Financial factors, firm size and potential*

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Abstract

Using a unique dataset covering the universe of Portuguese firms and their credit situation we revisit the relation between firm characteristics, financial factors and their sensitivity to business fluctuations. First, we provide three stylized facts: (1) Financially constrained firms react more to the business cycle and this mechanism is orthogonal to the size channel proposed by Crouzet & Mehrotra (2020). (2) Constrained firms are found across the entire size distribution, also in the top percentiles, which is in contrast to what existing financial friction models would predict. (3) Ex-ante heterogeneity of firms matters and persists over the firms' life cycle affecting constrained and unconstrained firms differently. We show that including permanent productivity heterogeneity in a financial frictions model can rationalise the stylized facts and larger constrained firms subsequently lead to stronger aggregate fluctuations.

Keywords: Firm size, business cycle, financial accelerator

JEL Codes: E62, E22, E23

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1 Introduction

A substantial amount of research in macroeconomics focuses on the propagation of aggregate shocks via financial factors and their relation to individual firm characteristics. In a seminal work on this topic, Gertler & Gilchrist (1994) propose firm size as an effective proxy for financial constraints. Smaller firms are arguably more risky, less liquid and face an elevated external finance premium. Accordingly, smaller firms are more sensitive to monetary policy shocks, as they tend to be in a weaker financial position. Revisiting this hypothesis with more granular data Crouzet & Mehrotra (2020) show that differences in firm cyclicality are not correlated with size, except for when one compares very large firms to the rest. Moreover, firm size remains informative, even when controlling for a number of balance sheet measures of financial conditions. This suggests that financial factors are not the deeper source of size-based cyclicality differences.

In this paper, we contribute to this debate by presenting both novel evidence and theoretical insights on the relationship between financial factors, size and sensitivity to the business cycle. Our empirical evidence comes from the Bank of Portugal's confidential credit registry database, matched with bank and firm balance sheet data between 2006 and 2017. Since all financial institutions are required to report both potential and actual credit above €50, we are able to construct detailed, firm-specific and credit-based measures for financial constraints. To the best of our knowledge, we are the first to utilize this level of empirical data on credit relationships in answering these questions and discipline the choice of our theoretical model. As such, our contribution is twofold.

First, using the firms' detailed credit information we present three stylized facts. (1) Financially constrained firms are more sensitive to the business cycle which is in line with the financial accelerator mechanism theory. This empirical result holds conditional on size and is robust across a battery of different specifications. (2) Size and other firm variables commonly used in the literature as proxies for financial constraints, are only weakly correlated to the firm's financial health. In fact, constrained firms can be found over the entire distribution. One factor that could explain this orthogonality and break the strong correlation between size and financial constraints is ex-ante firm heterogeneity. (3) We present evidence that ex-ante heterogeneity matters and persists over the firm's life cycle differently affecting constrained and unconstrained firms.

Second, we aim to reconcile these stylized facts in a heterogeneous firm model with financial frictions. We mainly build on Khan & Thomas (2013) but add a permanent component to the firms' productivity process. This introduces ex-ante heterogeneity and could be interpreted as the firm's business case potential, while the transitory shocks reflect execution successes or setbacks developing the business. We show that the model is capable of explaining the stylized facts. Furthermore, we explore how the existence of large constrained firms and ex-ante heterogeneity shape the aggregate response following a MIT shock to total factor productivity. Relative to the standard Khan & Thomas (2013) model with only a transitory productivity component, the aggregate response is stronger due to the cyclicality of large constrained firms in the top percentiles of the size distribution.

Our work follows a large literature in macroeconomics that has developed and analysed models with heterogeneous firms and financial frictions.

One of the early contributions in this literature by Cooley & Quadrini (2001) shares many features with our current model. They augment an otherwise standard Hopenhayn (1992) model of heterogeneous firms with financial frictions and persistent shocks. In doing so, they are able to match the empirical facts that both smaller firms, conditional on age, and younger firms, conditional on size, are more dynamic (ie. job creation and destruction, growth, volatility of growth and exit are all higher). Our theoretical contribution also emphasizes the importance of permanent productivity differences for rationalising empirical facts. However, instead of rationalising conditional size and age dependence, we instead focus on financial conditions shaping the cyclicality of firms, conditional on size.

Another, more recent instance where permanent productivity differences plays a crucial role is Mehrotra & Sergeyev (2020). They argue that financial frictions played a relatively minor role in unemployment increases associated with the Great Recession and that employment was reduced due to shocks that affected unconstrained and constrained firms alike. The seminal paper by Bernanke & Gertler (1989) on the financial accelerator mechanism develops a simple neoclassical model where the condition of the borrowers' balance sheets is a source for output dynamics. Positive business cycle shocks increase borrowers' net worth, lower agency costs, and thus increase investment, amplifying the business cycles.

Moreover, there is a broad strand of empirical work estimating the effect of financial frictions on firm behaviour. Gertler & Gilchrist (1994) find empirical evidence for the financial accelerator mechanism. They analyse the cyclical behaviour of small vs. large manufacturing firms and interpret this as evidence for the financial accelerator. Their main assumption is that size is a good proxy for financial constraints. They argue that size is strongly correlated with financial conditions due to the fact that informational frictions adding to the costs of external finance mainly apply to young firms, with a high degree of idiosyncratic risk and little collateral which on average are smaller firms. Sharpe (1994) finds a statistically significant relationship between a firm's financial leverage and the cyclicality of its labour force. Employment growth at highly leveraged firms is more sensitive - they are less likely to hoard labour. And this cyclicality also hold for the size dimension - confirming Gertler & Gilchrist (1994). Gilchrist & Himmelberg (1995) find that investment still responds to cash flow even after controlling for its role for forecasting future investment opportunities. And the effect is stronger for financially constrained firms. Our empirical contribution also provides evidence for the financial accelerator mechanism, yet we show that size is a much weaker proxy than anticipated by several papers. In fact, the financial accelerator mechanism is orthogonal to a separate size-related channel of reduced cyclicality for large firms, with the latter being the focus of Crouzet & Mehrotra (2020). The paper is structured as follows. First, we present the data we use for the empirical section as well as to discipline our theoretical model. Second, we present the three stylized facts outlined above. Then we set out the model to incorporate these stylized facts and discuss model predictions of aggregate effects. Finally, we conclude.

2 Data

We draw on a unique combination of datasets that cover the Portuguese economy between 2006 and 2017, all managed by the Bank of Portugal.

First, we use the *Informação Empresarial Simplificada* Central Balance Sheet Database (CBSD) that is based on annual accounting data of individual firms. Portuguese firms have to fill out mandatory financial statements in order to comply with their statutory obligation. Consequently, this dataset virtually covers the population of all non-financial corporations in Portugal from 2006 onward. We combine this with the Central Credit Register (CCR), that contains monthly[1] information on the actual and potential credit above 50 euros extended to individuals and non-financial corporations, reported by all financial institutions in Portugal. Actual credit includes loans that are truly taken up, such as mortgages, consumer loans, overdrafts and others. Potential credit encompasses all irrevocable commitments to the subject that have not materialised into actual credit, such as available credit on credit cards, credit lines, pledges granted by participants and other credit facilities. We then merge these two databases on the firm level. Moreover, we then also add the Monetary Financial Institutions Balance Sheet Database in order to gain information on the balance sheets of banks that extend credit to non-financial institutions. We combine this on a firm level using the bank identifier and the share of loans extended by one firm to arrive at our detailed dataset.

We restrict the set of firm in this panel dataset to those with at least five consecutive observations and to firms which are in business at the time of reporting. Furthermore, we

[1] Given that the firm balance sheet data is of yearly frequency, we consider the month in which the balance sheet data was reported. Results were robust to shifting and averaging the monthly credit data.

only consider privately or publicly held firms and drop micro firms, i.e. those with overall credit amounts of less than 10,000 €. Descriptive statistics for the relevant variables can be found in table 7 and 8 in the appendix. Based on the detailed credit information in the data set, we construct several binary and continuous measures indicating whether a firm is financially constrained. A detailed description of the individual measures and their descriptive statistics can be found in the appendix.

3 Stylized facts

The rich dataset covering the universe of Portuguese firms and including both balance sheet and credit information, allows us to generate novel evidence on the characteristics of constrained firms. This section presents three stylized empirical facts which form the basis for our theoretical model. First, we revisit the financial accelerator mechanism. Similar to Crouzet & Mehrotra (2020), we find firms' cyclicality to differ only at the top 1% of the size distribution. Moreover, we find evidence that supports the financial accelerator mechanism theory, with constrained firms being more cyclical, and this to be orthogonal to the size cyclicality. Second, we proceed by illustrating that size, among other variables commonly used in the literature as proxies for financial constraints, are weakly correlated to the firm's financial health. In fact, constrained firms can be found over the entire firm distribution. One factor that could explain this orthogonality and break the strong correlation between size and financial constraints, usually found in firm dynamic models, is ex-ante heterogeneity. Third, we present evidence that ex-ante components matter and persist over the firm's life cycle differently affecting constrained and unconstrained firms.

3.1 Size and financial factors matter for cyclicality

This section aims to test the financial accelerator mechanism empirically. In particular, we want to test whether constrained firms are more cyclical than unconstrained firms. In order to make our results comparable to Crouzet & Mehrotra (2020), we first replicate their results for the universe of Portugese firms and then augment their estimation strategy, using our set of firm-specific and time-varying measures for being financially constrained. Following Crouzet & Mehrotra (2020), the model estimated is:

$$
\begin{aligned}
g_{i,t} = \Delta GDP_t &+ \sum_{j \in \mathcal{J}} (\alpha_j + \beta_j \Delta GDP_t) \mathbf{1}_{i \in \mathcal{S}_t^{(j)}} + (\zeta + \eta \Delta GDP_t) fin\_health_{i,t} \\
&+ \sum_{l \in \mathcal{L}} (\gamma_l + \delta_l \Delta GDP_t) \mathbf{1}_{i \in \mathcal{L}} + \epsilon_{i,t},
\end{aligned} \tag{1}
$$

where i identifies a firm and t identifies a year. The dependent variable $g_{i,t}$ is the year-on-year log change in turnover or employees. The set $\mathcal{S}_t^{(j)}$ is a jth size group, e.g. all firms above the 90th but below the 99th percentile. Furthermore, ΔGDP_t is the year-on-year growth rate of GDP, and $\mathcal{L}$ is a set of industry dummies. fin_health refers to the variable measuring the strength of financial constraints.

Table 1 reports estimates of the semi-elasticity of firm-level growth in turnover to GDP growth. The first column reports estimates for the size groups $j \in \{[90, 99], [99, 99.5], [99.5, 100]\}$ with $[0, 90]$ as the reference group. On average, small firms have a semi-elasticity of roughly 2.5, meaning for any percent change in GDP growth, their turnover changes by 2.5%. Although, the coefficient for the size group $[90, 99]$ is insignificant, results are consistent with the view that larger firms are less sensitive to aggregate fluctuations which is in line with the findings of Crouzet & Mehrotra (2020) and earlier work. The difference is particularly notable at the very top of the firm distribution. The second column reports results including the baseline binary measure for being constrained. First, it is worth noting, that the estimation coefficient with respect to size hardly change. This is indicative that the mechanism going through size is orthogonal to any financial accelerator mechanism, and hence size might not be a good proxy as already pointed out by Crouzet & Mehrotra (2020). The coefficient for the interaction between the constrained measure and aggregate fluctuations is significant, offering support for the financial accelerator mechanism. Con-

	Turnover growth						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
GDP growth	2.505***	2.428***	2.44***	2.45***	2.362***	2.518***	2.510***
	(0.023)	(0.026)	(0.023)	(0.025)	(0.030)	(0.023)	(0.0250)
[90,99] × GDP growth	0.035	0.068	0.063	0.056	0.059	0.031	0.041
	(0.107)	(0.107)	(0.107)	(0.121)	(0.107)	(0.107)	(0.121)
[99,99.5] × GDP growth	-0.689**	-0.656**	-0.660**	-0.754**	-0.652**	-0.697**	-0.761**
	(0.300)	(0.301)	(0.300)	(0.341)	(0.301)	(0.300)	(0.342)
[99.5,100] × GDP growth	-1.469***	-1.426***	-1.434***	-1.646***	-1.427***	-1.473***	-1.686***
	(0.293)	(0.293)	(0.293)	(0.359)	(0.293)	(0.293)	(0.359)
Constrained_1 × GDP growth		0.119**					
		(0.055)					
Constrained_2 × GDP growth			0.572***				
			(0.135)				
Constrained_4 × GDP growth				1.202***			
				(0.251)			
Constrained_5 × GDP growth					0.159***		
					(0.048)		
Constrained_6 × GDP growth						-0.067***	
						(0.025)	
Constrained_7 × GDP growth							-0.172***
							(0.031)
Observations	1,323,660	1,323,660	1,323,660	1,159,037	1,323,660	1,322,316	1,158,059
R-squared	0.029	0.030	0.030	0.032	0.029	0.030	0.030
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE × GDP growth	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Clustering	Firm	Firm	Firm	Firm	Firm	Firm	Firm

Notes: Robust standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table 1: Cyclicality estimates for size bins and measures of financial constraints

strained firms have a 0.1 higher semi-elasticity relative to unconstrained firms according to the baseline measure. However, as already pointed out when introducing the different measures for being constrained, the baseline measure might capture firms for which potential credit is zero, but in fact are unconstrained. Hence, the baseline measure offers a lower bound of the increased sensitivity of constrained firms. We therefore consider other binary measures trying to overcome those drawbacks which are reported in columns (3) to (5). Estimation results are supportive of the notion that the baseline measure acts as a lower bound and sensitivity might be up to 50% higher for constrained firms as measured by constrained_5. Estimates for the standardized continuous measures are also in line with the findings so far. The negative sign is due to their definition. The higher their value, the less constrained is a firm. Results for sales growth and growth in employees are closely in line, see tables 9 and 10 in the appendix. Besides using different measures, we considered a battery of robustness checks. First, we included time fixed effects to account for broader macroeconomic circumstances. Second, we estimated the specification using firm fixed effects. Third, we estimated the model excluding those firms for which potential credit is zero throughout. Fourth, we controlled for supply effects using aggregated bank data. Estimates were robust across all specifications.

3.2 Constrained firms are found across the firm distribution

Figure 1 plots the share of constrained firms over percentiles of various common proxies for being constrained. Clearly, constrained firms can be found in every bin of the firm distribution[2]. While correlations are in line with the existing literature, they are not as strong as existing models assume. In fact, when running a linear probability model[3], the probability of being constrained only reduces by about 5% for one standard deviation increase in total assets. Standard firm models typically produce small constrained firms

[2] This is robust for different measures of being constraint. Correlations depend on the restrictiveness of the measure.

[3] Table 11 in the appendix.

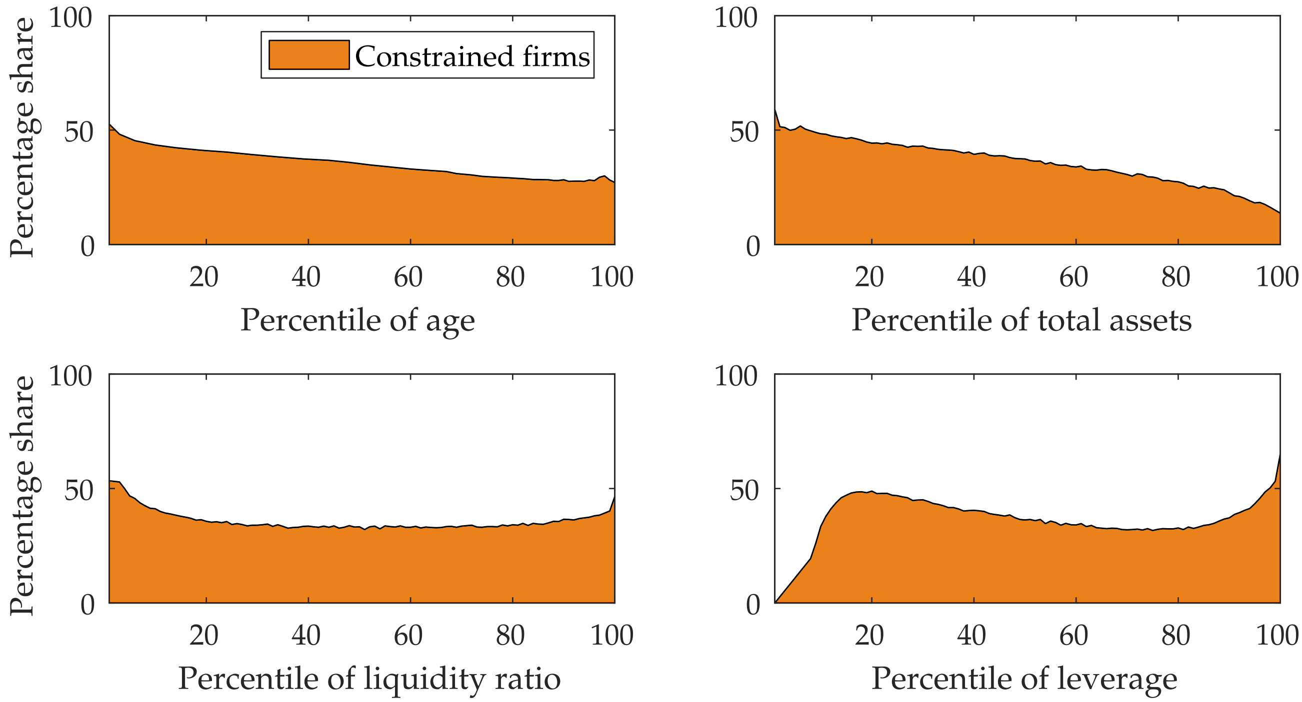

Figure 1: Decomposition of constrained and unconstrained firms across percentiles of firm variables

and large unconstrained firms, yet this model prediction evidently appears too simplistic. Moreover, even when controlling for a battery of financial variables the explanatory power to predict whether a firm is constrained is relatively low compared to the firms' fixed effects. Hence, existing proxies of financial constraints may be unable to capture this idiosyncrasy, which seems to play a role in credit decisions.

3.3 Ex-ante heterogeneity matters

Following the previous empirical results the question becomes how to make sense of a weak correlation between size and constraints? Standard financial frictions, firm dynamics models à la Khan & Thomas (2013) predict very strong correlation between the two, with smaller firms being the more constrained ones. One factor that could potentially break this strong correlation are ex-ante conditions. Small firms may be unconstrained as they already reached their potential - i.e. optimal size - while large firms may still be growing and may still be constrained. Equally, different potential would create a dispersion of unconstrained firms across the entire firm size distribution, similar to the results we presented in the previous sections. But is it reasonable to assume ex-ante heterogeneity?

Figure 2 presents the standard deviation of log employment by age in the left panel and the autocorrelation of log employment between age a and $h \leq a$.[4] The left panel plot showcases that even at early ages the standard deviation of log employment is already relatively high - above 0.8 - which indicates large size differences even at early ages. Also, the standard deviation of log employment is increasing with age. This supports the idea that there exists ex-ante heterogeneity, as firms at birth are not all equal, and that firms will settle at different levels of employment in the long run, as the standard deviation is increasing with age.

The right panel plot indicates that the long run autocorrelations appear to stabilize at relatively high levels. This may be indicative that ex-ante conditions are persistent and affect the firm even in the long run. This results are in line with the Pugsley et al. (2018) findings, who showcase the importance of ex-ante conditions in explaining the firm size distribution and firm dynamics.

As we are interested in understanding if differences between constrained and unconstrained firms can, partly, be explained by ex-ante heterogeneity, we plot in Figure 7 in Appendix B.2 the standard deviation and autocorrelation of log employment for both constrained and unconstrained firms.[5] It is possible to notice that both these statistics are lower for constrained firms than unconstrained ones. One could have expected the opposite to be true, as constrained firms would potentially have less resources to grow and so their employment tomorrow could have a stronger correlation with employment

[4]To prevent differences across sectors and business cycle conditions from explaining the majority of the standard deviation and autocorrelation, we first control for sector and year fixed effects and then use the residuals of log employment.

[5]Here we are using the measure that exclusively takes the amount of potential credit available into account to determine which are the constrained firms. A firm is considered constrained if at age $a - h$ has potential credit equal to zero.

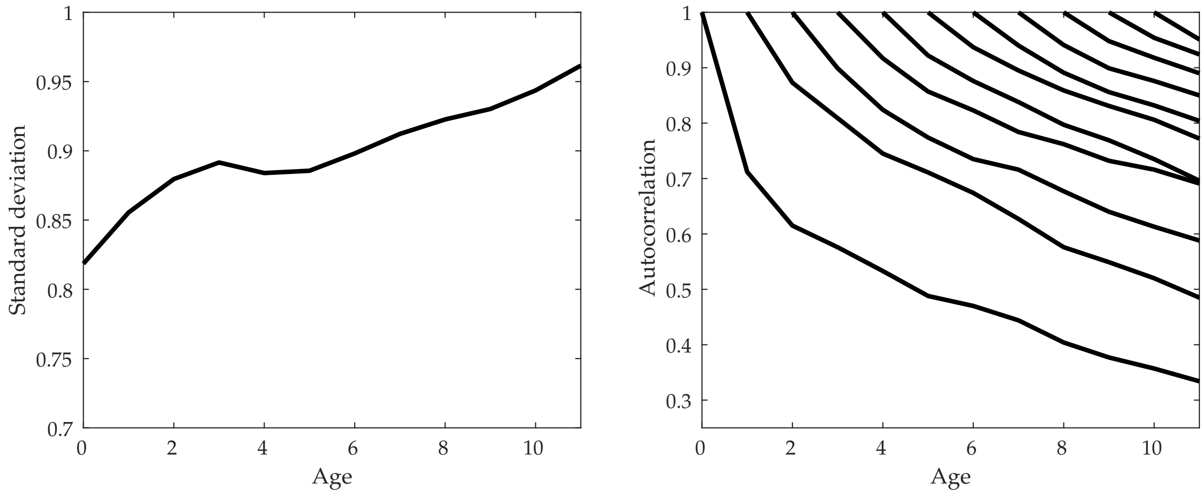

Figure 2: The left panel presents the standard deviation of log employment by age, after controlling for sector and year fixed effects. The right panel presents the autocorrelation of log employment between ages a and $h \leq a$. Across lines h changes, while a changes along the lines.

today. The fact that the autocorrelation is higher across the life-cycle for unconstrained firms may be indicative that they are born closer to their optimal size, when comparing to constrained firms. This may then explain, in part, why some young firms are constrained and others are not: the ones born closer to their optimal size have lower investments and do not become constrained, while firms that need to grow to reach the optimal size exhaust their credit lines.

To better understand the importance of ex-ante vs ex-post heterogeneity for constrained and unconstrained firms, we adopt the statistical model by Pugsley et al. (2018). This model will use the information provided by the autocovariance structure of log employment to capture the importance of both types of heterogeneity.

Consider the following process for employment n by firm i at age a

$$\underbrace{\ln n_{i,a}}_{\text{log employment}} = \underbrace{u_{i,a} + v_{i,a}}_{\text{Ex-ante component}} + \underbrace{w_{i,a} + z_{i,a}}_{\text{Ex-post component}} \tag{2}$$

where

	ρ_u	ρ_v	ρ_w	σ_θ	σ_u	σ_v	σ_ϵ	σ_z
Constrained	0.485	0.760	0.707	0.339	0.009	0.671	0.287	0.147
Unconstrained	0.447	0.694	0.780	0.452	0.760	0.841	0.305	0.110

Table 2: Static model parameters for constrained and unconstrained firms.

$$\begin{aligned}
u_{i,a} &= \rho_u u_{i,a-1} + \theta_i, & u_{i,-1} &\sim iid\left(\mu_{\tilde{u}}, \sigma^2_{\tilde{u}}\right), & \theta_i &\sim iid\left(\mu_\theta, \sigma^2_\theta\right), & |\rho_u| &\leq 1 \\
v_{i,a} &= \rho_v v_{i,a-1}, & v_{i,-1} &\sim iid\left(\mu_{\tilde{v}}, \sigma^2_{\tilde{v}}\right), & & & |\rho_v| &\leq 1 \\
w_{i,a} &= \rho_w w_{i,a-1} + \varepsilon_{i,a}, & w_{i,-1} &= 0, & \varepsilon_{i,a} &\sim iid\left(0, \sigma^2_\varepsilon\right), & |\rho_w| &\leq 1 \\
z_{i,a} &\sim iid\left(0, \sigma^2_z\right)
\end{aligned}$$

In this employment process, $u_{i,a} + v_{i,a}$ capture the ex-ante profile, while $w_{i,a} + z_{i,a}$ capture the ex-post one. The ex-ante component is determined by three shocks that are drawn just prior to the birth year, at $a = -1$. The shocks $v_{i,-1}$ and $u_{i,-1}$ represent the initial conditions of the firm, which allow for rich heterogeneity even at birth. θ_i is the permanent component, which will accumulate over the life-cycle at speed ρ_u. In particular, with $\rho_u < 1$, the long-run steady state level of employment will be given by $\frac{\theta_i}{1-\rho_u}$.

This specification will allow for rich heterogeneity not only in terms of optimal size of the firms, depending on the distribution of θ_i, but also in terms of the speed at which firms reach the steady state. As firms start at different points depending on $u_{i,-1}$ and $v_{i,-1}$, each shock with its own persistence parameter, the path from initial to steady state employment will highly differ across firms.

The ex-post component is formed of two different shocks, one i.i.d. shock with expected value of zero, and a persistent one that follows an AR1 process with i.i.d. innovations $\epsilon_{i,a}$ and persistence ρ_w. To abstract the ex-post component from affecting the ex-ante one, we set the initial conditions of the persistent shock to $w_{i,-1} = 0$.

To more clearly identify the ex-post and ex-ante contributions we follow Pugsley et al. (2018) and derive the formula for the autocovariance, which using the key parameters of the statistical model allows for a clear identification of the contribution of both components. The autocovariance formula is given by

$$Cov[\ln n_{i,a}, \ln n_{i,a-j}] = \underbrace{\left(\sum_{k=0}^{a} \rho_u^k\right)\left(\sum_{k=0}^{a-j} \rho_u^k\right)\sigma_\theta^2 + \rho_u^{2(a+1)-j}\sigma_{\hat{u}}^2 + \rho_v^{2(a+1)-j}\sigma_{\hat{v}}^2}_{\text{Ex-ante component}}$$

$$\underbrace{+\,\sigma_\epsilon^2 \rho_w^j \sum_{k=0}^{a-j} \rho_w^{2k} + \sigma_z^2 \mathbf{1}_{j=0}}_{\text{Ex-post component}}$$

Derivation of the autocovariance formula is presented in Appendix C. The autocovariance is a function of variance and persistence parameters of both ex-ante and ex-post shocks. We calibrate the model for constrained and unconstrained firms by minimizing the sum of squared differences between the model and empirical autocovariance. Figure 9 in Appendix B.2 plots the model fit to the data for both types of firms. Table 2 presents the parameters resulting from the calibration strategy. Two key parameters of the model are ρ_u and σ_θ, as, together, they imply steady state heterogeneity exists. The point estimates imply a standard deviation of steady state employment for constrained firms of 0.66 and of 0.82 for unconstrained ones. This, again, highlights that there are differences between both types of firms that come from ex-ante conditions.

Figure 3 quantifies the importance of the ex-ante component for the variance of both constrained and unconstrained firms. For both types of firms, the ex-ante component contribution is above 70% even in the long-run. At birth, the ex-ante component contribution to the standard deviation is around 86% for unconstrained firms and 78% for constrained ones.

All the empirical evidence in this subsection suggests that ex-ante heterogeneity: 1) matters both in the short and in the long-run; 2) more strongly affects unconstrained than constrained firms. Moreover, the standard deviation of steady state level of employment

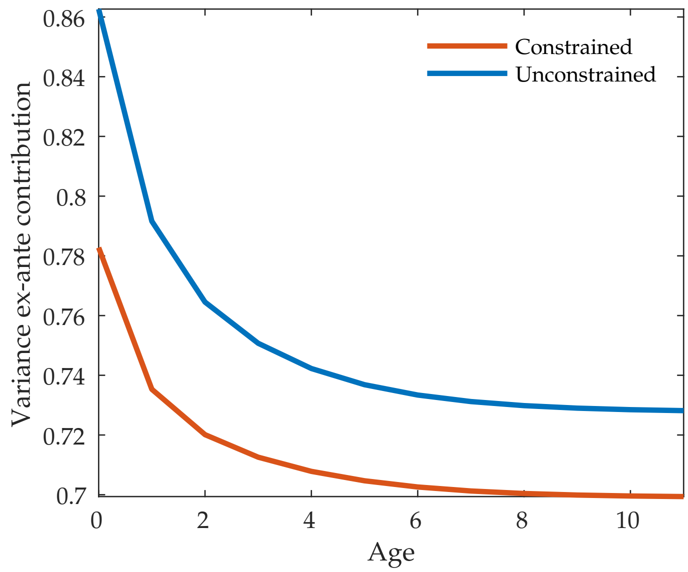

Figure 3: Variance ex-ante contribution. Values for constrained firms presented in orange, while blue stands for the unconstrained firms.

is higher for unconstrained firms. All this evidence may be indicative that unconstrained firms start closer to their steady state level of employment, while firms that still need to grow, exhaust their credit lines to reach their optimal size and so become constrained. To have a better understanding of these facts, we proceed to a general equilibrium, firm dynamics model.

4 Model

In this section we present a heterogeneous firms model with financial frictions which aims to reconcile the stylized facts above. We built on Khan & Thomas (2013) and introduce ex-ante heterogeneity through a permanent productivity component which can be interpreted as the firm's business potential. The rational is that this will break up the strong correlation between size and being financially constrained. Firms with lower permanent productivity will reach their optimal amount of capital earlier and will be unconstrained from then on, while firms with a higher potential will be constrained much longer as they take longer to grow into their potential.

4.1 Households

Households set their sequence of consumption, savings and labor choices according to the following maximization problem:

$$V(k) = \max_{c,l,k'} \left\{ U(c,l) + \beta \mathbb{E} V(k') \right\}$$

subject to:

$$k' + c = (1+r)k + \omega l + D$$

The first-order conditions will provide the prices that guarantee the markets clear:

$$U_c(c,l) = \beta \mathbb{E} \left[(1+r')U_c(c',l') \right]$$

$$U_l(c,l) = \omega U_c(c,l)$$

or, in the absence of aggregate risk,

$$(1+r) = \frac{1}{\beta}$$
$$\omega = \frac{U_l(c,l)}{U_c(c,l)}$$

We define the representative household utility function as

$$U(C,N) = \log(C) + \psi(1-N)$$

This way, the first order conditions become

$$(1+r) = \frac{1}{\beta}$$
$$\omega = \psi C$$

4.2 Production

4.2.1 Incumbents

Here we model the production sector of the economy. Firms maximize their value subject to collateral and no equity issuance constraints. Firms produce according to the following production function

$$y = \varphi k^{\alpha} l^{v}$$

where k and l are the inputs capital and labor used in the firm's production function and φ is the idiosyncratic productivity shock. We follow Pugsley et al. (2018) and assume the idiosyncratic shock has both a permanent and a transitory component

$$\ln \varphi_{i,t} = w_{i,t} + \theta_i$$

where $w_{i,t}$ is firm i's idiosyncratic transitory productivity shock, which follows an AR(1) process with persistence ρ_w and variance of innovations σ_ϵ^2. θ_i is the permanent productivity component, which firms draw at birth from a normall distribution with mean μ_θ and variance σ_θ^2

$$\theta_i \sim iid\left(\mu_\theta, \sigma_\theta^2\right)$$
$$w_{i,t} = \rho_w w_{i,t-1} + \varepsilon_{i,t} \quad \varepsilon_{i,t} \sim iid\left(0, \sigma_\varepsilon^2\right), \quad |\rho_w| \leq 1$$

Firm's total profits are given by

$$\pi = y - \omega l$$

where ω is the wage per unit of labor. The firm's labor decision is a static choice that can be found through the firm's first order condition

$$l(k, \varphi; \omega) = \left(\frac{\upsilon\varphi}{\omega}k^\alpha\right)^{\frac{1}{1-\upsilon}}$$

After the production stage, the firm may suffer an exogenous exit shock. The shock happens with probability π_d. So, after the production stage, the value of the firm is given by

$$V^1(x, \varphi) = \pi_d x + (1-\pi_d)V^2(x, \varphi)$$

If the firm survives the exit shock, at the end of the period it chooses debt b' and capital k'

to take to the next period and dividends to distribute this period D to maximize its value

$$V^2(x,\varphi) = \max_{k',b',D} \left[D + \mathbb{E}_{\varphi'|\varphi}\Lambda V^1(x',\varphi')\right] \tag{3}$$

s.t.:

$$D \equiv x + qb' - k' \geq 0$$

$$b' \leq \xi k'$$

$$x' \equiv x(k',b',\varphi') = y(l(k',z'),k',\varphi') - wl(k',\varphi') + (1-\delta)k' - b'$$

where ξ is the financial parameter that captures the financial frictions in the economy, x is the cash on hand that the firm starts the period today, which is given as the sum of profits plus the value of the non-depreciated capital minus the debt the firm has to pay back. q is the price of the bonds firms issue, with $\frac{1}{q} - 1$ being the equilibrium interest rate in the economy r. Λ is the firm discount factor. As the representative household is the owner of the firm, we assume $\Lambda = \beta$.

4.2.2 Entrants

We assume there is a fixed measure, M_e, of potential entrants. The potential entrants are uniformly distributed over their initial capital and debt combination, (k_0, b_0), and the initial productivity of a potential entrant, φ_0, follows the same process as the incumbents productivity. When a potential entrant decides to enter, it needs to pay a fixed entry cost, f_e. In this way, we are able to set up a simple binary problem of endogenous entry. Note that firm entry takes place at the end of a period, and actual entrants start operating in the next period, given their initial state, (k_0, b_0, φ_0).

4.3 Firm Level Decisions

To characterize the firms' decisions we divide the firms into three groups, following Khan and Thomas (2013):

1. **Unconstrained firms.** Firms that can implement the optimal amount of capital and guarantee that in the future they will never be constrained again.

2. **Constrained firms, type 1.** Firms that can implement the optimal amount of capital but not the minimums savings policy that guarantees they will never be constrained again in the future.

3. **Constrained firms, type 2.** Firms that are constrained and cannot implement the optimal amount of capital nor the minimums savings policy.

4.3.1 Unconstrained Firms

This group of firms can implement both the optimal amount of capital and the minimum savings policy that guarantees these firms will never be constrained in the future again. Given the absence of adjustment costs and the stochastic process for φ the optimal amount of capital is given by

$$\max_{k'} -k' + \beta \mathbb{E}_{\varphi'|\varphi} \left[(\pi(k', \varphi') + (1-\delta)k' \right]$$

So the optimal amount of capital solves the following equation

$$\beta \mathbb{E}_{\varphi'|\varphi} \left[\frac{\partial \pi}{\partial k'}(k', \varphi') \right] = 1 + \beta\delta - \beta$$

which is when the expected marginal productivity of capital is equal to the marginal cost of an extra unit. The minimum savings policy these firms implement guarantees they will never be constrained again. It is given by

$$B^*(\varphi_i) = \min_{\varphi_j} \tilde{B}(k^*(\varphi_i), \varphi_j)$$

where $\tilde{B}(k^*(\varphi_i), \varphi_j)$ is the minimum savings that guarantees that going from state φ_i to φ_j the firm is still able to implement the optimal amount of capital. It is given by

$$\tilde{B}(k^*(\varphi_i), \varphi_j) = \pi(k^*(\varphi_i), \varphi_j) + (1-\delta)k^*(\varphi_i) - k^*(\varphi_j) + q \min\left(B^*(\varphi_j), \xi k^*(\varphi_j)\right)$$

Given the optimal amount of capital and the minimum savings policy, the dividends distributed by the unconstrained firms are given by

$$D = x - k^* + qB^*$$

From the dividend constraint $D \geq 0$ we can extract the the minimum threshold for cash-on-hand that guarantees the firm is not constrained

$$\tilde{x} = k^* - qB^*$$

and the firms is constrained if $x \leq \tilde{x}$

4.3.2 Constrained Firms: Type 1

This firms can implement the optimal amount of capital, k^*, but not the optimal savings policy. As they may still be constrained in future states, they value internal financing more than households value dividends. As a results, for this type of firms, $D = 0$. The amount of debt is given by

$$b' = q(k^* - x)$$

It is type 1 if it can adopt the above amount of debt and capital and at the same time guaranteeing that it does not default in the next period.

4.3.3 Constrained Firms Type 2

For this firms we have to solve the problem numerically and find the optimal capital k', debt b' and dividends D policies.

4.4 Calibration

For now, we choose standard values from the literature, mostly following Khan & Thomas (2013). The purpose is mainly to illustrate the mechanism, particularly, the effect of the permanent component without yet taking a quantitative stance. The chosen set of parameters values are presented in table 3. All results in Section 5 are based on those parameters. Table 4 presents a preliminary model fit, when internally calibrating the standard deviation of both productivity components and the persistence of the transitory component. Using the simulated method of moments (SMM) we target a set of empirical moments of the firm distribution. Also, we keep some moments as external validity checks. Both models match the targeted moments relatively well, with the two productivity component model matching the size distribution slightly better. However, the out-of-sample model predictions are vastly different. The model with only a transitory component does not predict any constrained firms in the upper percentiles of the size distribution. On the other hand, the model with both, a transitory and a permanent component, manages to fit the untargeted moments quite well. This illustrates the ability of a model with two productivity components to produce large constrained firms, as well as small unconstrained firms as we find in the data.

Table 3: Parameter values

Parameter	Description	Value	Source
β	Discount factor	0.96	Khan & Thomas (2013)
α	Returns on capital	0.30	Khan & Thomas (2013)
η	Returns on labor	0.60	Khan & Thomas (2013)
δ	Depreciation rate	0.065	Khan & Thomas (2013)
ψ	Labour preference	2.15	Khan & Thomas (2013)
π_d	Exogenous probability of exit	0.1	Khan & Thomas (2013)
μ_{ke}	Relative size of entrants	0.10	Khan & Thomas (2013)
μ_θ	Average of permanent productivity	0	
μ_w	Average of transitory shock	0	
ξ	Collateral constraint	0.82	Jo & Senga (2019)
σ_θ	Standard deviation of permanent productivity	0.118	
ρ_w	Persistence of transitory shock	0.659	Khan & Thomas (2013)
σ_w	Standard deviation of transitory shock	0.118	Khan & Thomas (2013)
σ_{ke}	Standard deviation of entrants	0	Khan & Thomas (2013)

Table 4: Preliminary calibrated model fit

Moment	Data	Model trans. + perm. shock	Model transitory shock
Targeted moments			
Percentage of constrained firms	0.23	0.21	0.22
Size of 90th-percentile vs. median	9.44	9.32	11.09
Size of constrained firms 90th-percentile vs. median	7.35	7.75	8.34
Size of unconstrained firms 90th-percentile vs. median	9.67	9.25	8.65
Capital share of constrained firms	0.12	0.17	0.12
Untargeted moments			
Share of constrained firms in top 10% vs. bottom 20%	0.36	0.23	0
Percentage of constrained firms in top 1%	0.09	0.09	0

5 Discussion

In this section we start by discussing how the model fits the three stylized facts presented in Section 3 and then explore the impacts of accounting for different permanent productivity components across the distribution of firms.

We first start by illustrating that in a standard heterogeneous firms model with only a transitory productivity shock, the model cannot account for the independent effects of size and financial constraints on the investment response to a shock. When adding the permanent productivity shock to a standard model, we can now replicate the empirical findings from Section 3.1, with both size and financial frictions playing an important role in explaining the firms' response to an aggregate productivity shock.

Second, we show that, while a model that incorporates both transitory and permanent components of the productivity process can generate constrained firms across the entire distribution, similar to what Figure 1 suggests, a model with only a transitory component fails to account for this.

We then proceed to access the importance of accounting for the permanent productivity component and compare the aggregate effects of a 1% increase in productivity. Overall, the model with the permanent and transitory productivity components is going to generate a larger share of capital in constrained firms as well as a higher elasticity of constrained firms to the productivity shock. These two effects together, generate a larger increase of capital when compared to a standard model with only a transitory productivity shock.

5.1 Replicating the stylized facts

Size and financial constraints

In Section 3.1 we showcase how size and financial frictions both affect firms' response to aggregate shocks and how the two channels appear to be independent of each other.

We now check how our benchmark model performs in replicating this stylized fact, and compare it to a similar model without the permanent component of the idiosyncratic productivity process.

To do this, we shock our steady state equilibrium by increasing aggregate productivity by 1%. This shock is temporary, reverting to its steady state value, with a persistence of 0.8. We then use the model generated data to estimate the following regression

$$\Delta \ln K_{it} = \beta_0 + \beta_1 \ln K_{it-1} + \beta_2 constrained_{it-1} + \epsilon_{it} \tag{4}$$

where $\Delta \ln K_{it}$ is firm i growth rate of capital from the steady to the period following the shock, $\ln K_{it-1}$ is the log of capital in the steady state, our proxy for size of the firm in the model, and $constrained_{it-1}$ is a dummy variable equal to one if the firm was type 2 constrained firm in the steady state.[6]

Results are presented in Table 5. When adding a permanent productivity component to the model the coefficient associated with the size of the firm is reduced by half and the effect of constrained becomes stronger. This is explained by the much weaker correlation between constrained and size. In fact, while in the model with only a transitory productivity component the correlation is -0.02, indicating that smaller firms are more likely to be constrained, the correlation coefficient becomes 0.05 when adding the permanent component. This is justified by the large mass of smaller unconstrained firms and larger constrained firms generated by the addition of this component.

	Model transitory shock		Model trans. + perm. shock	
	(1)	(2)	(3)	(4)
Size	0.014	0.019	0.007	0.009
Constraint	-	0.010	-	0.013

Table 5: Regression with model data.

[6]We use the type two constrained firms in the model as these are the hard constrained firms and more comparable to the ones with zero potential credit in the data. Although, we equally test the regression accounting for both type 1 and type 2 constrained firms, and results are robust to it.

Constrained firms across the distribution

In Section 3.2 we highlight that constrained firms are found across the entire distribution of firms. As illustrated in Figure 1, even at the top of the distribution in terms of size, close to 10% of the firms are financially constrained.

Figure 4 presents the model generated share of constrained firms at each percentile of the size distribution. While the transitory model concentrates the constrained firms more at the bottom of the distribution, the model with both the permanent and transitory components generates larger constrained firms and smaller unconstrained firms, more in line with our second stylized fact.

This is explained by the fact that we have larger firms that are still growing to reach their steady state capital and that are still constrained. At the same time, the model with the two components, accounts for a larger share of small firms that are born at or close to their steady state level of capital. This justifies the lower share of small constrained firms than in the model with only the transitory shock, and also more in line with the data.

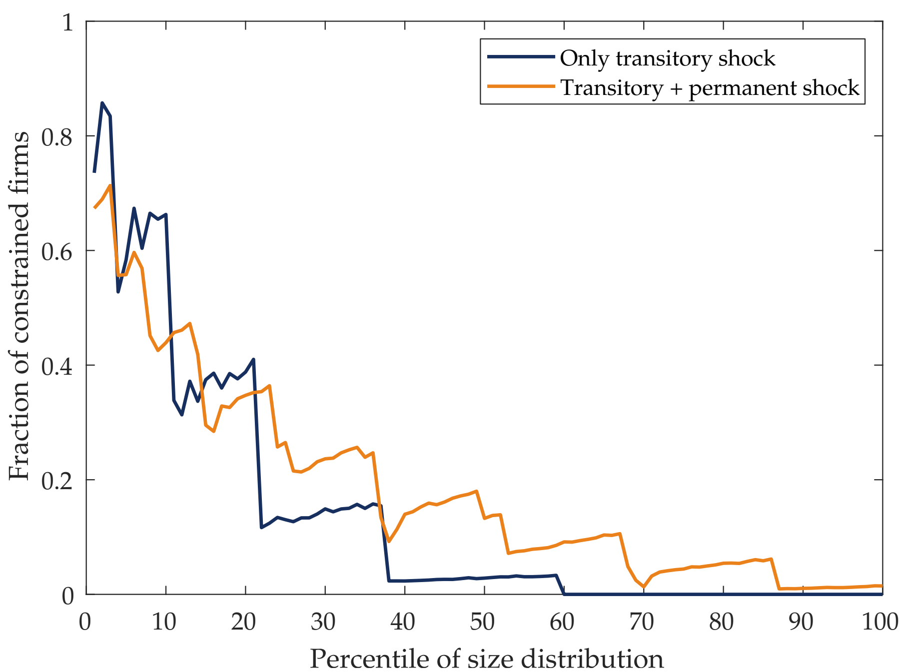

Figure 4: Share of constrained firms across the distribution.

5.2 Aggregate effects

We now proceed to access the aggregate consequences of having a distribution of constrained firms more in line with the data. In Figure 5 we have the evolution of the aggregate values of consumption, capital and output in response to a aggregate productivity shock. From period 1, when the economy is in the steady state, to period 2, aggregate productivity goes up by 1%. It then decays and goes back to its steady state level.

In response to the shock, capital goes up as firms see their profits increase and anticipate productivity to stay higher than the steady state value for the following periods. As capital goes up, output equally increases. Consumption falls at first, due to the strong increase in investment in period 2, going up in period 3.

In Figure 5 it is also presented the comparison between the benchmark model with

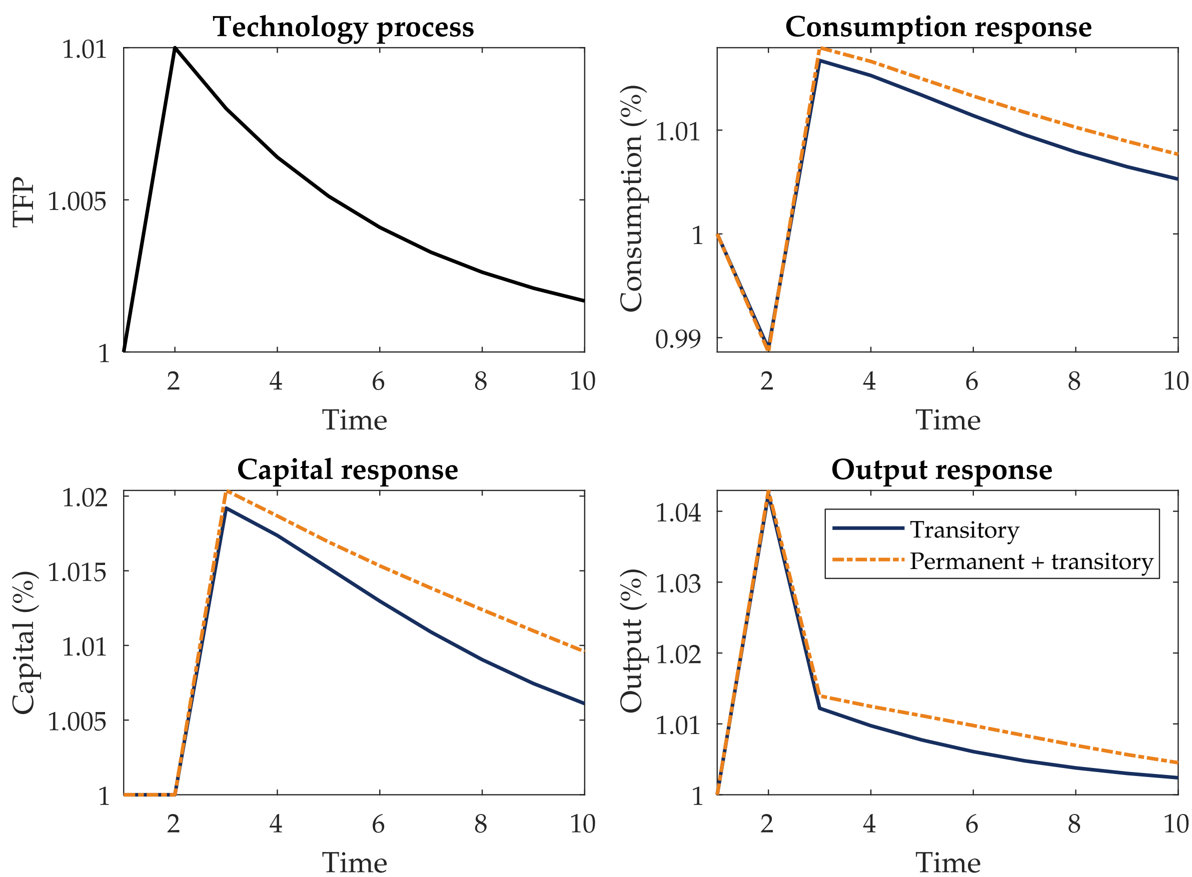

Figure 5: IRFs to the aggregate productivity shock.

both the permanent and transitory productivity components and a model with just the transitory component. In our benchmark model capital response is 6% stronger and more persistent when comparing to the model with only the transitory productivity component. This translates into a stronger output response, being 14% higher in period 3 in our benchmark economy.

Two mechanisms explain the stronger response of capital in the economy with the permanent productivity component. First, as can be seen in the right panel of Figure 6, constrained firms have a larger share of total capital. This is a direct consequence of the differences in the distribution of constrained firms, as highlighted in Figure 4, with the model with the permanent component being able to generate larger constrained firms. As constrained firms have a stronger response to the shock than unconstrained ones, this explains in part the stronger response of capital.

The second mechanism that explains the stronger capital response can be seen in the left panel of Figure 6. Despite constrained firms having a higher elasticity to the shock than unconstrained firms, the response is not equal for all constrained firms. On average, larger constrained firms have a stronger response to the shock. As the model with a per-

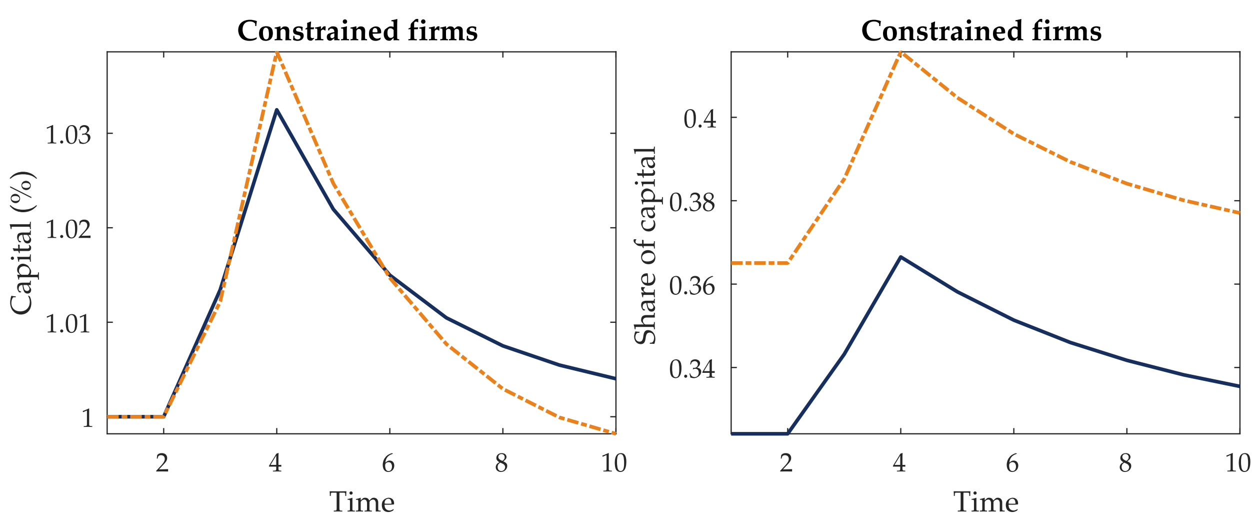

Figure 6: The left panel presents the constrained firms capital response to the shock.. The right panel presents the constrained firms share of total capital..

manent productivity component generates larger constrained firms, the average capital response by constrained firms becomes higher, contributing to a larger aggregate capital response.

6 Conclusion

In this paper we begin by documenting the importance of financial frictions for firms cyclicality and how this channel is independent of the size effect highlighted in recent literature. Empirically we show that at any point of the firm distribution there are both constrained and unconstrained firms, which justifies the weak correlation between size and financial frictions. We conclude the empirical section by showing that ex-ante conditions matter differently for constrained and unconstrained firms.

This motivates us to build a standard firm dynamic model, adding a permanent productivity component. We showcase that by adding this extra component to the productivity process helps us match the distribution of constrained firms across the size distribution, breaking the typical strong correlation between financial constraints and size, generating a large mass of small unconstrained and large constrained firms.

We conclude the paper by illustrating that this mechanism has implications for aggregate responses to shocks. We find aggregate capital and output to respond more to a productivity shock in a model that accounts for ex-ante firm heterogeneity than a model where idiosyncratic productivity is purely driven by a transitory component. This is explained by two mechanisms: 1) the larger share of capital in constrained firms, who have a higher elasticity to the shock; 2) the larger elasticity of large constrained firms relative to small constrained ones.

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A Construction of constrained measures

Financial constraints are most commonly conceived as a supply side phenomenon. Firms that could potentially obtain credit in perfect credit markets are unable to do so due to asymmetric information considerations on the supply side. For example, a firm that has a profitable investment project that requires external financing cannot realise it as the bank is not satisfied with the creditworthiness of that firm. This may happen either via the price dimension - an interest rate that is too high - or on the quantity dimension - the credit is denied altogether. We aim to identify constrained firms along the quantity dimension, using the credit information for each firm. Given that credit allowances are changing over time, this provides us with a time-varying and firm-specific measure for being financially constrained. It shall be noted however, that while credit information offers a far more detailed notion of a firm being constrained, relative to financial characteristics such as leverage or liquidity, it is still a proxy. Hence, taking this into account, we consider a wide range of binary and continuous measures for identifying whether a firm is constrained.

Binary measures In our baseline definition a firm is credit constrained at time t, if it has no potential credit available at time t:

$$constrained_1 := \mathbf{1}_{\text{Potential credit}_t = 0}.$$

As outlined above, potential credit summarizes all the irrevocable commitments by credit-granting institutions. Even though this measure enables an understanding of whether firms have drawn down their credit lines and are thus potentially constrained it also encompasses a lot of noise. One problem might be that specific banks may not grant credit lines. This would show up as potential credit = 0 in the data. But this does by no means make these firms credit constrained. They just simply do not have credit lines, which is quite common. In order to address this issue, we consider three approaches. First, we test the robustness of our results dropping all cases in which potential credit is zero throughout. Second, we estimate the panel regression using firm fixed effects, which will control for those cases by construction. Third, we introduce the following adjusted measure, demanding that potential credit was positive in the previous period $t-1$ and hence, the firm seem to have hit the credit line:

$$constrained_2 := \mathbf{1}_{\text{Pot. credit}_t = 0 \text{ \& Pot. credit}_{t-1} > 0}.$$

Another issue when relying only on potential credit, might be that although, firms have exhausted their committed credit line, they could still successfully apply for a short- or long-term loan. In order to cope with this issue, we introduce two further measures. The third measure re-defines those firms as unconstrained, which managed to secure short- or long-term credit, while potential credit was zero:

$$constrained_3 := \mathbf{1}_{\text{Pot. credit}_t = 0 \text{ \& } \Delta\text{Effective credit}_{t+1} < 0}.$$

The fourth measure augments the baseline definition by specifically considering those firms as being constrained for which overdue credit is growing:

$$constrained_4 := \mathbf{1}_{\text{Pot. credit}_t = 0 \text{ \& } \Delta\text{Overdue credit}_{t+1} > 0}.$$

The rational behind this definition is that growth in overdue credit is likely a signal for a firm in bad financial shape.

While the measures presented so far are conceptually in the spirit of a firm hitting the credit constraint and thus being strictly constrained, it might also be that a firm is granted credit, yet the amount is not sufficient to finance any planned investment. The following measure aims to capture this notion of potentially constrained firms:

$$constrained_5 := \mathbf{1}_{\text{Total Credit}_{t+1} > \text{Total Credit}_t + \text{Potential Credit}_t}.$$

Continuous measures In order to account for different levels of severity of financial constraints, we also introduce two continuous measures. The first one is defined as the ratio of potential credit and cash to total liabilities of the firm, thereby accounting for the fact that a firm might have enough cash making it financially unconstrained, despite being credit constrained:

$$constrained_6 := \frac{\text{Potential credit}_t + \text{ Cash}_t}{\text{Liabilities}_t}.$$

Our final measure can be interpreted as the continuous counterpart to the third measure taking into account potential credit and ex-post changes to short- and long-term credits:

$$constrained_7 := \frac{\text{Potential credit}_t + \Delta\text{Short- and long-term credit}_{t+1}}{\text{Liabilities}_t}.$$

Table 6 reports correlations between all measures. By construction, some correlation coefficients can be relatively low, as some measures represent subsets of others. In fact, correlation coefficients between those binary measures augmenting the baseline measure can be interpreted as the fraction of the subset relative to the baseline definition of being strictly constrained. The correlation coefficients between binary and continuous measures are close to zero, as the variance in the continuous measure is too high to be captured by a binary measure, underlining the importance of including continuous measures in the analysis. The negative sign is simply due to the definition of the continuous measures, as higher values are an indication for being less constrained.

In Table 7 we report medians for different size related variables for the universe of firms in our panel dataset. Following Crouzet & Mehrotra (2020), we split firms in size

Constrained measure	(1)	(2)	(3)	(4)	(5)	(6)	(7)
(1) $\mathbf{1}_{\text{Potential credit}_t=0}$	1						
(2) $\mathbf{1}_{\text{Pot. credit}_t=0\ \&\ \text{Pot. credit}_{t-1}>0}$	0.31***	1					
(3) $\mathbf{1}_{\text{Pot. credit}_t=0\ \&\ \Delta\text{Effective credit}_{t+1}<0}$	0.82***	0.24***	1				
(4) $\mathbf{1}_{\text{Pot. credit}_t=0\ \&\ \Delta\text{Overdue credit}_{t+1}>0}$	0.26***	0.12***	0.28***	1			
(5) $\mathbf{1}_{\text{Pot. credit}_t=0} + ...$	0.73***	0.23***	0.60***	0.19***	1		
(6) $\frac{\text{Pot. credit}_t + \text{Cash}_t}{\text{Liabilities}_t}$	-0.00	-0.00	-0.00	-0.00	-0.00	1	
(7) $\frac{\text{Pot. credit}_t + \Delta\text{Short/Long term credit}_{t+1}}{\text{Liabilities}_t}$	-0.00*	-0.00	-0.00**	-0.00	-0.00	0.33***	1

Table 6: Correlation between different measures for being constrained

bins based on their total asset amount. Furthermore, we document descriptive statistics for the mutually exclusive subsets of constrained and unconstrained firms, based on the baseline measure. We can observe that the median financially constrained firm has less assets, less turnover, is younger and employs less people compared to the median unconstrained firm. Table 8 reports medians of financial variables for size bins and the subset of constrained and unconstrained firms. Financial variables are expressed in terms of ratios relative to the total amount of assets a firm owns. While the median of smaller firms tends to have a higher leverage ratio, yet also a higher liquidity ratio, there is hardly any difference between the median constrained and the median unconstrained firm. This is suggestive of the fact, that when controlling for size, there are constrained and unconstrained firms across the distribution of financial variables.

Additional Tables

Size group	0 - 90th	90th-99th	99-99.5th	>99.5th	constrained	unconstrained
Assets (€ mio.)	0.25	4.64	23.07	77.09	0.21	0.39
Sales (€ mio.)	0.02	1.01	2.06	0.11	0.00	0.07
Turnover (€ mio.)	0.02	2.50	9.81	15.91	0.12	0.31
Age	12	20	22	21	11	14
Employees	4	18	50	56	2	5

Table 7: Size and age

Size group	0 - 90th	90th-99th	99-99.5th	>99.5th	constrained	unconstrained
Dividends (€ mio.)	0.00	0.04	0.34	1.28	0.00	0.00
Fixed tangible (€ mio.)	0.04	0.92	4.20	8.23	0.03	0.06
Investment (€ mio.)	-0.00	-0.01	-0.03	-0.02	-0.00	-0.00
Financial investments (€ mio.)	0.00	0.00	0.08	4.77	0.00	0.00
Equity (€ mio.)	0.06	1.43	7.42	26.18	0.04	0.10
Liabilities (€ mio.)	0.18	3.06	15.28	47.67	0.16	0.25
Total income (€ mio.)	0.19	2.63	10.93	25.18	0.12	0.33
EBIT (€ mio.)	0.01	0.12	0.65	2.30	0.00	0.01
Leverage	0.20	0.25	0.21	0.12	0.20	0.20
Liquidity ratio	0.06	0.02	0.01	0.01	0.04	0.05
Potential credit (€ mio.)	0.00	0.10	0.51	1.36	0.00	0.02
Effective credit (€ mio.)	0.04	1.12	4.90	9.98	0.03	0.06
Bank relationships	2	3	4	4	1	2

Table 8: Financial characteristics

	Sales growth						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
[90,99] × GDP growth	0.220	0.267*	0.247*	0.308*	0.238	0.211	0.296*
	(0.150)	(0.151)	(0.150)	(0.170)	(0.151)	(0.150)	(0.170)
[99,99.5] × GDP growth	-1.012**	-0.968**	-0.986**	-1.419***	-0.985**	-1.023**	-1.407***
	(0.446)	(0.446)	(0.446)	(0.526)	(0.446)	(0.446)	(0.526)
[99.5,100] × GDP growth	-1.708***	-1.650***	-1.674***	-1.775***	-1.678***	-1.725***	-1.824***
	(0.472)	(0.473)	(0.472)	(0.570)	(0.472)	(0.472)	(0.571)
Constrained × GDP growth		0.186*					
		(0.098)					
Constrained Adj × GDP growth			0.296				
			(0.222)				
Constrained Due× GDP growth				0.936**			
				(0.432)			
Const + potential const × GDP growth					0.111		
					(0.086)		
Constrained continuous × GDP growth						-0.155***	
						(0.056)	
Constrained continuous 2 × GDP growth							-0.186***
							(0.061)
Observations	803,742	803,742	803,742	708,881	803,742	803,326	708,598
R-squared	0.023	0.024	0.024	0.025	0.023	0.023	0.024
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE × GDP growth	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 9

	Employees growth						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
[90,99] × GDP growth	0.003	0.007	0.014	-0.107**	0.001	-0.001	-0.110**
	(0.046)	(0.047)	(0.046)	(0.052)	(0.047)	(0.046)	(0.052)
[99,99.5] × GDP growth	-0.003	-0.003	0.006	-0.154	0.005	-0.011	-0.151
	(0.130)	(0.131)	(0.130)	(0.152)	(0.130)	(0.130)	(0.152)
[99.5,100] × GDP growth	-0.328**	-0.327**	-0.316**	-0.395**	-0.312**	-0.339**	-0.411**
	(0.150)	(0.150)	(0.150)	(0.176)	(0.150)	(0.150)	(0.176)
Constrained × GDP growth		-0.021					
		(0.029)					
Constrained Adj × GDP growth			0.131*				
			(0.067)				
Constrained Due× GDP growth				0.318***			
				(0.121)			
Const + potential const × GDP growth					0.066**		
					(0.026)		
Constrained continuous × GDP growth						-0.80***	
						(0.013)	
Constrained continuous 2 × GDP growth							-0.088***
							(0.016)
Observations	1,287,296	1,287,296	1,287,296	1,126,902	1,287,296	1,286,004	1,125,990
R-squared	0.024	0.024	0.025	0.026	0.025	0.024	0.025
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE × GDP growth	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 10

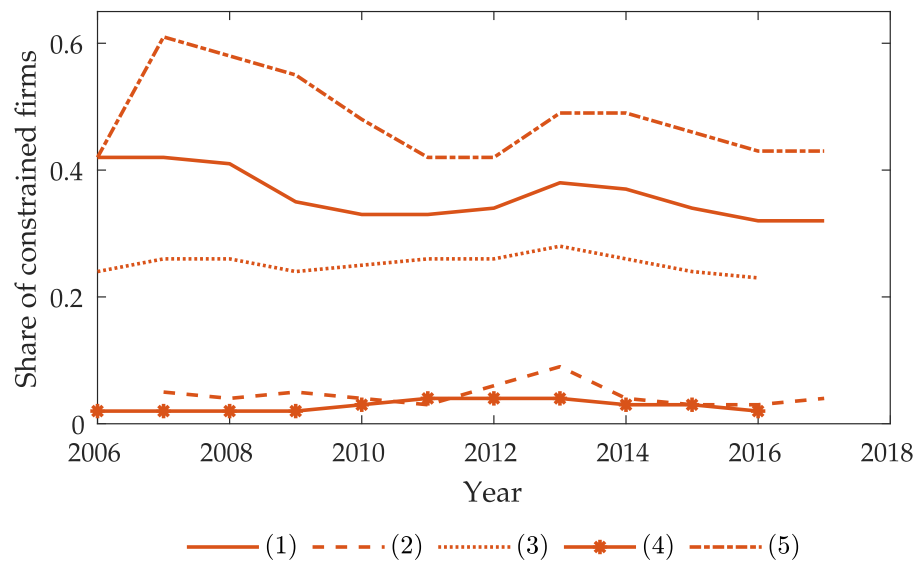
	Constrained binary			
	(1)	(2)	(3)	(4)
Age	-0.05***			
	(0.000)			
Total assets		-0.05***		
		(0.001)		
Leverage			0.03***	
			(0.000)	
Liquidity ratio				0.01***
				(0.000)
Constant	0.36***	0.36***	0.36***	0.36***
Observations	1,765,288	1,765,288	1,764,947	1,764,947
R-squared	0.011	0.000	0.000	0.000

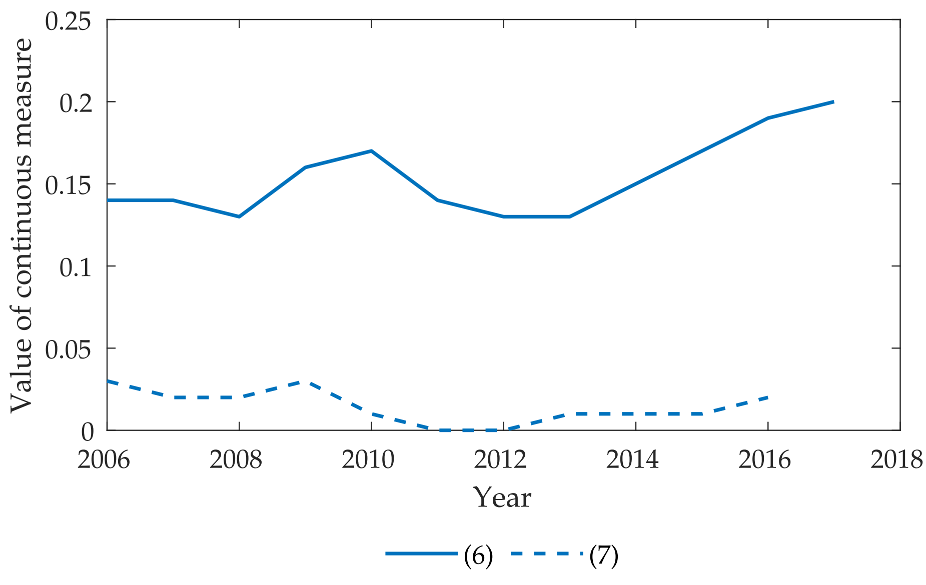
Notes: Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

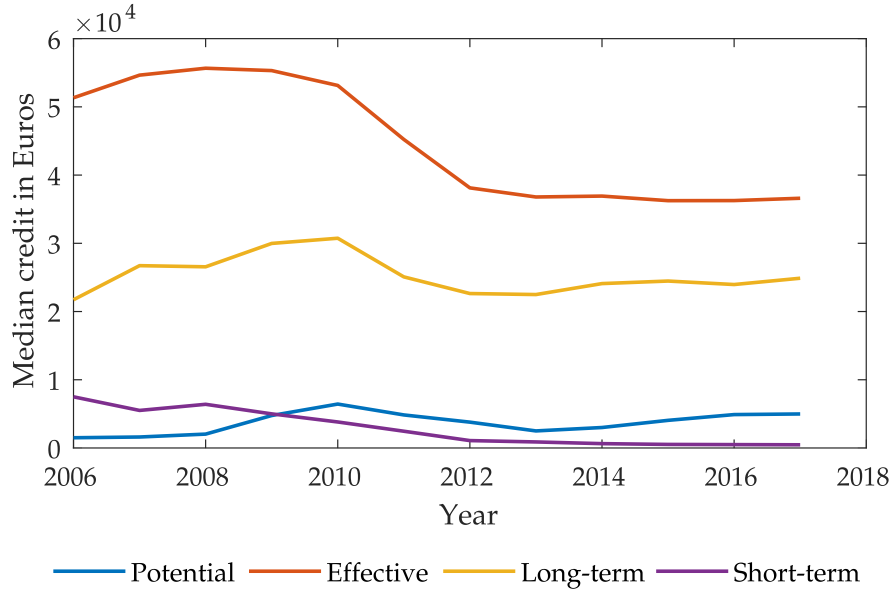
Table 11

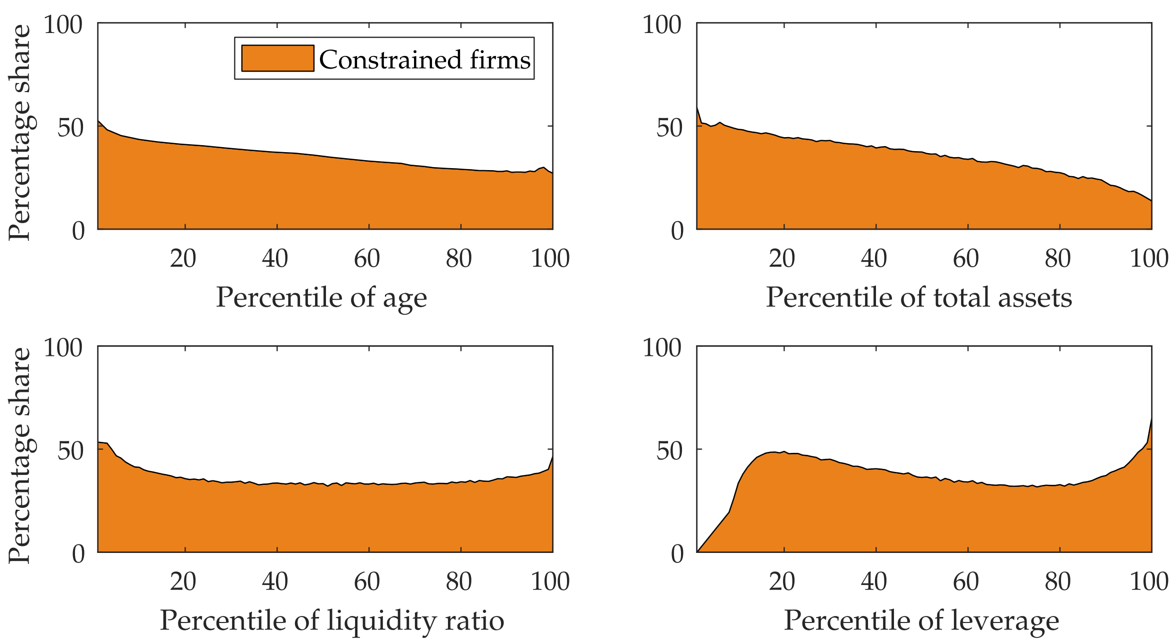
B Additional Figures

B.1 Descriptives






×10^4
6
5
4
3
2
1
0
Median credit in Euros
2006
2008
2010
2012
2014
2016
2018
Year
Potential
Effective
Long-term
Short-term


Percentage share
100
50
0
Constrained firms
20
40
60
80
100
Percentile of age
Percentile of total assets
Percentile of liquidity ratio
Percentile of leverage

B.2 Statistical Model

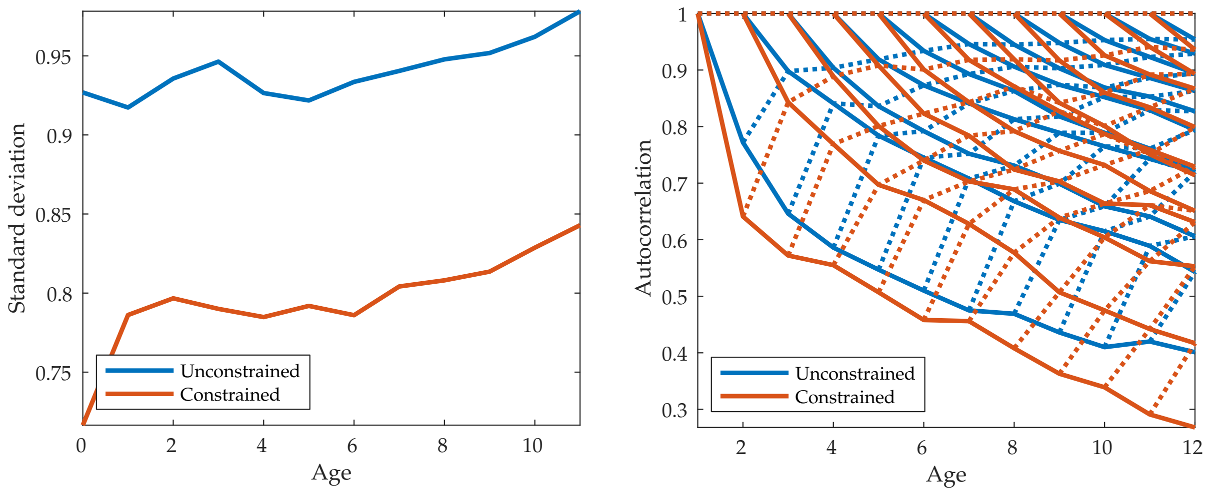
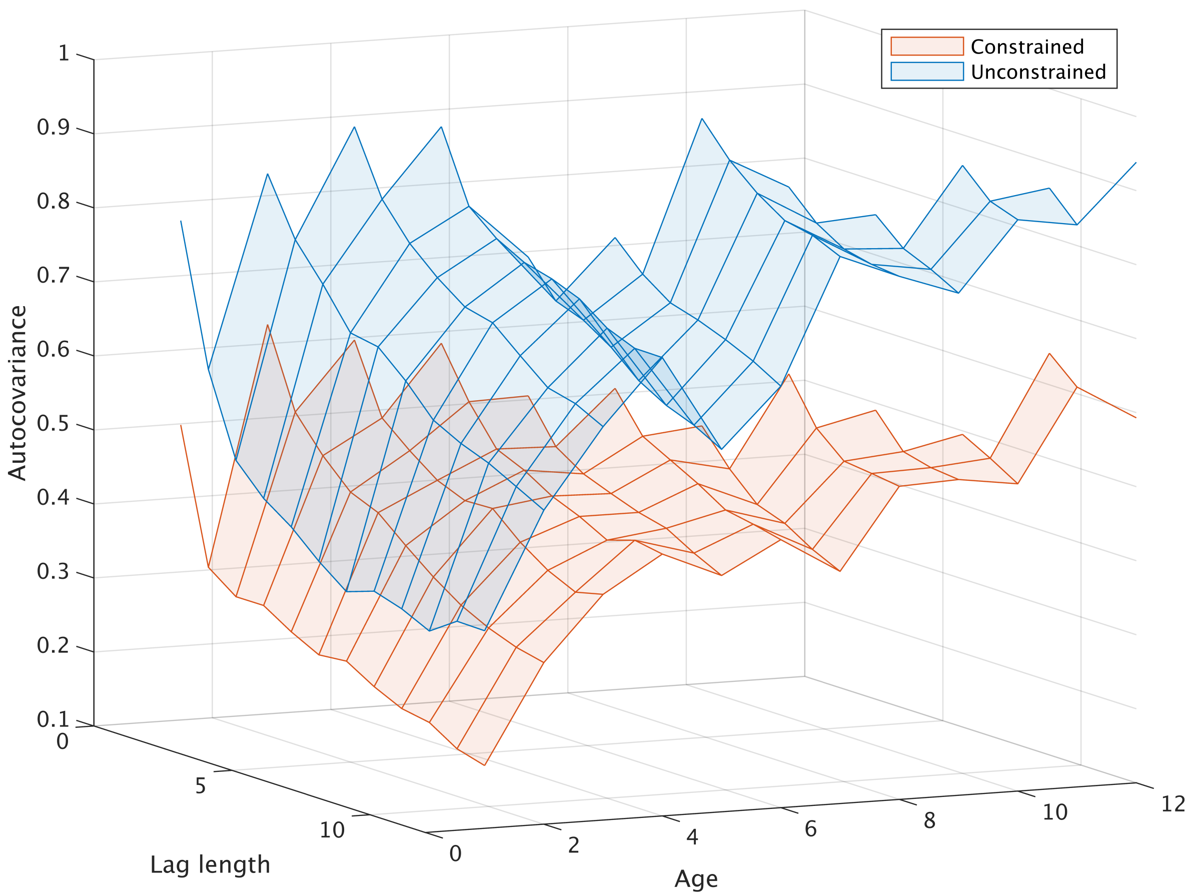


Figure 7: The left panel presents the standard deviation of log employment by age, after controlling for sector and year fixed effects. The right panel presents the autocorrelation of log employment between ages a and $h \leq a$. Across lines h changes, while a changes along the lines. Orange lines stand for constrained firms, while blue lines represent unconstrained firms.



Constrained
Unconstrained
Autocovariance
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
5
10
Lag length
0
2
4
6
8
10
12
Age

Figure 8

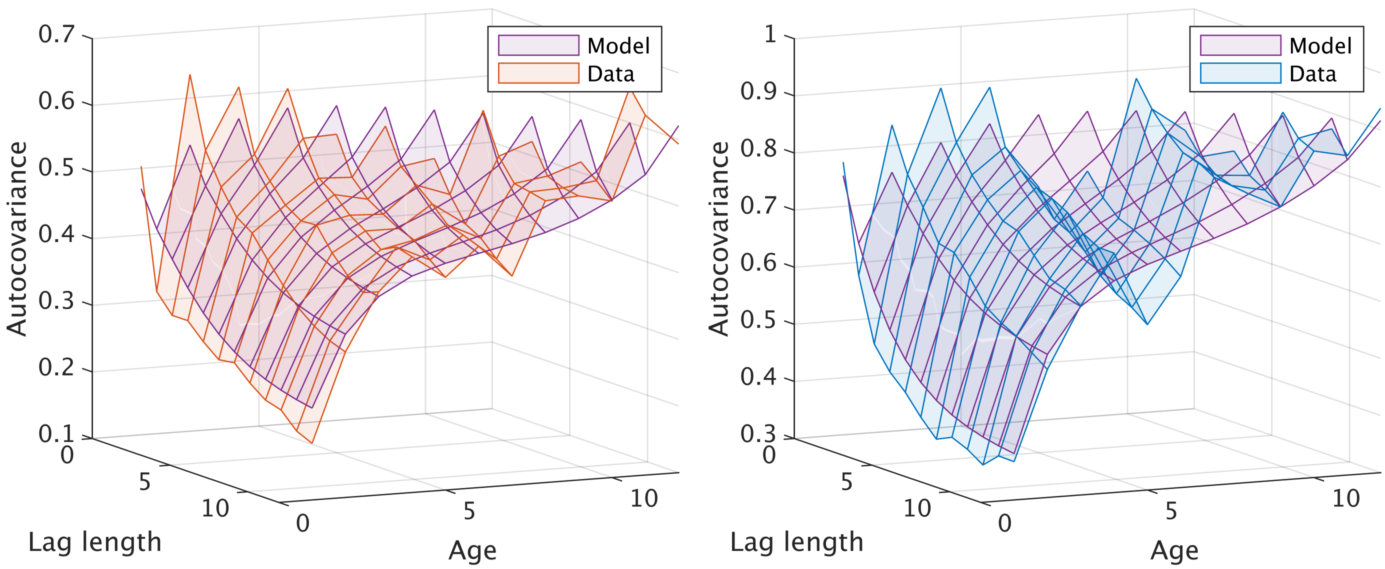


Figure 9: Empirical and model autocovariance for constrained firms (left panel) and unconstrained firms (right panel). Age a in the x-axis and lag length h in the y-axis.

C Statistical Model Derivation

Write stochastic processes in MA representation:

$$
\begin{aligned}
u_{i,t} &= \rho_u^{t+1} u_{i,-1} + \sum_{k=0}^{a} \rho_u^k \theta_i \\
v_{i,a} &= \rho_v^{a+1} v_{i,-1} \\
w_{i,a} &= \sum_{k=0}^{a} \rho_w^k \varepsilon_{i,a-k} = \sum_{k=0}^{j-1} \rho^k \varepsilon_{i,a-k} + \rho_v^j \sum_{k=0}^{a-j} \rho_v^k \varepsilon_{i,a-j-k} \quad 0 \leq j \leq a
\end{aligned}
$$

So the level of log employment of firm i at age a is:

$$
\ln n_{i,a} = \rho_u^{a+1} u_{i,-1} + \sum^{a} \rho_u^k \theta_i + \rho_v^{a+1} v_{i,-1} + \sum_{i=1}^{j-1} \rho^k \varepsilon_{i,a-k} + \rho_v^j \sum_{i=1}^{a-j} \rho_v^k \varepsilon_{i,a-j-k} + z_{i,a}
$$

Then the autocovariance of log employment at age a and $a-j$ for $j \geq 0$ is:

$$
\begin{aligned}
\text{Cov}\left[\log n_{i,a}, \log n_{i,a-j}\right] = \left(\sum_{k=0}^{a} \rho_u^k\right) \sigma_\theta^2 \left(\sum_{k=0}^{a-j} \rho_u^k\right) + \rho_u^{a+1} \sigma_{\tilde{u}}^2 \rho_u^{a-j+1} + \rho_v^{a+1} \sigma_{\tilde{v}}^2 \rho_v^{a-j+1} \\
+ \text{Cov}\left[\rho_v^j \sum_{k=0}^{a-j} \rho_v^k \varepsilon_{i,a-j-k}, \sum_{k=0}^{a-j} \rho_v^k \varepsilon_{i,a-j-k}\right] + \mathbf{1}_{\{j=0\}} \sigma_z^2 \\
= \sigma_\theta^2 \left(\sum_{k=0}^{a} \rho_u^k\right) \left(\sum_{k=0}^{a-j} \rho_u^k\right) + \sigma_{\tilde{u}}^2 \rho_u^{2(a+1)-j} + \sigma_{\tilde{v}}^2 \rho_v^{2(a+1)-j} + \sigma_\varepsilon^2 \rho_w^j \sum_{k=0}^{a-j} \rho_w^{2k} + \mathbf{1}_{\{j=0\}} \sigma_z^2
\end{aligned}
$$