An example of using LaTeX and Gnuplot: the Euler Spiral

Christian Lange

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An Euler spiral is a curve whose curvature changes linearly with its curve length (the curvature of a circular curve is equal to the reciprocal of the radius). Euler spirals are also commonly referred to as 'spiros', 'clothoids', or 'Cornu' spirals.

Euler spirals have applications to diffraction computations. They are also widely used as transition curves in railroad engineering/highway engineering for connecting and transitioning the geometry between a tangentand a circular curve. A similar application is also found in photonic integrated circuits. The principle of linear variation of the curvature of the transition curve between a tangent and a circular curve defines the geometry of the Euler spiral:

- Its curvature begins with zero at the straight section (the tangent) and increases linearly with its curve length.
- Where the Euler spiral meets the circular curve, its curvature becomes equal to that of the latter.

If $a = 1$, which is the case for normalized Euler curve, then the Cartesian coordinates are given by Fresnel integrals (or Euler integrals):

$$C(L) = \int_0^L \cos(s^2)ds \tag{1}$$

$$S(L) = \int_0^L \sin(s^2)ds \tag{2}$$

This is found at: `https://en.wikipedia.org/wiki/Euler_spiral`.

In fig. 1 C(L) is plotted as the y-coordinate, and S(L) is plotted as the x-coordinate. Here $L \in [-10, 10]$ and with increments of $dL = 0.05$.

Figure 1: Illustration of the Euler Spiral