Introduction to Matrix Algebra

J. Alexander Branham

Scalars

- Let's start with something familiar, with a new word
- One number (12, for example) is referred to as a *scalar*
- This can be thought of as a 1x1 matrix
  - more on that in a bit...

$$\begin{bmatrix} 12 \end{bmatrix} = c$$

Vectors

We can put several scalars together to make a vector

Vectors

We can put several scalars together to make a vector For example

$$\begin{bmatrix} 12 \\ 14 \\ 15 \end{bmatrix}$$

Vectors

We can put several scalars together to make a vector For example

$$\begin{bmatrix} 12 \\ 14 \\ 15 \end{bmatrix} = b$$

Vectors

We can put several scalars together to make a vector For example

$$\begin{bmatrix} 12 \\ 14 \\ 15 \end{bmatrix} = b$$

Since this is a column of numbers, we cleverly refer to it as a *column vector*

Row vectors

If we take b and arrange it so that it it a row of numbers instead of a column, we refer to it as a *row vector*:

$$\begin{bmatrix} 12 & 14 & 15 \end{bmatrix} = d$$

Matrix

We can put multiple vectors together to get a *matrix*:

$$\begin{bmatrix} 12 & 14 & 15 \\ 115 & 22 & 127 \\ 193 & 29 & 219 \end{bmatrix} = A$$

Matrices, cntd

- We refer to the *dimensions* of matrices by *row* x *column*
- So A is a 3x3 matrix.
- Note that matrices are usually designated by capital letters (and sometimes bolded as well)

Dimensions

ROW x COLUMN

Matrix Indices

- But how do we refer to specific elements of the matrix?

Matrix Indices

- But how do we refer to specific elements of the matrix?
- Come up with a clever index!

Matrix Indices

- But how do we refer to specific elements of the matrix?
- Come up with a clever index!
- Matrix A is an mxn matrix where $m = n = 3$.

Matrix Indices

- But how do we refer to specific elements of the matrix?
- Come up with a clever index!
- Matrix A is an mxn matrix where $m = n = 3$.
- More generally, matrix B is an mxn matrix where the elements look like this:

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} & \dots & b_{1m} \\ b_{21} & b_{22} & b_{23} & \dots & b_{2m} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ b_{n1} & b_{n2} & b_{n3} & \dots & b_{nm} \end{bmatrix}$$

Addition and subtraction are EASY

- Requirement: Must have *exactly* the same dimensions
- To do the operation, just add or subtract each element with the corresponding element from the other matrix:

$$A \pm B$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \pm \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} \pm b_{11} & a_{12} \pm b_{12} & a_{13} \pm b_{13} \\ a_{21} \pm b_{21} & a_{22} \pm b_{22} & a_{23} \pm b_{23} \\ a_{31} \pm b_{31} & a_{32} \pm b_{32} & a_{33} \pm b_{33} \end{bmatrix}$$

Scalar Multiplication

Easy - just multiply each element of the matrix by the scalar

$$cA = \begin{bmatrix} ca_{11} & ca_{12} & ca_{13} \\ ca_{21} & ca_{22} & ca_{23} \\ ca_{31} & ca_{32} & ca_{33} \end{bmatrix}$$

Matrix Multiplication

- Requirement: the two matrices must be *conformable*
- This means that the number of columns in the first matrix equals the number of rows in the second
- The resulting matrix will have the number of rows in the first, and the number of columns in the second!

Question time

Which can we multiply? What will the resulting dimensions be?

$$b = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix} \; M = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 2 & 4 \\ 2 & 3 & 2 \end{bmatrix} \; L = \begin{bmatrix} 6 & 5 & -1 \\ 1 & 4 & 3 \end{bmatrix}$$

Question time

Which can we multiply? What will the resulting dimensions be?

$$b = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix} \; M = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 2 & 4 \\ 2 & 3 & 2 \end{bmatrix} \; L = \begin{bmatrix} 6 & 5 & -1 \\ 1 & 4 & 3 \end{bmatrix}$$

ONLY LM and NOT ML

The dimensions will be 2x3

How to actually do this?

Multiply each row by each column
(Board examples)

How to actually do this?

Multiply each row by each column
(Board examples)

```
L <- matrix(c(6,5,-1, 1,4,3),
            nrow=2, byrow=TRUE)
M <- matrix(c(1,0,2, 1,2,4, 2,3,2),
            nrow=3, byrow=TRUE)
L%*%M

##      [,1] [,2] [,3]
## [1,]    9    7   30
## [2,]   11   17   24
```