Introduction to Math for Political Scientists

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Fall 2015

Let's start real slow. . .

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- But addition and subtraction have some cool (grool?) rules

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  - $(a \pm b) \pm c = a \pm (b \pm c)$

Multiplication

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```
4+4+4+4+4+4+4+4+4+4
```

```
## [1] 40
```

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```
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```

```
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```

Or I could just do

```
4*10
```

```
## [1] 40
```

Division

- Just fancy multiplication.

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- Just fancy multiplication.
- I have these four things one of ten times.

```
4*(1/10)
```

```
## [1] 0.4
```

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  - $a * b = b * a$

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  - $(ab)c = a(bc)$
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  - $a(b + c) = ab + ac$
  - Note that this works for division: $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

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    - $a > b$ and $b > c \Rightarrow a > c$

PEMDAS

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- Addition and subtraction (left to right)

$$(10 - 48 \div 12 * 2)^2 + 3^2 * (8 - 6)$$

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  - Othertimes, it means $log_e(n) = ln(n)$

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- $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n \qquad \forall a, b \neq 0$

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