Set Theory & Combinations

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Intro to Set Theory

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- A branch of mathematics
- Collects objects into *sets* and studies the properties

What's a set?

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- The objects can be anything
- We usually use variables or units of observation

Elements in or not

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- Or not:

$$q_1 \notin S$$

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- We can also define non-proper subsets:

$$L \subseteq S$$

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- Though Austin might be close...

$$Z = \{\emptyset\}$$

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- R represents all the possibilities of a (single) roll of a die
- We can define sets for the even possibilities and the odd possibilities

$$E = \{2, 4, 6\} \qquad O = \{1, 3, 5\}$$

Compliments

- A *compliment* is that together, they contain all the elements of the relevant universe

$$E = O^C \qquad ; \qquad O = E^C$$

Universe

- Board examples of how to draw sets

Properties

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- How many different combinations of 3 dice rolls are there?

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- think "permutation" = "position"

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  - $16 * 15 * 14 * 13 \ldots = 16!$