

Homework 1

Due 09/01/15

August 25, 2015

1. Prove that the following algorithm sorts its input data; i.e., that $data[1] \leq data[2] \leq \dots \leq data[n]$ when the algorithm terminates. You may assume that $data$ contains at least one element. Also, $\lfloor x \rfloor$ represents the *floor* function, which returns the largest integer less than or equal to the given value (e.g., $\lfloor 3.1415 \rfloor = 3$).

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Input: data: an array of integers
Input: n: the length of data
Output: a reordering of data in (ascending) sorted order
1 Algorithm: ThirdSort
2 if  $n = 1$  then
3   | return data
4 else if  $n = 2$  then
5   | if  $data[1] > data[2]$  then
6   |   | Swap  $data[1]$  and  $data[2]$ 
7   | end
8   | return data
9 else
10  |  $third = \lfloor n/3 \rfloor$ 
11  | Call ThirdSort on  $data[1..n-third]$ 
12  | Call ThirdSort on  $data[third+1..n]$ 
13  | Call ThirdSort on  $data[1..n-third]$ 
14  | return data
15 end
```

Hint: use strong induction on the length of $data$. You may find it useful to assign names (like A , B , and C) to the three “thirds” of the $data$ array given by $data[1..third]$, $data[third+1..n-third]$, and $data[n-third+1..n]$. You may also find it helpful to simulate the algorithm on some small inputs to understand what it is doing.