Calculus 1 B/C

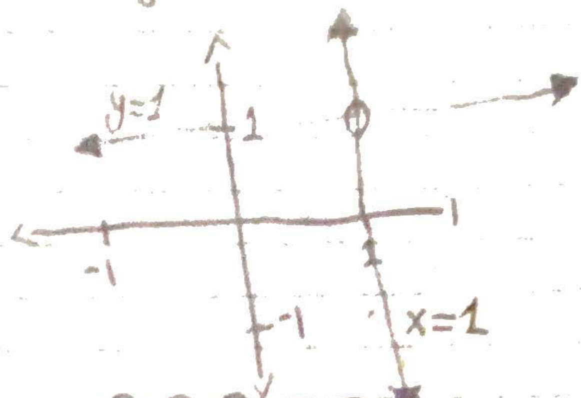
Limits and Continuity

A* Limit* is an important idea that all of Calculus is based upon. Even though it's an important idea, it's quite simple.

★ anything divided by zero including zero divided by zero is undefined

$f(x) = \frac{x-1}{x-1}$

$f(1) = \frac{0}{0}$ = undefined

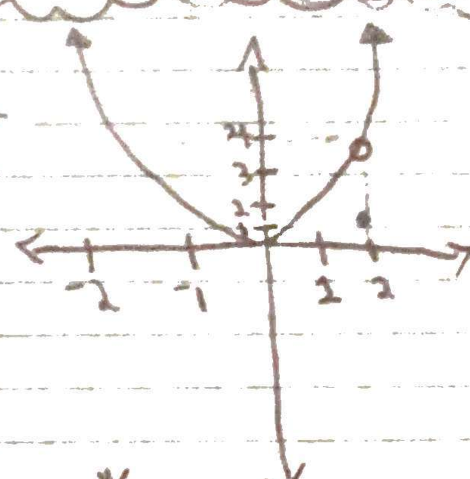


HA: 1
VA: 1

$\lim_{x \to 1} f(x) = 1$

A limit simply states that when x is approaches the y becomes undefined

$g(x) = \begin{cases} x^2, & x \neq 2 \\ 1, & x = 2 \end{cases}$



$\lim_{x \to 2} f(x) = 4$

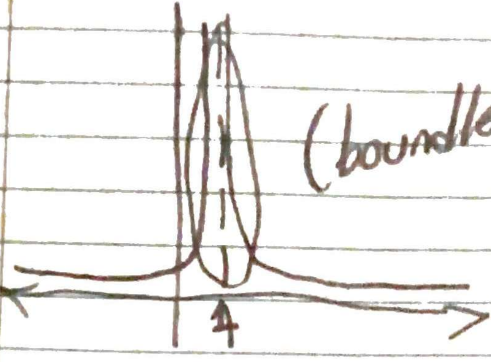
* The limit doesn't exist when the function doesn't approach the same value from both directions. *

Estimating Limit Values From Graphs

$\lim_{x \to 6} f(x) = 1$ - equal

$\lim_{x \to 4} f(x) = 3$ } undefined

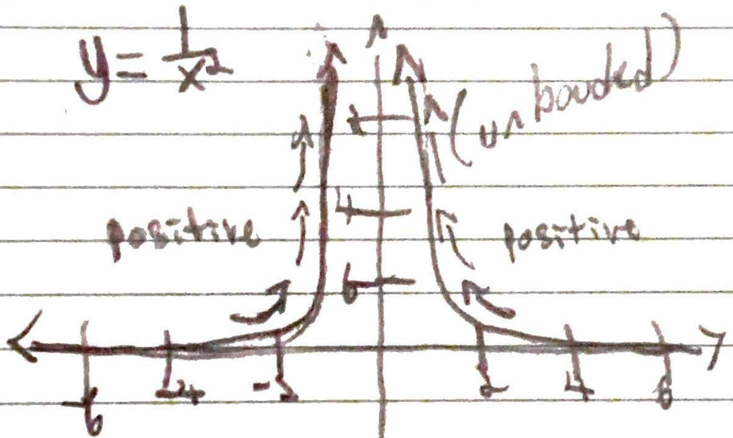
$\lim_{x \to 2} f(x) =$ Doesn't exist — defined



(boundless)

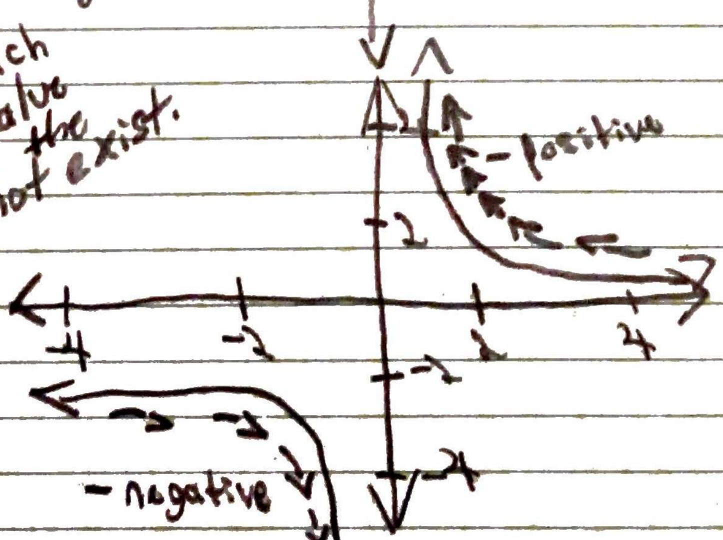
$\lim_{x \to 1}$ f(x) = doesn't exist

$y = \frac{1}{x^2}$



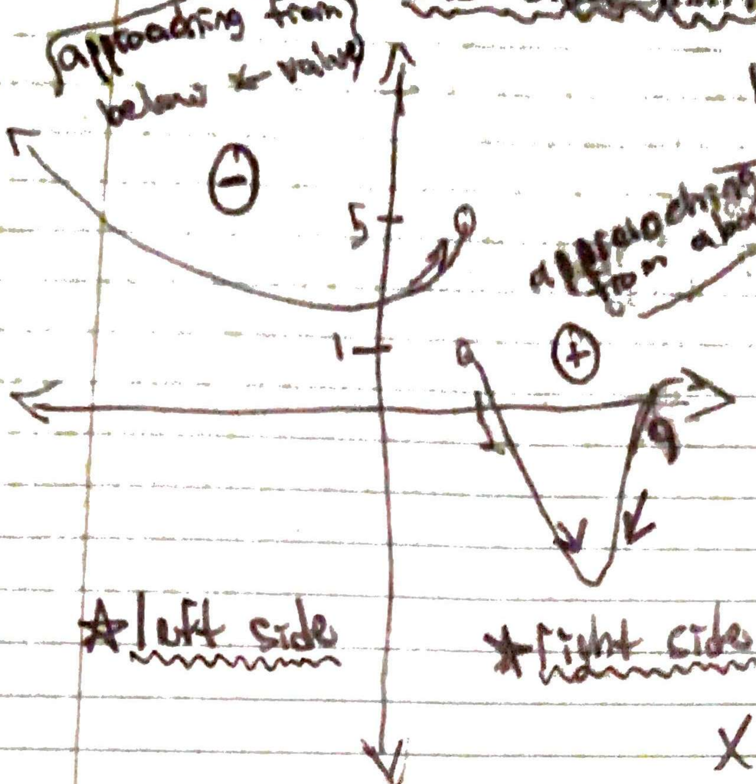
$\lim_{x \to 0} \frac{1}{x^2}$ = doesn't exist or Unbounded

If it doesn't approach the same value or direction the limit does not exist.



$\lim_{x \to 0} \frac{1}{x}$ = does not exist or unbounded

One-sided limits of Graphs



approaching from above value

$\lim_{x \to 2^-} f(x) =$ doesn't exist

← superscript

Not equal

$\lim_{x \to 2^+} f(x) =$ doesn't exist

$\lim_{x \to 4^-} f(x) = -5$

are equal

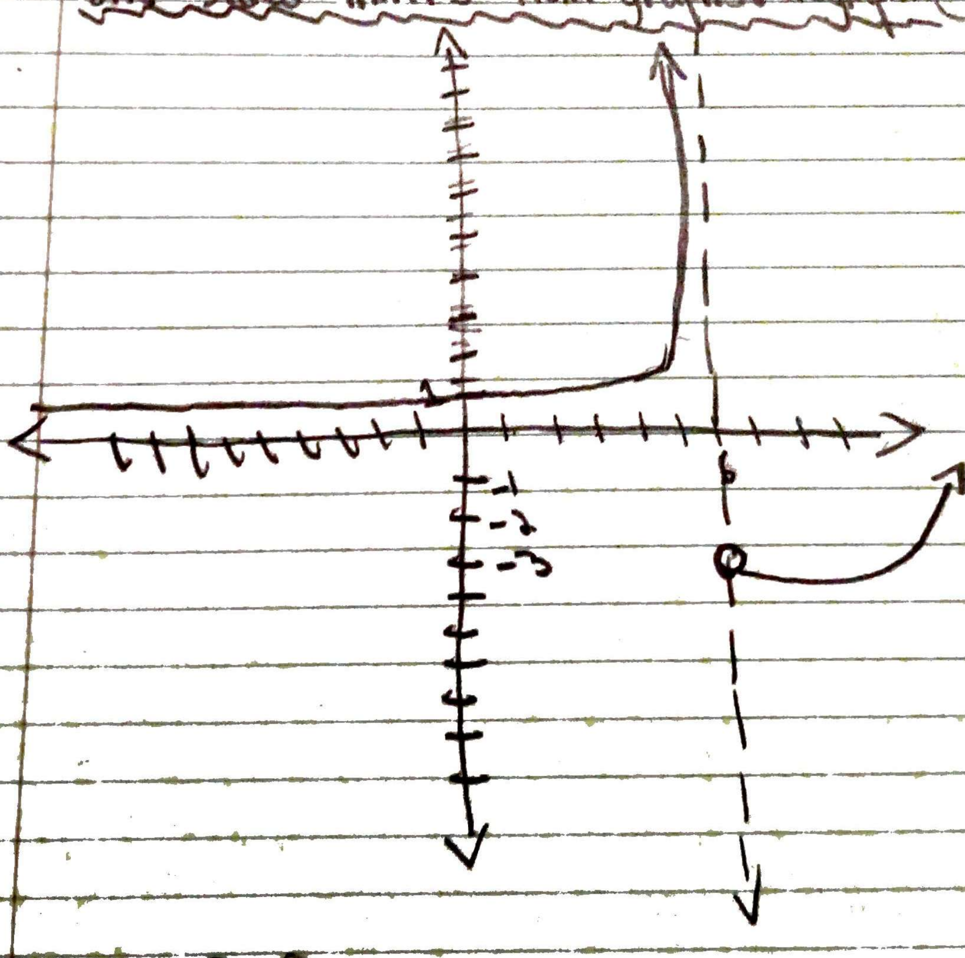
$\lim_{x \to 4^+} f(x) = -5$

★left side

★right side

$\boxed{\lim_{x \to 4} f(x) = -5}$ ✓

One sided limits from graphs: asymptote



$\lim_{x \to 6^-} g(x) =$ limit doesn't exist

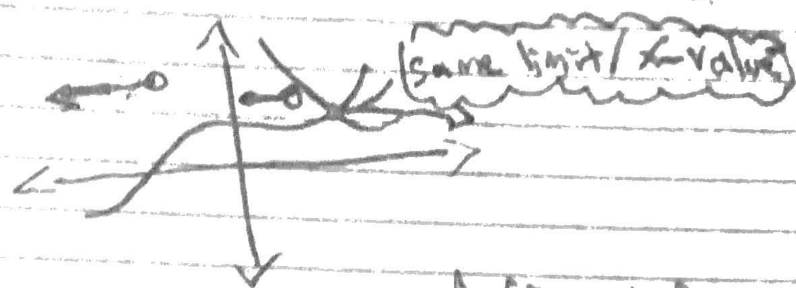
$\lim_{x \to 6^+} g(x) = -3$

Connecting limits and graphical behavior

A limit tells you the behavior of a function as it approaches a point but it does not tell you exactly what's going on at that specific point.

Appreciate the Limit

* Any line that intersects the limit and its x-value of the function can be defined the same like so. *



The Big takeaway in this section is that you can form many functions to have the same limit at a point and for a given function you can take the limit from an infinite amount of points.

Different functions can have the same limit and x-value.

Estimating Limit Values From Tables

$$\lim_{x \to 3} \frac{x^3 - 3x^2}{5x - 15} = 1.8$$

$$\frac{3^3 - 3 \cdot 3^2}{5 \cdot 3 - 15} = \frac{0}{0}$$

limit from left

x	$\frac{x^3-3x^2}{5x-15}$
2.9	1.682
2.99	1.788
2.999	1.4988

limit from right

x	$\frac{x^3-3x^2}{5x-15}$
3.1	1.922
3.01	1.812
3.001	1.801

X-Value must increase and decrease towards the same y to have a limit

The function is approaching 1.8 based on the two tables thus the limit is 1.8.

Determining limits using algebraic properties of limits & limit properties

Goes for All from ★ observing both sides of each function

$\lim_{x \to c} f(x) = L \qquad \lim_{x \to c} g(x) = M$

(Sum Property)

$$\lim_{x \to c} (f(x) + g(x)) = \overset{L}{\lim_{x \to c} f(x)} + \overset{M}{\lim_{x \to c} g(x)} = \boxed{L + M}$$

$$\lim_{x \to c} (f(x) - g(x)) = \lim_{x \to c} f(x) - \lim_{x \to c} g(x) = \boxed{L - M}$$

(Difference Property)

positive / neg

$$\lim_{x \to c} (f(x) \cdot g(x)) = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x) = \boxed{L \cdot M}$$

(Multiplication property)

$$\lim_{x \to c} K f(x) = K \overset{L}{\lim_{x \to c} f(x)} = \boxed{K \cdot L}$$

(Constant property)

Can't be zero for division

denom can't be zero

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} = \boxed{\frac{L}{M}}$$

(quotient property) except when $f(x) = 0$

$$\lim_{x \to c} (f(x))^{\frac{r}{s}} = \left(\lim_{x \to c} f(x)\right)^{\frac{r}{s}}$$

$$\boxed{(L)^{\frac{r}{s}}}$$

(exponent property)

Huge Tip: evaluate left side than the right ★

★ Composite function

Also, anything unbounded doesn't have a limit

You can't Jump from a discontinuity in a composite function but you can Jump from

~~[illegible]~~

Your finding out if a function is continuous with a specific input. If the input is (undefined) unbound or results in a discontinuity it is not continuous, thus a limit does not exist.

Determining limits using algebraic properties of limits: direct substitution

Limits by direct Substitution

$$\lim_{x \to -1} (6x^2 + 5x - 1) = 6(-1)^2 + 5(-1) - 1 = 0$$

All parabolas are continuous on domain

f is cont at $x = a$

iff!

$$\lim_{x \to a} f(x) = f(a)$$

Undefined limits by direct substitution

$$\lim_{x \to 1} \frac{x}{\ln(x)} = \text{Doesn't exist} = \frac{1}{0}$$

$\boxed{\text{differences}}$

Limits of trigonometric functions

defined for all real numbers continuosly

$$\lim_{x \to \pi} \sin(x) = 0 \qquad \lim_{x \to a} \sin(x) = \sin(a)$$

$$\lim_{x \to \frac{\pi}{4}} \cos(x) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \qquad \lim_{x \to a} \cos(x) = \cos(a)$$

more complex

not defined continuosly and can be undefinud

$$\lim_{x \to \pi} \tan(x) = \lim_{x \to \pi} \frac{\sin(x)}{\cos(x)} = \frac{\sin(\pi)}{\cos(\pi)} = \frac{0}{1} = 0 \checkmark$$

$$\lim_{x \to \frac{\pi}{2}} \tan(x) = \frac{\sin\left(\frac{\pi}{2}\right)}{\cos\left(\frac{\pi}{2}\right)} = \frac{1}{0} = \text{no limit}$$

$$\lim_{x \to \pi} \cot(x) = \frac{\cos(\pi)}{\sin(\pi)} = \frac{-1}{0} = \text{no limit}$$

Limits of Piecewise Functions

$$f(x) = \begin{cases} \frac{x+2}{x-1} & \text{for } 0 < x \leq 4 \\ \sqrt{x} & \text{for } > 4 \end{cases}$$

$$\lim_{x \to 4^+} f(x) = \sqrt{4} = 2$$

$$\lim_{x \to 4^-} f(x) = \frac{4+2}{4-1} = \frac{6}{3} = 2$$

$$\lim_{x \to 4} f(x) = 2$$

$$\lim_{x \to 2} f(x) = \frac{2+2}{2-1} = 4$$

$$g(x) = \begin{cases} \sin(x+1) & \text{for } x < -1 \\ 2^x & \text{for } \underline{-1 \leq x < 5} \end{cases}$$

if the requirements don't include what's being approach exclude it. Then, focus on the one that does.

$$\lim_{x \to -1^+} g(x) = \frac{1}{2}$$

$$\lim_{x \to -1^-} g(x) = \sin(-1+1) = \sin(0) = 0$$

$$\lim_{x \to -1} g(x) = \text{doesn't exist}$$

plus, they must have the same output to have a limit

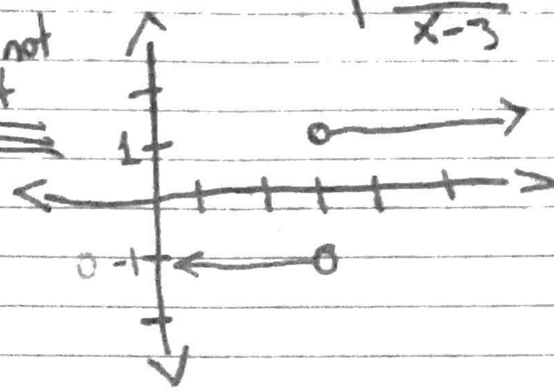
$$\lim_{x \to 0^+} g(x) = 2^0 = 1$$

Limits of piecewise functions: absolute value

$f(x) = \frac{|x-3|}{x-3}$

$f(x) = \begin{cases} \frac{x-3}{x-3}, & x>3 \\ \frac{-(x-3)}{x-3} & x<3 \end{cases} = \begin{cases} 1, x>3 \\ -1, x<3 \end{cases}$

$\lim_{x \to 3} f(x)$ does not exist



$\lim_{x \to 3^-} f(x) = -1$

$\lim_{x \to 3^+} f(x) = 1$

The Limit of a function

$\lim_{x \to c} f(x) = L \iff \lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = L$

Selecting procedures for determining Limits

$\frac{1}{x^2 - x}$ is undefined for any x

f(a)

- $f(a) = \frac{b}{0}$ — vertical asymptote
- $f(a) = b$ — we're done
- $f(a) = \frac{0}{0}$ — intermediate
  - try removing discontinuity
    - factoring
    - conjugate
    - trig
  - Make approximations with table

Determining Limits Using algebraic Manipulation

Limits by factoring

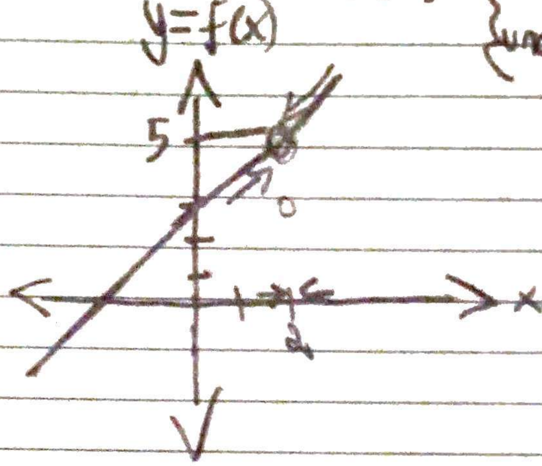
1. $f(x)=\frac{x^2+x-6}{x-2} = \frac{(x+3)(x-2)}{(x-2)} = x+3,\ x\neq 2$

$\lim_{x\to 2} f(x) = 5$

$f(x)=\begin{cases} x+3, & x\neq 2 \\ \text{undefined}, & x=2\end{cases}$

$f(2)=\frac{0}{0}$

not defined



Limits by rationalizing

$\lim_{x\to -1} \boxed{\frac{x+1}{\sqrt{x+5}-2}} = \frac{\lim_{x\to-1} x+1}{\lim_{x\to-1}(\sqrt{x+5}-2)} = \frac{-1+1}{\sqrt{-1+5}-2} = \frac{0}{0}$

They both have the same limit you just can't see it because of the discontinuity so you have to take it out

① $g(x) = \frac{(x+1)}{(\sqrt{x+5}-2)}\cdot\frac{(\sqrt{x+5}+2)}{(\sqrt{x+5}+2)}$

② $g(x)=\frac{(x+1)(\sqrt{x+5}+2)}{x+5-2^2} =$ ③ $\frac{\cancel{(x+1)}\sqrt{x+5}+2}{\cancel{(x+1)}}=$

④ $\sqrt{x+5}+2$, for $x\neq -1$

⑤ $f(x)=\sqrt{x+5}+2$

$f(x)=g(x)$ for all $x\neq-1$

⑥ $\lim_{x\to-1}(\sqrt{x+5}+2)$

Trig limit using Pythagorean identity

$$\lim_{\theta \to 0} \frac{1-\cos\theta}{2\sin^2\theta} = \frac{\lim_{\theta\to 0} 1-\cos\theta}{\lim_{\theta \to 0} 2\sin^2\theta} \quad \frac{1-\cos(0)}{2\sin^2(0)} = \boxed{\frac{0}{0}}$$

Indeterminant

$$f(x) = \frac{1-\cos\theta}{2(1-\cos^2\theta)} = \frac{1-\cos\theta}{2(1-\cos\theta)(1+\cos\theta)}$$

$$\boxed{f(x) = \frac{1}{2+2\cos\theta} \quad \theta \neq 0}$$

$$\sin^2\theta + \cos^2\theta = 1$$
$$-\cos^2\theta$$

$$\lim_{\theta\to 0} f(x) = \lim_{\theta \to 0} g(x)$$

$$g(x) = \frac{1}{2+2\cos\theta} \qquad g(0) = \frac{1}{2+2} = \boxed{\frac{1}{4}}$$

Trig limit using double angle Identity

$$\lim_{\theta \to -\frac{\pi}{4}} \frac{1+\sqrt{2}\sin\theta}{\cos 2\theta} = \frac{\lim_{\theta\to -\frac{\pi}{4}} 1+\sqrt{2}\sin\theta}{\lim_{\theta\to-\frac{\pi}{4}} \cos 2\theta} = \frac{1+\sqrt{2}\sin\left(-\frac{\pi}{4}\right)}{\cos 2\cdot\left(-\frac{\pi}{4}\right)}$$

$$-\frac{\sqrt{2}}{2}$$

$$\frac{\cancel{1+\sqrt{2}\sin\theta}}{\cancel{\cos 2\theta}}$$
$$(\cancel{1+\sqrt{2}\sin\theta})(1-\sqrt{2}\sin\theta)$$

$$= \frac{1-1}{0} = \boxed{\frac{0}{0}}$$

indeterminant

$$= \frac{1}{(1-\sqrt{2}\sin\theta)} \quad \theta \neq -\frac{\pi}{4}$$

in open interval of $(-1,1)$

$$\lim_{\theta \to -\frac{\pi}{4}} \frac{1}{1-\sqrt{2}\sin\theta} = \frac{1}{1-\sqrt{2}\sin\left(-\frac{\pi}{4}\right)} = \frac{1}{1+1} = \boxed{\frac{1}{2}}$$

$$-\frac{\sqrt{2}}{2}$$

f and g equal for all $x \neq a$ and f cont at a then the $\lim_{x\to a} f(x) = \lim_{x \to a} G(x)$

Determining Limits Using the Squeeze Theorem

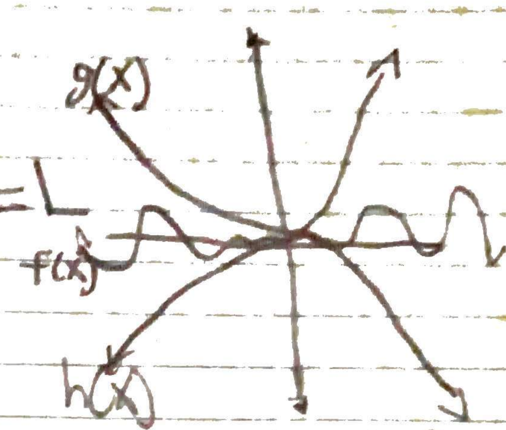
Suppose: for x near a but $x \neq a$

If,

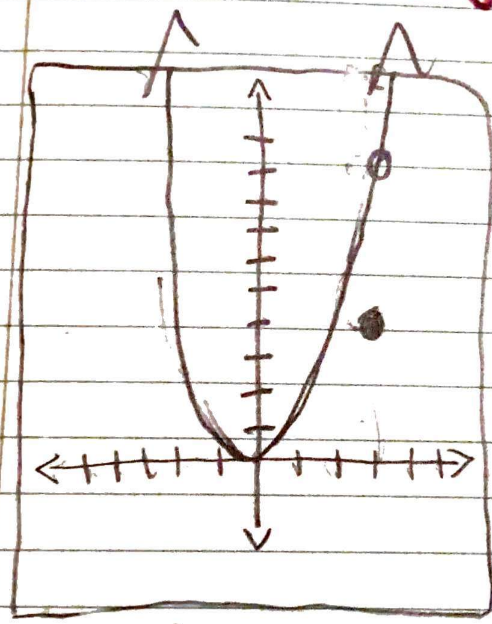
① $g(x) \leq f(x) \leq h(x)$

② $\lim\limits_{x \to a} g(x) = \lim\limits_{x \to a} h(x) = L$

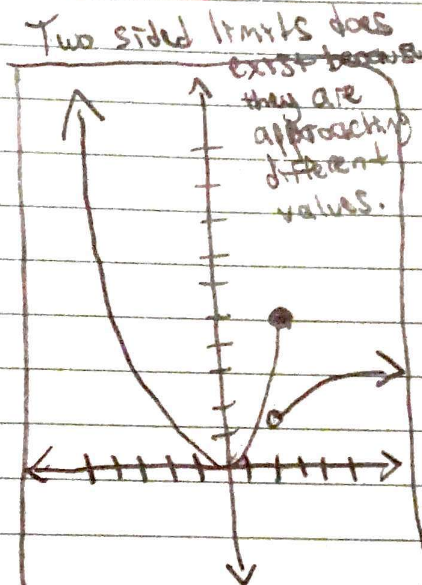
Then $\lim\limits_{x \to a} f(x) = L$



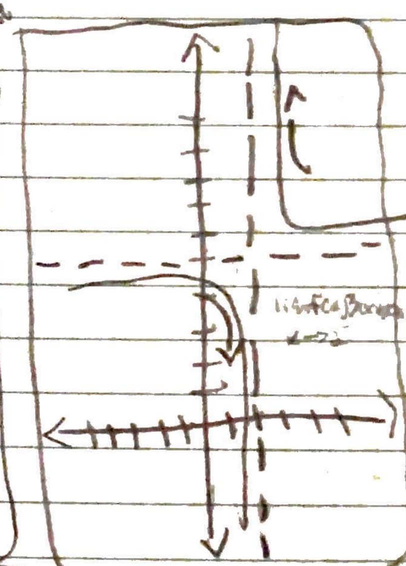
Types of discontinuities



{Point/Removable discontinuity}



{Jump discontinuity
$\lim\limits_{x \to 5} f(x) = 4$
$\lim\limits_{x \to 5^+} f(x) = \sqrt{2}$}



{Asymptotic Discontinuity (unbounded)}

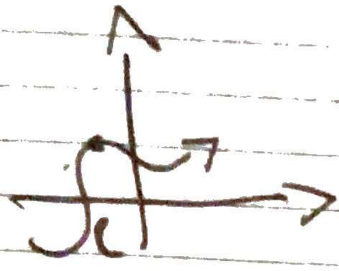
definition of Continuity → limit does exist

f is continuous at $x = c$ if and only if $\lim\limits_{x \to c} f(x) = f(c)$

The limit of the point that's being approached must be equal to the point substituted in the function.

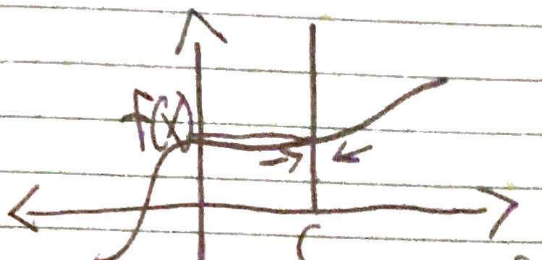
Defining Continuity at a Point

Continuity

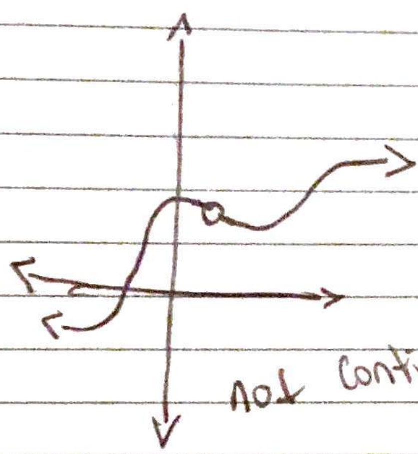


f is cont' at x=c

$$\lim_{x \to c} f(x) = f(c)$$

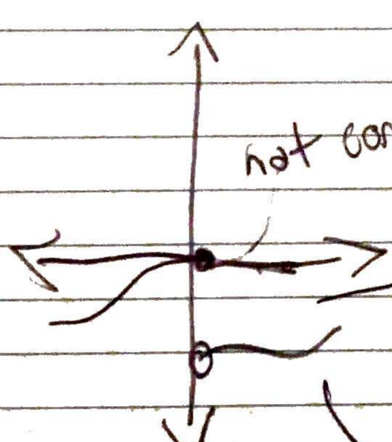


continuous



not continuous

important

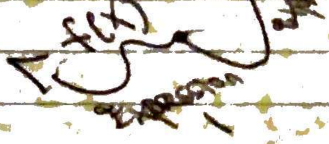


not continuous

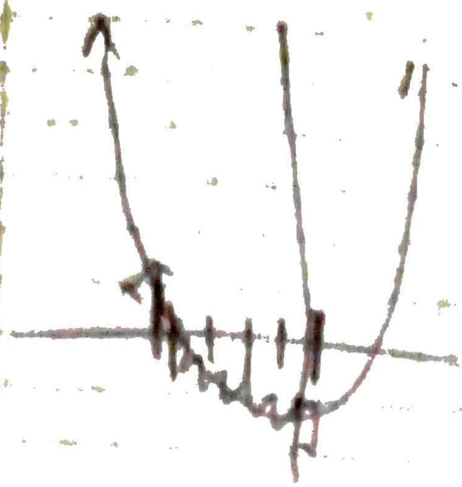
Two different outputs

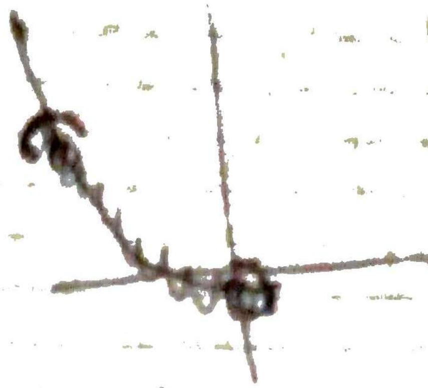
There must be one concise one for continuity. To know that they are joint and heading to one another like this

Since they don't have the same output for the same input, it's not continuous.



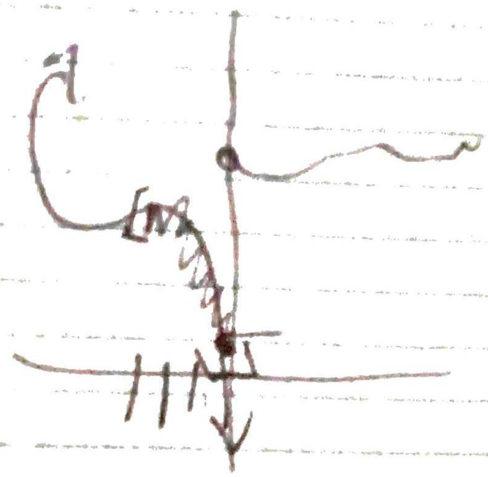
Determining Continuity over an Interval



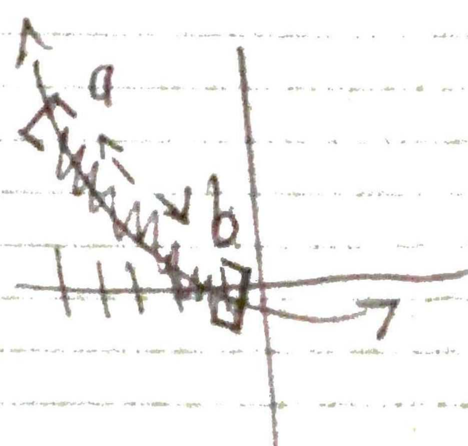


$[4, 0]$ continuous

Continuous because the parenthesis does not include initial starting points



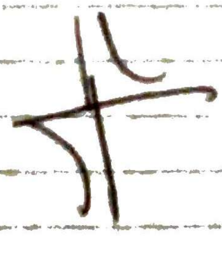
$[-3, 0]$



Its continuous but you make sure the whole interval is continuous through

$\lim_{x \to a^+} f(x) = f(a)$ $\lim_{x \to b^-} f(x) = f(b)$

Thus, the one sided interval must approach the value of the function



this function is heading opposite infinity but is weirdly connected and going back from infinity

Removing Discontinuity

I already did this.

$$f(x)\ \frac{6(x+1)(x+2)}{(x+2)(x-2)} = \boxed{\frac{6(x+1)}{x-2}}$$

Overall, either use factoring, conjugate(rationalizing), or trigonometric functions to find your f(x) or limit.

$$\frac{(x+2)\cancel{(x+3)}}{\cancel{(x+3)}}$$

$$\frac{x+1\ \sqrt{x+5}+2}{x+5-4}$$

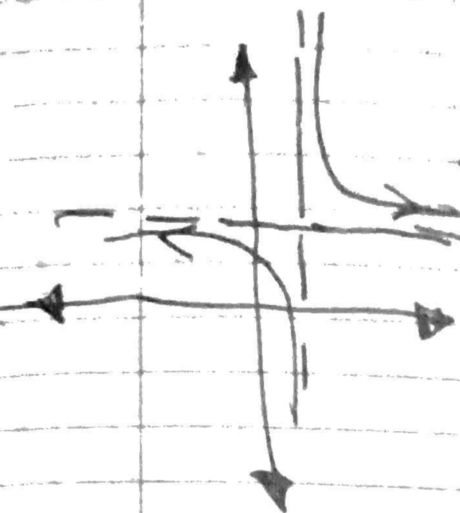
$$\frac{\cancel{(x-2)}(\sqrt{x+7}+3)}{\cancel{(x-2)}} \qquad \sqrt{x+6}+2$$

$$2$$

$$x-7 \qquad \frac{(x-2)(\sqrt{x+7}+3)}{x+7-9}$$

$$\frac{\cancel{(x+7)}(x-7)}{\cancel{(x+7)}}$$

Introduction to Infinite limits



$\lim_{\boxed{x \to a}} f(x) = L$ - finite

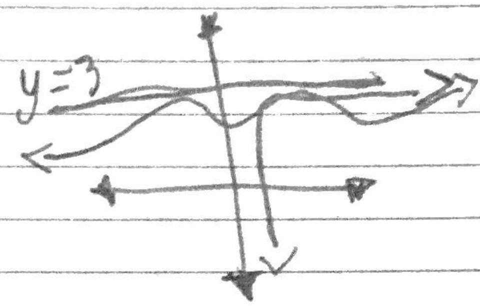
★ limits at Infinity have a different formal definition compared to finite values.

$\lim_{x \to \infty} f(x) = 2$ - infinite $\quad \lim_{x \to -\infty} f(x) = 2$

$\lim_{x \to 1} f(x) = \infty$

Functions with same limit infinity

$\lim_{x \to \infty} f(x) = 3$



Limits at Infinity of quotients

part 1

dominant and will change fast — will approximate its ∞

$f(x) = \frac{4x^5 - 3x^2 + 3}{6x^5 - 100x^2 - 10} \approx \frac{4x^5}{6x^5} = \frac{2}{3}$ for very large x $(x \to \infty)$ $(x \to -\infty)$

$\lim_{x \to \infty} f(x) = \frac{2}{3}$ = Horizontal asymptote

part 2

$\lim_{x \to -\infty} f(x) = \frac{2}{3}$

$\lim_{x \to -\infty} \frac{3x^3 - 2x^2 + 7}{6x^4 - x^3 + 2x - 80} = \lim_{x \to -\infty} \frac{3x^3}{6x^4} = \lim_{x \to -\infty} \frac{1}{2x} = 0$

or infinity

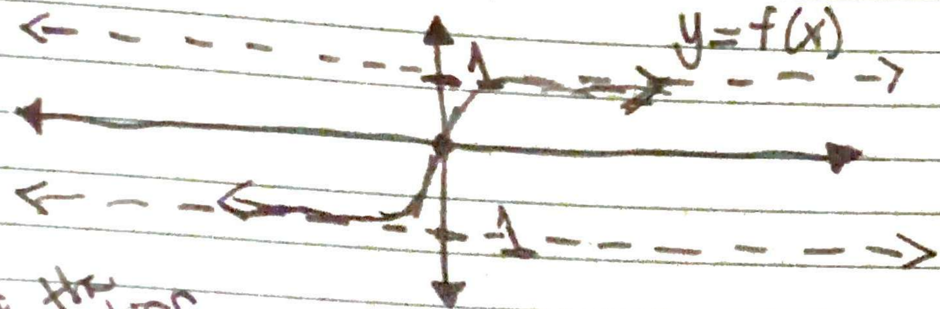
$\lim_{x \to \infty} \frac{4x^4 - 3x^3 + 7x^2 + 10}{250x^3 + 5x^2 - x + 1000} = \lim_{x \to \infty} \frac{4x^4}{250} = \lim_{x \to \infty} \frac{4}{250}x$

limits at Infinite quotients with Square roots (odd power)

$$f(x) = \frac{x}{\sqrt{x^2+1}} \approx \frac{x}{\sqrt{x^2}} = \frac{x}{|x|} \text{ for } \begin{matrix} x \to \infty \\ x \to -\infty \end{matrix}$$

$$\lim_{x \to \infty} f(x) = = \lim_{x \to \infty} \frac{x}{|x|} = 1$$

$$\lim_{x \to -\infty} f(x) = = \lim_{x \to -\infty} \frac{x}{|x|} = -1$$



★ if the rational function is bottom heavy its zero

The key is to see what terms dominate as $x \to \pm\infty$. The larger terms are important. This applied to rational functions with high degrees.

$$\lim_{x \to -\infty} \frac{(\frac{1}{x^2})\sqrt{4x^4 - x}}{(\frac{1}{x^2})2x^2 + 3} = \lim_{x \to -\infty} \frac{\sqrt{4 - \frac{1}{x^3}}^{\,0}}{2 + \frac{3}{x^2}^{\,0}} = \frac{\sqrt{4}}{2} = \frac{2}{2}$$

answer 1

$$\frac{1}{x^2}\sqrt{4x^4 - x} = \frac{1}{\sqrt{x^2}}\sqrt{4x^4 - x} = \sqrt{\frac{4x^4 - x}{x^4}} =$$

$$\sqrt{4 - \frac{1}{x^3}}$$

$$\frac{\frac{x^5}{x^5} + \frac{4x^2}{x^5}}{\frac{x^5}{x^5} + 0}$$

$$\frac{\frac{4x}{x} \quad \frac{4x}{x}}{\frac{4x}{x} \quad \frac{1}{x}} \qquad \frac{\frac{2\cancel{x}}{\cancel{x}} \quad \frac{2\cancel{x}}{\cancel{x}}}{\frac{4\cancel{x}}{\cancel{x}} \quad \frac{\cancel{x}}{\cancel{x}}\,0} \qquad \frac{2}{4} \qquad \frac{1}{2}$$

limits at infinity with quotients and square roots.

$$\sqrt{x^2} \quad |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

∞	$-\infty$
$\sqrt{x^2} = x$	$\sqrt{x^2} = -x$
$-\sqrt{x^2} = -x$	$-\sqrt{x^2} = x$

$-\sqrt{x^2}$

$-\sqrt{x^2}$

$$\frac{\frac{x^5}{x^5} + \frac{4x^2}{x^5}}{\frac{\sqrt{x^{10} + 8x^7}}{-\sqrt{x^{10}}}}$$

$$\frac{\frac{-3x^2}{x^2}+\frac{7}{x^2}\ 0}{\frac{x}{x^2}} \qquad \frac{-3}{x}$$

$$-\frac{9}{2} \qquad \frac{\frac{4x^2}{x^2}-\frac{3x}{x^2}}{\frac{x^3}{x^2}} \qquad \frac{}{x}$$

$$\frac{\frac{\sqrt{x^8-5x^3}}{1} \to 0}{x^8}$$

$$\sqrt{x^2}=|x| \begin{cases} x\geq 0 \\ x<0 \end{cases}$$

$$\frac{\frac{3x^4}{x^4}+\frac{24}{x^4}\ 0}{} \nearrow$$

$$\frac{4x-1}{\sqrt{x^2}-6}$$

$-\infty$	∞
$\sqrt{x^2}=-x$	$x=\sqrt{x^2}$
$-\sqrt{x^2}=x$	$-x=-\sqrt{x^2}$

$$\frac{1}{-\sqrt{\frac{4x^2}{x^2}-\frac{3x}{x^2}}}\ 0$$

+ +
− −

$$\frac{3}{-1}$$

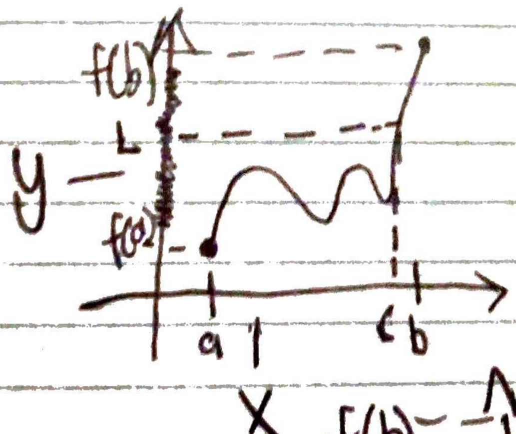
$$-\frac{2}{4} \qquad -\frac{1}{2}$$

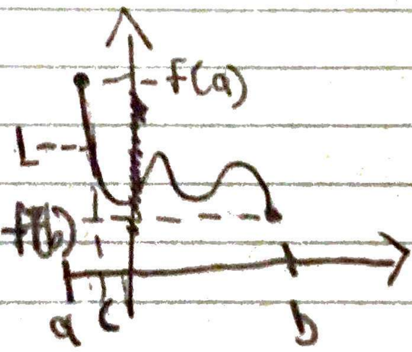
Working With the Intermediate Value Theorem

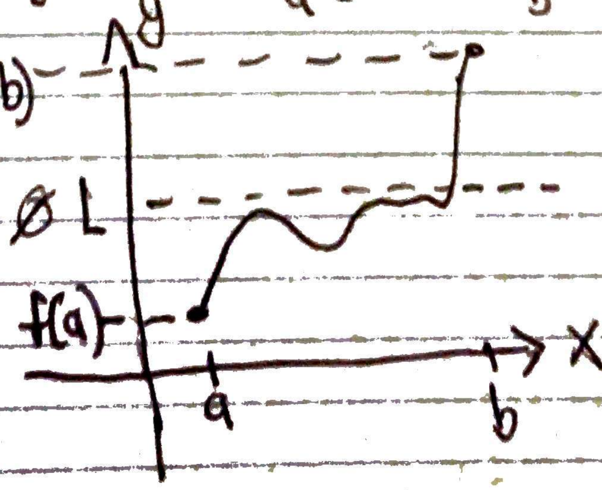
Intermediate Value Theorem

Suppose f is a function continuous at every point of the interval $[a,b]$

- f will take on every value between $f(a)$ and $f(b)$ over the interval
- For any L between the values $f(a)$ and $f(b)$, there exists a number c in $[a,b]$ for which $f(c) = L$.





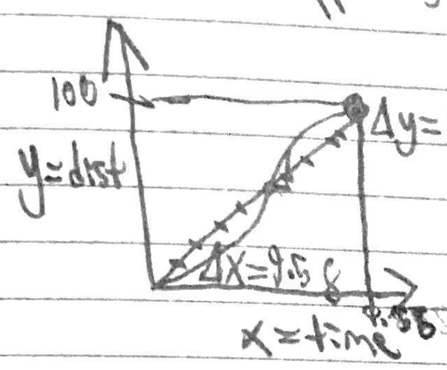


Diffrentiation: Definition and Basic Derivative rules

Defining average and instantaneous Rate of change at a point

Newton, Leibniz and Usain Bolt

Differential Calculus is focused on what's happening in this instant!!



avg speed $= \frac{\Delta \text{dist}}{\Delta \text{time}} = \frac{\Delta y}{\Delta x}$

$\frac{100m}{9.58 sec} \approx 10.4 \frac{m}{s}$

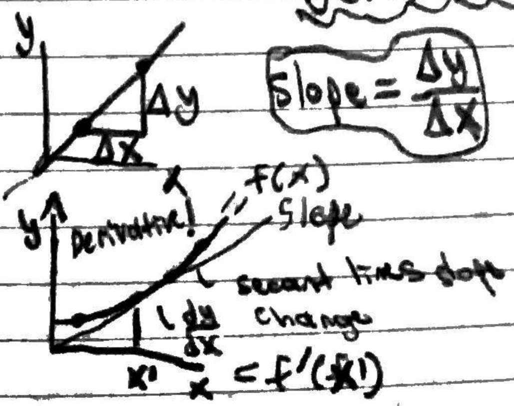
$\approx 23.5 \frac{\text{miles}}{\text{hour}}$

* Average speed is different than instantaneous speed

$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \to \frac{dy}{dx}$

whenever you use average your aproximating the actual slope. So, you use derivatives to get the precise measurements of change in $\frac{y}{x}$.

Derivative as a Concept



$= f'(x')$

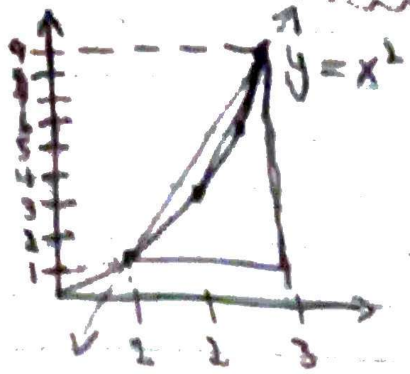
$= \dot{y} = y'$

$\boxed{\text{Slope} = \frac{\Delta y}{\Delta x}}$

- The instantaneous rate of change of a curve, which rate constantly changes.

* secant lines — lines that Intersect a curve.

Secant line & average rate of change



Average rate of change over $[1,3]$

x	y
1	1
3	9

$\Delta x = 2$ $\quad \Delta y = 8 \quad \frac{\Delta y}{\Delta x} = \frac{8}{2} = 4$

Lagrange's notation: $\boxed{f'} = y = f(x)$

Leibniz's notation: $\boxed{\frac{dy}{dx}} = y = f(x) = \frac{d}{dx}(f(x))$

Newton's notation: $\boxed{\dot{y}} = y = f(x)$

Derivative as Slope of Curve

$(1,2)$
$(5,32)$ $\quad \frac{30}{4} = \frac{7.5}{1} = \frac{15}{2}$

$(2,8)$
$(4,32)$ $\quad \frac{24}{2} = \boxed{\frac{12}{1}}$

$(2, \frac{2}{5})$
$(3, \frac{3}{10})$ $\quad \frac{\frac{3}{10} - \frac{4}{10}}{3-2} = \frac{-\frac{1}{10}}{1} \quad -\frac{1}{10}$

Point Slope Formula for Tangent Line

$$y - y_1 = m(x - x_1)$$

slope

y-coordinate

x-coordinate

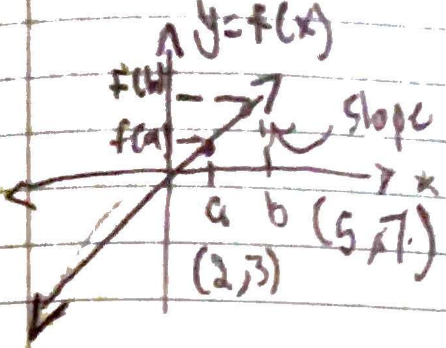
$\frac{4-5}{3-1} = \boxed{-\frac{1}{2}}$

$y = -\frac{1}{2}(1) \quad y = -\frac{1}{2}x + 5\frac{1}{2}$

$5 = -\frac{1}{2}$

Defining the derivative of a function and using derivative notation

Formal definition of a derivative as a limit

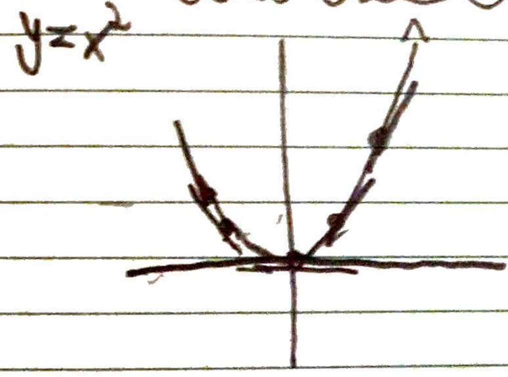


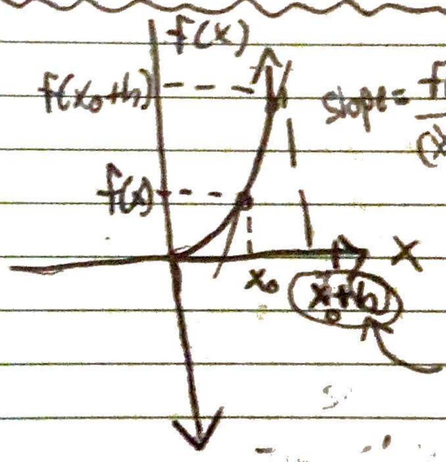
$\frac{\text{rise}}{\text{run}}$

$$\frac{y_2 - y_1}{x_2 - x_1} = \text{slope}$$

$$\frac{7-3}{5-2} = \boxed{\frac{4}{3}} \qquad \frac{\Delta y}{\Delta x} = \boxed{\frac{f(b)-f(a)}{b-a}}$$

let's generalize this to a curve





$$\text{slope} = \frac{f(x_0+h) - f(x_0)}{(x_0+h) - (x_0)} \frac{\Delta y}{\Delta x}$$

* when h approaches zero you'll find the slope of a particular point

The slope of the secant line $= \frac{f(x_0+h) - f(x_0)}{h} = \frac{\Delta y}{\Delta x}$

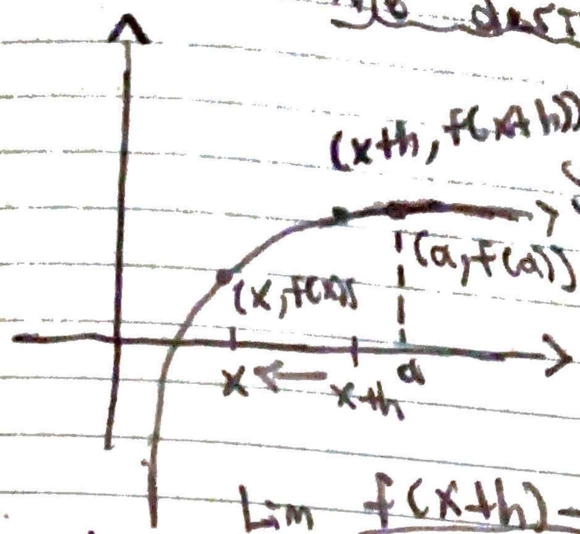
$h = \Delta X$

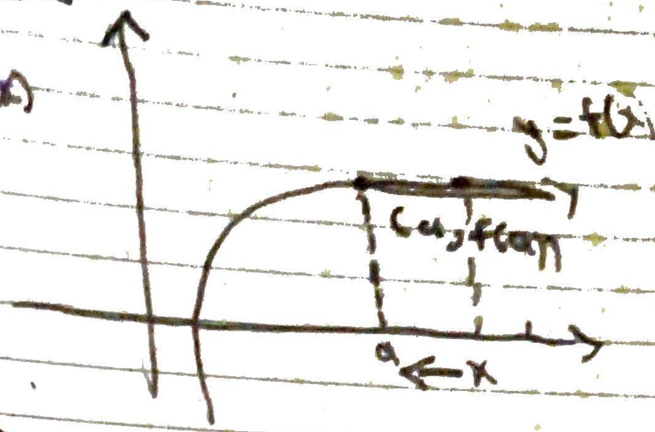
$$\boxed{\text{The slope of the tangent line}} = \boxed{\lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}}$$

Derivative of f

$\boxed{f'(x)}$

Formal and alternate form of the derivative





$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{x+h-x}$$

$$\lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$

Use the slope of the secant line to find the slope of the tangent line

vertical

$$f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

horizontal

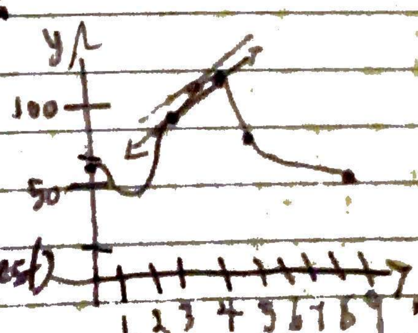
x	0	3	5	6	9
f(x)/y	72	95	112	77	54

This table values of the differentiable function

What is the best [redacted] estimate for $f'(4)$ we can [redacted] make based on this table?

when you substitute 4 for the derivative you find the slope for the point

Use slope of secant line [redacted] to between tangent point to get estimate (best) of tangent slope of point.



Connecting differentiability and continuity: determining when derivatives do not exist.

There are three cases where a function is not differentiable

1. Wherever the function isn't continuous
2. Wherever the graph has a vertical tangent line
3. Wherever the graph of the function has a sharp turn

~~[illegible]~~

Continuous –

$\frac{-1x^2 + 1BA(-3)^2 + 3)}{x-3}$ $-x^2 + 12$

$(-x + 4)$ $x \to 3$