MIS Compensation: Optimizing Sampling Techniques in Multiple Importance Sampling

Seminar-Ausarbeitung von

Christian Navolskyi, B. Sc.

An der Fakultät für Informatik
Institut für Visualisierung und Datenanalyse,
Lehrstuhl für Computergrafik

November 14, 2019

Contents

1. Abstract

(Write abstract (at the end)) **ToDo**

2. Introduction

ToDo **(Write introduction. Structure as in the paper (overview, problem, others, solution proposal))**

3. Related Work

(Sum up what others did in this field.) **ToDo**

4. Multiple Importance Sampling

ToDo **(Match to actual structure of this chapter)** In this chapter the fundamentals of Monte Carlo integration and importance sampling will be introduced. After that Multiple Importance Sampling is explained which is the fundament for the next chapter 5.

4.1 Probability Basics

For our use case we need to define random variables, probability distribution functions (pdf) and cumulative distribution functions (cdf). A random variable maps an event to a real number $X : \Omega \to \mathbb{R}$. In a discrete scenario our random variable corresponds to one exact event e.g. a dice throw showing three pips on the top. The probability for this can be expressed as $P(X = 3) = \frac{1}{6}$ or as $X(3) = \frac{1}{6}$ since the random variable X can take six different values which are all equally likely. To express the probability that an event is within a range of values we can use the cdf which is defined as $F(x) = P(X \leq x), x \in \mathbb{R}$. For the probability of a random variable in an interval $[a, b)$ we can write $P(a \leq X < b) = F(b) - F(a)$.[Sch]

When we are talking about continuous events e.g. turning a wheel of fortune, every exact angle has a probability of zero. But we can still express our probability for an angle interval with the cdf from above. In the continuous case we have a pdf that is used to calculate the cdf with $F(x) = \int_{-\infty}^{x} p(\tilde{x})d\tilde{x}$. The pdf has to be non-negative ($\forall x : p(x) \geq 0$) and integrate to one ($\int_{-\infty}^{\infty} p(x)dx = 1$) to be valid and "[...] describes the relative probability of a random variable taking on a particular value."[PJH18] ToDo **(How to cite when the content of this section was created with a reference?)**

4.2 Generating Samples after a specific Function

This will be very important for the next section 4.3, since this is our adjustment wheel to manipulate the quality of our rendering (for a given amount of time). Most of the time we don't need evenly distributed samples, rather our samples should follow a specific characteristic of a material or a scene. For this we need to know how to generate samples that correspond to that characteristic.

Given a function $f(x)$ after which we would like to draw samples the first step is to make sure it fulfills the constraints for a pdf in an interval $[a, b)$ we want to use. First we check that $\forall x \in [a, b) : f(x) \geq 0$ and then calculate the integral with $F = \int_{a}^{b} f(x)dx$. F

can be used to normalize our goal function which yields us $\tilde{f}(x) = \frac{f(x)}{F}$ as a valid pdf. The next step is to calculate our cdf as $F(x) = \int_a^x \tilde{f}(t)dt$ which we will then invert to get $F^{-1}(x)$. Now we only need to draw an equally distributed random number (which most programming languages have a library for) in the interval $[0,1)$ which we call ξ and evaluate $F^{-1}(\xi) = X$ to get X which is distributed after $f(.)$.[Sch]

4.3 Monte Carlo and Importance Sampling

Monte Carlo integration is a technique to approximate the integral of an arbitrary function $f(x)$ by taking N random samples X_i from a probability density function (pdf) $p(X_i)$. X_i is a random variable that belongs to one particular event (e.g. $X_0 = 1$ when throwing a dice and it shows 1) or in the

To draw samples in a Monte Carlo renderer a sampling technique has to be chosen. This choice has a significant impact on the variance and therefore the noise in the final image. We need a probability density function (pdf) $p(x)$ that integrates to one ($1 = \int_{-\infty}^{\infty} p(x)dx$) and is non-negative ($\forall x : p(x) \geq 0$). Now we can draw samples

The general formula for the Monte Carlo integration is $F \approx \frac{1}{n} \sum_{i=1}^{n} \frac{f(X_i)}{p(X_i)}$, with $i = 1, ..., n$ X_i is a random sa

(Explain MIS in more detail than in the paper) **ToDo**

(check what happends, when one technique says probability is 0) **ToDo**

5. Multiple Importance Sampling Compensation

5.1 Optimality

ToDo (Explain their work. Formulas should be clearly explained.)

6. Application: Image Based Lighting

(Explain the contents of this chapter) ToDo

6.1 Image Based Lighting

(Explain Image based lighting shortly) ToDo

6.2 MIS Compensation in Image Based Lighting

(Explain how their approach improves ibl variance) ToDo

(If space left show results) ToDo

7. Application: Path Guiding

7.1 Path Guiding

ToDo **(Explain the concept of path guiding)**

7.2 MIS Compensation in Path Guiding

ToDo **(Explain how their approach improves path guiding)**

8. Summary

(Write a shor summary of what they achieved) **ToDo**

8.1 Future Work

(Mention what could be done next based on this work) **ToDo**

Bibliography

[PJH18] M. Pharr, W. Jakob, and G. Humphreys, *Physically Based Rendering: From Theory To Implementation*, 3rd ed. MK, Morgen Kaufmann, 2018.

[Sch] J. Schudeiske, "Fotorealistische bildsynthese," course slides from lecture at the KIT.

Erklärung

Ich versichere, dass ich die Arbeit selbstständig verfasst habe und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe, die wörtlich oder inhaltlich übernommenen Stellen als solche kenntlich gemacht und die Satzung des KIT zur Sicherung guter wissenschaftlicher Praxis in der jeweils gültigen Fassung beachtet habe. Die Arbeit wurde in gleicher oder ähnlicher Form noch keiner anderen Prüfungsbehörde vorgelegt und von dieser als Teil einer Prüfungsleistung angenommen.

Karlsruhe, den November 14, 2019

(Christian Navolskyi, B. Sc.)