Matrix Elements in lattice QCD

Christian Shultz

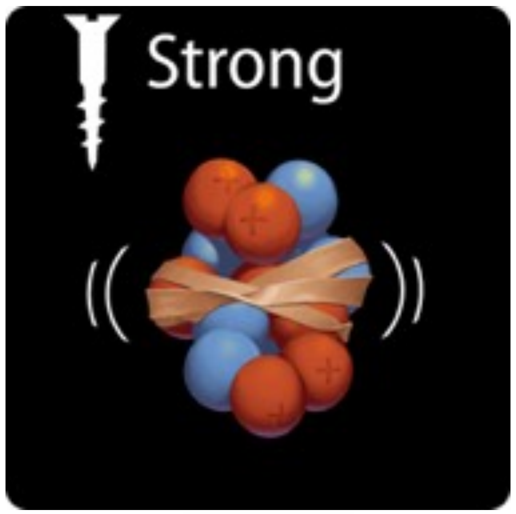
Quantum Chromodynamics

this is the Lagrangian

$$\mathcal{L}_{QCD} = \bar{\Psi}\left(i\not{D} - m\right)\Psi - \frac{1}{4}\mathrm{Tr}\left(GG\right)$$

quarks & gluons
are 'colored'

color is a type of charge
just like + and -



nuclei

muon magnetic moment

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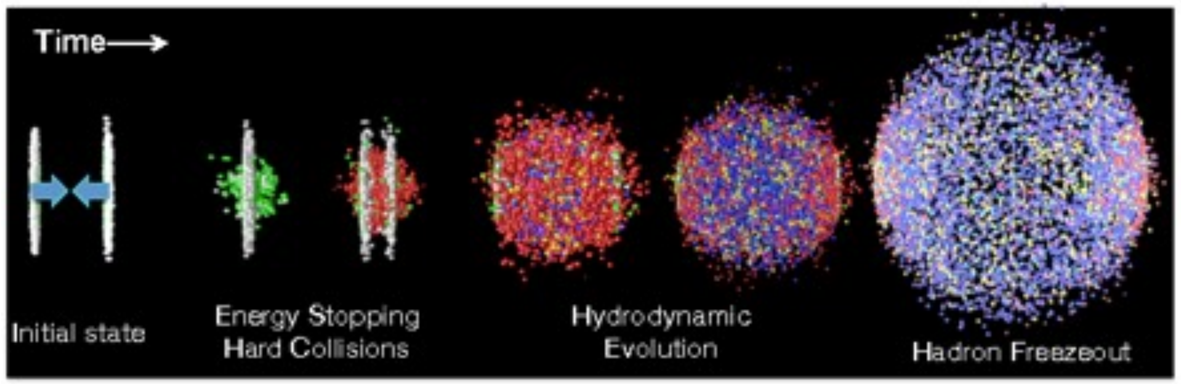
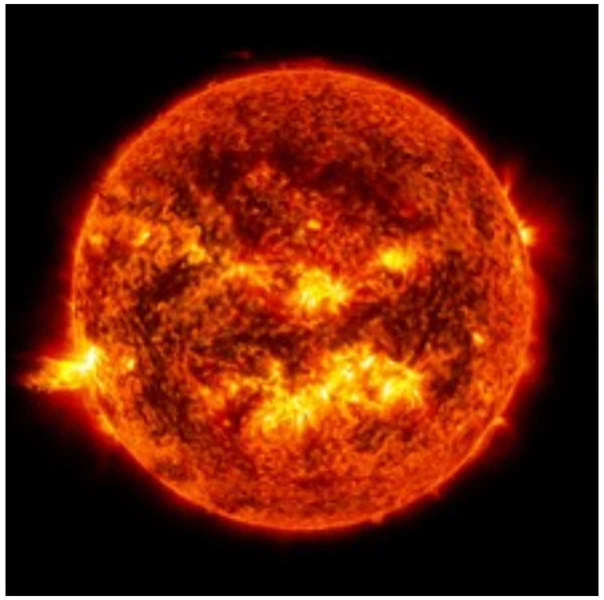


heavy ion collisions



stellar evolution

some background

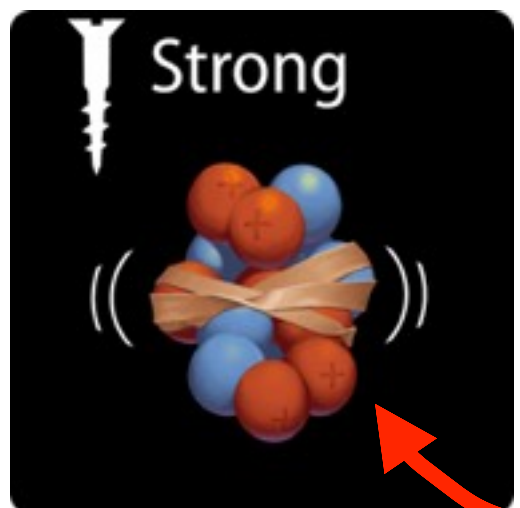
$$\mathcal{L}_{QCD} = \bar{\Psi}\left(i\not{D} - m\right)\Psi - \frac{1}{4}\mathrm{Tr}\left(GG\right)$$

colorless states are called hadrons

We only observe colorless states in nature

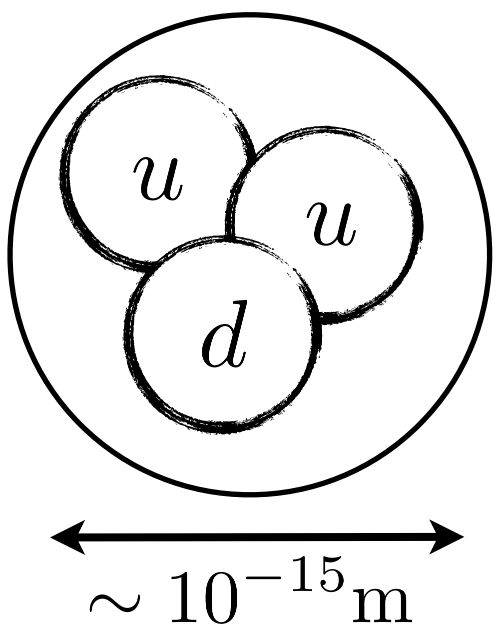
quarks come in six flavors

u up	***c*** charm	***t*** top
d down	***s*** strange	***b*** bottom



nuclei

a proton



the force between quarks increases linearly with distance

yet to be proven analytically

this talk

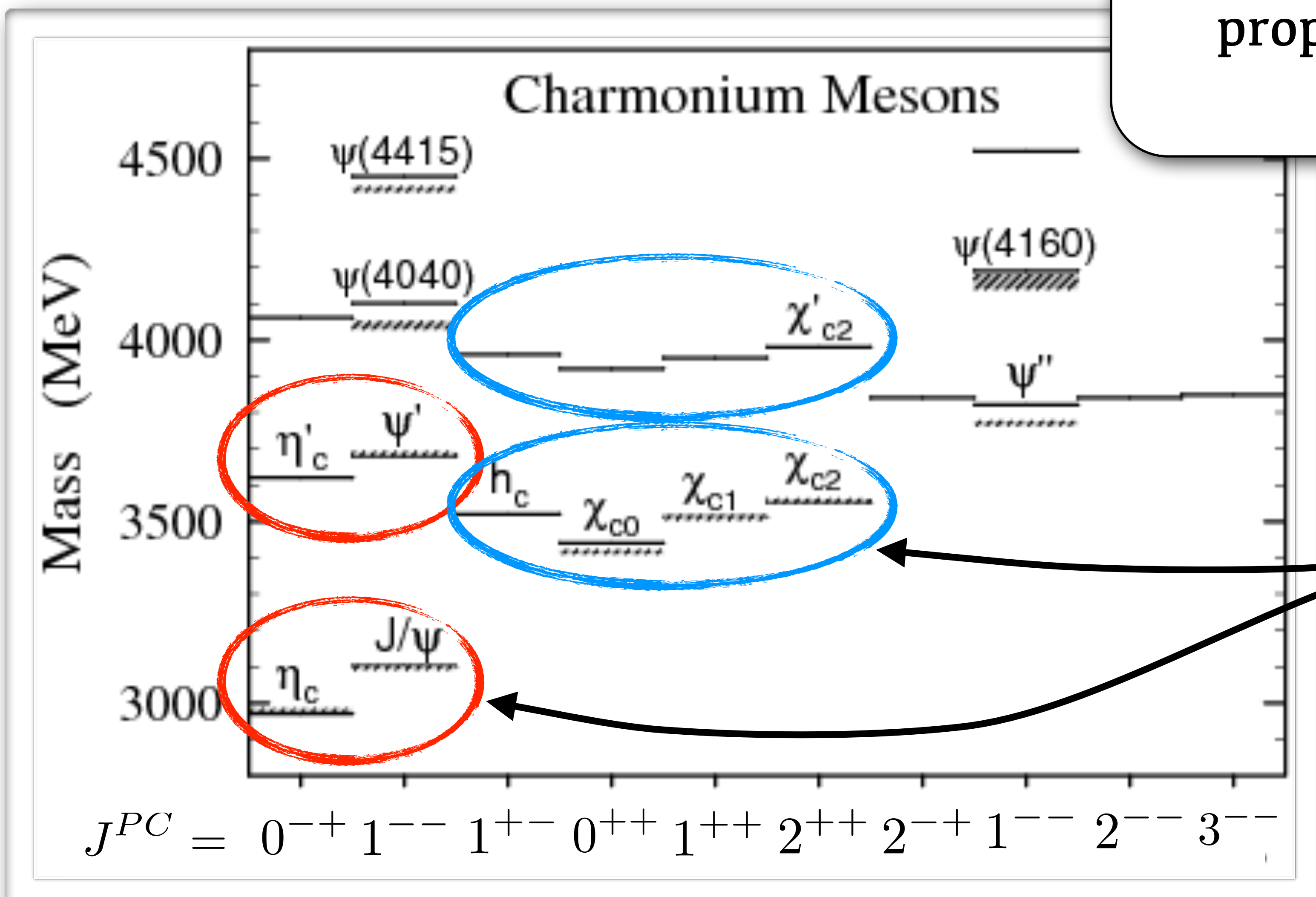
QCD → lattice QCD → observables

- ☐ teach you some physics
- ☐ present a puzzle
- ☐ show some progress

What do we know?

$$\mathcal{L}_{QCD} = \bar{\Psi}\left(i\not{D} - m\right)\Psi - \frac{1}{4}\mathrm{Tr}\left(GG\right)$$

meson refers to the transformation properties of the particle under rotation

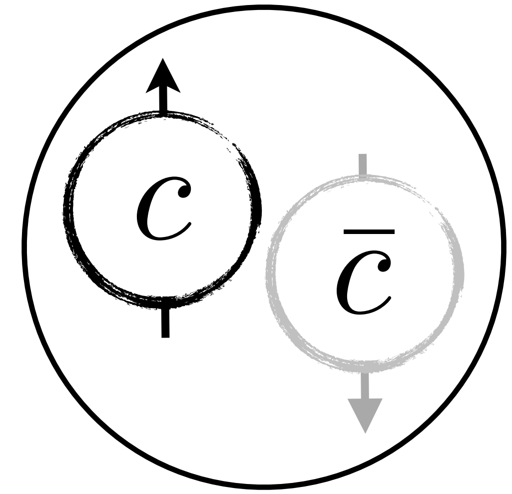


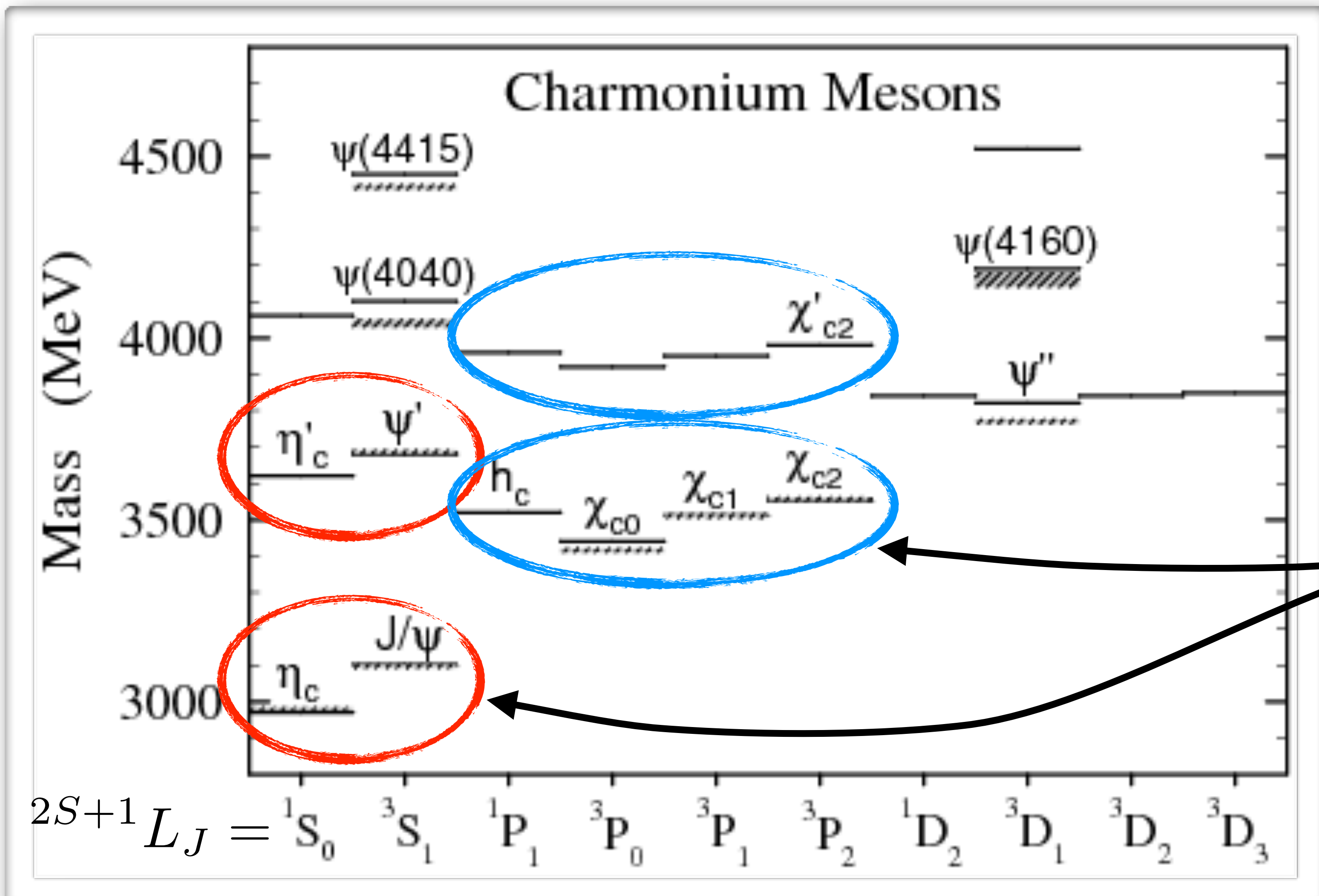
degeneracy patterns?

a quark picture of mesons

$$\boxed{\mathcal{L}_{QCD} = \bar{\Psi}\left(i\not{D} - m\right)\Psi - \frac{1}{4}\mathrm{Tr}\left(GG\right)}$$

mesons as two quark objects?





degeneracy patterns!

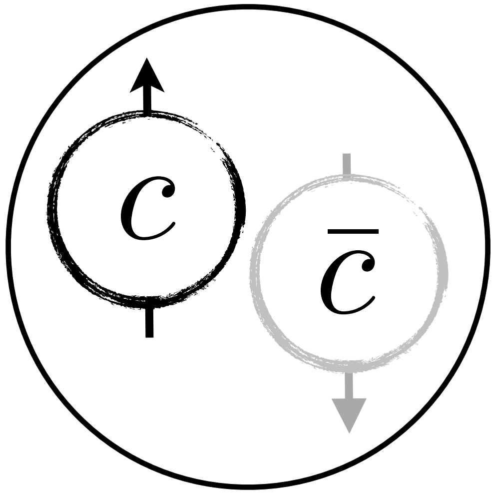
the quark model

☑ **teach you some physics**

quarks have an intrinsic property called spin

→ two quarks can have total spin S=0,1

mesons as two quark objects?



the quarks can orbit each other - get angular momentum L=0,1,2,...

form eigenstates of total spin (J), parity (P), charge conjugation (C)

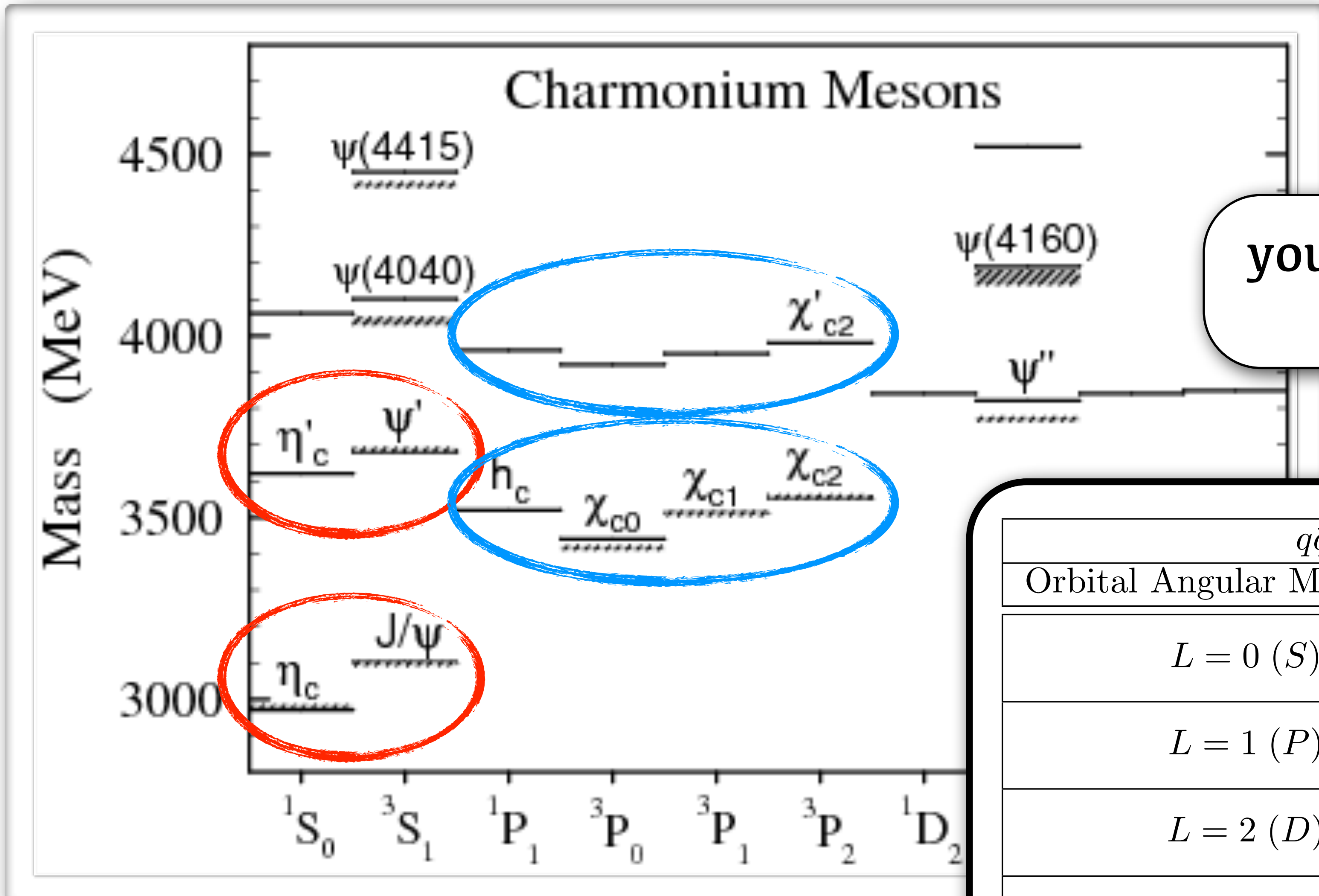
$$|L-S| \leq J \leq L+S$$

$$P=(-1)^{L+1}$$
$$C=(-1)^{L+S}$$

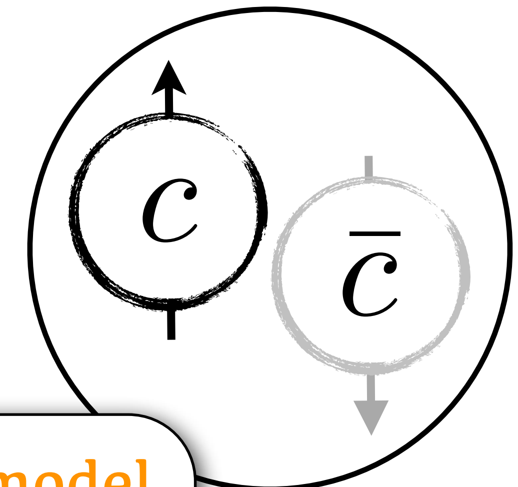
$q\bar{q}$ Orbital Angular Momentum	Spin	Supermultiplet (J^{PC})
$L=0\ (S)$	$S=0$	0^{-+}
	$S=1$	1^{--}
$L=1\ (P)$	$S=0$	1^{+-}
	$S=1$	$(0,1,2)^{++}$
$L=2\ (D)$	$S=0$	2^{-+}
	$S=1$	$(1,2,3)^{--}$
$L=3\ (F)$	$S=0$	3^{+-}
	$S=1$	$(2,3,4)^{++}$
$L=4\ (G)$	$S=0$	4^{-+}
	$S=1$	$(3,4,5)^{--}$

the quark model

$$\mathcal{L}_{QCD} = \bar{\Psi}\left(i\not{D} - m\right)\Psi - \frac{1}{4}\mathrm{Tr}\left(GG\right)$$



mesons as two quark objects?



you can take this model much further

$q\bar{q}$ Orbital Angular Momentum	Spin	Supermultiplet (J^{PC})
$L = 0\ (S)$	$S = 0$	0^{-+}
	$S = 1$	1^{--}
$L = 1\ (P)$	$S = 0$	1^{+-}
	$S = 1$	$(0, 1, 2)^{++}$
$L = 2\ (D)$	$S = 0$	2^{-+}
	$S = 1$	$(1, 2, 3)^{--}$
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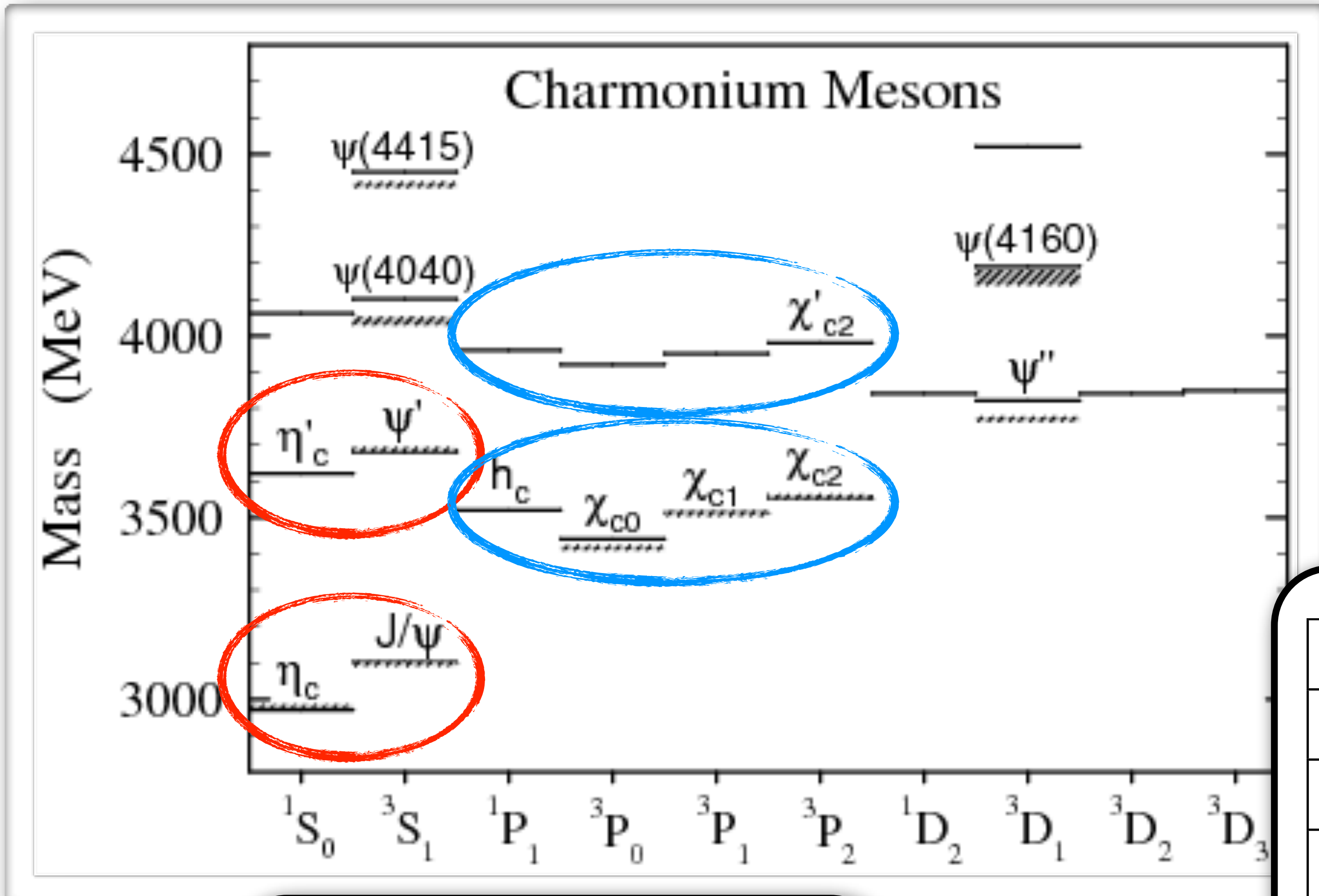
	$S = 1$	$(2, 3, 4)^{++}$
$L = 4\ (G)$	$S = 0$	4^{-+}
	$S = 1$	$(3, 4, 5)^{--}$

the puzzle

$$\mathcal{L}_{QCD} = \bar{\Psi}\left(i\not{D} - m\right)\Psi - \frac{1}{4}\mathrm{Tr}\left(GG\right)$$

☑ **present a puzzle**

the effective degrees of freedom don't 'look' like the Lagrangian



where are the gluons

missing quantum numbers

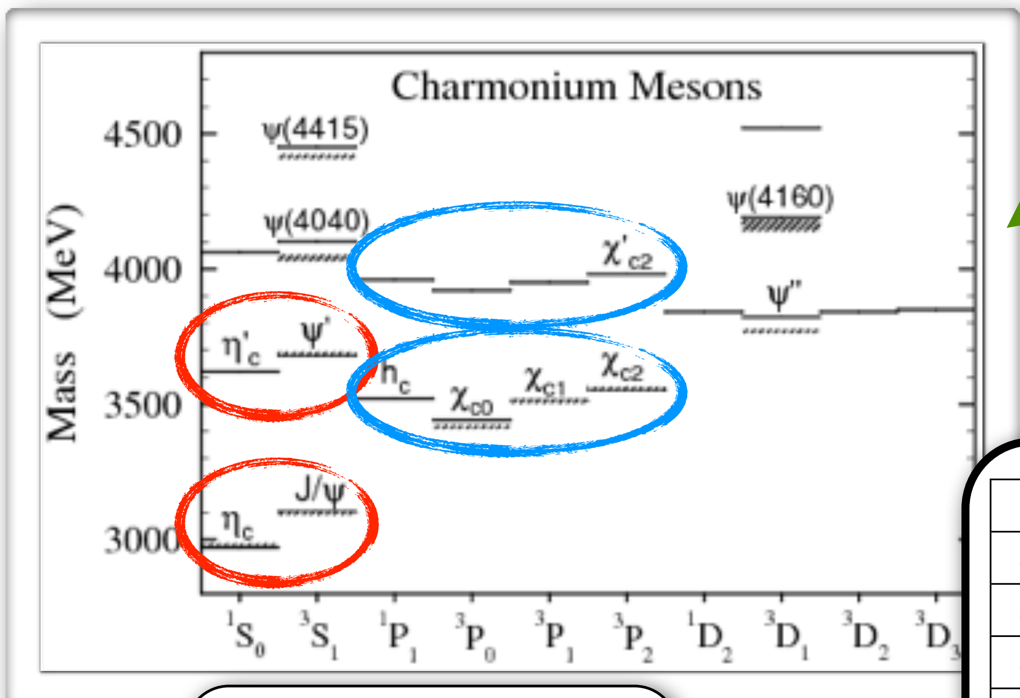
J^{PC}	$--$	$-+$	$++$	$+-$
$J=0$	$\otimes$	0^{-+}	0^{++}	$\otimes$
$J=1$	1^{--}	$\otimes$	1^{++}	1^{+-}
$J=2$	2^{--}	2^{-+}	2^{++}	$\otimes$
$J=3$	3^{--}	$\otimes$	3^{++}	3^{+-}

but QCD is ‘hard’

We are interested in light quark hadronic physics . . .

. . . QCD is non-perturbative here

$$\mathcal{L}_{QCD} = \bar{\Psi}\left(i\not{D} - m\right)\Psi - \frac{1}{4}\mathrm{Tr}\left(GG\right)$$



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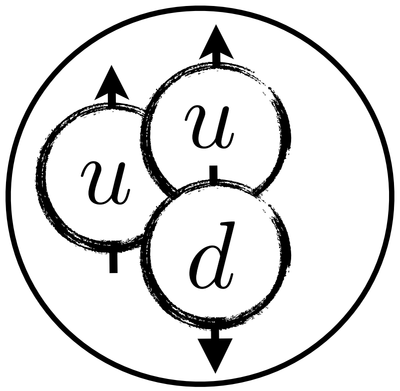

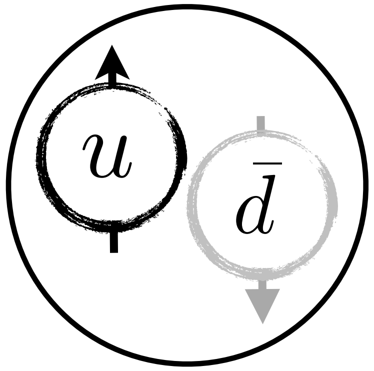

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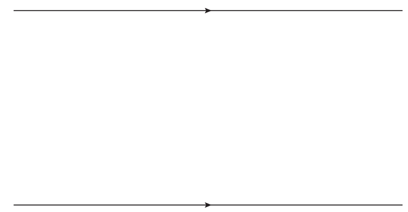
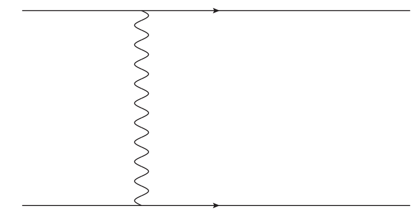
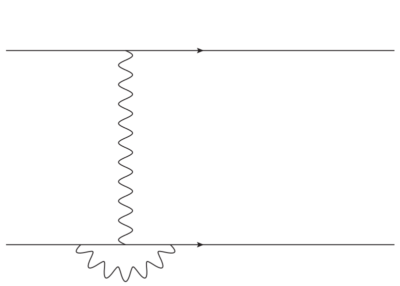
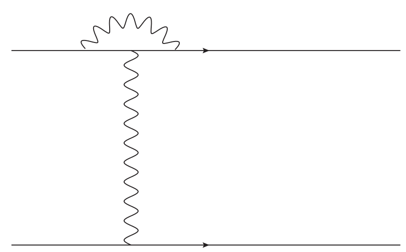
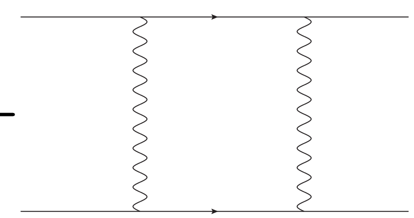
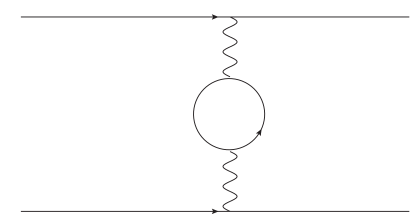
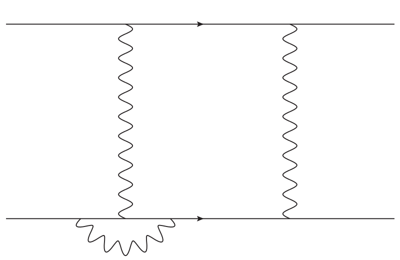
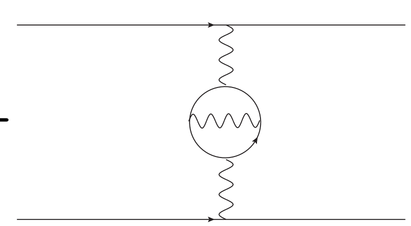
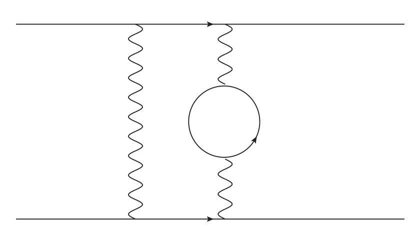
perturbative field theory

Quantum Electrodynamics

$e^- e^- \to e^- e^- \quad \sim \quad \alpha^0$  $+ \ \alpha$ 

$+ \ \alpha^2 \Big($  $+$  $+$  $+$  $\Big)$

$+ \ \alpha^3 \Big($  $+$  $+$  $\cdots \Big)$

$\alpha \sim \dfrac{1}{137}$

the roadmap

QCD → 

→ observables

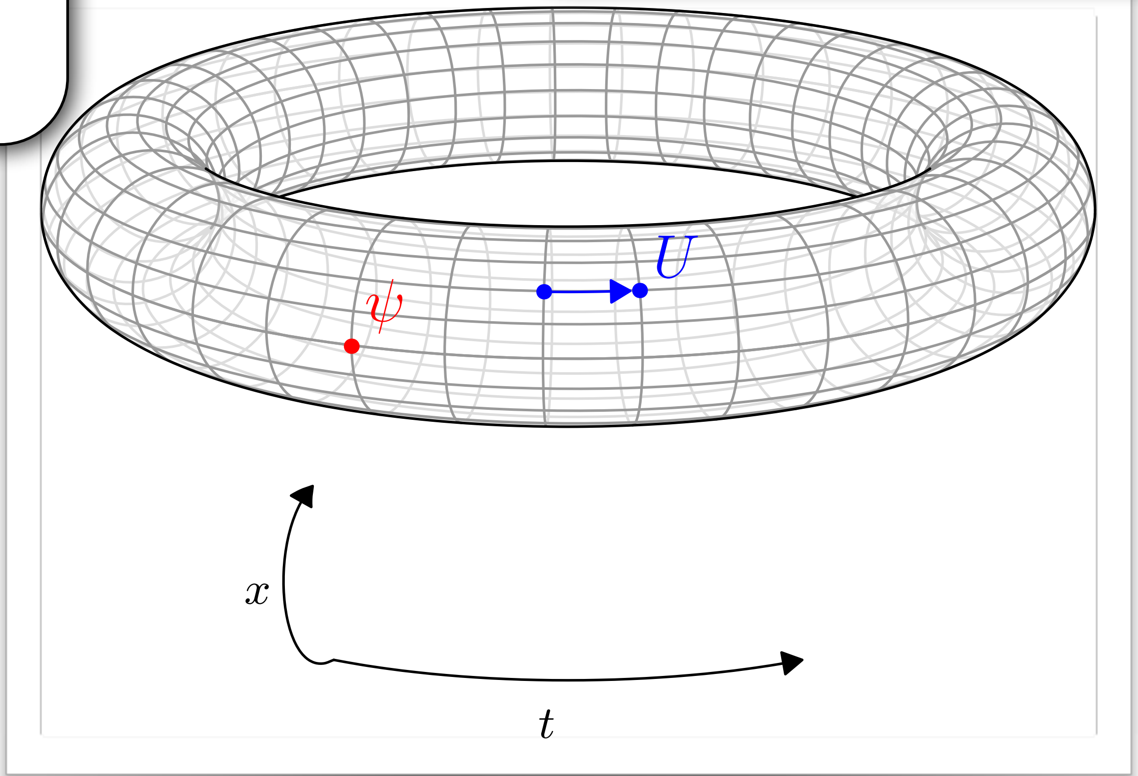
- ☑ teach you some physics
- ☑ present a puzzle
- ☐ show some progress
  - crack open the black box
    - spectroscopy
    - matrix elements
  - results

the lattice part

$$\mathcal{L}_{QCD} = \bar{\Psi}\left(i\not{D} - m\right)\Psi - \frac{1}{4}\mathrm{Tr}\left(GG\right)$$

- ☑ buy a really big computer
- ☑ chop up a patch of spacetime
- ☑ run numeric simulations

You can only solve this problem on a computer

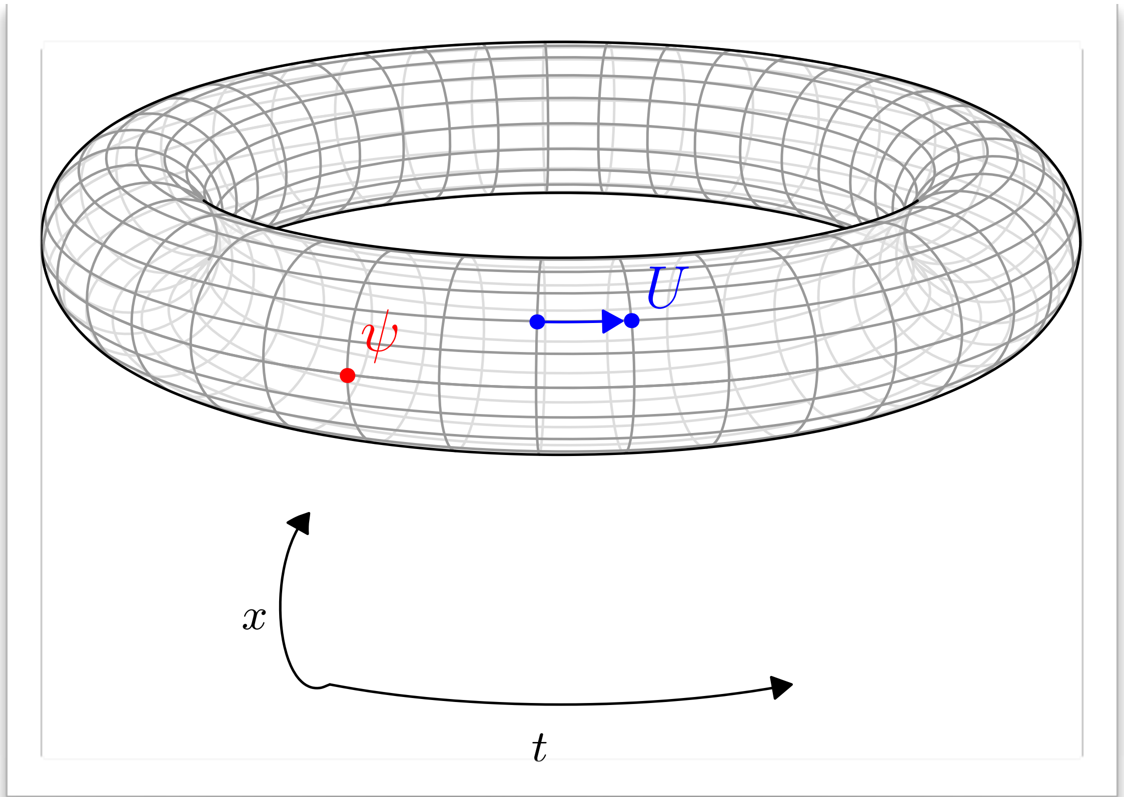


Virginia’s ‘fastest’ supercomputer

observables

$$\mathcal{L}_{QCD} = \bar{\Psi}\left(i\not{D} - m\right)\Psi - \frac{1}{4}\mathrm{Tr}\left(GG\right)$$

the theoretical quantities are correlation functions



$$C(t) = \langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle$$

just functions of the quark and gluon fields

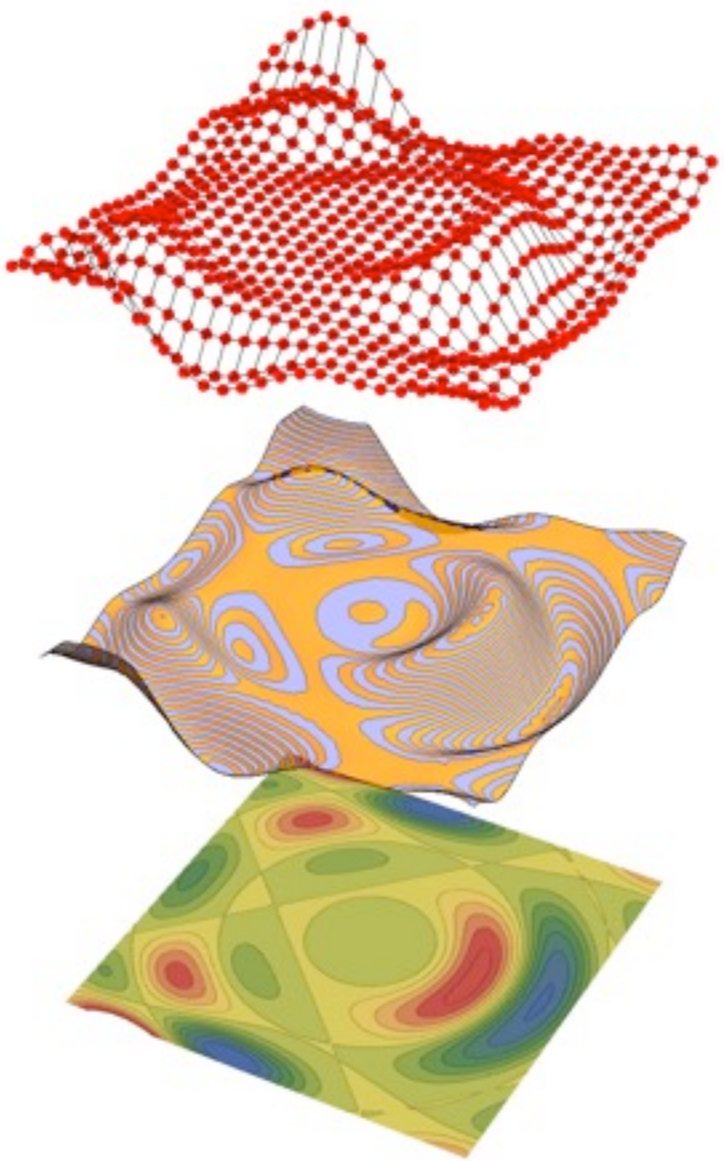
a two-point function tells us about the spectrum

$$\mathcal{O} \sim \sum_{\vec{x},\vec{y}} \bar{\Psi}_{\vec{x}}\Gamma_{\vec{x}\vec{y}}\Psi_{\vec{y}}$$

a look under the hood..

this is the path integral

$$e^- e^- \to e^- e^- \sim \int D[\psi, \bar{\psi}, U] f[\psi, \bar{\psi}] e^{-\bar{\psi} M[U] \psi - S[U]}$$



'snapshots' of the gauge field

$$\Rightarrow \int D[U] e^{-S[U]} \det(M[U]) f(M[U])$$

probability density

Markov Chain Monte Carlo

$$\langle f \rangle \sim \frac{1}{N} \sum_i^N f[M[U_i]]$$

$$P(U_i) \propto e^{-S[U]} \det(M[U])$$

spectroscopy

$$\mathcal{O}(t) = e^{Ht}\mathcal{O}(0)e^{-Ht}$$

time evolve the operator

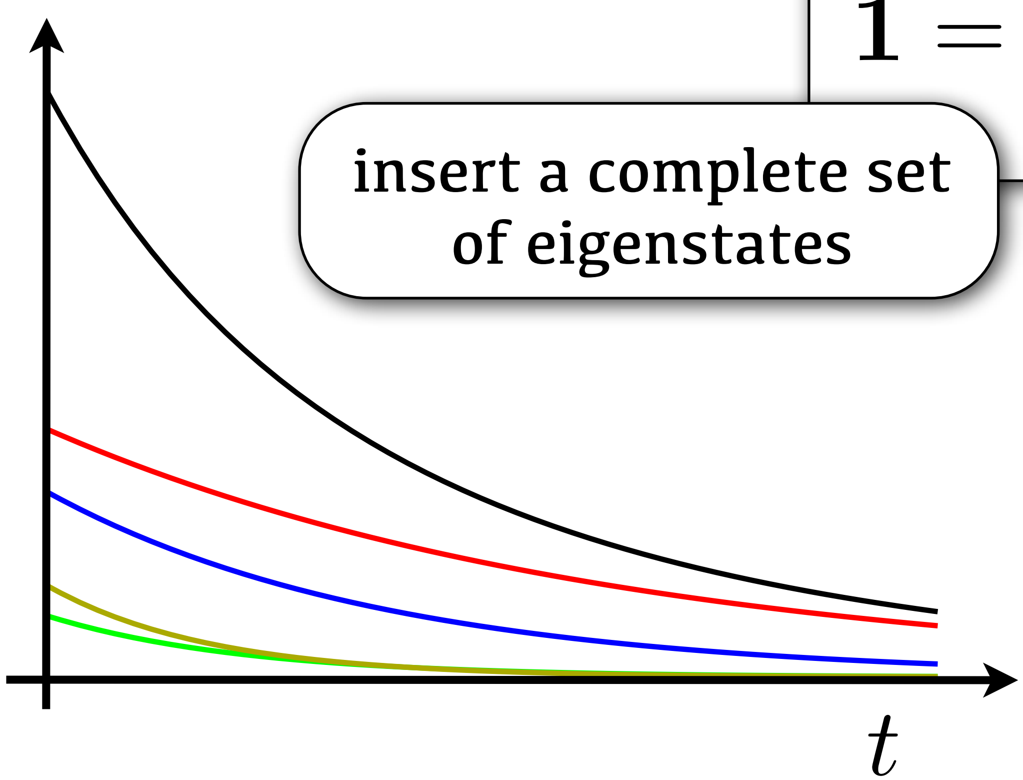
$$C(t) = \langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle = \sum_{\mathfrak{n}} \frac{|\langle \mathfrak{n}|\mathcal{O}(0)^\dagger|0\rangle|^2}{2E_{\mathfrak{n}}} e^{-E_{\mathfrak{n}}t}$$

the spectrum

$$\mathbf{1} = \sum_{\mathfrak{n}} \frac{1}{2E_{\mathfrak{n}}}|\mathfrak{n}\rangle\langle\mathfrak{n}|$$

insert a complete set of eigenstates

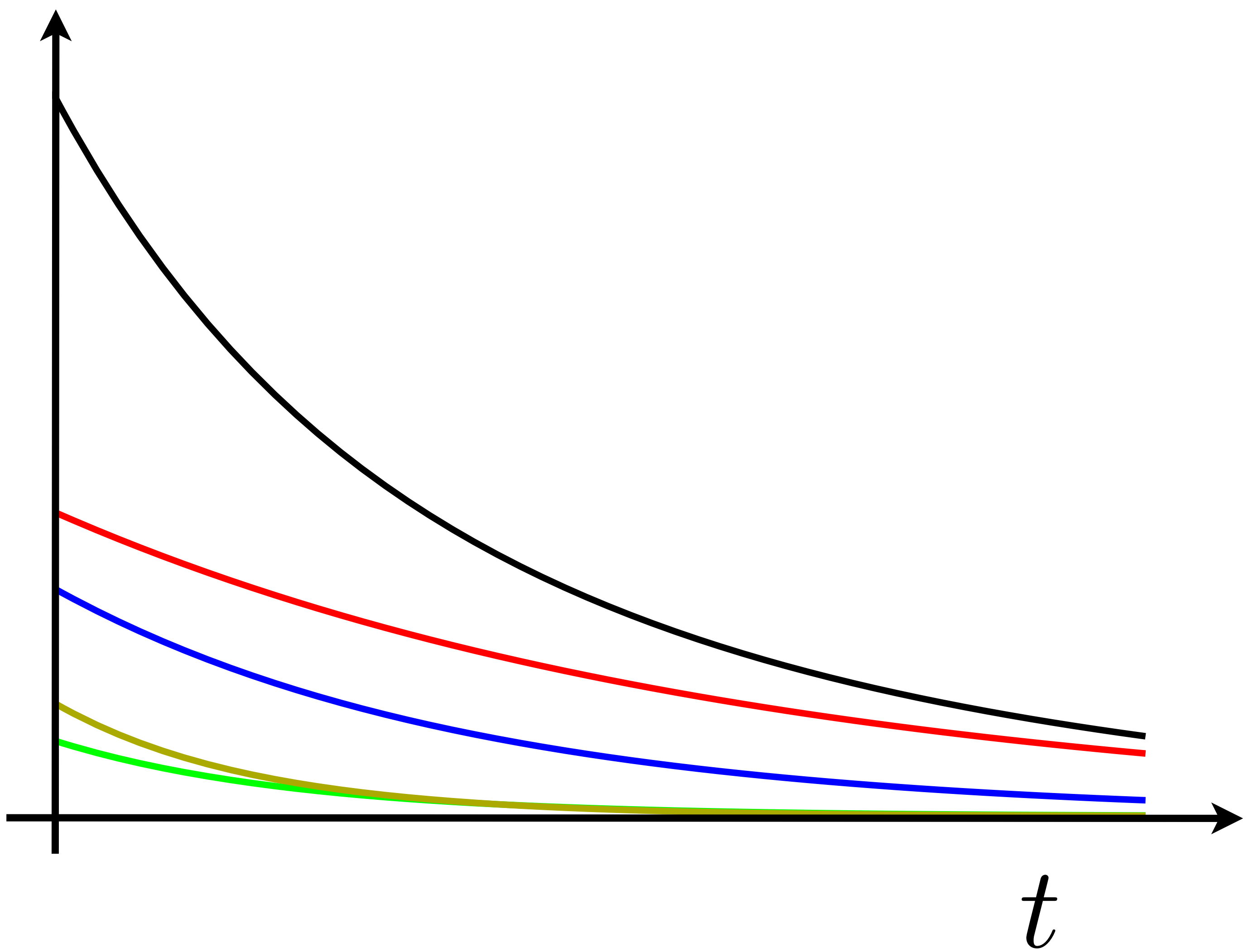
$$H|\mathfrak{n}\rangle = E_{\mathfrak{n}}|\mathfrak{n}\rangle$$

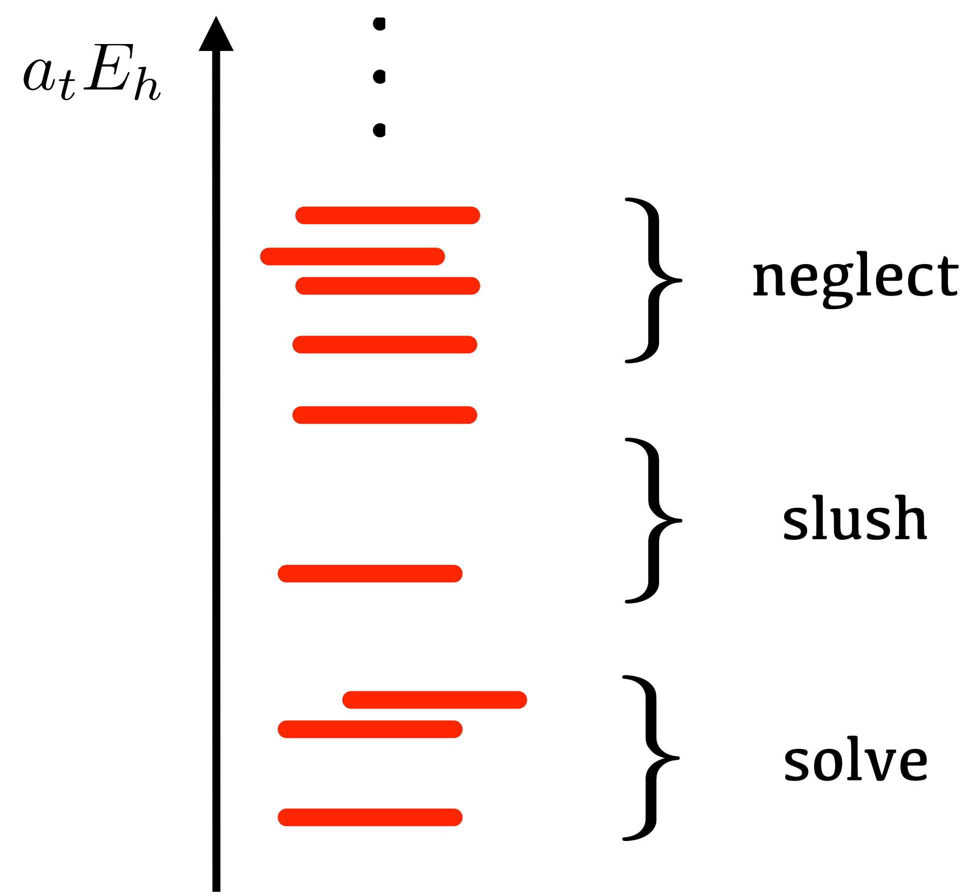


variational spectroscopy

$$C(t) = \langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle = \sum_{\mathfrak{n}} \frac{|\langle \mathfrak{n}|\mathcal{O}(0)^\dagger|0\rangle|^2}{2E_{\mathfrak{n}}} e^{-E_{\mathfrak{n}} t}$$

isolate a single state





variational spectroscopy

an American approach:

One operator = good
More operators = better

$$C(t) = \langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle = \sum_{\mathfrak{n}} \frac{|\langle \mathfrak{n}|\mathcal{O}(0)^\dagger|0\rangle|^2}{2E_{\mathfrak{n}}} e^{-E_{\mathfrak{n}} t}$$

isolate a single state

a solution

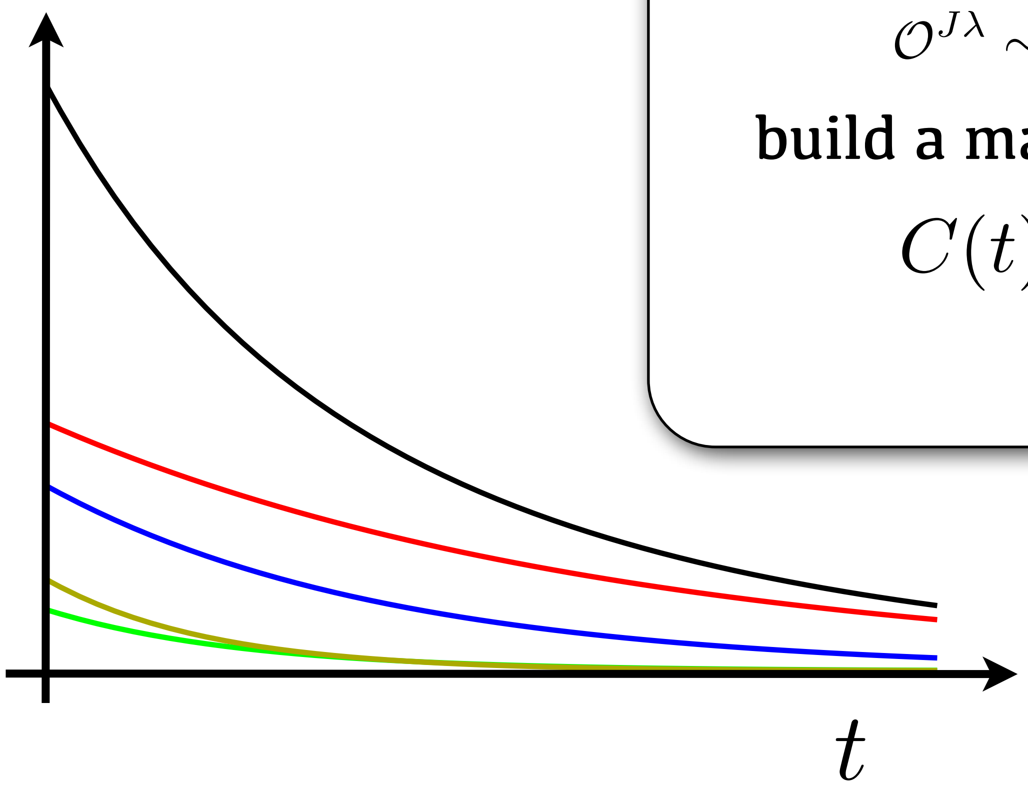
use a basis of operators

$$\mathcal{O}^{J\lambda} \sim \bar{\psi}\Gamma_a \overleftrightarrow{D}_b \overleftrightarrow{D}_c \cdots \psi \times \mathrm{CG}(a,b,c \to J\lambda)$$

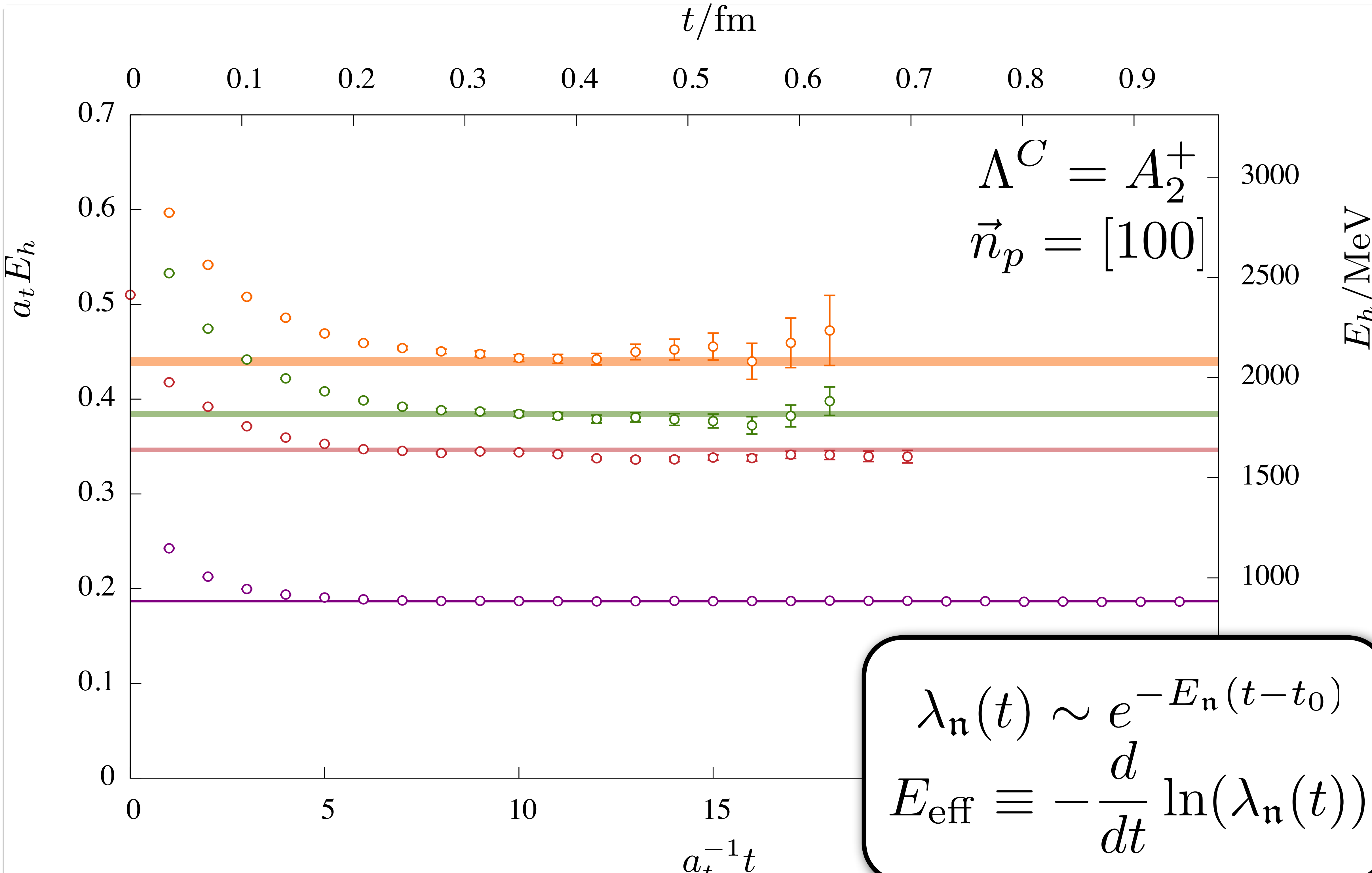
build a matrix of correlators

$$C(t)v^{(\mathfrak{n})} = \lambda_{\mathfrak{n}}(t) C(t_0) v^{(\mathfrak{n})}$$

diagonalize the matrix



variational spectroscopy

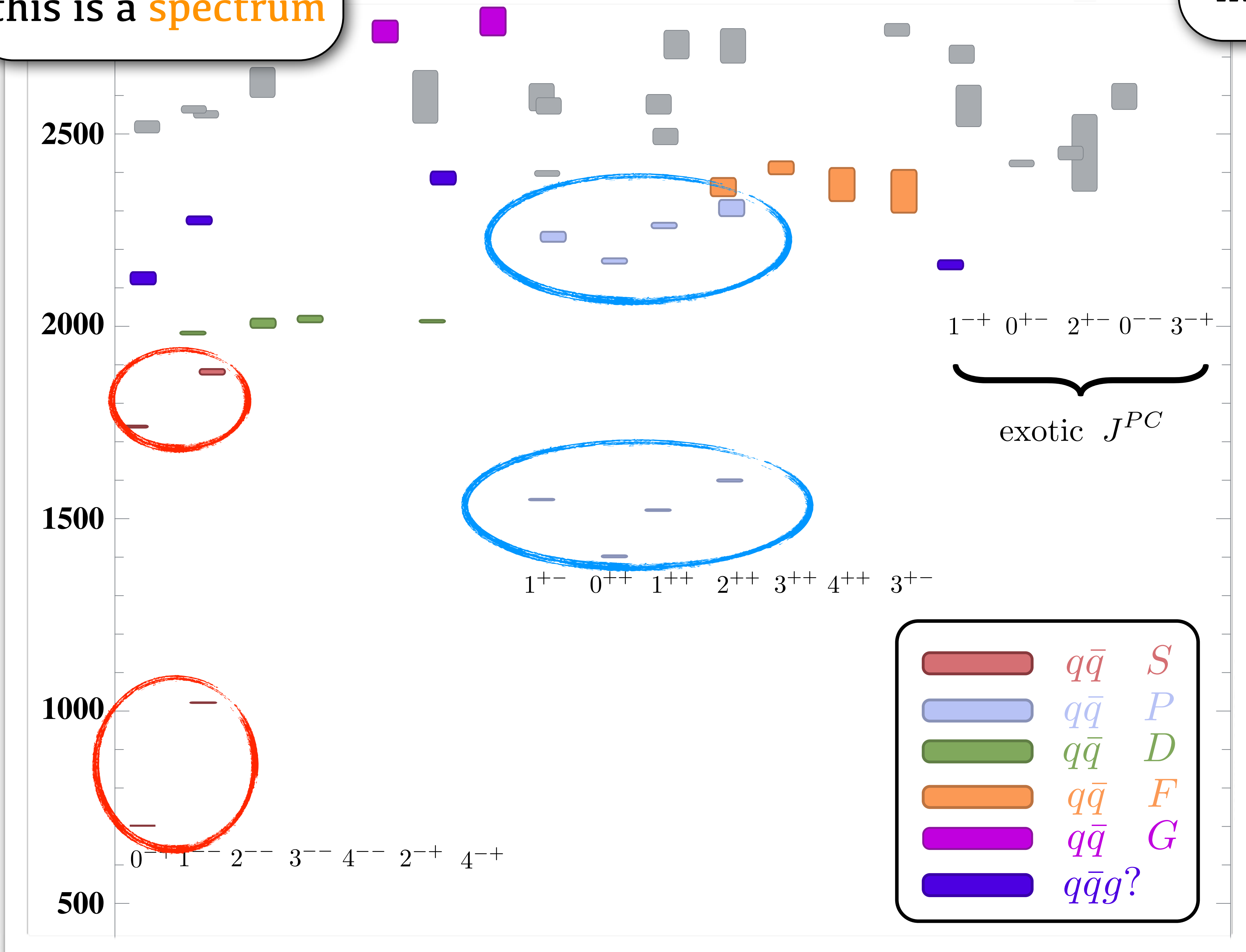


$$\lambda_{\mathfrak{n}}(t) \sim e^{-E_{\mathfrak{n}}(t-t_0)}$$

$$E_{\text{eff}} \equiv -\frac{d}{dt}\ln(\lambda_{\mathfrak{n}}(t))$$

a lattice calculation

this is a spectrum



~~missing~~ quantum numbers

J^{PC}	$--$	$-+$	$++$	$+-$
$J=0$	$\otimes$	0^{-+}	0^{++}	$\otimes$
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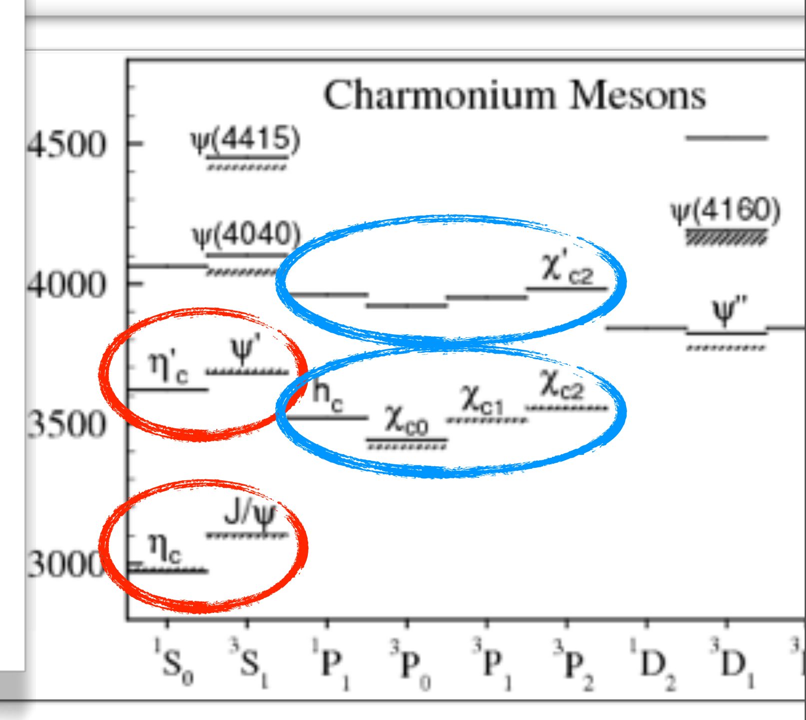
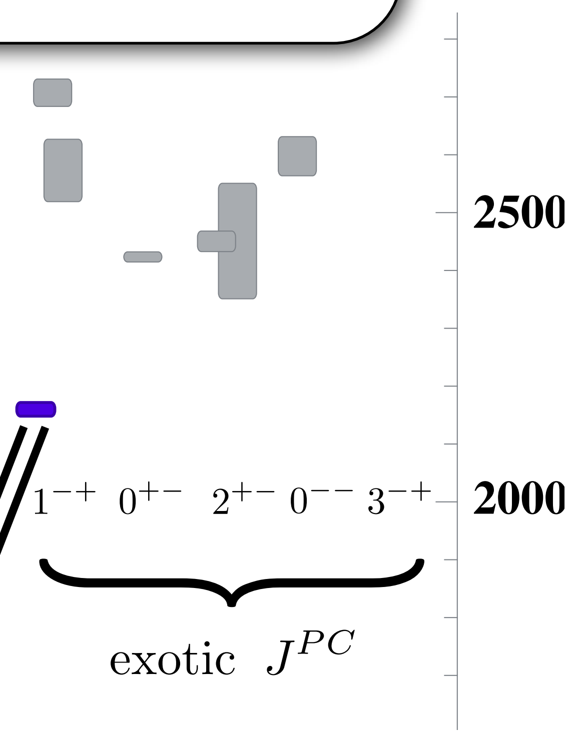
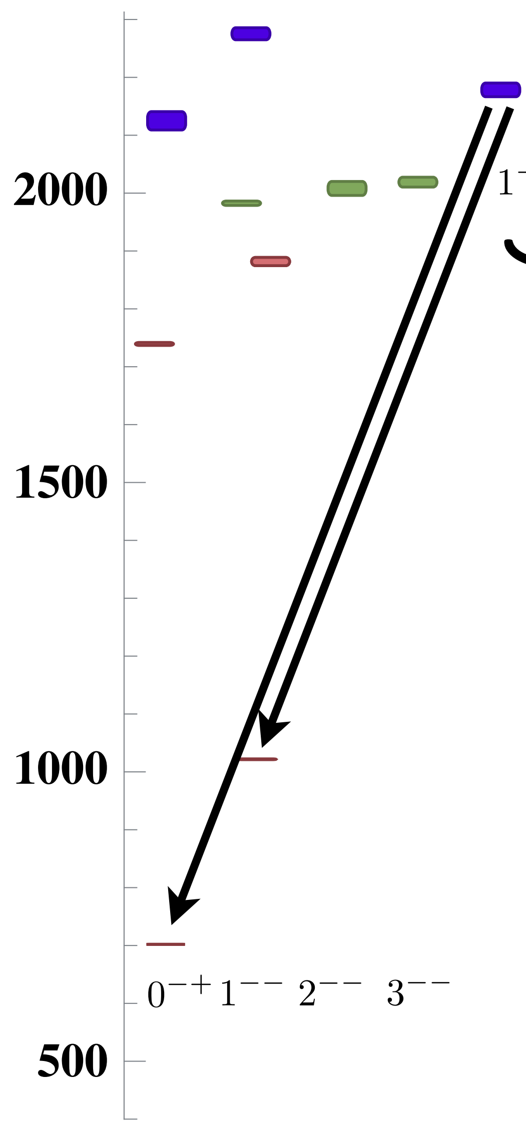


the problem at hand

~~missing~~ quantum numbers

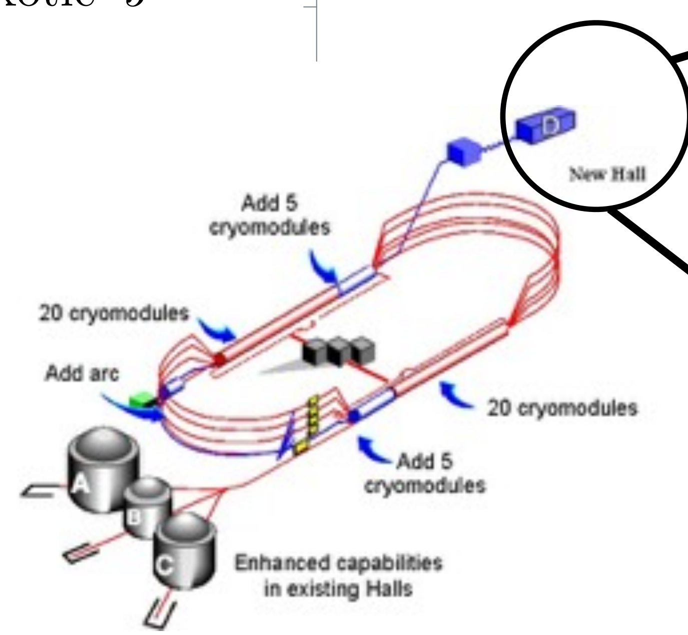


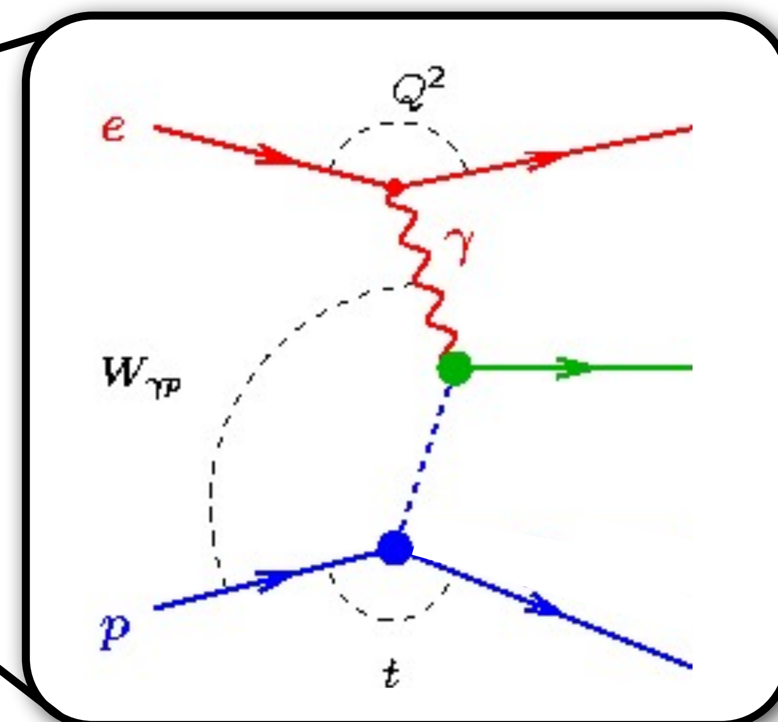


☑ present (another) puzzle

we want to study exotic mesons

we need to make them





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matrix elements

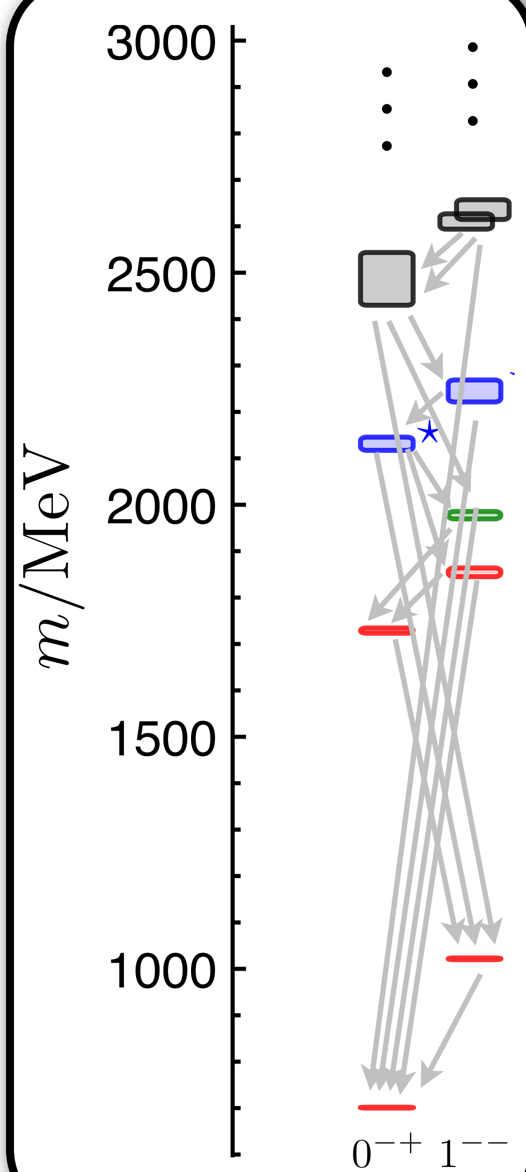
matrix elements ‘hide’ in three-point functions

$$j^\mu = \sum_q e_q \bar{\psi}_q \gamma^\mu \psi_q$$

$$C^\mu_{\mathfrak{nm}} = \langle 0 | \Omega_{\mathfrak{n}}(t) j^\mu(t_\gamma) \Omega^\dagger_{\mathfrak{m}}(0) | 0 \rangle$$

$$= \sum_{\mathfrak{n}'\mathfrak{m}'} \langle \mathfrak{n}' | j^\mu | \mathfrak{m}' \rangle e^{-E_{\mathfrak{n}'}(t-t_\gamma)} e^{-E_{\mathfrak{m}'} t_\gamma}$$

$$\times \frac{\langle 0 | \Omega_{\mathfrak{n}}(0) | \mathfrak{n}' \rangle}{2E_{\mathfrak{n}'}} \frac{\langle \mathfrak{m}' | \Omega^\dagger_{\mathfrak{m}}(0) | 0 \rangle}{2E_{\mathfrak{m}'}}$$



a solution

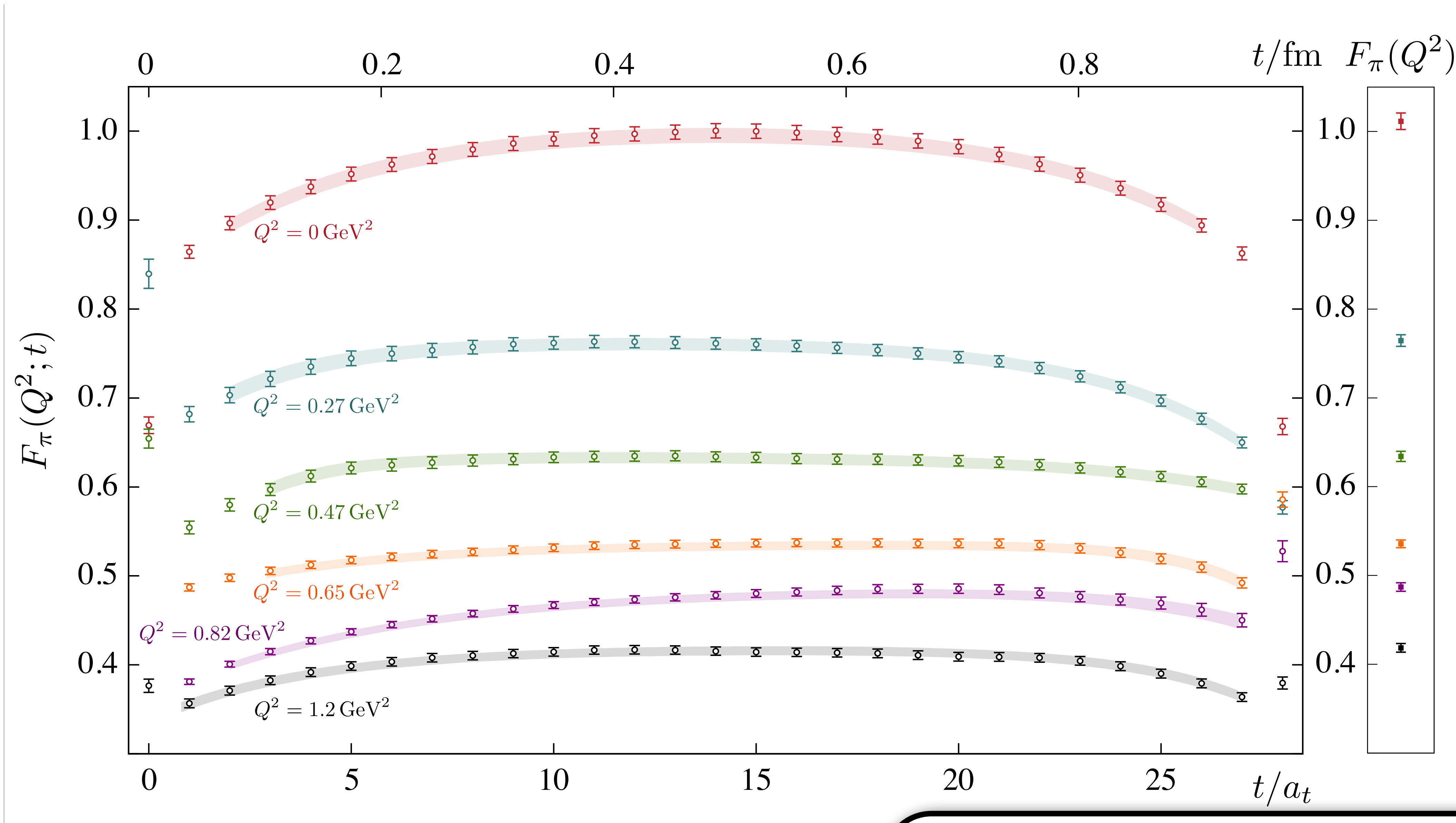
use a basis of operators

$$\mathcal{O}^{J\lambda} \sim \bar{\psi} \Gamma_a \overleftrightarrow{D}_b \overleftrightarrow{D}_c \cdots \psi \times \mathrm{CG}(a, b, c \to J\lambda)$$

generate an optimal set of weights

$$C(t) v^{(\mathfrak{n})} = \lambda_{\mathfrak{n}}(t) C(t_0) v^{(\mathfrak{n})}$$

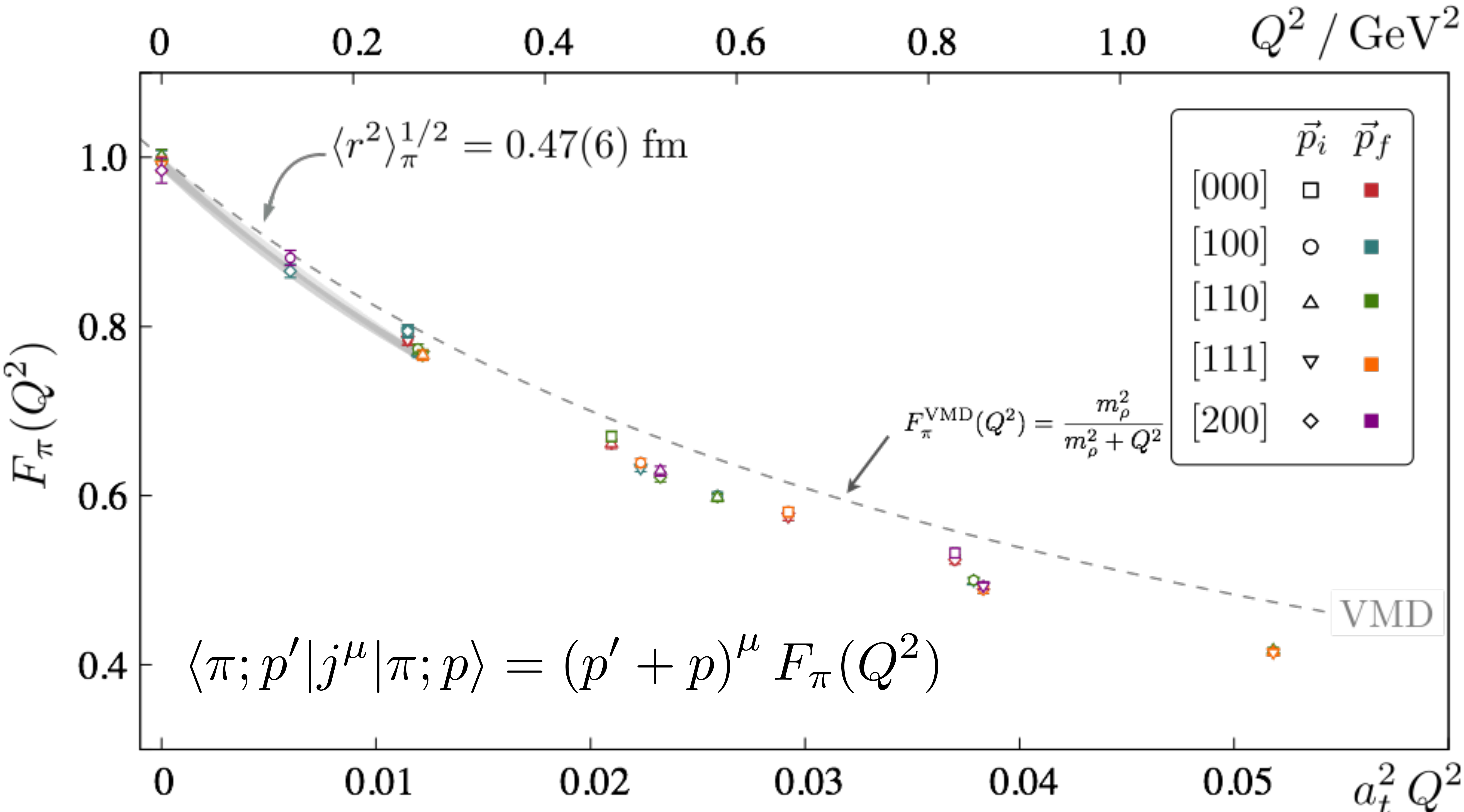
signal isolation



$$\begin{aligned} C^{\mu}_{\mathfrak{nm}} &= \langle 0|\Omega_{\mathfrak{n}}(t) j^{\mu}(t_\gamma) \Omega^{\dagger}_{\mathfrak{m}}(0)|0\rangle \\ &= \sum_{\mathfrak{n}'\mathfrak{m}'} \langle \mathfrak{n}'|j^{\mu}|\mathfrak{m}'\rangle e^{-E_{\mathfrak{n}'}(t-t_\gamma)} e^{-E_{\mathfrak{m}'}t_\gamma} \\ &\quad \times \frac{\langle 0|\Omega_{\mathfrak{n}}(0)|\mathfrak{n}'\rangle}{2E_{\mathfrak{n}'}} \frac{\langle \mathfrak{m}'|\Omega^{\dagger}_{\mathfrak{m}}(0)|0\rangle}{2E_{\mathfrak{m}'}} \end{aligned}$$

$$\Omega^{\dagger}_{\pi} \sim \sum_i v_i^{(\pi)} \mathcal{O}_i^{\dagger}$$

a result

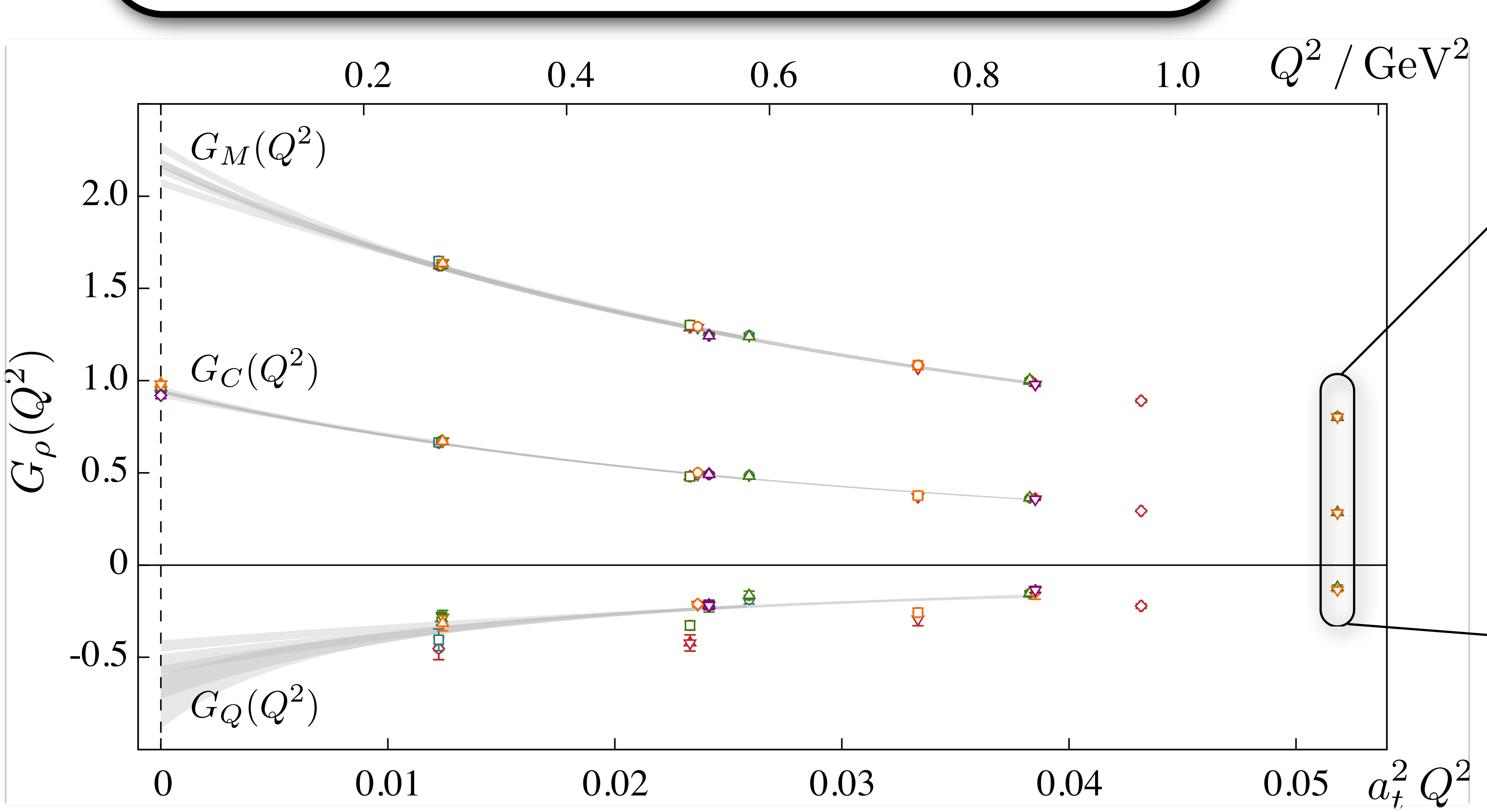


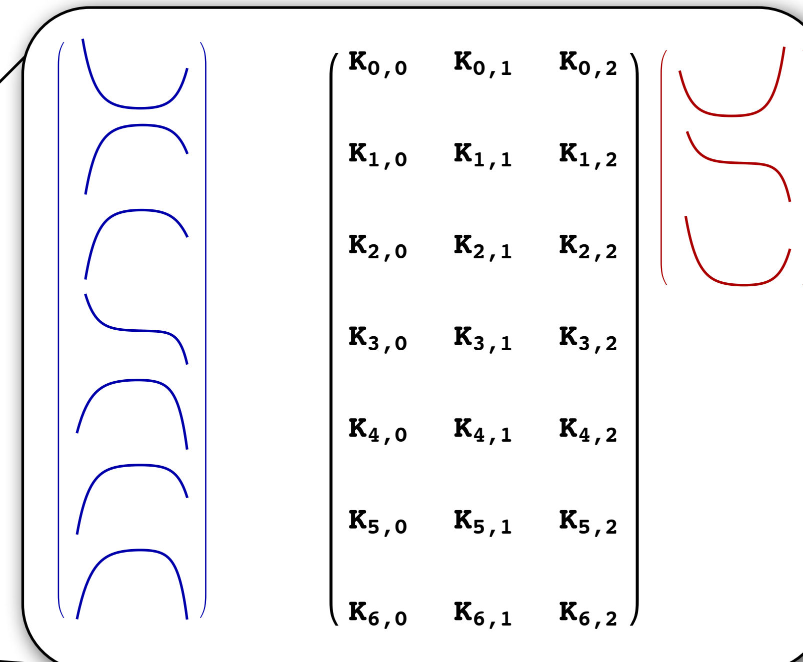
$$\langle r^2 \rangle = -6 \frac{d}{dQ^2} F(Q^2)|_{Q^2=0}$$

something more complicated

$$\langle \mathfrak{n} | j^\mu | \mathfrak{m} \rangle = \sum_k K_k^\mu(\mathfrak{n}, \mathfrak{n}) F_k(Q^2)$$

there can be more than one invariant

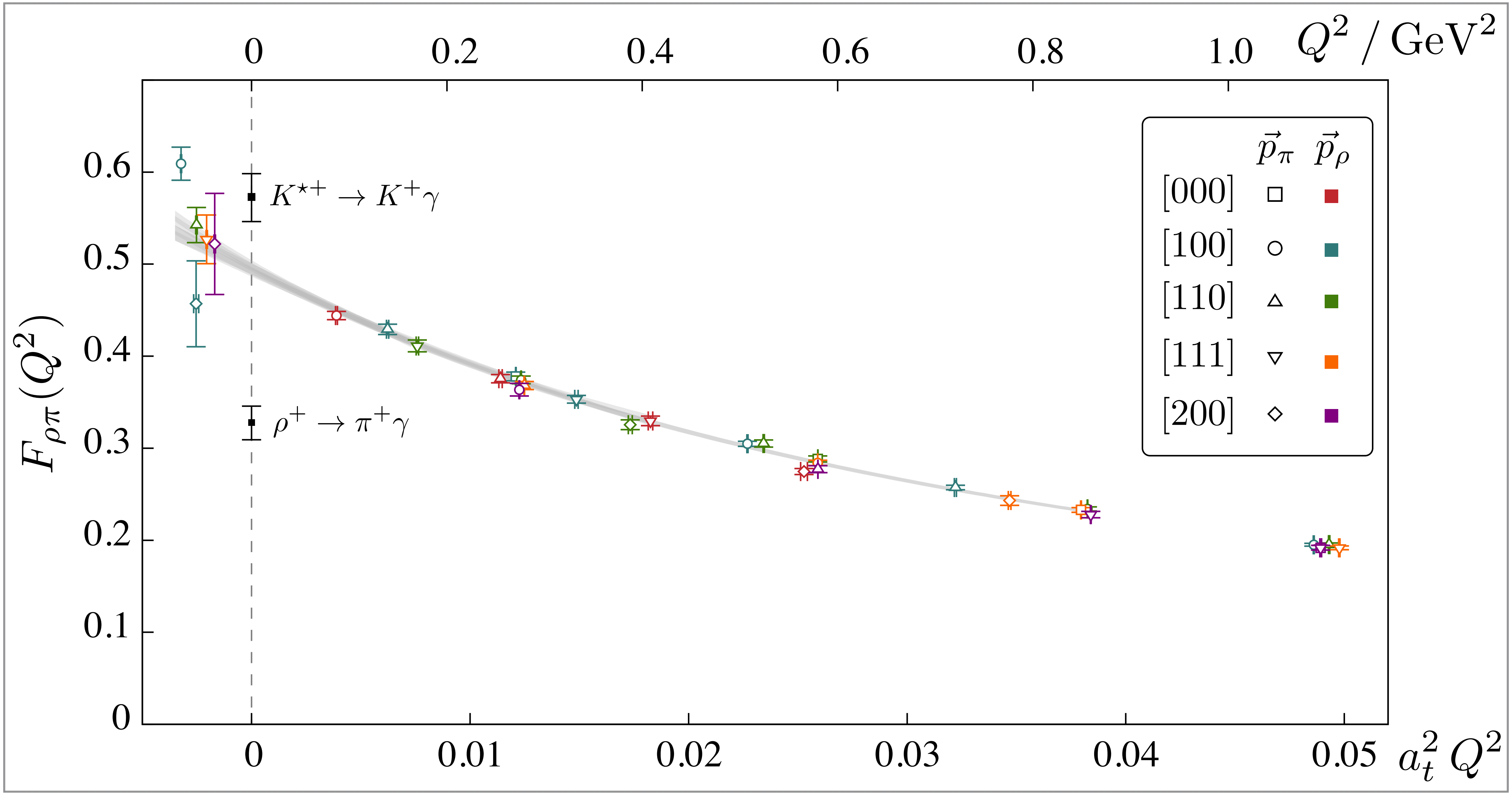




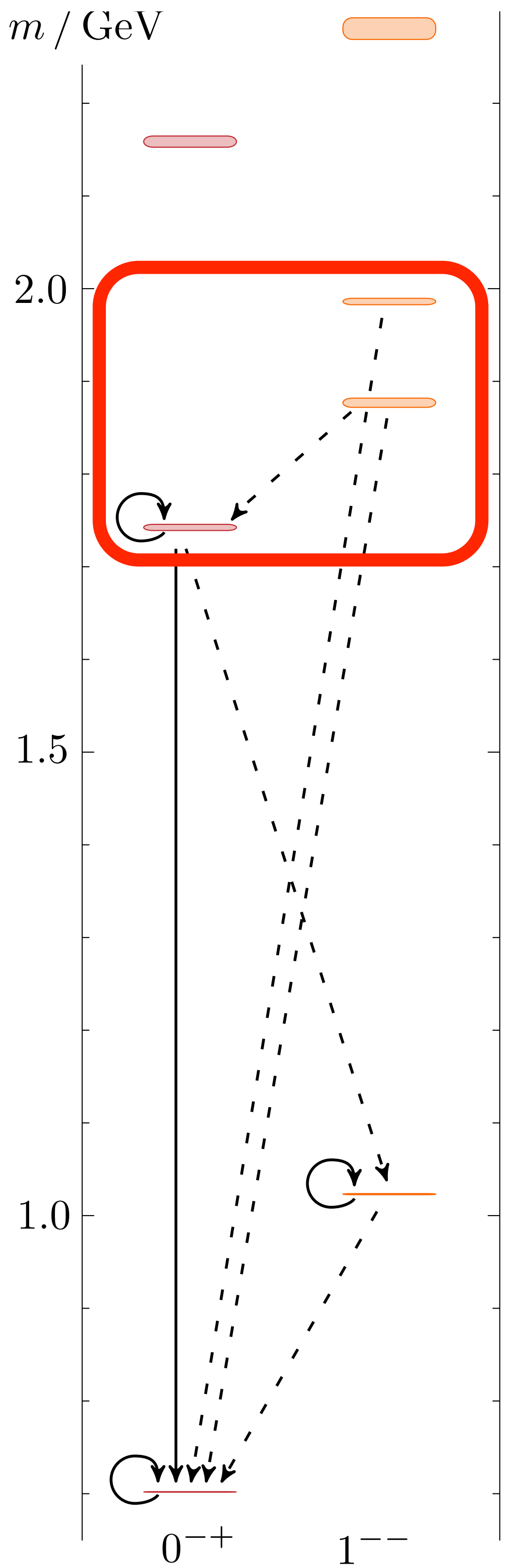
$$\begin{aligned}\langle \rho, p'\lambda' | j^\mu | \rho, p\lambda \rangle = & - (p'+p)^\mu \epsilon^*_\alpha(p'\lambda')\epsilon^\alpha(p\lambda) G_1(Q^2) \\ & + \left[\epsilon^\mu(p\lambda)\epsilon^*_\alpha(p'\lambda')p^\alpha + \epsilon^{\mu *}(p'\lambda')\epsilon^\alpha(p\lambda)p'_\alpha\right] G_2(Q^2) \\ & - (p'+p)^\mu \epsilon^*_\alpha(p'\lambda')p^\alpha \epsilon^\beta(p\lambda)p'_\beta \frac{G_3(Q^2)}{2m_\rho^2}\end{aligned}$$

a transition

$$\langle \rho; p'\lambda | j^\mu | \pi; p\rangle = \frac{2F(Q^2)}{m_\rho + m_\pi}\epsilon^{\mu\nu\eta\sigma} p'_\nu p_\eta \varepsilon^*_\sigma(p',\lambda)$$



the remainder of the talk



we made operators to project excited states ..

.. these would be subleading exponentially damped contributions

$$C^{\mu}_{\mathfrak{nm}} = \langle 0 | \Omega_{\mathfrak{n}}(t) j^{\mu}(t_\gamma) \Omega^{\dagger}_{\mathfrak{m}}(0) | 0 \rangle$$

$$= \sum_{\mathfrak{n}'\mathfrak{m}'} \langle \mathfrak{n}' | j^{\mu} | \mathfrak{m}' \rangle e^{-E_{\mathfrak{n}'}(t-t_\gamma)} e^{-E_{\mathfrak{m}'} t_\gamma}$$

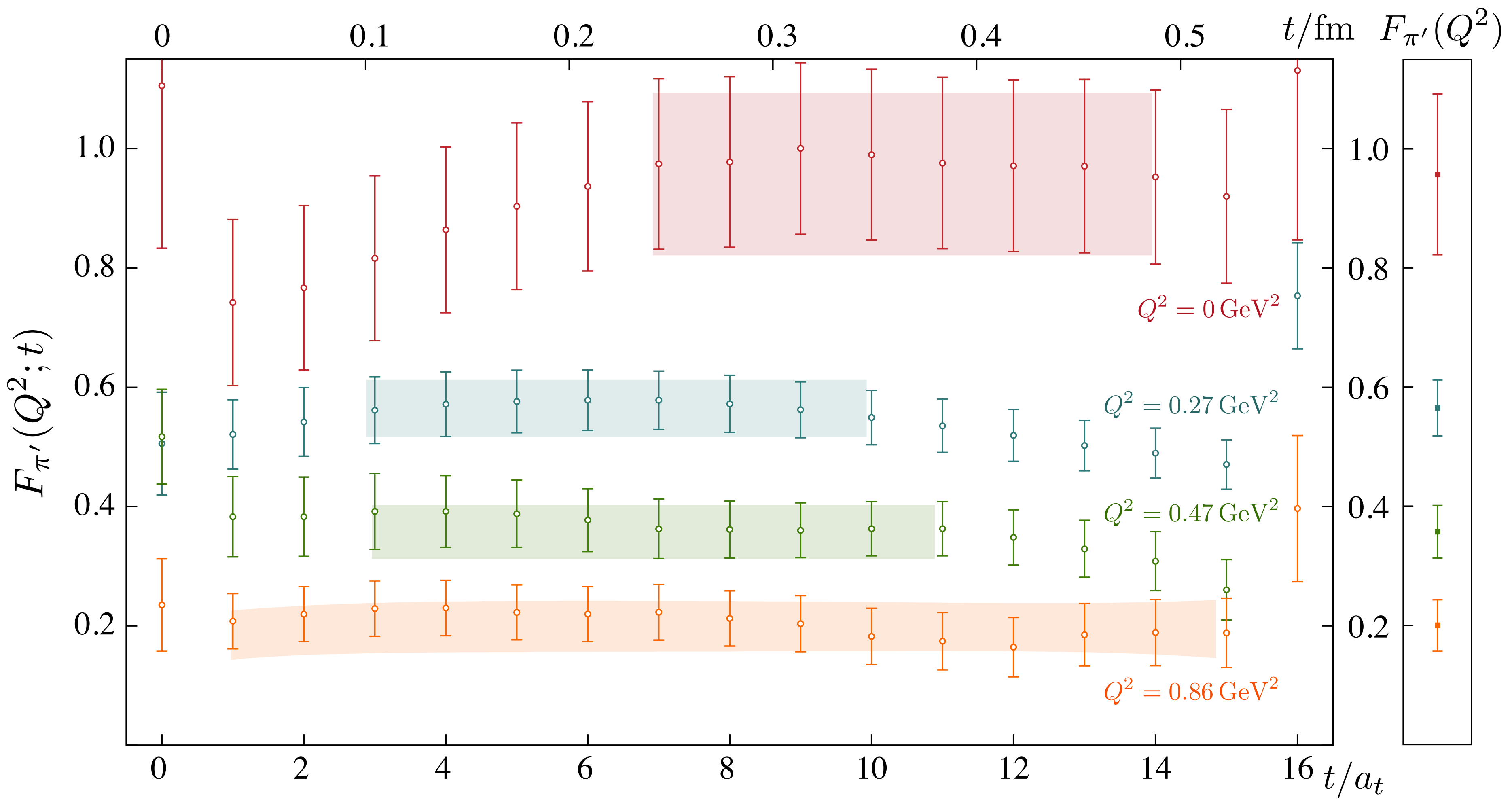
$$\times \frac{\langle 0 | \Omega_{\mathfrak{n}}(0) | \mathfrak{n}' \rangle}{2E_{\mathfrak{n}'}} \frac{\langle \mathfrak{m}' | \Omega^{\dagger}_{\mathfrak{m}}(0) | 0 \rangle}{2E_{\mathfrak{m}'}}$$

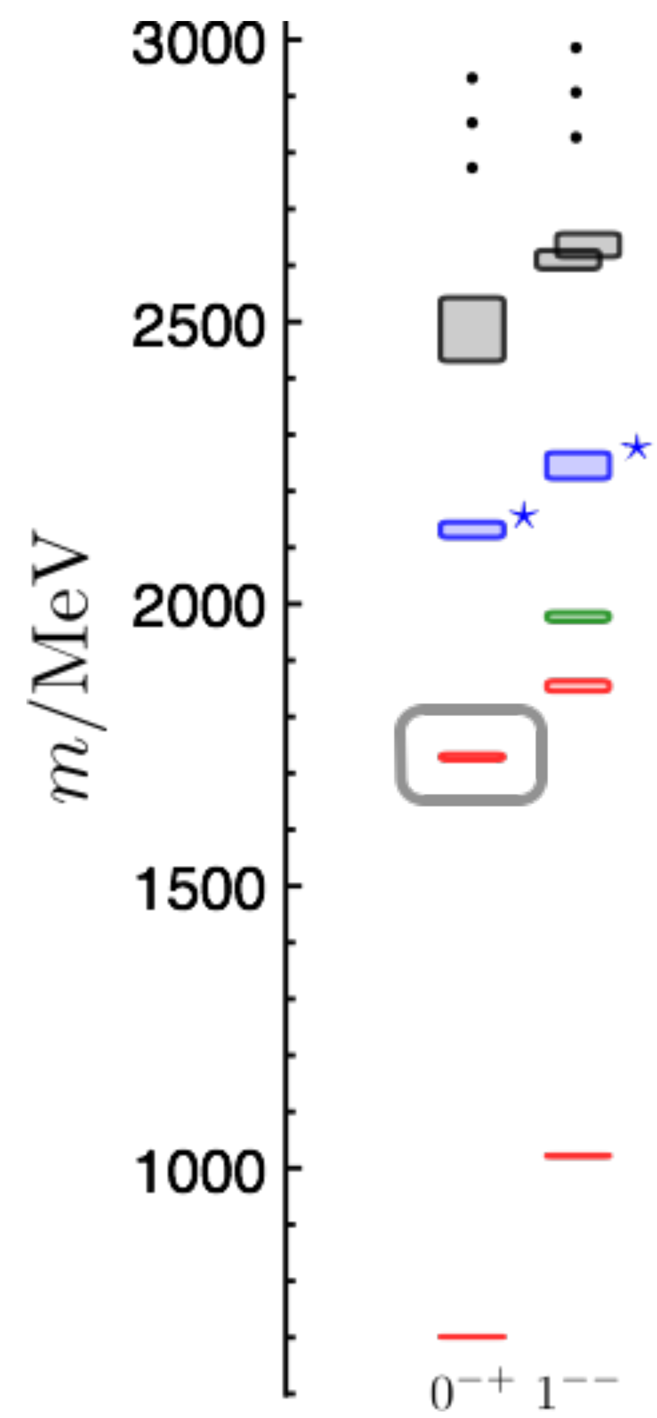
a look under the hood

$$C^{\mu}_{\mathfrak{nm}} = \langle 0|\Omega_{\mathfrak{n}}(t) j^{\mu}(t_\gamma)\Omega^{\dagger}_{\mathfrak{m}}(0)|0\rangle$$

$$= \sum_{\mathfrak{n}'\mathfrak{m}'} \langle \mathfrak{n}'|j^{\mu}|\mathfrak{m}'\rangle e^{-E_{\mathfrak{n}'}(t-t_\gamma)} e^{-E_{\mathfrak{m}'}t_\gamma}$$

$$\times \frac{\langle 0|\Omega_{\mathfrak{n}}(0)|\mathfrak{n}'\rangle}{2E_{\mathfrak{n}'}} \frac{\langle \mathfrak{m}'|\Omega^{\dagger}_{\mathfrak{m}}(0)|0\rangle}{2E_{\mathfrak{m}'}}$$





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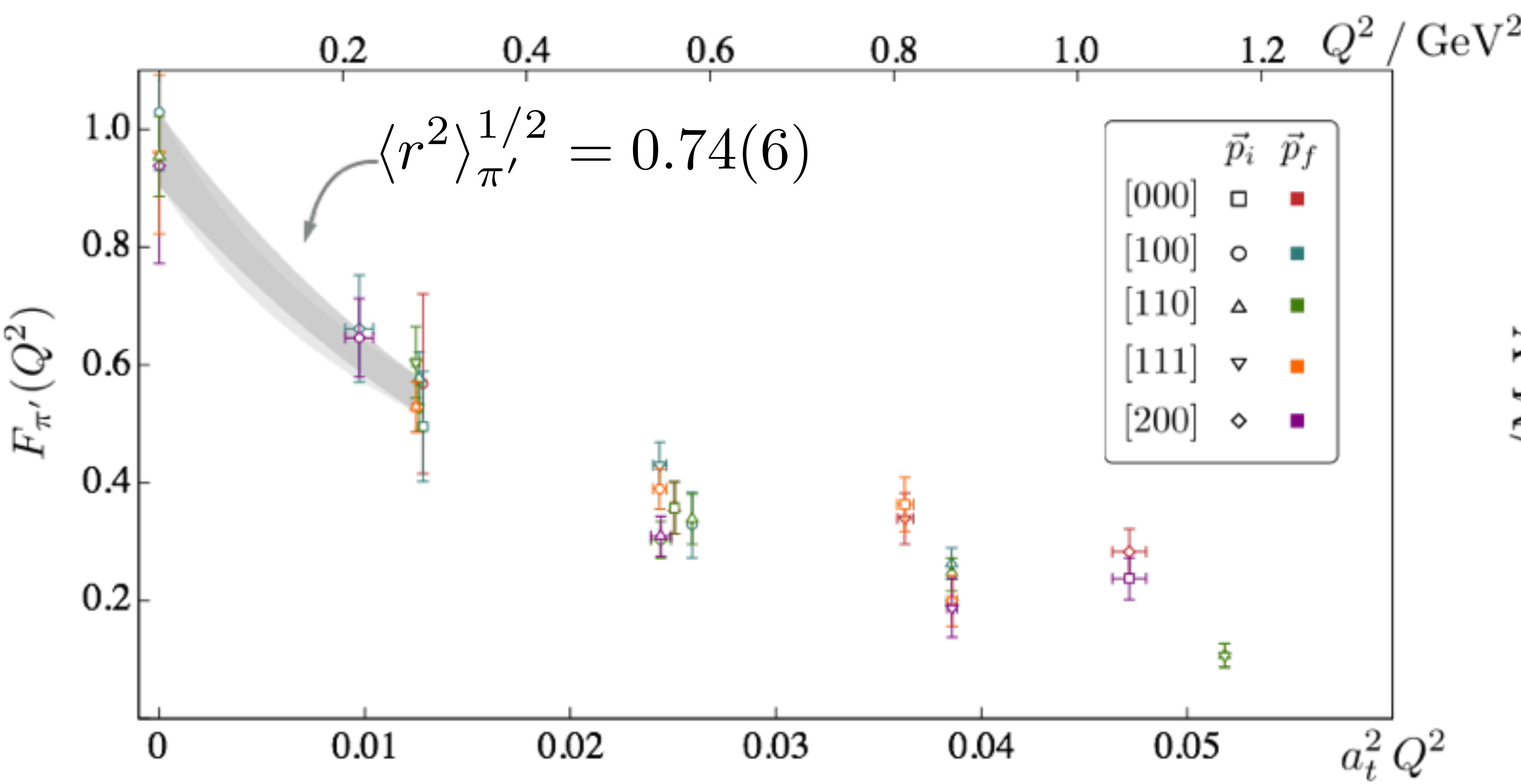
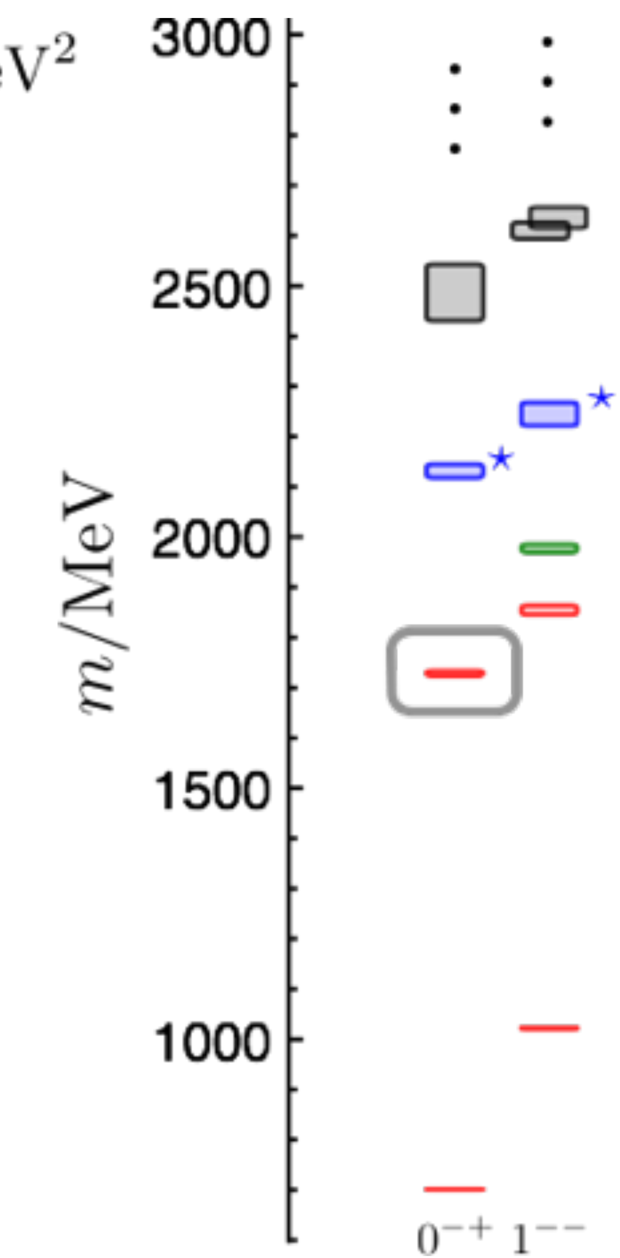
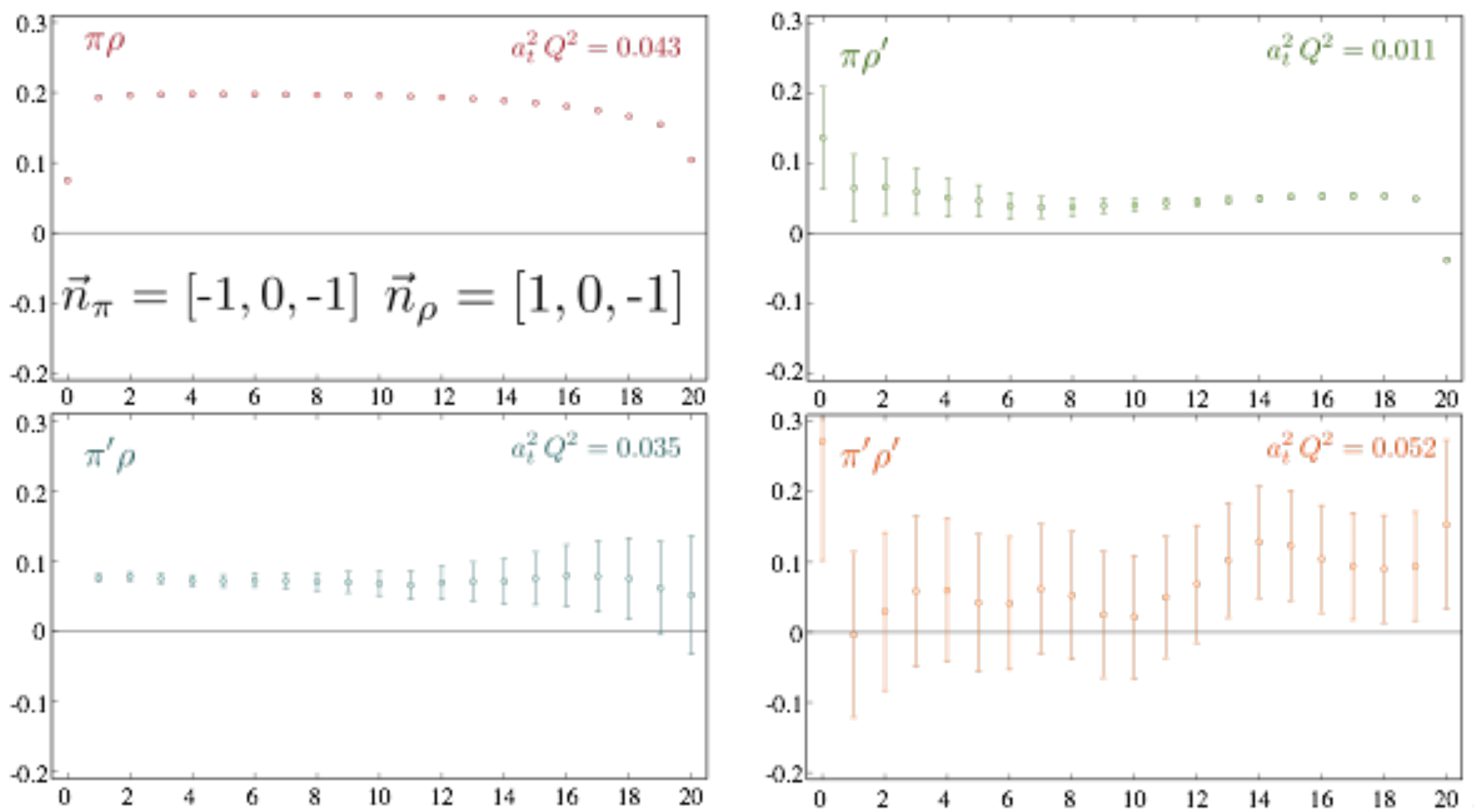
an excited state form factor

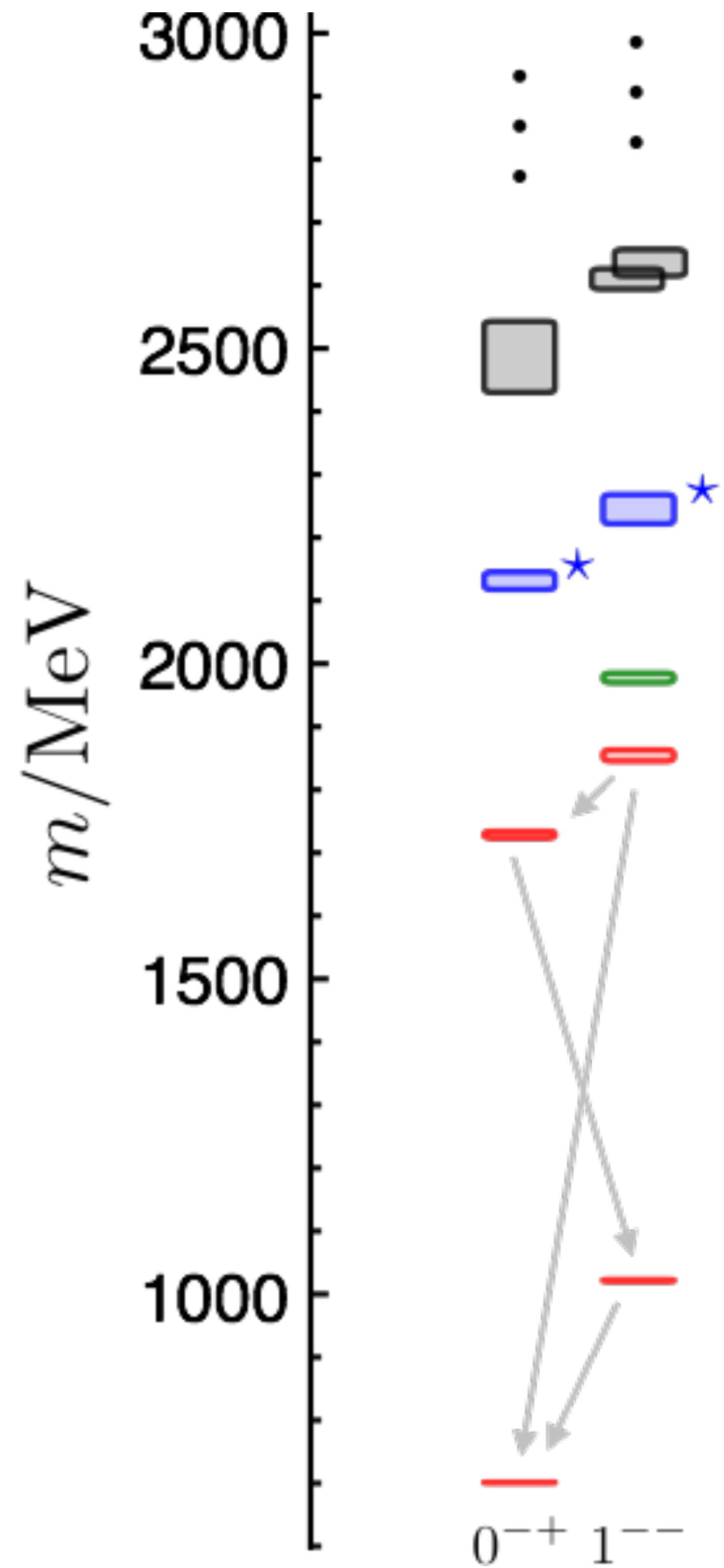




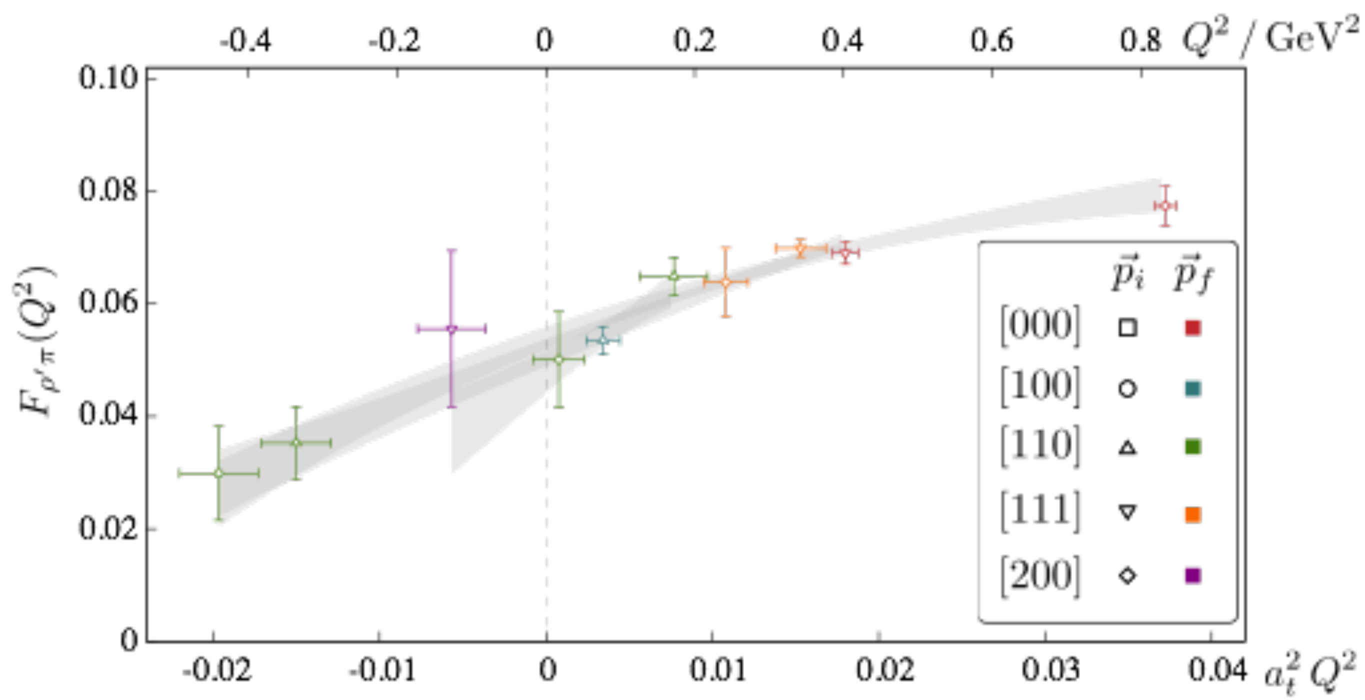
a look under the hood

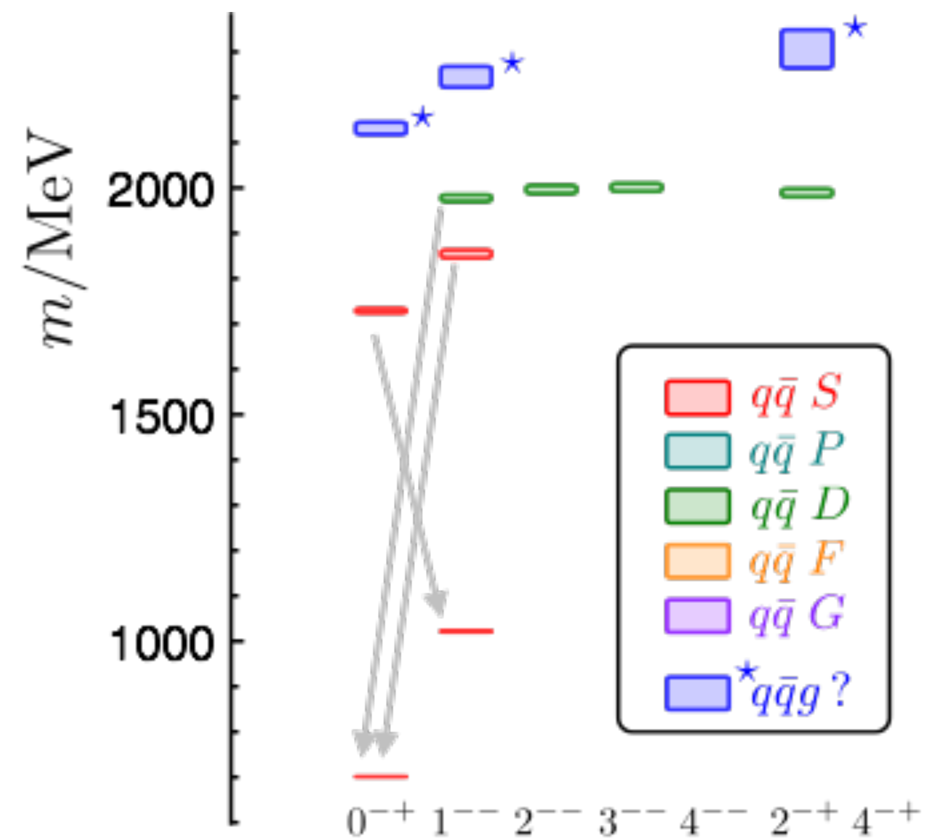
$$
\begin{aligned}
C^{\mu}_{\mathfrak{nm}} &= \langle 0|\Omega_{\mathfrak{n}}(t)\, j^{\mu}(t_\gamma)\,\Omega^{\dagger}_{\mathfrak{m}}(0)|0\rangle \\
&= \sum_{\mathfrak{n}'\mathfrak{m}'} \langle \mathfrak{n}'|j^{\mu}|\mathfrak{m}'\rangle\, e^{-E_{\mathfrak{n}'}(t-t_\gamma)}\, e^{-E_{\mathfrak{m}'}t_\gamma} \\
&\qquad \times \frac{\langle 0|\Omega_{\mathfrak{n}}(0)|\mathfrak{n}'\rangle}{2E_{\mathfrak{n}'}}\, \frac{\langle \mathfrak{m}'|\Omega^{\dagger}_{\mathfrak{m}}(0)|0\rangle}{2E_{\mathfrak{m}'}}
\end{aligned}
$$

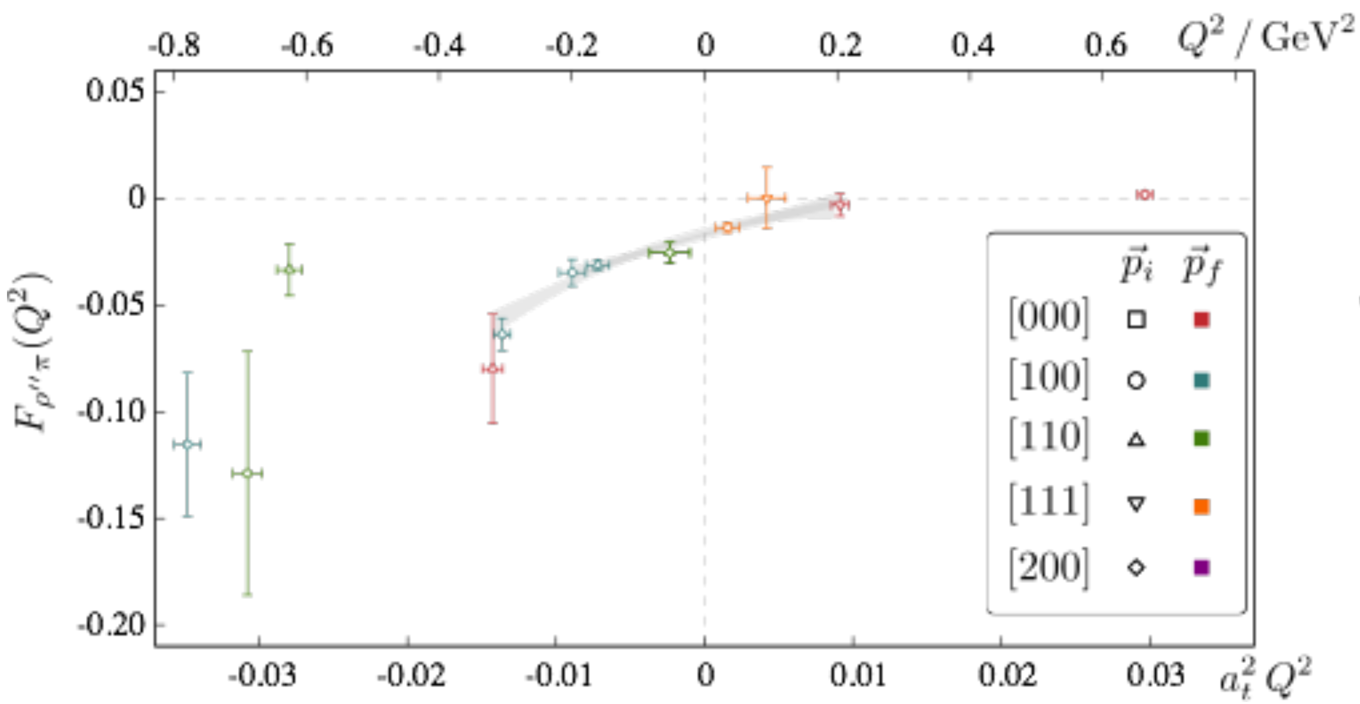


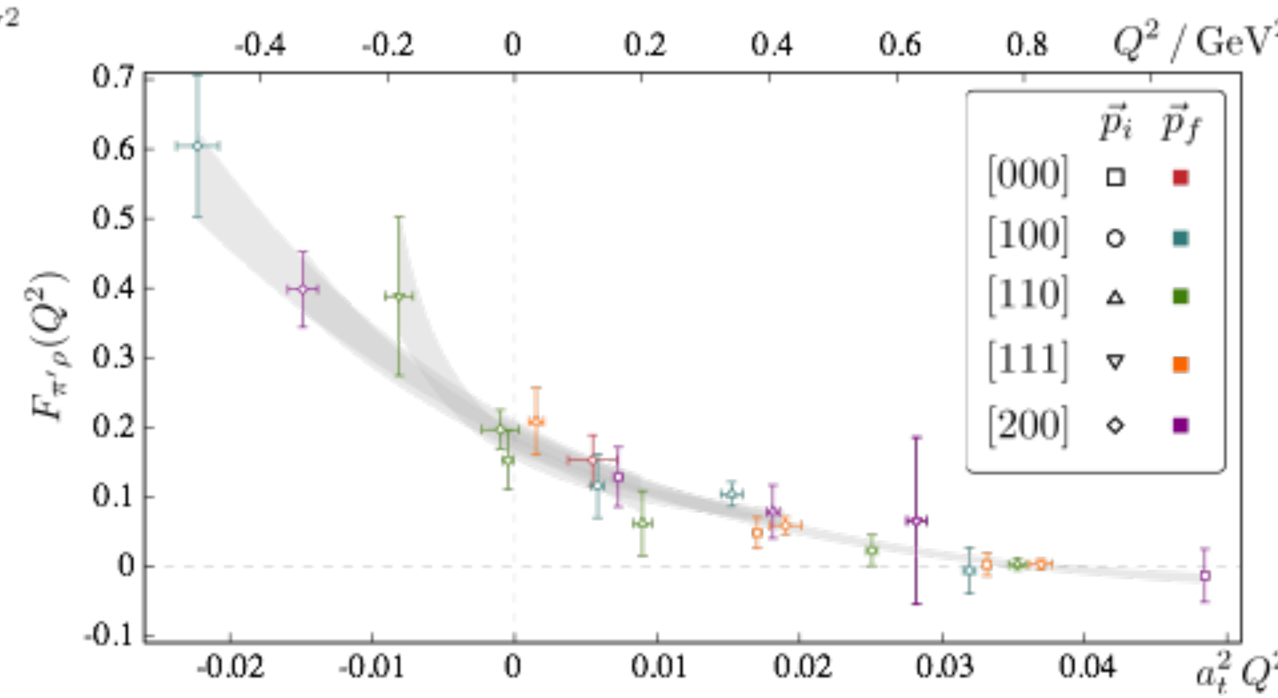


‘hindered’ transitions









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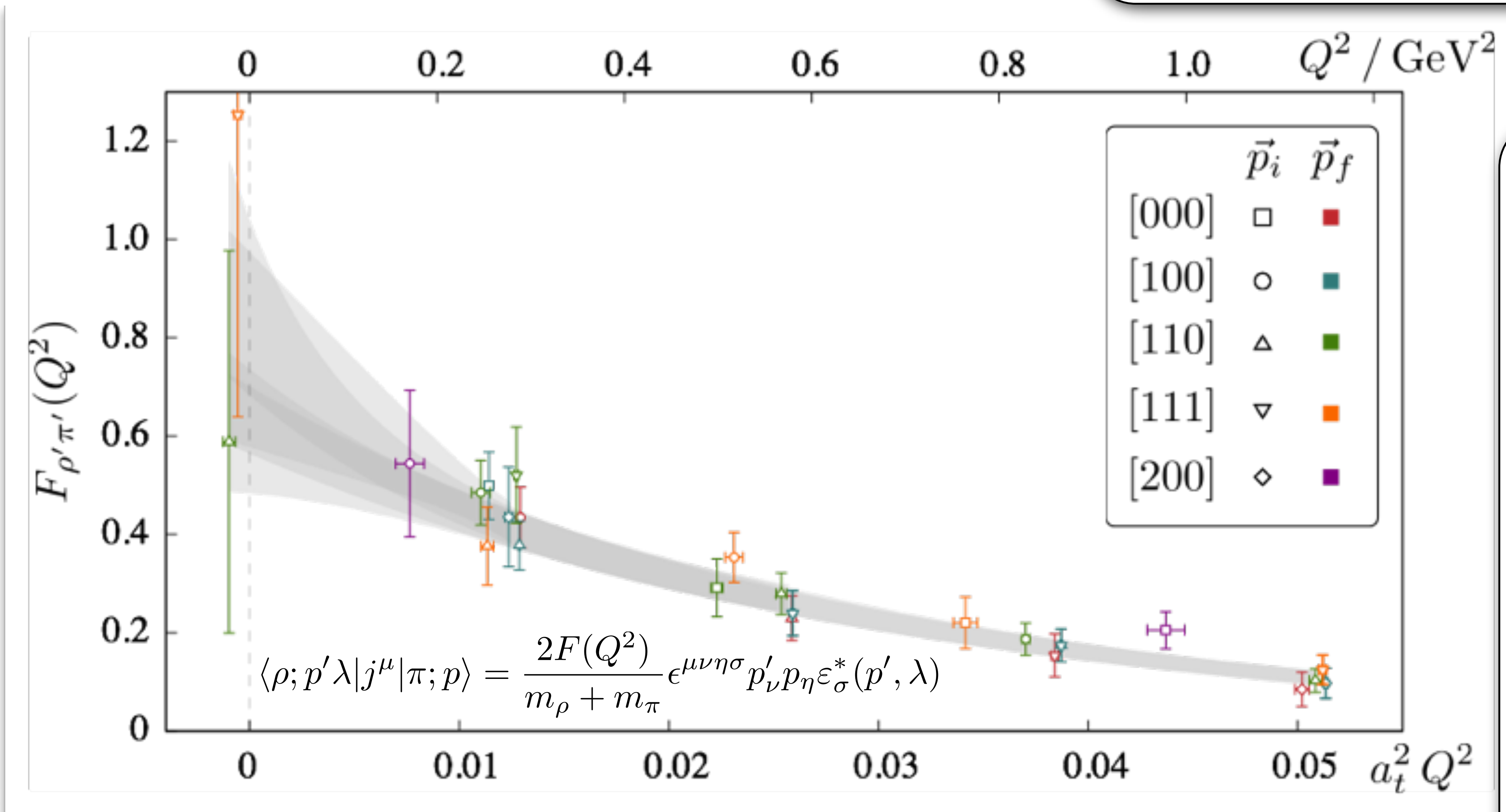
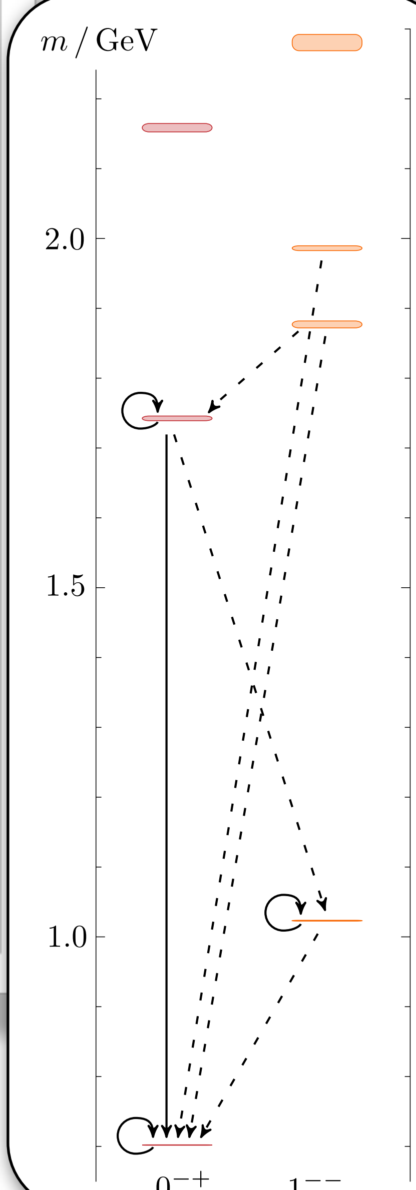
excited state radiative transitions

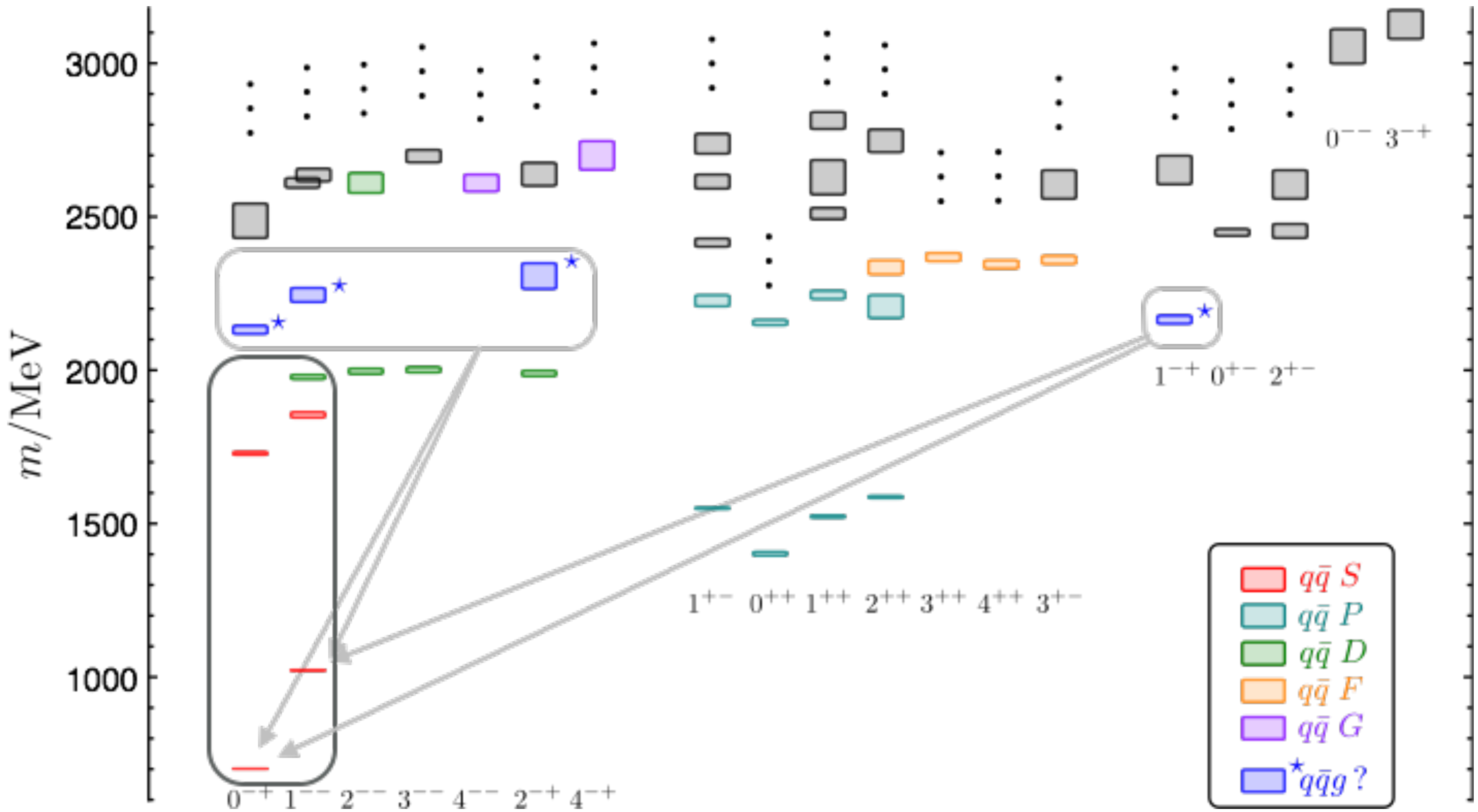
☑ **show some progress**

this was the 'first' study of excited state matrix elements in lQCD





the horizon



what have we done?

- implemented the formalism
- moved beyond ground states
- explored systematics
- seen real physics

In the works

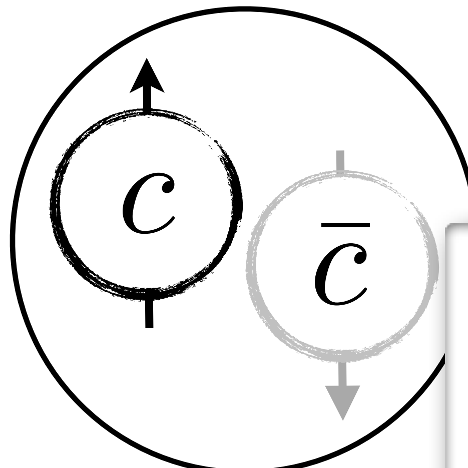
- explore more of the spectrum
- unstable to stable transition

other directions

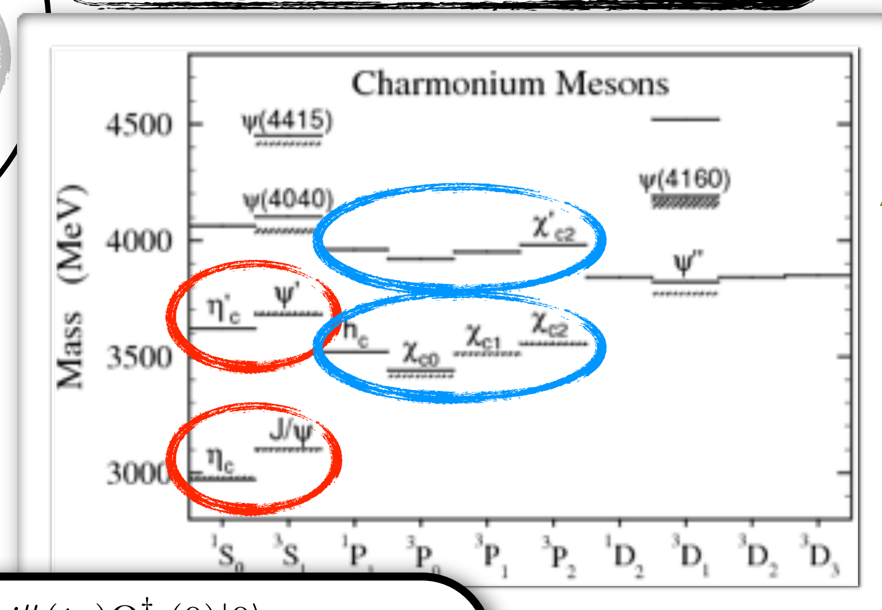
charm, tetraquarks, baryons?

thanks

QCD → **lattice QCD** → **observables**



$$\mathcal{L}_{QCD} = \bar{\Psi}\left(i\not{D} - m\right)\Psi - \frac{1}{4}\mathrm{Tr}\left(GG\right)$$



- ☑ **teach you some physics**
- ☑ **present a puzzle**
- ☑ **show some progress**

$$\begin{aligned} C^{\mu}_{\mathfrak{n}\mathfrak{m}} &= \langle 0|\Omega_{\mathfrak{n}}(t)\, j^{\mu}(t_\gamma)\,\Omega^{\dagger}_{\mathfrak{m}}(0)|0\rangle \\ &= \sum_{\mathfrak{n}'\mathfrak{m}'} \langle \mathfrak{n}'|j^{\mu}|\mathfrak{m}'\rangle e^{-E_{\mathfrak{n}'}(t-t_\gamma)} e^{-E_{\mathfrak{m}'}t_\gamma} \\ &\qquad \times \frac{\langle 0|\Omega_{\mathfrak{n}}(0)|\mathfrak{n}'\rangle}{2E_{\mathfrak{n}'}} \frac{\langle \mathfrak{m}'|\Omega^{\dagger}_{\mathfrak{m}}(0)|0\rangle}{2E_{\mathfrak{m}'}} \end{aligned}$$

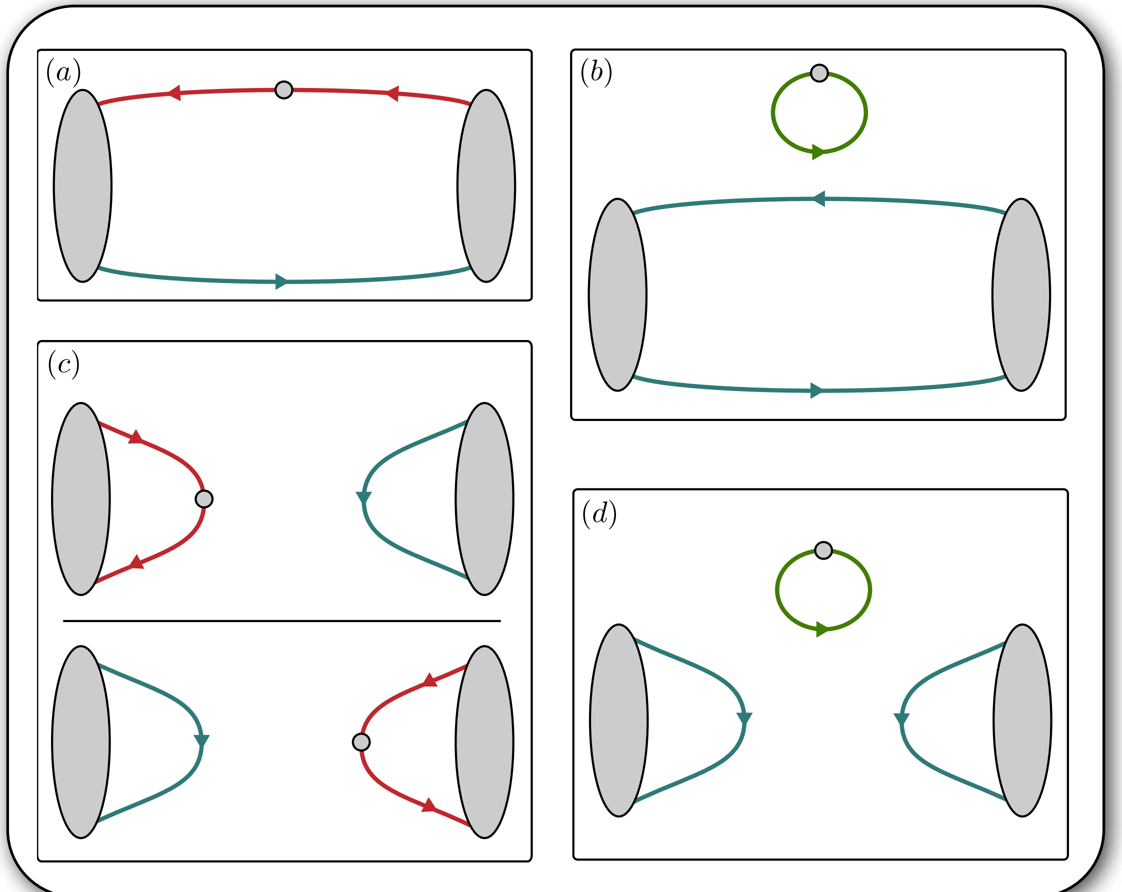
$$\Omega^{\dagger}_{\pi} \sim \sum_i v_i^{(\pi)} \mathcal{O}_i^{\dagger}$$

based on: http://arxiv.org/pdf/1501.07457v1.pdf

Matrix Elements in lattice QCD

Christian Shultz

my biggest contributions



a solution

use a basis of operators

$$\mathcal{O}^{J\lambda} \sim \bar{\psi}\Gamma_a \overleftrightarrow{D}_b \overleftrightarrow{D}_c \cdots \psi \times \mathrm{CG}(a,b,c \to J\lambda)$$

build a matrix of correlators

$$C(t)v^{(\mathfrak{n})} = \lambda_{\mathfrak{n}}(t) C(t_0) v^{(\mathfrak{n})}$$

diagonalize the matrix

$$\begin{aligned} C^{\mu}_{\mathfrak{n}\mathfrak{m}} &= \langle 0|\Omega_{\mathfrak{n}}(t) j^{\mu}(t_\gamma)\Omega^{\dagger}_{\mathfrak{m}}(0)|0\rangle \\ &= \sum_{\mathfrak{n}'\mathfrak{m}'} \langle \mathfrak{n}'|j^{\mu}|\mathfrak{m}'\rangle e^{-E_{\mathfrak{n}'}(t-t_\gamma)} e^{-E_{\mathfrak{m}'}t_\gamma} \\ &\quad \times \frac{\langle 0|\Omega_{\mathfrak{n}}(0)|\mathfrak{n}'\rangle}{2E_{\mathfrak{n}'}} \frac{\langle \mathfrak{m}'|\Omega^{\dagger}_{\mathfrak{m}}(0)|0\rangle}{2E_{\mathfrak{m}'}} \end{aligned}$$

$$\Omega^{\dagger}_{\pi} \sim \sum_i v_i^{(\pi)} \mathcal{O}_i^{\dagger}$$

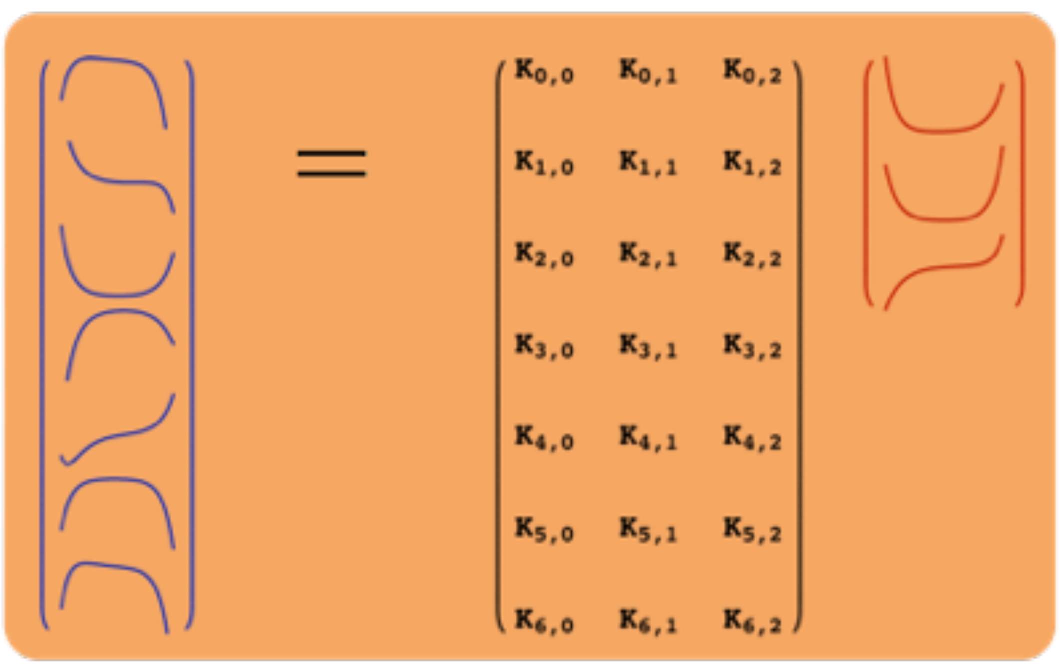
linear least squares

Optimized Operators
'relax' quickly

pollution term

$$C^{\mu}_{n,m}(\delta t, t_\gamma) \sim \frac{e^{-E_n(\delta t - t_\gamma)}}{2E_n}\frac{e^{-E_m t_\gamma}}{2E_m}\langle n|j^\mu|m\rangle + f(\delta t, t_\gamma)$$

Matrix Element Decompositions



Kinematic Factor

Form Factor

$$\langle n|j^\mu|m\rangle = \sum_k K^\mu_k(n,m) F_k(Q^2)$$

Some Examples

$$\langle \pi, p'|j^\mu|\pi, p\rangle = (p'+p)^\mu F_\pi(Q^2)$$

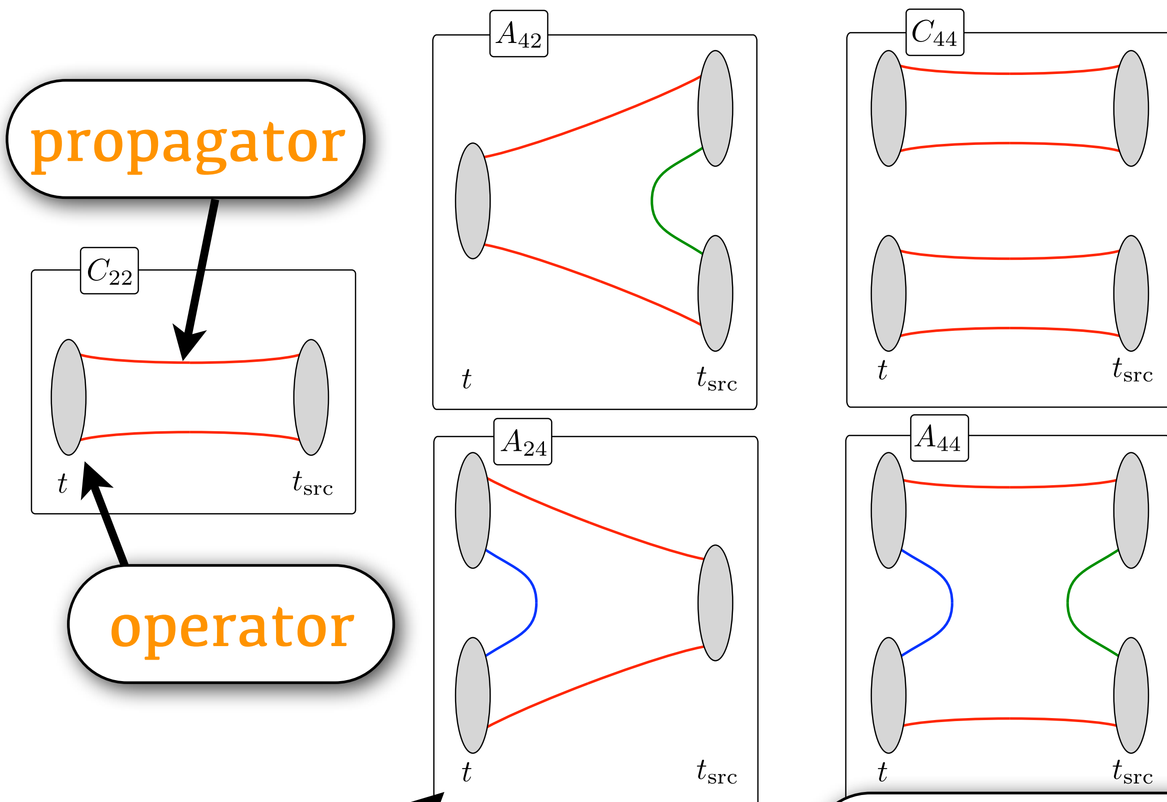
$$\langle \pi, p'|j^\mu|\rho, p\lambda\rangle = \frac{2}{m_\rho + m_\pi}\epsilon^{\mu\nu\sigma\eta}p'_\nu p_\sigma \varepsilon_\eta(p,\lambda) F_{\rho\pi}(Q^2)$$

variational spectroscopy (numeric side)

$$C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0\rangle$$

estimated numerically

propagator



operator

traces over products of matrices

a solution

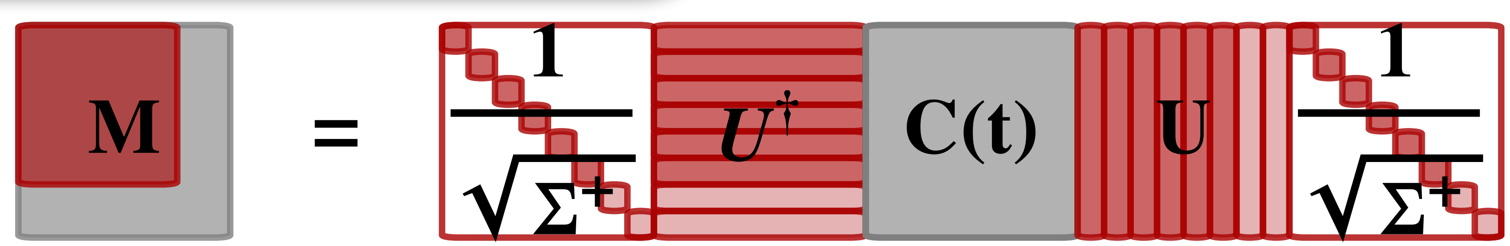
use a basis of operators

$$\mathcal{O}^{J\lambda} \sim \bar{\psi}\Gamma_a \overleftrightarrow{D}_b \overleftrightarrow{D}_c \cdots \psi \times \mathrm{CG}(a,b,c \to J\lambda)$$

build a matrix of correlators

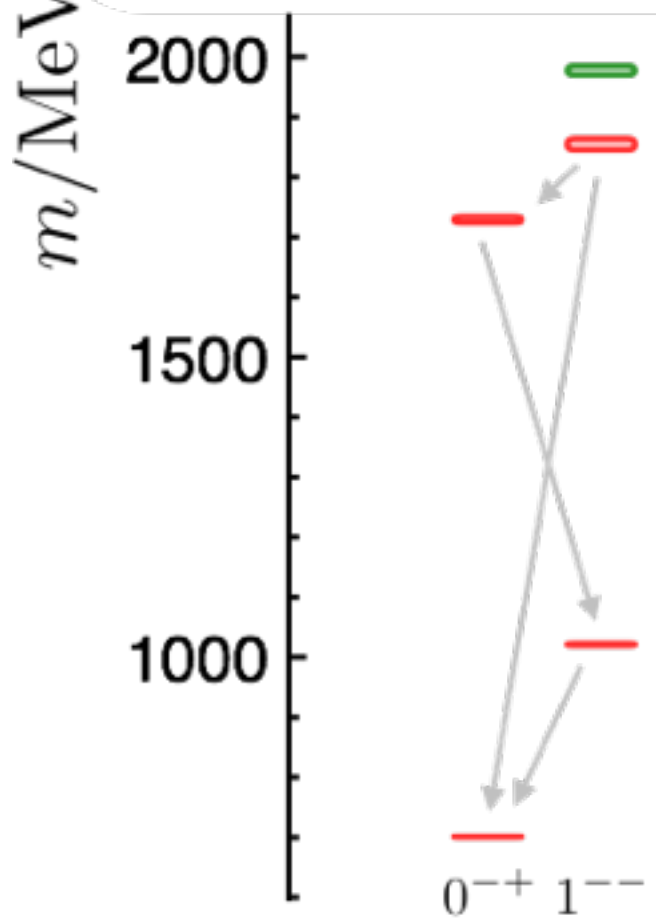
$$C_{ij}(t)v_j^{(\mathfrak{n})} = \lambda_{\mathfrak{n}}(t)C_{ij}(t_0)v_j^{(\mathfrak{n})}$$

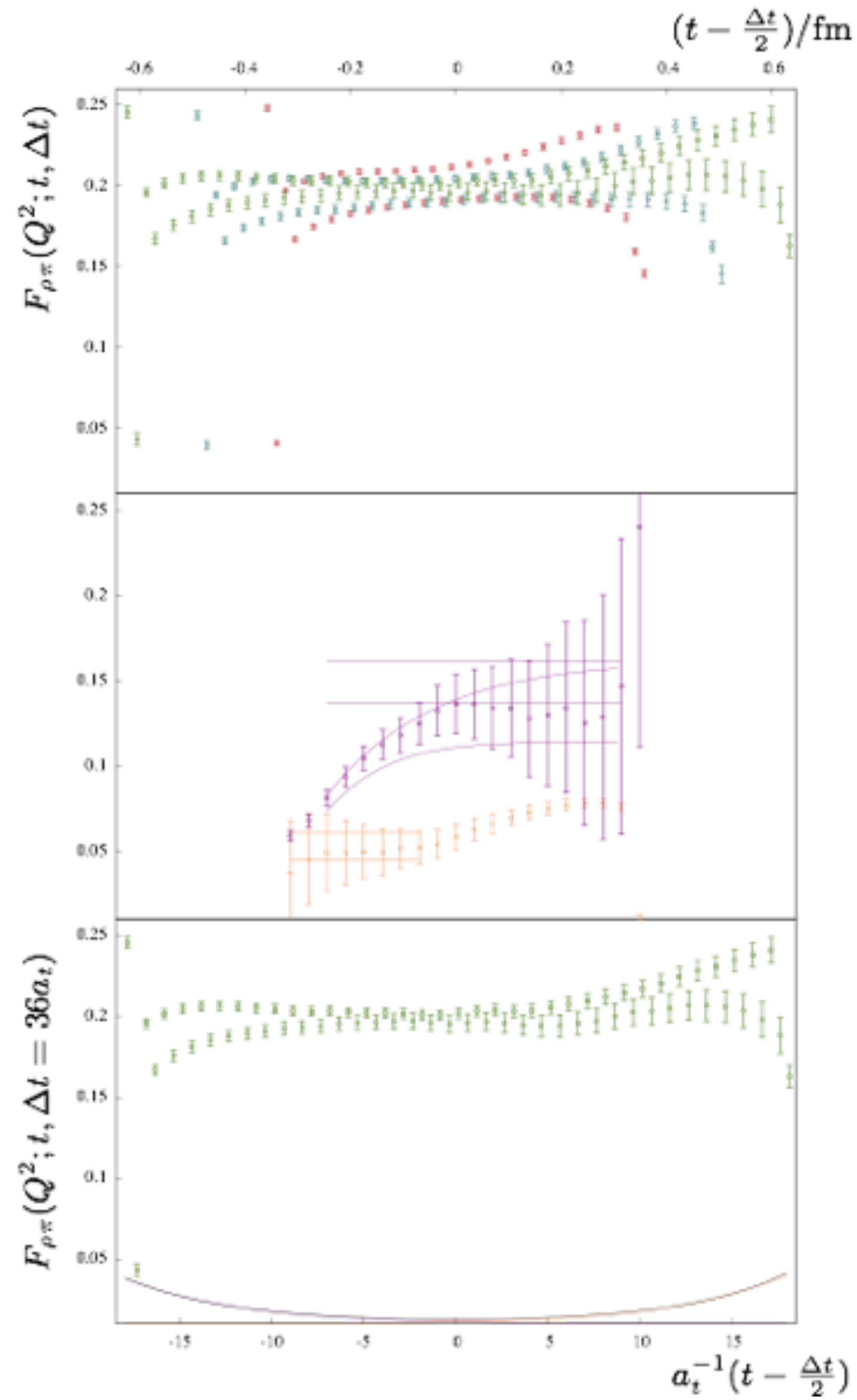
remove null space with SVD



reconstruction

$$
\begin{aligned}
C^{\mu}_{\mathfrak{n}\mathfrak{m}} &= \langle 0|\Omega_{\mathfrak{n}}(t)\, j^{\mu}(t_\gamma)\, \Omega^{\dagger}_{\mathfrak{m}}(0)|0\rangle \\
&= \sum_{\mathfrak{n}'\mathfrak{m}'} \langle \mathfrak{n}'|j^{\mu}|\mathfrak{m}'\rangle\, e^{-E_{\mathfrak{n}'}(t-t_\gamma)}\, e^{-E_{\mathfrak{m}'}t_\gamma} \\
&\qquad \times \frac{\langle 0|\Omega_{\mathfrak{n}}(0)|\mathfrak{n}'\rangle}{2E_{\mathfrak{n}'}}\, \frac{\langle \mathfrak{m}'|\Omega^{\dagger}_{\mathfrak{m}}(0)|0\rangle}{2E_{\mathfrak{m}'}}
\end{aligned}
$$





clover action

Add irrelevant operators

$$S_C \equiv a_t a_s^3 \sum_x \bar{\psi}(x) \left[m_0 + \nu_t \not{D}_t^W + \nu_s \sum_i \not{D}_s^W - \frac{a_s}{2} \left(c_t \sum_i \sigma_{4i} F_{4i} - c_s \sum_{i<i'} \sigma_{ii'} F_{ii'} \right) \right] \psi(x)$$

$$S_C = \bar{\psi} \overleftarrow{\Omega} \left[m + \not{\nabla} \right] \overrightarrow{\Omega} \psi$$

$$\overrightarrow{\Omega} = 1 + a_t \Omega_m m + a_t \Omega_t \overrightarrow{\not{\nabla}}_t + a_s \Omega_s \sum_i \overrightarrow{\not{\nabla}}_i$$

Redefine QED fields

$$\tilde{V}^\mu = \bar{\psi} \overleftarrow{\Omega} \gamma^\mu \overrightarrow{\Omega} \psi$$

Tree Level

$$\mathcal{V}_0 = \bar{\psi} \gamma_0 \psi - \frac{1}{4} a_s \nu_s (1 - \frac{1}{\xi}) \partial_k \bar{\psi} \sigma_{0k} \psi$$

$$\mathcal{V}_k = \bar{\psi} \gamma_k \psi - \frac{1}{4} a_t \left(1 - \xi\right) \partial_0 \bar{\psi} \sigma_{0k} \psi$$

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