_g)} \mid \sum_{g=1}^{\gamma}(1-D_g)>0\right]}{\pi\xi(\alpha_{uc})\sqrt{\gamma-1}\sqrt{K}} - 1\\ &= \Delta E_{\text{heterog-complete}}^{\propto\text{gross}} \end{aligned}$$

and

$$\min\left(\Delta E^{\propto\text{gross}}_{\text{periphery}}, \Delta E^{\propto\text{gross}}_{\text{core}}\right) = \Delta E^{\propto\text{gross}}_{\text{core (flat)}}$$
$$= \frac{\sqrt{K-1}}{\sqrt{K}} + \frac{\xi(\alpha_{CCP})\mathbb{E}\left[\frac{(1-D_i)}{\sum_{g=1}^{\gamma}(1-D_g)v^K_{g*}}\sum_{j=1}^{\gamma} D_j \bar{\sigma}^{CCP}_j \mid \sum_{g=1}^{\gamma}(1-D_g)>0\right]}{\pi\xi(\alpha_{uc})\sigma\sqrt{K}} - 1$$

and

$$\Delta E^{\propto\text{gross}}_{\text{flat-complete}} < \Delta E^{\propto\text{gross}}_{\text{core (flat)}}$$
$$\Leftrightarrow \frac{\mathbb{E}\left[\frac{(1-D_i)\sum_{j=1}^{\gamma} D_j}{\sum_{g=1}^{\gamma}(1-D_g)} \mid \sum_{g=1}^{\gamma}(1-D_g)>0\right]}{\sqrt{\gamma-1}} < \frac{\mathbb{E}\left[\frac{(1-D_i)}{\sum_{g=1}^{\gamma}(1-D_g)v^K_{g*}}\sum_{j=1}^{\gamma} D_j \bar{\sigma}^{CCP}_j \mid \sum_{g=1}^{\gamma}(1-D_g)>0\right]}{\sigma}$$

and using that $\sigma \leq \bar{\sigma}^{CCP}_g \leq \sigma\sqrt{\frac{\gamma+1}{2}}$, the inequality holds if

$$\frac{1}{\sqrt{\gamma-1}}\mathbb{E}\left[\frac{(1-D_i)\sum_{j=1}^{\gamma} D_j}{\sum_{g=1}^{\gamma}(1-D_g)} \mid \sum_{g=1}^{\gamma}(1-D_g)>0\right] < \mathbb{E}\left[\frac{(1-D_i)\sum_{j=1}^{\gamma} D_j}{\sum_{g=1}^{\gamma}(1-D_g)v^K_{g*}} \mid \sum_{g=1}^{\gamma}(1-D_g)>0\right]$$
$$\Leftrightarrow \gamma > 2.$$

□

B.4. *Proof of Proposition 4.*

Proof. Assume that $\rho^2_{X,M} > 0$. Let $v^*_i = \frac{|\sum_{j=1,j\neq i}^{\gamma} v^K_{ji}|}{\sum_{j=1,j\neq i}^{\gamma}|v^K_{ji}|}$ entity i's relative net class-K position, and let

$$H = \mathbb{E}\left[\left(\frac{\sum_{g=1}^{\gamma}\bar{\sigma}^{CCP}_g}{\sum_{g=1}^{\gamma}(1-D_g)\bar{\sigma}^{CCP}_g} - 1\right)\xi(\alpha_{CCP})(1-D_i) \mid \sum_{g=1}^{\gamma}(1-D_g)>0\right], \tag{B.21}$$

where $\bar{\sigma}^{CCP}_i = \sqrt{\sum_{j=1,j\neq i}^{\gamma}(v^K_{ji})^2}\sqrt{\sigma^2_M\beta^2(v^*_i)^2\sum_{j=1,j\neq i}^{\gamma}(v^K_{ji})^2+\sigma^2}$. If cleared losses are allocated proportionally to net risk, $\mathbb{E}[LSC_i^{\propto\text{net}}] = \bar{\sigma}^{CCP}_i H$. Then,

$$v^*_i = \begin{cases} \frac{|\sum_{j=1,j\neq i}^{\gamma} v^K_{ji}|}{\frac{\gamma-1}{2}} \leq 1, & \text{if } i \text{ is core (directional)},\\ 0, & \text{if } i \text{ is core (flat)},\\ 1, & \text{if } i \text{ is periphery.}\end{cases} \tag{B.22}$$

and thus

$$\mathbb{E}[LSC_i^{\propto\text{net}}] = \begin{cases} \sqrt{\frac{\gamma-1}{2}}\sqrt{\sigma_M^2\beta^2\left(v_i^*\right)^2\frac{\gamma-1}{2}+\sigma^2}H, & \text{if } i \text{ is core (directional)},\\ \sqrt{\frac{\gamma+1}{2}}\sigma H, & \text{if } i \text{ is core (flat)},\\ \sqrt{\sigma_M^2\beta^2+\sigma^2}H, & \text{if } i \text{ is periphery} \end{cases} \tag{B.23}$$

and

$$\mathbb{E}[\Delta E_i^{\propto\text{net}}] = \begin{cases} \frac{\sqrt{\frac{\gamma-1}{2}}\sqrt{\sigma_M^2\beta^2\left(v_i^*\right)^2\frac{\gamma-1}{2}+\sigma^2}H+\frac{\gamma-1}{2}\pi\xi(\alpha_{uc})\sqrt{\sigma_M^2\beta^2(K-1)^2+(K-1)\sigma^2}}{\pi\xi(\alpha_{uc})\frac{\gamma-1}{2}\sqrt{\sigma_M^2\beta^2K^2+K\sigma^2}}, & \text{if } i \text{ is core (directional)},\\ \frac{\sqrt{\frac{\gamma+1}{2}}\sigma H+\pi\xi(\alpha_{uc})\frac{\gamma+1}{2}\sqrt{\sigma_M^2\beta^2(K-1)^2+(K-1)\sigma^2}}{\pi\xi(\alpha_{uc})\frac{\gamma+1}{2}\sqrt{\sigma_M^2\beta^2K^2+(K)\sigma^2}}, & \text{if } i \text{ is core (flat)},\\ \frac{\sqrt{\sigma_M^2\beta^2+\sigma^2}H+\pi\xi(\alpha_{uc})\sqrt{\sigma_M^2\beta^2(K-1)^2+(K-1)\sigma^2}}{\pi\xi(\alpha_{uc})\sqrt{\sigma_M^2\beta^2K^2+(K)\sigma^2}}, & \text{if } i \text{ is periphery} \end{cases} \tag{B.24}$$

$$= \begin{cases} \frac{\sqrt{\sigma_M^2\beta^2\left(v_i^*\right)^2+\sigma^2\frac{1}{\sqrt{\frac{\gamma-1}{2}}}}H+\pi\xi(\alpha_{uc})\sqrt{\sigma_M^2\beta^2(K-1)^2+(K-1)\sigma^2}}{\pi\xi(\alpha_{uc})\sqrt{\sigma_M^2\beta^2K^2+K\sigma^2}}, & \text{if } i \text{ is core (directional)},\\ \frac{\frac{\sigma}{\sqrt{\frac{\gamma+1}{2}}}H+\pi\xi(\alpha_{uc})\sqrt{\sigma_M^2\beta^2(K-1)^2+(K-1)\sigma^2}}{\pi\xi(\alpha_{uc})\sqrt{\sigma_M^2\beta^2K^2+(K)\sigma^2}}, & \text{if } i \text{ is core (flat)},\\ \frac{\sqrt{\sigma_M^2\beta^2+\sigma^2}H+\pi\xi(\alpha_{uc})\sqrt{\sigma_M^2\beta^2(K-1)^2+(K-1)\sigma^2}}{\pi\xi(\alpha_{uc})\sqrt{\sigma_M^2\beta^2K^2+(K)\sigma^2}}, & \text{if } i \text{ is periphery} \end{cases} \tag{B.25}$$

Since $v_i^* \in [0,1]$ and $\gamma \geq 3$, it is

$$\sqrt{\sigma_M^2\beta^2+\sigma^2} \geq \sqrt{\sigma_M^2\beta^2\left(v_i^*\right)^2+\sigma^2\frac{1}{\sqrt{\frac{\gamma-1}{2}}}} \geq \frac{\sigma}{\sqrt{\frac{\gamma+1}{2}}}$$

implying that

$$\Delta E_{\text{periphery}}^{\propto\text{net}} \geq \Delta E_{\text{core (directional)}}^{\propto\text{net}} \geq \Delta E_{\text{core (flat)}}^{\propto\text{net}}.$$

The first inequality binds only if $v_i^* = 0$ and $\gamma = 3$, and is strict otherwise. The second inequality is strict for any $v_i^* \in [0,1]$ and $\gamma \geq 3$. Moreover,

$$\frac{d}{dv_i^*}\Delta E_{\text{core (directional)}}^{\propto\text{net}} > 0,$$

i.e., core entities with more directional portfolios have a smaller reduction in default losses (holding portfolios' gross notional fixed).

Define

$$H' = \mathbb{E}\left[\frac{(1-D_i)}{\sum_{g=1}^{\gamma}(1-D_g)v_{g*}^K}\xi(\alpha_{CCP})\sum_{j=1}^{\gamma}D_j\bar{\sigma}_j^{CCP} \mid \sum_{g=1}^{\gamma}(1-D_g)>0\right], \tag{B.26}$$

If cleared losses are allocated proportionally to gross notional, $\mathbb{E}[LSC_i^{\propto\text{gross}}] = v_{i*}^K H$, where $v_{i*}^K = \sum_{j=1,j\neq i}^{\gamma}|v_{ij}^K|$. Then,

$$v_{i*}^K = \begin{cases} \frac{\gamma-1}{2}, & \text{if } i \text{ is core (directional),} \\ \frac{\gamma+1}{2}, & \text{if } i \text{ is core (flat),} \\ 1, & \text{if } i \text{ is periphery.} \end{cases} \tag{B.27}$$

and thus

$$\mathbb{E}[LSC_i^{\propto\text{gross}}] = \begin{cases} \frac{\gamma-1}{2}H', & \text{if } i \text{ is core (directional),} \\ \frac{\gamma+1}{2}H', & \text{if } i \text{ is core (flat),} \\ H', & \text{if } i \text{ is periphery} \end{cases} \tag{B.28}$$

and

$$\mathbb{E}[\Delta E_i^{\propto\text{gross}}] = \begin{cases} \frac{\frac{\gamma-1}{2}H'+\frac{\gamma-1}{2}\pi\xi(\alpha_{uc})\sqrt{\sigma_M^2\beta^2(K-1)^2+(K-1)\sigma^2}}{\pi\xi(\alpha_{uc})\frac{\gamma-1}{2}\sqrt{\sigma_M^2\beta^2K^2+K\sigma^2}}, & \text{if } i \text{ is core (directional),} \\ \frac{\frac{\gamma+1}{2}H'+\pi\xi(\alpha_{uc})\frac{\gamma+1}{2}\sqrt{\sigma_M^2\beta^2(K-1)^2+(K-1)\sigma^2}}{\pi\xi(\alpha_{uc})\frac{\gamma+1}{2}\sqrt{\sigma_M^2\beta^2K^2+(K)\sigma^2}}, & \text{if } i \text{ is core (flat),} \\ \frac{H'+\pi\xi(\alpha_{uc})\sqrt{\sigma_M^2\beta^2(K-1)^2+(K-1)\sigma^2}}{\pi\xi(\alpha_{uc})\sqrt{\sigma_M^2\beta^2K^2+(K)\sigma^2}}, & \text{if } i \text{ is periphery} \end{cases} \tag{B.29}$$

$$= \begin{cases} \frac{H'+\pi\xi(\alpha_{uc})\sqrt{\sigma_M^2\beta^2(K-1)^2+(K-1)\sigma^2}}{\pi\xi(\alpha_{uc})\sqrt{\sigma_M^2\beta^2K^2+K\sigma^2}}, & \text{if } i \text{ is core (directional),} \\ \frac{H'+\pi\xi(\alpha_{uc})\sqrt{\sigma_M^2\beta^2(K-1)^2+(K-1)\sigma^2}}{\pi\xi(\alpha_{uc})\sqrt{\sigma_M^2\beta^2K^2+(K)\sigma^2}}, & \text{if } i \text{ is core (flat),} \\ \frac{H'+\pi\xi(\alpha_{uc})\sqrt{\sigma_M^2\beta^2(K-1)^2+(K-1)\sigma^2}}{\pi\xi(\alpha_{uc})\sqrt{\sigma_M^2\beta^2K^2+(K)\sigma^2}}, & \text{if } i \text{ is periphery} \end{cases} \tag{B.30}$$

and thus

$$\Delta E_{\text{periphery}}^{\propto\text{net}} = \Delta E_{\text{core (directional)}}^{\propto\text{net}} = \Delta E_{\text{core (flat)}}^{\propto\text{net}}$$

independent of portfolio directionality and γ. $\square$

B.5. Proof of Proposition 5.

Proof. First, consider the case without central clearing. Let $C_{ij}^K = \Phi^{-1}(\alpha_{uc})\sqrt{\left(\sum_{k=1}^K v_{ij}^k\right)^2 \beta^2\sigma_M^2 + K\sigma^2}$ the bilateral collateral posted by j to i, and define

$$\bar{\mu}_{ij|M}^{BN} = \mathbb{E}\left[\sum_{k=1}^K X_{ij}^k - C_{ij}^K \mid M\right] = M\sum_{k=1}^K v_{ij}^k\beta_{ij}^k - C_{ij}^K \tag{B.31}$$

$$\left(\bar{\sigma}_{ij|M}^{BN}\right)^2 = \text{var}\left(\sum_{k=1}^K X_{ij}^k\right) = \text{var}\left(\sum_{k=1}^K (v_{ij}^k)^2((\beta_{ij}^k)^2 M + (\sigma_{ij}^k)^2(\varepsilon_{ij}^k)^2)\right) \tag{B.32}$$

$$= \text{var}\left(\sum_{k=1}^K (v_{ij}^k)^2(\sigma_{ij}^k)^2(\varepsilon_{ij}^k)^2\right) = \sigma^2\sum_{k=1}^K \left(v_{ij}^k\right)^2. \tag{B.33}$$

Then, the expected default loss of i given default of j and conditional on M is given by

$$\mathbb{E}[e_{ij}^{BN} \mid M] = \bar{\mu}_{ij|M}\Phi(\bar{\mu}_{ij|M}/\bar{\sigma}_{ij|M}) + \bar{\sigma}_{ij|M}\varphi(-\bar{\mu}_{ij|M}/\bar{\sigma}_{ij|M}) \tag{B.34}$$

$$= \left(M\beta\sum_{k=1}^K v_{ij}^k - C_{ij}^K\right)\Phi\left(\frac{M\beta\sum_{k=1}^K v_{ij}^k - C_{ij}^K}{\sigma\sqrt{K}}\right) \tag{B.35}$$

$$+\sigma\sqrt{K}\varphi\left(-\frac{M\beta\sum_{k=1}^K v_{ij}^k - C_{ij}^K}{\sigma\sqrt{K}}\right),$$

using in the last step that $(v_{ij}^k)^2 \equiv 1$, $\beta_{ij}^k \equiv \beta$ and $\sigma_{ij}^k \equiv \sigma$. The overall expected default loss is then given by the sum of exposures across counterparties

$$\mathbb{E}[DL_i^K \mid M] = \pi\sum_{j=1,j\neq i}^{\gamma} 1_{\{\sum_{k=1}^K |v_{ij}^k|>0\}}\mathbb{E}[e_{ij}^{BN} \mid M]. \tag{B.36}$$

Second, consider the case that class-K is centrally cleared. Define $C_i^{CCP} = VaR_{\alpha_{CCP}}\left(\sum_{j=1,j\neq i}^{\gamma} X_{ji}^k\right)$ the collateral posted by i to the CCP and

$$\bar{\mu}_{j|M}^{CCP} = \mathbb{E}\left[\sum_{h=1,h\neq j}^{\gamma} X_{hj}^K - C_j^{CCP}\right] = M\beta\sum_{h=1,h\neq j}^{\gamma} v_{hj}^K - C_j^{CCP} \tag{B.37}$$

$$(\bar{\sigma}_{j|M}^{CCP})^2 = \text{var}\left(\sum_{h=1,h\neq j}^{\gamma} X_{hj}^K\right) = \text{var}\left(\sum_{h=1,h\neq j}^{\gamma} v_{hj}^K\sigma\varepsilon_{hj}^K\right) = \sigma^2\sum_{h=1,h\neq j}^{\gamma}(v_{hj}^K)^2. \tag{B.38}$$

Thus, defining by $e_j^{CCP} = \max\left(\sum_{\substack{h=1,\\h\neq j}}^{\gamma} X_{hj}^K - C_j^{CCP}, 0\right)$ the CCP's loss upon j's default, it is

$$\begin{aligned}
\mathbb{E}[e_j^{CCP} \mid M] &= \bar{\mu}_{j|M}^{CCP}\Phi(\bar{\mu}_{j|M}^{CCP}/\bar{\sigma}_{j|M}^{CCP}) + \bar{\sigma}_{j|M}^{CCP}\varphi(-\bar{\mu}_{j|M}^{CCP}/\bar{\sigma}_{j|M}^{CCP}) && \text{(B.39)}\\
&= \left(M\beta\sum_{h=1,h\neq j}^{\gamma} v_{hj}^K - C_j^{CCP}\right)\Phi\left(\frac{M\beta\sum_{h=1,h\neq j}^{\gamma} v_{hj}^K - C_j^{CCP}}{\sigma\sqrt{\sum_{h=1,h\neq j}^{\gamma}(v_{hj}^K)^2}}\right) && \text{(B.40)}\\
&\quad + \sigma\sqrt{\sum_{h=1,h\neq j}^{\gamma}(v_{hj}^K)^2}\varphi\left(-\frac{M\beta\sum_{h=1,h\neq j}^{\gamma} v_{hj}^K - C_j^{CCP}}{\sigma\sqrt{\sum_{h=1,h\neq j}^{\gamma}(v_{hj}^K)^2}}\right). && \text{(B.41)}
\end{aligned}$$

If losses are shared proportionally to initial margin, using the law of iterated expectation and independence between M and D_j, i's expected loss sharing contribution conditional on M is given by

$$\begin{aligned}
\mathbb{E}[LSC_i^{\propto\text{net}} \mid M] &= \mathbb{E}\left[\frac{(1-D_i)C_i^{CCP}}{\sum_{g=1}^{\gamma}(1-D_g)C_g^{CCP}} DL^{CCP} \mid \sum_{g=1}^{\gamma}(1-D_g) > 0, M\right] && \text{(B.42)}\\
&= \mathbb{E}\left[\frac{(1-D_i)\bar{\sigma}_i^{CCP}}{\sum_{g=1}^{\gamma}(1-D_g)\bar{\sigma}_g^{CCP}} \sum_{j=1}^{\gamma} D_j\mathbb{E}[e_j^{CCP} \mid M] \mid \sum_{g=1}^{\gamma}(1-D_g) > 0\right], && \text{(B.43)}
\end{aligned}$$

where $\bar{\sigma}_i^{CCP} = \sqrt{\sigma_M^2\beta^2\left(\sum_{j=1,j\neq i}^{\gamma} v_{ji}^K\right)^2 + \sigma^2\sum_{j=1,j\neq i}^{\gamma}(v_{ji}^K)^2}$ as above.

If losses are instead shared proportionally to gross notional, using the law of iterated expectation and independence between M and D_j, i's expected loss sharing contribution conditional on M is given by

$$\begin{aligned}
&\mathbb{E}[LSC_i^{\propto\text{gross}} \mid M]\\
&= \mathbb{E}\left[\frac{(1-D_i)\sum_{j=1,j\neq i}^{\gamma}|v_{ij}^K|}{\sum_{g=1}^{\gamma}(1-D_g)\sum_{j=1,j\neq g}^{\gamma}|v_{gj}^K|} DL^{CCP} \mid \sum_{g=1}^{\gamma}(1-D_g) > 0, M\right]\\
&= \mathbb{E}\left[\frac{(1-D_i)\sum_{j=1,j\neq i}^{\gamma}|v_{ij}^K|}{\sum_{g=1}^{\gamma}(1-D_g)\sum_{j=1,j\neq g}^{\gamma}|v_{gj}^K|} \sum_{j=1}^{\gamma} D_j \max\left(\sum_{\substack{h=1,\\h\neq j}}^{\gamma} X_{hj}^K - C_j^{CCP}, 0\right) \mid \sum_{g=1}^{\gamma}(1-D_g) > 0, M\right]\\
&= \mathbb{E}\left[\frac{(1-D_i)\sum_{j=1,j\neq i}^{\gamma}|v_{ij}^K|}{\sum_{g=1}^{\gamma}(1-D_g)\sum_{j=1,j\neq g}^{\gamma}|v_{gj}^K|} \sum_{j=1}^{\gamma} D_j\mathbb{E}[e_j^{CCP} \mid M] \mid \sum_{g=1}^{\gamma}(1-D_g) > 0\right].
\end{aligned}$$

$\square$

B.6. *Proof of Proposition A.1.*

Proof. Define

$$
\begin{aligned}
&(\bar{\sigma}_{ij}^{BN})^2 = \operatorname{var}\left(\sum_{k=1}^{K} X_{ij}^k\right) = \sigma_M^2 \left(\sum_{k=1}^{K} v_{ij}^k \beta_{ij}^k\right)^2 + \sum_{k=1}^{K} (v_{ij}^k \sigma_{ij}^k)^2 = \sigma_M^2 \beta^2 \left(\sum_{k=1}^{K} v_{ij}^k\right)^2 + K\sigma^2,\\
&\bar{\mu}_{ij}^{BN} = -C_{ij}^K = -\bar{\sigma}_{ij}^{BN} \Phi^{-1}(\alpha_{uc}).
\end{aligned}
$$

Then, it is

$$
\begin{aligned}
\mathbb{E}\left[DL_i^K\right] &= \sum_{j=1, j\neq i}^{\gamma} \pi 1_{\{\sum_{k=1}^{K} |v_{ij}^k|>0\}} \left(\bar{\mu}_{ij}^{BN} \Phi(\bar{\mu}_{ij}^{BN}/\bar{\sigma}_{ij}^{BN}) + \bar{\sigma}_{ij}^{BN} \varphi(-\bar{\mu}_{ij}^{BN}/\bar{\sigma}_{ij}^{BN})\right)\\
&= \sum_{j=1, j\neq i}^{\gamma} \pi 1_{\{\sum_{k=1}^{K} |v_{ij}^k|>0\}} \left(-\Phi^{-1}(\alpha_{uc}) \bar{\sigma}_{ij}^{BN} \Phi\left(\frac{-\Phi^{-1}(\alpha_{uc}) \bar{\sigma}_{ij}^{BN}}{\sigma_{ij}^{\bar{B}N}}\right) + \bar{\sigma}_{ij}^{BN} \varphi\left(-\frac{-\Phi^{-1}(\alpha_{uc}) \bar{\sigma}_{ij}^{BN}}{\sigma_{ij}^{\bar{B}N}}\right)\right)\\
&= \sum_{j=1, j\neq i}^{\gamma} \pi 1_{\{\sum_{k=1}^{K} |v_{ij}^k|>0\}} \left(-\Phi^{-1}(\alpha_{uc}) \bar{\sigma}_{ij}^{BN} \Phi\left(\Phi^{-1}(1-\alpha_{uc})\right) + \bar{\sigma}_{ij}^{BN} \varphi\left(\Phi^{-1}(\alpha_{uc})\right)\right) \qquad \text{(B.44)}\\
&= \sum_{j=1, j\neq i}^{\gamma} 1_{\{\sum_{k=1}^{K} |v_{ij}^k|>0\}} \bar{\sigma}_{ij}^{BN} \underbrace{\left((1-\alpha_{uc})\Phi^{-1}(1-\alpha_{uc}) + \varphi\left(\Phi^{-1}(\alpha_{uc})\right)\right)}_{=\xi(\alpha_{uc})}. \qquad \text{(B.45)}
\end{aligned}
$$

$\square$

B.7. *Proof of Proposition A.2.*

Proof. Define

$$
\begin{aligned}
(\bar{\sigma}_i^{CCP})^2 =&\operatorname{var}\left(\sum_{j=1, j\neq i}^{\gamma} X_{ji}^K\right) = \sigma_M^2 \left(\sum_{j=1, j\neq i}^{\gamma} v_{ji}^K \beta_{ji}^K\right)^2 + \sum_{j=1, j\neq i}^{\gamma} (v_{ji}^K \sigma_{ji}^K)^2\\
=&\sigma_M^2 \beta^2 \left(\sum_{j=1, j\neq i}^{\gamma} v_{ji}^K\right)^2 + \sigma^2 \sum_{j=1, j\neq i}^{\gamma} (v_{ji}^K)^2,\\
\bar{\mu}_i^{CCP} =& -C_i^{CCP} = -\bar{\sigma}_i^{CCP} \Phi^{-1}(\alpha_{CCP}).
\end{aligned}
$$

Then, i's expected loss sharing contribution is

$$\begin{aligned}
\mathbb{E}[LSC_i{}^{\propto\text{net}}] &= \mathbb{E}\left[\frac{(1-D_i)C_i^{CCP}}{\sum_{g=1}^{\gamma}(1-D_g)C_g^{CCP}} DL^{CCP} \mid \sum_{g=1}^{\gamma}(1-D_g)>0\right] \qquad \text{(B.46)}\\
&= \mathbb{E}\left[\frac{(1-D_i)\bar{\sigma}_i^{CCP}\Phi^{-1}(\alpha_{CCP})}{\sum_{g=1}^{\gamma}(1-D_g)\bar{\sigma}_g^{CCP}\Phi^{-1}(\alpha_{CCP})}\sum_{j=1}^{\gamma} D_j \max\left(\sum_{\substack{h=1,\\h\neq j}}^{\gamma} X_{hj}^K - C_j^{CCP}, 0\right) \mid \sum_{g=1}^{\gamma}(1-D_g)>0\right]\\
&= \mathbb{E}\left[\frac{(1-D_i)\bar{\sigma}_i^{CCP}}{\sum_{g=1}^{\gamma}(1-D_g)\bar{\sigma}_g^{CCP}}\xi(\alpha_{CCP})\sum_{j=1}^{\gamma} D_j\bar{\sigma}_j^{CCP} \mid \sum_{g=1}^{\gamma}(1-D_g)>0\right]\\
&= \mathbb{E}\left[\frac{\sum_{g=1}^{\gamma} D_g\bar{\sigma}_g^{CCP}}{\sum_{g=1}^{\gamma}(1-D_g)\bar{\sigma}_g^{CCP}}\xi(\alpha_{CCP})(1-D_i)\bar{\sigma}_i^{CCP} \mid \sum_{g=1}^{\gamma}(1-D_g)>0\right]\\
&= \mathbb{E}\left[\frac{\sum_{g=1}^{\gamma}\bar{\sigma}_g^{CCP} - \sum_{g=1}^{\gamma}(1-D_g)\bar{\sigma}_g^{CCP}}{\sum_{g=1}^{\gamma}(1-D_g)\bar{\sigma}_g^{CCP}}\xi(\alpha_{CCP})(1-D_i)\bar{\sigma}_i^{CCP} \mid \sum_{g=1}^{\gamma}(1-D_g)>0\right]\\
&= \mathbb{E}\left[\left(\frac{\sum_{g=1}^{\gamma}\bar{\sigma}_g^{CCP}}{\sum_{g=1}^{\gamma}(1-D_g)\bar{\sigma}_g^{CCP}}-1\right)\xi(\alpha_{CCP})(1-D_i)\bar{\sigma}_i^{CCP} \mid \sum_{g=1}^{\gamma}(1-D_g)>0\right],
\end{aligned}$$

where we use the law of iterated expectation. □

B.8. *Proof of Proposition A.3.*

Proof. Define

$$\begin{aligned}
v_{i*}^K &= \sum_{j=1,j\neq i}^{\gamma}|v_{ij}^K|\\
(\bar{\sigma}_i^{CCP})^2 &= \text{var}\left(\sum_{j=1,j\neq i}^{\gamma} X_{ji}^K\right) == \sigma_M^2\beta^2\left(\sum_{j=1,j\neq i}^{\gamma} v_{ji}^K\right)^2 + \sigma^2\sum_{j=1,j\neq i}^{\gamma}(v_{ji}^K)^2,\\
\bar{\mu}_i^{CCP} &= -\,C_i^{CCP} = -\bar{\sigma}_i^{CCP}\Phi^{-1}(\alpha_{CCP}).
\end{aligned}$$

Then, i's expected loss sharing contribution is

$$\begin{aligned}
\mathbb{E}[LSC_i{}^{\propto\text{gross}}] &= \mathbb{E}\left[\frac{(1-D_i)v_{i*}^K}{\sum_{g=1}^{\gamma}(1-D_g)v_{g*}^K} DL^{CCP} \mid \sum_{g=1}^{\gamma}(1-D_g)>0\right] \qquad \text{(B.47)}\\
&= \mathbb{E}\left[\frac{(1-D_i)v_{i*}^K}{\sum_{g=1}^{\gamma}(1-D_g)v_{g*}^K}\xi(\alpha_{CCP})\sum_{j=1}^{\gamma} D_j\bar{\sigma}_j^{CCP} \mid \sum_{g=1}^{\gamma}(1-D_g)>0\right],
\end{aligned}$$

where we use the law of iterated expectation. □

C. Calibration

We calibrate the volatility of contract values based on index CDS, since these are already subject to clearing obligations in the US and EU. For this purpose, we retrieve data about the performance of the North American family of index CDS, the *CDX family*, from January 2006 to 2010, from Markit. We choose this period because it covers the 2007-08 financial crisis. Table C.1 reports the names of index CDS included in our sample. Starting with the assumption of a five-day settlement period, the descriptive statistics in Table C.2 show that the average standard deviation of index CDS prices' five-day log returns roughly equals $\sigma_X = 0.01$, which we use as an estimate for total contract volatility. During the same time period, the standard deviation of S&P 500 five-day log returns is roughly $\sigma_M = 0.03$, which we use as an estimate for volatility of the systematic risk factor.[35]

CDX name	Description
CDX NA.HY	North American High Yield CDSs
CDX NA.HY.B	Rating sub-index of CDX NA.HY
CDX NA.HY.BB	Rating sub-index of CDX NA.HY
CDX NA.HY.HB	Sub-index of CDX NA.HY (high beta)
CDX NA.IG	North American investment-grade CDSs
CDX NA.IG.CONS	Sub-index of CDX NA.IG (consumer cyclical)
CDX NA.IG.ENRG	Sub-index of CDX NA.IG (energy)
CDX NA.IG.FIN	Sub-index of CDX NA.IG (financials)
CDX NA.IG.TMT	Sub-index of CDX NA.IG (telecom, media and technology)
CDX NA.IG.INDU	Sub-index of CDX NA.IG (industrial)
CDX NA.IG.HVOL	Sub-index of CDX NA.IG (high volatility)
CDX NA.XO	Sub-index of CDX NA.IG (crossover between grade and junk)
CDX.EM	Emerging market CDSs
CDX EM.DIV	Emerging market CDSs (diversified)

Table C.1: Names and descriptions of index CDS included in our data sample. *Source*: Markit (2015).

To calibrate the correlation between contract returns and systematic risk, we employ a one-factor model, regressing CDS index returns on five-day S&P 500 log returns between 2006 to 2010,

$$CDX_{name,tenor,series,version,t} = \alpha + \beta SP_t + \varepsilon_{name,tenor,series,version,t}, \tag{C.1}$$

where $CDX_{name,tenor,series,version,t}$ is the five-day CDS index log returns for different family names,

[35] We approximate the discrete returns r_{ij}^k in our model by using empirically calibrated log returns $\tilde{r}_{ij}^k$ (i.e., $\log(1 + r_{ij}^k) \approx \tilde{r}_{ij}^k$). The calibration, in particular the standard deviation and correlation of S&P 500 and index CDS returns, is robust to using either empirical discrete returns or log returns.

Statistic	N	Min	Pctl(25)	Median	Pctl(75)	Max	Mean	St. Dev.
S&P 500	1,021	−0.203	−0.013	0.002	0.015	0.175	−0.001	0.031
CDX (all)	590,706	−0.288	−0.002	0.0003	0.004	0.291	0.001	0.012
CDX (CDX.NA.HY)	131,945	−0.096	−0.004	0.002	0.010	0.095	0.003	0.015
CDX (CDX.NA.HY.B)	27,921	−0.090	−0.003	0.0005	0.005	0.146	0.002	0.013
CDX (CDX.NA.HY.BB)	19,474	−0.064	−0.003	0.0004	0.003	0.056	0.0005	0.009
CDX (CDX.NA.HY.HB)	38,254	−0.163	−0.005	0.002	0.011	0.215	0.005	0.024
CDX (CDX.NA.IG)	83,264	−0.288	−0.001	0.0001	0.002	0.291	0.0002	0.006
CDX (CDX.NA.IG.CONS)	29,007	−0.046	−0.001	0.000	0.001	0.027	−0.0001	0.005
CDX (CDX.NA.IG.ENRG)	29,007	−0.039	−0.001	−0.00001	0.001	0.032	−0.00003	0.004
CDX (CDX.NA.IG.FIN)	47,653	−0.095	−0.003	0.0003	0.005	0.045	0.0003	0.011
CDX (CDX.NA.IG.TMT)	31,953	−0.056	−0.002	0.00001	0.002	0.078	0.0001	0.006
CDX (CDX.NA.IG.INDU)	35,790	−0.049	−0.002	0.0001	0.002	0.037	0.00002	0.005
CDX (CDX.NA.IG.HVOL)	56,996	−0.073	−0.002	0.0001	0.002	0.048	0.0001	0.008
CDX (CDX.NA.XO)	30,508	−0.081	−0.005	0.001	0.006	0.067	0.001	0.012
CDX (CDX.EM)	14,372	−0.180	−0.003	−0.00001	0.004	0.192	−0.0002	0.018
CDX (CDX.EM.DIV)	14,562	−0.144	−0.002	0.0002	0.003	0.149	0.0002	0.014

Table C.2: Descriptive statistics of five-day log returns of index CDS and the S&P 500.
The statistics are based on date-tenor-series-version observations for different index CDS families (see Table C.1 for descriptions), all family-date-tenor-series-version observations for CDS (all), and date observations for the S&P 500 from January 2006 to December 2009. *Source*: Markit.

	Dependent variable: five-day CDX return		
	Full	On-the-run	Off-the-run
	(1)	(2)	(3)
S&P 500	0.148	0.235	0.148
	t = 370.284***	t = 23.845***	t = 369.824***
Observations	590,706	856	589,850
R^2	0.188	0.400	0.188
Adjusted R^2	0.188	0.399	0.188
Residual Std. Error	0.011 (df = 590704)	0.007 (df = 854)	0.011 (df = 589848)
Implied correlation $\rho_{X,M}$	0.43	0.63	0.43
Note:			*p<0.1; **p<0.05; ***p<0.01

Table C.3: Calibration of the correlation of contract values with systematic risk.
OLS regression of five-day index CDS log returns on S&P 500 five-day returns between January 2006 and December 2009: $CDX_{name,t,tenor,series,version} = \alpha + \beta SP_t + \varepsilon_{name,t,tenor,series,version}$ for all index CDS at days t. The methodology is equivalent to estimating a single-factor model for an equally weighted basket of all index CDS. $\rho_{X,M}$ is the implied correlation coefficient between index CDS and S&P 500 returns. *Source:* Markit and own calculations.

tenors, series, and versions at day t, and SP_t is the five-day S&P 500 log return at day t. The estimated OLS coefficients are in Table C.3. The implied correlation between CDS and S&P 500 returns roughly equals $\rho_{X,M} = 0.43$, which we use as a baseline calibration. It is larger for indices that are on-the-run (0.63) and slightly smaller for indices that are off-the-run (0.4).[36] The methodology is equivalent to estimating the correlation between an equally weighted basket of index CDS and the S&P 500. We do not allow for different factor loadings β for different indices, since we are interested in only one parameter for the correlation $\rho_{X,M}$. The level of correlation is similar when estimating the single-factor model for individual index CDS for the baseline period from 2006 to 2010 as well as for the period from 2010 to 2018, confirming the robustness of our estimate.[37]

Based on these empirical results, Table C.4 and C.5 describe the final calibration of our model.

Variable	**Value**	**Description**
γ	16	Number of counterparties
K	10	Number of derivative classes
σ_X	0.01	Total contract volatility
$\rho_{X,M}$	0.43	Correlation between contract value and systematic risk M
σ_M	0.03	volatility of the systematic risk factor
β	0.1433	Implied beta-factor contracts
σ	0.009	Implied idiosyncratic contract volatility
v	1	Initial market value
$\text{cor}\left(r_{ij}^k, r_{hl}^m\right)$	0.185	Implied pair-wise correlation of contracts

Table C.4: Baseline calibration. We assume the same calibration for each entity and derivative class.

[36] index CDS are frequently updated. The most recently updated index is called *on-the-run* and typically exhibits the highest liquidity. Older versions of the indices are called *off-the-run* and are often still traded but exhibit lower liquidity.

[37] Correlation estimates are available on request. The correlation can be substantially smaller for single reference entities, as these do not diversify across entities' idiosyncratic default risk. For example, the correlation of the S&P 500 with Wells Fargo's five-year tenor spreads is -0.06; with Goldman Sachs's, it is -0.12; with Deutsche Bank's, it is -0.1; with General Electric's, it is -0.18; with AIG's, it is -0.16, and with Metlife's, it is -0.42. The correlation is almost identical when using three-year tenors. Note that the negative sign of the correlation coefficient reflects the protection buyer's perspective in spreads, while we account for the difference between buyer and seller with the sign of the contract size v. Thus, we use the absolute value of the correlation.

Variable	Value	Description
pd	0.1	Individual probability of default
$\rho_{A,A}$	0.25	Correlation of log assets conditional on M
σ_A	1	Log-asset value volatility
α_{uc}	0.99	Bilateral margin level
α_{CCP}	0.99	Multilateral (clearing) margin level

Table C.5: Baseline calibration of the default model. We assume the same calibration for each entity and derivative class.

D. Model for correlated defaults

In order to allow for correlation of entity defaults, in our sensitivity analysis we employ a credit model based on the Merton model (Merton (1974)). In particular, we assume that each counterparty i defaults at the start of the settlement period if the random value of its assets is below a given bankruptcy threshold, $A_i < B_i$.

The value of assets at the start of the settlement period is given by

$$A_i = \exp\left(\mu_{A_i} - \frac{\sigma_{A_i}^2}{2} + \sigma_{A_i} W_i\right), \tag{D.1}$$

where $(W_1, ..., W_\gamma)$ are jointly standard normally distributed and correlated with pairwise correlation ρ_{A_i,A_j}. The log value of assets is normally distributed with $\log A_i \sim \mathcal{N}\left(\mu_{A_i} - \frac{\sigma_{A_i}^2}{2}, \sigma_{A_i}^2\right)$. The pairwise correlation of two entities' log assets is given by

$$\tilde{\rho}_{A_i,A_j} = cor\left(\log A_i, \log A_j\right) = \frac{\sigma_{A_i}\sigma_{A_j}\rho_{A_i,A_j}}{\sigma_{A_i}\sigma_{A_j}}. \tag{D.2}$$

The individual (unconditional) default probability of entity i is given by

$$\pi_i = \mathbb{P}\left(A_i < B_i\right) = \Phi\left(\frac{\log B_i - \mu_{A_i} + \frac{\sigma_{A_i}^2}{2}}{\sigma_{A_i}}\right). \tag{D.3}$$

Without loss of generality, we assume that $\mu_{A_i} \equiv 0$. Then, the default intensity is given by $\bar{d}_i = \frac{\log B_i}{\sigma_{A_i}} + \frac{\sigma_{A_i}}{2}$. We define by $D = (D_1, ..., D_\gamma)$ a vector of binary random variables $D_i = \delta_{A_i < B_i}$ that signal the default of entity $i \in \{1, ..., \gamma\}$. The joint distribution of two entities' default state is determined by

$$\mathbb{P}\left(D_i = 1, D_j = 1\right) = \mathbb{P}\left(\bar{Z}_i < \bar{d}_i, \bar{Z}_j < \bar{d}_j\right) = \Phi_{2,\Sigma}(\bar{d}_i, \bar{d}_j), \tag{D.4}$$

where (Z_i, Z_j) are multi-normally distributed with zero mean, unit variance, and correlation matrix

Σ, with $\Sigma_{ij} = \rho$, $i \neq j$, and $\Sigma_{ii} = 1$, and

$$\mathbb{P}\left(D_i = 1, D_j = 0\right) = \mathbb{P}\left(Z_i < \bar{d}_i, Z_j \geq \bar{d}_j\right) = \mathbb{P}\left(Z_i < \bar{d}_i, -Z_j < -\bar{d}_j\right) \tag{D.5}$$

$$= \mathbb{P}\left(Z_i < \bar{d}_i, \tilde{Z}_j < -\bar{d}_j\right) = \Phi_{2,\tilde{\Sigma}}(\bar{d}_i, -\bar{d}_j) \tag{D.6}$$

where $(Z_i, \tilde{Z}_j)$ is multi-normally distributed, $(Z_i, \tilde{Z}_j) \sim \mathcal{N}_2(0, \tilde{\Sigma})$ with correlation matrix $\tilde{\Sigma}_{ij} = -\tilde{\rho}$, $i \neq j$ and $\tilde{\Sigma}_{ii} = 1$, $i, j \in \{1, 2\}$. Iteration yields the general distribution of default states as

$$\mathbb{P}(D = d) = \Phi_{\gamma,\tilde{\Sigma}}\left(\tilde{d}\right), \tag{D.7}$$

where $\tilde{d}_i = \begin{cases} \bar{d}_i, & d_i = 1 \\ -\bar{d}_i, & d_i = 0 \end{cases}$, $\tilde{\Sigma}_{ii} = 1$, and $\tilde{\Sigma}_{ij} = \begin{cases} \tilde{\rho}, & d_i = d_j \\ -\tilde{\rho}, & d_i \neq d_j \end{cases}$, $i \neq j$. Thus, $\tilde{\Sigma}$ has a unit diagonal and 4 blocks of $\tilde{\rho}$ and $-\tilde{\rho}$:

$$\tilde{\Sigma} = \begin{pmatrix} 1 & \tilde{\rho} & ... & \tilde{\rho} & -\tilde{\rho} & ... & ... & -\tilde{\rho} \\ \tilde{\rho} & 1 & \tilde{\rho} & \tilde{\rho} & -\tilde{\rho} & ... & ... & -\tilde{\rho} \\ & & \ddots & & -\tilde{\rho} & ... & ... & -\tilde{\rho} \\ \tilde{\rho} & ... & \tilde{\rho} & 1 & -\tilde{\rho} & ... & ... & -\tilde{\rho} \\ & & & & & & & \\ -\tilde{\rho} & ... & ... & -\tilde{\rho} & 1 & \tilde{\rho} & ... & \tilde{\rho} \\ -\tilde{\rho} & ... & ... & -\tilde{\rho} & \tilde{\rho} & 1 & ... & \tilde{\rho} \\ -\tilde{\rho} & ... & ... & -\tilde{\rho} & & & \ddots & \\ -\tilde{\rho} & ... & ... & -\tilde{\rho} & \tilde{\rho} & ... & \tilde{\rho} & 1 \end{pmatrix} \tag{D.8}$$

Assuming homogeneous counterparties (i.e., $\bar{d} \equiv \bar{d}_i$), the number of defaulting counterparties, $N_D = \sum_{i=1}^{\gamma} D_i$, is distributed as

$$\mathbb{P}\left(N_D = k\right) = \binom{\gamma}{k} \Phi_{\gamma,\tilde{\Sigma}}(\underbrace{\bar{d}, ..., \bar{d}}_{k}, \underbrace{-\bar{d}, ..., -\bar{d}}_{\gamma-k}), \tag{D.9}$$

where $\bar{d} > 0$ is the individual default intensity, and $\tilde{\Sigma}$ is defined as before.

As a benchmark case, consider independent defaults (i.e., $\tilde{\rho} = 0$). Then, the distribution of joint defaults is given by

$$\Phi_{\gamma,\tilde{\Sigma}}(\underbrace{\bar{d},...,\bar{d}}_{k},\underbrace{-\bar{d},...,-\bar{d}}_{\gamma-k}) = \Phi(\bar{d})^k\Phi(-\bar{d})^{\gamma-k} = \Phi(\bar{d})^k(1-\Phi(\bar{d}))^{\gamma-k}. \tag{D.10}$$

Thus, if defaults are independent, the number of defaults is binomially distributed, $N_D \sim Binom(\gamma, \Phi(\bar{d}))$. As Figure D.1 shows, increasing the correlation $\tilde{\rho}$ yields larger tails of the distribution of N_D. Then, it is more likely that counterparties default together (i.e., a large or small number of counterparties defaults).

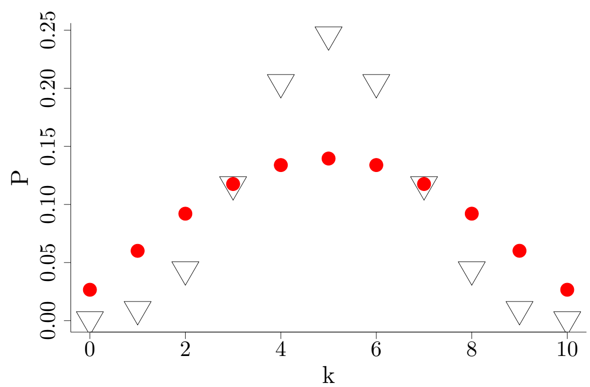


Fig. D.1. Probability distribution of the number of defaults, N_D, for $\gamma = 10$ entities and individual probability of default $\pi = 0.5$ if defaults are uncorrelated (triangles) or correlated with $\tilde{\rho} = 0.25$ (filled dots).

Figure D.1 depicts the distribution of N_D for exemplary parameters. Clearly, increasing the overall correlation $\tilde{\rho}$ yields larger tails of the distribution. Then it becomes more likely that entities default jointly.