Use the definition of Big-Oh to prove that $T(n) = O(n^2)$, where

$$T(n) = \begin{cases} O(1), & \text{if } n = 1 \\ T(n-1) + O(n), & \text{if } n > 1 \end{cases}$$

Proof. Since $T(n) = T(n-1) + O(n)$ if $n > 1$, $T(n) \le T(n-1) + cn$, for all $n \ge n_0$. Let $c_2 = \max\{T(n_0), c\}$. We prove that $T(n) \le c_2 n^2$ for all $n \ge n_0$ by induction.
(*Base case*) Since $n_0 \ge 1$ and $c_2 = \max\{T(n_0), c\}$, $c_2 {n_0}^2 \ge c_2 \ge T(n_0)$, so $T(n) \le c_2 n^2$ when $n = n_0$.
(*Inductive step*) Suppose that $T(n) \le c_2 n^2$ for $n = k$, and consider $n = k+1$. $T(k+1) \le T(k) + c(k+1)$. By the inductive hypothesis, $T(k) \le c_2 k^2$, so $T(k+1) \le c_2 k^2 + ck + c$. Since $c \le c_2$, $T(k+1) \le c_2 k^2 + c_2 k + c_2 \le c_2 k^2 + 2c_2 k + c_2 = c_2(k+1)^2$. Thus, $T(k+1) \le c_2(k+1)^2$, so $T(n) \le c_2 n^2$ for all $n \ge n_0$ by induction.
Since there exist constants c_2 and n_0 such that $T(n) \le c_2 n^2$ for all $n \ge n_0$, $T(n) = O(n^2)$ by the definition of Big-Oh. $\square$