Introduction to Probability

J. Alexander Branham

Fall 2015

Probability

- Frequency with which an event occurs

Probability

- Frequency with which an event occurs
- Typically:

$$Pr(A) = P(A) = \pi(A) = \frac{\text{Number of ways an event can occur}}{\text{Total number of possible outcomes}}$$

Probability

- Frequency with which an event occurs
- Typically:

$$Pr(A) = P(A) = \pi(A) = \frac{\text{Number of ways an event can occur}}{\text{Total number of possible outcomes}}$$

- There are other ways of thinking about probability, but we'll stick with this one

Kolmogorov's Axioms

- First: $Pr(E) \in \mathbb{R}, Pr(E) \geq 0 \qquad \forall E \in F$

Kolmogorov's Axioms

- First: $Pr(E) \in \mathbb{R}, Pr(E) \geq 0 \qquad \forall E \in F$
  - Probabilities must be non-negative

Kolmogorov's Axioms

- First: $Pr(E) \in \mathbb{R}, Pr(E) \geq 0 \qquad \forall E \in F$
  - Probabilities must be non-negative
- Second: $Pr(\Omega) = 1$

Kolmogorov's Axioms

- First: $Pr(E) \in \mathbb{R}, Pr(E) \geq 0 \qquad \forall E \in F$
  - Probabilities must be non-negative
- Second: $Pr(\Omega) = 1$
  - Something has to happen

Kolmogorov's Axioms

- First: $Pr(E) \in \mathbb{R}, Pr(E) \geq 0 \qquad \forall E \in F$
  - Probabilities must be non-negative
- Second: $Pr(\Omega) = 1$
  - Something has to happen
  - Probabilities sum/integrate to 1

Kolmogorov's Axioms

- First: $Pr(E) \in \mathbb{R}, Pr(E) \geq 0 \qquad \forall E \in F$
  - Probabilities must be non-negative
- Second: $Pr(\Omega) = 1$
  - Something has to happen
  - Probabilities sum/integrate to 1
- Third: $Pr\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} Pr(E_i)$

Kolmogorov's Axioms

- First: $Pr(E) \in \mathbb{R}, Pr(E) \geq 0 \qquad \forall E \in F$
  - Probabilities must be non-negative
- Second: $Pr(\Omega) = 1$
  - Something has to happen
  - Probabilities sum/integrate to 1
- Third: $Pr\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} Pr(E_i)$
  - The probability of disjoint (mutually exclusive) sets is equal to their sums

Discrete probabilities

- What's the probability that we'll roll a 3 on one die roll:

$$Pr(y = 3) = \frac{1}{6}$$

Discrete probabilities

- What's the probability that we'll roll a 3 on one die roll:

$$Pr(y = 3) = \frac{1}{6}$$

- If one roll of the die is an "experiment"

Discrete probabilities

- What's the probability that we'll roll a 3 on one die roll:

$$Pr(y = 3) = \frac{1}{6}$$

- If one roll of the die is an "experiment"
- We can think of a 3 as a "success"

Discrete probabilities

- What's the probability that we'll roll a 3 on one die roll:

$$Pr(y = 3) = \frac{1}{6}$$

- If one roll of the die is an "experiment"
- We can think of a 3 as a "success"
- And $Y \sim Bern(\frac{1}{6})$

Discrete probabilities

- What's the probability that we'll roll a 3 on one die roll:

$$Pr(y = 3) = \frac{1}{6}$$

- If one roll of the die is an "experiment"
- We can think of a 3 as a "success"
- And $Y \sim Bern(\frac{1}{6})$
- Fair coins are $\sim Bern(.5)$ for example

Discrete probabilities

- What's the probability that we'll roll a 3 on one die roll:

$$Pr(y = 3) = \frac{1}{6}$$

- If one roll of the die is an "experiment"
- We can think of a 3 as a "success"
- And $Y \sim Bern(\frac{1}{6})$
- Fair coins are $\sim Bern(.5)$ for example
- More generally $Bern(\pi)$

Discrete probabilities

- What's the probability that we'll roll a 3 on one die roll:

$$Pr(y = 3) = \frac{1}{6}$$

- If one roll of the die is an "experiment"
- We can think of a 3 as a "success"
- And $Y \sim Bern(\frac{1}{6})$
- Fair coins are $\sim Bern(.5)$ for example
- More generally $Bern(\pi)$
  - π represents the probability of success

Binomial distributions

- Before we ran the experiment just once

Binomial distributions

- Before we ran the experiment just once
- What happens if we run it multiple times?

$$Pr(y_1 = 3, y_2 = 3) = \frac{1}{6} * \frac{1}{6}$$

Binomial distributions

- Before we ran the experiment just once
- What happens if we run it multiple times?

$$Pr(y_1 = 3, y_2 = 3) = \frac{1}{6} * \frac{1}{6}$$

- Now $Y \sim Binom(2, \frac{1}{6})$

Binomial distributions

- Before we ran the experiment just once
- What happens if we run it multiple times?

$$Pr(y_1 = 3, y_2 = 3) = \frac{1}{6} * \frac{1}{6}$$

- Now $Y \sim Binom(2, \frac{1}{6})$
  - Generally, $Y \sim Binom(n, p)$

Binomial distributions

- Before we ran the experiment just once
- What happens if we run it multiple times?

$$Pr(y_1 = 3, y_2 = 3) = \frac{1}{6} * \frac{1}{6}$$

- Now $Y \sim Binom(2, \frac{1}{6})$
  - Generally, $Y \sim Binom(n, p)$
  - n = number of trials, p = probability of success

Binomial distributions

- Before we ran the experiment just once
- What happens if we run it multiple times?

$$Pr(y_1 = 3, y_2 = 3) = \frac{1}{6} * \frac{1}{6}$$

- Now $Y \sim Binom(2, \frac{1}{6})$
  - Generally, $Y \sim Binom(n, p)$
  - n = number of trials, p = probability of success
- PMF:

$$\binom{n}{k} p^k (1-6)^{n-k}$$

The basics

- What happens when our outcome is continous

The basics

- What happens when our outcome is continous
- Much harder to think about...

The basics

- What happens when our outcome is continous
- Much harder to think about. . .
- There are infinity possible outcomes

The basics

- What happens when our outcome is continous
- Much harder to think about...
- There are infinity possible outcomes
- Makes the denominator of our fraction difficult to work with

The basics

- What happens when our outcome is continous
- Much harder to think about. . .
- There are infinity possible outcomes
- Makes the denominator of our fraction difficult to work with
- What's the probability that you make $15,293.19/year?

The basics

- What happens when our outcome is continous
- Much harder to think about...
- There are infinity possible outcomes
- Makes the denominator of our fraction difficult to work with
- What's the probability that you make \$15,293.19/year?
- Probability of the whole space must equal 1

The basics

- What happens when our outcome is continous
- Much harder to think about...
- There are infinity possible outcomes
- Makes the denominator of our fraction difficult to work with
- What's the probability that you make \$15,293.19/year?
- Probability of the whole space must equal 1
- Even if all events equally likely, $\frac{1}{\infty} = 0$

The basics

- What happens when our outcome is continous
- Much harder to think about...
- There are infinity possible outcomes
- Makes the denominator of our fraction difficult to work with
- What's the probability that you make $15,293.19/year?
- Probability of the whole space must equal 1
- Even if all events equally likely, $\frac{1}{\infty} = 0$
  - Kinda...

Continuous distributions

```
ggplot(data.frame(x = c(-5, 5)), aes(x)) +
  stat_function(fun = dnorm)
```