Chapter 4

The Turbojet cycle

4.1 Thermal efficiency of the ideal turbojet

Recalling our discussion in Chapter 2, the thermal efficiency of a jet engine propulsion system is defined as

$$\eta_{th} = \frac{power\,to\,the\,vehicle + \frac{\Delta kinetic\,energy\,of\,air}{second} + \frac{\Delta kinetic\,energy\,of\,fuel}{second}}{\dot{m}_f h_f} \tag{4.1}$$

or

$$\eta_{th} = \frac{\mathrm{TU}_0 + \left[\frac{\dot{m}_a (U_e - U_0)^2}{2} - \frac{\dot{m}_a (0)^2}{2}\right] + \left[\frac{\dot{m}_f (U_e - U_0)^2}{2} - \frac{\dot{m}_f (U_0)^2}{2}\right]}{\dot{m}_f h_f}. \tag{4.2}$$

If the exhaust is fully expanded so that $P_e = P_0$ the thermal efficiency reduces to

$$\eta_{th} = \frac{\frac{\left(\dot{m}_a + \dot{m}_f\right) {U_e}^2}{2} - \frac{\dot{m}_a {U_0}^2}{2}}{\dot{m}_f h_f}. \tag{4.3}$$

For the ideal ramjet we were able to rearrange the thermal efficiency as follows.

$$\eta_{th} = \frac{\frac{(\dot{m}_a+\dot{m}_f)U_e{}^2}{2} - \frac{\dot{m}_a U_0{}^2}{2}}{\dot{m}_f h_f} = \frac{(\dot{m}_a + \dot{m}_f)(h_{te} - h_e) - \dot{m}_a (h_{t0} - h_0)}{(\dot{m}_a + \dot{m}_f) h_{te} - \dot{m}_a h_{t0}}$$

$$\eta_{th} = 1 - \frac{Q_{rejected\,during\,the\,cycle}}{Q_{input\,during\,the\,cycle}} = 1 - \frac{(\dot{m}_a + \dot{m}_f) h_e - \dot{m}_a h_0}{(\dot{m}_a + \dot{m}_f) h_{te} - \dot{m}_a h_{t0}} \tag{4.4}$$

$$\eta_{th} = 1 - \frac{T_0}{T_{t0}} \left(\frac{(1+f)\frac{T_e}{T_0} - 1}{(1+f)\frac{T_{te}}{T_{t0}} - 1} \right)$$

Noting that for the ideal ramjet $T_e/T_0 = T_{te}/T_{t0}$, the term in brackets is one, and the thermal efficiency of the ideal ramjet becomes

$$\eta_{th} = 1 - \frac{T_0}{T_{t0}} = 1 - \frac{1}{\tau_r} = \frac{\left(\frac{\gamma-1}{2}\right) M_0{}^2}{1 + \left(\frac{\gamma-1}{2}\right) M_0{}^2}. \tag{4.5}$$

The thermal efficiency of the ideal ramjet is entirely determined by the flight Mach number. As the Mach number goes to zero the thermal efficiency goes to zero and the engine produces no thrust.

To overcome this, we need an engine cycle that produces its own compression at zero Mach number. This is achieved through the use of a compressor driven by a turbine. A sketch of a turbojet engine is shown in Figure 4.1.

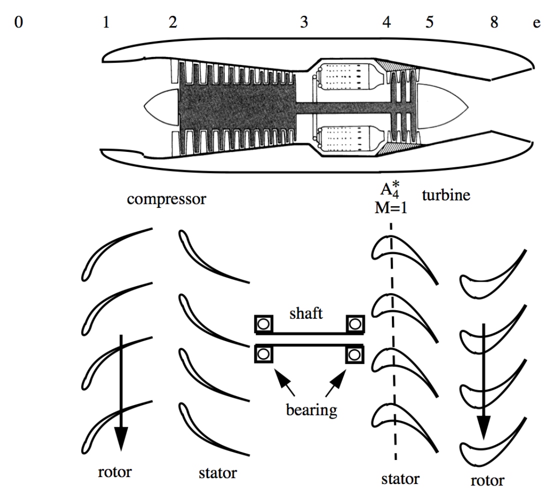

Figure 4.1: *Turbojet engine and compressor-turbine blade diagram.*

In an adiabatic system with no shaft bearing losses the work done by the gas on the turbine matches the work done by the compressor on the gas. This is expressed as a simple enthalpy balance.

$$(\dot{m}_a + \dot{m}_f)(h_{t4} - h_{t5}) = \dot{m}_a (h_{t3} - h_{t2}) \tag{4.6}$$

The enthalpy balance across the burner is

$$(\dot{m}_a + \dot{m}_f)\, h_{t4} = \dot{m}_a h_{t3} + \dot{m}_f h_f . \tag{4.7}$$

Subtract (4.6) from (4.7). The enthalpy balance across the engine is

$$(\dot{m}_a + \dot{m}_f)\, h_{t5} = \dot{m}_a h_{t2} + \dot{m}_f h_f . \tag{4.8}$$

Assume that the inlet and nozzle flow are adiabatic. Then (4.8) is equivalent to

$$(\dot{m}_a + \dot{m}_f)\, h_{te} = \dot{m}_a h_{t0} + \dot{m}_f h_f . \tag{4.9}$$

Now the thermal efficiency (4.3) can be written as

$$\eta_{th} = \frac{(\dot{m}_a + \dot{m}_f)(h_{te} - h_e) - \dot{m}_a (h_{t0} - h_0)}{(\dot{m}_a + \dot{m}_f)\, h_{t4} - \dot{m}_a h_{t3}} . \tag{4.10}$$

Using (4.9) and (4.7), equation (4.10) becomes

$$\eta_{th} = \frac{(\dot{m}_a + \dot{m}_f)\, h_{t4} - \dot{m}_a h_{t3} - (\dot{m}_a + \dot{m}_f)\, h_e + \dot{m}_a h_0}{(\dot{m}_a + \dot{m}_f)\, h_{t4} - \dot{m}_a h_{t3}} \tag{4.11}$$

or

$$\begin{aligned} \eta_{th} &= 1 - \frac{Q_{rejected\,during\,the\,cycle}}{Q_{input\,during\,the\,cycle}} = 1 - \left(\frac{(\dot{m}_a + \dot{m}_f)\, h_e - \dot{m}_a h_0}{(\dot{m}_a + \dot{m}_f)\, h_{t4} - \dot{m}_a h_{t3}} \right) \\ \eta_{th} &= 1 - \frac{h_0}{h_{t3}} \left(\frac{(1+f)\frac{h_e}{h_0} - 1}{(1+f)\frac{h_{t4}}{h_{t3}} - 1} \right) . \end{aligned} \tag{4.12}$$

If the gas is calorically perfect then (4.12) can be expressed in terms of the temperature.

$$\eta_{th} = 1 - \frac{T_0}{T_{t3}}\left(\frac{(1+f)\frac{T_e}{T_0} - 1}{(1+f)\frac{T_{t4}}{T_{t3}} - 1}\right) \tag{4.13}$$

In the ideal Brayton cycle the compression process from the free stream to station 3 is assumed to be adiabatic and isentropic. Similarly the expansion from station 4 to the exit is assumed to be isentropic. Thus

$$\begin{aligned} \frac{T_{t3}}{T_0} &= \left(\frac{P_{t3}}{P_0}\right)^{\frac{\gamma-1}{\gamma}} \\ \frac{T_{t4}}{T_e} &= \left(\frac{P_{t4}}{P_e}\right)^{\frac{\gamma-1}{\gamma}}. \end{aligned} \tag{4.14}$$

Also in the ideal Brayton cycle the heat adition and removal is assumed to occur at constant pressure. Therefore $P_{t4} = P_{t3}$, $P_e = P_0$ and we can write

$$\frac{T_{t4}}{T_e} = \frac{T_{t3}}{T_0}. \tag{4.15}$$

Therefore the expression in brackets in (4.13) is equal to one for the ideal turbojet cycle and the thermal efficiency is

$$\eta_{th_{idealturbojet}} = 1 - \frac{T_0}{T_{t3}} = 1 - \frac{1}{\tau_r \tau_c}. \tag{4.16}$$

When the Mach number is zero ($\tau_r = 1$), the thermal efficiency is positive and determined by the stagnation temperature ratio $\tau_c = T_{t3}/T_{t2}$ of the compressor.Thermodynamic diagrams of the turbojet cycle are shown in Figures 4.2 and 4.3.

The important impact of the compression process on thermal efficiency is a major factor behind the historical trend toward higher compression engines for both commercial and military applications.

4.2 Thrust of an ideal turbojet engine

The thrust equation for a fully expanded nozzle is

$$\frac{T}{P_0 A_0} = \gamma {M_0}^2 \left((1+f)\frac{U_e}{U_0} - 1\right). \tag{4.17}$$

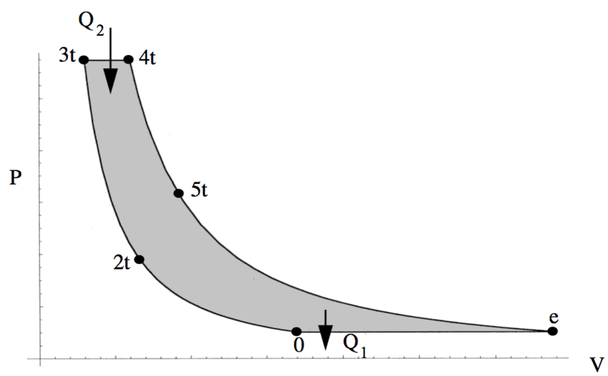

Figure 4.2: *P-V diagram of the ideal turbojet cycle. Station number with a "t" refers to the stagnation state of the gas at that point.*

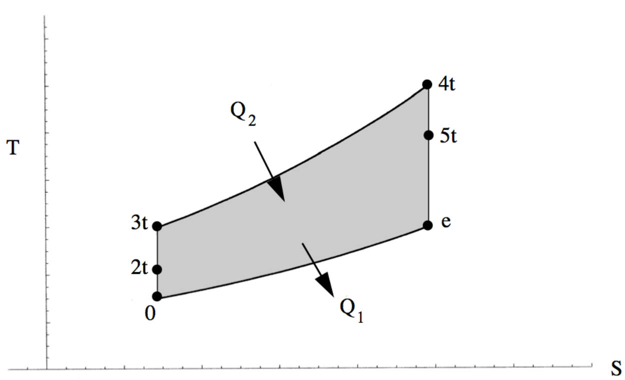

Figure 4.3: *T-S diagram of the ideal turbojet cycle.*

To determine the thrust we need to work out the velocity ratio.

$$\frac{U_e}{U_0} = \frac{M_e}{M_0}\sqrt{\frac{T_e}{T_0}} \tag{4.18}$$

To determine the Mach numbers we focus on the variation of stagnation pressure through the engine. Begin with the following identity.

$$P_{te} = P_0\left(\frac{P_{t0}}{P_0}\right)\left(\frac{P_{t2}}{P_{t0}}\right)\left(\frac{P_{t3}}{P_{t2}}\right)\left(\frac{P_{t4}}{P_{t3}}\right)\left(\frac{P_{t5}}{P_{t4}}\right)\left(\frac{P_{te}}{P_{t5}}\right) \tag{4.19}$$

Using our engine parameters this would be written as

$$P_{te} = P_0\pi_r\pi_d\pi_c\pi_b\pi_t\pi_n. \tag{4.20}$$

Under the assumptions of the ideal cycle the stagnation pressure losses in the diffuser and nozzle are negligible. In other words, skin friction and shock losses are negligible.

$$\begin{aligned}\pi_d &= 1\\ \pi_n &= 1\end{aligned} \tag{4.21}$$

Similarly, the Mach number through the burner is assumed to be so low that the stagnation pressure losses due to heat addition and aerodynamic drag are assumed to be negligible.

$$\pi_b = 1 \tag{4.22}$$

Therefore

$$P_{te} = P_0\pi_r\pi_c\pi_t = P_e\left(1+\frac{\gamma-1}{2}{M_e}^2\right)^{\frac{\gamma}{\gamma-1}}. \tag{4.23}$$

Another assumption of the ideal cycle is that the nozzle is fully expanded.

$$\pi_r\pi_c\pi_t = \left(1+\frac{\gamma-1}{2}{M_e}^2\right)^{\frac{\gamma}{\gamma-1}} \tag{4.24}$$

The final assumption of the ideal turbojet is that the compressor and turbine behave isentropically.

$$\begin{aligned} \pi_c &= \tau_c^{\frac{\gamma}{\gamma-1}} \\ \pi_t &= \tau_t^{\frac{\gamma}{\gamma-1}} \end{aligned} \tag{4.25}$$

Using (4.25) in (4.24) and the relation $\pi_r = \tau_r^{\gamma/(\gamma-1)}$, the exit Mach number can be determined from

$$M_e^2 = \frac{2}{\gamma-1}\left(\tau_r\tau_c\tau_t - 1\right) \tag{4.26}$$

and the Mach number ratio is

$$\frac{M_e^2}{M_0^2} = \left(\frac{\tau_r\tau_c\tau_t - 1}{\tau_r - 1}\right). \tag{4.27}$$

We take a similar approach to determining the temperature ratio across the engine. Begin with the identity

$$T_{te} = T_0\left(\frac{T_{t0}}{T_0}\right)\left(\frac{T_{t2}}{T_{t0}}\right)\left(\frac{T_{t3}}{T_{t2}}\right)\left(\frac{T_{t4}}{T_{t3}}\right)\left(\frac{T_{t5}}{T_{t4}}\right)\left(\frac{T_{te}}{T_{t5}}\right) \tag{4.28}$$

or, in terms of component temperature parameters

$$T_{te} = T_0\tau_r\tau_d\tau_c\tau_b\tau_t\tau_n. \tag{4.29}$$

In the ideal turbojet we assume that the diffuser and nozzle flows are adiabatic and so

$$T_{te} = T_0\tau_r\tau_c\tau_b\tau_t = T_e\left(1 + \frac{\gamma-1}{2}M_e^2\right) = T_e\tau_r\tau_c\tau_t. \tag{4.30}$$

From (4.30) we have the result

$$\frac{T_e}{T_0} = \tau_b = \frac{T_{t4}}{T_{t3}}. \tag{4.31}$$

This is the same result we deduced earlier in (4.15) when we analyzed the thermal efficiency. Actually it is more convenient to express the temperature ratio in terms of the all-important parameter $\tau_\lambda = T_{t4}/T_0$.

$$\frac{T_e}{T_0} = \frac{\tau_\lambda}{\tau_r \tau_c} \tag{4.32}$$

The reason is that τ_λ is a parameter that we would like to make as large as possible, but is limited by the highest temperature that can be tolerated by the turbine materials before they begin to lose strength and undergo creep. The maximum allowable turbine inlet temperature (and therefore the maximum design operating temperature) is one of the cycle variables that is essentially fixed when an engine manufacturer begins the development of a new engine. Enormous sums of money have been invested in turbine materials technology and turbine cooling schemes in an effort to enable jet engines to operate with as high a turbine inlet temperature as possible.

Our thrust formula is now

$$\frac{T}{P_0 A_0} = \frac{2\gamma}{\gamma - 1} \left(\tau_r - 1\right) \left(\left(1 + f\right) \left(\left(\frac{\tau_r \tau_c \tau_t - 1}{\tau_r - 1} \right) \frac{\tau_\lambda}{\tau_r \tau_c} \right)^{1/2} - 1 \right). \tag{4.33}$$

The fuel/air ratio is found from

$$f = \frac{\tau_\lambda - \tau_r \tau_c}{\tau_f - \tau_\lambda}. \tag{4.34}$$

At this point it would appear that for fixed γ, and τ_f the thrust is a function of four variables.

$$\frac{T}{P_0 A_0} = F\left(\tau_r, \tau_c, \tau_\lambda, \tau_t\right) \tag{4.35}$$

But the turbine and compressor are not independent components. They are connected by a shaft and the work done across the compressor is the same as the work done across the turbine. They are related by the work matching condition (4.6) repeated here in terms of the temperatures.

$$\left(\dot{m}_a + \dot{m}_f\right) C_P \left(T_{t4} - T_{t5}\right) = \dot{m}_a C_P \left(T_{t3} - T_{t2}\right) \tag{4.36}$$

For simplicity we have assumed the same value of C_p for the compressor and turbine. If we divide (4.36) by $C_p T_0$ then it becomes

$$(1+f)\,\tau_\lambda\,(1-\tau_t) = \tau_r\,(\tau_c - 1) \tag{4.37}$$

or

$$\tau_t = 1 - \frac{\tau_r\,(\tau_c - 1)}{(1+f)\,\tau_\lambda} \tag{4.38}$$

where we have assumed $T_{t2} = T_{t0}$. The result (4.38) only assumes adiabatic flow in the inlet and no shaft losses. It is not tied to the other assumptions of the ideal cycle. The velocity ratio across the engine is now

$$\left(\frac{U_e}{U_0}\right)^2 = \frac{1}{(\tau_r - 1)}\left(\tau_\lambda - \tau_r\,(\tau_c - 1) - \frac{\tau_\lambda}{\tau_r \tau_c}\right) \tag{4.39}$$

where the fuel/air ratio has been neglected.

4.3 Maximum thrust ideal turbojet

How much compression should we use? If there is too little the engine is like a ramjet and may not produce enough thrust at low Mach number. If we use too much then the fuel flow has to be reduced to avoid raising the temperature above the "do not exceed" (redline) value of T_{t4}. The thrust and specific impulse of a typical ideal turbojet is shown in Figure 4.4. Recall that

$$\frac{I_{sp}g}{a_0} = \left(\frac{1}{f}\right)\left(\frac{1}{\gamma M_0}\right)\left(\frac{T}{P_0 A_0}\right). \tag{4.40}$$

It is clear from the upper left graph in Figure 4.4 that there is a choice of τ_c that maximizes the thrust for fixed values of the other three engine parameters. We can determine this compression ratio by maximizing $(U_e/U_0)^2$.

$$\frac{\partial}{\partial \tau_c}\left(\frac{U_e}{U_0}\right)^2 = \frac{1}{(\tau_r - 1)}\left(-\tau_r + \frac{\tau_\lambda}{\tau_r {\tau_c}^2}\right) = 0 \tag{4.41}$$

The maximum velocity ratio for the ideal turbojet occurs when

$$\tau_{c_{\mathrm{maxthrust}}} = \frac{\sqrt{\tau_\lambda}}{\tau_r}. \tag{4.42}$$

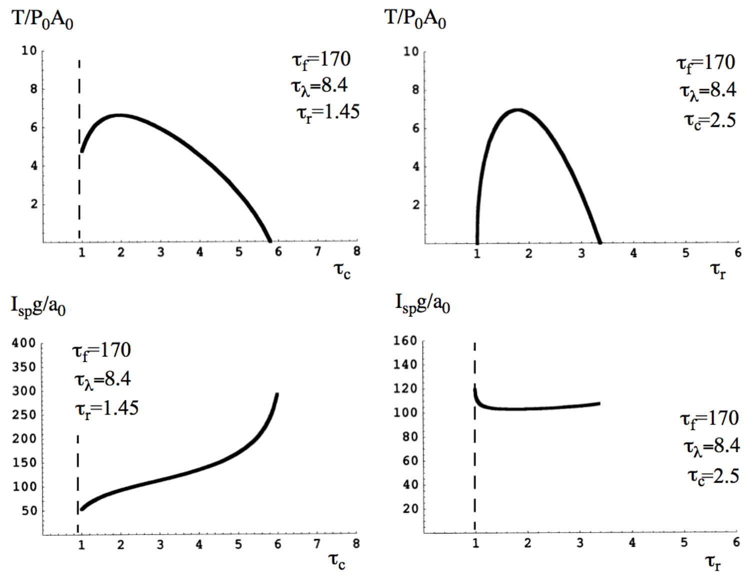

Figure 4.4: *Thrust and specific impulse curves for an ideal turbojet.*

Note that fuel shut-off and zero thrust occurs when $\tau_c = \tau_\lambda/\tau_r$. The relation (4.42) tells us a great deal about why engines look the way they do. An engine designed to cruise at low Mach number (a low value of τ_r) will be designed with a relatively large compressor generating a relatively high value of τ_c as indicated by (4.42). But as the flight Mach number increases the optimum compression decreases until at $\tau_r = \sqrt{\tau_\lambda}$ one would like to get rid of the compressor altogether and convert the engine to a ramjet. This also tells us something about the general trend of engine design with history. As higher temperature turbine materials and better cooling schemes have been developed over the years, newer engines tend to be designed with correspondingly higher compression ratios leading to higher specific impulse and better fuel efficiency. Over the 40 year period since the introduction of the JT9D, allowable turbine inlet temperatures have increased over $1000F$.

One should note that (4.42) is not a particularly useful relationship for the design of an actual engine. This is partly because, strictly speaking, it only applies to the ideal cycle but mostly due to the fact that any real engine must operate effectively from take-off to cruise. If the compressor is rigorously designed to satisfy (4.42) at cruise then the engine will be seriously underpowered and inefficient at take-off when the desired compression is much larger. On the other hand if the compressor is too large, then the engine will tend to be over-designed and over-weight for cruise.

As (4.42) would indicate, this problem becomes more and more difficult to solve as the cruise Mach number of the engine increases. The J58 powered SR71 Blackbird cruises at

Mach numbers greater than 3.

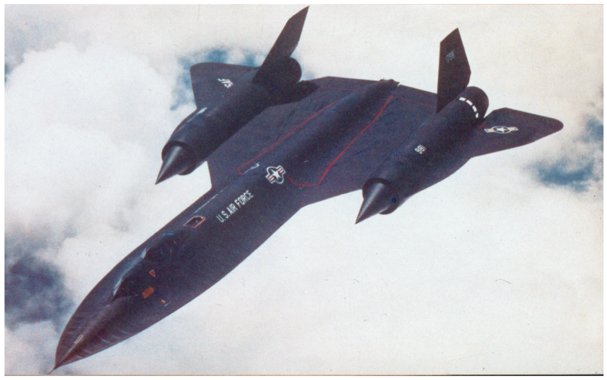

Figure 4.5: *The Mach 3+ SR71 Blackbird.*

The engine is designed to be a variable cycle system so that at cruise a large fraction of the inlet air bypasses the rotating machinery and enters the afterburner directly as it would in a ramjet. Despite this design the aircraft cannot take-off with a full load of fuel and has to be refueled in flight before beginning a mission. Another, unrelated reason for the partial fuel load at takeoff is that the braking power of the landing gear is too small for an emergency take-off abort with a full fuel load.

Designing a high Mach number engine for a supersonic transport faces the same problem. In an engine out situation the aircraft must be able to cruise subsonically to the nearest landing field and so the engine must be able to supply adequate thrust for long distances at subsonic Mach numbers.

4.4 Turbine-nozzle mass flow matching

The mass balance between the entrance of the turbine and the nozzle throat is

$$\begin{aligned} &\dot{m}_4 = \dot{m}_e \\ &\frac{P_{t4}A_4}{\sqrt{\gamma R T_{t4}}} f\left(M_4\right) = \frac{P_{t8}A_8}{\sqrt{\gamma R T_{t8}}} f\left(M_8\right). \end{aligned} \tag{4.43}$$

In general, the turbine is designed to provide a large pressure drop per stage. This is possible because of the favorable pressure gradient that stabilizes the boundary layers on the turbine airfoils. The result is a large amount of work per stage and this can be seen in the highly cambered, high lift shape of typical turbine airfoils shown in Figure 4.1. The large pressure drop across each stage implies that at some point near the entrance to the first stage turbine stator (also called the turbine nozzle) the flow is choked as indicated in Figure 4.1. At this point $A_4 f(M_4) = A_4^*$. The choked area occurs somewhere in the stator passage. Similarly for the vast range of practical engine operations the nozzle throat is also choked. Therefore the mass balance (4.43) can be written

$$\frac{P_{t4}A_4^*}{\sqrt{T_{t4}}} = \frac{P_{t8}A_8}{\sqrt{T_{t8}}}. \tag{4.44}$$

Under the assumption of an ideal cycle the turbine operates isentropically.

$$\frac{P_{t5}}{P_{t4}} = \left(\frac{T_{t5}}{T_{t4}}\right)^{\frac{\gamma}{\gamma-1}} \tag{4.45}$$

The skin friction losses in the nozzle duct are assumed to be negligible and the duct is adiabatic ($P_{t5} = P_{t8}$) and ($T_{t5} = T_{t8}$). Therefore

$$\tau_t = \frac{T_{t5}}{T_{t4}} = \left(\frac{A_4^*}{A_8}\right)^{\frac{2(\gamma-1)}{\gamma+1}}. \tag{4.46}$$

The temperature and pressure ratio across the turbine is determined entirely by the area ratio from the turbine inlet to the nozzle throat. As the fuel flow to the engine is increased or decreased with the areas fixed, the temperature drop across the turbine may increase or decrease changing the amount of work done while the temperature ratio remains constant. The turbine inlet and nozzle throat are choked over almost the entire practical range of engine operating conditions except during brief transients at start-up and shut-down.

4.5 Free-stream-compressor inlet flow matching

The mass balance between the free stream and the compressor face is

$$\begin{aligned} \dot{m}_a &= \dot{m}_2 \\ \frac{P_{t0}A_0}{\sqrt{T_{t0}}} f(M_0) &= \frac{P_{t2}A_2}{\sqrt{T_{t2}}} f(M_2). \end{aligned} \tag{4.47}$$

The flow from the free-stream to the compressor face is assumed to be adiabatic so that $T_{t2} = T_{t0}$. Thus the mass balance is

$$P_{t0}A_0 f(M_0) = P_{t2}A_2 f(M_2) \tag{4.48}$$

which we write as follows

$$f(M_2) = \frac{P_{t0}A_0 f(M_0)}{P_{t2}A_2}. \tag{4.49}$$

In terms of our engine parameters (4.49) is

$$f(M_2) = \left(\frac{1}{\pi_d}\right)\left(\frac{A_0}{A_2}\right) f(M_0). \tag{4.50}$$

We shall see that the fuel setting and nozzle throat area determine the value of $f(M_2)$ independently of what is happening in the free stream and inlet. In other words the engine demands a certain value of $f(M_2)$ and the gas dynamics of the inlet adjust A_0 and/or π_d in (4.50) to supply this value.

4.6 Compressor-turbine mass flow matching

The mass balance between the compressor face and the turbine inlet is

$$\begin{aligned} &\dot{m}_2(1+f) = \dot{m}_4 \\ &(1+f)\frac{P_{t2}A_2}{\sqrt{T_{t2}}} f(M_2) = \frac{P_{t4}A_4^*}{\sqrt{T_{t4}}}. \end{aligned} \tag{4.51}$$

We can write (4.51) in terms of our flow parameters as follows.

$$f(M_2) = \left(\frac{1}{1+f}\right)\frac{\pi_c\pi_b}{\sqrt{\tau_\lambda/\tau_r}}\left(\frac{A_4^*}{A_2}\right) \tag{4.52}$$

Under the ideal cycle assumption $\pi_b = 1$. Neglecting the fuel-air ratio, (4.52) becomes

$$f(M_2) = \frac{\pi_c}{\sqrt{\tau_\lambda/\tau_r}}\left(\frac{A_4^*}{A_2}\right). \tag{4.53}$$

Notice that we have written $f(M_2)$ on the left hand side of (4.53). In this point of view $f(M_2)$ is an outcome of the interaction of the nozzle with the turbine and compressor. The inlet behavior is then determined from (4.50).

4.7 Summary - engine matching conditions

In summary, the various component matching conditions needed to understand the operation of the turbojet in order, from the nozzle to the inlet are as follows.

$$\tau_t = \left(\frac{A_4^*}{A_8}\right)^{\frac{2(\gamma-1)}{\gamma+1}} \tag{4.54}$$

$$\tau_c - 1 = \frac{\tau_\lambda}{\tau_r}\left(1 - \tau_t\right) \tag{4.55}$$

$$f\left(M_2\right) = \frac{\pi_c}{\sqrt{\tau_\lambda/\tau_r}}\left(\frac{A_4^*}{A_2}\right) \tag{4.56}$$

$$f\left(M_2\right) = \left(\frac{1}{\pi_d}\right)\left(\frac{A_0}{A_2}\right) f\left(M_0\right) \tag{4.57}$$

The quantity A_0 in (4.57) is the area of the external stream tube of air captured by the engine. At first this seems like a vaguely defined quantity. In fact it is precisely determined by the engine pumping characteristics as we shall see shortly.

4.7.1 Example - turbojet in supersonic flow with an inlet shock

A turbojet operates supersonically at $M_0 = 3$ and $T_{t4} = 1944\,K$. The compressor and turbine polytropic efficiencies are $\eta_{pc} = \eta_{pt} = 1$. At the condition shown, the engine operates semi-ideally with $\pi_b = \pi_n = 1$ but $\pi_d \neq 1$ and with a simple convergent nozzle. The relevant areas are $A_1/A_2 = 2$, $A_2/A_4^* = 14$ and $A_e/A_4^* = 4$. Supersonic flow is established at the entrance to the inlet with a normal shock downstream of the inlet throat. This type of inlet operation is called supercritical and will be discussed further in a later section.

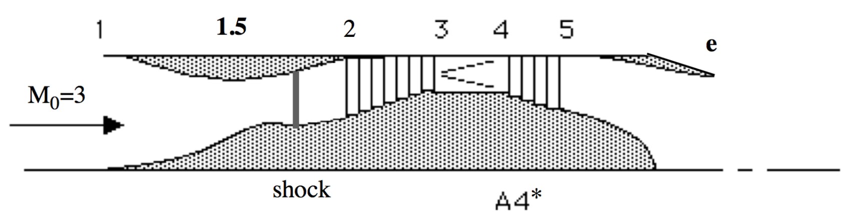

Figure 4.6: *Supersonic turbojet with inlet shock.*

1) Sketch the distribution of stagnation pressure, P_t/P_{t0} and stagnation temperature, T_t/T_{t0} through the engine. Assign numerical values at each station.

Solution - Note that $f(3) = 0.236$, $T_{t0} = 605\,K$ and

$$\frac{A_e}{A_1} = \left(\frac{A_e}{A_4^*}\right)\left(\frac{A_4^*}{A_2}\right)\left(\frac{A_2}{A_1}\right) = \frac{4}{14}\left(\frac{1}{2}\right) = 0.143. \tag{4.58}$$

We need to determine π_c, $f(M_2)$ and π_d. The analysis begins at the nozzle where the flow is choked. Choking at the turbine inlet and nozzle determines the turbine temperature and pressure ratio.

$$\tau_t = \left(\frac{A_4^*}{A_e}\right)^{\frac{2(\gamma-1)}{\gamma+1}} = \left(\frac{1}{4}\right)^{\frac{1}{3}} = 0.63 \tag{4.59}$$

$$\pi_t = \tau_t^{\frac{\gamma}{\gamma-1}} = 0.63^{3.5} = 0.198 \tag{4.60}$$

Matching turbine and compressor work gives the compressor temperature and pressure ratio.

$$\tau_c = 1 + \frac{\tau_\lambda}{\tau_r}\left(1-\tau_t\right) = 1 + \frac{1944}{605}\left(1-0.63\right) = 2.19 \tag{4.61}$$

$$\pi_c = \tau_c^{\frac{\gamma}{\gamma-1}} = 2.19^{3.5} = 15.54 \tag{4.62}$$

Now the Mach number at the compressor face is determined.

$$f(M_2) = \frac{A_4^*}{A_2}\left(\frac{\tau_r}{\tau_\lambda}\right)^{1/2}\pi_c\pi_b = \frac{A_4^*}{A_2}\left(\frac{605}{1944}\right)^{1/2}15.54 = \frac{A_4^*}{A_2}8.67 = \frac{8.67}{14} = 0.62 \tag{4.63}$$

Use free-stream-compressor-mass-flow matching to determine the stagnation pressure loss across the inlet.

$$\pi_d = \frac{A_0 f(M_0)}{A_2 f(M_2)} = 2 \times \frac{0.236}{0.62} = 0.76 \tag{4.64}$$

Now determine the stagnation pressure ratio across the engine.

$$\frac{P_{te}}{P_{t0}} = \pi_d\pi_c\pi_t = 0.76\,(15.54)\,(0.198) = 2.34 \tag{4.65}$$

Now the exit static pressure ratio is determined

$$\frac{P_e}{P_0} = \frac{P_{te}}{P_{t0}} \left(\frac{1 + \frac{\gamma-1}{2} {M_0}^2}{1 + \frac{\gamma-1}{2} {M_e}^2} \right)^{\frac{\gamma}{\gamma-1}} = 2.34 \left(\frac{2.8}{1.2} \right)^{3.5} = 45.4 \tag{4.66}$$

as is the stagnation temperature ratio,

$$\frac{T_{te}}{T_{t0}} = \frac{\tau_\lambda}{\tau_r} \tau_t = \frac{1944}{605} 0.63 = 2.02 \tag{4.67}$$

static temperature ratio,

$$\frac{T_e}{T_0} = \frac{T_{te}}{T_{t0}} \left(\frac{1 + \frac{\gamma-1}{2} {M_0}^2}{1 + \frac{\gamma-1}{2} {M_e}^2} \right) = 2.02 \left(\frac{2.8}{1.2} \right) = 4.71 \tag{4.68}$$

velocity ratio,

$$\frac{U_e}{U_0} = \frac{M_e}{M_0} \left(\frac{T_e}{T_0} \right)^{1/2} = \frac{1}{3} (4.71)^{1/2} = 0.723 \tag{4.69}$$

and thrust

$$\begin{gathered}\frac{T}{P_0 A_0} = \gamma {M_0}^2 \left(\frac{U_e}{U_0} - 1 \right) + \frac{A_e}{A_0} \left(\frac{P_e}{P_0} - 1 \right) \\ \frac{T}{P_0 A_0} = 1.4 \times 9 \times (0.723 - 1) + 0.143\,(45.4 - 1) = -3.49 + 6.35 = 2.86.\end{gathered} \tag{4.70}$$

At this point we have all the information we need (and then some) to answer the problem. The pressure ratios are

$$
\begin{aligned}
\frac{P_{t2}}{P_{t0}} &= \pi_d = 0.76 \\
\frac{P_{t3}}{P_{t0}} &= \pi_d \pi_c = 11.8 \\
\frac{P_{t4}}{P_{t0}} &= \pi_d \pi_c \pi_b = 11.8 \\
\frac{P_{te}}{P_{t0}} &= \pi_d \pi_c \pi_b \pi_t = 2.34
\end{aligned}
\tag{4.71}
$$

and the relevant temperature ratios are

$$
\begin{aligned}
\frac{T_{t2}}{T_{t0}} &= \tau_d = 1 \\
\frac{T_{t3}}{T_{t0}} &= \tau_d \tau_c = 2.19 \\
\frac{T_{t4}}{T_{t0}} &= \tau_d \tau_c \tau_b = \frac{1944}{605} = 3.21 \\
\frac{T_{te}}{T_{t0}} &= \tau_d \tau_c \tau_b \tau_t = 2.02.
\end{aligned}
\tag{4.72}
$$

The stagnation pressure and temperature ratios through the engine are sketched in Figure 4.7.

At this relatively high Mach number, the nozzle exit pressure is far higher than the ambient pressure. That suggests that it should be possible to increase the thrust by adding an expansion section to the nozzle. Let's see how much improvement might be possible. The thrust is

$$
\frac{T}{P_0 A_0} = \gamma {M_0}^2 \left(\frac{M_e}{M_0} \sqrt{\frac{T_e}{T_0}} - 1 \right) + \frac{A_e}{A_0} \left(\frac{P_e}{P_0} - 1 \right). \tag{4.73}
$$

Let's express (4.73) in terms of the nozzle exit Mach number assuming isentropic flow in the nozzle.

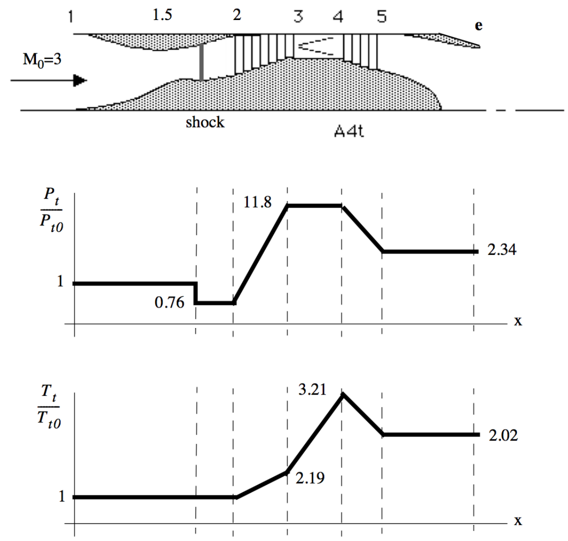

Figure 4.7: *Stagnation temperature and pressure through a supersonic turbojet.*

$$\frac{T}{P_0 A_0} = \gamma {M_0}^2 \left(\frac{M_e}{M_0} \sqrt{\frac{T_{te}}{T_0} \left(\frac{T_e}{T_{te}} \right)} - 1 \right) + \frac{A_8}{A_0} \frac{A_e}{A_8} \left(\frac{P_{te}}{P_0} \frac{P_e}{P_{te}} - 1 \right)$$

or

$$\frac{T}{P_0 A_0} (M_e) = \gamma {M_0}^2 \left(\frac{M_e}{M_0} \sqrt{\frac{T_{te}}{T_0} \left(\frac{1}{1 + \frac{\gamma - 1}{2} {M_e}^2} \right)} - 1 \right) + \frac{A_8}{A_0} \frac{1}{f(M_e)} \left(\frac{P_{te}}{P_0} \left(\frac{1}{1 + \frac{\gamma - 1}{2} {M_e}^2} \right)^{\frac{\gamma}{\gamma - 1}} - 1 \right) \tag{4.74}$$

The latter version of the thrust equation in (4.74) can be considered to be just a function of the Mach number for fixed stagnation pressure and temperature leaving the turbine. Equation (4.74) is plotted in Figure 4.8 for selected values of pressure, temperature, and area ratio.

At this flight Mach number the thrust can be nearly doubled using the nozzle. The maximum thrust occurs when the nozzle is fully expanded to $P_e = P_0$. The corresponding exit Mach number is $M_e = 3.585$ at a nozzle area ratio of $A_e/A_8 = 7.34$. The overall engine area ratio is $A_e/A_1 = (A_e/A_8)(A_8/A_0) = 1.05$ which suggests that the expansion could be added without much increase in the frontal area that the engine presents to the flow and therefore without much drag penalty.

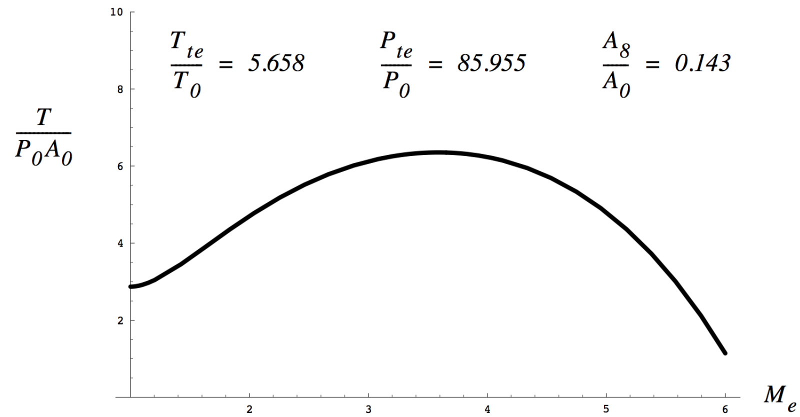

Figure 4.8: *Thrust variation with nozzle exit Mach number for example 4.7.1*

For an engine designed for a lower flight Mach number the performance gain by fully expanding the nozzle is relatively less. For a commercial engine designed to fly at subsonic Mach numbers the improvement is quite small and usually not worth the additional weight required to fully expand the nozzle.

4.8 How does a turbojet work?

The answer to this question lies in the various matching conditions that must be satisfied between engine components. These are mainly the requirements that the mass flow in and out of a component must be accommodated by the neighboring components and the work taken out of the flow by the turbine must equal the work done on the flow by the compressor. The analysis is simplified by the fact that under most practical operating conditions the nozzle and turbine inlet are choked. The rule of thumb, when trying to understand engine behavior, is to begin at the nozzle and work forward finishing with the inlet.

The total temperature ratio across the turbine is fixed by the turbine and nozzle choked areas. As a result, the turbine tends to operate at a single point. For example, if the pilot pushes the throttle forward, the turbine inlet temperature will go up and the temperature exiting the turbine will go up, but not as much. This leads to an increase in the temperature drop across the turbine. More work is taken out of the flow and the engine revs up while τ_t remains fixed. There are some variable cycle engine concepts that use a variable area turbine (VAT) to improve performance but the temperature and materials problems associated with movement of the turbine inlet vane make this very difficult to implement.

Many engines, especially military engines, designed to operate over a wide altitude and Mach number flight envelope, do incorporate variable area nozzles.

4.8.1 The compressor operating line

Now eliminate τ_λ/τ_r between (4.55) and (4.56) , using

$$\pi_c = \tau_c^{\frac{\gamma}{\gamma-1}}. \tag{4.75}$$

The result is

$$\frac{\pi_c}{\left(\pi_c^{\frac{\gamma-1}{\gamma}} - 1\right)^{1/2}} = \left(\frac{1}{1-\left(\frac{A_4^*}{A_8}\right)^{\frac{2(\gamma-1)}{\gamma+1}}}\right)^{1/2} \left(\frac{A_2}{A_4^*}\right) f\left(M_2\right). \tag{4.76}$$

Equation (4.76) defines the compressor operating line on a plot of π_c versus $f(M_2)$. Note that the denominator on the left hand side represents a relatively weak dependence on π_c except at unrealistically low values of π_c where the denominator can become singular. So, to a rough approximation (4.76) defines a nearly straight line relationship between π_c and $f(M_2)$. Equation (4.76) is sketched below.

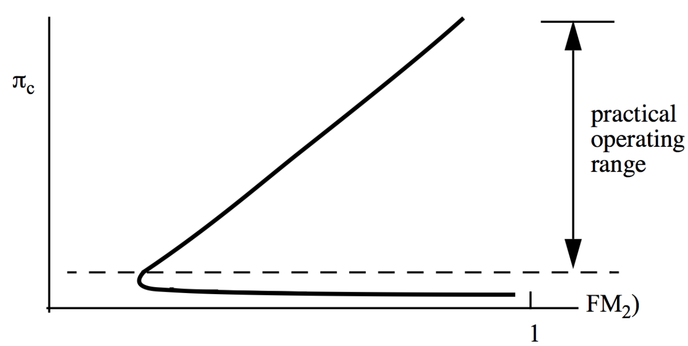

Figure 4.9: *Schematic of the compressor operating line Equation* (4.76).

Still missing from our understanding of turbojet operation is the relationship between the compressor temperature and pressure rise and the actual compressor speed. To determine this we will need to develop a model of the compressor aerodynamics.

4.8.2 The gas generator

The combination of compressor, burner and turbine shown below is called the gas generator.

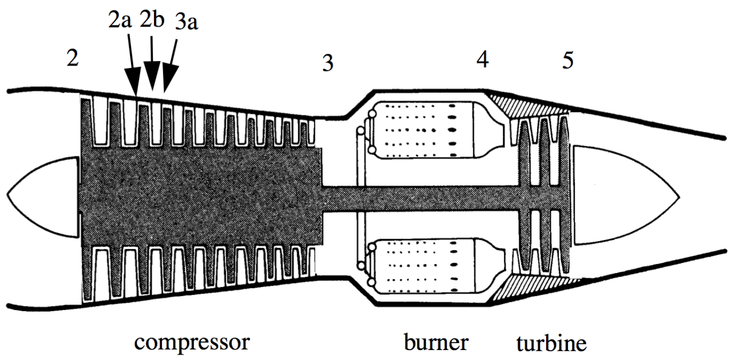

Figure 4.10: *Gas generator with imbedded station numbers.*

Compressor performance is characterized in terms of the compressor map which describes the functional relationship between compressor pressure ratio, mass flow and compressor speed. The compressor map from a J85 turbojet is shown below.

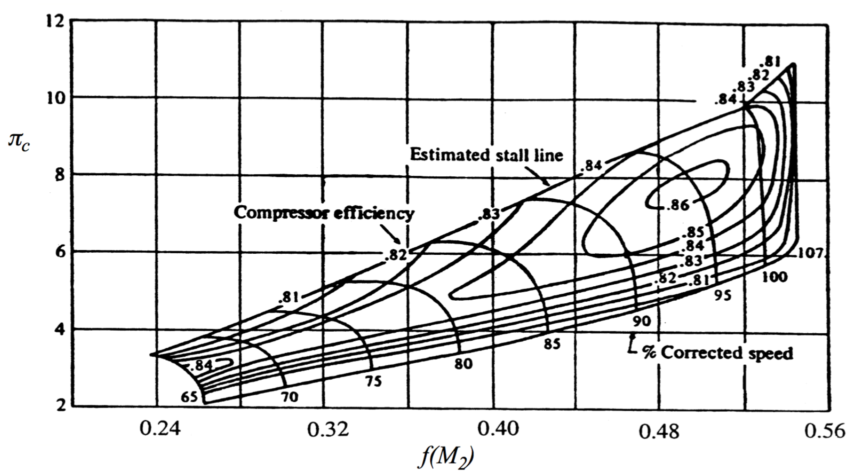

Figure 4.11: *Compressor map from a J-85 turbojet.*

In general the pressure ratio increases with increasing compressor rotation speed. At a given rotation speed the pressure ratio goes up as $f(M_2)$ is decreased. This latter behavior can be understood in terms of increasing relative angle of attack of the air flowing over the compressor blades leading to increased blade lift as the axial speed of the flow decreases.

More will be said on this point later.

4.8.3 Corrected weight flow is related to $f(M_2)$.

Industry practice is to correct the mass flow for the effects of altitude and flight speed. One defines the corrected weight flow as

$$\dot{w}_c = \dot{m}_a g \frac{\sqrt{\theta}}{\delta} \tag{4.77}$$

where

$$\begin{aligned} \theta &= \frac{T_{t2}}{T_{SL}} \\ \delta &= \frac{P_{t2}}{P_{SL}}. \end{aligned} \tag{4.78}$$

The quantities T_{SL} and P_{SL} refer to sea level standard pressure and temperature. In English units

$$\begin{aligned} T_{SL} &= 518.67R \\ P_{SL} &= 2116.22\text{pounds/ft}^2. \end{aligned} \tag{4.79}$$

The gas constant for air is

$$R_{air} = 1710.2\text{ft}^2/\left(\sec^2 - R\right). \tag{4.80}$$

We can write (4.77) as

$$\dot{w}_c = \dot{m} g \frac{\sqrt{\theta}}{\delta} = \left(\frac{1}{\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}} \frac{\gamma g P_{SL}}{\sqrt{\gamma R T_{SL}}} \right) A f(M). \tag{4.81}$$

Note that the quantity in parentheses is a constant. Thus the corrected mass flow is proportional to $f(M_2)$. At the compressor face

$$\dot{w}_c = 49.459 A_2 f\left(M_2\right) \text{pounds/sec} \tag{4.82}$$

where A_2 is expressed in terms of square feet. Throughout this course $f\left(M_2\right)$ will be the preferred measure of reduced mass flow through the compressor instead of the usual corrected weight flow. This quantity has several significant advantages. It is dimensionless, independent of compressor size and for practical purposes lies in a fairly narrow range of values that is more or less the same for all engines. The compressor entrance Mach number, M_2, is generally restricted to lie in the range between 0.2 and 0.6. Unusually low values of $f\left(M_2\right)$ imply that the engine diameter is too large. If $f\left(M_2\right)$ gets too large ($f\left(M_2\right)$ approaches one), the compressor blade passages begin to choke and stagnation pressure losses increase dramatically.

The compressor map can be regarded as a cross plot of three independent functions. The first is determined by the compressor-turbine inlet matching function (4.52). Rearranging variables (4.52) becomes

$$\pi_c = F_1\left(\frac{\tau_\lambda}{\tau_r}, f\left(M_2\right)\right) = \left(\frac{(1+f)}{\pi_b}\frac{A_2}{A_4^*}\right)\sqrt{\frac{\tau_\lambda}{\tau_r}} f\left(M_2\right) \tag{4.83}$$

where the contribution of the fuel/air ratio and burner pressure loss has been included. The factor in parentheses in (4.83) is approximately constant.

The second function relates the compressor efficiency to the pressure ratio and mass flow.

$$\eta_c = F_2\left(\pi_c, f\left(M_2\right)\right) \tag{4.84}$$

This is a function that can only be determined empirically through extensive compressor testing. The contours of constant efficiency in Figure 4.11 illustrate a typical case.

The third function relates the pressure ratio and mass flow to the rotational speed of the compressor. This function is of the form

$$\pi_c = F_3\left(\frac{M_{b0}}{\sqrt{\tau_r}}, f\left(M_2\right)\right) \tag{4.85}$$

where

$$M_{b0} = \frac{U_{blade}}{\sqrt{\gamma R T_0}} \tag{4.86}$$

is the compressor blade Mach number based on the free stream speed of sound and U_{blade} is the blade speed. Equation (4.85) is shown as lines of constant percent corrected speed in Figure 4.11.

4.8.4 A simple model of compressor blade aerodynamics

An accurate model of (4.85) can be derived from a detailed computation of the aerodynamics of the flow over the individual compressor blade elements. This is beyond the scope of this course but we can develop a simplified model of blade aerodynamics that reproduces the most important features of the relation between pressure ratio, mass flow and blade speed illustrated by the family of speed curves in Figure 4.11. Figure 4.12 shows the flow through a typical compressor stage called an imbedded stage. The stations labeled in Figure 4.12 are 2a (the space just ahead of the compressor rotor), 2b (the space between the rotor and stator) and 3a (the space after the stator and just ahead of the next rotor). The velocity vectors at various points in the stage are indicated in Figure 4.12. The vector relationships are

$$\begin{aligned} W_{2a} &= C_{2a} - U_{blade} \\ W_{2b} &= C_{2b} - U_{blade}. \end{aligned} \tag{4.87}$$

The axial velocity component is c_z. Tangential components are $c_{2a\theta}$ and $c_{2b\theta}$. Flow angles in non-moving coordinates are α_{2a}, α_{2b} and α_{3a}. Flow angles in moving coordinates are β_{2a}, β_{2b} and β_{3a}. The assumptions of the model are

$$\begin{aligned} &c_z \; is\, constant\, through\, the\, engine \\ &All\, stages\, are\, identical \\ &\alpha_{2a} = \alpha_{3a} \; and \; C_{2a} = C_{3a}. \end{aligned} \tag{4.88}$$

In addition, radial variations in the flow along the compressor blade elements are ignored (c_r is negligible). This is called a strip model of the compressor where the blades are approximated by an infinite 2-D cascade.

The basic aerodynamic principle utilized in this model is that the flow coming off the trailing edge of the compressor airfoils is guided by the wing surface and leaves the wing at the angle of the trailing edge. In contrast, the flow angle at the leading edge varies with the axial flow speed and blade speed while the airfoil lift varies accordingly as suggested in Figure 4.13.

When the airfoil is one element of a cascade the guiding effect of the trailing edge is enhanced. One of the design parameters of a compressor cascade is the solidity which

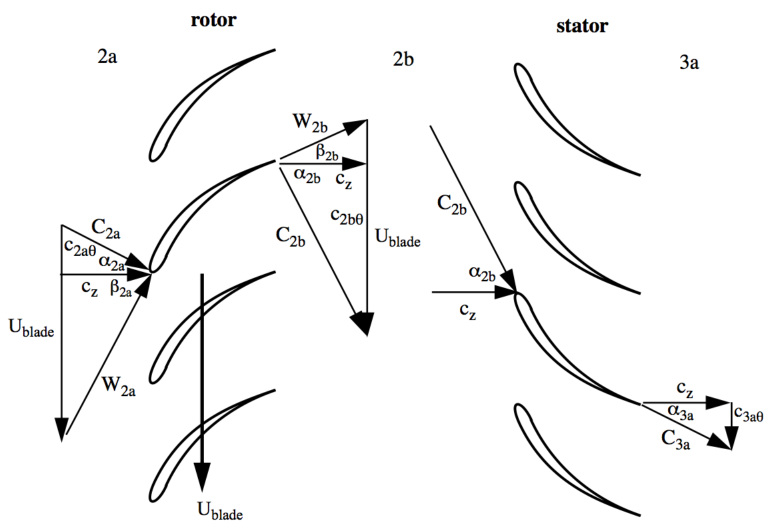

Figure 4.12: *Flow geometry in an imbedded stage.*

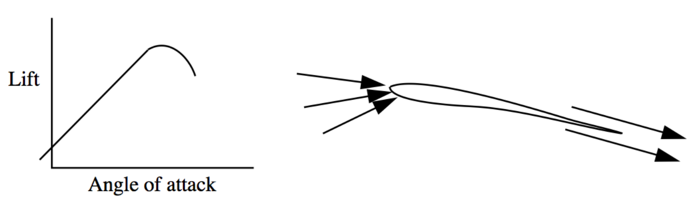

Figure 4.13: *Effect of angle of attack on airfoil lift.*

is defined as the blade chord divided by the vertical distance between compressor blade trailing edges. If the solidity is low (blades far apart) then the guiding effect of the cascade on the flow is reduced, the trailing edge flow is susceptible to stall (flow separation) and the work capability of the compressor is reduced. If the solidity gets too high then the drag losses of the compressor become excessive as does the compressor weight. A solidity of approximately one is fairly typical.

The tangential velocity components are

$$\begin{aligned} c_{2a\theta} &= c_z Tan\left(\alpha_{2a}\right) \\ c_{2b\theta} &= U_{blade} - c_z Tan\left(\beta_{2b}\right). \end{aligned} \tag{4.89}$$

The tangential velocity change of the flow induced by the tangential component of the lift force acting on the compressor blades is

$$\Delta c_\theta = c_{2b\theta} - c_{2a\theta} = U_{blade} - c_z Tan\left(\beta_{2b}\right) - c_z Tan\left(\alpha_{2a}\right). \tag{4.90}$$

Note that there is a considerable axial force component on the stage due to the pressure rise that the flow experiences as the stator removes the tangential velocity change. An energy balance on a control volume that encloses the rotor can be used to show that the work done across the rotor is

$$\dot{m}_a \left(h_{t2b} - h_{t2a}\right) = \bar{F} \cdot \bar{U}_{blade}. \tag{4.91}$$

In terms of the tangential velocity change

$$\dot{m}_a \left(h_{t2b} - h_{t2a}\right) = \dot{m}_a \Delta c_\theta U_{blade}. \tag{4.92}$$

This is a key equation that connects the work done across a cascade with the speed of the blade and the tangential velocity change. Note that all of the stage work is done by the rotor and so we can write

$$\left(h_{t3a} - h_{t2a}\right) = \Delta c_\theta U_{blade}. \tag{4.93}$$

Assume there are n identical stages. Then the enthalpy rise across the compressor is

$$\left(h_{t3} - h_{t2}\right) = n\left(\Delta c_\theta\right) U_{blade}. \tag{4.94}$$

Assume constant heat capacity and divide (4.94) by C_pT_0.

$$\tau_r\left(\tau_c-1\right)=n\left(\gamma-1\right)\frac{U_{blade}^2}{\gamma RT_0}\left(\frac{\Delta c_\theta}{U_{blade}}\right) \tag{4.95}$$

Solve for τ_c

$$\tau_c=1+n\left(\gamma-1\right)\left(\frac{M_{b0}}{\sqrt{\tau_r}}\right)^2\psi \tag{4.96}$$

where the stage load factor

$$\psi=\frac{\Delta c_\theta}{U_{blade}} \tag{4.97}$$

is introduced. The stage load factor compares the tangential velocity change of the flow across the rotor to the rotor speed. The upper limit of this parameter is about 1/4 and is a measure of the maximum pressure rise achievable in a stage. Equation (4.96) is expressed in terms of the basic compressor speed parameter introduced in (4.85). Now we need to express the stage load factor in terms of this speed parameter and $f\left(M\right)$. From (4.90).

$$\psi=1-\frac{c_z}{U_{blade}}\left(Tan\left(\beta_{2b}\right)+Tan\left(\alpha_{2a}\right)\right) \tag{4.98}$$

Equation (4.98) brings into play a second dimensionless velocity ratio, the flow coefficient which compares the flow axial speed to the blade speed.

$$\phi=\frac{c_z}{U_{blade}} \tag{4.99}$$

The basic aerodynamic design of the compressor boils down to two dimensionless velocity ratios, the flow coefficient and the stage load factor. Note that (4.98) is written in terms of the trailing edge flow angles. In our simple model these angles are assumed to be constant and so the stage load factor is a simple linear function of the flow coefficient. Now

$$\tau_c=1+n\left(\gamma-1\right)\left(\frac{M_{b0}}{\sqrt{\tau_r}}\right)^2\left(1-\phi\left(Tan\left(\beta_{2b}\right)+Tan\left(\alpha_{2a}\right)\right)\right). \tag{4.100}$$

At station 2 where the Mach number is relatively low $f\left(M_2\right)$ can be approximated by

$$f\left(M_2\right) \cong \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \frac{c_z}{\sqrt{\gamma R T_2}} \cong \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \frac{U_{blade}}{\sqrt{\gamma R T_2}} \left(\frac{c_z}{U_{blade}}\right). \tag{4.101}$$

So to a reasonable approximation

$$\phi = \frac{1}{\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}} \frac{f\left(M_2\right)}{\left(\frac{M_{b0}}{\sqrt{\tau_r}}\right)}. \tag{4.102}$$

Finally our aerodynamic model of the compressor is

$$\tau_c = 1 + n\left(\gamma-1\right)\left(\frac{M_{b0}}{\sqrt{\tau_r}}\right)^2 - \frac{n\left(\gamma-1\right)}{\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}} \left(Tan\left(\beta_{2b}\right) + Tan\left(\alpha_{2a}\right)\right)\left(\frac{M_{b0}}{\sqrt{\tau_r}}\right) f\left(M_2\right). \tag{4.103}$$

The pressure ratio is generated from (4.103) using $\pi_c = {\tau_c}^{\gamma(\gamma-1)}$. A polytropic efficiency of compression (defined below) can also be included. Figure 4.14 shows a cross plot of (4.83) and (4.103) for a typical case.

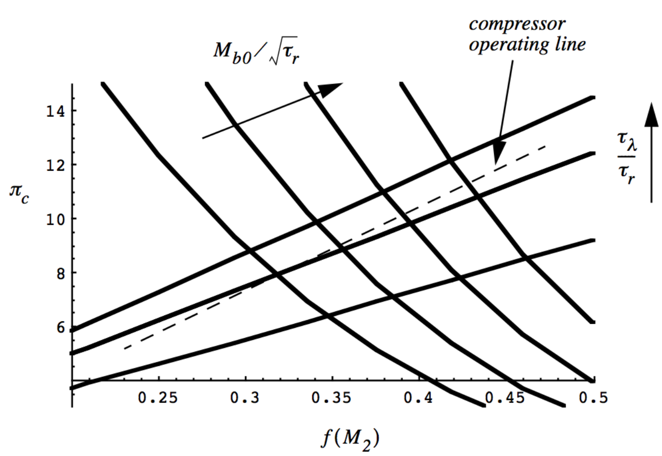

Figure 4.14: *Compressor map generated by the strip model.*

The model does a reasonable job of reproducing the inverse relationship between pressure ratio and blade speed although the curvature of the speed characteristics (lines of constant $M_{b0}/\sqrt{\tau_r}$) is opposite to that observed in Figure 4.9. This is because the model does not include viscous effects at all. Such effects as the deviation in trailing edge flow angle due to boundary layer thickening at high pressure ratio are not accounted for. At high pressure ratio and low flow speed the trailing edge flow eventually separates and the compressor stalls. This is indicated in Figure 4.11 as the estimated stall line. Modern engine control systems are designed to prevent the engine from stalling although some trailing edge separation can be tolerated. Substantial stall can lead to a condition called surge where large flow oscillations can do substantial damage to the engine. In the most extreme case the high pressure in the engine cannot be maintained and the internal gas may be released in an nearly explosive manner similar to the release from a burst pressure vessel with gas (and possibly engine parts) coming out of the inlet.

4.8.5 Turbojet engine control

The two main inputs to the control of the engine are:

1) The throttle, which we can regard as controlling T_{t4}, or equivalently at a fixed altitude, τ_λ and,

2) the nozzle throat area A_8.

The logic of the engine operation is as follows.

Case 1- Increase A_8 keeping τ_λ constant. Equation (4.54) determines τ_t which is used in (4.55) to determine τ_c. This determines π_c through (4.75) and $f(M_2)$ through (4.56). Given $f(M_2)$, the combination $(1/\pi_d)(A_0/A_2)$ is now known. This quantity completely specifies the inlet operation. The increase of A_8 leads to an increase in both $f(M_2)$ and π_c due to an increase in compressor speed as indicated on the compressor map. The compressor operating point moves along a constant τ_λ/τ_r characteristic.

Case 2 - Increase τ_λ keeping A_8 constant. The logic in this case is very similar to case 1 except that the compressor-turbine work matching condition, has $\tau_t = constant$. This determines π_c through (4.75) and $f(M_2)$ through the (in this case fixed) compressor operating line (4.76). Given $f(M_2)$ then $(1/\pi_d)(A_0/A_2)$ is known and the inlet operation is defined. As in case 1 the change of $f(M_2)$ and π_c is achieved by an increase in compressor speed according to the compressor map. The compressor operating point moves along the operating line (4.76) which crosses the constant τ_λ/τ_r characteristics as shown in Figure 4.12.

4.8.6 Inlet operation

There are two main points to take away from the previous discussion of engine control. The first is that to understand engine operation one begins at the nozzle and works forward. The other is that the inlet flow is essentially defined by the engine operating point through the value of $f(M_2)$. In effect, the engine sets the back pressure for the inlet. This is the fundamental purpose of the inlet; to provide the air mass flow to the engine at the Mach number dictated by the engine operating point with as small a stagnation pressure loss as possible. This whole mechanism is referred to as the pumping characteristic of the engine.

Lets look at the various possible modes of inlet operation recalling the discussion of capture area in Chapter 2. Figure 4.15 depicts an engine in subsonic flow. Shown is the variation in inlet flow as the nozzle throat A_8 area is increased with τ_λ held constant.

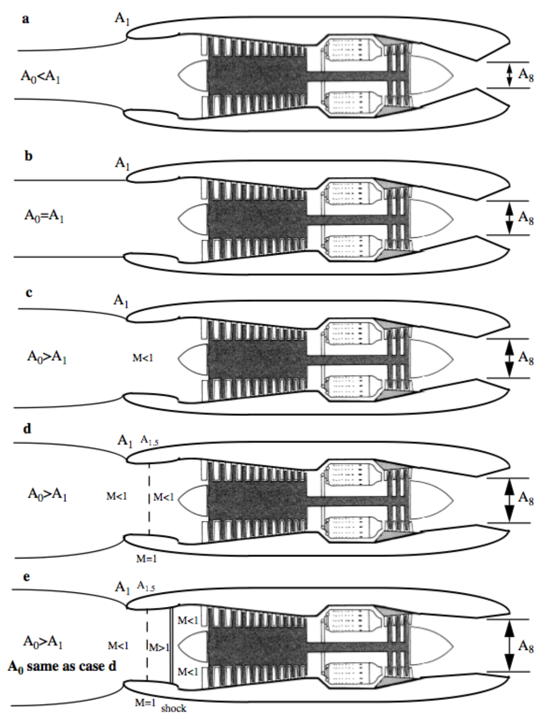

Figure 4.15: *Inlet behavior with increasing nozzle throat area in subsonic flow.*

The Mach number at station 2 entering the compressor increases from top to bottom in Figure 4.15. In the top four figures (a, b, c, d) there is no inlet shock and so, neglecting skin friction, the only way the increase in the Mach number at station 2 can be accommodated according to the matching condition (4.57) is for the capture area A_0 to increase leading to an increase in the air mass flow through the engine with $\pi_d = 1$.

As A_8 is increased further the inlet eventually chokes (this is the situation shown as case d). The condition for inlet choking is determined from the mass balance between the inlet throat $A_{1.5}$ and compressor face A_2.

$$\frac{P_{t2}A_2}{\sqrt{T_{t2}}}f\left(M_2\right)=\frac{P_{t1.5}A_{1.5}}{\sqrt{T_{t1.5}}}f\left(M_{1.5}\right) \tag{4.104}$$

Neglecting skin friction and heat transfer, the flow from $A_{1.5}$ to A_2 is adiabatic and isentropic. The mass balance (4.104) becomes

$$A_2 f\left(M_2\right)=A_{1.5} f\left(M_{1.5}\right). \tag{4.105}$$

The inlet chokes when $f\left(M_{1.5}\right)=1$. This occurs when

$$f\left(M_2\right)\big|_{inlet\,choking}=\frac{A_{1.5}}{A_2}. \tag{4.106}$$

If the nozzle area is increased beyond this point there is no change in A_0, the air mass flow remains fixed and a shock wave forms downstream of the inlet throat (this is depicted as case e in Figure 4.15). The matching condition (4.57) is satisfied by increasing stagnation pressure loss across the shock ($\pi_d < 1$). The shock becomes stronger as the Mach number at the compressor face is further increased. This whole mechanism is referred to as the pumping characteristic of the engine. Once a shock begins to form in the inlet, the engine performance (thrust and efficiency) begins to drop off rather rapidly. A well designed system is designed to avoid shock formation.

In supersonic flow the inlet is routinely designed to accommodate an inlet shock and/or a system of external shocks that may be needed to decelerate a high Mach number flow to the subsonic value at the compressor face dictated by the engine pumping characteristics. The basic operation of the inlet throat in supersonic flow is similar to that shown in Figure 4.15. Stagnation pressure losses may include fixed losses due to the external shock system as well as variable losses, due to the movement of the inlet shock. The figure below depicts the effect of increasing A_8 on the inlet flow for an engine operating in a supersonic stream.

In case a the Mach number at station 2 is low enough so that the Mach number at station 1 is less than the Mach number behind the normal shock ahead of the inlet as determined by A_2/A_1. The inlet operation is said to be sub-critical and after the system of oblique and normal shocks over the center body, the flow into the inlet is all subsonic . The inlet pressure ratio π_d is less than one due to the oblique and normal shocks. As the nozzle is opened up, the air mass flow into the engine increases with π_d approximately constant (exactly constant if the inlet is planar as opposed to axisymmetric). When the Mach number at station 2 has increased to the point where the Mach number at station 1 is just slightly less than the Mach number behind the normal shock, the normal shock will

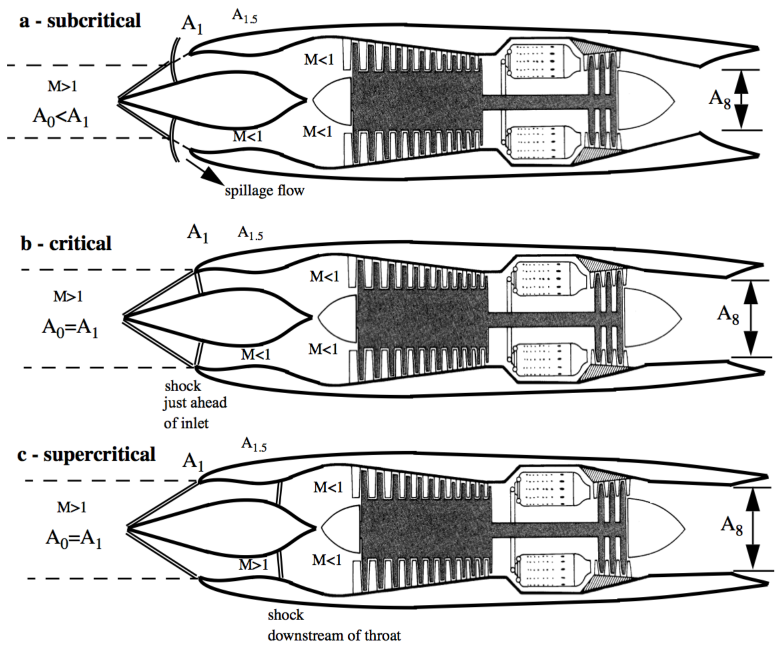

Figure 4.16: *Inlet behavior with increasing nozzle throat area in supersonic flow.*

be positioned just ahead of the inlet lip and the inlet operation is said to be critical. The Mach number between stations 1 and 2 is all subsonic.

Further increasing A_8 leads to *starting* of the inlet flow and shock formation downstream of $A_{1.5}$. If the nozzle is opened up still more the engine will demand increasing values of $f(M_2)$. In this case the mass flow through the inlet can no longer increase and the mass flow balance between the free stream and compressor face (4.57) is satisfied through decreasing values of π_d (supercritical operation) due to downstream movement of the shock to higher shock Mach numbers. Similar inlet behavior occurs with fuel throttling. The shadowgraph photos in Figure 4.17 illustrate sub and supercritical flow on an axisymmetric spike inlet. A final point to be made here is to remind ourselves of the artificial nature of the ideal turbojet cycle which assumes $\pi_d = 1$. The first step toward a more realistic supersonic engine is to allow the inlet the freedom to accommodate some stagnation pressure loss. Just as in the ramjet cycle the inlet plays a crucial role in the stable operation of the engine.

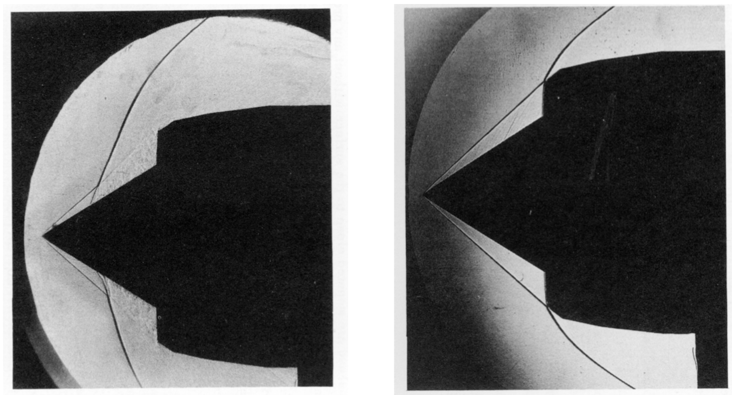

Figure 4.17: *Flow over a Mach 3 spike inlet, left photo subcritical behavior, right photo supercritical behavior.*

4.9 The non-ideal turbojet cycle

We have already studied one of the most important mechanisms for non-ideal behavior; namely the formation of an inlet shock.

Less than full expansion of the nozzle $P_e < P_0$ generally leads to less than maximum thrust. The loss in performance of a nozzle is a strong function of flight Mach number and becomes less important as the flight Mach number falls below one. Most military engines employ a converging-diverging nozzle for good supersonic performance whereas most commercial engines use a purely convergent nozzle for subsonic flight where the emphasis is on reducing weight and complexity. Most nozzle stagnation pressure losses are associated with viscous skin friction although some shock loss can occur at off design conditions. The nozzle operates in a strongly favorable pressure gradient environment and so stagnation pressure losses tend to be small and flow separation usually does not occur unless the nozzle becomes highly over-expanded. Flow separation can be an important issue in rocket nozzles when operating in the lower atmosphere but the problem is less severe in jet engines.

Stagnation pressure losses across the burner due to heat addition cause π_b to be always less than one. Additional reduction of π_b occurs due to wall friction and injector drag. Recall that the stagnation pressure loss due to heat addition and friction is proportional to , $\gamma M^2/2$. A rule of thumb is

$$\pi_b = 1 - \text{constant} \times \gamma \mathrm{M_3}^2 \tag{4.107}$$

where the constant is between one and two. In addition to the loss of stagnation pressure it is necessary to account for incomplete combustion as well as radiation and conduction of heat to the combustor walls. The combustor efficiency is defined directly from the energy balance across the burner.

$$\eta_b = \frac{(1+f)\,h_{t4} - h_{t3}}{f h_f} \tag{4.108}$$

The burner efficiency in a modern gas turbine engine is generally very close to one; Typically the efficiency is 0.99 or better.

The shaft that connects the turbine and compressor is subject to frictional losses in the bearings that support the shaft and a shaft mechanical efficiency is defined using the work balance across the compressor and turbine.

$$\eta_m = \frac{h_{t3} - h_{t2}}{(1+f)\,(h_{t4} - h_{t5})} \tag{4.109}$$

Typical shaft efficiencies are also very close to one.

4.9.1 The polytropic efficiency of compression

In the ideal turbojet cycle the compressor is assumed to operate isentropically. But this ignores the viscous frictional losses that are always present. An $h - s$ diagram illustrating the non-ideal operation of the compressor and turbine in an otherwise ideal turbojet cycle is shown in Figure 4.18.

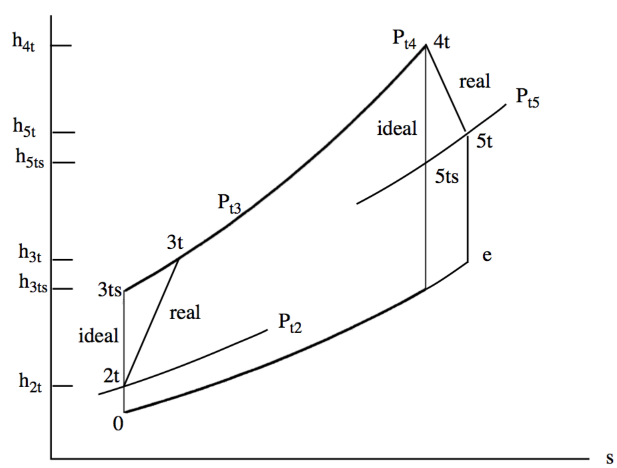

Figure 4.18: *Path of a turbojet in h-s coordinates with non-ideal compressor and turbine.*

The diagram shows the thermodynamic path of the gas flowing through a turbojet with ideal inlet, burner and nozzle but non-ideal compressor and turbine. As we consider the non-ideal cycle it is well to keep in mind that the engine is designed to produce thrust first and be efficient second. As an engine ages and various components begin to degrade, the engine control system is designed to increase fuel flow and the turbine inlet temperature so as to maintain the design thrust at the expense of efficiency.

A compressor is expected to reach the design pressure ratio regardless of its efficiency. and the same goes for the turbine. With this in mind Figure 4.18 suggests a reasonable definition of compressor and turbine efficiency

$$\begin{aligned}\eta_c &= \frac{\textit{The work input needed to reach } P_{t3}/P_{t2} \textit{ in an isentropic compression process}}{\textit{The work input needed to reach } P_{t3}/P_{t2} \textit{ in the real compression process}} \\ \eta_c &= \frac{h_{t3s} - h_{t2}}{h_{t3} - h_{t2}}\end{aligned} \tag{4.110}$$

and

$$\begin{aligned}\eta_e &= \frac{\textit{The work output needed to reach } P_{t5}/P_{t4} \textit{ in the real expansion process}}{\textit{The work output needed to reach } P_{t5}/P_{t4} \textit{ in an isentropic expansion process}} \\ \eta_e &= \frac{h_{t5} - h_{t4}}{h_{t5s} - h_{t4}}\end{aligned} \tag{4.111}$$

In terms of the temperature for a calorically perfect gas these definitions become

$$\begin{aligned}\eta_c &= \frac{T_{t3s} - T_{t2}}{T_{t3} - T_{t2}} \\ \eta_e &= \frac{T_{t5} - T_{t4}}{T_{t5s} - T_{t4}}.\end{aligned} \tag{4.112}$$

These definitions are useful but a little hard to interpret when comparing one compressor with another if the compression ratios are not the same. We can use the approach suggested by Figure 4.18 to define an efficiency that characterizes the compression process itself. Consider an infinitesimal compression process defined by the $T - s$ diagram shown in Figure 4.19.

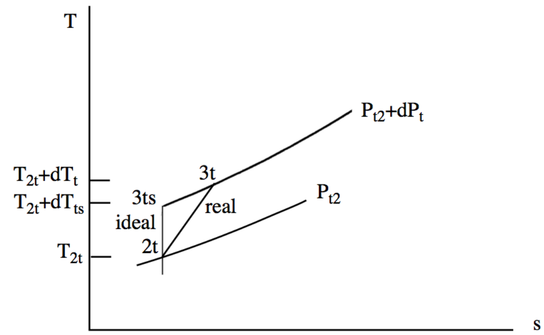

Figure 4.19: *Infinitesimal compression process.*

Define the polytropic efficiency of compression as the efficiency of an infinitesimal compression process.

$$\eta_{pc} = \left(\frac{T_{t3s} - T_{t2}}{T_{t3} - T_{t2}} \right)_{infinitesimal\,compression} = \frac{dT_{ts}}{dT_t} \tag{4.113}$$

For an isentropic process of a calorically perfect gas the Gibbs equation is

$$\frac{dT_{ts}}{T_t} = \left(\frac{\gamma - 1}{\gamma} \right) \frac{dP_t}{P_t}. \tag{4.114}$$

Using (4.113) the differential change in stagnation temperature for the real process is

$$\frac{dT_t}{T_t} = \left(\frac{\gamma - 1}{\gamma \eta_{pc}} \right) \frac{dP_t}{P_t}. \tag{4.115}$$

Now assume the polytropic efficiency is constant over the real finite compression from station 2 to station 3. Integrating (4.115) from 2 to 3 we get

$$\frac{P_{t3}}{P_{t2}} = \left(\frac{T_{t3}}{T_{t2}} \right)^{\frac{\gamma \eta_{pc}}{\gamma - 1}}. \tag{4.116}$$

The polytropic efficiency of compression allows us to analyze the flow through the compressor in terms of a relation that retains the simplicity of the isentropic relation. A lot of poorly understood physics is buried in the specification of η_{pc}. Modern compressors are designed to have values of η_{pc} in the range 0.88 to 0.92. Now the overall compressor efficiency becomes

$$\eta_c = \frac{\frac{T_{t3s}}{T_{t2}} - 1}{\frac{T_{t3}}{T_{t2}} - 1} = \frac{\left(\frac{P_{t3}}{P_{t2}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\left(\frac{P_{t3}}{P_{t2}}\right)^{\frac{\gamma-1}{\gamma\eta_{pc}}} - 1}. \tag{4.117}$$

Note that for pressure ratios close to one $\eta_c \cong \eta_{pc}$. The polytropic efficiency is a fundamental measure of the degree to which the compression process is isentropic. Given η_{pc} the overall compression efficiency is determined for any given pressure ratio.

4.10 The polytropic efficiency of expansion

Consider an infinitesimal expansion process defined by the $T-s$ diagram shown in Figure 4.20.

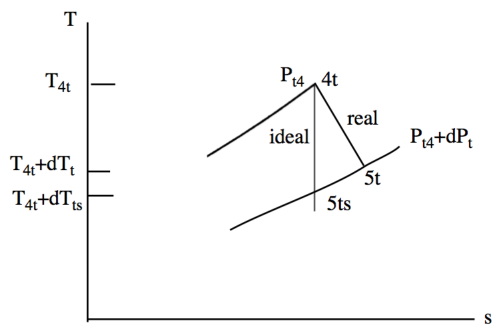

Figure 4.20: *Infinitesimal compression process.*

Define the polytropic efficiency of expansion as

$$\eta_{pe} = \left(\frac{T_{t5} - T_{t4}}{T_{t5s} - T_{t4}}\right)_{\text{infinitesimal expansion}} = \frac{dT_t}{dT_{ts}}. \tag{4.118}$$

For an isentropic process the Gibbs equation for an ideal, calorically perfect gas is

$$\frac{dT_{ts}}{T_t} = \left(\frac{\gamma - 1}{\gamma}\right)\frac{dP_t}{P_t}. \tag{4.119}$$

Using (4.118) the differential change in stagnation temperature for the real process is

$$\frac{dT_t}{T_t} = \left(\frac{(\gamma-1)\,\eta_{pe}}{\gamma}\right)\frac{dP_t}{P_t}. \tag{4.120}$$

Now if we assume the polytropic efficiency is constant over the real finite expansion from station 4 to station 5, then integrating (4.115) from 4 to 5

$$\frac{P_{t5}}{P_{t4}} = \left(\frac{T_{t5}}{T_{t4}}\right)^{\frac{\gamma}{(\gamma-1)\eta_{pe}}}. \tag{4.121}$$

The polytropic efficiency of expansion allows us to analyze the flow through the turbine in terms of a relation that retains the simplicity of the isentropic relation. Similar to the compression case, a lot of ignorance regarding the viscous turbulent flow through the turbine is buried in the specification of η_{pe}. Modern turbines are designed to values of η_{pe} in the range 0.91 to 0.94. The overall turbine efficiency is

$$\eta_e = \frac{\frac{T_{t5}}{T_{t4}} - 1}{\frac{T_{t5s}}{T_{t4}} - 1} = \frac{\left(\frac{P_{t5}}{P_{t4}}\right)^{\frac{(\gamma-1)\eta_{pe}}{\gamma}} - 1}{\left(\frac{P_{t5}}{P_{t4}}\right)^{\frac{\gamma-1}{\gamma}} - 1}. \tag{4.122}$$

Note that for pressure ratios close to one $\eta_e \cong \eta_{pe}$. Generally speaking turbine efficiencies are somewhat greater than compressor efficiencies because of the strongly favorable pressure gradient in the turbine.

4.11 The effect of afterburning

Figure 4.21 depicts a turbojet with an afterburner (also called an augmentor). The afterburner is a relatively simple device that includes a spray bar where fuel is injected and a flame holder designed to provide a low speed wake where combustion takes place. Note my drawing is not to scale. Usually the afterburner is considerably longer than the engine itself to permit complete mixing and combustion between the injected fuel and the vitiated air coming out of the turbine, before the flow reaches the exhaust nozzle.

The main effect of the afterburner is to add a lot of heat to the turbine exhaust gases while producing relatively little stagnation loss since the heat addition is at relatively low Mach number. The exhaust Mach number is determined by the nozzle area ratio and for the same exit Mach number the exit velocity is increased in proportion to the increase in the square root of exhaust temperature. In terms of engine parameters

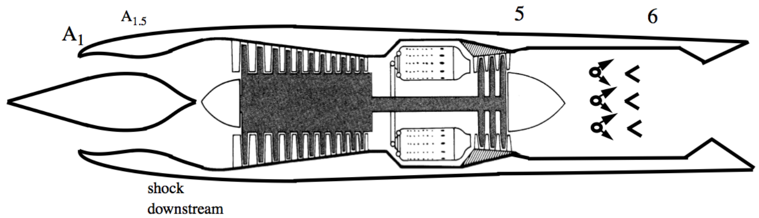

Figure 4.21: *Turbojet with afterburner.*

$$\pi_a = \frac{P_{t6}}{P_{t5}} \cong 1. \tag{4.123}$$

The fact that the nozzle area ratio is fixed (M_e is constant) and the stagnation pressure is the same, implies that the pressure contribution to the thrust is the same. The velocity ratio is

$$\frac{U_e}{U_0} = \frac{M_e}{M_0}\sqrt{\frac{T_e}{T_0}} = \frac{M_e}{M_0}\sqrt{\frac{1}{T_0}}\left(\frac{T_{te}}{1+\left(\frac{\gamma-1}{2}\right)M_e{}^2}\right)^{1/2}. \tag{4.124}$$

The exit stagnation temperature is

$$T_{te} = T_{t5}\left(\frac{T_{te}}{T_{t5}}\right) = T_{t5}\tau_a. \tag{4.125}$$

The bottom line is that $U_e \approx \sqrt{\tau_a}$. The afterburner provides a rapid increase in thrust on demand allowing the aircraft to respond quickly to changing mission circumstances; perhaps to escape a suddenly emerging threat. The price is a substantial increase in fuel burn rate. Most military engines only spend a few hundred hours in the after-burning mode over a typical engine lifetime of $3-4000$ hours before a major overhaul.

4.12 Nozzle operation

Commercial engines generally operate with fixed, purely convergent nozzles. There is a penalty for not fully expanding the flow but at low Mach numbers the performance loss is relatively small and the saving in weight and complexity is well worth it. For a commercial engine operating at $M_0 = 0.8$, π_n is on the order of 0.97 or better.

On the other hand military engines almost always employ some sort of variable area nozzle and in several modern systems the nozzle is also designed to be vectored. The most well known example is the planar nozzle of the F22. After-burning engines especially require a variable area nozzle. When the afterburner is turned on, and the exit gas temperature is increased according to (4.125), the nozzle throat area must be increased in a coordinated way to preserve the mass flow through the engine without putting an undue load on the turbine. Remember the exhaust nozzle is choked. With the augmentor on, the turbine temperature ratio is

$$\tau_t = \left(\frac{A_4^*}{A_8}\sqrt{\tau_a}\right)^{\frac{2(\gamma-1)}{\gamma+1}}. \tag{4.126}$$

In order to keep the turbine temperature ratio unchanged and the rest of the engine at the same operating point when the augmentor is turned on, it is necessary to program the nozzle area so that $\sqrt{\tau_a}/A_8$ remains constant. If this is not done the dimensionless mass flow through the engine will decrease, the actual mass flow may decrease and the desired thrust increment will not occur, or worse, the compressor might stall.

4.13 Problems

Problem 1 - Consider the turbojet engine shown in Figure 4.22.

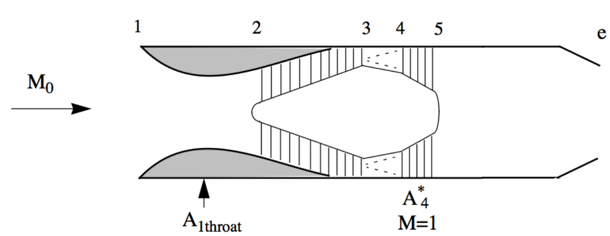

Figure 4.22: *Turbojet in subsonic flow.*

The engine operates at a free stream Mach number $M_0 = 0.8$. The ambient temperature is $T_0 = 216\,K$. The turbine inlet temperature is $T_{t4} = 1944\,K$ and $\pi_c = 20$. Relevant area ratios are $A_2/A_4^* = 10$ and $A_2/A_{1throat} = 1.2$. Assume the compressor, burner and turbine all operate ideally. The nozzle is of a simple converging type and stagnation pressure losses due to wall friction in the inlet and nozzle are negligible. Determine $f(M_2)$. Sketch the compressor operating line. Suppose T_{t4} is increased. What value of T_{t4} would cause the inlet to choke? Assume $f << 1$.

Problem 2 - A turbojet engine operates at a Mach number of 2.0 with a normal shock ahead of the inlet as shown in the sketch in Figure 4.23. The flow between the shock and station 2 is all subsonic. Assume $f << 1$ where appropriate and assume the static pressure outside the nozzle exit has recovered to the free stream value as indicated in the sketch. The ambient temperature and pressure are $T_0 = 216\,K$ and $P_0 = 2 \times 10^4\,N/m^2$.

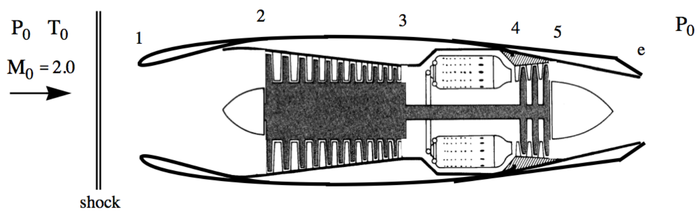

Figure 4.23: *Turbojet with upstream normal shock.*

The turbine inlet temperature is $T_{t4} = 1512\,K$, the compressor pressure ratio is $\pi_c = 20$ and $A_2/A_4^* = 18$. Assume the compressor, burner and turbine all operate ideally and stagnation pressure losses due to wall friction in the inlet and nozzle are negligible. Assume $f << 1$. Determine A_2/A_0, the pressure ratio P_e/P_0, temperature ratio T_e/T_0 and dimensionless thrust T/P_0A_0.

Problem 3 - Consider the turbojet engine shown in Figure 4.24.

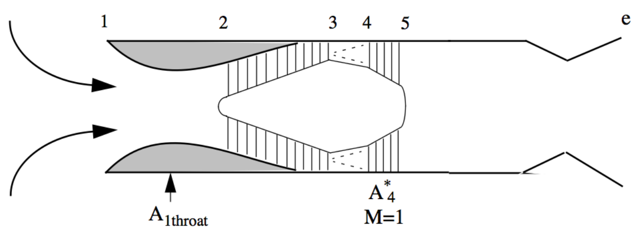

Figure 4.24: *Operating turbojet at rest.*

The engine operates at zero free stream Mach number $M_0 = 0$. The ambient temperature and pressure are $T_0 = 273K$ and $P_0 = 1.01 \times 10^5 N/m^2$. The turbine inlet temperature is $T_{t4} = 1638K$ and $\pi_c = 20$. Relevant area ratios are $A_2/A_4^* = 10$ and $A_2/A_{1throat} = 1.2$. Assume the compressor, burner and turbine all operate ideally. The nozzle is fully expanded $P_e = P_0$ and stagnation pressure losses due to wall friction in the inlet and nozzle are negligible. Assume $f << 1$. Determine the overall pressure ratio P_{te}/P_0 and dimensionless thrust T/P_0A_2.

Problem 4 - Figure 4.25 shows a typical turbojet engine flying supersonically. In Figure 4.26 are typical stagnation pressure and stagnation temperature ratios at various points inside the engine (the figures are not drawn to scale).

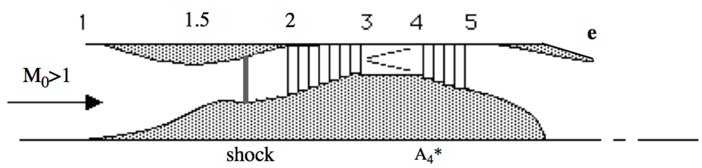

Figure 4.25: *Turbojet in supersonic flow.*

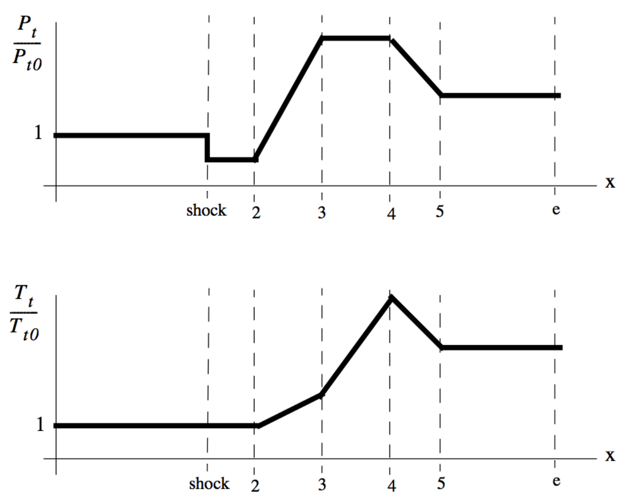

Figure 4.26: *Stagnation pressure and stagnation temperature through a turbojet engine with inlet shock.*

The turbine inlet and nozzle exit are choked, and the compressor, burner and turbine operate ideally. Supersonic flow is established in the inlet and a normal shock is positioned downstream of the inlet throat. The inlet and nozzle are adiabatic. Neglect wall friction and assume $f << 1$.

Suppose τ_λ is increased while the flight Mach number and engine areas including the nozzle throat area are constant.

1) Show whether P_{t3}/P_{t0} increases, decreases or remains the same.

2) At each of the stations indicated above explain how the stagnation pressure and stagnation temperature change in response to the increase in τ_λ.

Problem 5 - A turbojet operates supersonically at $M_0 = 2$ with $f(M_2) = 0.5$, $\pi_c = 20$ and $T_{t4} = 2160\,K$. The compressor and turbine polytropic efficiencies are $\eta_{pc} = \eta_{pt} = 1$. At the condition shown in Figure 4.27, the engine operates semi-ideally with $\pi_d = \pi_b = \pi_n = 1$ but with a simple convergent nozzle. The relevant inlet areas are $A_1/A_{1.5} = 1.688$ and $A_2/A_{1.5} = 2$. Assume $\gamma = 1.4$, $R = 287\,m^2/\left(\sec^2 - K\right)$, $C_p = 1005\,m^2/\left(\sec^2 - K\right)$. The fuel heating value is $h_f = 4.28 \times 10^7\,J/kg$. The ambient temperature and pressure are $T_0 = 216\,K$ and $P_0 = 2 \times 10^4\,N/m^2$. These are typical values in the atmosphere at an altitude of about $12,000$ meters.

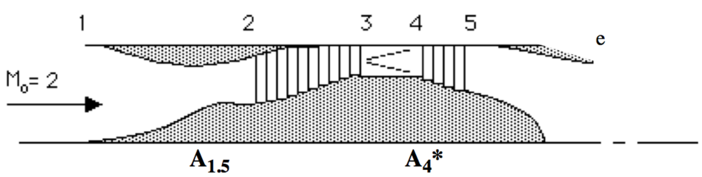

Figure 4.27: *Turbojet at Mach 2.0.*

Assume throughout that the fuel/air ratio is much less than one and that all areas of the engine structure remain fixed.

1) Determine A_2/A_4^* and A_4^*/A_e.

2) For each of the following three cases determine $f(M_2)$, $f(M_{1.5})$, $f(M_1)$, A_0/A_1 and T/P_0A_1.

Case I - First T_{t4} is slowly raised to $2376\,K$.

Case II - Then T_{t4} is reduced to $1944\,K$.

Case III - Finally T_{t4} is increased back to $2160\,K$.

Problem 6 - Consider the turbojet engine shown in Figure 4.28.

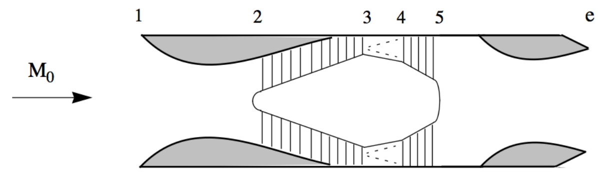

Figure 4.28: *Generic turbojet engine.*

The engine has a converging-diverging nozzle and operates at a free stream Mach number $M_0 = 0.8$. The turbine inlet temperature is $T_{t4} = 1800\,K$. Instead of me giving you a lot of information from which you can determine engine thrust, I would like to turn the question around and have you supply me with the information necessary to design the engine. Since this is a very preliminary design you may assume ideal behavior where appropriate. Note however that the polytropic efficiency of the compressor is $\eta_{pc} = 0.85$ and that of the turbine is $\eta_{pe} = 0.90$. Note also that my crude engine drawing is not to scale! After you finish the design, make a sketch for yourself that is more to scale.

The goal of the design is to produce as much thrust per unit area as possible for the given operating conditions. You are asked to supply the following.

1) The compressor pressure ratio, π_c.

2) The fuel/air ratio.

3)The compressor face mass flow parameter, $f(M_2)$.

4) All relevant area ratios, $A_{4_{throat}}/A_{5_{throat}}$, $A_{5_{throat}}/A_e$, $A_2/A_{4_{throat}}$, $A_0/A_{4_{throat}}$, $A_{1_{throat}}/A_2$, $A_{1_{throat}}/A_1$

5) The engine thrust, T/P_0A_0.

You may find that not every quantity that you are asked to supply is fixed by specifying the engine operating point. Where this is the case, you will need to use your experience to choose reasonable values. Be sure to explain your choices.

Assume $\gamma = 1.4$, $R = 287\,m^2/\left(\sec^2 - K\right)$, $C_p = 1005\,m^2/\left(\sec^2 - K\right)$. The fuel heating value is $h_f = 4.28 \times 10^7 J/kg$. The ambient temperature and pressure are $T_0 = 216\,K$ and $P_0 = 2 \times 10^4\,N/m^2$. These are typical values in the atmosphere at an altitude of about $12,000$ meters.

Problem 7 - A test facility designed to measure the mass flow and pressure characteristics of a jet engine compressor is shown in Figure 4.29. An electric motor is used to power the compressor. The facility draws air in from the surroundings which is at a pressure of one atmosphere and a temperature of $300\,K$. The air passes through the inlet throat at station

1, is compressed from 2 to 3 and then exhausted through a simple convergent nozzle at station e. Assume the compressor (2-3) has a polytropic efficiency of $\eta_{pc} = 0.95$.

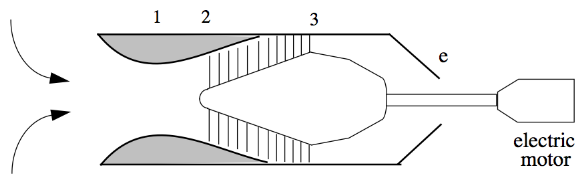

Figure 4.29: *A compressor test facility.*

Relevant area ratios of the rig are $A_1/A_e = 8$ and $A_1/A_2 = 0.5$. Suppose the power to the compressor is slowly increased from zero.

1) Determine the compressor pressure ratio P_{t3}/P_{t2} at which the nozzle chokes.

2) Determine the compressor pressure ratio P_{t3}/P_{t2} at which the inlet throat chokes.

3) Plot the overall pressure ratio P_{te}/P_0 versus the temperature ratio T_{te}/T_0 over the full range from less than sonic flow at e to beyond the point where a normal shock forms in the inlet.

Problem 8 - Because of their incredible reliability, surplus jet engines are sometimes used for power generation in remote locations. By de-rating the engine a bit and operating at lower than normal temperatures, the system can run twenty four hours a day for many years with little or no servicing. Figure 4.30 shows such an engine supplying shaft power P to an electric generator. Assume that there are no mechanical losses in the shaft.

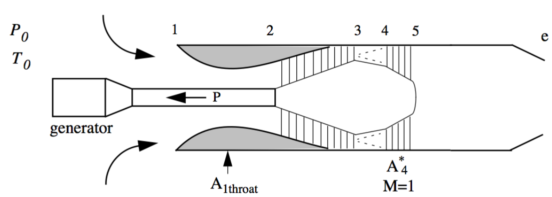

Figure 4.30: *A gas-turbine based power plant.*

The ambient temperature and pressure are $T_0 = 273\,K$ and $P_0 = 1.01 \times 10^5\,N/m^2$. The turbine inlet temperature is $T_{t4} = 1638\,K$ and $\pi_c = 20$. Relevant area ratios are $A_2/A_4^* = 15$ and $A_2/A_{1_{throat}} = 1.5$. Assume the compressor, burner and turbine all operate ideally. The nozzle is a simple convergent design and stagnation pressure losses due to wall friction in the inlet and nozzle are negligible. Assume $f << 1$. Let the nozzle area be set so that $P_{t5}/P_0 = 2$.

1) Is there a shock in the inlet?

2) How much dimensionless shaft power $P/(\dot{m}_a C_p T_0)$ is generated at this operating condition?

Problem 9 - Consider the afterburning turbojet shown in Figure 4.31. The inlet operates with a normal shock in the diverging section. The nozzle is of simple convergent type.

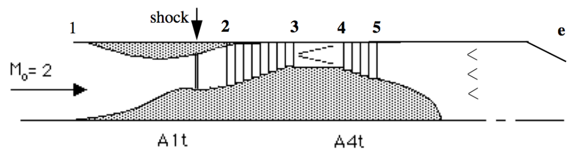

Figure 4.31: *Turbojet with afterburner.*

Initially the afterburner is off so that $P_{te} = P_{t5}$ and $T_{te} = T_{t5}$. At the condition shown the overall engine pressure ratio is $P_{te}/P_{t1} = 5.6$ and the temperature ratio is $T_{te}/T_{t1} = 3.1$.

1) Determine A_e/A_1.

2) Determine the thrust $T/P_0 A_0$.

3) Suppose the afterburner is turned on increasing the exit temperature to $T_{te}/T_{t5} = 1.5$. Assume that, as the afterburner is turned on, the nozzle area is increased so that P_{t5} remains constant thus avoiding any disturbance to the rest of the engine. Determine the new value of $T/P_0 A_0$. State any assumptions used to solve the problem.

Problem 10 - For this problem assume the properties of air are $\gamma = 1.4$, $R = 287\, m^2/\left(\sec^2 - K\right)$, $C_p = 1005\, m^2/\left(\sec^2 - K\right)$. Where appropriate assume $f << 1$. Figure 4.32 shows a flow facility used to test a small turbojet engine. The facility is designed to simulate various flight Mach numbers by setting the value of $P_{t0}/P_0 > 1$.

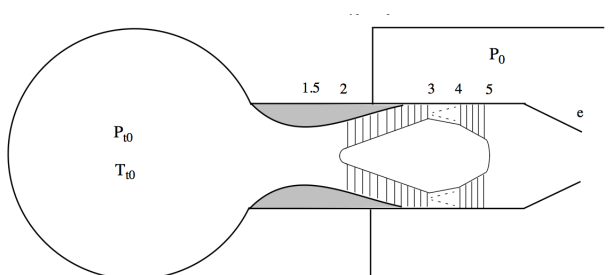

Figure 4.32: *A turbojet test facility.*

The relevant areas are $A_{4t}/A_e = 1/2$, $A_{1.5}/A_{4t} = 5$, $A_2/A_{1.5} = 2$. Assume that the compressor, burner and turbine operate ideally and that there is no stagnation pressure or stagnation temperature loss in the nozzle.

1)Let $T_{t4}/T_{t0} = 6$. Determine, π_c, π_d, $f(M_2)$ and the engine stagnation pressure ratio, $\pi_d \pi_c \pi_t$. Assuming $P_{t0}/P_0 > 1$ can you be certain that A_e is choked?

2) Let $P_{t0}/P_0 = 7.82$ and $T_{t4}/T_{t0} = 6$. What flight Mach number is being simulated at this facility pressure ratio? Determine P_e/P_0.

3) On the compressor map, (π_c versus $f(M_2)$ indicate the operating point for part 1. Sketch the change in engine operating point when A_e is increased with T_{t4}/T_{t0} held fixed.

Problem 11 - Consider an ideal turbojet with after-burning. Show that for a given total fuel flow, part to the main burner and part to the afterburner, the compressor temperature ratio for maximum thrust is

$$\tau_c\big|_{max\ thrust\ turbojet\ with\ afterburning} = \frac{1}{2}\left(1 + \frac{\tau_\lambda}{\tau_r}\right). \tag{4.127}$$

Recall that for a non-afterburning turbojet $\tau_c\big|_{max\ thrust\ turbojet} = \sqrt{\tau_\lambda}/\tau_r$. For typical values of τ_λ and τ_r the compressor pressure ratio with after-burning will be somewhat larger than that without after-burning.

Problem 12 - In the movie Top Gun there is depicted a fairly realistic sequence where two F-14s are engaged in a dogfight at subsonic Mach numbers with another aircraft. During a maneuver, one F-14 inadvertently flies through the hot wake of the other. This causes both engines of the trailing F-14 to experience compressor stall and subsequently flame-out leading to loss of the aircraft and crew. Can you explain what happened? Why might the sudden ingestion of hot air cause the compressor to stall?

Problem 13 - Figure 4.33 below depicts the flow across a compressor rotor. The axial speed is $c_z = 200\,m/\sec$ and the blade speed is $U_{blade} = 300\,m/\sec$. Relevant angles are $\alpha_{2a} = 30°$ and $\beta_{2b} = 30°$. Determine T_{t2_b}/T_{t2_a} where $T_{t2_a} = 260\,K$.

Problem 14 - Figure 4.34 depicts the flow across a compressor stage composed of two counter-rotating rotors.

The axial speed is $c_z = 200\,m/\sec$ and the blade speed is $U_{blade} = 300\,m/\sec$. Tangential velocities are $c_{2a\theta} = 50\,m/\sec$, $c_{2b\theta} = -50\,m/\sec$ and $c_{3b\theta} = 50\,m/\sec$.

1)!Determine T_{t3_b}/T_{t2_a} where $T_{t2_a} = 300\,K$.

2)Let the polytropic efficiency of compression be $\eta_{pc} = 0.85$. Determine P_{t3_b}/P_{t2_a}.

3) What benefits can you see in this design, what disadvantages?

Problem 15 - Figure 4.35 depicts the flow across a compressor rotor. The axial speed is $c_z = 150\,m/\sec$ and the blade speed is $U_{blade} = 250\,m/\sec$.

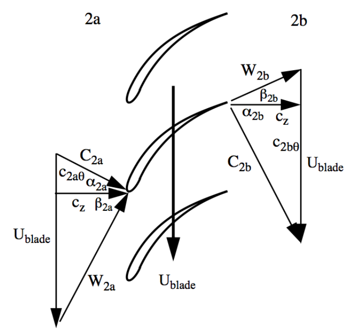

Figure 4.33: *Compressor rotor flow diagram.*

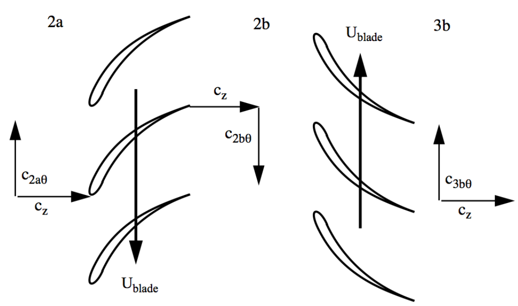

Figure 4.34: *Compressor stage flow diagram.*

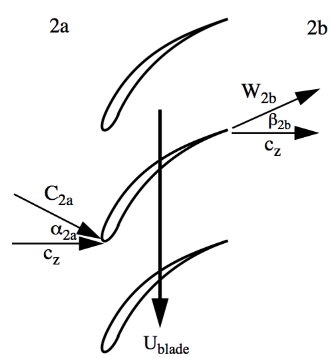

Figure 4.35: *Compressor rotor.*

Relevant angles are $\alpha_{2a} = 30^\circ$ and $\beta_{2b} = 30^\circ$.

1) Determine T_{t2_b}/T_{t2_a} where $T_{t2_a} = 350\,K$.

2) Let the polytopic efficiency of compression be $\eta_{pc} = 0.9$. Determine P_{t2_b}/P_{t2a}.

Problem 16 - Consider the turbojet engine shown in Figure 4.36.

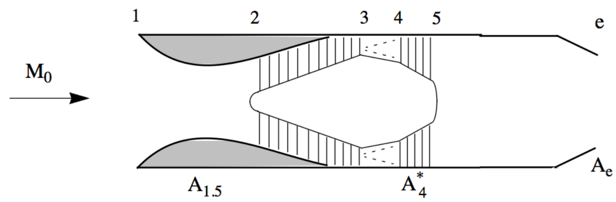

Figure 4.36: *Turbojet schematic.*

The engine operates at a free stream Mach number $M_0 = 0.8$. The turbine inlet temperature is $T_{t4} = 1296\,K$ and $\pi_c = 15$. The compressor face to turbine inlet area ratio is $A_2/A_4^* = 10$. Assume the compressor, and turbine operate ideally and there is no stagnation pressure loss across the burner. The nozzle is of a simple converging type and stagnation pressure losses due to wall friction in the inlet and nozzle are negligible. The nozzle throat area A_e can be varied. The ambient temperature and pressure are $T_0 = 216\,K$ and $P_0 = 2 \times 10^4\,N/m^2$.

1) Determine τ_t, A_4^*/A_e, and $f\,(M_2)$.

2) Now suppose that the compressor operates non-ideally with $\eta_{pc} = 0.8$. The turbine inlet temperature is kept at $T_{t4} = 1296\,K$. What value of A_4^*/A_e is required to maintain the same value of $\pi_c = 15$?

Problem 17 - An aircraft powered by a turbojet engine shown in Figure 4.37 is ready for take-off. The ambient temperature and pressure are $T_0 = 300\,K$ and $P_0 = 10^5\,N/m^2$. The turbine inlet temperature is $T_{t4} = 1500\,K$. The compressor pressure ratio is $\pi_c = 25$ and $A_2/A_4^* = 15$. The compressor polytropic efficiency is $\eta_{pc} = 0.85$ and the turbine polytropic efficiency is $\eta_{pe} = 0.9$. Assume that stagnation pressure losses in the inlet, burner and nozzle are negligible. Determine the pressure ratio P_e/P_0, temperature ratio T_e/T_0 and dimensionless thrust T/P_0A_2. The fuel heating value is $h_f = 4.28 \times 10^7\,J/kg$. The nozzle is of simple converging type as shown.

Problem 18 - Consider the turbojet engine shown in Figure 4.38.

The engine operates at a free stream Mach number $M_0 = 0.6$. The turbine inlet temperature is $T_{t4} = 1296\,K$ and $\pi_c = 15$. The compressor face to turbine inlet area ratio is $A_2/A_4^* = 10$ and $A_{1.5}/A_2 = 0.8$. Assume the compressor, burner and turbine operate

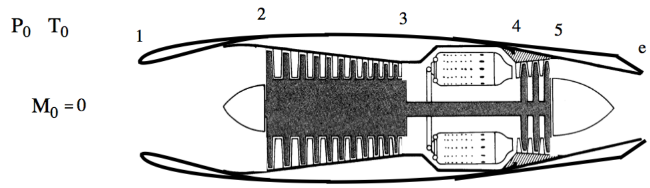

Figure 4.37: *Turbojet schematic.*

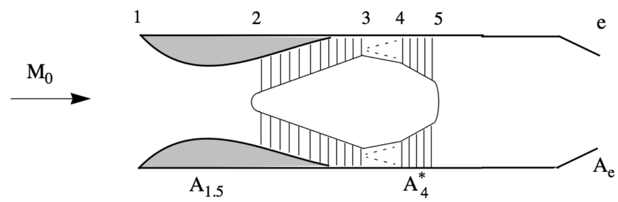

Figure 4.38: *Turbojet schematic.*

ideally. The nozzle is of a simple converging type and stagnation pressure losses due to wall friction in the inlet and nozzle are negligible.

1) Determine τ_t, A_4^*/A_e, and $f(M_2)$.

2) Suppose T_{t4} is increased. What value of T_{t4} would cause the inlet to choke?

Problem 19 - In the 1950s engine designers sought to decrease engine weight by increasing the compression achieved per stage of a jet engine compressor. In their endeavors they toyed with the idea of greatly increasing the relative Mach number of the flow entering the compressor to values exceeding Mach one. Thus was born the concept of a supersonically operating compressor and today the fans of most turbofan engines do in fact operate with blade tip Mach numbers that are greater than one. Axial Mach numbers still remain well below one. One of the most innovative design ideas during this period came from Arthur Kantrowitz of Cornell University. He conceived the idea of a *shock in rotor* compressor that could operate at Mach numbers considerably greater than one. The idea is illustrated in Figure 4.39.

Let $T_{2a} = 354\,K$, $U_{blade} = 800\,m/\sec$ and $C_{2a_z} = 800\,m/\sec$. The velocity vector entering the compressor is exactly aligned with the leading edge of the rotor blade as seen by an observer attached to the rotor. This is shown in the figure above. The tangential (swirl) velocity entering the stage is zero. The flow through the rotor passes through a $M = 2.0$ shock wave and then exits the rotor at the same angle it entered. In other words there is no turning of the flow by the blade in the frame of reference of the blade.

a) In a frame of reference attached to the rotor determine

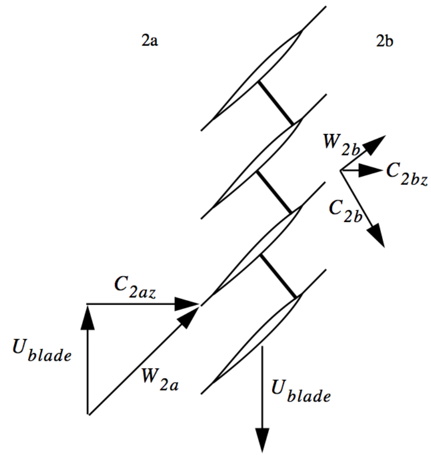

Figure 4.39: *Shock-in-rotor supersonic compressor.*

1) M_{2a}, P_{t2b}/P_{t2a} and M_{2b}

2) T_{t2b}/T_{t2a} and P_{t2b}/P_{t2a}.

b) In the frame of a non-moving observer determine

1) $T_{t2b_{rest\,frame}}/T_{t2a_{rest\,frame}}$

2) $P_{t2b_{rest\,frame}}/P_{t2a_{rest\,frame}}$.

c) Determine the polytropic efficiency of the compression process in the non-moving frame.