Solve for a side in right Triangles

$(7)\ \tan(10) = \frac{x}{7}\ (7)$

$(5)\ \tan(25) = \frac{x}{\cancel{5}}\ (\cancel{5})$

$(\tan^{-1})\ \tan(\theta) = \frac{5}{6}\ (\tan^{-1})$

$(\sin^{-1})\ \sin(\theta) = \frac{2}{3} (\sin^{-1})$

$(2)\ \cos(20) = \frac{x}{2}\ (2)$

$(7)\ \cos(65) = \frac{x}{7} (7)$ $\cancel{\frac{\cos(50)}{2} = \frac{2}{x} 2}$ $(\cos^{-1}) \cos(\theta) = \frac{3}{7} (\cos^{-1})$

$(x)\ \cos(50) = \frac{2}{x}\ (x)$

$\frac{\cancel{(x)} \sin(20)}{\sin(20)} = \frac{\frac{3}{\cancel{x}} \cancel{(x)}}{\sin(20)}$ $\frac{(x)\cos(50)}{\cos(50)} = \frac{2}{\cos(50)}$

$x = \frac{2}{\cos(50)}$

$(6)\ \sin(65) = \frac{x}{6}\ (6)$

$(\sin^{-1})\ \sin(\theta) = \frac{8}{9}\ (\sin^{-1})$

$\theta =$

Relate Ratios in Right Triangles

$\cos(\theta) = \frac{21}{29}$

$(6)\ \frac{x}{6} = \cos(55^\circ)(6)$

$(5)\ \sin(29) = \frac{x}{5}\ (5)$

$\sin(\theta) = \frac{42}{100} = .42$

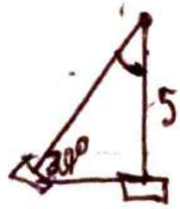
$\tan(\theta) = \frac{443}{18}$

$(10)\ \tan(40) = \frac{x}{10}\ (10)$

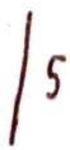
$\cos^{-1}\ \cos(\theta) = \frac{15}{17}\ \cos^{-1}$

$(2.1)\ \tan(37) = \frac{x}{2.1}\ (2.1)$

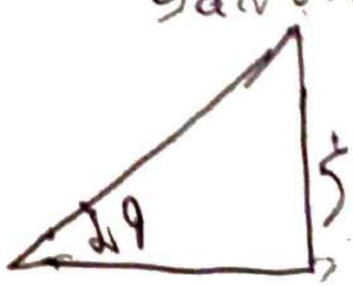
$(x)\ \cos(29) = \frac{5}{x}\ (x)$



$\frac{6.6}{10} = \tan(\theta)$



$\frac{(x)\ \tan(31) = \frac{5}{\cancel{x}}(x)}{\tan(31)} \quad \frac{}{\tan(31)}$



$(4)\ \cos(23) = \frac{x}{\cancel{4}}\ \cancel{(4)}$

Reciprocal trig ratios

CSC = Sine

$\frac{\pi}{5} \times \frac{180}{\pi}$ Secant = Cosine $260 * \frac{\pi}{180}$

Cotangent = tangent

$180 \times \frac{\pi}{180}$

opposite to one another

$\frac{130\pi}{90}$

Radians and Degrees

$\frac{25\pi}{360} \times \frac{180}{\pi}$

$\frac{14\cancel{\pi}}{9} \times \frac{180}{\cancel{\pi}}$ $\frac{8\cancel{\pi}}{9}$ $\frac{180}{\cancel{\pi}}$ $\frac{65\pi}{45}$

$\frac{12\pi}{18}$ $\frac{6\pi}{9}$

$\frac{230}{1} \times \frac{\pi}{180}$ 36

$\frac{36\pi}{30}$ $\frac{29\pi}{15} \cdot \frac{180}{\pi}$

$\boxed{\frac{216\pi}{180}}$ $\frac{27\pi}{30}$ $\frac{9\pi}{10}$ $290 * \frac{\pi}{180}$ $\frac{145\pi}{90}$

$$\frac{\pi}{5} \times \frac{180}{\pi}$$

$$\cancel{\frac{3\pi}{5}} \times \frac{180}{\pi}$$

$$\frac{3\pi}{7} \times \frac{180}{\pi}$$

$$\frac{4\pi}{5} \times \frac{180}{\cancel{\pi}}$$

$$\sin\left(\frac{3\pi}{2}\right)$$

$$\frac{270\pi}{180}$$

$$\frac{110\pi}{180}$$

$$\frac{2\pi}{3} \quad \frac{4\pi}{6} \quad \frac{\div 12\pi}{\div 18}$$

Use The Pythagorean Identity

$$\boxed{-\frac{3\pi}{2} < \theta < -\pi}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\left(\frac{1}{2}\right)^2 + \cos^2\theta = 1$$

$$\frac{1}{4} + \cos^2\theta = 1$$

$$\cos^2\theta = \frac{3}{4}$$

$$\cos\theta = \pm\sqrt{\frac{3}{4}} \quad \pm\frac{\sqrt{3}}{2}$$

$-,+$ | $+,+$

$-,-$ | $+,-$

$$\sin^2\theta + \cancel{}\left(\frac{10}{14}\right)^2 = 1$$

$-\cancel{\sin^2\theta}$ $\quad -\sin^2\theta$

$\cancel{\left(\frac{10}{14}\right)^2 = 1 - \sin^2\theta}$

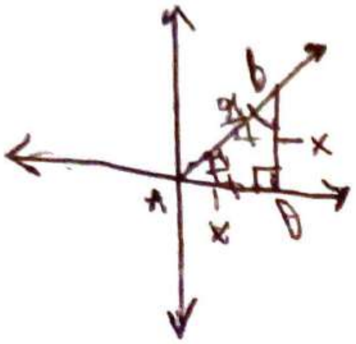
$\frac{9}{25}$ $\quad \frac{\sqrt{56}}{225}$

$$\sin^2\theta = \frac{189}{289}$$

$-\frac{169}{225}$ $\quad -\frac{\sqrt{56}}{15}$ $\quad \frac{11}{61},\ \frac{11}{61}$ $\quad \frac{121}{3721}$

$$\frac{\sqrt{189}}{17}$$

Trig values of Special angles



The beauty of the unit circle is that the hypotenuse is always one.

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

$240 \quad \frac{\pi}{180}$

$\frac{40\pi}{30}$

$\frac{80}{60}$

$\frac{8\pi}{6}$

$\frac{4\pi}{3}$

$135 \times \frac{\pi}{180}$

Midline of sinusoidal functions

From graph

find Amplitude with absolute value

(1×d) $y = 50 \sin x$

Amplitude is the distance from midpoint

The Amplitude is always positive. So, its always the absolute value

The amplitude is half of the distance from the Minimum/Maximum of function.

$\frac{60}{1} = \frac{5}{x}$

$\frac{5}{60} = \frac{60x}{60}$

$\frac{1}{30}$ $\frac{x}{2100}$

Period of Sinusoidal functions from graph

* The period is the length of the smallest interval that contains at least one copy of the repeating pattern of the periodic function.

To find the period

Midline info

~~whether positive or negative~~ tells ~~max or min~~

$$\frac{2\pi}{|x|} \qquad y = (a)\sin(x) + \boxed{?}$$

this is your midline when there is nothing assume it's zero

Shows whether or not it's inverted

120−180

$$\boxed{\frac{2}{3}\cancel{\pi} \times \frac{180}{\cancel{\pi}}}$$

180 60

×

$$\frac{2}{120}$$

$$\frac{7}{8} - \frac{1\times4}{2\times4}$$

$$\frac{7}{8} - \frac{4}{8} = \boxed{\frac{3}{8}}\times 4 \qquad \boxed{\frac{12}{34}\pi}$$

Graph sinusoidal functions

$\frac{2\pi}{4}$ $\frac{2\pi}{3}$

$\frac{2\pi}{\frac{\pi}{10}}$ $\frac{2\pi}{.45}$ $\frac{4}{3}$ $\boxed{\frac{8\pi}{3}}$ 6π

2.5

$\frac{2\pi}{\frac{\pi}{6}}$ $\frac{6}{\pi}$ 12 $\frac{2\pi}{10}$ $4\overline{)10}$

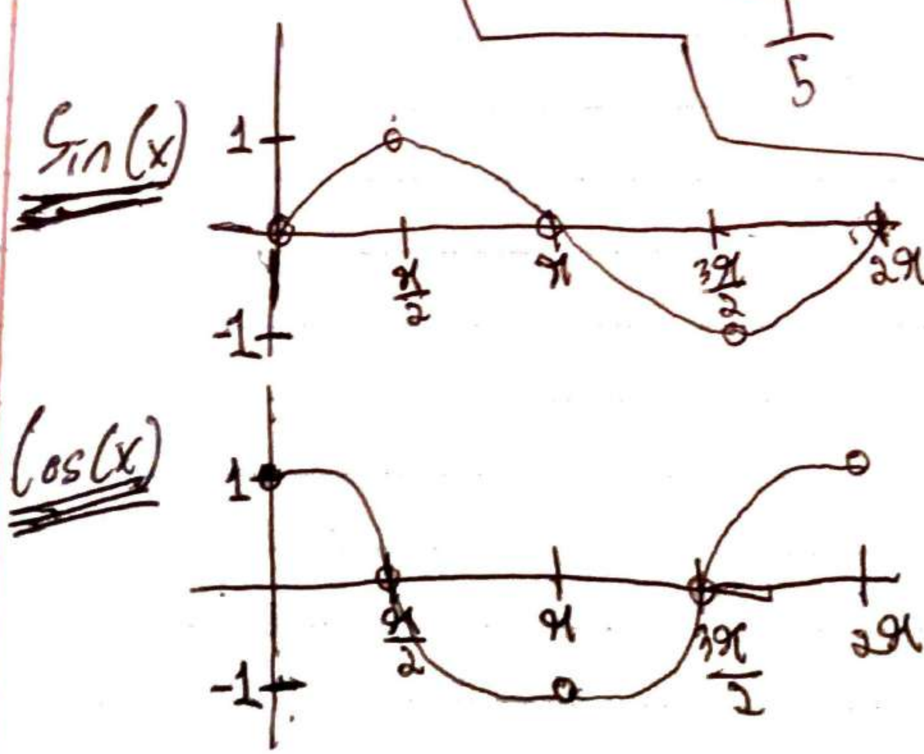
-8

20

$\frac{2\pi \cdot 10}{\pi}$ 20

2.5π

$\frac{2\pi}{\frac{1}{5}}$



6

$\frac{2\pi}{1} \cdot \frac{5}{1}$ $\frac{10\pi}{4}$

$\frac{5\pi}{2}$

12π

3π

Graph sinusoidal functions: phase shift

$\boxed{\frac{\pi}{2}}$ c

factor out the phase that shifts the x-axis.

$\boxed{2.5}$

$\left(\frac{1}{2}x + \frac{3}{4}\pi\right)$

2π 2

$\frac{10}{4}$ $\frac{5}{2}$ $\frac{2\pi 2}{1}$

$\frac{1}{2}\left(x + \frac{\frac{3}{4}\pi}{\frac{1}{2}}\right)$

$\boxed{4\pi}$

$\frac{1}{2}\left(x + \frac{3}{4} \times \frac{2}{1}\right)$

$\frac{6\pi}{4}$ $\frac{3\pi}{2}$ 4π

$\frac{1}{2}\left(x + \frac{6}{4}\pi\right)$

$\frac{2\pi}{\frac{1}{3}}$

$\boxed{1.5\pi}$ P

$\frac{3}{2}\pi$

$\cancel{2}$ A

$\cancel{\frac{7\pi}{4} \times 4}$ 7π

$-\frac{3}{4}\pi$

$\boxed{+\pi}$ S

$\cancel{2}$ M

$4\sin(2x+3\pi)+3$

~~4sin~~

$4\sin(2 + \frac{3}{4}\pi$

$(0,3)$ mid $(.25\pi, 7)$ max

Interpreting trigonometric graphs

$\cancel{60\cos(\frac{\pi}{6}x)}$

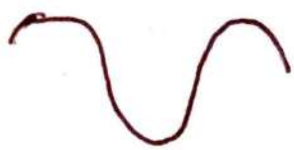
$-5\cos(\frac{1}{14}x)+2$

12

T W/L

S- $(0, 2)$

$(7\pi, 7)$



$7\pi \times 4$

$\frac{2\pi}{28\pi}$

$\boxed{\frac{1}{14}}$

T W

S- $(0, 9.1)$ $.9\cos(\frac{2\pi}{365}t)+8.2$

$(\boxed{91.25}, 8.2)$ $.9\cos$

$\frac{2\pi}{365}$

$35\sin(10t)+35$

T D

$(0, 35)$

$\max\left(\frac{\pi}{20}, 70\right)$

$\frac{2\pi}{\frac{\pi}{5}}$ 10

$2\sin\left(\frac{\pi}{2}t\right)+5$

t $\cancel{D}$

$(0, 5)$

$\min(1, 3)$

$\frac{2\pi}{4}$ $\frac{\pi}{2}$

$-1\cos(3t)+0$ $\frac{6}{2}$

t D

$(0, -1)$

$\left(\frac{\pi}{6}, 0\right)$

$\frac{4\pi}{6}$ $\frac{2\pi}{\frac{2\pi}{3}}$

$-10\cos\left(\frac{1}{20}t\right)+20$

t D

$(0, 10)$

([illegible], 30)

20π

$\frac{1}{20}$ [illegible] [illegible]

$-1.5\sin(\frac{2\pi}{365}t) + 3.47$

$+ \quad S$

mid $(0, 3.47)$

min $(91.25, 1.47)$

① ②

T D

$-5\cos(\frac{1}{14}t) + 7$

min$(0, 2)$

mid$(7\pi, 7)$

$\cancel{\frac{2\pi}{28\pi}}$

②

T D

$(0, 15)$

$-6\cos(\frac{\pi}{3}t) + 21$

$(1.5, 21)$

$\frac{\pi}{3}$ $\frac{2\pi}{6}$ $\frac{1}{3}$

T, D

$5\sin(\pi t) + 7$

mid$(0, 7)$

max$(.5, 12)$

$.5 \times 4$

$\frac{2\pi}{2}$

Algebra 2 (struck out)

Modeling with sinusoidal functions: phase shift

~~The phase~~

The phase shift restricts the function to where it starts

$15\cos(\ )+63$

$-\frac{51.875}{}\cos\left(\frac{2\pi}{365}(t+7)\right)$

$+72.625$

$80.75 = 91.25 + 365\pi$

$\cos(x)$

$A-b)\cos(x)$

low

$\frac{7}{4} + 7\pi n$

$-\cos\left(\frac{2\pi}{29.53}(t-7)\right)$

$-\frac{1}{8}\cos\left(\frac{2\pi}{29.53}(t-7)\right)+\frac{1}{8}$

s for one input

time passes

mid line

$\pm\frac{\pi}{2}+2\pi n$

$8.5\sin(\ )-35.5$ max$(1.3,-27)$

max$(3,-27)$

$8.5\sin\left(\frac{2\pi}{5.2}(t)\right)-35.5$

min$(?,-44)$

Max

$\frac{\pi}{2}+2\pi$

peak

$=2+2\pi n$

Min

$f(x)=\frac{\pi}{2}+2\pi$ 1

2.1

1.3

Solve triangles using the law of sines

leg length over (sin) adjacent (angle) is equal to leg length over (sin) adjacent (angle)

$$\frac{18}{\sin(68)} = \frac{7}{\sin(x)}$$

$$\frac{x}{\sin(37)} = \frac{15}{\sin(98)}$$

$$\frac{12}{\sin(40)} = \frac{x}{\sin(110)}$$

$$\frac{34}{\sin(x)} = \frac{19}{\sin(38)}$$

$$(\sin(26))\frac{x}{\sin(26)} = \frac{5}{\sin(24)}(\sin(26))$$

$$\frac{25}{\sin(x)} = \frac{10}{\sin(22)}$$

$$(25)\frac{\sin(x)}{25} = \frac{\sin(22)}{10}(25)$$

$$(\sin^{-1})\sin(x) = .9365\,(\sin^{-1})$$

$$x = 69^\circ$$

★ if the result contradicts whether the angle is acute or obtuse subtract 180° to your result.

Solve triangles using the law of cosines

(Leg) a^2 = (leg) b^2 + (leg) c^2 − (legs) $2bc\cos(\angle A)$

$$13^2 = \underbrace{6^2+10^2}_{\text{subtract}}\ \underbrace{-2(6)(10)}_{\text{divide}}\ \underbrace{\cos(x)}_{\text{inverse}}$$

General Triangle word problems

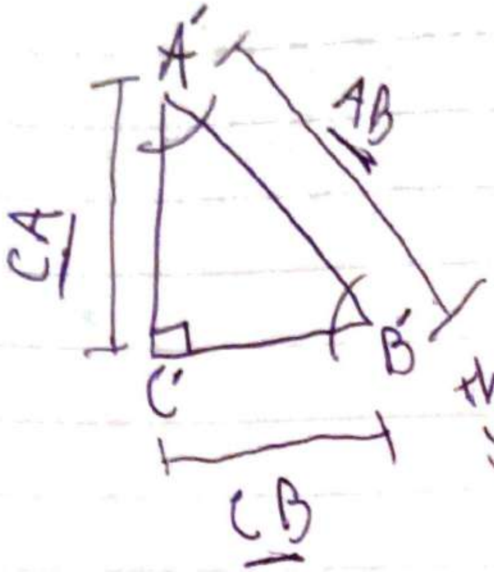
$$861^2 = 673^2 + 989^2 - 2(673)(989)\cos(\theta)$$

$x^2 =$

$$(\sin(75))\frac{900}{\sin(45)} = \frac{x}{\sin(45)}(\sin(75))$$

$$(25)\frac{30}{\sin(41)} \overset{\text{switch}}{=} \frac{25}{\sin(x)}(25) \quad (25)\frac{\sin(41)}{30} = \frac{\sin(x)}{25}(25)\ (\sin^{-1})$$

Relate rattios in right triangles



Ex:

~~SOH~~ ~~CAH~~ ~~TOA~~

~~sin(x)~~ ~~cos(x)~~ ~~tan(x)~~

the length you want → $\frac{AB}{X} = \sin(\theta)$

if a right triangle has 2 acute angles then it equals 90°.

equal but one is in fraction form and the other in trigonometric.

Evaluate inverse trig functions

~~When your answer is negative subtract from the interval that this in~~

★ **Always check if it wants the Answer in degrees / Radians**

Solve sinusoidal equations (basic)

$-180 \leq x \leq 180$

- $\sin(\theta) = \sin(-180° - \theta)$

$\sin(x) = -.7$ $(\sin^{-1})$ $(\sin^{-1})$

add

opposite

$x = -180 + 44.43$

$x = -135.57$

Sin

$x = -44.43 + n \cdot 360$

$x = -135.54 + n \cdot 360$

$$\boxed{\begin{gathered} x = \theta + 2\pi n \\ x = \theta + 2\pi n \end{gathered}}$$

$-\frac{\pi}{2} + 2\pi n$

$\frac{3\pi}{2} + 2\pi n$

$14 + 2\pi n$

$165.522 + 2\pi n$

$\sin(\theta) = \sin(180 - \theta)$ The peak/min " " has only one solution.

The peak/min means one solution

- $\cos(\theta) = \cos(-\theta)$

$\cos(x) = .15$

$x = \cos^{-1}(.15)$

$x = \underline{1.42}$

(second solution to interval)

$\cos(\theta) = \cos(\theta + 2\pi)$

$x = -1.42 + n \cdot 2\pi$

$x = 1.42 + n \cdot 2\pi$

Cosine

$$\boxed{\begin{gathered} \cos(\theta) = \cos(\theta) \\ x = \theta + 2\pi n \\ x = \theta + 2\pi n \end{gathered}}$$

$\cos(\theta) = \cos(2\pi - \theta)$

(second solution

$-1.5 < x < 7.8$

Solve sinusoidal equations

$20\sin(10x) - 10 = 5$

$+10 \quad +10$

$$\frac{20\sin(10x)}{20} = \frac{15}{20}$$

$$\sin(10x) = \frac{3}{4}$$

$.089 + \frac{\pi}{5}$ $\qquad \frac{.84 + 2\pi n}{10}$

$.229 + \frac{\pi}{5} = \frac{2.29 + 2\pi n}{10}$

$\cos(30x) = \frac{-2}{30} = -\frac{1}{15}$

$30x = \frac{1.6 + 2\pi n}{30}$

$30x = \frac{-1.6 + 2\pi n}{30}$

$\cos(9x) = \frac{2}{7}$

$\frac{1.28 + 2\pi n}{9}$

$\boxed{\pm .1423 + \frac{\pi}{4.5} n}$ $\frac{-1.28 + 2\pi n}{9}$

$8.155 + \frac{2\pi}{9} n$ or 40°

$-8.155 + \frac{2\pi}{9} n$ or 40°

$5\sin(3x) - 1 = 3$

$+1 \quad +1$

$\frac{}{5} \quad -5$

$\sin(3x) = \frac{4}{5}$

$53.13 \div 17.71 + \frac{2\pi}{3} n$ or 120°

$126.86 \div 42.28 + \frac{2\pi}{3} n$

$$75 = -15\cos\left(\frac{2\pi}{15}t\right) + 65$$
$$-65 \qquad\qquad -65$$

$$\frac{10}{-15} = \frac{-15\cos\left(\frac{2\pi}{15}t\right)}{-15}$$

$$(\cos^{-1})\,\frac{10}{-15} = \cos\left(\frac{2\pi}{15}t\right)(\cos^{-1})$$

$$= \frac{2\pi}{15}t\left(\frac{15}{2\pi}\right)$$

$$8400 = 624\sin\left(\frac{2\pi}{365}t\right) + 8736$$
$$-8736 \qquad\qquad -8736$$

$$\frac{-336}{624} = \frac{624\sin\left(\frac{2\pi}{365}t\right)}{624}$$

$$-\frac{7}{13} = \sin\left(\frac{2\pi}{365}t\right)$$

Using the trig angle addition identities

Deriving double angle identity

$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$

$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$

$\cos(a - b) = \cos(a)\cos(b) + \sin(a)\sin(b)$

$\sin(a+b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$

$\sin(a+b) = \sin(\theta+\theta) = \sin\theta\cos\theta + \cos\theta\sin\theta$

$\cos(2\theta) = \cos(\theta)^2 - \sin(\theta)^2$ $\quad \sin(2\theta) = 1\sin\theta\cos\theta + 1\sin\theta\cos\theta$

only use ratio of function

$\sin(2\theta) = 2\sin\theta\cos\theta$

Practice

[redacted]

$\frac{12}{13} \cdot \cos(60) + \cos\left(\frac{5}{13}\right) \cdot \sin(60)$ $\quad 2 \cdot \frac{4}{2\sqrt{5}} \cdot \frac{2}{2\sqrt{5}} =$

$-2.285 + (-.288)$

[redacted]

$\frac{6}{1?} + .866$

$\frac{16}{20} - \frac{4}{20}$ $\left(\frac{4}{2\sqrt{5}}\right)^2 - \left(\frac{2}{2\sqrt{5}}\right)^2$

$2 \times \frac{2}{\sqrt{3}} \times \frac{3}{\sqrt{3}}$ $\frac{12}{3} = 4$ $\frac{12}{20} \times \frac{\sqrt{3}}{2} -$

$2 \times \frac{8}{10} \times \frac{8}{10}$ 100 $\frac{16}{20} \times \frac{\sqrt{3}}{2} + \frac{12}{20}$

$\sin(\angle ABC + 60) = \sin(\frac{4}{5})\cos(60) + \cos(\frac{3}{5})\sin(60)$

$\frac{4}{5} \times \frac{1}{2} + \frac{3}{5} \cdot \frac{\sqrt{3}}{2}$

$\boxed{\frac{4}{10} + \frac{3\sqrt{3}}{10}}$

$\frac{2}{5} +)$

$2\cos\left(\frac{2}{\sqrt{13}}\right)^2 - \sin\left(\frac{3}{\sqrt{13}}\right)^2$

$\frac{5}{13} \times \frac{1}{2} - \frac{12}{13} + \frac{\sqrt{3}}{2}$ $2 \times \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2$

$\cos(\angle ABC)\cos(60) + \sin(\angle ABC)\sin(60)$

$\frac{8}{10} \times \frac{\pi}{3}$ $\frac{6}{10} \times \frac{\pi}{3}$ $2 \times \frac{9}{25} - \frac{16}{25}$

$\frac{3}{\sqrt{10}}\ \frac{1}{\sqrt{10}}$ $+$ $\frac{18}{25} - \frac{16}{25}$

$\cdot \left(\frac{8}{10}\right)^2 - \left(\frac{6}{10}\right)^2$

$\cdot \frac{80}{100} - \frac{60}{100}$ $\frac{6}{10} \times \frac{1}{2} - \frac{8}{10} + \frac{\sqrt{3}}{2}$ $60 \times \frac{\pi}{180} = \frac{60\pi}{180}$ $\frac{6\pi}{18}$

$-$ $\frac{2\pi}{6}$ $\frac{\pi}{3}$

$\frac{160}{100} - \frac{60}{100}^2 =$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{24}{25}$

$\frac{3\pi}{2}$

$\cos(\frac{12}{13})\cos(60) + \sin(\frac{5}{13})\sin(60)$ $\sin\frac{4}{5}\cos 60 + \sin 60 \cos$

$\frac{12}{13} * \frac{1}{2} + \frac{5}{13} * \frac{\sqrt{3}}{2}$ $\frac{4}{10}$ $\frac{3}{10}\sqrt{3}$ $\frac{1}{2}$ $=$ $\frac{\sqrt{3}}{2}$

Find trig values using angle addition identities

$\sin(\phi+\theta) = \sin(\phi)\cos(\theta) + \cos(\phi)\sin(\theta)$

$\cos(\phi+\theta) = \cos(\phi)\cos(\theta) - \sin(\phi)\sin(\theta)$

$$\tan(\phi+\theta) = \frac{\sin(\phi+\theta)}{\cos(\phi+\theta)} = \frac{\tan(\phi)+\tan(\theta)}{1-\tan(\phi)\tan(\theta)}$$

$\sin(\phi-\theta) = \sin(\phi)\cos(\theta) - \cos(\phi)\sin(\theta)$

$\cos(\phi-\theta) = \cos(\phi)\cos(\theta) + \sin(\phi)\sin(\theta)$

$$\tan(\phi-\theta) = \frac{\tan(\phi)+\tan(-\theta)}{1+\tan(\phi)\tan(\theta)}$$

or

$$\frac{\tan(\phi)+\tan(\theta)}{1-\tan(\phi)\tan(\theta)}$$

Reciprocal and Quotient identities

$\sec(\theta) = \frac{1}{\cos(\theta)}$ $\quad \csc(\theta) = \frac{1}{\sin(\theta)}$

$\cot(\theta) = \frac{1}{\tan(\theta)}$ $\quad \tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$

$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$

Angle	terminal
$0 = 0$	$(1, 0)$
$30^\circ = \frac{\pi}{6}$	$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$45^\circ = \frac{\pi}{4}$	$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$60^\circ = \frac{\pi}{3}$	$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$90^\circ = \frac{\pi}{2}$	$(0, 1)$

When it's tangent of difference divide outcome with a positive integer.

When it's tangent of sun divide outcome with negative integer.

$\cos\left(\frac{3\pi}{4} + \frac{5\pi}{12}\right)$

$\cos\left(\frac{9\pi}{12} + \frac{5\pi}{12}\right)$

$\cos\left(\frac{14\pi}{12}\right)$

$\cos\left(\frac{7\pi}{6}\right)$

$\cos\left(\pi + \frac{\pi}{6}\right)$

$-\cos\left(\frac{\pi}{6}\right)$

$-\frac{\sqrt{3}}{2}$

$\cos\left(\frac{7\pi}{4} + \frac{5\pi}{12}\right)$

$\cos\left(\frac{21\pi}{12} + \frac{5\pi}{12}\right)$

$\cos\left(\frac{26\pi}{12}\right) \frac{\div 2}{\div 2}$

$\cos\left(\frac{13\pi}{6}\right)$

$\cos\left(\frac{\pi}{6}\right)$

$\boxed{\frac{\sqrt{3}}{2}}$

$\tan\left(\frac{20\pi}{12} + \frac{3}{12}\pi\right)$

$$\frac{\tan\left(\frac{20\pi}{12}\right) + \tan\left(\frac{3}{12}\pi\right)}{1 - \tan\left(\frac{20\pi}{12}\right)\tan\left(\frac{3}{12}\pi\right)}$$

$\left(\frac{-\sqrt{3} + 1}{\quad}\right)$

$\sin\left(\frac{10\pi}{12} + \frac{9\pi}{12}\right)$

$\sin\left(\frac{5\pi}{6} + \frac{3\pi}{4}\right)$

$\sin\left(\frac{5\pi}{6}\right)\cos\left(\frac{3\pi}{4}\right) + \cos\left(\frac{5\pi}{6}\right)\sin$

$\frac{1}{2} \cdot \frac{-\sqrt{2}}{2} + \frac{-\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$\frac{-\sqrt{2}}{2} + \frac{\cancel{\sqrt{6}}}{2}$

$\boxed{\frac{-\sqrt{2} - 2}{\cancel{4}}}$

$\cos\left(\frac{5\pi}{12}+\frac{2\pi}{12}\right)$

$\cos\left(\frac{5\pi}{12}\right)\cos\left(\frac{\pi}{6}\right) - (5$

$\cos\left(\frac{5\pi}{12}\right)\cos\left(\frac{\pi}{6}\right) - \sin\left(\frac{5\pi}{12}\right)\sin\left(\frac{\pi}{6}\right)$

like terms

$5\sqrt{3}+4\sqrt{3} = 11\sqrt{3}$

This means they can be combined

$\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)\left(\frac{1}{2}\right)$

Powers of the imaginary Unit

$i^0 = 1$

$i^1 = \sqrt{-1} = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

Simplify roots of negative numbers

1) $\pm\sqrt{49}$

$\pm 7i$

2) $\pm\sqrt{-37}$

$\pm\sqrt{-1}\cdot\sqrt{37}$

$\pm i\cdot\sqrt{37}$

$\pm\sqrt{37}\,i$

$\pm\sqrt{-66}$

3) $\pm i\sqrt{66}$

4) $\pm\sqrt{-36}$

$\pm 6i$

5) $\pm\sqrt{-55}$

$\pm\sqrt{55}\,i$

6) $\pm\sqrt{-72}$

$\sqrt{8}\,\sqrt{9}\,\sqrt{-1}$

$3\sqrt{4}\,\sqrt{2}$

$6\sqrt{2}\,i$

Parts of complex numbers

$z = \underbrace{-19i}_{\text{imaginary}} + \underbrace{14}_{\text{real}}$

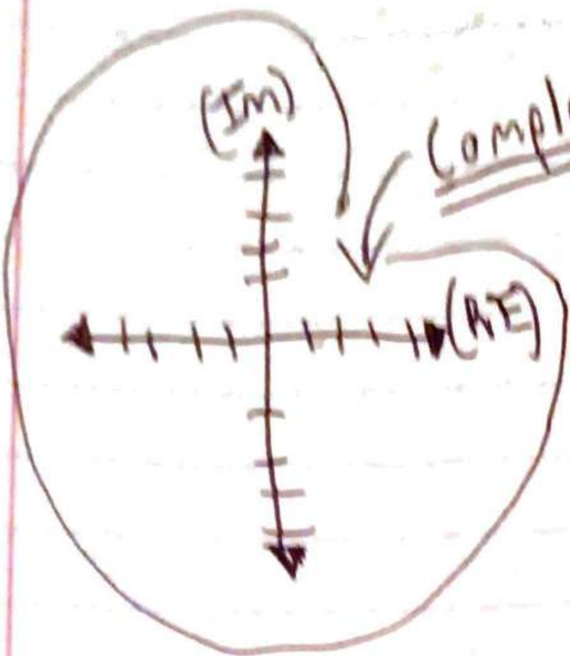
Classify Complex Numbers

$7.5 + 52i$ — complex

7.5 — real/complex

$52i$ — imaginary/complex

Plot numbers on the Complex Plane



Complex plane

Y-axis = imaginary axis

X-axis = Real axis

Add & Subtract complex numbers

$(-90-27i)-(66+34i) = \boxed{-156-61i}$

$-90-27i-66-34i$

$-156 - 61i$ [redacted]

Graphically add & subtract complex numbers

simple

Distance of Complex Numbers

$a^2+b^2=c^2$

Midpoint of Complex Numbers

L = Leg

$\frac{L^1+L^2}{2}$ $\boxed{\frac{a+b}{2}}$ ~~$z^1 + z^2$~~

Multiply complex numbers (basics)

Simple

Multiply complex numbers

$(1+5i)\cdot(-3-i) = -3 - i - 15i + 5$

$2 - 16i$

$(3-3i)(-2+2i) = -\cancel{6} + 6i + 6i + \cancel{6}$

$(4+4i)(-2-5i)$

$-8-20i-8i-20i^2$

$-8-28i+20$

$12-28i$

$(5-5i)\cdot(-3+5i) = -15+25i+15i-25i^2$

$-15+40i+25$

$10+40i$

$(-4+2i)\cdot(4-4i) =$

$-16+16i+8i-8i^2$

$-16+24i+8$

$-8+24i$

$(4-4i)(-5-3i) =$

$20+12i+20i+12i^2$

$20+32i-12$

$8+32i$

Complex Number Conjugates

$a + \underline{bi} =$ Conjugate Form $a - \underline{bi}$

The conjugate is just the reflection over the Real-axis.

Divide Complex Numbers

Conjugate is just the opposite form of denominator's central sign $a + bi$ (before) mutiplied by the fraction at hand.

$a - bi$ conjugate/After

before $2 + \sqrt{4}$

$2 - \sqrt{4}$ conjugate/After

To divide complex numbers you must multiply by the conjugate

Factor Polynomials: Complex Numbers

~~[illegible]~~

- Square of sum
- Difference of squares
- Sum of cubes
- greatest common facto
- Grouping
- Completing square
- Quadratic method

Factor Polynomials: Complex Numbers Continuation

$x^4 + 3x^2 + 2$

$u^2 + 3u + 2$ $\qquad x^4 = u^2$

$(u+2)(u+1)$ $\qquad x^2 = u$

① $(x^2+2)(x^2+1)$

② $(x+\sqrt{2})(x-\sqrt{2})(x+\sqrt{1})(x-\sqrt{1})$

③ $(x^2+2i)(x^2-i)$

Foil

$-x^2 i + x^4 + 2ix^2 - 2i^2 - x^2 i$

$+ \cancel{\quad} + x^4 + \cancel{\quad} - 2i^2$

$x^4 - 2i^2$

$x^6 + 9$

$(x^2+3)^3$

$(x+\sqrt{3}\,i)(x-\sqrt{3}\,i)$

$x^1 \cdot x^2 = x^{1+2}$

$\frac{x^1}{x^2} = x^{1-2}$

$(x^1)^3 = x^{1\times 3}$

$\sqrt[3]{x^2} = x^{\frac{2}{3}}$

$(x^2+3xi)(x-2i)$

$x^3-2ix^2+3x^2i-6xi^2$

perfect square

★ first factor is not always i or I

$4(x^4+4)$

$4(x^4+2i)$

★ two variable

dif of square

second variable is I or i

$(x+\sqrt{20}i)(x-\sqrt{20}i)$

$(x+2\sqrt{5}i)(x-2\sqrt{5}i)$ ✓

$x^8=v^2$

$x^4=v$

$(x+4i)(x-5i)$

$x^2-x5i+4ix-20i^2$

(-1)

$x^2-1xi+20$

v^2+5v+4

$(v+4)(v+1)$

$(x^4+4)(x^4+1)$

$(x^2+\sqrt{1})(x^2-\sqrt{1})$

v^2+4v+4

$x^4=v^2$

$(v+2)(v+2)$

$x^2=v$

$(x^2+2)(x^2+2)$

Absolute Value of Complex Numbers

☆ Note to self distance is always positive.

The absolute value of a complex number is just it's distance from zero. So, you use the distance formula, and remember the numbers must be positive.

$D = |\sqrt{a^2+b^2}|$

complex number

Absoulute Value — distance from zero which is hypotenuse

Continuation of Top portion

Ex: $-6-8i$

$|\sqrt{36+64}|$

$\sqrt{100}$

10

$5-8i$

$|\sqrt{25+64}|$

$\sqrt{89}$

Quick Time! $9+49=58$ $\sqrt{58}$

$25+36=61$ $\sqrt{61}$

Angle of Complex Numbers

Use the complex number's negative and positive position to reason the quadrant

This formula gives you the interval $-180, 18$

$$\boxed{\tan\theta = \frac{Im}{Re}}$$

Formula gives solution

Angle of Complex Numbers Continuation

2 Solutions to tangent

depends on the interval

$\tan\theta = \frac{Im}{Re}$ First

$\tan(180+\theta) = \tan\theta$ Second

$\tan(-180+\theta) = \tan\theta$ Third

when it say round to nearest tenth and it's neat, leave it!!

* **Reference Angle:** The positive acute angle between the terminal side and the x-axis

Complex numbers from absolute Value and Angle

* Magnitude in front of function or $\boxed{\text{distance from zero}}$

* Angle inside of function or $\boxed{\tan\theta = \frac{y}{x}}$

Polar & Rectangular forms of Complex Numbers

* use degree or radian depending on the type of

Multiply and divide complex numbers in polar form

✱ Multiply	✱ Divide
- multiply Magnitude	- Divide Magnitude
- Add Angles	- Subtract Angles

Powers of Complex Numbers

$2(\cos(18)$

$\frac{3\times\theta}{3} = \frac{360}{3}$

$1\times\theta = k\cdot 360$

9^3

$\frac{\theta\cdot 5}{5} = \frac{k\cdot 90}{5}$ $\theta = k\cdot 18$

$54+k\cdot 7$

$729($

$r^3 = -512$

$3($

$\frac{\theta\cdot 5}{5} = \frac{270+k\cdot 360}{5}$

$64\sin(300)$ $r = \boxed{8}$

$8\sin 330$

$64\cos(300)$

$8\cos 330$

$\theta\times 3 = \underline{270\cdot k\cdot 360}$

$90+k\cdot 120$

$4+$ 4096 $\theta\cdot 3 = (243$

Add & Subtract polynomials: two variables (intro)

Easy (Canceling and combining) Shit!!!

Add and Subtract polynomials: two Variable

Easy (Same Shit)

Expand binomial

You can use binomial theorem or pascal's triangle

Each Row represents the coefficients of each Variable. The number of the row is equivalent to the number of the exponent.

Pascal's triangle

Row 0: 1

Row 1: 1 1

Row 2: 1 2 1

Row 3: 1 3 3 1

Row 4: 1 4 6 4 1

Row 5: 1 5 10 10 5 1

Binomial theorem

$(a+b)^n = (x+y)^3$

$1x^3 + 3x^2y + 3x^1y^2 + y^3$

$x^3 + 3x^2y + 3xy^2 + y^3$

Exponent power

$\binom{n}{r} = \frac{n!}{r!(n-r)!}$

Counting up to power

$16z^4$ $\quad 8z^3\ -3k \quad 4z^2\ \ 9k^2 \quad 2z\ \ -27k^3 \quad 81k^4$

$\frac{(2z)^4}{1} \quad \frac{(2z)^3(-3k)^1}{4} \quad \frac{(2z)^2(-3k)^2}{6} \quad \frac{(2z)^1(-3k)^3}{4} \quad \frac{(-3k)^4}{1}$

$-24z^3k$

$36z^2k^2$

$1(5x)^3 + 3(5x)^2(-y)^1 + 3(5x)^1(-y)^2 + 1(-y)^3$

$125x^3$

$-75x^2y + 15xy^2 - y^3$

$\boxed{0}$ 1

$\boxed{1}$ 1 1

$\boxed{2}$ 1 2 1

$\boxed{3}$ 1 3 3 1

$\boxed{4}$ 1 4 6 4 1

$\boxed{5}$ 1 5 10 10 5

$(r)^4 \quad (r)^3(-t)^1 \quad (r)^2(-t)^2 \quad (r)^1(-t)^3 \quad (-t)^4$

$(r)^4 + 4(r)^3(-t)^1 + 6(r)^2(-t)^2 + 4(r)^1(-t)^3 + (-t)^4$

$r^4 - 4r^3t + 6r^2t^2 - 4rt^3 + t^4$

$125x^3 - 150x^2y + 60xy^2 - 8y^3$

$16z^4 - 24z^3k + 36z^2k^2 - 54zk^3 + 81k^4$

$\times 4 \quad \times 6 \quad \times 4$

$-96 \quad 216 \quad -216$

$\frac{(5x)^3}{125x^3} \quad \frac{(5x)^2(-2y)^1}{-50x^2y} \quad \frac{(5x)^1(-2y)^2}{20xy^2} \quad \frac{(-2y)^3}{-8y^3}$

(3) (3)

Divide Polynomials by monomials (with remainders)

$$\boxed{\frac{a(x)}{b(x)} = q(x) + \frac{r(x)}{b(x)}}$$

$$\frac{a(x)}{b(x)} = \frac{3x^3+1}{x^2} = \frac{3x^3}{x^2} + \frac{1}{x^2}$$

$3x + \frac{1}{x^2}$ — remainder

quotient

OR

$$x^2 \overline{)\,3x^3+1}$$

$3x$ — quotient

$-3x^3$

$+1$ — remainder

← Easiest Way

$$x^3 \overline{)\,4x^6+5x^5-6x^2+2x+1}$$

quotient: $4x^3+5x^2$

$4x^6 \quad 5x^5$

$$x^4 \overline{)\,-2x^7+x^5+3x^4+2x-7}$$

quotient: $-2x^3+x+3$

$-2x^7 \quad x^5 \quad +3x^4$

$$x \overline{)\,-12x^5-7x^3-4x^2+6}$$

quotient: $-12x^4-7x^2-4x$

$+12x^5+7x^3+4x^2$

$\boxed{+6}$

$$x^4 \overline{)\,5x^9-8x^5+4x^3+8x}$$

quotient: $5x^5-8x$

$5x^9 - 8x^5$

$4x^3+8x$

Divide polynomials with remainders

$$-2x+7$$
$$3x^2+x+1 \overline{\big) -6x^3+19x^2+8x+12}$$
$$-(-6x^3-2x^2-2x)$$
$$21x^2+10x+12$$
$$-(21x^2+7x+7)$$
$$\underline{3x+5}$$

$$5$$
$$x^3+x+1 \overline{\big) 5x^3+2x^2+x+2}$$
$$-(5x^3+5x+5)$$
$$2x^2-4x-3$$

If what is being divided has a lower value than the quotient is zero and the remainder is what was being divided

$$8x-5$$
$$x^2+5x+3 \overline{\big) 8x^3+35x^2-17x-5}$$
$$-(8x^3+40x^2+24x)$$
$$-5x^2-41x-5$$
$$-(-5x^2-25x-15)$$
$$\underline{-16x+10}$$

$$2$$
$$2x^5+x^2 \overline{\big) 4x^5-x^3+7}$$
$$-(4x^5+2x^2)$$
$$-x^3-2x^2+7$$

$$
\begin{array}{r}
2x^2 + x + 1 \\
x^2+3x-1 \overline{\big)\, 2x^4 + 7x^3 + 2x^2 + 5x - 4} \\
-(2x^4 + 6x^3 - 2x^2) \\
x^3 + 4x^2 + 5x - 4 \\
(x^3 + 3x^2 - x) \\
x^2 + 6x - 4 \\
-(x^2 + 3x - 1) \\
3x - 3
\end{array}
$$

Absolute value function

opposite
(X) (Y)

$g(x) = |x-4| - 3$

Interpret Equations Graphically

Finding minimum / maximum of quadratic

$max = a < 0, \frac{-b}{2a}$

$min = a > 0, f\left(\frac{-b}{2a}\right)$

Side Shift I Forgot

Graphing Functions

Graphing Exponential Functions

$y = 2^x$

X	Y
0	1
1	2
2	4

Exponents have horizontal Asymptotes

$2^x + 1$

indicates the Asymptote

An Asymptote indicates that it's meaning the function is heading towards a value infinitely.

Domain generally infinite for exponents

$D(-\infty, \infty)$ $R(0, \infty)$

Range is generally infinite for logarithms and natural logs

* If this was a piecewise there would be some restrictions.

(An Asymptote is undefined)

For exponents with no altered range there is no value of x to make $y=0$

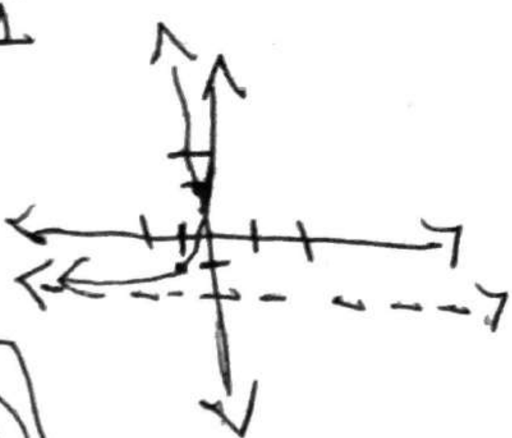
y

set this equal to zero and 1 to find values.

shifted to the left by 1

$Y = 3^{(x+1)} - 2$

shifted down 2



HA: -2

x	y
-1	-1
0	1

$\boxed{\begin{array}{l} x+1=0 = x=-1 \\ x+1=1 = x=0 \end{array}}$

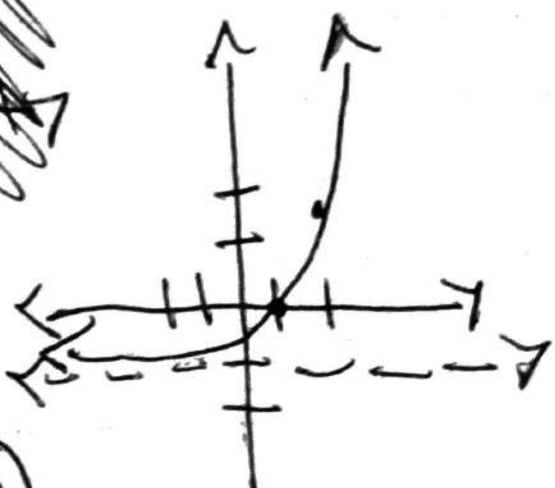
D$(-\infty, \infty)$

R$(-2, \infty)$

Anything to the zero power is 1

$Y = e^{x-1} - 1$

HA: -1



$x-1=0 = 1$

$x+1=1 = 2$

x	y
1	0
2	1.7

D$(-\infty, \infty)$

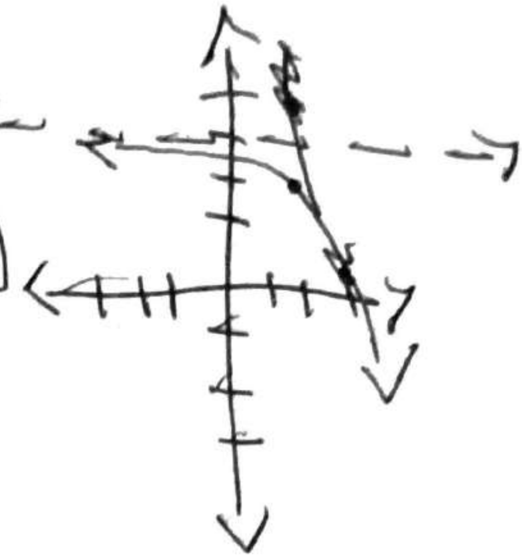
R$(-1, \infty)$

$y = 3 - e^{x-2}$

x	y
2	2
3	3

$x - 2 = 0 \quad \boxed{x = 2}$

$x - 2 = 1 \quad \boxed{x = 3}$



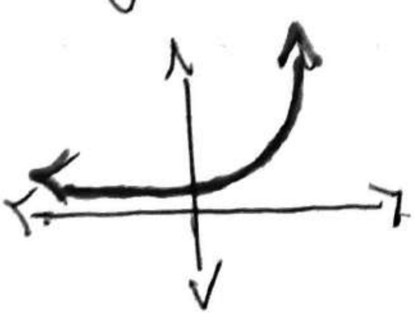
H.A.: 3

D$(-\infty, \infty)$ R$(3, -\infty)$

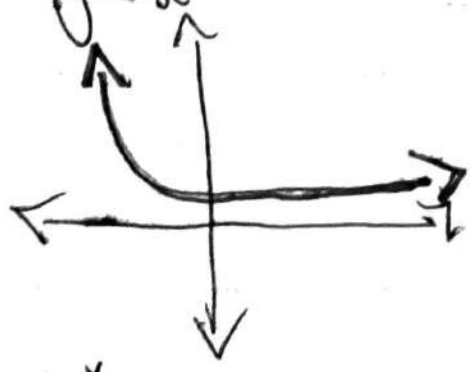
Graphs of Exponential Functions

Q1

$y = 2^x$



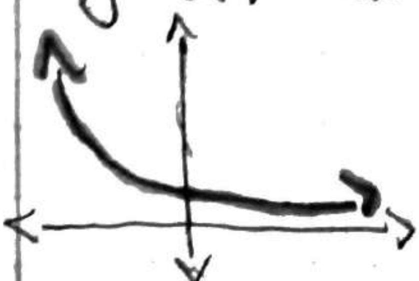
$y = 2^{-x}$ Q2



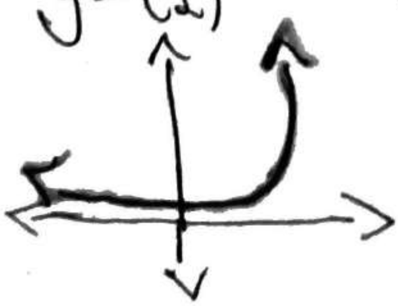
$y = -2^x$

4th Q

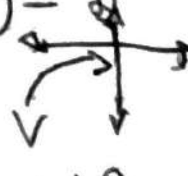
$y = \left(\frac{1}{2}\right)^x$ Q2



$y = \left(\frac{1}{2}\right)^{-x}$ Q1



$y = -2^{-x}$

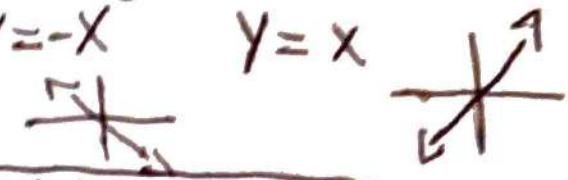


3rd Q

logs are the inverse of an exponential function

Graphing Logarithmic Functions

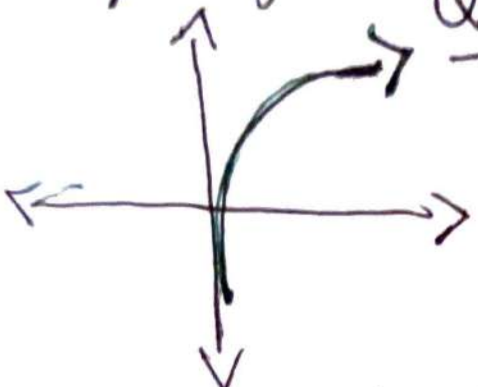
$y=-x$ $y=x$



Side Shit I Forgot

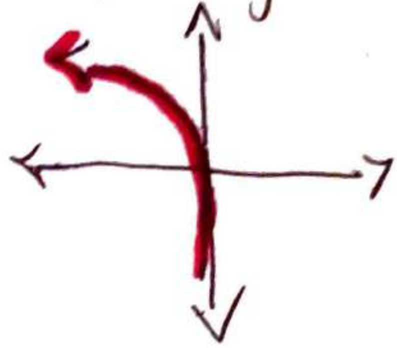
$y=\log(x)$

Q1



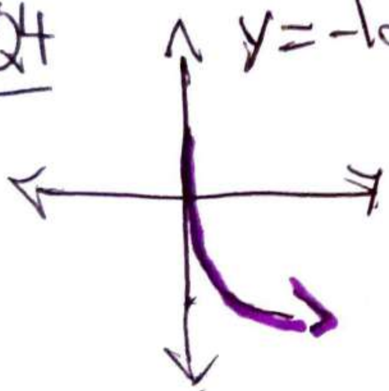
$y=\log(-x)$

Q2



Q4

$y=-\log(x)$



$y=-\log(-x)$

Q3

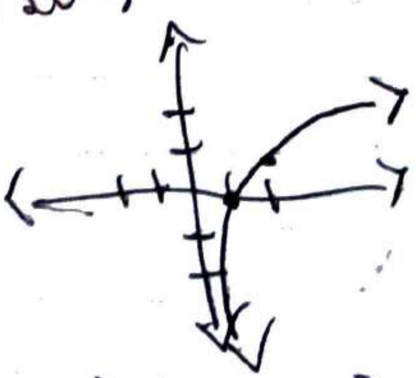
zero to find asymptote

$y=\log_2(x)$

$x=0$

$x=1$

$x=2$

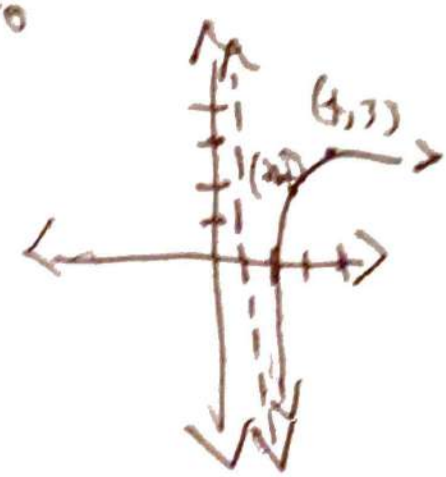


x	y
1	0
2	1

$D(0,\infty)$ $R(-\infty,\infty$

$\log_3(3)=1$ when they're same it's 1

$\log(1)=0$ when log is 1 it's zero



$y=\log_3(x-1)+2$

$x-1=3=\boxed{4}$

$x-1=1=\boxed{2}$

$x-1=0=\boxed{1}$ — VA

x	y
4	3
2	2

R $(-\infty, \infty)$

D $(1, \infty)$

$\log_3(2-1)+2$

$\log_3(1)+2=0+2$

$\log_3(4-1)+2$

$\log_3(3)+2$

$1+2=3$

Logarithms

$\log_2(8)=3$ $\quad$ $\log_6(36)=2$

$\log_3(9)=2$ $\quad$ $\log_2(16)=4$

$\boxed{\log_2(\tfrac{1}{16})=-4}$ $\quad$ $\boxed{2^{-4}=\tfrac{1}{16}}$

$\log_3(27)=3$

$\boxed{16^{\frac{1}{4}}=\sqrt[4]{16}}$ $\quad$ $\boxed{\sqrt[4]{16}=2}$

$\log_2(32)=5$ $\quad$ $\log_{16}(\tfrac{1}{2})=-\tfrac{1}{4}$

$\log_5(25)=2$

$\log_2 8 = 3$

$\log_8 2 = \frac{1}{3}$

* Fraction makes base small (cause it roots it)

* Negative makes it a fraction

$\log_a(B) =$ integer

$\log_A(b) =$ Fraction

$\log_A(\frac{1}{b}) =$ negative Fraction

log 10 = 1 because every standard log has a bas of 10 and every Natural log has a base of 3.

$\log 10^5 = 5$

$\log 1000 = 3$

$\log .001 = -3$

$\log(0) =$ Does Not Exist

$\log(-4) =$ Does Not Exist

Any Negative Number

$\log_2(128^{\frac{1}{2}}) = 9$

Next page

* $\sqrt{2^x} = \sqrt{128^{\frac{1}{2}}}$

$\frac{1}{2}$ $\sqrt{2^x} = 128$ wrong $2^7 = 128$

If your argument is raised to a power, then move it to the front

$$\log_2 128^{1/2} = \frac{1}{2}\underbrace{\log_2 128}_{7} = \frac{1}{2}\times 7 = \boxed{3.5}$$

$$\log_3 \sqrt{27} = \log_3 27^{\frac{1}{2}} = \frac{1}{2}\log_3 27 = \frac{1}{2}\times 3 = \boxed{\frac{3}{2}}$$

Quick Natural logs

$$\frac{14 = 50e^{\frac{-t}{250}}}{50}$$

$$e^{\frac{-t}{250}} = \frac{7}{25}$$

$$\ln_e e^{\frac{-t}{250}} = \ln_e \frac{7}{25}$$

$$-250\left(\ln \frac{-t}{250} = \ln\frac{7}{25}\right) - 250$$

$$t = \left(-250 \ln \frac{7}{25}\right)$$

Compound interest Formula

$$\boxed{A = be^{m}}$$

future result; days

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

interest rate; compounding periods; years; present value

$$\frac{5000e^{.025t}}{5000} = \frac{10000}{5000}$$

$$\ln_e e^{.025t} = \ln_e 2$$

$$.025t\ln_e e = \ln_e 2$$

$$\frac{.025t(1)}{.025} = \frac{\ln e(2)}{.025}$$

★ interpolate — predict inside of pattern

★ extrapolate — predict outside of the pattern

$$\boxed{\log_a (a)^x = x}$$

Trying to get rid of exponent try to find log of its base with same number

$$\boxed{a^{\log_a (x)} = x}$$

Trying to get rid of log, raise log to its base

$$\log_3 (.5x) = \frac{11}{6}$$

$$8.5 = .67 \operatorname{Log}(.37E) + 1.46$$
$$-1.46 \qquad\qquad -1.46$$

$$\frac{7.04}{.67} = \frac{\cancel{.67} \operatorname{Log}(.37E)}{\cancel{.67}}$$

$$10^{\left(\frac{7.04}{.67}\right)} = 10^{\left(\operatorname{Log}(.37E)\right)}$$

$$\frac{10^{\left(\frac{7.04}{.67}\right)}}{.37} = \frac{.37E}{.37}$$

Back to Logarithms

$\log_3 \sqrt[3]{81} = \log_3 (81)^{\frac{1}{3}} = \frac{1}{3} \log_3 (81) = \frac{1}{3} \times 4 = \boxed{\frac{4}{3}}$

log Identities

$\log_A B = B \log A$

$+ \log A + \log B = \log(AB)$

$\log_A A^b = 1 \times b$

$A^{\log_A b} = 1 \times b$

$\log A - \log B = \log\left(\frac{A}{B}\right)$

$\log(x^2 y^3 z^4)$

$\log(x^{(2)}) + \log(y^{(3)}) + \log(z^{(4)})$

$2\log(x) + 3\log(y) + 4\log(z)$

$\log\left(\frac{x^5 y^2}{z^3 r^6}\right) = \log(x)^5 + \log(y)^2 - \log(z)^3 - \log(r)^6$

$5\log(x) + 2\log(y) - 3\log(z) - 6\log(r)$

Expansion Form

$$\log\left(\sqrt[3]{\frac{x^2 y^5}{z^4}}\right) = \log(x)^{\frac{2}{3}} + \log(y)^{\frac{5}{3}} - \log(z)^{\frac{4}{3}}$$

$$\frac{2}{3}\log(x) + \frac{5}{3}\log(y) - \frac{4}{3}\log(z)$$

or

$$\frac{1}{3}\left(2\log(x) + 5\log(y) - 4\log(z)\right)$$

$$\log\left(\frac{\sqrt[5]{x^3}}{y^4}\right) \quad \log(x)^{\frac{3}{5}} + \log(y)^4 = \frac{3}{5}\log(x) - 4\log(y)$$

Compressed Form

$$3\log(x) - 2\log(y) + \frac{1}{3}\log(z)$$

$$\log\left(\frac{x^3\sqrt[3]{z}}{y^2}\right)$$

log Change of base Formula

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$3^{2x} = 5$

$\log_3(3)^{2x} = \log_3(5)$

$\frac{2x}{2} = \frac{\log_3(5)}{2}$ ← Change of base

$x = \frac{\log_3(5)}{2}$

Back to some compression Shit!!

$\frac{1}{4}\log x - \frac{5}{4}\log y + \frac{3}{4}\log z$

$\log\left(\frac{\sqrt[4]{x}\ \sqrt[4]{z^3}}{\sqrt[4]{y^5}}\right) = \boxed{\log\left(\frac{x\ z^3}{y^5}\right)^{\frac{1}{4}}}$

Back to Change of Base

$\log_5(100) = \frac{\log 100}{\log 5}$

$\log_z x^2 y^3 = \frac{\log(x^2y^3)}{\log(z)}$ or $\frac{\ln(x^2y^3)}{\ln(z)}$

Convurt exponent to log

[redacted]

$3^4 = 81 = \log_3 81 = 4$

[redacted]

$5^3 = 125 = \log_5(125) = 3$

Convert log to exponent

$\log_9 9 = 2 = 3^2 = 9$

$\log_3(243) = 5 = 3^5 = 243$

$\log_7(x) = 3$ $\quad\quad$ $\log_4(49) = 2$

$7^3 = x$ $\quad\quad$ $4^2 = 49$

Solving Log Equations

$\log_5(10x-5)=2$

$$5^2 = 10x-5$$
$$+5 \qquad +5$$

$$\frac{30}{10} = \frac{10x}{10}$$

$$\boxed{3 = x}$$

$\ln(\sqrt{x-8})=5$

Add

$x^2 \cdot x^2 = x^4$

multiply

$(x^2)^3 = x^6$

$$(e^5)^2 = (\sqrt{x-8})^2$$

$$e^{10} = x-8$$
$$+8 \qquad +8$$

$$\boxed{e^{10}+8 = x}$$

$\log_3(5x+2) = \log_3(7x-4)$ = If bases are the same then set them equal to each other

$5x+2 = 7x-4$
$-2 \qquad -2$

$5x = 7x-6$
$-7x \quad -7x$

$\frac{-2x}{-2} = \frac{-6}{-2}$

$\boxed{x=3}$

$\text{Log}_2(x-1) + \text{Log}_2(x+1) = 3$

$\log_2(x-1)(x+1) = 3$

$(x+1)(x-1)$

$x^2 - \cancel{1x+1x} - 1$

x^2-1

$\log_2(x^2-1) = 3$

$2^3 = x^2-1$
$+1 \qquad +1$

$\sqrt{9} = \sqrt{x^2}$

$\boxed{3=x}$

$$\log_3\left(\frac{8x+3}{x}\right) = 2$$

$$(x)\, 3^2 = \frac{8x+3}{x}(x)$$

$$9x = 8x+3$$
$$-8x \quad -8x$$

$$\boxed{x = 3}$$

$$\log_2(x) + \log_2(x+5) = \log_2(x+4)$$

$$\log_2(x)(x+5) = \log_2(x+4)$$

$x^2+4x-4=0$ $\quad$ $\log_2(x^2+5x) = \log_2(x+4)$

$x^2+4x=4$ $\quad$ $x^2+5x = x+4$

$\quad\quad -x \quad -x$

Completing square

$x^2+4x+4=4+4$ $\quad$ $x^2+4x = 4$

$\quad\quad -4$

$x^2+4x+4=8$ $\quad$ $x^2+4x-4=0$

$\sqrt{(x+2)^2} = \sqrt{8}$ $\quad$ $\boxed{-2 \pm \sqrt{8}}$ $\quad$ $\boxed{-2+\sqrt{8}}$

$\quad -2 \quad -2$

$$4^{2x-1} = 8^{3x+2}$$

$$(2)^{2(2x-1)} = (2)^{3(3x+2)}$$

$$4x-2 = 9x+6$$
$$+2 \qquad +2$$
$$4x = 9x+8$$
$$-9x \quad -9x$$
$$\frac{-5x}{-5} = \frac{8}{-5}$$
$$\boxed{x = -\frac{8}{5}}$$

$$9^{x-2} = 27^{\frac{1}{3}x-4}$$

$$(3)^{2(x-2)} = (3)^{3(\frac{1}{3}x-4)}$$

$$2x-4 = x-12$$
$$-x \quad +4 \quad -x \quad +4$$
$$\boxed{x = -8}$$

Change base to one

divide with any base

$5^x = 8$

$\log_5 5^x = \log_5 8$ or $\ln 5^x = \ln 8$

$x = \log_5 8$

$\frac{x\ln 5}{\ln 5} = \frac{\ln 8}{\ln 5}$

$\boxed{x = \frac{\ln 8}{\ln 5}}$

$\log 5^x = \log 8$

$\frac{x\log 5}{\log 5} = \frac{\log 8}{\log 5}$

$\boxed{x = \frac{\log 8}{\log 5}}$

$\log 3^{2x+4} = \log 7^{x-3}$

$3^{2x+4} = 7^{x-3}$

$\ln 3^{2x+4} = \ln 7^{x-3}$

$(2x+4)\ln 3 = (x-3)\ln 7$

$2x\ln(3) + 4\ln(3) = x\ln(7) - 3\ln(7)$

$x(2x\ln 3 - x\ln(7)) \stackrel{\div}{=} -4\ln(3)$

$3\ln c$

factor then divide

Solving For X Base "e" Problems

$$e^{2x} = 7$$

$$\ln_e(e)^{2x} = \ln_e(7)$$

$$(2x)\ln_e(e) = \ln_e(7)$$

$$(2x)(1) = \ln_e(7)$$

$$\frac{2x}{2} = \frac{\ln_e(7)}{2}$$

$$x = \frac{\ln_e(7)}{2}$$

$$e^{2x} - 4e^{x} - 5 = 0$$

$$A^2 - 4A - 5 = 0 \qquad A^2 = e^{2x}$$

$$(A-5)(A+1) \qquad A = e^{x}$$

$$(e^{x}-5)(e^{x}+1)$$

$$e^{x} = +5 \qquad e^{x} = -1$$

$$\ln(e)^{x} = \ln(5)$$

$$x = \ln(5)$$

$$\ln(e)^{x} = \ln(-1)$$

$$x = \ln(-1)$$ X can't find log of negative

$$\cancel{(1+e^{-x})}\ \frac{525}{\cancel{1+e^{-x}}} = 275\,(1+e^{-x}$$

$$525 = 275 + 275e^{-x}$$
$$-275 \quad -275$$

$$\frac{250}{275} = \frac{275e^{-x}}{275}$$

$$\frac{250}{275} = e^{-x}$$

$$\ln\left(\frac{250}{275}\right) = \ln(e)^{-x}$$

$$\frac{\ln\left(\frac{250}{275}\right)}{-1} = \frac{(-x)}{-1}$$

$$\boxed{-\ln\left(\frac{250}{275}\right) = x}$$

This can be further simplified but this is the Answer

5000 double 5 hours

$$\left(5000(2)^{\frac{t}{5}}\right)$$

$$3000(3)^{\frac{t}{4}}$$

5000 8% every year, $t=5$

Exponential or Population Growth Shift!!

$$5000(1.08)^{1\cdot t}$$

$$5000(e^{.08t})$$

Growth: $50000\left(1+\frac{.03}{1}\right)^{1\times 10}$

Decay: $5000\left(1-\frac{.03}{1}\right)^{1\times 10}$

increasing

$$50000(.97)^{10}$$

negative means decreasing

logarithms

Natural logs and Regular logs are Always greater than zero. So, if you see an equation make it greater than zero and choose the Answer that is positive

How to Find the inverse of a function

$f(x) = 2x - 7$

$y = 2x - 7$

$x = 2y - 7$
$+7 \qquad +7$

$\frac{x+7}{2} = \frac{2y}{2}$

$\boxed{f^{-1}(x) = \frac{x+7}{2}}$

$f(x) = x^3 + 8$

$y = x^3 + 8$

$x = y^3 + 8$
$-8 \qquad -8$

$\sqrt[3]{x-8} = \sqrt[3]{y^3}$

$\boxed{\sqrt[3]{x-8}} = y$

$f^{-1}(x) =$ [redacted] $= y$

$f(x) = \sqrt{x+2} - 5$

$y = \sqrt{x+2} - 5$

$x = \sqrt{y+2} - 5$
$+5 \qquad\qquad +5$

$(x+5)^2 = \sqrt{y+2}^{\,2}$

$(x+5)^2 = y+2$
$-2 \qquad\qquad -2$

$\boxed{(x+5)^2 - 2} = y = f^{-1}(x)$

or

$\boxed{x^2 + 10x + 23}$

$f(x) = \sqrt[3]{x+4} - 2$

$y = \sqrt[3]{x+4} - 2$

$x = \sqrt[3]{y+4} - 2$
$+2 \qquad\qquad +2$

$(x+2)^3 = \left(\sqrt[3]{y+4}\right)^3$

$(x+2)^3 = y+4$
$-4 \qquad\qquad -4$

$(x+2)^3 - 4 = y$

$\boxed{(x+2)^3 - 4 = f^{-1}(x)}$

$$f(x) = \frac{3x-7}{4x+3}$$

$$y = \frac{3x-7}{4x+3}$$

$$\frac{x}{1} = \frac{3y-7}{4y+3}$$

$$3y-7 = 4yx+3x$$

$$-4yx$$

$$+7$$

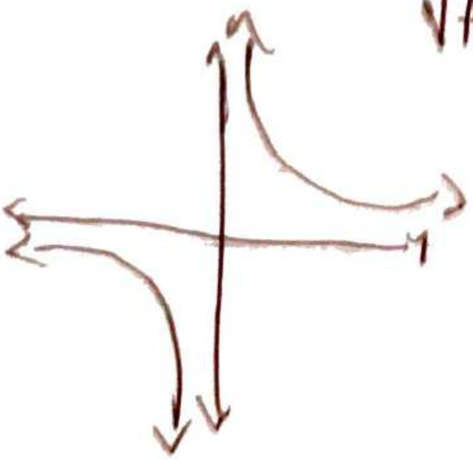
$$3y-4yx = 7+3x$$

$$\frac{y(3-4x)}{3-4x} = \frac{7+3x}{3-4x}$$

$$y = \frac{7+3x}{3-4x} = f^{-1}(x)$$

Graphing Rational Functions with Vertical, Horizontal & Slant Asymptotes, Holes, Domains and Ranges

$y = \frac{1}{x}$ HA: $y=0$ VA: $x=0$



$D(-\infty,0)\cup(0,\infty)$

$R(-\infty,0)\cup(0,\infty)$

$\frac{1}{x^2}$ VA: $x=0$ HA: $y=0$

when bottom heavy $y=0$

[- closed

(- open

$D(-\infty,0)(0,\infty)$

$R(0,\infty)$

$f(x) = \frac{1}{x-1}$ - vertical asymp

HA=0

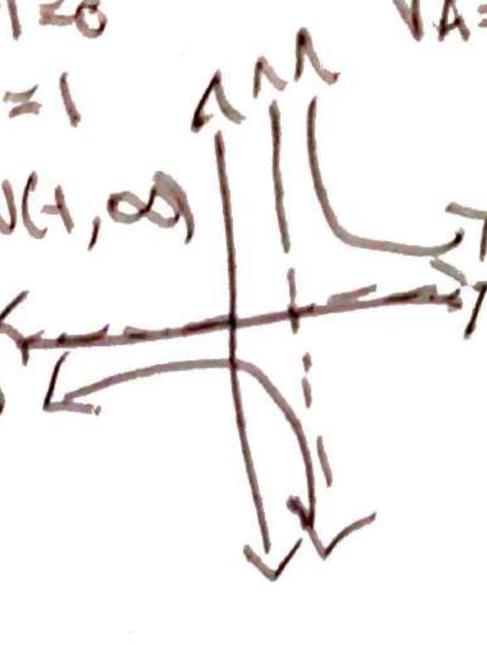
VA=1

$x-1=0$

$x=1$

$D(-\infty,-1)\cup(1,\infty)$

$R(-\infty,0)(0,\infty)$

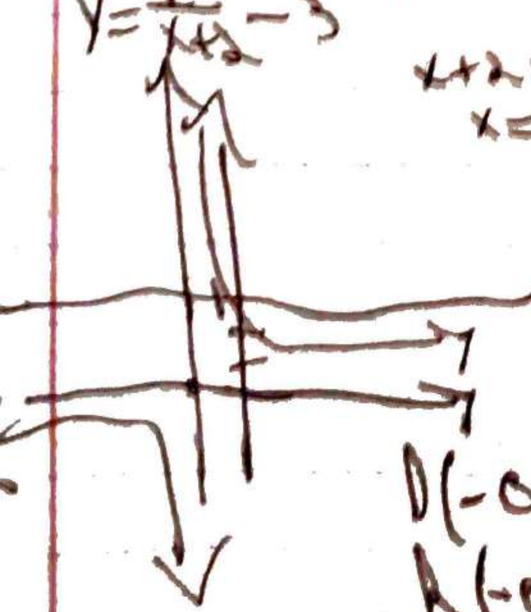


$y=\frac{1}{x+2}-3$ HA Direction

$x+2=0$

$x=-2$

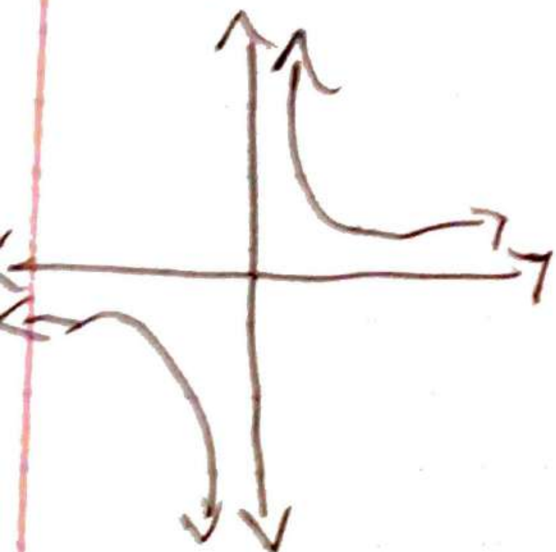
VA: -2



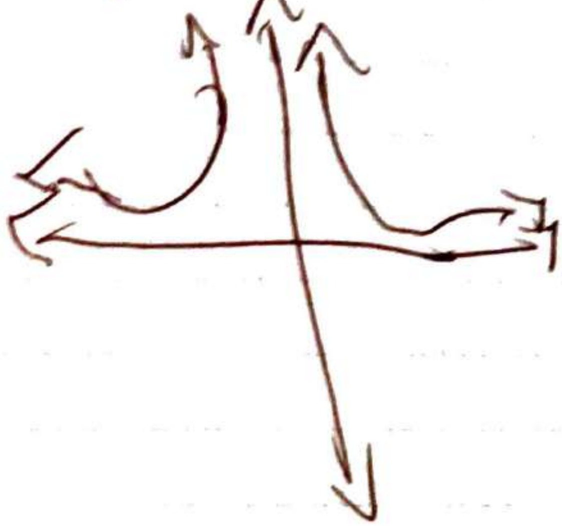
$D(-\infty,-2)\cup(-2,\infty)$

$R(-\infty,-3)\cup(-3,$

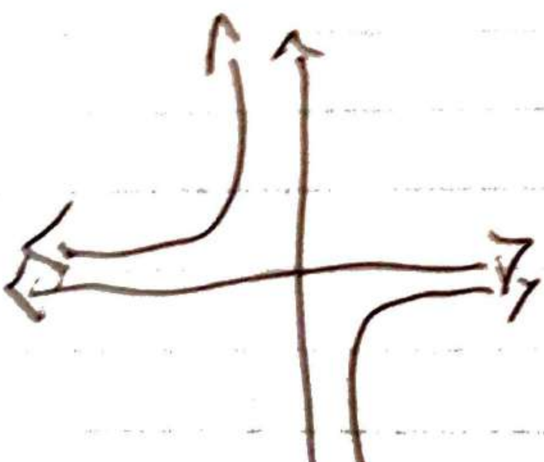
$y = \frac{1}{x}$



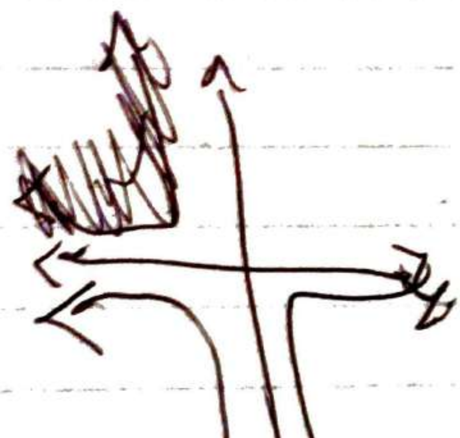
$y = \frac{1}{x^2}$



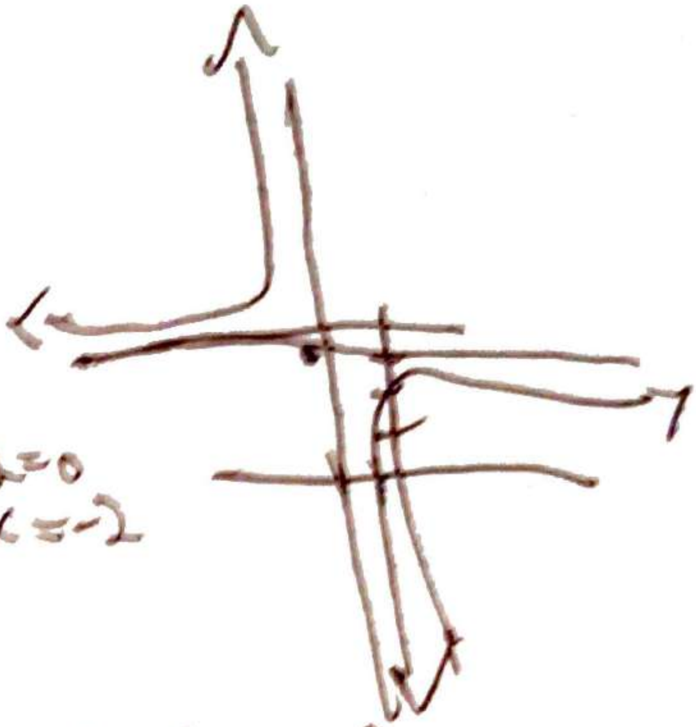
$y = \frac{-1}{x}$



$y = -\frac{1}{x^2}$



$y = -\frac{1}{x+2} + 3$



$x+2=0$

$x=-2$

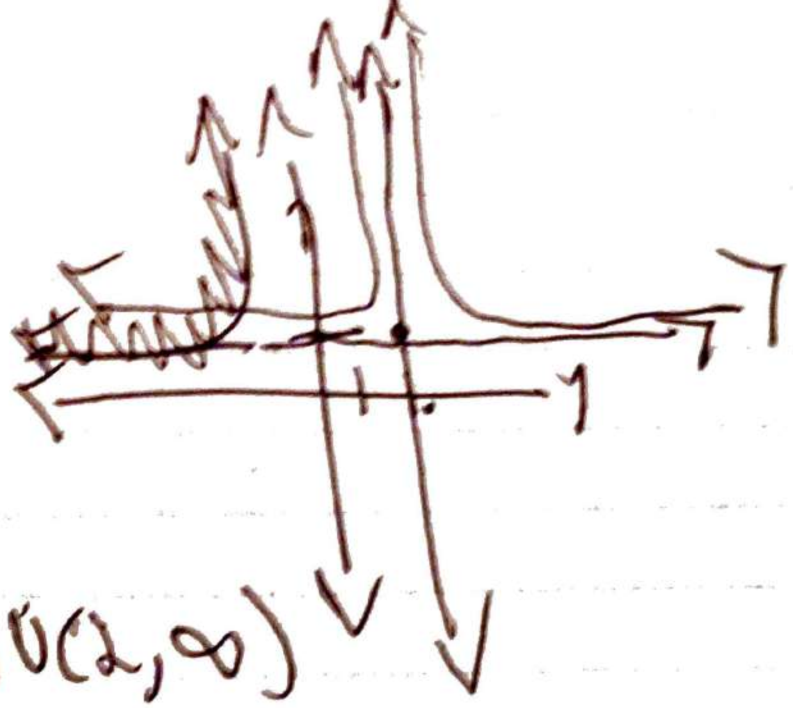
VA: -2

HA: 3

D $(-\infty, -2) \cup (-2, \infty)$

R $(-\infty, 3) \cup (3, \infty)$

$y = \frac{1}{(x-2)^2} + 1$



$x-2=0$

$x=2$

VA: 2

HA: 1

D $(-\infty, 2) \cup (2, \infty)$

R $(1, \infty)$

$y = -\frac{1}{(x+1)^2} + 2$

HA: 2

VA: -1

3^{x+2} -2

$y = \frac{2x-4}{1x+3}$ with $+5$

HA: 8

$\boxed{HA = 2}$

$y = \frac{3x^2}{x^2-4}$

$\boxed{HA: 3}$

$y = \frac{7-8x^2}{2x+4x^2} + 7$

$\boxed{HA: 5}$

$\frac{6x^2}{x^2} + 4$

HA: $6 + 4 = 10$

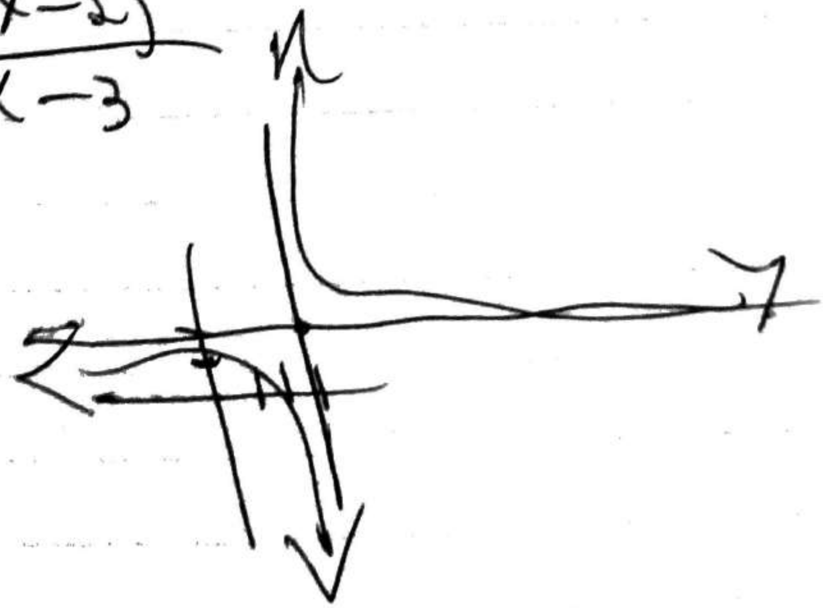
Bigger / Slant Asymptote

$$y = \frac{2x^{3}+4x}{5x^{2}+6}$$

$$\begin{array}{r} \frac{2}{5}x \\ 5x^{2}+6 \overline{\big) 2x^{3}+4x} \\ -\left(2x^{3}+\frac{12}{5}\right. \end{array}$$

$\frac{2}{5}x \quad R\left(4x-\frac{12}{5}\right)$

$$y = \frac{2(x-2)}{x-3}$$



VA: 3

HA: 2

Bottom is not heavy

$$f(x)=\frac{3x^2+9x-12}{x^2+x-2}-4$$

whole function to zero to find

NOM = to zero If this is not there

HA: $3+(-4) = -1$

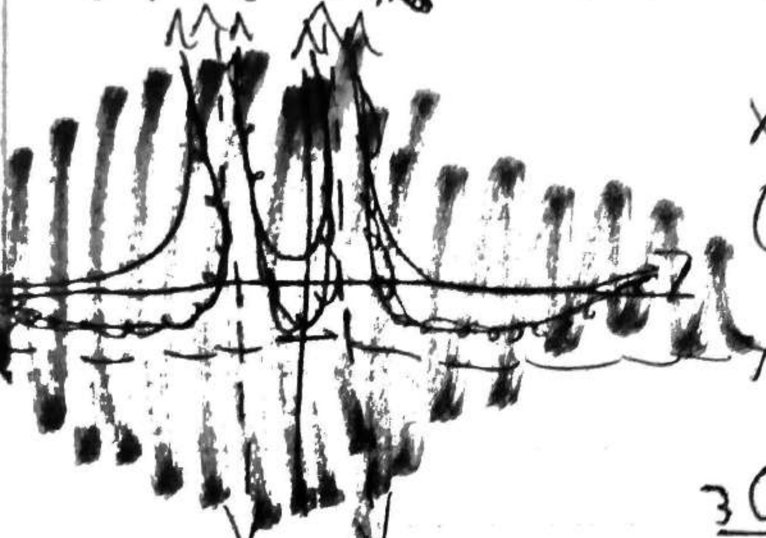
VA: -2

Hole: $(1,1)$

x intercept: ~~(4,0)~~

y intercept: $(0,2)$

whole function x to zero



x^2+1x-2

$(x+2)(x-1)$

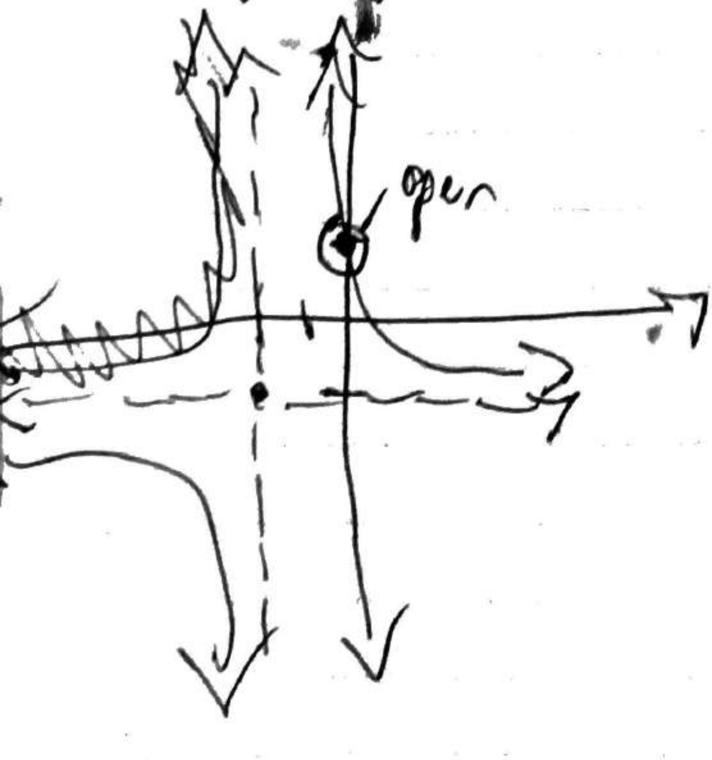
$x+2=0$ $\boxed{x=-2}$

$x-1=0$ $\boxed{x=1}$

$$\frac{3(x+4)\cancel{(x-1)}}{(x+2)\cancel{(x-1)}}-4$$

not included and its y too

$$\frac{3(x+4)}{x+2}-4$$



$$\frac{3(1+4)}{1+2}-4$$

$$\frac{15}{3}-4$$

$5-4=\boxed{1}$

$y=1$

flug in points
to find where
the function
lies

$$y = \frac{2x^2+6x-8}{x-2}$$

$$\begin{array}{r l}
 & \boxed{2x+10} \\
x-2 \,\big) & 2x^2+6x-8 \\
 & -(2x^2-4x\quad) \\
 & \qquad 10x-8 \\
 & \quad -(10x-20) \\
 & \qquad R(12)
\end{array}$$

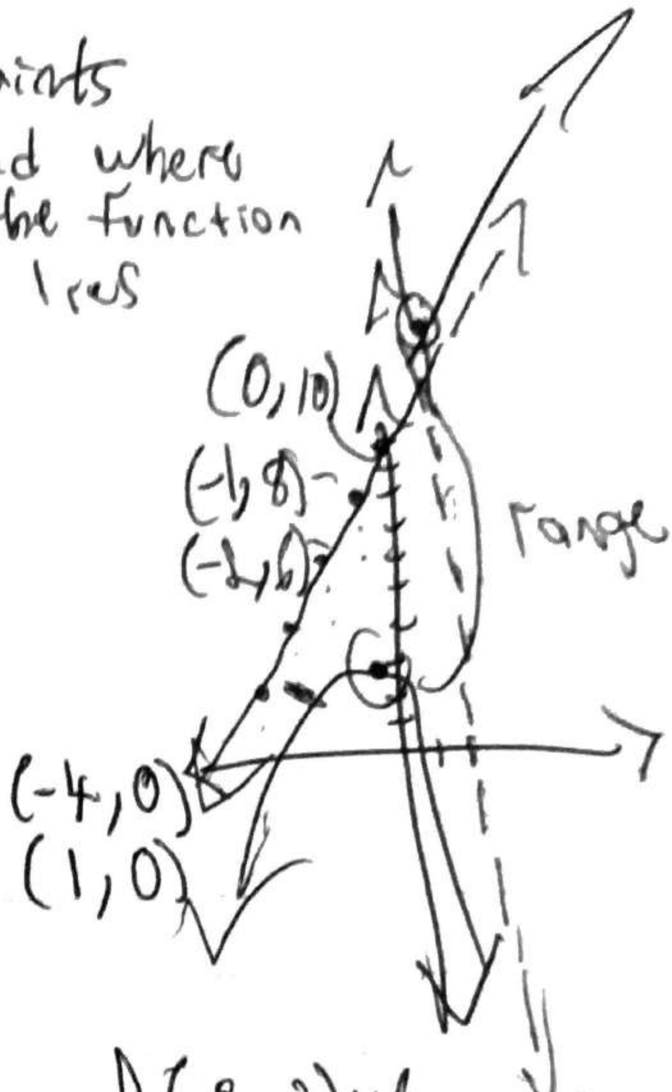
slant asymptote

SA: $y = 2x+10$

VA: $x = 2$

$$y = \frac{2(x^2+3x-4)}{x-2}$$

$$y = \frac{2(x+4)(x-1)}{x-2}$$



$D\ (-\infty,2)\cup(2,\infty)$

$R\ (-\infty,4.2)\cup(23.8,\infty)$

☆ The Type of function depends on the surviving equation

How to Find the Domain of a function - Radicals, Fractions & Square Roots - Interval Notation

$f(x) = 2x - 7$

$D(-\infty, \infty)$

$f(x) = x^2 + 3x - 5$

$D(-\infty, \infty)$

$f(x) = 2x^3 - 5x^2 + 7x - 3$

$D(-\infty, \infty)$

All Real If there are no Fractions OR Square Roots

$f(x) = \frac{5}{x-2}$ ~ x can be anything except a value that will produce a zero in the denominator

$x - 2 \neq 0$

$x \neq 2$

Set the denominator $\neq$ to zero for rational functions to get domain

$D(-\infty, 2) \cup (2, \infty)$

$$f(x) = \frac{3x-8}{x^2-9x+20}$$

$$(x-4)(x-5) \neq 0$$

$\boxed{x \neq 4}$ $\boxed{x \neq 5}$

$D(-\infty, 4) \cup (4, 5) \cup (5, \infty)$

$$f(x) = \frac{(2x-3)}{x^2+4} =$$

All Real

$D(-\infty, \infty)$

$$x^2 + 4 \neq 0$$

It will never equal zero so all Real Numbers

$$x^2 \neq -4$$

If uneven then anything

for even Radicals

$f(x) = \sqrt[2]{x-4}$ $x - 4 \geq 0$

$x \geq +4$

$D[4, \infty)$

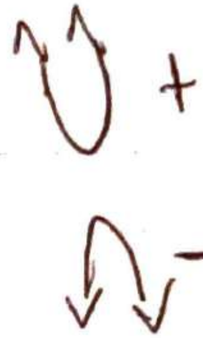
$f(x) = \sqrt{x^2 + 3x - 28}$

$x^2 + 3x - 28 \geq 0$

[redacted] $(x-4)(x+7) \geq 0$

$x - 4 \geq 0 \quad x + 7 \geq 0$
$+4 \quad +4 \quad -7 \quad -7$

[redacted] $x \geq 4 \quad x \geq -7$



$(x \leq 7)$

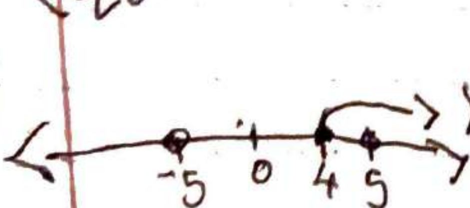
$D(-\infty, -7] \cup [4, \infty) \; (x \geq 4)$

can't have the bottom be zero

$f(x) = \dfrac{2x - 7}{\sqrt{x+3}} \qquad x + 3 > 0$

$f(x) = \dfrac{\sqrt{x - 4}}{x^2 - 25} \qquad \dfrac{x - 4 \geq 0}{x \geq 4}$

$x \neq 5$



$x^2 - 25 \neq 0$

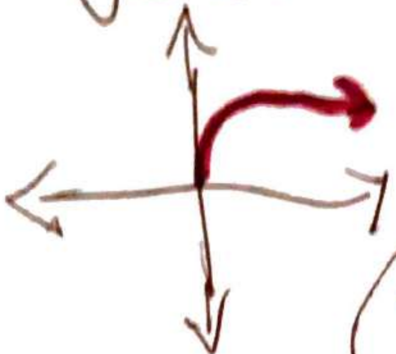
$(x+5)(x-5)$

$x \neq -5$

$D[4,5) \cup (5, \infty$

Determines ~~[illegible]~~ Vertical Shift

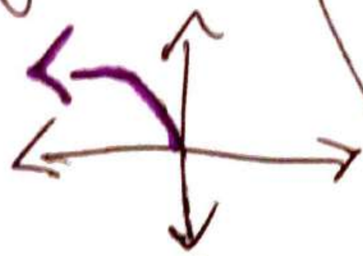
$y = \sqrt{x}$



$y = -\sqrt{x}$

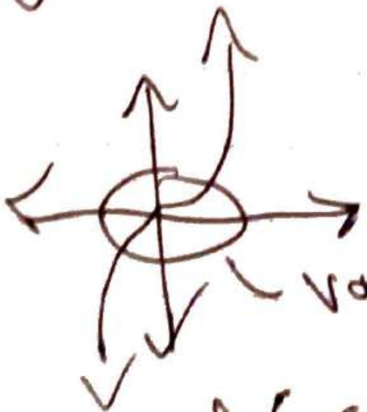
Roots are similar to logarithms in terms of reflections

$y = \sqrt{-x}$



$y = -\sqrt{-x}$

$y = x^3$



odd example

varies

$R(-\infty, \infty)$

odd ~~exponent~~/power

$R(\infty, -\infty)$

Even ~~[illegible]~~/power

Solve rational equation

Simplify ~~Solve~~ Complex Fractions

multiply rational expressions

Simplify rational expression

Divide rational Expressions

Solve poly inequalities

Difference Quotient

P.S. You rationalize by multiplying by the conjugate

Solving Rational Equations

or

$$\frac{5\times5}{8\times5} - \frac{3\times8}{5\times8} = \frac{x}{10}$$

Find least common multiple of 8, 5 and 10.

$$\frac{25}{40} - \frac{24}{40}$$

$$\frac{1}{40} = \frac{x}{10}$$

$$\frac{40x}{40} = \frac{10}{40}$$

$$\boxed{x = \frac{1}{4}}$$

$$\left(\frac{5}{8} - \frac{3}{5} = \frac{x}{10}\right) 40$$

$$25 - 24 = 4x$$

$$\frac{1}{4} = \frac{4x}{4}$$

$$\frac{1}{4} = x$$

$$\frac{9}{x} = \frac{x}{4}$$

$$\sqrt{x^2} = \sqrt{36}$$

$$x = \pm 6$$

$$\frac{4}{x-3} = \frac{9}{x+2}$$

$$4x + 8 = 9x - 27$$
$$-4x + 27 \quad -4x + 27$$

$$\frac{35}{5} = \frac{5x}{5}$$

$$\boxed{7 = x}$$

$$\left(\frac{x+2}{3} + 4 = \frac{x+9}{2}\right)6$$

$$2x + 4 + 24 = 3x + 27$$

$$2x + 28 = 3x + 27$$
$$-2x - 27 \quad -2x - 27$$

$$\boxed{x = 1}$$

$$\left(\frac{4}{x} + \frac{8}{x+2} = 4\right)^{x(x+2)}$$

$$4x + 8 + 8x = 4x(x+2)$$

$$12x + 8 = 4x^2 + 8x$$

$$0 = 4x^2 + 8x - 12x - 8$$

$$0 = 4x^2 - 4x - 8$$

$$0 = 4(x^2 - x - 2)$$

$$4(x-2)(x+1)$$

$$x = -1$$

The LCD is a numeral or expression that can be perfectly divided into the denominator, to remove fraction component

First see if the numerator can be factored then, worry about denominator

$$\frac{5-x}{x-5} = \frac{-1(-5+x)}{x-5} \Rightarrow \boxed{-1}$$

$$\frac{72x^8y^7}{64x^5y^4} = \frac{8\cdot 9\cdot x^5\cdot x^3\cdot y^4\cdot y^3}{8\cdot 8\cdot x^5\cdot y^4} = \boxed{\frac{9x^3\cdot y^3}{8}}$$

$$\frac{5x^2-15x}{8x-24} = \frac{5(x^2-3x)}{8(x-3)} = \frac{5x(x-3)}{8(x-3)}$$

$$\boxed{\frac{5x}{8}}$$

$$\frac{7\cdot 6-6x}{3(x-7)} \quad \frac{6\cdot 7\cdot 2-6\cdot x}{3(x-7)} \quad \boxed{\frac{12-6x}{x-7}}$$

$$\frac{42-6x}{3x-21} = \frac{-6(7-x)}{3(x-7)} = -\frac{6}{3} \Rightarrow 2$$

The LCD is a number or expression that can be perfectly divided into the denominator, to remove fraction component

$$\frac{5-x}{x-5} = \frac{-1(-5+\cancel{x})}{\cancel{x}-5} = \boxed{-1}$$

First see if the numerator can be factored then worry about denominator

$$\frac{72x^8y^7}{64x^5y^4} = \frac{\cancel{8}\cdot 9\cdot \cancel{x^5}\cdot x^3\cdot \cancel{y^4}\cdot y^3}{\cancel{8}\cdot 8\cdot \cancel{x^5}\cdot \cancel{y^4}} = \boxed{\frac{9x^3\cdot y^3}{8}}$$

$$\frac{5x^2-15x}{8x-24} = \frac{5(x^2-3x)}{8(x-3)} = \frac{5x(x-\cancel{3})}{8(x\cancel{-3})}$$

$$\boxed{\frac{5x}{8}}$$

$$\cancel{\frac{7\cdot 6-6x}{3(x-7)}} \quad \cancel{\frac{6\cdot 7\cdot 2-6\cdot x}{7(x-7)}} \quad \boxed{\cancel{\frac{12-6x}{x-7}}}$$

$$\frac{42-6x}{3x-21} = \frac{-6(7-\cancel{x})}{3(\cancel{x}-7)} = -\frac{6}{3} = -2$$

$\boxed{\text{grouping}}$

$(4y^3 + 8y^2)(-9y - 18)$

$4y^2(4y+2) - 9(4y+2)$

$$\boxed{(4y+2)(4y^2-9)}$$

$3x^2 - 16x - 12$ $\quad \begin{array}{c} 36 \\ 2 \quad -18 \end{array}$

$(3x+2)\left(\frac{3x}{3} - \frac{18}{3}\right)$ or

$(3x+2)(x-6)$

AC method

$6x^2 + x - 15$ $\quad \begin{array}{ccc} & -90 & \\ 10 & \times & -9 \\ & 1 & \end{array}$

$6x^2 + 10x - 9x - 15$

$2x(3x+5) - 3(3x+5)$

$\left(\frac{4y}{2} + \frac{10}{2}\right)\left(\frac{4y}{2} - \frac{6}{2}\right)$

$(3x+5)(2x-3)$

$3x + 5 = 0 \qquad 2x - 3 = 0$

$\quad -5 \; -5 \qquad\quad +3 \; +3$

$\frac{3x}{3} = \frac{-5}{3} \quad \boxed{x = \frac{-5}{3}} \qquad \frac{2x}{2} = \frac{3}{2} \quad \boxed{x =}$

Cubic Polynomials

$1x^3 - 4x^2 + x + 6$ - factors

$\frac{-4}{1} \neq \frac{6}{1}$ so we cannot group

Synthetic division

$1^{\pm}\ 2^{\pm}\ 3^{\pm}\ 6^{\pm}$

See which is equal to zero

$$\begin{array}{r|rrrr} 2 & 1 & -4 & 1 & 6 \\ & \downarrow & 2 & -4 & -6 \\ \hline & 1 & -2 & -3 & 0 \end{array}$$

$(1x^2 - 2x - 3)(x-2)$

$(x-3)(x-1)(x-2)$ = Answers

Multiply rational Functions

To multiply factor out the expressions and combine them to cancel them out through division

$$\frac{7x + 14}{2x^2 - 8} \cdot x^2 + 3x - 10 = \frac{7(x+2)}{2(x^2-4)} \cdot (x+5)(x-2)$$

Difference Of Cubes

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$(a+b)^2 = a^2 + 2ab + b^2$ – Square of Sum

$(a-b)^2 = (a+b)(a-b)$ — Difference of Squares

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ – Sum of Cube

How to divide Rational Expressions

1 Keep change flip, 2 factor it out, 4 multiply across, 3 get rid of like terms

Addition and Subtraction of Complex Fractions

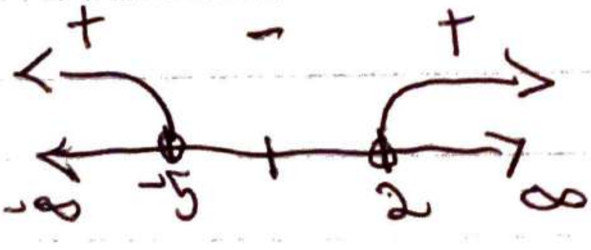
First factor the fractions out first and then find the LCD.

Solving Polynomial Inequalities

$x^2 + 3x > 10$

$x^2 + 3x - 10 > 0$

$(x-2)(x+5) > 0$



D$(-\infty, -5) \cup (2, \infty)$

$x < -5$

$x > 2$

$x^3 - x \leq 3x^2 - 3$

$x^3 - 3x^2 - x + 3 \leq 0$

$x^2(x-3) - 1(x-3)$

$(x^2-1)(x-3)$

$(x+1)(x-1)(x-3)$

$\boxed{D(-\infty, -1] \cup [1, 3]}$

$\boxed{1 \leq x \leq 3 \quad x \leq -1}$

$x^3-2x^2-5x+6 \geq 0$

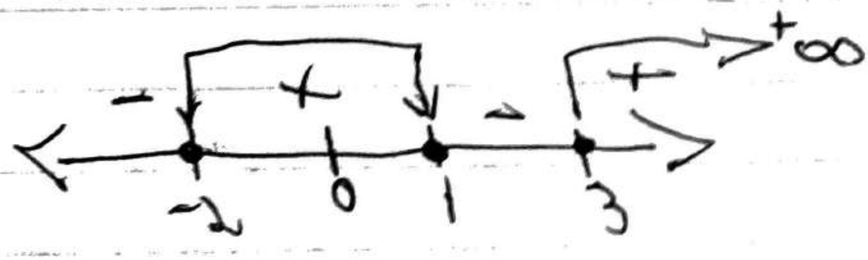
$1^{\pm}\ 2^{\pm}\ 3^{\pm}\ 6^{\pm}$

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -5 & 6 \\ & \downarrow & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 3 & 1 & -2 & -5 & 6 \\ & \downarrow & 3 & 3 & -6 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$(x^2+x-2)(x-3)$

$\underline{(x+2)}\,\underline{(x-1)}\,\underline{(x-3)} \geq 0$



$D\ [-2,1] \cup [3,\infty)$

$$\left\{ \begin{array}{c} -2 \leq x \leq 1 \\ \text{or} \\ x \geq 3 \end{array} \right\}$$

open circle → can't equal zero

$$\frac{x+2}{x-1} \le 3$$

$$\frac{x+2}{x-1} - \frac{3(x-1)}{x-1} \le 0$$

$$\frac{x+2-3x-3}{x-1} \le 0$$

$$\frac{-2x+5}{x-1} \le 0$$

cause $\frac{-2x}{-2} \le \frac{-5}{-2}$

$x \ge \frac{5}{2}$

alway plug in points between interest regions

$x \ne 1$

$x = \frac{5}{2}$

closed if there's an equal sign.

$$\boxed{\frac{x-3}{x+2}}$$

always open

Absolute Value with Inequality

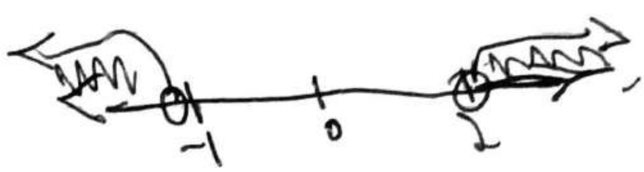
$|3x-1|>5$

$3x-1>5$ $\qquad$ $3x-1<-5$

$+ \quad +1$ $\qquad$ $+1 \quad +1$

$\frac{3x}{3}>\frac{6}{3}$ $\qquad$ $\frac{3x}{3}<\frac{-4}{3}$

$x>2$ $\qquad$ $x<-4/3$



D $(-\infty,-\frac{4}{3})\cup(2,\infty)$

$3-2|2x-1|\leq -7$

$-3 \qquad\qquad -3$

$\frac{-2|2x-1|}{-2}\leq\frac{-10}{-2}$

$|2x-1|\geq +5$

$2x-1\geq 5$ $\qquad$ $2x-1\leq 5$

$\boxed{x\geq 3}$ $\qquad$ $\boxed{x\leq -2}$

Difference Quotient

replace x with x+h inside function

$$f(x) = 7x \quad = \quad \frac{7(x+h) - 7x}{h}$$

$$\frac{f(x+h) - f(x)}{h} \qquad 7x + 7h - 7x$$

$$\frac{7h}{h} = 7$$

$$f(x) = 5x + 4$$

$$\frac{5(x+h) + 4 - (5x+4)}{h}$$

$$\frac{\cancel{5x} + 5h + 4 - (\cancel{5x} + 4)}{h}$$

$$\frac{5h}{h} = \boxed{5}$$

$f(x)=x^2$

$$\frac{(x+h)^2-(x^2)}{h}$$

$$\frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h}$$

$$\boxed{2x+h}$$

$f(x)=\sqrt{x}$

$\sqrt{x}\cdot\sqrt{x}=x$

$$\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{(\sqrt{x+h}+\sqrt{x})}{(\sqrt{x+h}+\sqrt{x})}$$

$$\frac{\cancel{x}+h+\cancel{\sqrt{x}\sqrt{x+h}}-\cancel{\sqrt{x}\sqrt{x+h}}-\cancel{x}}{h(\sqrt{x+h}+\sqrt{x})}$$

$$\frac{h}{h(\sqrt{x+h}+\sqrt{x})}$$

$$\frac{1}{\sqrt{x+h}+\sqrt{x}}$$

$f(x) = \frac{1}{x}$

① $\left(\dfrac{\frac{1}{x+h} - \frac{1}{x}}{h}\right)$

② $\dfrac{\left(\frac{1}{x+h} - \frac{1}{x}\right)(x+h)(x)}{\left(h\right)(x+h)(x)}$

$\dfrac{x-(x+h)}{h(x+h)(x)} =$

$\dfrac{\cancel{x} - \cancel{x} - h}{h(x+h)(x)}$

$\dfrac{-h}{h(x+h)(x)}$

$\boxed{\dfrac{-1}{(x+h)(x)}}$

$f(x) = 3x^2 + 4x - 5$

$$\frac{3(x+h)^2 + 4(x+h) - 5}{h} - f(x)$$

$$\frac{3(x^2 + 2xh + h^2) + 4(x+h) - 5 - f(x)}{h}$$

$$\frac{3x^2 + 6xh + 3h^2 + 4x + 4h - 5 - f(x)}{h}$$

$3x^2 + 6x + 3h + 4x + 4 - 5 - f(x)$

$\cancel{3x^2} + 3h + 10x - 1 - \cancel{3x^2} - 4x + 5$ [redacted]

$\boxed{3h + 6x + 4}$

[redacted]

Integrating Equations Graphically

* When the question generally asks for the solution most of the time they mean X.

Solve Equations Graphically

* Vertex Form

$\boxed{\text{Slope}}$

$f(-\frac{b}{2a})$

$$y=(x-3)^2+5$$

only affects $\boxed{\text{x-coordinate}}$ $\boxed{\text{y-coordinate}}$

1 Slope Opposite

of either the minimum or maximum

$$-2|x-3|+4 = \left|\frac{x}{-2}-3\right|+4$$

when finding the y of an absolute value you treat || as ( ) EX: $-2(x-3)+4$

when graphing not and I repeat not when solving for Y.

Evaluate Composition Functions

$g(y) = \frac{y-15}{y}$ what's $h(g(5))$

$h(y) = -3y$

$\frac{5-15}{5} = -\frac{10}{5} = -2$

$-3(-2) = 6$

$(f \circ g)(-11) = f(g(-11))$

Evaluate Composite Function: graph & tables

$(I^1 \to O^2) \Rightarrow (I^2 \to O^2)$

Find Composite functions

$2(\sqrt[3]{12-5x})^3 + 8$

$2(12-5x)+8$

$24-10x+8$

$32-10x$

1)

$$\sqrt[3]{(8x^3+5)-5}$$

$$\sqrt[3]{8x^3}$$

$$2x$$

2)

$$15-4(\tfrac{3}{2}x+8)$$

$$15-6x-32$$

3)

$$\frac{(9+4x)+21}{5}$$

$$\frac{30+4x}{5}$$

4)

$$(\sqrt[3]{2x-15})^3-6$$

$$2x-15-6$$

$$2x-21$$

Model with Composite Functions

Involves: $I^1 \to D^1 = I^2 \to D^2$

This can be a function of this now

Ex: $D^2(D^1(I^1))$

Verify Inverse functions

* you verify if a number is an inverse by multiplying the number to its supposed inverse and see if it's 1.

☆ use composite functions to see if your given x.

$\cancel{f(g(x))}$
$f(g(x))$

$3(\frac{1}{3}x^2-9)^2+9$

$3(\frac{1}{3}x^2-9)(\frac{1}{3}x^2-9)+9$

$3(\frac{1}{9}x^4-3x^2-3x^2+81)+9$

$\cancel{(\frac{1}{3}x^4-9x^2-9x^2+81)}$

$3(\frac{1}{9}x^4-6x^2+81)+9$

$(\frac{1}{3}x^4-18x^2+243)+9$

$\boxed{\frac{1}{3}x^4-18x^2+252}$

$g(f(x))$

$\frac{1}{3}(3x^2+9)^2-9$

$\frac{1}{3}(3x^2+9)(3x^2+9)-9$

$\frac{1}{3}(9x^4+27x^2+27x^2+81)-9$

$\frac{1}{3}(9x^4+54x^2+81)-9$

$(3x^4+18x^2+27)-9$

$\boxed{3x^4+18x^2+18}$

$g(h(x))$

$\left(2\left(\frac{\sqrt[3]{x}+5}{2}\right)-5\right)^3$

$((\sqrt[3]{x}+5)-5)^3$

x

$h(g(x))$

$\frac{\sqrt[3]{(2x-5)^3}+5}{2}$

x

$f(h(x))$

$$\left(\frac{\sqrt[3]{8x}}{8}\right)^3$$

$$\frac{8x}{512}$$

$$\boxed{\frac{x}{64}}$$

$h(f(x))$

$$\sqrt[3]{8\left(\frac{x^3}{8}\right)}$$

$$2\left(\frac{x}{8}\right)$$

$$\frac{2x}{8}$$

$$\boxed{\frac{x}{4}}$$

$f(g(x))$

$$\frac{2\left(\frac{4x+5}{2}\right)-5}{4}$$

$$\frac{2\left(2x+\frac{5}{2}\right)-5}{4}$$

$$\frac{4x+5-5}{4}$$

$$x$$

$g(f(x))$

$$\frac{4\left(\frac{2x-5}{4}\right)+5}{2}$$

$$\cancel{\frac{4\left(x-\frac{5}{4}\right)+5}{2}}$$

$$\cancel{\frac{(4x-5)+5}{2}}$$

$$2x$$

Determining If A Function Is Invertible

An Invertible function is a function you can find the inverse mapping with.

(☆ What does it mean to be a function.)

your input has only one output. your inputs could share the same out put. However, your outputs can't share the same input.

Restrict Domains of Functions to make them Invertible

★ Just make sure the (range or f(x)/y) doesn't repeat.

Vectors

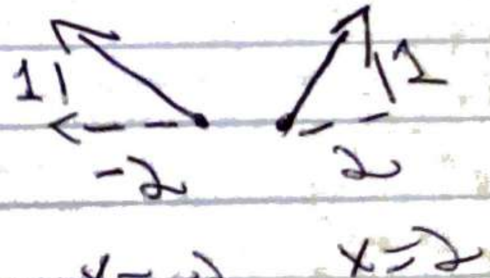
Equivalent Vectors

Now were focusing on directions and magnitudes

Magnitude: $A^2+b^2=c^2 \to \boxed{\sqrt{A^2+b^2}}$

Components of Vectors

components are distance x or y in a particular direction



(-3, -9)

$x=-2$ $y=1$

$x=2$ $y=1$

3^2+9^2 $9+81$ $\sqrt{100}$

Magnitude of Vectors

Simple $M = \sqrt{|a|^2 + |b|^2}$

Scalar Multiplication

Simple Multiply Vector

Analyze Scalar Multiplication

$\vec{V} = (\overset{+}{x}, \overset{+}{y})$ $||V|| = 4$

Shows the current of direction

$(-2x, -2y)$, So $||W|| = 8$

Similar to factoring

$|-2||(4)| = ||\vec{w}||$

If its different sign different direction

$-2(x, y)$

$(|-2|)(|4|) = 8$

$8 = ||\vec{w}||$

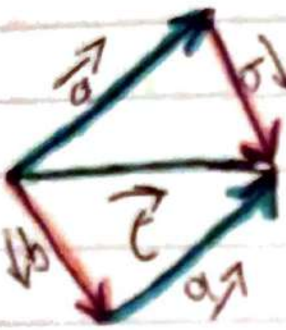
Add & Subtract Vectors

Simple

Graphically Add & Subtract Vectors

$\vec{a} + \vec{b} = \vec{c}$

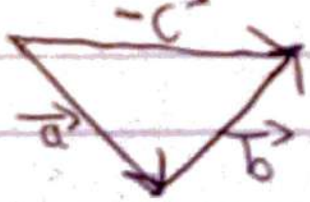
when adding 2 vectors the tail of one is at the head of the other



as long as it's the same direction and magnitude you can reorient vector

$\vec{a} - \vec{b} = \vec{d}$

The Absolute Value of any expression is never negative. So, no solution exists.



$\vec{a} + \vec{b} = -\vec{c}$

when a vector goes in a circle change it.

you can rewrite a vector as long as you get same vector result ~~as long~~

* Depending on question (Maybe just Khan Academy) can be rewritten as positive or neg

Combined Vector Operations

$\vec{u} = (-6,3)$

$\vec{w} = (9,-5)$

$\frac{1}{3}\vec{u} + 2\vec{w} = (16,-9)$

$(-2,1)+(18,-10)=$

$(16,-9)$

Practice

Unit Vectors

Unit Vector has Magnitude of 1

$\vec{a} = (3,4)$

$||\vec{a}|| = \sqrt{3^2+4^2}$

$\sqrt{25}$

5

Magnitude is this length

unit

$\hat{u} = \left(\frac{3}{||\vec{a}||}, \frac{4}{||\vec{a}||}\right)$

uc , cause not.

Direction of Vectors

Find Direction (Angle) of Vector with: $\tan^{-1}(\frac{y}{x})$ – remember to factor in the quadrant it resides in

Vector Components from magnitude & directions

Magnitude $\sin$(Angle) = Y-component
Magnitude $\cos$(Angle) = X-component

Adding Vectors Part 1

First find components, whether X or Y and then add them together.

Vector Word Problems

$\vec{a} + \vec{d} = \vec{p}$

★ $a+b=c$ – important $\vec{a} - \vec{p} = -\vec{d}$

(~~20~~, ~~15~~) (15, 19) ★ $\hat{i} = x$ $\hat{j} = y$

(20, 15) look at Vector diagram Ex: $30\hat{i} + 2\hat{j}$

Matrix Introduction

☆ Pro-Tip for systems of equations

R x C = R x C

must be the same to multiply

New matrix Dimension

Always line variable constants accordingly

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 3 & 2 & 8 \\ 1 & 8 & 9 \\ 5 & 6 & 3 \end{bmatrix} \end{array} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

constant variable product

Matrix Dimensions

$$\begin{bmatrix} 3 & 3 \\ -6 & -6 \end{bmatrix} = \underset{\text{row}}{2} \times \underset{\text{column}}{2}$$

Matrix Elements

$$0 = A^{\text{element}}_{3,4}$$

$$\begin{array}{c|c} & \begin{array}{cccc} & & & 4 \end{array} \\ \hline \begin{array}{c} \\ \\ 3 \\ \\ \end{array} & \begin{bmatrix} 10 & 2 & -8 & 9 \\ 3 & 6 & 1 & -4 \\ -2 & 1 & 9 & 0 \\ 8 & -3 & -6 & \end{bmatrix} \end{array}$$

This is the element

Represent linear Systems with matrices

$4x + 7y = 15$
$6x - 8y = 4$

$$\begin{bmatrix} 4 & 7 \\ 6 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ 4 \end{bmatrix}$$

Matrix Row Operations

Switch rows
$\longleftrightarrow$

$$\begin{bmatrix} 2 & 1 & 2 & 0 \\ -4 & 4 & -7 & -1 \\ 0 & 9 & 1 & 9 \end{bmatrix} \quad 3R_1 \rightarrow R_1 \quad = \begin{bmatrix} 6 & 3 & 6 & 0 \\ -4 & 4 & -7 & -1 \\ 0 & 9 & 1 & 9 \end{bmatrix}$$

into this row

Result

Add & Subtract Matrices

$$\begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 0 & -2 \end{bmatrix}$$

Multiply Matrices by Scalar

$$4 \cdot \begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} -4 & -8 \\ -8 & -4 \end{bmatrix}$$

Multiply Matrices

multiply and add

$$E \begin{bmatrix} 0 & 3 & 5 \\ 5 & 5 & 2 \end{bmatrix} \quad D \begin{bmatrix} 3 & 4 \\ 3 & -2 \\ 4 & -2 \end{bmatrix} \quad \begin{matrix} 29 & -16 \\ 38 & 6 \end{matrix}$$

2×3 $\quad$ 3×2

Transforming Vectors using Matrices

Just Matrix multiplication

Transform Polygons using matrices

Just plot points in matrix and multiply

See them as two separate vectors with x & y

$\begin{array}{c} x \\ y \end{array}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ standard matrix for dialation

How to Find determinant of 2X2 Matrix

L - R = Determinant

$D\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = (1)(4)-(2)(3)$

$= 4-6$

$= -2$

if the determinant is 0 then there is no invertability or inverse & it's [illegible]

Determining Inverse Matrices

Matrices that are inverse when multiplied look like $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Series: Recursive formulas, Arithmatic Sequence

$D(1)=9$

$9, -1, -11, -21$

$D(n)=D(n-1)+(-10)$

Arithmatic Sequence formulas

$d(1)=13$

n = term

$d(n)$ = term result

$d(n)=d(n-1)+17$

Ex: $d(2)=13+17$

$d(2)=13+$ [illegible] $+17$

$d(2)=$ [illegible]

$d(3)=29+$ [illegible] $+17$

$d(3)=$ [illegible]

$d(4)=45+$ [illegible] $+17$

Extend Arithmetic Sequences

$-6, -3, 0, 3, \underline{6}$

Example: Explained

$-2-12(n-1)$

1 change

$\frac{32}{128} \times 100$

Extend geometric Sequences

128, 32, 8, 2

Geometric Sequence Formulas

$c(1) = \frac{3}{16}$

$c(n) = c(n-1) \cdot 4$ or $d(n) = 3(-2)^{n-1}$

Sum of Geometric Series

First term

$$\boxed{S_n = \frac{a(1-r^n)}{1-r}}$$

$$15624 = \frac{a(1-5^6)}{(1-5)}$$

$$15624 = \frac{a(-15624)}{-4}$$

Summation notation intro

Sum

2 numbers

$$\sum_{j=1}^{2}(5j)$$

$5(1) + 5(2) = 15$

81 numbers

$$\sum_{j=0}^{80}$$

change

Multiply terms − 1

Sub ~~add~~ first term

Arithmatic Series

first term

last term

~~terms after first term × rate, subtracted from first term~~

$$S_n = \frac{(a_1 + a_n)}{2} \cdot n \text{ - terms}$$

change × terms − 1 + first term

Conic Sections:

Distance/Radius

$$= \sqrt{(x^1 - x^2)^2 + (y^1 - y^2)^2}$$

(x, y) = Center

Graph a Circle from its features

Features of a Circle from its Standard equation

$$(x-h)^2 + (y+k)^2 = R^2$$

when there is nothing it's zero

Features of a Circle from its expanded form

$$x^2 + y^2 + 12x + 4y = -15$$

$$(x^2 + 12x + 36) + (y^2 + 4y + 4) = -15 + 36 + 4$$

$$x^2+4x+y^2-18y=-81+4+81$$

$$(x^2+4x+4)+(y^2-18y+81)$$

$$(x+2)^2+(y-9)^2=\sqrt{4}$$

$$2$$

$$x^2+y^2-2y-99=0$$

$$x^2+y^2-2y=99$$

$$x^2+(y^2-2y+1)=99+1$$

$$(x)^2+(y-1)^2=100$$

$$x^2+6x+y^2+2y=6+9+1$$

$$(x^2+6x+9)+(y^2+2y+1)$$

$$(x+3)^2+(y+1)^2=\sqrt{16}$$

$x^2-6x \quad y^2-6y=-2+9++9$

$(x^2-6x+9)+(y^2-6y+9)=16$

$(x-3)^2+(y-3)^2=16$

$x^2-4x+y^2-12y=9+4+36$

$(x^2-4x+4)+(y^2-12y+36)=49$

$(x-2)^2+(y-6)^2=49$

$x^2+y^2+4y=60+4$

$x^2-14x \quad y^2+2y=-41+49+1$

$7 \qquad -1 \qquad 3$

$x^2-4x \quad y^2+8y=5+4+16$

$2 \qquad -4 \qquad 5$

$x^2+4x \qquad y^2+8y=-16+4+16$

$-2 \qquad -4 \qquad 2$

$x^2+6x \quad y^2-10y=-18+9+25$

-3 5 4

$\cancel{x}$
$\cancel{y}$

$y \quad x^2-2x=80+1$

$0 \quad (x-1)^2=81$

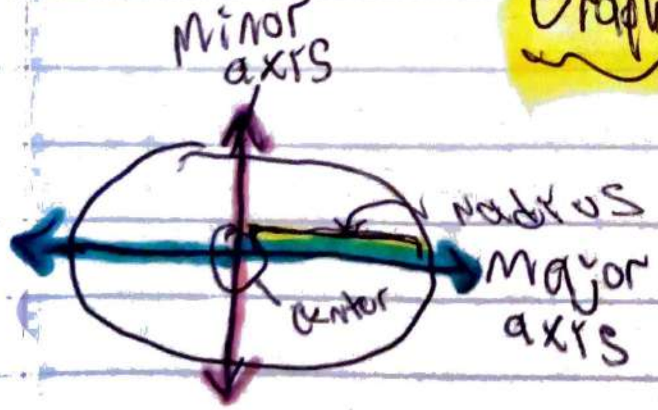
1 9

$x^2+8x \quad y^2-2y=19+1+16$

$(x+4) \quad (y-1) \quad 36$

-4 +1 6

Graph & Features of Ellipses



center

$$\frac{(x-8)^2}{16}+\frac{y^2}{64}=1$$

minor axis

major axis

Foci of an ellipse from radii

on major axis $\pm f^2 = p^2 - q^2$

Foci — Major axis — Minor axis

Foci lies on major axis and is always placed on the left and right.

$-4, 7$

$(-16, 7)\ (8, 7)$

Equation of an Ellipse from features

$\sqrt{37}\ |\ \sqrt{12}\ \sqrt{37}$

$\sqrt{12}^2 = \sqrt{37}^2 + q^2$

$\sqrt{q^2} = \sqrt{\sqrt{37}^2 - \sqrt{12}^2}$

$q = \sqrt{37 - 12}$

$q = \sqrt{25}$

$q = 5$

$\frac{(x \quad)^2}{37} + \frac{(y \quad)^2}{25} = 1$

$\frac{x^2}{2} + \frac{y^2}{25} = 1$

$3^2 = 2^2 - q^2$

$\sqrt{3^2} = \sqrt{2^2 - q^2}$

$3 = q - 2 \quad 3 = 2 - q \quad +q$

$q = -1 \quad q = 2 - 3$

$$\frac{x^2}{20}+\frac{y^2}{25}=1$$

$$15^2 = 25^2 - q^2$$

$$225 = 625 - q^2$$

$$q^2 = 625 - 225$$

$$\sqrt{q^2} = \sqrt{400}$$

$$q = 20$$

$$\sqrt{55}^2 = \sqrt{89}^2 - q^2$$

$$55 = 89 - q^2$$

$$q^2 = 89 - 55$$

$$\sqrt{q^2} = \sqrt{34}$$

$$q = \sqrt{3}$$

$$12^2 = 13^2 - q^2$$

$$q^2 = 169 - 144$$

$$q^2 = 25$$

$$3^2 = p^2 - 4^2$$

$$3^2 + 4^2 = p^2$$

$$9 + 16 = p^2$$

$$25 = p^2$$

$$\sqrt{15}^2 = p^2 - \sqrt{21}^2$$

$$15 + 21 = p^2$$

$$36 =$$

The distance between focus point and parabola is equavalent to the distance of any point on parabola and directrix

Solve variable that the directrix represents. [redacted] Also, only factor out that variable to find your parabola equation.

$$(x-3)^2 = (x+2)^2 + (y-5)^2$$

$$\cancel{x^2} - 6x + 9 = \cancel{x^2} + 4x + 4 + (y-5)^2$$

$$-4x \qquad\qquad -4x$$

$$-10x + 9 = 4 + (y-5)^2$$

$$-9 \qquad -9$$

$$\frac{-10x}{-10} = \frac{-5 + (y-5)^2}{-10}$$

$$(y-5)^2 \quad \frac{-5}{-10}$$

$$x = \frac{(y-5)^2}{-10} + \frac{1}{2}$$

$$(y+2)^2 = (x-1)^2 + (y-2)^2$$

$$\cancel{y^2} + 4y + \cancel{4} = (x-1)^2 + \cancel{y^2} - 4y + \cancel{4}$$

$$+4y \qquad\qquad\qquad +4y$$

$$8y + \cancel{4} =$$

$$\frac{8y}{8} = \frac{(x-1)^2}{8}$$

$$\boxed{y = \frac{(x-1)^2}{8}}$$

$$(y+4)^2 = (x-9)^2 + (\cancel{y})$$

$$\cancel{y^2} + 8y + 16 = \text{[illegible]} + (y)^2$$

$$\frac{8y}{8} = \frac{(x-9)^2 - 16}{8}$$

$$\frac{(x-9)^2}{8} - 2$$

$$(x-5)^2 = (x-7)^2 + (y+6)^2$$

$$\cancel{x^2} - 10x + 25 = \cancel{x^2} - 14x + 49 + (y+6)^2$$

$$+14x - 25 \qquad +14x - 25$$

$$\frac{4x}{4} = \frac{(y+6)^2 + 24}{4}$$

$$x = \frac{(y+6)^2}{4} + 6$$

$$(x+4)^2 = (x+7)^2 + (y+5)^2$$

$$\cancel{x^2} + 8x + 16 = \cancel{x^2} + \cancel{14x} + 49 + (y+5)^2$$

$$-14x - 16 \qquad\qquad -\cancel{14x} - 16$$

$$\frac{-6x}{-6} = \frac{33 + (y+5)^2}{-6}$$

$$(y-6)^2 = (x-1)^2 + (y-2)^2$$

$$\cancel{y^2} - 12y + 36 = (x-1)^2 + \cancel{y^2} - 4y + 4$$

$$+4y \quad -36 \qquad\qquad +4y \quad -36$$

$$\frac{-8y}{-8} = \frac{(x-1)^2 - 32}{-8}$$

$$y = \frac{(x-1)^2}{-8} + 4$$

$$(x-1)^2 = (x-6)^2 + (y-4)^2$$

$$\cancel{x^2} - 2x + 1 = (\cancel{x^2} - 12x + 36) + (y-4)^2$$

$$+12x - 1 \qquad\qquad +12x \quad -1$$

$$\frac{10x}{10} = \frac{35 + (y-4)^2}{10}$$

$$(y+4)^2 = (x-9) + (\cancel{y^2})$$

$$\cancel{y^2} + 8y + 16 = x^2 - 18x + 81 +$$

$$\underset{-16}{8y+16} = \underset{-16}{(x-9)-16}$$

$$\frac{8y}{8} = \frac{(x-9)^2-16}{8}$$

$$y = \frac{(x-9)^2}{8} - 2$$

$$(x-8)^2 = (x-2)^2 + (y-2)^2$$

$$\underset{+4x\ -64}{\cancel{x^2} - 8x + 64} = \underset{+4x\ -64}{\cancel{x^2} - 4x + 4} + (y-2)^2$$

$$\frac{-4x}{-4} = \frac{-60 + (y-2)^2}{-4}$$

$$x = 15 + \frac{(y-2)}{-4}$$

Vertices & direction of a hyperbola

$$\frac{y^2}{9} - \frac{x^2}{4} = 1$$

$$\frac{}{4}$$

$x = 0$

$\frac{y^2}{9} = 1$

$y^2 = 9$

$y = \pm 3$

Positive term shows you how the parabola will open.

$98 = 61 + q^2$

$\sqrt{37} = \sqrt{q^2}$

Finding Foci of Hyperbola

$$\boxed{f^2 = 36 + 64}$$

$$\frac{x^2}{625} - \frac{y^2}{3600} = 1$$

$$\frac{x^2}{33} - \frac{y^2}{26} = 1$$

5

$$\frac{x^2}{5625} - \frac{y^2}{1600} = 1$$

Probability and Combinatorics

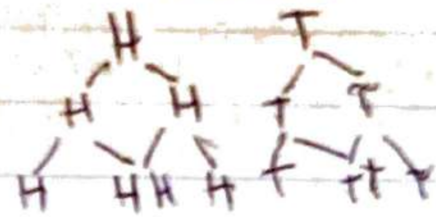
Simple Probability

Fair Coin = <u>50%</u>

Easy

Probabilities of Compound Events

$\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2}$ $\quad 5 \times 4 = \boxed{20}$



$\frac{1}{5} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{5} \quad \frac{1}{400}$

$\frac{1}{8}$

Probability = $\dfrac{\text{Favorable combination}}{\text{total possible combinations}}$

	Clay	plastic	metal	wood
tulip	Ct	Pt	Mt	Wt
fern	CF	Pf	MF	WF
Cactus	CC	PC	MC	WC
Ficus	CF	PF	MF	WF

B T C

BT CB B

T

C

Permutations of Letters

$$\frac{\text{Total leters (!)}}{\text{repeating letters (!)}}$$

Between numbers
1) round when divided and add
2) Divide result by multiple of digits and that's your answer

$$\boxed{{}_nP_k = \frac{n!}{(n-k)!}}$$ — permutation formula

Multiply 2 num then ÷ 98 then − from 1

$$\boxed{{}_nC_k = \frac{\frac{n!}{(n-k)!}}{\text{\# of ways to arrange k things in k spots}}}$$

$$\boxed{\frac{n!}{k!(n-k)!}}$$

$$\frac{k}{1}\frac{(k-1)}{2}\frac{(k-2)}{3}\cdots\frac{1}{k} = (k!)$$

Total group
Total group
Can pick
Times
~~Flip Each~~

~~Total outcomes~~

Flip 6 times
has 2 tails

B T C

BB TB CB B

BT TT CT T

BC TC CC C

35

1 2 3 4 5 6

1 2 3 4 5 6

$2^4 = 1296$