Christian G Bardan

<u>Calc 2</u>

- more area oriented
- Volume with integral
- Other integral techniques
- Sequence/Series

<u>Calc 1 Review</u>

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

I. If $f(x) = \int_a^x g(t)\,dt$, - Derivative relationship
Then $F'(x) = g(x)$ (you get function back)

II $\int_a^b f(x)\,dx = F(b) - F(a)$, when $F' = f$

<u>examples of computing integrals</u>

$$\int_0^2 (2x-7)\,dx =$$

$$\frac{2x^2}{2} - \cancel{\ } 7x$$

$$\int \sin x\,dx$$

$$-\cos x + C,\ C \in \mathbb{R}$$

§ 5.5 The Substitution Rule §

"Chain Rule in Reverse"

Chain Rule ex

$$\frac{d}{dx}(f(g(x)) = f'(g(x)) * g'(x)$$

ex: $h(x) = \sqrt{2x+1}$

$$(2x+1)^{1/2}$$

$$h'(x) = \frac{1}{2}(2x+1)^{-1/2} * \left(\frac{d}{dx}(2x+1)\right)$$

$$= (2x+1)^{-\frac{1}{2}}$$

Substitution Ex

$u = 1+x^2$ $\quad \int 2x\sqrt{1+x^2}\,dx$

$\frac{du}{dx} = 2x$ $\quad \int \sqrt{u}\,du$

$du = 2x\,dx$ $\quad \boxed{\frac{2}{3}(1+x^2)^{3/2}+C}$ $\quad \boxed{\frac{2u^{1.5}}{3}+C}$

$$\boxed{\int_0^4 \sqrt{\cos + T}\, dx}$$

$9^{3/2} = \sqrt{9}^3 = \sqrt{\ }$

$\int e^{7x}\, dx$

$u = 7x$

$\frac{du}{7} = \frac{7dx}{7}$

$\int e^u \frac{du}{7}$

$\frac{1}{7}\int e^u$

$\frac{1}{7} e^u + C$

$\frac{1}{7} e^{7x} + C$

$\int \tan x\, dx$

$u = x$

$\int \tan u \frac{du}{x} + C$

$\frac{du}{x} = \frac{x\,dx}{x}$

$$\boxed{\frac{1}{x} \sec x^2 + C}$$

$\int_1^2 \frac{dx}{(3-5x)^2}$

$\frac{dx}{(u)^2} + C$

$u = 3-5x$

d

$\int_0^4 \sqrt{2x+1}\, dx$

$\int \frac{\sin x}{\cos x} dx$

$\frac{du}{dx} = -\sin x$

$du = -\sin x\, dx$

$-du = \sin x\, dx$

$= \int \sin x \cdot \frac{1}{\cos x} dx$

$= -\int u^{-1} du$

$\ln(u) = u'$

$= -\ln|\cos x| + C$

$$\boxed{-(\ln|u|) + C}$$

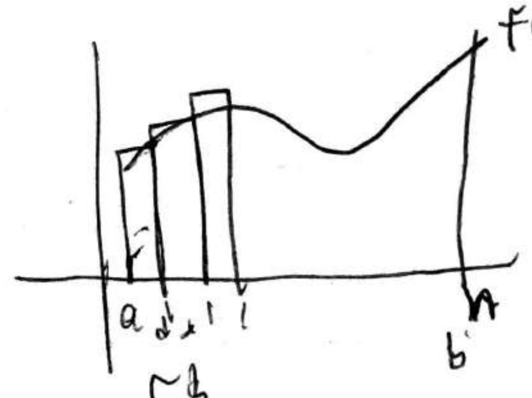
$= \ln(|\cos x|^{-1})$

$$\ln|x| = \begin{cases} \ln x & \text{if } x>0 \\ \ln(-x) & \text{if } x<0 \end{cases}$$

$$\frac{d}{dx}(\ln|x|) = \left\{ \begin{array}{ll} \frac{1}{x} & \text{if } x>0 \\ \frac{-1}{x} & \text{if } x<0 \end{array} \right\}$$

$$= \frac{1}{x} \quad x \neq 0$$

$$\int \tan x\,dx$$



$$\int_1^2 \frac{dx}{(3-5x)^2} = \frac{1}{14}$$

$$\int_0^4 \sqrt{2x+1}\,dx = \frac{26}{3}$$

$$\int_a^b f(x)\,dx \approx$$

As $dx \to 0$

better approximation

write an C cause
it could be killed

$$\int_0^4 \sqrt{2x+1}\,dx$$

$$\int_0^4 \sqrt{2x+1}\,dx = \frac{1}{2}\int_1^9 u^{\frac{1}{2}}\,du =$$

$$\frac{1}{2}\left(\frac{2}{3}u^{\frac{3}{2}}\right|_1^9$$

$$= \frac{1}{3}\left(u^{\frac{3}{2}}\right|_1^9$$

$$= \frac{1}{3}\left(9^{\frac{3}{2}} - 1^{\frac{3}{2}}\right) = \boxed{\frac{26}{3}}$$

$u = 2x+1$

$u(0) = 1$

$du = 2dx$ $\quad u(4) = 9$

$dx = \frac{1}{2}du$

$\int x^n dx$ $\qquad$ $\int \cancel{x}^n = \frac{x^{n+1}}{n+1}$

$\int_0^4 \sqrt{2x+1}\, dx =$ $\qquad$ $u = 2x+1$

$\int_0^4 \sqrt{u}\, dx$ $\qquad$ $\frac{du}{2} = \frac{\cancel{2}dx}{\cancel{2}}$

$\frac{du}{2} = dx$

$\int_0^4 \frac{\sqrt{u}\, du}{2}$

$\frac{1}{2}\int_{x=0}^{x=4} u^{\frac{1}{2}}\, du$

$\frac{1}{2}\int_0^4 \sqrt{u}\, du$

$\frac{1}{2}\left(\frac{2}{3}u^{\frac{3}{2}}\right|_{x=0}^{x=4}$

$\frac{1}{2}\int_0^4 u^{\frac{1}{2}}\, du$

$= \frac{1}{2}\left(\frac{2}{3}(2x+1)^{\frac{3}{2}}\right.$

$\frac{1}{2}\int_0^4 \frac{2u^{\frac{3}{2}}}{3}\, du$ $\qquad$ (

$= \frac{1}{3}(2x+1)^{\frac{3}{2}}$

$\frac{1}{2}\cdot\frac{2}{3}\int_0^4 u^{\frac{3}{2}}\, du$

$F(b) - F(A)$

$\frac{1}{3}(2x+1)^{\frac{3}{2}}$ $\qquad$ $\left(\frac{1}{3}(2(4)+1)^{\frac{3}{2}}\right) - \left(\frac{1}{3}(2(0)+1\right.$

$$-\frac{1}{5}\int_{-2}^{-7} u^{-2}\,du$$

$$\int_1^2 \frac{dx}{(3-5x)^2}$$

$$\int_1^2 \frac{1}{(3-5x)^2}\,dx$$

$u = 3-5x$

$$\int_1^2 \frac{1}{(u)^2}\,\frac{du}{-5}$$

$u' = -5$

$du = -5\,dx$

$$-\frac{1}{5}\int_1^2 u^{-2}\,du$$

$\frac{du}{-5} = dx$

$$-\frac{1}{5}\int_1^2 -u^{-1}\,du$$

$= \frac{1}{5}\left(-\frac{1}{7}\right.$

$$-\frac{1}{5}\left(-(3-5x)^{-1}\right)$$

$$-\frac{1}{5}(-3+5x)^{-1}$$

ex: $\int_1^6 \frac{(\ln x)^3}{x} dx$

$u = x$

$u' = 1$

ex: $\int \frac{2^x}{2^x+3} dx$

$\frac{du}{1} = \frac{1\,dx}{1}$

$\frac{1}{x} \times \ln x$

$\int_1^e \frac{(\ln x)^3}{u} du$

$\int \frac{2^x}{2^x+3}$

$\int_1^6 (\ln x)^3 \cdot \frac{1}{u} du$

$\int 2x \cdot \frac{1}{2^x+3}$

$\int_1^6 (\ln x)^3 \cdot \ln u$

$\frac{d}{dx}(a^x) = a^x \ln(a)$

$\int_1^e (\ln x)^3 \cdot \ln x$

$$\int_0^{\frac{\pi}{2}} \cos x \cdot \sin(\sin x)\,dx = 1-\cos 1$$

$$\int_1^{e} (\ln x)^3 \cdot \frac{1}{x}\,dx$$

let $u = \ln x$

$du = \frac{1}{x}\,dx$

$\underline{u(1) = 0}$

$\underline{u(e) = 1}$

$\ln_e 1 = 0$

$\log_a 1 = 0$

$\ln e = 1$

$\int_0^1 u^3\,du$

$(\frac{1}{4}u^4 \Big|_0^1$

$\frac{1}{4}(u^4 \Big|_0^1$

$\frac{1}{4}(1-0)$

$= \frac{1}{4}$

$\frac{d}{dx}(a^x) = (\ln a)(a^x)$

$u = 2^x + 3$

$du = (\ln 2)(2^x)\,dx$

$\frac{1}{\ln 2}\,du = 2^x\,dx$

$\int \frac{2^x}{2^x+3}\,dx =$

$\frac{1}{\ln 2}\int u^{-1}\,du$

$= \frac{1}{\ln 2}\ln|u| + C$

$= \frac{\ln|u|}{\ln 2} + C$

$= \frac{\ln|2^x+3| + C}{\ln 2}$

$$\sum_{n=1}^{5} n = 1+2+3+4+5 = 15$$

$$\sum_{n=1}^{100} n$$

$$\sum_{i=1}^{n} i = 1+2+3 \dots + n = \frac{n(n+1)}{2}$$

$$S = 1+2+3+\dots+(n-1)+n$$

$$S = n+(n-1)+(n-2)\dots 2+1$$

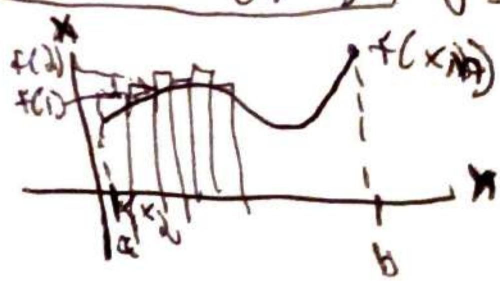
$$2S = (n+1)+(n+1)+(n+1)+\dots(n+1)+(n+1)$$

$$2S = n(n+1)$$

$$S = \frac{n(n+1)}{2}$$

Gauss

$y = f$



$$A = \int_a^b f(x)\,dx$$

$$\lim_{n \to 100} \sum_{n=1}^{n} f(x_i)\,dx$$

Also true area is found (Area = A)

As $n \to \infty$, $dx \to 0$

Distance is going to zero

Approximate Area:

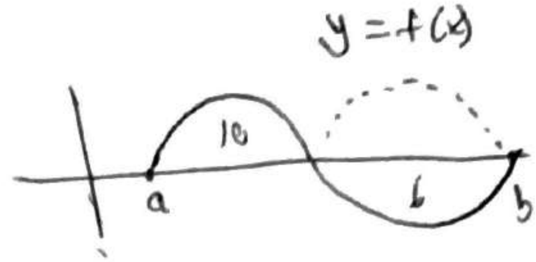
$$f(x_1)dx + f(x_2)dx + \dots f(x_n)d$$

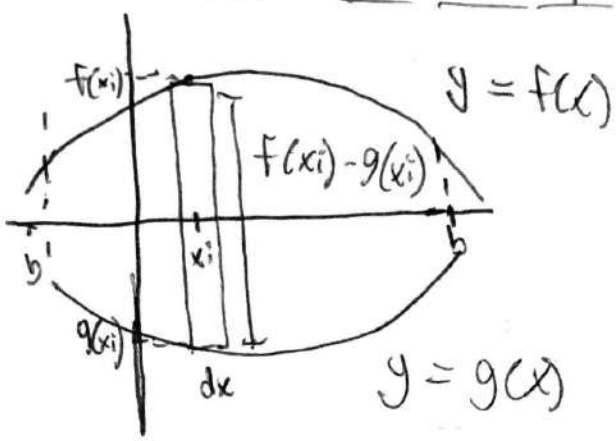
$$\sum_{n=1}^{n} f(x_n)\,dx$$

6.1) Area between 2 curves

$$\int_a^b f(x)\,dx = 10 + (-6) = 4$$

$$\int_a^b |f(x)|\,dx = 16$$





$d(a,b) = |b-a|$

Approximate area:

$(f(x_i) - g(x_i))\,dx$

$$\sum_{i=1}^{n} (f(x_i) - g(x_i))\,dx$$

The area between the curves $y = f(x)$ and $y = g(x)$, with $f(x) \geq g(x)$, from $x = a$ to $x = b$

$$\int_a^b (f(x) - g(x))\,dx$$

Exact Area: $\lim_{n \to \infty} \sum_{n=1}^{n} (f(x_i) - (g(x_i))$

if you don't know which is larger

$$\int_a^b |f(x) - g(x)|\,dx = A_1 + A_2 + A_3 = \sum_{i=1}^{3} A_i = \int_a^b (f(x) - g(x))$$

higher lower

Christian T Barden

1.1) Gaussian elimination

1.2) Vectors

Define: A vector is a list of real of real numbers

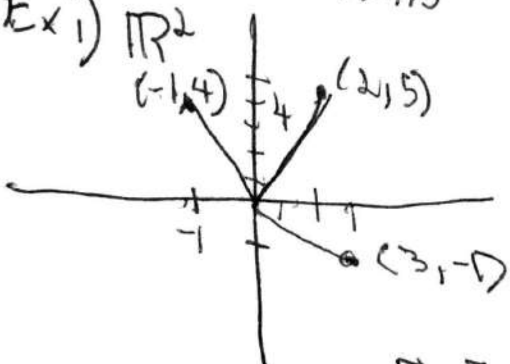
in print vector is boldface → $\bar{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$ lives in $\mathbb{R}^n$

write it vertically →

(time goes in 1 direction)

n dimensions

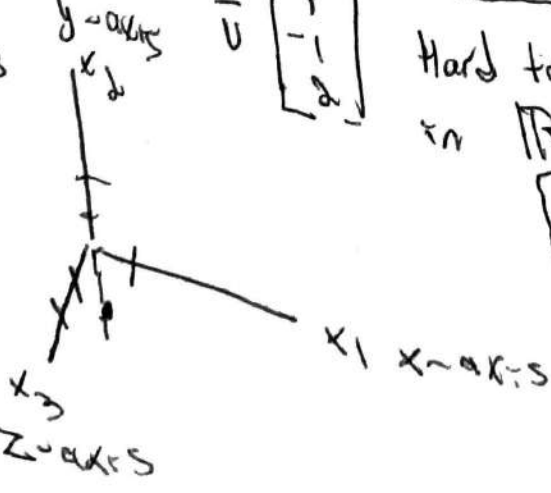
Ex 1) $\mathbb{R}^2$ $\quad \bar{u} = \begin{bmatrix} -1 \\ 4 \end{bmatrix} \quad \bar{v} = \begin{bmatrix} 2 \\ 5 \end{bmatrix} \quad w = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$



3) $\mathbb{R}^4$ $\quad \bar{v}\begin{bmatrix} 1 \\ 2 \\ 1 \\ 5 \end{bmatrix}$ point (1,2,1,5) in 4 space

Vector Addition and Scalar Multiplication

2) $\mathbb{R}^3$ $\quad \bar{u}\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ Hard to draw in $\mathbb{R}^3$



ex 1) $\bar{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \bar{v} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

diagonal of the parallelogram

*Check to see if intersection is outside interval of interest

$(15-x^2)-(2x-20)$

$15-x^2-2x+20$

$-x^2-2x+35$

$(22x+114)-(18-x^2)$

$x^2-18+22x+114$

$-5x-11=13-x^2$
$-13+x^2$

$x^2-5x-24=0$

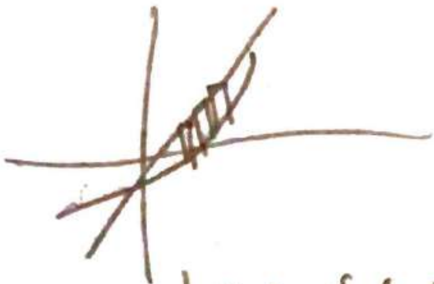
$-18+x^2+22x+114$

$$\frac{-(-5)\pm\sqrt{(5)^2-4(1)(-24)}}{2(1)}$$

$-3 \quad 8$

Christian G Barden

Find the area between the graph of $y = x^3 - x^2 + 3x$ and $y = 5x$



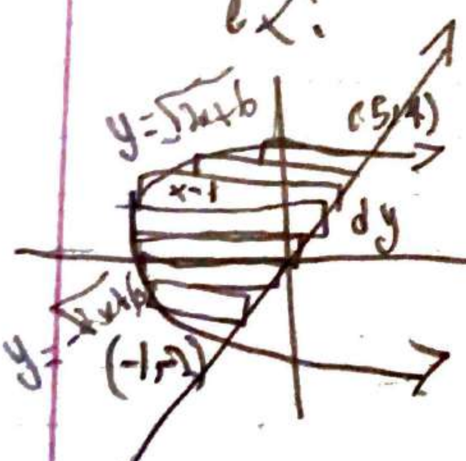
$$\int_{-1}^{2} \left| (x^3 - x^2 - 3x) - 5x \right| dx$$

terms of x $= \int_{-1}^{0} (x^3 - x^2 + 3x - 5x)\,dx + \int_{0}^{2} (5x - (x^3 - x^2 + 3x)$

$$= \int_{-1}^{0} (x^3 - x^2 - 2x)\,dx + \int_{0}^{2} (-x^3 + x^2 + 2x)\,dx$$

---

ex: Find the area enclosed by $y = x - 1$ and $y^2 = 2x + 6$



$y^2 = 2x + 6$

place in terms of y

$= \left(-\frac{1}{6} y^3 + \frac{1}{2} y^2 + 4\right.$

$= \left(-\frac{1}{6}(64) + \frac{1}{2}(16) + 16\right) -$

terms of y

$y = x - 1$

$x = y + 1$

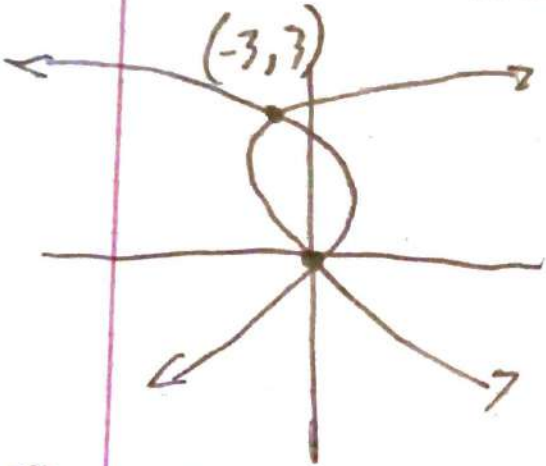
$$\int_{y=-2}^{y=4} (y+1) - \left(\frac{y^2 - 6}{2}\right) d_y$$

$$\int_{-2}^{4} \left(y + 1 - \frac{1}{2} y^2 + 3\right) dy$$

$$\int_{-2}^{4} \left(-\frac{1}{2} y^2 + y + 4\right) dy$$

Christian & Bardon

$x = y^2 - 4y$ and $x = 2y - y^2$



$x = 2y - y^2$

$2y - y^2 = y^2 - 4y$

$0 = 2y^2 - 6y$

$2y(y-3) = 0$

$$\int_0^3 (2y - y^2) - (y^2 - 4y)\, dx$$

higher x values in terms of y

$$\int_0^3 (6y - 2y^2)\, dy$$

$$\int_0^3 (3y - y^2)\, dy$$

higher y value in terms of x

$$2\left(\frac{3}{2}y^2 - \frac{1}{3}y^3\right)\Big|_0^3$$

$$2\left(\frac{27}{2} - 9\right) = 2\left(\frac{9}{2}\right) = \boxed{9}$$

§ $\boxed{6.2 \text{ \& } 6.3}$ Volumes (by disk and shell method)

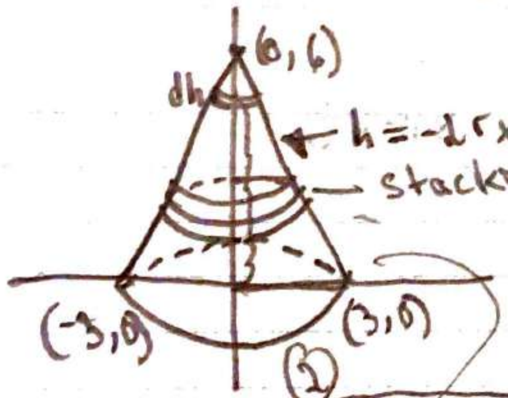
Ex: Find the volume of a cone with radius 3 and height 6

From high school, we can write

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi(9)(6) = 18\pi$$

Now, let's try using calculus:



Disk Method:

$\pi r^2 \times h$

taking many cylinders

r

use slope of line to find r

Summing these discs gives:

line joining (0,6) and (3,0) has slope

$\frac{6-0}{0-3} = -2$ and

eqn $h = -2rx + 6$

$\frac{6-h}{2} = r$

$$\int_{h=0}^{h=6} \pi r^2 \, dh \quad ① \quad \frac{\pi}{4}\int_0^6 (36h - 6h^2 + \frac{1}{3} \text{[illegible]}$$

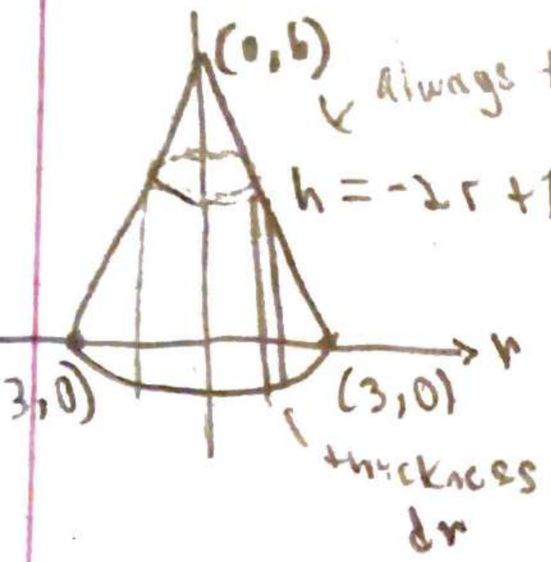
$$= \int_{h=0}^{h=6} \pi\left(\frac{6-h}{2}\right)^2 dh \qquad \frac{\pi}{4}(6^3 - \frac{6}{6} + \text{[illegible]}$$

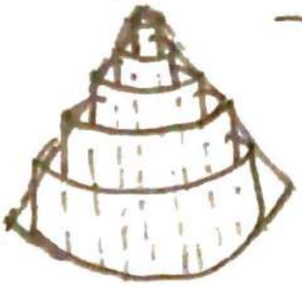
$\boxed{2^{\circ}}$

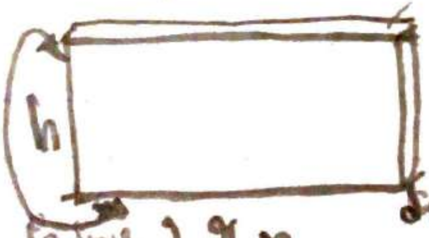
$$= \frac{\pi}{4}\int_0^6 (36 - 12h + h^2)\, dh \qquad \frac{\pi}{4}(6 \text{[illegible]}$$

Christian Bardun

Method: Shelves







~~$h = 6 - 2r$~~

volume of each shell is:

$$2\pi r \cdot h \cdot dr$$

Summing over all shells (and letting) $dr \to 0$

Shelves are already extending in negative

$$V = \int_{r=0}^{r=3} 2\pi r h \, dr$$

$$= 2\pi \int_0^3 r(-2r+6)\, dr$$

$$= 2\pi \cdot \left(-\frac{2}{3} r^3 + 3r^2 \right|_0^3$$

$$2\pi\left(-\frac{2}{3}(3)^3 + 3^3\right)$$

$$2\pi(-18+27)$$

$$= \boxed{18\pi}$$

$\int \sin(x)\,dx = -\cos x + C$

$\int \cos(x)\,dx = \sin x + C$

$\int \tan x \, dx$

$\int \frac{\sin x}{\cos x}\,dx$

$u = \cos x$

$\int \frac{\sin x}{u}\,dx$

$\frac{du}{-\sin x} = \frac{-\sin x \, dx}{\cancel{\sin x}}$

$\int \frac{\sin x}{u} \times \frac{du}{\sin x}$

$\int -\frac{1}{u}\,du$

$\int -\ln(u)\,du$

$\boxed{-\ln|\cos x| + C}$

$$\text{Volume} = \int_{h=0}^{h=6} \pi r^2 \, dh$$ constant

* your summing up all the height thus you're integrating h

$$\int_0^{90} w \cdot (-720h + 720)\, dh$$

$$\int_0^{90} (-720h + 720)^2 \, dh$$

$$\int_0^{90} w \times L \, dh \qquad \int_0^{90} 720(-h+1)^2 \, dh =$$

$$\frac{720(-h+1)^3}{3}\Big|_0^{100}$$

$$h = -\frac{90}{720} L + 90$$

$$-\frac{90}{720}$$

$$\left(\frac{720}{90}\right) h - 90 = \frac{-90}{720} L \left(\frac{720}{90}\right)$$

$$y = \frac{-90}{720} x + 90$$

$$x = -720y + 720 \qquad -\frac{(720)(h-90)}{90} = L$$

$$-720h + 720 = L \qquad \frac{(-720)(h-\cancel{90})}{\cancel{90}} = L$$

(0,90)

(720,0)

$r = x$

$h = y$

$\int_0^{90} (2r)^2 \, dh$

$y = -\frac{90}{720}x + 90 \quad \left(\frac{720}{90}\right) y - \cancel{90} = \frac{-}{720}$

-90

$\int_0^{90} (2(720y - 720))^2 \, dh$

$720y - 720 = x$

$(1440y - 1440)^2$

---

(2)

(1)

(0,100)

$\left(\frac{600}{2}, 0\right)$

$\frac{0-100}{300-0} = -\frac{100}{300}x + 100$

$\int_0^{100} 2x^2 \, dh$

(3)

$\left(\frac{300}{100}\right)$

$(-3)(y-100) = \frac{-100}{300}x$

(4)

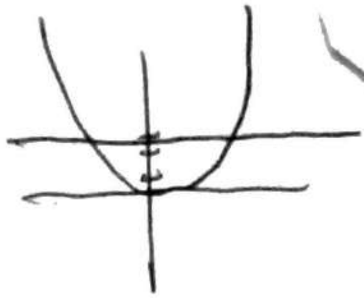
$\int_0^{90} 36(h-100)^2 \, dh$

$((-3)(y-100))^2 =$

$36(y-100)^2$

$\frac{36(h-100)^3}{3} \Big|^{100}$

$36(h-100)^2$



$y = x^2$

$y = 3$

$\int_0^3 \pi y \, dy$ $\sqrt{x^2} = \sqrt{3}$

$x = \sqrt{3}$

$\int_0^3 \pi y \, dy = \frac{\pi}{2} y^2$ πx^2

$\int_0^3 \pi (\sqrt{3})^2 \, dy$

$\int$

$x^2 - 9 = 0$

$(3, 0)$ $\frac{2y^2}{2} - 4y$

$(-3, 0)$

$y = 9 - x^2$

$\sqrt{y-9} = \sqrt{x^2}$ $2(\sqrt{y-9})^2$

$\sqrt{y-9} = x$ $\int_0^3 2y - 9 \, dy$

Quadratic Revolvining about y-axis

$$V=\int_a^b 2\pi x(f(x)-g(x))\,dx$$

Shelving Method

$$\int_0^5 2\pi x(9-x^2)\,dx$$

$$2\pi\int_0^5 x(9-x^2)\,dx$$

$$2\pi\int_0^5 (9x-x^3)\,dx$$

$$2\pi\int_0^5\left(\frac{9x^2}{2}-\frac{x^4}{4}\right)dx$$

$$2\pi\left(\frac{9(3)^2}{2}-\frac{(3)^4}{4}\right)-(0)$$

$$2\pi\int_0^5 25x-x^3$$

$$2\pi\int_0^5\left(\frac{25x^2}{2}-\frac{x^4}{4}\right)dx$$

$$2\pi\left(\frac{25(5)^2}{2}-\frac{(5)^4}{4}\right) \quad 2\pi\int_0^5\left(\frac{25(5)^2}{2}-\frac{(5)^4}{4}\right)dx$$

Shelving just radius interval
disc takes the entire diameter

Disk Method revolved around x-axis

$$V = \int_a^b \pi [f(x)]^2 dx$$

$$\pi \int_{-5}^{5} (25 - x^2)^2$$

$$\pi \int_{-5}^{5} (25 - x^2)(25 - x^2)$$

$$625 + x^4 - 25x^2 + x^4$$

$$625 - 25x^2 - 25x^2 + x^4$$

$$625 - 50x^2 + x^4$$

$$\pi \int_{-5}^{5} (625 - 50x^2 + x^4)\, dx$$

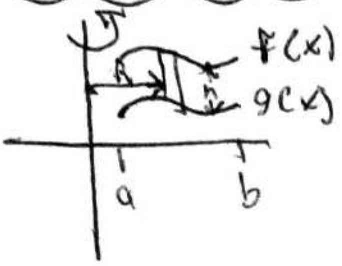
$$625x - \frac{50x^3}{3} + \frac{x^5}{5}$$

$$\left(\pi\left(625(5) - \frac{50(5)^3}{3} + \frac{(5)^5}{5}\right)\right) - \left(\pi\left(625(-5) - 5\right.\right.$$

Shell Method: $V = 2\pi \int_a^b R(x)\, h(x)\, dx$

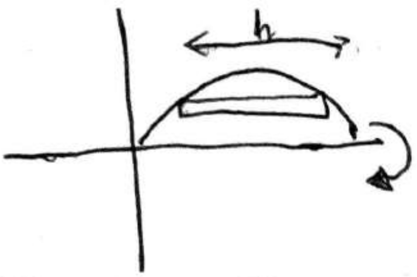
1a)

1b)



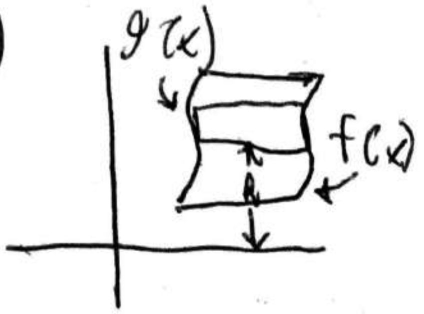
$h(x) = f(x) - g(x)$

2a)



$V = 2\pi \int_c^d R(y)\, h(y)\, dy$

2b)



$h = f(x) - g(x)$

Disk Method:

$$\int_0^6 \pi \left(\frac{6-h}{2}\right)^2 dh$$

$$\frac{\pi}{\cancel{4}} \int_0^6 (6-h)^2$$

$$\frac{\pi}{4} \int_0^6 36-(h)^2 \, dh$$

$$\frac{\pi}{4} \int_0^6 36x-\frac{(h)^3}{3} \, dh$$

Yo [redacted]
I've got
Kambucha
at the crib

Rotate y-axis sum up dy

Rotate x-axis sum up dx

$2\pi x \int (f(x)-g(x))\,dx$ - Shell

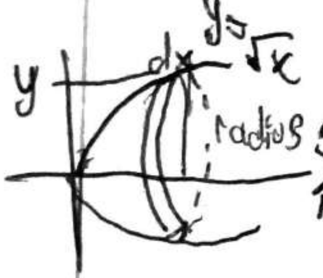
summation

$$\int_{x=0}^{x=1} A(x)\,dx$$

$\pi \int ((g(x))^2-(h(x))^2)\,dx$ — Disc

ex: Find the volume of the solid obtained by rotating about the x-axis, the region under $y=\sqrt{x}$ from $x=0$ to $x=1$

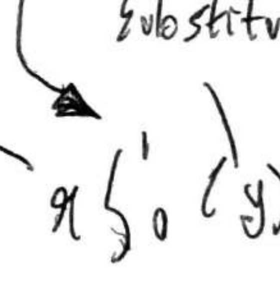
Substitute your x



$\pi\int_{x=0}^{x=1}(\sqrt{x})^2\,dx$, or $\pi\int_0^1 (y)^2\,dx$

$\pi\int_0^1 x\,dx$

Volume of each disk:



y-coordinate is your radius

$\pi\int_0^1 \frac{x^2}{2}\,dx$

$A = \pi r^2 = \pi y^2$

$\pi \frac{1^2}{2}$ $\boxed{\frac{\pi}{2}}$

$$\int_0^4 dy \qquad \sqrt{y} = x$$

$$\int_0^4 2\pi y \left(f(y) - g(y) \right) dy$$

$$\qquad\qquad \sqrt{y} \qquad\quad 0$$

$$2\pi \int_0^4 \frac{y(\sqrt{y})^2}{2} dy$$

$$2\pi \int_0^4 \frac{y^2}{2} dy$$

$$\int_{y=0}^{y=}$$

$$(y)^2 = (\sqrt{x-9})^2$$

$$\int_{x=0}^{x=3} \pi (x^2)^2 \, dx$$

$$y^2 = x - 9$$
$$+9 \qquad +9$$

$$\int_{x=0}^{x=3} \pi x^4 \, dx$$

$$\boxed{y^2 + 9 = x}$$

$$\int_{x=0}^{x=3} \pi \frac{x^5}{5} \, dx$$

$$y = \tfrac{1}{5}(x-9)$$

$$y = \tfrac{1}{5}x - \tfrac{9}{5}$$
$$+\tfrac{9}{5} \qquad +\tfrac{9}{5}$$

$$\pi \frac{(3)^5}{5} - (0)$$

$$\left(\tfrac{5}{1}\right)\left(y + \tfrac{9}{5}\right) = \tfrac{1}{5}x \left(\tfrac{5}{1}\right)$$

$$\boxed{5y + 9 = x}$$

$y = 0$

$y = 5$

$$y(y-5) = 0$$

$$y^2 + 9 = 5y + 9$$
$$-9 \qquad -9$$

$$y - 5 = 0 \qquad y = 5 \qquad y^2 - 5y = 0$$
$$+5 \quad +5$$

$3\cdot\sqrt{x-9} = \cancel{\tfrac{1}{3}}(x-9)\cdot\cancel{3}$

$(3\cdot\sqrt{x-9})^2 = (x-9)^2$ remember

$9\cdot(x-9) = x^2-81$

$9x-81 = \cancel{\quad}\ (x-9)(x-9)$

$x^2-9x-9x+81$

$9x-81 = x^2-18x+81$

$+81 \qquad\qquad +81$

$9x = x^2-18x+162$

$-9x \qquad -9x$

$\sqrt{}\ \boxed{0 = x^2-27x+162}$

$\pi\int_9^{18}(\sqrt{x-9})^2\,dx$

$\pi\int_9^{18} x-9\,dx$

$\pi\frac{81}{2}$

$\pi\int_9^{18}\left(\frac{x^2}{2}-9x\right)dx$

$\left(\frac{(18)^2}{2}-9(18)\right)-\left(\frac{(9)^2}{2}-9(9)\right)$

$$\pi \int_9^{18} \left(\frac{1}{3}(x-9)\right)^2 dx$$

$$\frac{\pi}{9} \int_9^{18} (x-9)^2 dx$$

$$\frac{\pi}{9} \int_9^{18} (x-9)(x-9)\, dx$$

$$\frac{\pi}{9} \int_9^{18} (x^2-9x-9x+81)\, dx$$

$$\frac{\pi}{9} \int_9^{18} (x^2-18x+81)\, dx$$

$$\frac{\pi}{9} \int_9^{18} \left(\frac{x^3}{3}-\frac{18x^2}{2}+81x\right) dx$$

$$3\cdot\left(\sqrt{x-9}\right) = \left(\tfrac{1}{3}(x-9)\right)\cdot 3$$

$$\left(3\cdot\left(\sqrt{x-9}\right)\right)^2 = \left((x-9)\right)^2$$

$$9\cdot(x-9) = (x-9)(x-9)$$

$$9x-81 = x^2-9x-9x+81$$

$$-9x+81 \qquad -9x \quad +81$$

$$0 = x^2-27x+162$$

18

9

$$\pi\int_9^{18}\left(x^2-27x+162\right)dx$$

$$\pi\int_9^{18}\left(\frac{x^3}{3}-\frac{27x^2}{2}+162x\right)dx$$

$$\frac{(18)^3}{3}-\frac{27(18)^2}{2}+162(18)-\left(\frac{(9)^3}{3}-\frac{27(3)^2}{2}+16(9)\right)$$

$y = x^2$, $y = 2x$, rotated around y-axis

$$x^2 = 2x$$
$$-2x \quad -2x$$

$$x^2 - 2x = 0$$

$$x = 2$$
$$x = -2$$

$$2\pi \int_{-2}^{2} x\left(2x - (x^2)\right) dx$$

$$2\pi \int_{-2}^{2} (2x^2 - x^3)\, dx$$

$$\boxed{\frac{64\pi}{3}} \checkmark$$

$$2\pi \int_{-2}^{2} \left(\frac{2x^3}{3} - \frac{x^4}{4}\right) dx$$

$$2\pi\left(\left(\frac{2(2)^3}{3} - \frac{(2)^4}{4}\right) - \left(\frac{2(-2)^3}{3} - \frac{(-2)^4}{4}\right)\right)$$

high

$y = 6 - x^2$, $y = 2$; about the x-axis

Whenever there's a whole function squared Factor

$6 - x^2 = 2$
$-2 \quad -2$
$-x^2 + 4 = 0$

$\boxed{x = 2}$
$\boxed{x = -2}$

$(6 - x^2)(6 - x^2)$

$\pi \int_{-2}^{2} ((6 - x^2)^2 - (2)^2)\,dx$

$\pi \int_{-2}^{2} (36 - x^4 - 4)\,dx$

$\pi \int_{-2}^{2} (-x^4 + 32)\,dx$

$\pi \int_{-2}^{2} \left(-\frac{x^5}{5} + 32x\right)dx$

$\pi \left(\left(-\frac{(2)^5}{5} + 32(2) \right) - \left(-\frac{(-2)^5}{5} + 32(-2) \right) \right)$

$\boxed{\dfrac{606\pi}{5}}$

1) find limit of integration

2) find highest curve

3) determine whether to use shell or disk method

4) integrate

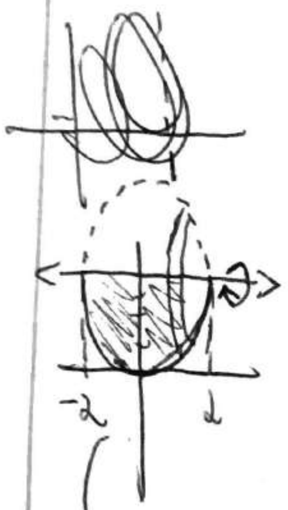
$y = x^2$, $y = 4$, rotate around $y = 4$

$\cancel{x^2-4=}$

$x^2-4=0$

$x=2$

$x=-2$



$(4-x^2)^2$

$\pi\int_{-2}^{2}((4)^2-(x^2)^2)\,dx$

semetrical

$\pi\int_{-2}^{2}(16-x^4)\,dx$

$=2$ forl

$=2\int_0^2 \pi(4-x^2)^2\,dx$

$\pi\int_{-2}^{2}\left(16x-\frac{x^5}{5}\right)dx$

$2\pi\int_0^2(16-8x^2+x^4)\,dx$

$\pi\left(\left(16(2)-\frac{(2)^5}{5}\right)-\left(16(-2)-\frac{(-2)^5}{5}\right)\right)$

$\int_0^2(16-8x^2+x^4)\,dx$

$\boxed{\cancel{\frac{256\pi}{5}}} = \frac{512\pi}{15}$

Christian G Barden

$\boxed{\text{Disc in Reference to } y}$

$y - y^2 = 0$

$y(1-y) = 0$ $\quad \boxed{y=0}$ $\boxed{y=1}$

$x = y - y^2$

$x = 0$, rotated about y

$$\pi \int_0^1 (y-y^2)^2 \, dy$$

$$\pi \int_0^1 (y-y^2)(y-y^2) \, dy$$

$$\pi \int_0^1 (y^2 - y^3 - y^3 + y^4) \, dy$$

$$\pi \int_0^1 \left(\frac{y^3}{3} - \frac{2y^4}{4} + \frac{y^5}{5}\right) dy$$

$$\pi \left(\frac{(1)^3}{3} - \frac{2(1)^4}{4} + \frac{(1)^5}{5}\right) dy$$

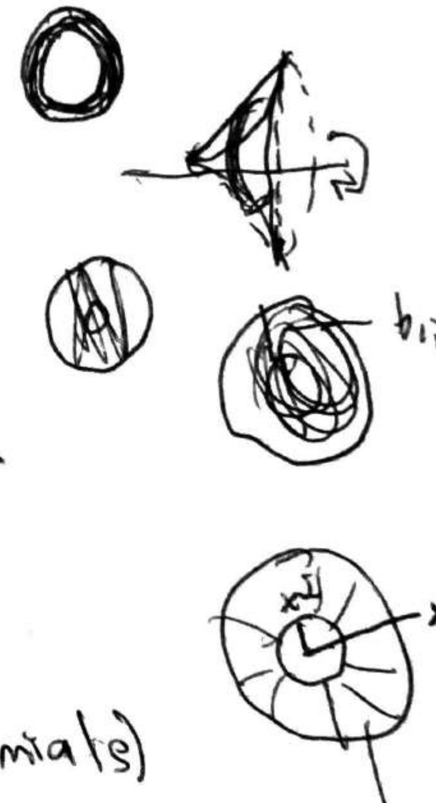
$$V = \boxed{\frac{\pi}{30}}$$

Christian T Bardon

Washers (Difference of disks)

The region enclosed by the
$y = x$ and $y = x^2$ is rotated

$$\pi \int_0^1 \left((x)^2 - (x^2)^2 \right) dx$$



$$\cancel{\pi \int_0^1 (x - x^2)^2 \, dx}$$

don't foil with washers

$$\cancel{\pi \int_0^1 (x - x^2)(x - x^2) \, dx}$$

$$A(x) = \pi x^2 - \pi (x^2)^2$$

Only foil (binomials)

$(a + b)^n$

want to fi

$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

ex: The region between $y = \sin x$ & $y = \cos x$ for $0 \leq x \leq \frac{\pi}{4}$

rotate y

$\frac{180}{}$

$2\pi x \int$

$\cos y = a$

$\cos(x+y)$

$\pi \int_0^{\frac{\pi}{4}} \left((\cos x)^2 - (\sin x)^2\right) dx$

$\pi \int_0^{\frac{\pi}{4}} (\cos^2 x - \sin^2 x)\, dx$

f even

$f(-x) = f(x)$ ex: $-2, 2$

Semetrical relative to y-axis

$\pi \int_0^{\frac{\pi}{4}} \left(\frac{\cos^3 x}{3} - \frac{\sin^3 x}{3} \right) dx$

f odd

$\pi \left(\frac{\cos^3(\frac{\pi}{4})}{3} - \frac{\sin^3(\frac{\pi}{4})}{3} \right)$

$f(-x) = -f($

$A(x) = \pi(\cos x + 1)^2 - \pi(\sin x + 1)^2$

$\sin x - (-1) = \sin x + 1$ $V = 1.04689929$

$\cos x - (-1) = \cos x + 1$

$$\cos(2x) = \cos(x+x$$

$$= \cos x \cos x - \sin x \sin x$$

$$\cos^2 x - \sin^2 x$$

$$A(x) = \pi(\cos 2x + 2\cos x - 2\sin x)$$

$$= \pi \int_0^{\frac{\pi}{4}} (\cos 2x + 2\cos x - 2\sin x)\,dx$$

$$= \pi \int_0^{\frac{\pi}{4}} \left(\frac{1}{2}\sin 2x + 2\sin x + 2\cos x\right|_0^{\frac{\pi}{4}}$$

$$2\pi\sqrt{2} - \frac{3\pi}{2}$$

ex: $\int \sec x \, dx$

$$= \int \sec x \cdot \frac{\sec x + \tan x}{\sec x + \tan x} \, dx$$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \, dx$$

$u = \sec x + \tan x$

$\frac{du}{dx} = \sec \tan x + \tan x$

$\frac{du}{dx} = \sec x \tan x + \sec^2 x$

$du = (\sec x \tan x + \sec^2 x) dx$

$= \int u^{-1} du$

$= \ln |u| + C$

$= \ln |\sec x + \tan x| + C$

Friday 10/7 @ 2:00pm

Test 2 trig integrals

$\int \csc x \, dx$

$= \int \csc x \frac{\csc x - \cot x}{\csc x - \cot x} dx$

Answer $= \ln|\csc x - \cot x| + C$

$\int \cot^n x \csc^m x \, dx$

$1 + \cot^2 x = \csc^2 x$

$= \frac{-\cos x}{\sin^2 x} = \frac{-\cos x}{\sin x} \cdot \frac{1}{\sin x}$

$= -\cot x \cdot \csc x$

$\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)$

$\boxed{\frac{d}{dx}(\sec x) = \sec x \tan x}$

$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$

$= \frac{-1}{\sin^2 x} = -\csc^2 x$

Christian Warden

radius is x, dx

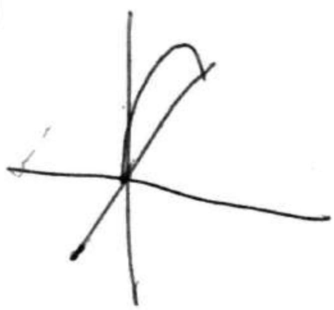
Ex: Find the volume of the solid bounded by $y=x$ and $y=4x-x^2$ about y

$4x-x^2=x$

$3x-x^2=0$

$x=0$

$x=3$



$2\pi r = 2\pi x$

$\boxed{A(x)}$

$A(x) = 2\pi x$

$$2\pi \int_0^3 x((4x-x^2)-(x))\,dx$$

$$2\pi\left(x^3-\frac{1}{4}x^4\right)$$

$$2\pi \int_0^3 ((4x^2-x^3)-(x^2))\,dx$$

$$2\pi \int_0^3 (3x^2-x^3)\,dx$$

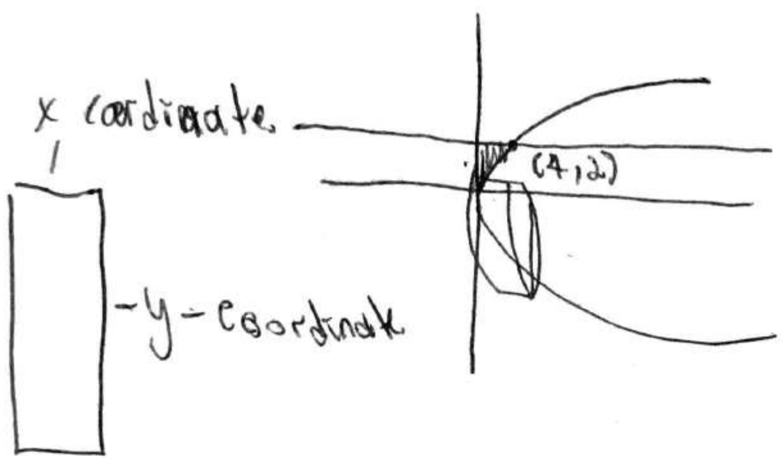
$$2\pi \int_0^3 \left(\frac{3x^3}{3}-\frac{x^4}{4}\right)dx$$

$$2\pi\left(\frac{3(3)^3}{3}-\frac{(3)^4}{4}\right)\Big|_0^3$$

$$\boxed{\frac{27\pi}{2}}$$

$y^2 = \sqrt{x}^2$ $\quad y = 2$

$\boxed{y^2 = x}$ $\quad y = 2$



2π

$2\pi \int_0^2 y\,((2) - (y^2))\,dy$

$2\pi \int_0^2 (2y - y^3)\,dy$

$2\pi \int_0^2 \left(\frac{2y^2}{2} - \frac{y^4}{4}\right)dy$

$2\pi\left(\frac{2(2)^2}{2} - \frac{(2)^4}{4}\right)$

Volume $= \boxed{0}$

bound by $x+y=3$

$x=-y+3$

$x=4-(y-1)^2$ x-axis

$2\pi\int_0^3 y((4-(y-1)^2)-(-y+3))\,dy$ need to delete

$(-y^2+3y)\,dy$

$2\pi\int_0^3(-y^3+3y^2)\,dy$

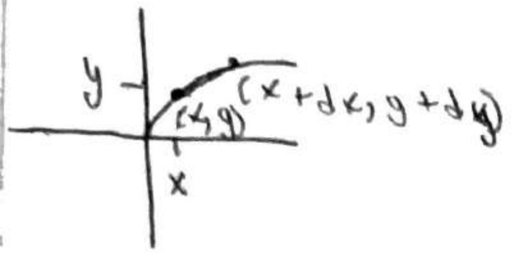
dont need to delete cuz it's flushed

$2\pi\left(-\frac{1}{4}y^4+y^3\right)dy$

$=\boxed{\frac{27\pi}{2}}$

Flushed don't negate

8.1 Arc length : How to Find the length of a curve



distance between (x, y) and $(x+dx, y+dy)$ is

$$L = \sqrt{(x+dx-x)^2 + (y+dy-y)^2}$$

$$= \sqrt{(dx)^2 + (dy)^2}$$

$$= \sqrt{(dx)^2 + (dy)^2}\ \frac{dx}{dx}$$

$$= \sqrt{\frac{(dx)^2 + (dy)^2}{1}}\ \frac{dx}{\sqrt{dx}}$$

$$= \sqrt{\frac{(dx)^2}{(dx)^2} + \frac{(dy)^2}{(dx)^2}}\ dx$$

$$= \sqrt{1 + \left(\frac{dy}{dx}\right)}\ dx$$

Let $y = f(x)$ be a curve that is continuos on $[a, b]$ and differentiable on (a, b). Then the length of the curve over the interval $[a, b]$ is given by

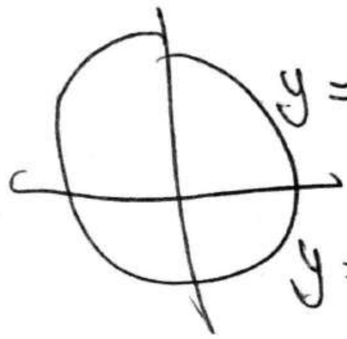
$$\int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

next week weds/thurs

Christian 9 Barden

$$\text{arc length} = \sqrt{1 + (f'(x))^2}$$

$$x^2 + y^2 = 1$$



$y = \sqrt{1-x^2}$

$y = -\sqrt{1-x^2}$

Putnam- Competition Exam

(Open stax calculus volume 2)

(Test 1)

10/10, 10/12 } possible Exam dates

On test

- U subtitution
- Finding Area
- volumes
- arc length
- derivatives of invers funct
- hypberbolic funct
- integration by parts
- integration of trig functions
- products of sine and cosine

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$$

$y' = \frac{1}{\cos y}$ $y = \frac{1}{\sqrt{1-\sin^2 y}}$

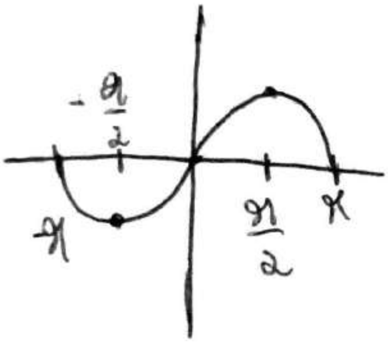
$y' = \frac{1}{\sqrt{1-x^2}}$

$\int \cos = \sin c$

$\int \sin = -\cos$

$$L = \int_a^b \sqrt{1 + (f'(x))^2}\,dx$$

Want: $\frac{d}{dx}(\sin^{-1}x)$



$y = \sin x,\ -\frac{\pi}{2} \le x \le \frac{\pi}{2}$ a one to one function

function

Every 1 to 1 has an inverse

let $y = \sin^{-1}x$

If f is a 1:1 funct, it has an inverse f^{-1} define by

$= \sin^{-1}(x) \longleftrightarrow \sin y = x$ & $y \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ $f^{-1}(x) = y \iff f(y) = x$

$\frac{d}{dx}(\sin y) = \frac{d}{dx}(x)$

$\frac{y' \cos y}{\cos y} = \frac{1}{\cos y}$

$y' = \frac{1}{\cos y}$

$\cos y > 0$ when $y \in (-\frac{\pi}{2}, \frac{\pi}{2})$

$\sin^2 y + \cos^2 y = 1$

$\cos^2 y = 1 - \sin^2 y$

$\cos y = \pm\sqrt{1-\sin^2 y}$

$\cos y = \sqrt{1-\sin^2 y}$

arc length = $\sqrt{1 + (f'(x))^2}$

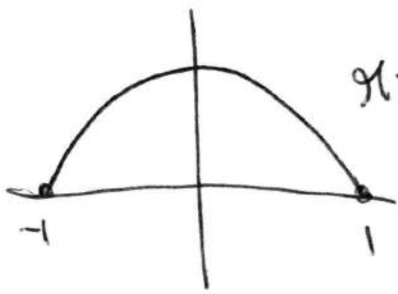
ex: Find the length of the curve

$x^2 + y^2 = 1$

$y^2 = 1 - x^2$

$y = \pm\sqrt{1-x^2}$

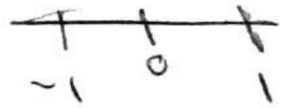
$y = \sqrt{1-x^2}$ – top half of unit circle



πr

$1 - x^2 \geq 0$

$(1-x)(1+x) \geq 0$



$r = 1$ cause unit circle

$\frac{1}{2}(2\pi r) = \frac{1}{2}(2\pi) = \pi$

$x = -1$

$x = 1$

$$L = \int_{-1}^{1} \sqrt{1 + \left[\frac{d}{dx}(1-x^2)\right]^2}\, d$$

$$L = 2\int_0^1 \sqrt{1 + \left(\frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)\right.}$$

$$L = 2\int_0^1 \sqrt{1 + \left(\frac{-x}{\sqrt{1-x^2}}\right)^2}\, dx$$

$$= 2\int_0^1 \sqrt{1 + \frac{x^2}{1-x^2}}\, dx$$

$$= 2\int_0^1 \sqrt{\frac{1-x^2}{1-x^2} + \frac{x^2}{1-x^2}}\, dx$$

$$= 2\int_0^1 \sqrt{\frac{1}{1-x^2}}\, dx$$

$$= 2\int_0^1 \frac{1}{\sqrt{1-x^2}}\, dx$$

$$= 2(\sin^{-1}x|_0^1$$

$$2(\sin^{-1}(1) - \sin^{-1}(0))$$

ex: Find the length of the curve $y=\frac{1}{6}(x^2+4)^{3/2}$
over the interval $0 \leq x \leq 3$

$$y' = \frac{1}{6}\cdot\frac{3}{2}(x^2+4)^{1/2}(2x)$$

$$= \frac{x}{2}(x^2+4)^{1/2}$$

$$= \int_0^3 \sqrt{1+\left(\frac{x}{2}(x^2+4)^{\frac{1}{2}}\right)^2}\; dx$$

$\sqrt{x^2} = |x|$

$$= \int_0^3 \sqrt{1+\frac{x^2}{4}(x^2+4)}\; dx$$

$(ab)^2 = a^2b^2$

$$= \int_0^3 \sqrt{1+\frac{x^4}{4}+x^2}$$ factor that out

$$= \int_0^3 \sqrt{\left(\frac{1}{2}x^2+1\right)\left(\frac{1}{2}x^2+1\right)}$$

$$= \left(\frac{1}{6}x^3+x\right|_0^3$$

Net Change Theorem:

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

" the integral of a rate of change is a net change "

recall: If $s(t)$ is the position that for a a object moving along a straight

$$v(t) = s'(t)$$

$$= \int_{t=a}^{t=b} v(t)\,dt = s(b) - s(a)$$ displacement

net change position

$$\int_{t=a}^{t=b} |v(t)|\,dt$$ is total distance traveled between a and b

ex: A particle moves so its velocity is 6 meters per second at time
$v(t) = t^2 - t - 6$

(a) a find displacement during time period 1 to 4.

(b) Find the total distance traveled in the time period

$$\int_1^4 v(t)\,dt$$

$$\int_1^4 (t^2 - t - 6)\,dt$$

$$= \left(\frac{1}{3}t^3 - \frac{1}{2}t^2 - 6t\right)\Big|_1^4$$

$$= \left(\frac{64}{3} - 8 - 24\right) - \left(\frac{1}{3} - \frac{1}{2} - 6\right)$$

$$= -\frac{9}{2}\text{ m}$$

$v(t) = t^2 - t - 6$

negate

$= (t-3)(t+2)$

negate integrate

$$= -\int_1^3 v(t)\,dt + \int_3^4 v(t$$

$\frac{61}{6}$ m $= 10.1\overline{6}$

Integrate a rate of change and you get a net-change

ex: If $V(t)$ is the volume of water in a reservoir at time t, then its derivative $V'(t)$ is the rate at which water flows into reservoir at time t

mile per hour, get miles

gallons per hour, get gallons

Suppose a leak develops in a pipe and water leaks out at the rate $L(t) = .7t + 5$ liters per hour, t hours after the leak begins

return the numerator

a) how much water leaks out after 3 hours

$\int_0^3 (.7t + 5)\,dt =$

$\int_0^t (.7t+5)$

$\int_0^3 .7x$

21 liter

$$\sqrt{1+\tan^2 y} = \sqrt{\sec^2 y} \quad x \quad x$$

$$\boxed{\sqrt{1+\tan^2 y} = \sec y}$$

$$\boxed{x, \sqrt{x^2-1}}$$

$$1 + \tan^2 y = \sec^2 y$$
$$-1 \qquad\qquad -1$$

$$\sqrt{\tan^2 y} = \boxed{\sqrt{\sec^2 y - 1}}$$

$$\boxed{\tan y = \sqrt{\sec^2 y - 1}}$$

$$\left(\sqrt{\sec^2 y - 1}\right)\left(\sqrt{1+\tan^2 y}\right)$$

$$\left(\sqrt{\left(\sqrt{1+\tan^2 y}\right)^2 - 1}\right)\left(\sqrt{1+\tan^2 y}\right)$$

$$\frac{1}{x\sqrt{1+x^2}} \qquad \left(\sqrt{1+\tan^2 y - 1}\right)\left(\sqrt{1+\tan^2 y}\right)$$

Christian 9 Bardesh

HW 1.3 HELP

ex:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Must be a square matrix

$$AI = A \quad \text{and} \quad IA = A$$

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ -4 & -3 & 8 \end{bmatrix}$$

$$\boxed{AI = IA}$$

$$(AB)^T = [$$

$$AA^{-1} = I \quad \text{and} \quad A^{-1}A = I$$

$$\text{So } AA^{-1} = A^{-1}A$$

$$A^T = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -3 \\ 3 & 2 & 8 \end{bmatrix} \quad B^T = \begin{bmatrix} -3 & 2 & 5 \\ 1 & 0 & 3 \\ 2 & 2 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$B^T A^T$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Verify that $(AB)^T = B^T A^T$

1.5 : LU factorization

1.5 is a long section, 10 pages
it ties up all the practice sections
& plains what exactly elementary row
operations are.

Grand picture: In math we take large
objects and break it down to smaller
easier objects.

$24 = 2 \times 2 \times 2 \times 3$ "factor"

In this section we will take
a complicated matrix and factor it
into a lower triangular matrix and
an upper matrix

Define An upper triangular matrix has
zeros below diagonal

ex $U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$

A lower triangular matrix has
zeros above the diagonal

ex $L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 5 & 6 \end{bmatrix}$

LU factorization is easy if you have been following the Gaussian Elimination Algorithm

Ex Factor $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & -1 \\ 3 & -1 & -1 \end{bmatrix}$ as LU

Note: You check to confirm

$LU = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & \frac{2}{3} & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -5 \\ 0 & 0 & -\frac{11}{3} \end{bmatrix}$

$\begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & -1 \\ 3 & -1 & -1 \end{bmatrix}$

numx 6

denom x

Solve $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & -1 \\ 3 & -1 & -1 \end{bmatrix}$

$\longrightarrow$

$R_2 - 2R_1$, $R_3 - 3R_1$: $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -5 \\ 0 & 2 & -7 \end{bmatrix}$

$R_3 - \frac{2}{3}R_2$: $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -5 \\ 0 & 0 & -\frac{11}{3} \end{bmatrix}$

$-7 - 2(-5) -$

U

This is your L

or multipliers

Coefficients $\frac{-21+10}{3} = \frac{-11}{3}$

$U = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -5 \\ 0 & 0 & -\frac{11}{3} \end{bmatrix}$ $L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & \frac{2}{3} & 1 \end{bmatrix}$

NOTE: One of the reasons LU factorisation is important is when you have the same A and different $\bar{b}$

We will solve $A\bar{x} = b$ with two back substitutions

$$A\bar{x} = \bar{b}$$

$$\underbrace{U\bar{x}}_{\bar{y}}$$

$$L U\bar{x} = \bar{b}$$

$\bar{y}$ let $\bar{y} = U\bar{x}$

$$L\bar{y} = b$$

Find $\bar{y}$ by solving $L\bar{y} = \bar{b}$ using back substitution then we use $\bar{y}$ to solve $U\bar{x} = \bar{y}$ also by back sub

Given $A = LU$ where $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & -1 \\ 3 & -1 & -2 \end{bmatrix}$ $U = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -5 \\ 0 & 0 & - \end{bmatrix}$

and $L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & \frac{2}{3} & 1 \end{bmatrix}$, solve $A\bar{x} = \underbrace{\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}}_{\text{this is } \bar{b}}$

Solve Step 1 $L\bar{y} = \bar{b}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & \frac{2}{3} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$$\bar{y} = \begin{bmatrix} 1 \\ 1 \\ \frac{4}{3} \end{bmatrix}$$

Step 2 Solve $U\bar{x} = \vec{y}$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -5 \\ 0 & 0 & -\frac{11}{3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \frac{4}{3} \end{bmatrix}$$

Second part must do H.W! by 28th

$$\bar{x} = \begin{bmatrix} \frac{16}{11} \\ -\frac{3}{11} \\ -\frac{4}{11} \end{bmatrix}$$

faster

$\boxed{n^2 \text{ is faster}}$

why 2 back-substitution
Gaussian Elimination requires n^3 steps and back substitution takes n^2 steps

$n = 10^6$

$n^3 = (10^6)^3 = 10^{18}$

$n^2 = (10^6)^2 = 10^{12}$

$y = \arctan(2x^2 - 3x + 4)$, $\frac{dy}{dx}$

$x = \tan y$ $\qquad$ $\cancel{1} + \tan y^2$

$\frac{d}{dx}(x) = \frac{d}{dx}(\tan(y))$ $\qquad$ $1 + \tan^2 x = \sec^2 x$

$- 1$

$\frac{1}{\frac{d}{dx}(\tan(y))} = \frac{\frac{d}{dx}(\tan(y)) \cdot \frac{dy}{dx}}{\frac{d}{dx}(\tan(y))}$ $\qquad$ $\sqrt{\tan^2 x} = \sqrt{\sec^2 x - 1}$

$\tan x = \sqrt{\sec^2 x - 1}$

$\frac{1}{\sec^2 y - 1}$

sine x =	[redacted] cos x
cos x =	− sin x
tan x =	$\sec^2 x$
sec x =	sec x tan x

$4x^2+9y^2-36=0$
$-4x^2 \quad +36 \quad +36$

inverse $\tan^{-1}$

$-4x^2$

$x = \tan(y)$ remember implicit dif / chain

$\frac{9y^2}{9} = \frac{-4x^2+36}{9}$

$\frac{1}{\sec^2 y} = \frac{\sec^2 \cancel{y} \frac{d\cancel{y}}{dx}}{\sec^2 y}$

$\sqrt{y^2} = \sqrt{\frac{4x^2+36}{\cancel{9}}}$

$1 + \tan^2 \cancel{y} = \sec^2 y$

$\frac{1}{\sec^2 y} = \frac{dy}{dx}$

$y = \sqrt{\frac{4x^2+36}{3}}$

$\boxed{\frac{1}{1+x^2}}$

$\boxed{y = \frac{2x+6}{3}}$

$\frac{1}{1+(e^{3x})^2} \cdot (e^{3x} \times 3)$

$y = \frac{2}{3}x + 2$

$\frac{16y^2}{16} = \sqrt{\frac{-9x^2-144}{16}}$

$\frac{d}{dx}(y) = \frac{d}{dx}\left(\frac{2}{3}x+2\right)$

$\frac{-18x}{32y}$ $\quad \sqrt{y^2} =$

$\frac{dy}{dx} = \frac{2}{3}$

$$\frac{d}{dx}(4x^2 + 9y^2 - 36) = (0)\frac{d}{dx}$$

$$8x + 9y\frac{dy}{dx} = 0$$

$$\frac{\cancel{9y}\frac{dy}{dx}}{\cancel{9y}} = \frac{-8x}{9y}$$

$$\frac{dy}{dx} = \frac{-8x}{9y}$$

$$\frac{-8x}{9\left(\frac{2x+6}{3}\right)}$$

$x = \sin y$

$\cos y^2 + \sin y^2 = 1$

$-\sin y^2$

$\frac{1}{\cos y} = \frac{\cancel{\cos y}\frac{dy}{dx}}{\cancel{\cos y}}$

$\sqrt{\cos^2 y} = \sqrt{1 - \sin^2 y}$

$\boxed{\frac{1}{\sqrt{1 - x^2}}}$

$\left(-\frac{98x}{32\left(\sqrt{\frac{-49x^2+784}{16}}\right)}\right)^2$

$\frac{-98x}{32y}$

$-\frac{(98x)^2}{32\left(\frac{-49x^2+784}{16}\right)}$

$\frac{1}{\sqrt{1-\left(\frac{8}{9}\right)^2}} \times 4$

$\frac{16y^2}{16} = \sqrt{\frac{-49x^2 + 784}{16}}$

$\frac{1}{1-x^2}$

$\frac{1}{1+(2)^2} \times -3$

$\frac{1}{1-(-3)^2} \times (-10)$

$\frac{-10}{8}$

$\boxed{-\frac{3}{5}}$ $\boxed{-\frac{3}{5}}$

$\cosh^2 x + \sinh^2 x = 1$

$x = \sec(y)$ $\qquad \frac{1}{\cosh x}$

$$\frac{1}{\sqrt{1-\sinh^2 x}}$$

$$\frac{1}{\sec(y)\tan(y)} = \frac{dy}{dx}$$

$$1 + \tan^2 y = \sec^2 y$$
$$-1 \qquad\qquad -1$$

$$\frac{1}{x\sqrt{x-1}}$$

$$\sqrt{\tan^2 y} = \sqrt{\sec^2 y - 1}$$

$$\sqrt{\sec^2 y - 1}$$

Hyperbolic Sine and Cosine Functions

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

$$\frac{d}{dx}(\cosh x) = \frac{d}{dx}(\sinh x)$$

$$\cosh^2 x - \sinh^2 x = 1$$

Def

$\boxed{y = \sinh^{-1} x \text{ if and only if } \sinh y = x}$

$$\frac{d}{dx}(\sinh x) = \frac{d}{dx}\left(\frac{e^x - e^{-x}}{2}\right) = \frac{e^x + e^{-x}}{2}$$

Def

$\boxed{y = \cosh^{-1} x \text{ if and only if } \cosh y = x \text{ and } y \geq 0}$

$$\frac{1}{\cosh(y)} = \frac{\cosh(y)\frac{dy}{dx}}{\cosh(y)}$$

$$\cosh^2 y - \sinh^2 y = 1$$

$$+\sinh^2 y$$

$$\sqrt{\cosh^2 y} = \sqrt{1 + \sinh^2 y}$$

$$\frac{1}{\sqrt{1 + (x^2 - x - 3)^2}} \times 2x - 1$$

$u = 3x + 4$

$du = 3\,dx$

$v = -\frac{1}{6}\cos(6x)$

$dv = \sin(6x)\,dx$

$(3x+4)\left(-\frac{1}{6}\cos(6x)\right) - \int (\cancel{v}\,du)$ $\quad\int \sin(6x)\,dx \quad u = 6x$

$\frac{du}{6} = \cancel{6}\,dx$

$(3x+4)\left(-\frac{1}{6}\cos(6x)\right) - \int -\frac{1}{6}\cos(6x)\,3\,dx \quad -\int \cos(u)\,dx$

$-\int \frac{\cos(u)\,du}{6}$

$-\left(-\frac{3}{6}\right)\int \cos(6x)$

$-\frac{1}{6}\int \cos(6x)$

$-\left(-\frac{3}{6}\right)\left(\frac{1}{6}\right)\int \sin(6x)\,d$

$-\frac{1}{6}\cos(6x)$

$\left((3x+4)\left(-\frac{1}{6}\cos(6x)\right)\right) - \left(\left(-\frac{3}{6}\right)\left(\frac{1}{6}\right)\sin(6x)\right)$

$$(8t-9)\left(-\frac{1}{3}e^{-3t}\right)-\int\left(-\frac{1}{3}e^{-3t}\right)8\,dt$$

$u = 8t-9$

$du = 8\,dt$

$$)-\left(-\frac{8}{3}\right)\int\left(e^{-3t}\right)$$

$$v=-\frac{1}{3}e^{-3t} \quad \left((8t-9)\left(-\frac{1}{3}e^{-3t}\right)-\left(\left(-\frac{8}{3}\right)\left(-\frac{1}{3}\right)\right)\left(e^{-3t}\right)\right)+C$$

$$\left(\frac{8}{9}\right)$$

$dv = e^{-3t}\,dt$

$$\int e^{-3t}\,dt$$

$$\int e^{u}\,dt \qquad u=-3t$$

$$\int \frac{e^{u}\,du}{-3} \qquad \frac{du}{-3}=\frac{-3dt}{-3}$$

$$\frac{1}{-3}\int e^{-3t}\,dt$$

$$-\frac{1}{3}e^{-3t}$$

$$\int_6^{11} \left(e^{-7x} f'(x)\right) dx = e^{-7x} f(x) - \int f(x) \left(-\frac{1}{7} e^{-7x}\right)$$

$$((e^{-7x} \times 2) - 15) - ((e^{-7x} \times 10)$$

15

$u = e^{-7x}$

e

$du = -\frac{1}{7} e^{-7x}$

$dv = f'(x)$

$v = f(x)$

$--5$

-62

$\downarrow$

$+5$

$v = e^{-7x}$

$(e^{-7x})(f(x)) + 7(15)$

$du = (e^{-7x}\ x - 7)$

Evaluate at 2 dif bounds

sub

$dv = f'(x)$

$v = (f(x))$

$+105$

$-(13)\int_6^9 \left(\frac{16x^2}{2} + 9x\right) dx$

$16x + 9$

$\frac{16x^2}{2} + 9x$

$-\int_6^9 \left(\frac{16x^2}{2} + 9x\right)(f(x))\, dx$

$\left(\frac{16x}{2} + 9\right) \times f(x)$

$-\int_6^9 \left(\frac{16x}{2} + 9\right) \times (f(x))\, dx$

$\left(\frac{16(9)}{2} + 9\right) \times 17\, dx$

17

$\left(\frac{16(6)}{2} + 9\right) \times 17$

$u = (5x^2 + 4x - 6)$

$du = 10x + 4\, dx$

$dv = f'(x)\, dx$

$v = f(x)$

$(5x^2+4x-6)(f(x)) - \int_7^8$ ~~(~~

$f(x)(10x+4$

$12(10x$

$-(12)\int_7^8 (\frac{10x^2}{}$

$f(5) = 3$

$f(8) = 10$

$u = f(x)$

$du = f'(x)\,dx$

$v = \frac{12x^2}{2} + 7x$

$dv = 12x + 7\,dx$

$\int_5^8 (12x + 7) f(x)\,dx =$

$f(x)\left(\frac{12x^2}{2} + 7x\right)$

$(10)\left(\frac{12(8)^2}{2} + 7(8)\right) - (3)\left(\frac{12(5)^2}{2} + 7(5)\right) -$

$\frac{12}{2} \int x^2 f'(x)$ $\left(\frac{12}{2}(18) + 7(12)\right)$

$7 \int x f'(x)$

$\int \frac{\ln x}{x^5} dx =$

$\int \frac{\ln x}{x^8} dx =$

$\int (5x-2)\sin(2x+8)\, dx =$

$\int \ln x \frac{1}{x^5}$

$\ln x\left(\frac{x^{-4}}{4}\right) - \left(-\frac{1}{4}\left(\frac{x^{-4}}{-4}\right)\right)$

$\ln x\left(\frac{x^{-4}}{-4}\right) - \int \left(\frac{x^{-4}}{-4}\right)\left(\frac{1}{x}\right) dx$

$\int \left(\frac{x^{-4}}{-4x}\right) dx$

$-\frac{1}{4}\int \frac{1}{x^5} dx$ $\quad \frac{x^{-4}}{-4}$

$\int x^{-5} dx$

$\frac{-4}{4}$

$5)$

rec

$\cancel{\int (\sin^2 x)(\sin x)(\cos^3 x)}$

$\int \sin^3 x \cos^3 x \, dx$

$\sin^2 x + \cos^2 x = 1$

$-\sin^2 x \qquad -\sin^2 x$

$\cos^2 x = 1 - \sin^2 x$

1 $\int \sin^3 x \cos x (1 - \sin^2 x) \, dx$

2 $\int \frac{u^3 \cancel{\cos x} (1 - u^2)}{\cancel{\cos x}} \, du$

$u = \sin x$

$\frac{du}{\cos x} = \frac{\cos x \, dx}{\cos x}$

3 $\int u^3 (1 - u^2) \, du$

4 $\int u^3 - u^5 \, du$

5 $\frac{u^{3+1}}{3+1} - \frac{u^{5+1}}{5+1}$

$\frac{\sin^4 x}{4} - \frac{\sin^6 x}{6}$

$$\int \cos^2 x \sin^3 x \, dx$$

$$\int (1-\sin^2 x)(\sin^3 x)\, dx$$

$\cos^2 x + \sin^2 x = 1$

$-\sin^2 x \quad -\sin^2 x$

$\cos^2 x = 1 - \sin^2 x$

$$\int (\sin^3 x - \sin^5 x)\, dx$$

$u = \sin x$

$\frac{du}{\cos x} = \frac{\cos x \, dx}{\cos x}$

$$\boxed{\frac{\sin^4 x}{4} - \frac{\sin^6 x}{6}}$$

$$\int \cos^2 x \sin^2 x (\sin x)\, dx$$

$$\int \underline{\cos^2 x (1-\cos^2 x)(\sin x)\, dx} \qquad \sin^2 x = 1-\cos^2 x$$

$$\int \frac{\cos^2 x (1-\cos^2 x)\cancel{(\sin x)}}{\cancel{\sin x}}\, du \qquad u = \cos x \qquad u' = -\sin x$$

$$-\frac{du}{\sin x} = \frac{\sin x\, \cancel{dx}}{\sin x}$$

$$\int u^2 (1-u^2)\, du$$

$$\int u^2 - u^4\, du$$

$$\int \frac{u^3}{3} - \frac{u^5}{5}\, du$$

$$\frac{\cos x^3}{3} - \frac{\cancel{\cos x}^5}{5}$$

sub

$$\int \cos^4 x \sin^2 x (\sin x)\, dx$$

$$\int \cos^4 x (1-\cos^2 x)(\sin x)\, dx \qquad \sin^2 x = 1-\cos^2 x$$

$$\int \frac{u^4(1-u^2)\cancel{(\sin x)}\, du}{-\cancel{\sin x}} \qquad u = \cos x$$

$$\frac{du}{-\sin x} = \frac{-\sin x\, dx}{-\sin x}$$

$$\int -u^4(1-u^2)\, du$$

$$\int (-u^4 + u^6)\, du$$

$$\int \left(\frac{-u^5}{5} + \frac{u^7}{7}\right) du$$

$$\boxed{-\frac{(\cos x)^5}{5} + \frac{(\cos x)^7}{7} + C}$$

$\int (\tan^2 x)(\tan^3 x)(\sec^2 x)\,dx$

~~$\int ((u^2)(\tan$~~

$\tan x = u$

$\sec^2 = u'$

$\int \frac{(u^2)(u^3)(\cancel{\sec^2 x})\,dx}{\cancel{\sec^2 x}}$

$\frac{du}{\sec^2 x} = \frac{\sec^2 x\,dx}{\sec^2 x}$

$\int (u^2)(u^3)\,du$

$\int \left(\frac{}{3} \frac{}{4}\right) du$

$\left(\frac{\tan^3 x}{3}\right)\left(\frac{\tan^4 x}{4}\right) + C$

$1 + \tan^2 x = \sec^2$

$\frac{u^7}{12} + C$

$\int \tan^5 x\,(1 + \tan^2 x)\,dx$

$\int u^5(1 + u^2)\,dx$

$u = \tan x$

$\int (u^5 + u^7)\,dx$

$dx =$

dx

$1 + \tan^2 x = \sec^2$

$\int \tan^2 x \tan^2 x \sec^2 x\,dx$

$u = \tan x$

$u' = \sec^2 x$

$\dfrac{du}{\sec^2 x} = \dfrac{\sec^2 x\,du}{\sec^2 x}$

$\int \tan^4 x \sec^2 x\,dx$

$\int \dfrac{u^4 \cancel{\sec^2 x}\,du}{\cancel{\sec^2 x}}$

$\int u^4\,du$

$\int \dfrac{u^5}{5}\,du$

$\boxed{\dfrac{\tan x^5}{5} + C}$

$\int \tan x \sec^3 x \, dx$

$\frac{d}{dx}(\sec x) = \tan x \sec x$

$\int (\tan x \sec x)(\sec^2 x) \, dx$

$\int (\tan x \sec x)(\sec x)(\sec x) \, dx$

$\int \frac{(\cancel{\tan x \sec x})(u)(u) \, du}{\cancel{\tan x \sec x}}$

$u = \sec x$

$\frac{du}{\sec x \tan x} = \frac{(\sec x \tan x) \, dx}{\sec x \tan x}$

$\int (u^2) \, du$

$\int \frac{u^3}{3} \, du$

$\boxed{\frac{\sec x^3}{3} + C}$ ✓

$1 - 2\cos^2 x + \cos^4 x$

$(1 - \cos^2 x)(1 - \cos^2 x)$

$1 - \cos^2 x - \cos^2 x + \cos^4 x$

$\frac{\tan x^9}{9}$ $\frac{\tan x^{10}}{}$ u^2

$\cancel{\tan^2 x(\tan^6 x + \tan^8 x)}$

$\tan^2 x(\tan^6 x + \tan^8 x)\sec^2$ $u = \tan x$

$\tan^8 x + \tan^8 x$

$u' = \sec^2$

$\frac{u^7}{7} + \frac{u^9}{9}$

$\int (1 - 2\sin^2 x + \sin^4 x)\cos x \, dx$

$\int \frac{(1 - 2u^2 + u^4)\cancel{\cos x}\, du}{\cancel{\cos x}}$

$u = \sin x$

$u' = \cos x$

$du = \cos x\, dx$

$\sin x - \frac{2\sin x^3}{3} + \frac{\sin x^5}{5}$

Christian Q Bardun

cotangent

$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csc}$

$\frac{d}{dx}(\operatorname{sech} x)$

find $f'(x)$ for

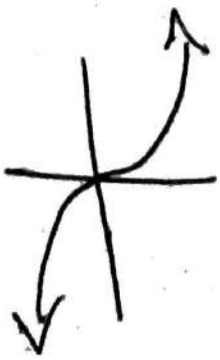
$$\boxed{f(x) = x\sinh x - \cosh x}$$

$$f'(x) = \sinh x \frac{d}{dx}(x) + x\frac{d}{dx}(\sinh x) - \sinh x$$

$$= \sinh x + x\cosh x - \sinh x$$

$$\boxed{x\cosh x}$$

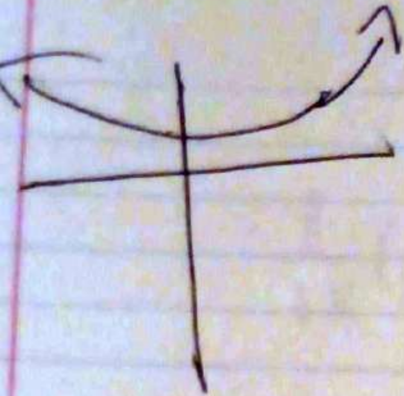
ex: Find y' when $y = e^{\cosh 3x}$



— hyperbolic sin

$\boxed{\text{derive inverse trig functions and hyperbolic functions}}$

cosh x



$\cancel{\cosh^2 x - \sinh^2 x = 1}$

$$\boxed{\cosh^2 y - \sinh^2 y = 1}$$

$$y = \sinh^{-1} x \iff \sinh y = x$$

$$\pm\sqrt{1 + \sinh^2 x}$$

$$\frac{d}{dx}(\sinh y) = \frac{d}{dx}(x)$$

$$\sqrt{1 + \sinh^2 x}$$

$$\frac{\left(\frac{dy}{dx}\right)\cancel{\cosh y}}{\cancel{\cosh y}} = \frac{1}{\cosh y}$$

$$\cosh y = \sqrt{1 + \sinh^2 y}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{\sqrt{1 + x^2}}}$$

$\frac{d}{dx}(\cosh^{-1}x)$

$y = \cosh^{-1}x \Longleftrightarrow x = \cosh y,\ y \geq 0$

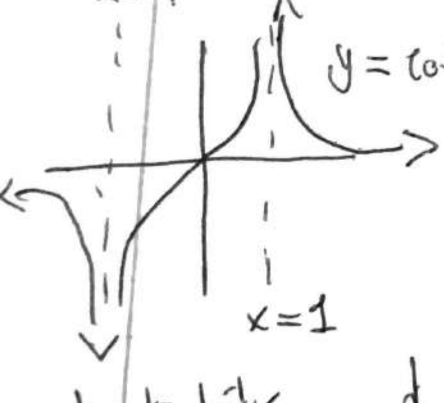
$\frac{d}{dx}(x) = \frac{d}{dx}(\cosh y)$

$\boxed{\frac{1}{\sinh y}} = \frac{\sinh y \frac{dy}{dx}}{\sinh y}$

$\sqrt{\cosh^2 y + 1} = \sqrt{\sinh^2 y}$

$\sqrt{\cosh^2 y - 1}$

$\sinh y = +\sqrt{\cosh^2 y - 1}$



$\boxed{\frac{1}{\sqrt{x^2 - 1}}}$

$\frac{d}{dx}(\sinh^{-1}x) = \frac{1}{\sqrt{1+x^2}}$ $\qquad$ $\frac{d}{dx}(\operatorname{csch}^{-1}x) = \frac{-1}{|x|\sqrt{x^2+1}}$

$\frac{d}{dx}(\cosh^{-1}x) = \frac{1}{\sqrt{x^2-1}}$ $\qquad$ $\frac{d}{dx}(\operatorname{sech}^{-1}x) = \frac{-1}{x\sqrt{1-x^2}}$

$\frac{d}{dx}(\tanh^{-1}x) = \frac{1}{1-x^2}$ $\qquad$ $\frac{d}{dx}(\coth^{-1}x) = \frac{1}{1-x^2}$

$\sinh(x+y)$ $\int \sinh(x) + \cosh(y)$

cosh even

sinh - odd

tanh odd

$$\int \frac{1}{1-x^2}\,dx = \begin{cases} \tanh^{-1}x + c & \text{if } |x| < 1 \\ \coth^{-1}x + c & \text{if } |x| > 1 \end{cases}$$

act : $\sinh^{-1}x = \ln\left(x + \sqrt{x^2+1}\right)$

ex : Find $\frac{dy}{dx}$ for $y = \tanh^{-1}(\sin x)$

$\frac{d}{dx}$

$\frac{d}{dx}$

$$\frac{dy}{dx} = \frac{1}{1-\sin^2 x} \times \frac{d}{dx}(\sin x)$$

$$= \frac{\cos x}{\cos^2 x}$$

$$= \frac{1}{\cos x} = \sec x$$

LIATE

Integration by Parts

(Product rule in reverse)

recall: If $y = f(x) \cdot g(x)$

$\cancel{f(x)g'(x) + f'(x)g(x)}$

$\frac{dy}{dx}$ libenitz

$f(x)g'(x) + f'(x)g(x)$

(omit constant for now)

$\frac{d}{dx}(f(x) \cdot g(x)) = f(x)g'(x) + g(x)f'(x)$

$\int (f(x)g'(x) + g(x)f'(x))\,dx = f(x) \cdot g(x)$

$(3x^2 + 2)\,dx$

$\int f(x)g'(x)\,dx + \int g(x)f'(x)\,dx = f(x)g(x)$

$= \int (3x^2)\,dx + \int (2)\,dx$

$\int f(x)g'(x)\,dx = f(x)g(x) - \int g(x)f'(x)\,dx$

sums and diff respected

Formula = $\boxed{\int u\,dv = uv - \int v\,du}$

. Integration does Not respect Products

- a simpler integral

ex: $\int x \sin x \, dx$

$u = x$

$dv = \sin x \, dx$

$du = 1 dx$

$v = -\cos x$

$-x\cos x - \int -\cos x \, dx$

$x\cos x - (-\sin x)$

$\boxed{-x\cos x + \sin x} + c$

$\boxed{-x\cos x + \sin x + C}$

ex: $\int t e^{-3t} \, dt$

$u = t$

$du = dt$

$dv = e^{-3t} dt$

$v = -\frac{1}{3} \cdot e^{-3t}$

$(t)\left(-\frac{e^{-3t}}{3}\right) - \int \left(-\frac{e^{-3t}}{3}\right) dt$

$(t)\left(-\frac{e^{-3t}}{3}\right) - \frac{1}{3}\int \left(-e^{-3t}\right) dt$

$\boxed{(t)\left(-\frac{e^{-3t}}{3}\right) - \frac{1}{3}\left(\left(\frac{e^{-3t}}{3}\right)\right) + C}$

Christian & Bardun

Oct 12 Exam
practice $_\wedge$ Friday 2pm
review

* We're meeting on blackboard

Trig Substitution

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\cos 2x = \cos(x+x) = \cos x \cos x - \sin x \sin x$

$\sin^2 x + \cos^2 x = 1$

$\boxed{\cos 2x = \cos^2 x - \sin^2 x}$

$\boxed{\cos 2x = 1 - 2\sin^2 x}$

$\cos 2x = \cos^2 x - (1 - \cos^2 x)$

$\boxed{\cos 2x = 2\cos^2 x - 1}$

$2\cos^2 x = \cos 2x + 1$

$\cos^2 x = \frac{1}{2}(\cos 2x + 1)$

$2\sin^2 x = 1 - \cos 2x$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$$\int \sin^n x \cos^m x \, dx$$

$$\int \cos^2 x \, dx$$

$$= \int \frac{1}{2}(1 + \cos 2x) \, dx$$

$$\frac{1}{2} \int (1 + \cos 2x) \, dx$$

$$\frac{1}{2} \int \left(x + \frac{1}{2} \sin(2x) \right) dx$$

$\frac{1}{4}(x +$ [redacted] $\sin(2x))$

Midterm is 4 Questions, then true/False
2 hour exam, Nov 2

1.5 : LU factorization (Continued)

Ex Find the LU factorization of

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 2 & 6 & 8 \\ 6 & 8 & 18 \end{bmatrix}$$

(st the
ltiplier)

$$\begin{array}{l} R_2 - \underline{1}R_1 \\ R_3 - \underline{3}R_1 \end{array} \begin{bmatrix} 2 & 1 & 3 \\ & 5 & 5 \\ & 5 & 9 \end{bmatrix} \xrightarrow{R_3 - \underline{1}R_2} \begin{bmatrix} 2 & 1 & 3 \\ 0 & 5 & 5 \\ 0 & 0 & 4 \end{bmatrix} \leftarrow U$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix}$$

(b)

$$\frac{dy}{dx} = \frac{d}{dx}(\cot^{-1} x) + \frac{d}{dx}(\cot^{-1} \frac{1}{x})$$

$$\frac{dy}{dx} = \frac{1}{-(-1+x^2)} + -(-1+x^2) \qquad 1 + \cot^2\theta = \csc^2\theta$$

$$\frac{dy}{dx} = 0$$

$$y = \cot^{-1}(x)$$

$$x = \cot(y)$$

$$\frac{1}{-\csc^2 x} = \frac{dy}{dx}$$

$$-1 + \cot^2 x$$

$$\frac{1}{-(-1+x^2)}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\ln(8x+5)\right)(\arctan 3x)$$

$$\ln(8x+5)\left(\frac{1}{3(1+x^2)}\right) + \frac{1}{8x+5}(8)(\arctan 3x)$$

$$x = \tan(3y)x$$

$$x = \frac{\sec^2(3y)\times 3}{\sec^2}$$

$$\frac{1}{3(\sec^2 x)}$$

$$1+\tan^2$$

$$\frac{1}{3(1+\tan^2 x)}$$

trig identity done

$y = \tanh^{-1}(x^2)$

$$\boxed{\frac{dy}{dx} = \frac{2x}{1-x^2}}$$

$x^2 = \tanh(y)$

$$2x = \frac{\operatorname{sech}^2(y)\frac{dy}{dx}}{\operatorname{sech}^2(y)}$$

$$\frac{2x}{\operatorname{sech}^2(y)}$$

$1 - \tanh^2(x) = \operatorname{sech}^2 x$

$$\frac{2x}{1-x^2}$$

cosh - pos
tanh - pos
sinh - pos

$y = \ln(\cosh x)$

$\frac{d}{dx}(\cosh x) = \sinh x$

$$\frac{dy}{dx} = \boxed{\frac{1}{\cosh(x)} \times \sinh(x)}$$

$y = \coth^{-1}(\sec x)$

$\coth^2 x - 1 = \csc^2 x$

$\sec x = \coth^{-1}(y)$

$\frac{d}{dx}(\sec x)$

$\sec x \tan x = \frac{1}{y^2-1}\left(\frac{dy}{dx}\right)$

$\sec x \tan x = \frac{1}{x^2-1}\left(\frac{dy}{dx}\right)(x^2-1)$

$x = \coth(y)$

$\frac{1}{-\csc^2 x} = -\csc^2 x$

$\cancel{\frac{dy}{dx} = \left(\frac{1}{(\sec x)^2+1}\right) \times \sec x \tan x}$

$\frac{1}{-(1-\coth^2 x)}$

$\frac{1}{x^2-1}$

$\frac{dy}{dx} = \frac{1}{(\sec x)^2-1} \times \sec x \tan x$

(g)

$y = \coth^{-1}(\sec x)$

$$\boxed{\frac{dy}{dx} = -\frac{1}{(\sec x)^2+1} \times (\sec^2 x)(\tan^2 x)}$$

$x = \coth(y)$

$$\frac{1}{-(\csc(x))^2} = \frac{-(\csc(x))^2 \frac{dy}{dx}}{-(\csc(x))^2}$$

$\cot^2 x - 1 = \csc^2 x$

$$\frac{1}{-(\cot^2 x - 1)}$$

$$\frac{1}{-x^2+1}$$

(h)

$y = x\sinh^{-1}\left(\frac{x}{3}\right) - \sqrt{9+x^2}$

$$\boxed{x\left(\frac{1}{\sqrt{1+\left(\frac{x}{3}\right)^2}}\right)\left(\frac{1}{3}\right) + \sinh^{-1}\left(\frac{x}{3}\right) - \left(\frac{1}{2}(9+x^2)^{-\frac{1}{2}} \times (2x)\right)}$$

$x = \sinh(y)$

$$\frac{1}{\cosh(y)} = \frac{\cosh(y)\frac{dy}{dx}}{\cosh(y)}$$

$$\frac{1}{\sqrt{1+\sinh^2 x}}$$

$\cosh^2 x - \sinh^2 x = 1$

$+\sinh^2 x$

$\sqrt{\cosh^2 x} = \sqrt{1+\sinh^2 x}$

$(9+x^2)^{\frac{1}{2}}$

$\frac{1}{2}(9+x^2)^{-\frac{1}{2}} \times (2x)$

$$\int_0^1 \frac{e^z+1}{e^z+2}\,dz$$

$$\int_0^1 \frac{e^z+1}{u}\,dz \qquad u = e^z+2$$

$$\frac{du}{e^z+1} = \frac{(e^z+1)\,dz}{e^z+1}$$

$$\int_0^1 \frac{e^z\cancel{+1}}{u(e^z\cancel{+1})}\,du$$

$$\int_0^1 \frac{1}{u}\,du$$

$$\ln|u| = \ln(e+1)$$

$$\ln|e^z+2|$$

$$\int_0^{\frac{\pi}{2}} \cos 2x - \sin x \, dr$$

$$\left(\frac{\sin(2x)}{2}\right) - f(\cos X)$$

$$\left(\frac{\sin(2x)}{2}\right) - (-\cos x) \Big|_0^{\frac{\pi}{2}}$$

$$\boxed{.027026649}$$

replace x with 2x

$1 + \cos 2(2x)$

Another positive case

$$\int \sin^4 x \, dx$$

$$(\sin^2 x)^2 = \sin^4 x$$

$$\downarrow$$

$$= \left(\tfrac{1}{2}(1-\cos^2 x)\right)^2$$

$$= \frac{1}{4}\left(1 - 2\cos 2x + \cos^2 2x\right)$$

$$\frac{1}{4}\int \left(1 - 2\cos 2x + \cos^2 2x\right) dx$$

$$\cos^2(2x) = \frac{1}{2}(1+\cos 4x)$$

$$\frac{1}{4}\int \left(1 - 2\cos 2x + \frac{1}{2}(1+\cos 4x)\right) dx$$

$$\frac{1}{4}\int \left(1 - 2\cos 2x + \frac{1}{2} + \frac{1}{2}\cos 4x\right) dx$$

$$\frac{1}{4}\int \left(\frac{3}{2} - 2\cos 2x + \frac{1}{2}\cos 4x\right) dx$$

$$= \boxed{\frac{1}{4}\left(\frac{3}{2}x - \sin 2x + \frac{1}{8}\sin 4x\right) + C}$$

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\sin 2x = \sin(x+x)$

$= \sin x \cos x + \cos x \sin x$

$\sin 2x = 2 \sin x \cos x$ ← just divide both sides by 2

$\sin x \cos x = \frac{1}{2} \sin 2x$

ex: $\int (\sin^2 x \cos^2 x) dx$

$= \int (\sin x \cos x)^2 dx$

$\boxed{\frac{1}{8}\left(x - \frac{\sin 4x}{4}\right) + C}$

$= \int (\frac{1}{2} \sin 2x)^2 dx$

$= \int (\frac{1}{4} \sin^2 2x) dx$

$= \frac{1}{4} \int \frac{1}{2}(1 - \cos 4x) dx$

$= \frac{1}{4} \cdot \frac{1}{2} \int (1 - \cos 4x) dx$

$= \frac{1}{8} \int (1 - \cos 4x) dx$

Separate integrals the include subtra

Christian T Bardon

$$\int \cos^4 x \sin^2 x\, dx = \frac{1}{16}x - \frac{1}{64}\sin 4x + \frac{1}{48}\sin^3 2x + C$$

$$= \int \cos^2 x(\cos x \sin x)^2\, dx$$

$$= \int \cos^2 x\left(\frac{1}{2}\sin 2x\right)^2 dx$$

$$= \int \frac{1}{2}(1+\cos 2x)\left(\frac{1}{4}\right)\sin^2 2x\, dx$$

$$= \frac{1}{8}\int (\sin^2 2x + \sin^2 2x \cos 2x)\, dx$$

$$\frac{1}{8}\int \sin^2 2x\, dx + \int \sin^2 2x \cos 2x\, dx$$

$$\frac{1}{8}\int \frac{1}{2}(1-\cos 4x)\, dx + \quad (\sin 2x)(\sin 2x)\cos 2x$$

$$-\sin 2x$$

$$\int(-\sin(2x))(\cos 2x)$$

$\int \sin^n x \cos^m dx$

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$\int \tan^n x \sec^m x \, dx$ only if the power is even it's possible

$\frac{d}{dx}(\tan x) = \sec^2 x$

$= \int \tan^2 x \sec^2 x \sec^2 x \, dx$

$= \int \tan^2 x \sec^2 x (\tan^2 x + 1) \, dx$

$= \int \tan^2 x (\tan^2 x + 1) \sec^2 x \, dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$= \int u^2 (u^2 + 1) \, du$

$\int (u^4 + u^2) du$

$\frac{1}{5} u^5 + \frac{1}{3} u^3 + C$

$\frac{1}{5} \tan^5 x + \frac{1}{3} \tan^3 x + C$

Both odd Case

ex: $\int \tan^5 x \sec^3 x \, dx$

$= \int \tan^4 x \sec^2 x (\tan x \sec x) \, dx$

$\int (\sec^2 x - 1)^2 \sec^2 x (\tan x \sec x) \, dx$

$\int (u^2-1)^2 u^2 \, du$

$u = \sec x$

$du = \sec x \tan x \, dx$

$\int (u^4 - 2u^2 + 1) u^2$

$\int (u^6 - 2u^4 + u^2) \, du$

$$\boxed{\frac{1}{7}u^7 - \frac{2}{5}u^5 + \frac{1}{3}u^3 + C}$$

$(1-\sin^2 x)(1-\sin^2 x)$

$\sin^2 x \sin^2 x$

$1-\sin^2 x-\sin^2 x+\sin^4 x$

$\sin^4 x(1-2\sin^2 x+\sin^4 x)$

$(-\cos^2 x+1)(-\cos^2 x+1)$

$\sin^4 x-2\sin^6 x+\sin^8 x)$

$$\left(\frac{1+\cos(2x)}{2}\right)\left(\frac{1+\cos(2x)}{2}\right)$$

$$\frac{1}{4}\left((1+\cos(2x))(1+\cos(2x))\right)$$

$$1+\cos(2x)+\cos(2x)+\cos^2(2x)$$

$$\frac{1}{4}\left(1+2\cos(2x)+\cos^2(2x)\right)$$

$\cancel{\frac{1}{4}\left(1+\cos^2 x\right.}$

$\cancel{\frac{1}{4}\left(\frac{1+\cos(2x)}{2}\right.}$

$$\frac{1}{4}\left((1+\cos^2 x)(1-\cos^2 x)\right)$$

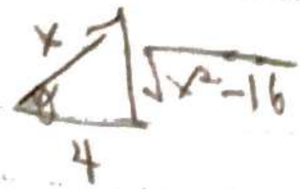
$$\frac{1}{4}\left(1-\cos^2 x\right) \qquad \frac{1}{4}\left(x-\frac{\sin^3 x}{3}\right)$$

$$\frac{1}{4}\left(x-\sin^2 x\right)$$

7.4 integration of rational functions by Partial Fractions

$x = 4\sec\theta$

$\sec\theta = \frac{x}{4}$



$\cos\theta = \frac{4}{x}$

$\theta = \cos^{-1}\left(\frac{4}{x}\right)$

$\frac{P(x)}{Q(x)}$ — polynomial / polynomial

$$\frac{2}{x-3} - \frac{1}{x-1}$$

$$= \frac{2(x-1)}{(x-3)(x-1)} - \frac{1(x-3)}{(x-1)(x-3)}$$

$$= \frac{2x-2}{(x-3)(x-1)} - \frac{x-3}{(x-1)(x-3)}$$

$$= \frac{x+1}{(x-3)(x-1)}$$

$$= \frac{x+1}{x^2-4x+3}$$

$$\int \frac{x+1}{x^2-4x+3}\,dx$$

$$= \int\left(\frac{2}{x-3} - \frac{1}{x-1}\right)dx$$

$$= 2\ln|x-3| - \ln|x-1| + C$$

$$\boxed{(x-1)(x-3)}\left(\frac{x+1}{(x-1)(x-3)}\right) = \left(\frac{A}{x-1} + \frac{B}{x-3}\right)(x-1)(x-3)$$

$$x+1 = A(x-3) + B(x-1)$$

$$x+1 = Ax - 3A + Bx - B$$

$$1x + 1 = x(A+B) - 3A - B$$

Solve coeff by sub

$$\begin{cases} A + B = 1 \\ 1 = -3A - B \end{cases}$$

$-2A = 2$ $\boxed{A = -1}$

$-1 + B = 1$ $\boxed{B = 2}$

$$x+1 = A(x-3) + b(x-1)$$

if $x=3$: $4 = 2b$

$$\boxed{b=2}$$

if $x=1$: $-2 = -2A$

$$\boxed{A=-1}$$

---

def: let $f(x)$ and $g(x)$ be polynomials Then the rational funct $\frac{f(x)}{g(x)}$ is called a proper rational funct if $\deg(f) < \deg(g)$, Otherwise it is an improper rational funct

theorem: (Partial Fractions Part I)

Suppose $f(x)/g(x)h(x)$ is a proper rational funct. then we can always produce a partial function. decomposition

$$\frac{f(x)}{g(x)h(x)} = \frac{A(x)}{g(x)} + \frac{b(x)}{h(x)}$$

where A & b are polynomials

bottom larger than top so decomp ↓

deg(A) < deg(g) B

deg(b) < deg(h)

ex: $\int \frac{5x^2+2x-13}{(x^2+x-4)(x-1)}\,dx$

$$\frac{5x^2+2x-13}{(x^2+x-4)(x-1)} = \frac{Ax+B}{x^2+x-4} + \frac{C}{x-1}$$

$$5x^2+2x-13 = (Ax+B)(x-1) + C(x^2+x-4)$$

if $x=1$

$$\frac{-6}{-2} = \frac{-2C}{-2}$$

$$\boxed{3 = C}$$

Compare x^2 coefficient

$$5 = A + C$$

$$5 = A + 3$$

$$\boxed{A = 2}$$

compare x coefficients

$$2 = B - A + C$$

$$2 = B - 2 + 3$$

$$\boxed{B = 1}$$

$$\int \frac{5x^2+2x-13}{(x^2+x-4)(x-1)}\,dx = \int \left(\frac{2x+1}{x^2+x-4}+\frac{3}{x-1}\right)dx$$

$$\int \frac{2x+1}{x^2+x-4}\,dx + \int \frac{3}{x-1}\,dx$$

$$= \ln|u| + 3\ln|x-1| + C$$

$$\ln|x^2+x-4| + 3\ln|x-1| + C$$

$(x-1)^5$ – can't find roots

galois theory

create crypto system

super computer

where $r_i \in \mathbb{C}$

---

coefficients of p are real and $z \in \mathbb{C}$ is a root, then $P(z)$ is also a root

$(x-(a+ib))(x-(a-ib))$

Partial Fractions (Part II)

Suppose $f(x)/g(x)$ is a proper rational fract w/ n N then we can always produce a partial fraction decomposition

$$\frac{f(x)}{[g(x)]^n} = \frac{A(x)}{g(x)} + \frac{A_2(x)}{[g(x)]^2} + \dots \frac{A_n(x)}{[g(x)]^n}$$

• where $\deg(A_n) < \deg(g)$

degree zero are constants

ex: $\int \frac{x}{(x+3)^2}\,dx = \frac{A}{x+3} + \frac{B}{(x+3)^2}$

must be less

$-3 \quad \frac{-1}{-1}$

$x = A(x+3) + B$

$-3\int \frac{1}{u^2}\,dx$

remember

$A = 1$

$-3\int u^{-2}\,dx$

du

$B = -3$

$\int \frac{1}{u}\,dx$

$$\int \frac{1}{x+3}\,dx + \int \frac{-3}{(x+3)^2}\,dx$$

$$\boxed{\ln(x+3) + \frac{-3(x+3)^{-1}}{-1} + C}$$

ex: $\int \frac{x^2 - 5x + 16}{(2x+1)(x-2)^2}\,dx$

$$\frac{x^2-5x+16}{(2x+1)(x-2)^2} = \frac{A}{2x+1} + \frac{\alpha(x)^b}{(x-2)^2}$$

$$= \frac{A}{2x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$x^2-5x+16 = A(x-2)^2 + B(x-2)(2x+1) + C(2x+1)$$

$10 = 5x$

$C = 2$

$A = 3 \qquad 1 + 10 + 64$

Comparing x^2 coefficients

$1 = A + 2B$

$$\int \left(\frac{3}{2x+1} + \frac{-1}{x-2} + \frac{2}{(x-2)^2}\right) dx$$

$$3\int \frac{1}{2x+1}\,dx$$

$$u = 2x+1$$

$$du = 2\,dx$$

$$\frac{1}{2}\,du = dx$$

$$I + J + K = \frac{3}{2}\ln|2x+1| - \ln|x-2| - \frac{2}{x-2} + C$$

$$\int \frac{x^3-4x-10}{x^2-x-6}\,dx$$

$$\begin{array}{r|l} & \quad x+1 \\ \hline x^2-x-6 & x^3+0x^2-4x-10 \\ & \underline{x^3-x^2-6x} \\ & \qquad x^2+2x-10 \\ & \qquad \underline{x^2-x-6} \\ & \qquad\qquad 3x-4 \end{array}$$

$$\frac{x^3-4x-10}{x^2-x-6} = x+1+\frac{3x-4}{x^2-x-6}$$

$$= x+1+\frac{3x-4}{(x-3)(x+2)}$$

$$\frac{3x-4}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}$$

$$A = 1,\ B = 2$$

$4\ln(|x+4|) + 3\left(\frac{(x+4)^{-1}}{-1}\right) + c$

just find right one

$\int \frac{4}{(x+4)}dx + \int \frac{3}{(x+4)^2}dx$

$19 = -4(B-4) - 4b$

$19 =$

$6 = A + 2$
$-2 \quad -2$

$\boxed{B = 3}$

$\boxed{B = 2}$

$\boxed{4 = A}$ $(x-4)^2$

$-B - 4 = A$

$A(x+1) + B$

$-4 = A + B$

$19 = -4A - 4B$

$-4x + 19 = A(x-4) + B$

$-4 = A + 3$
$-3 \quad -3$

$\boxed{-7 = A}$

$(x-4)(x-4)$

$-4x + 19 = A(x-4) + B(x-4)$

$-4x + 19 = Ax - 4A + Bx - B4$

$$\int \frac{3}{x+5}\,dx + \int \frac{2}{x-2}\,dx$$

$$3\int \frac{1}{x+5}\,dx + 2\int \frac{1}{x-2}\,dx$$

$u = x+5 \qquad u = x-2$

$du = 1dx \qquad du = 1dx$

$$3\int \frac{1}{u}\,du + 2\int \frac{1}{u}\,du$$

$$3\ln u + 2\ln u$$

$$3\ln(x+5) + 2\ln(x-2) + C$$

$$(t+4)(t+5)\left(\frac{18t+82}{t^2+9t+20}\right) = \left(\frac{A}{(t+4)} + \frac{B}{(t+5)}\right)(t+4)(t+5)$$

$$18t + 82 = A(t+5) + B(t+4)$$

$18(-4) + 82 = A \qquad \frac{-8}{-1} = \frac{B-1}{-1} \qquad 10\ln(t+4) + 8\ln(t+5)$

$A = 10 \qquad 8 = B \qquad \int \frac{10}{(t+4)} + \frac{8}{(t+5)}$

$$5x-13 = \frac{A}{\quad} + \frac{b}{\quad} + \frac{c}{\quad}$$

$$5x-13 = \frac{A}{(x+2)} + \frac{b}{(x^2+1)}$$

$$5x-13 = A(x^2+1) + b(x+2)$$

if -2:

$$-13 = A + 2b$$

$$-10-13 = A5$$

$$\boxed{\frac{-23}{5} = \frac{A\cancel{5}}{\cancel{5}}}$$

$$-13 = \left(\frac{-23}{5}\right) + 2b$$

$$\frac{-42}{5} = 2b$$

$$\frac{-5}{21}$$

$$\int \frac{5x-10}{(x+2)(x^2+1)}\,dx$$

$$\frac{A}{(x+2)} + \frac{B}{(x^2+1)}$$

$$5x-10 = A(x^2+1) + B(x+2)$$

$$5x-10 = Ax^2 + 1 + Bx + 2B$$

$$5 = B + 2$$
$$-2 \qquad -2$$

$$\boxed{3 = B} \qquad \boxed{0 = A}$$

$$\int \frac{5x-10}{(x+2)(x^2+1)}\,dx$$

$$\frac{A}{(x+2)} + \frac{B}{(x-1)} + \frac{C}{(x+1)}$$

$$5x-10 = A(x-1)(x+1) + B(x+2)(x+1) + C(x+2)(x-1)$$

$$-20 = A(-2)(1)$$

$$5-1 = B(\quad(\quad$$

$$\frac{-20}{-2} = \frac{-2A}{-2}$$

$$\boxed{-\frac{5}{6} = B}$$

$$\boxed{10 = A}$$

$$-5-10 = C(1)(-2)$$

$$\boxed{\frac{-15}{-2} = \frac{C-2}{-2}}$$

$$\frac{10}{(x+2)} + \frac{(-5)}{6(x-1)} + \frac{(15)}{2(x+1)}$$

$$10\int \frac{1}{x+2}\,dx + \frac{(-5)}{6}\int \frac{1}{x-1}\,dx + \frac{15}{2}\int \frac{1}{(x+1)}\,dx$$

$-6 \int \frac{\cancel{2x-6}}{u \cancel{2x+1}} \, du$ $\quad \frac{2x-5}{x^2+x+6} + \frac{6}{(x-4)}$

$-6 \ln(x^2+x+6) \quad + 6 \ln(x-4)$

$\cancel{\frac{A}{(x-4)} + \frac{B}{} + \frac{C}{}}$

$$\frac{8x^2-x+32}{(x-4)(x^2+x+6)} = \frac{Ax+b}{(x^2+x+6)} + \frac{C}{(x-4)}$$

$$8x^2-x+32 = (Ax+b)(x-4) + C(x^2+x+6)$$

$$\frac{156}{26} = \frac{C(26)}{26} \qquad -1 = B - A + C$$

$\boxed{6 = C}$ $\qquad -2+6$

$8 = A + C \qquad B + 4$
$\qquad\qquad\qquad -4$

$\boxed{2 = A}$ $\quad 8 = A + 6 \quad \boxed{-5 = B}$
$\qquad -6 \quad -6$

$$5x^2+72x+139 = \frac{A}{(x-4)} + \frac{B}{(x+9)} + \frac{C}{(x+9)^2}$$

$$5x^2+72x+139 = A(x+9)^2 + B(x-4)(x+9) + C(x-4)$$

if -9

$$\frac{-104}{-13} = \frac{C(-13)}{-13}$$

$\cancel{72 = 16A + 5b + C}$ $\boxed{8 = C}$

$(x+9)(x+9) =$

$A(x^2+9x+9x+81) + B(x^2+9x-4x-36)$

$16x$ $5x$

$C(x-4)$

Nov 9th 2nd test Wednesday

<u>whats On there</u>

int of trig funct $\boxtimes$

cocept $\square$
product
sine and
Cosine

trig sub $\boxtimes$

partial fract $\boxtimes$

(1) Approximate integration $\square$

l'hospital $\square$

$$\begin{array}{r} x \\ x^2+4 \overline{) x^3+0x^2+0x+4} \\ x^3 \qquad\quad +4 \\ \hline -4x+4 \end{array}$$

Improper integrals $\square$

$\frac{-4x+4}{x^2+4}$

degree upstairs Larger = improper fraction

(2) $\int \frac{x^3+4}{x^2+4}dx = \int \left(x + \frac{-4x+4}{x^2+4}\right)dx$

$\frac{-4x+4}{x^2-4} = \frac{-4x}{x^2-4} + \frac{4}{x^2-4}$

(3) $= \underbrace{\int x\,dx}_{I} + \underbrace{\int \frac{-4x}{x^2+4}dx}_{J} + \underbrace{\int \frac{4}{x^2+4}dx}_{K}$

$$\boxed{\frac{1}{2}x^2 - 2\ln(x^2+4) + 2\tan^{-1}\left(\frac{x}{2}\right) + C}$$

$$I = \int x\,dx = \boxed{\frac{1}{2}x^2 + C_1} \quad ①$$

$$J = -2\int \frac{2x}{x^2+4}\,dx$$

$$= -2\int \frac{1}{u}\,du$$

$$\theta = \tan^{-1}\left(\frac{x}{2}\right) \quad ②$$

$$= -2\ln|u| + C$$

$$\boxed{-2\ln(x^2+4) + C_2}$$

$$K = \int \frac{4}{x^2+4}\,dx \qquad x = 2\tan\theta$$

$$K = \int \frac{4}{x^2+4}\,dx \qquad x = 2\tan\theta$$

$$K = \int \frac{4}{(2\tan\theta)^2+4}(2\sec^2\theta)\,d\theta \qquad \frac{dx}{d\theta} = 2\sec^2\theta$$

$$dx = 2\sec^2\theta\,d\theta$$

$$2\int \frac{1}{4(\tan^2\theta+1)}\sec^2\theta\,d\theta$$

$$2\int d\theta = 2\theta + C_3 =$$

$$= 2\int \frac{1}{\sec^2\theta}\sec^2\theta\,d\theta \quad ③ \quad \boxed{2\left(\tan^{-1}\left(\frac{x}{2}\right)\right) + C}$$

$$\int \frac{1}{x^4-1}\,dx$$

$$\int \frac{1}{(x-1)(x+1)(x^2+1)}\,dx$$

$$\frac{1}{(x-1)(x+1)(x^2+1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{(x^2+1)}$$

$$A(x+1)(x^2+1) + B(x-1)(x^2+1) + C(x+D)(x-1)(x+1)$$

Compare coefficients

x^3

$0 = A + B + C$

$x=1$: $1=4A$

$A = \frac{1}{4}$

$x=-1$: $1=-4B$ $\quad b=-\frac{1}{4}$

$x=0$: $1 = A - B - D$ $\quad \boxed{D=-\frac{1}{2}}$

$$\int\left(\frac{\frac{1}{4}}{x-1}+\frac{-\frac{1}{4}}{x+1}+\frac{-\frac{1}{2}}{x^2+1}\right)dx$$

$$=\frac{1}{4}\int\frac{1}{x-1}dx-\frac{1}{4}\int\frac{1}{x+1}dx-\frac{1}{2}\int\frac{1}{x^2+1}dx$$

cause it's 1

$$=\frac{1}{4}\ln|x-1|-\frac{1}{4}\ln|x+1|-\frac{1}{2}\tan^{-1}x+C$$

$$\int\frac{1}{x^3-1}dx$$

complete the square

$$\frac{2x+1}{(x-2)(x+3)(x^2+7)(x+4)^2}+B(x-2)x^2+$$

$$\boxed{\begin{array}{l}\int e^{x^2}dx \\ \int\sqrt{x^3+1}\,dx\end{array}}$$

Not Elementary

$$\int\frac{x}{(x^2+8x+17)^2}dx$$

$$\frac{x}{(x^2+8x+17)^2}=\frac{Ax+B}{x^2-8x+17}+\frac{(x+)}{(x^2-8x+17)^2}$$

$$\frac{f(x)}{g(x)h(x)} = \frac{a(x)}{g(x)} + \frac{b(x)}{h(x)}$$

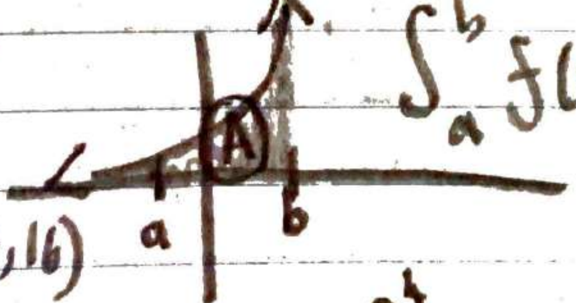
where deg(a)<

$$\frac{f(x)}{[g(x)]^n} = \frac{a_1}{g(x)} + \frac{a_2}{g(x)^2} + \frac{a_3}{g(x)^3}$$

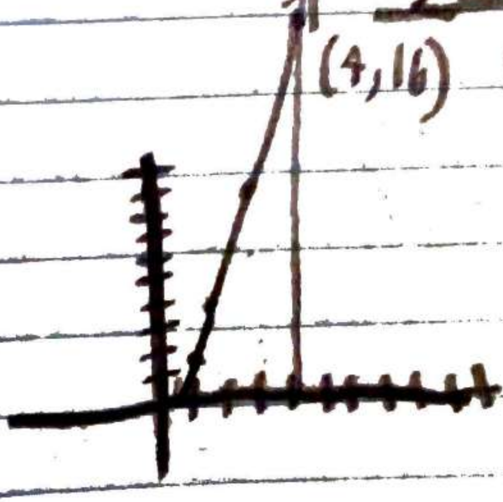
where deg (a_i) < deg(g)

§7.7 Approximate Integration:

infinite rectangle is integration



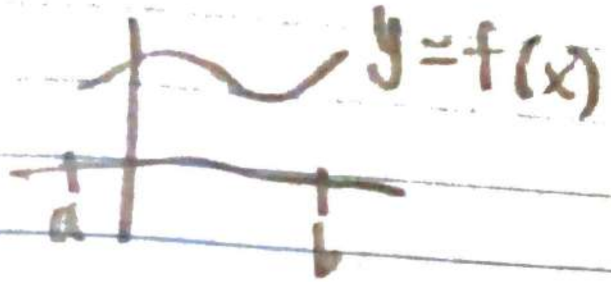
$$\int_a^b f(x)\,dx = A$$



$$\int_0^4 x^2\,dx$$

Midpoint Formula

$F(x) = A$



$$\int_a^b f(x)\,dx \approx M_n = \Delta x\left(f(\bar{x}_1)+f(\bar{x}_2)\cdots f(\bar{x}_n)\right)$$

where $\Delta x = \frac{b-a}{n}$ and $\bar{x_i} =$

21

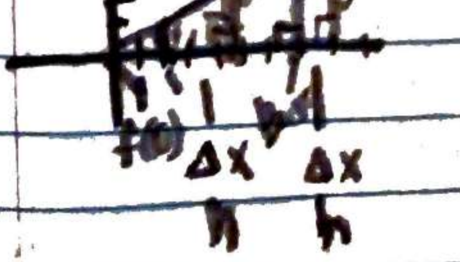
$$\frac{x_i + x_{i+1}}{2}$$

21 $\sqrt{3}$

Midpoint Rule

Trapezoid Rule

A() $= \left(\frac{b_1 b_2}{2}\right)h$



$$\int_0^4 x^2\,dx \approx T_4 = \overset{1}{\Delta x}\left(\frac{f(0)+f(1)}{2}\right)$$

$\Delta x \frac{1}{2}$

Trapazoid Rule

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}\left(\frac{b^1 + b^2}{2}\right)$$

§ 4.4 Indeterminante form and l'Hôp

ex: $\lim_{x \to 1} \frac{x-1}{x^2-1}$

Notice $\lim_{x \to 1} (x-1) = 0$

$\lim_{x \to 1} (x^2-1) = 0$

$\boxed{\frac{0}{0}}$ = inderminant

$= \lim_{x \to 1} \frac{x-1}{(x-1)(x+1)} = \lim_{x \to 1} \frac{1}{x+1} = \frac{1}{2}$

ex: $\lim_{x \to 1} \frac{x-1}{(x-1)^2} = \lim_{x \to 1} \frac{1}{x-1}$ DNE

$\lim_{x \to 1} (x-1) = 0$

$\lim_{x \to 1} (x-1)^2 = 0$

ex: $\lim_{x\to\infty} \frac{x-1}{x^2+2x-7} = \lim_{x\to\infty} \frac{x-1\left(\frac{1}{x^2}\right)}{x^2+2x-7\left(\frac{1}{x^2}\right)}$

$\frac{0}{0} = \text{DNE}$

$\boxed{\frac{0}{1} = 0}$ may or may not exist

$\boxed{\frac{0}{\infty}}$

notice:

$$\lim_{x\to 1} \frac{x-1}{x^2-1} = \lim_{x\to 1} \frac{\frac{d}{dx}(x-1)}{\frac{d}{dx}(x^2-1)}$$

$$= \lim_{x\to 1} \frac{1}{2x} = \boxed{\frac{1}{2}}$$

$$\lim_{x-1} \frac{x-1}{(x-1)^2} = \frac{\frac{d}{dx}(x-1)}{\frac{d}{dx}(x^2-2x+1)}$$

(divided by zero doesn't exist)

$$= \lim_{x\to 1} \frac{1}{2x-2} \text{ DNE } \boxed{\frac{1}{0}}$$

ex: $\lim_{x \to \infty} \frac{x-1}{x^2+2x-7} = \lim_{x \to \infty} \frac{x-1\left(\frac{1}{x^2}\right)}{x^2+2x-7\left(\frac{1}{x^2}\right)}$

$\frac{0}{0} = DNE$

$\boxed{\frac{0}{1} = 0}$ may or may not exist

$\boxed{\frac{0}{\infty}}$

notice:

$$\lim_{x \to 1} \frac{x-1}{x^2-1} = \lim_{x \to 1} \frac{\frac{d}{dx}(x-1)}{\frac{d}{dx}(x^2-1)}$$

$$= \lim_{x \to 1} \frac{1}{2x} = \boxed{\frac{1}{2}}$$

$$\lim_{x-1} \frac{x-1}{(x-1)^2} = \frac{\frac{d}{dx}(x-1)}{\frac{d}{dx}(x^2-2x+1)}$$

(divided by zero doesn't exist)

$$= \lim_{x \to 1} \frac{1}{2x-2} \text{ DNE } \boxed{\frac{1}{0}}$$

limit compares how fast 2 things are growing

$$\lim \frac{x^5}{e^x} = \text{approaching zero}$$

larger/faster

(Prime Numbers (credit cards))

factorization

exponential time (efficient)

If we have a limit of the form

$\lim_{x \to a} \frac{f(x)}{g(x)}$ where both

$f(x) \to \infty$ and $g(x) \to \infty$

$f \to 0$ and $g(x) \to 0$, $\frac{0}{0}$ $\frac{\infty}{\infty}$

~~as $x \to a$, then this is~~

then this limit may or may not exist and is called an indeterminate form of type $\frac{0}{0}$

[redacted] L'Hospital's Rule:

Suppose f and g are differentiable and $g'(x) \neq 0$ when x is near a

Suppose $\lim_{x \to a} f(x) = 0$ & $\lim_{x \to a} g(x) = 0$

or $\lim_{x \to \infty} f(x) = \pm\infty$ & $\lim_{x \to \infty} g(x) = \pm\infty$

Then $$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

ex: $$\lim_{x \to 1} \frac{\ln x}{x-1}$$

Note $$\lim_{x \to 1} \ln x = 0$$

$$\lim_{x \to 1} (x-1) = 0$$

So $\lim \frac{\ln x}{x-1}$ is of indeterminate form type $\frac{0}{0}$

$$\lim_{x \to 1} \frac{\ln x}{x-1} = \lim_{x \to 1} \frac{\frac{d}{dx}(\ln x)}{\frac{d}{dx}(x-1)}$$

$$\lim_{x \to 1} = \frac{\frac{1}{x}}{1}$$

$$\lim_{x \to 1} \frac{1}{x} = \boxed{1}$$

---

ex: $\lim\limits_{x \to \infty} \dfrac{x^2}{e^x}$ — should be zero

Note: $\lim\limits_{x \to \infty} (x^2) = \infty$

$\lim\limits_{x \to \infty} (e^x) = \infty$ This is a limit of type $\frac{\infty}{\infty}$,

we may apply L'Hospital's Rule:

$$\lim_{x \to \infty} \frac{x^2}{e^x} = \lim_{x \to \infty} \frac{\frac{d}{dx}(x^2)}{\frac{d}{dx}(e^x)}$$

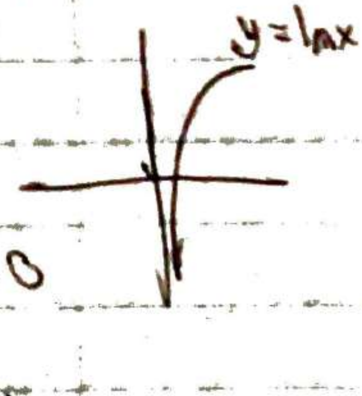
$$\lim_{x\to\infty} \frac{2x}{e^x}$$

Here: $\lim_{x\to\infty}(2x)=\infty$ and $\lim_{x\to\infty} e^x=\infty$

So $\lim \frac{2x}{e^x}$ is of indeterminate form $\frac{\infty}{\infty}$ i.e. we may apply L'H Rule again:

$$\lim_{x\to\infty}\frac{2x}{e^x} = \lim_{x\to\infty}\frac{\frac{d}{dx}(2x)}{\frac{d}{dx}(e^x)} = \lim_{x\to\infty}\frac{2}{e^x}=0$$

ex $\lim_{x\to 0^+}\frac{\ln x}{x} = -\infty$, $\lim_{x\to 0^+}\ln x = -\infty$



$\lim_{x\to 0^+}$ $x=0$

We cannot use L'H Rule

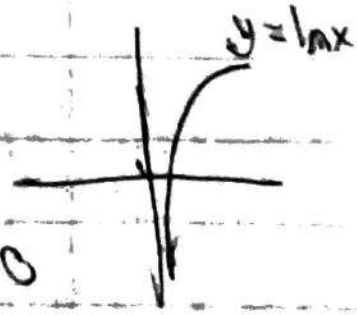
$$\lim_{x\to\infty} \frac{2x}{e^x}$$

Here: $\lim\limits_{x\to\infty}(2x)=\infty$ and $\lim\limits_{x\to\infty} e^x=\infty$

So $\lim \frac{2x}{e^x}$ is of indeterminate form $\frac{\infty}{\infty}$ i.e. we may apply L'H Rule again:

$$\lim_{x\to\infty}\frac{2x}{e^x}=\lim_{x\to\infty}\frac{\frac{d}{dx}(2x)}{\frac{d}{dx}(e^x)}=\lim_{x\to\infty}\frac{2}{e^x}=0$$

ex $\lim\limits_{x\to0^+}\frac{\ln x}{x}=-\infty$, $\lim\limits_{x\to0^+}\ln x=-\infty$



$\lim\limits_{x\to0^+} x=0$

We cannot use L'H Rule

$\left(\frac{-\infty}{-\infty}\right)$ or $\left(\frac{+\infty}{+\infty}\right)$

More indeterminate forms

$0 \cdot \infty$

$\infty - \infty$

ex: $\lim_{x \to 0^+} x \ln x \to \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x}}$

$\lim_{x \to 0^+} \ln x = -\infty$

$\lim_{x \to 0^+} x = 0$

$\lim_{x \to 0^+} \frac{1}{x} = \infty$

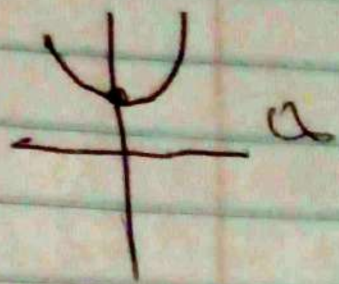
$\lim_{x \to 0^+} \ln x = -\infty$

4) $= \boxed{\lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x}} = \lim_{x \to 0^+} (-x) = 0}$

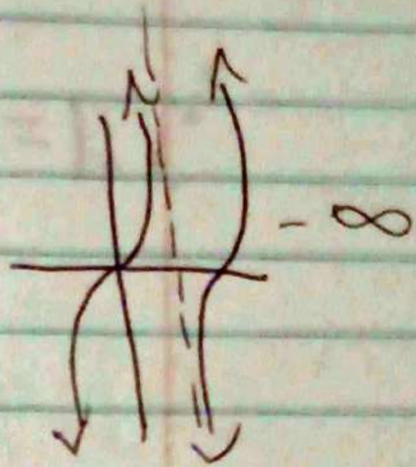
So have indeterminate form type $\frac{\infty}{\infty}$, so may use L'H Rule:

1) $\lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x}}$ 2) $= \lim_{x \to 0^+} \frac{\frac{d}{dx}(\ln x)}{\frac{d}{dx}(x^{-1})}$ 3) $= \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}}$

ex: $\lim\limits_{x \to (\frac{\pi}{2})^-} (\sec x - \tan x)$



$= \lim\limits_{x \to (\frac{\pi}{2})^-} \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x} \right)$



$= \lim\limits_{x \to (\frac{\pi}{2})^-} \left(\frac{1 - \sin x}{\cos x} \right)$

$= \lim\limits_{x \to (\frac{\pi}{2})} \frac{-\cos x}{-\sin x} = \lim\limits_{x \to (\frac{\pi}{2})} \cot x$

$= \boxed{0}$

ex:

$\lim\limits_{x \to 0} \overset{\mp\infty}{\overset{1}{x^{-1}}} \overset{0}{\sin x}$

li. $\lim\limits_{x \to 0} \left(\frac{\sin x}{x} \right)$

type $\frac{0}{0}$

$$\lim_{x \infty} (x^5 - \ln x)$$

$$= \lim_{x \to \infty} (\ln e^{x^5} - \ln x)$$

$x^5 = \ln e^{x^5}$

$$= \lim_{x \to \infty} \ln \frac{e^{x^5}}{x}$$

$$= \ln \lim_{x \to \infty} \frac{e^{x^5}}{x}$$

$\frac{\infty}{\infty}$

$$= \ln \lim_{x \infty} \frac{5x^4 e^{x^5}}{1}$$

$$= \boxed{\infty}$$

Indeterminate Powers:

0^0

1^∞

∞^0

ex: $\lim_{x\to\infty} \left(1+\frac{2}{x}\right)^{3x} \quad 3x - \infty$

type 1^∞

Let $L = \lim_{x\to\infty} \left(1+\frac{2}{x}\right)^{3x}$

$\ln L = \lim_{x\to\infty} 3x \ln\left(1+\frac{2}{x}\right)$

until you get to type $\frac{0}{0}$ / $\frac{\infty}{\infty}$

$\ln L = \lim_{x\to\infty} \dfrac{\ln\left(1+\frac{2}{x}\right)}{\left(\frac{1}{3x}\right)}$ type $\frac{0}{0}$

$\ln L = \lim_{x\to\infty} \dfrac{\frac{1}{1+\frac{2}{x}}(-2x^{-2})}{-\frac{1}{3}x^{-2}}$

$\ln L = \lim_{x\to\infty} \dfrac{\frac{2}{1+\frac{2}{x}}}{\frac{1}{3}}$

$\ln L = 3\lim_{x\to\infty} \frac{2}{1} = 3(2) = 6$

$\ln L = 6$

$\boxed{L = e^6}$

$$x = e^{\ln x}$$

$$\lim_{x \to 0^+} \left(e^{\ln x}\right)^x$$

$$\lim_{x \to 0^+} e^{x \ln x}$$

$$= e^{\lim_{x \to 0^+} x \ln x}$$

$$= e^{\lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x}}}$$

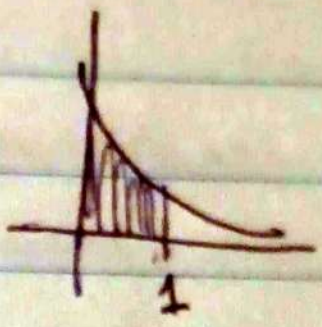
$$= e^{\lim_{x \to 0^+} (-x)}$$

$$= e^0 = \boxed{1}$$

7.8 Improper Integrals

II Type II: Discontinuos intervals.

$$\int_0^1 \frac{1}{\sqrt{x}}\,dx$$



$$= \lim_{a \to 0^+} \int_a^1 \frac{1}{\sqrt{x}}\,dx$$

$$= \lim_{a \to 0^+} \left(2x^{\frac{1}{2}}\right|_a^1$$

$$= \lim_{a \to 0^+} \left(2 - 2\sqrt{a}\right)$$

$$= 2 - 2(0)$$

$$= \boxed{2}$$

def: If f is continuos on [a,b)
and discontinuos at b,

then $\int_a^b f(x)\,dx = \lim_{t \to b^-} \int_a^t f(x)\,dx$

If f is continuos on (a,b]
and discontinuos at a, then

$\int_a^b f(x)\,dx = \lim_{t \to a^+} \int_t^b f(x)\,dx$

The improper integral $\int_a^b f(x)\,dx$ is
called convergent if the corresponding
limit exist and divergent if it
does not exist

- If f has a discontinuity at $c \in (a,b)$
and both $\int_a^c f(x)\,dx$ and $\int_c^b f(x)\,dx$ coverage,
then $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b (f(x))\,dx$

discont at 1

ex: $\int_0^9 \frac{1}{\sqrt[3]{x-1}}\,dx = \int_0^1 \frac{1}{\sqrt[3]{x-1}}\,dx + \int_1^9 \frac{1}{\sqrt[3]{x-1}}\,dx$

$= \lim\limits_{b \to -1} \int_0^b (x-1)^{-1/3}\,dx + \lim\limits_{a \to 1^+} \int_a^9 (x-1)^{-1/3}\,dx$

$\lim\limits_{b \to -1} \frac{3(x-1)^{2/3}}{2}\Big|_0^b + \lim\limits_{a \to +1} \frac{3(x-1)^{2/3}}{2}\Big|_a^9$

$\frac{3}{2}(-1) + \frac{3}{2}(4)$

$-\frac{3}{2}[(b-1)^{2/3} - 1]$ $\qquad$ $\frac{3}{2}[4-(a-1)^{2/3}]$

integral doesn't exist if it diverges to infinity ex: $\int_0^1 \ln x \, dx = \lim_{a \to 0^+} \int_a^1 \ln x \, dx$

discont

$$= \lim_{a \to 0^+} \left(x \ln x \Big|_a^1 - \int_a^1 \frac{x}{x} \, dx \right)$$

$$= \lim_{a \to 0^+} \left(x \ln x \Big|_a^1 - x \Big|_a^1 \right)$$

$$= \lim_{a \to 0^+} \left(\underbrace{-a \ln a}_{-\infty} - \underbrace{(1-a)}_{0} \right)$$

Use Hopital rule form $\frac{0}{0}$

$$\lim_{a \to 0^+} \left(\left(\frac{-\ln a}{\frac{1}{a}} \right) - 1 + a \right)$$

$$\lim_{a \to 0^+} \left(\frac{\ln a}{\frac{1}{a}} \right) \left(\left(\frac{-\ln a}{\frac{1}{a}} \right) 0 - 1 + a \right)$$

$$\lim_{a \to 0^+} \left(\frac{\frac{1}{a}}{-\frac{1}{a^2}} \right)$$

$$\lim_{a \to 0^+} (-a) = 0$$

(Improper integrals are either Infinite or have a discontinuity)

ex: Type I does n't converge

trig sub tangent

non - intensive

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$$

$$= \int_{-\infty}^{0} \frac{1}{1+x^2}\,dx + \int_{0}^{\infty} \frac{1}{1+x^2}\,dx$$

$$\lim_{a \to -\infty} \int_a^0 \frac{1}{1+x^2}\,dx + \lim_{b \to \infty} \int_0^b \frac{1}{1+x^2}\,dx$$

$$= \lim_{a \to -\infty} \tan^{-1} x \Big|_a^0 + \lim_{b \to \infty} \tan^{-1} x \Big|_0^b$$

$$= \lim_{a \to -\infty} (-\tan^{-1} a) + \lim_{b \to \infty} (\tan^{-1} b)$$

$$-\left(-\frac{\pi}{2}\right) + \frac{\pi}{2}$$

$$= \boxed{\pi}$$

undefined at 2 and
inbetween integral is problematic

ex: $\int_0^5 \frac{x}{x-2}\,dx$

$= \int_0^2 \frac{x}{x-2}\,dx + \int_2^5 \frac{x}{x-2}\,dx$

let's look at $\int_0^2 \frac{x}{x-2}\,dx$

$$x-2 \overline{\big)\, x - 0}$$
$$\frac{x-2}{2}$$

$\lim\limits_{b \to 2^-} \left(x + 2\ln|x-2|\right)\Big|_0^b \qquad \frac{x}{x-2} = \left(1 + \frac{2}{x-2}\right)$

infinite

$\int_0^2 \left(1 + \frac{2}{x-2}\right)dx$

$\lim\limits_{b \to 2^-} \left(b + 2\ln|b-2| - 2\ln 2\right) = \lim\limits_{b \to 2^-} \int_0^b \left(1 + \frac{2}{x-2}\right)dx$

$= -\infty$

$\Rightarrow \int_0^5 \frac{x}{x-2}\,dx$ diverges

(doesn't equal a finite value

$$\int_0^{\pi/2} \sin^7\theta \cos^5\theta \, d\theta$$

$$\int_0^{\pi/2} \sin\theta \cos\theta \left(\sin^6\theta \cos^4\theta\right) d\theta$$

$u = \sin\theta$

$$\frac{du}{\cos\theta} = \frac{\cos\theta \, d\theta}{\cos\theta}$$

$$u \cos\theta \left(u^6 (1-u^2)^2\right) du$$

$$\int_0^{\pi/2} \frac{u \cancel{\cos\theta} \left(u^6 (1-u^4)\right)}{\cancel{\cos\theta}} du$$

$1-\sin^2\theta$

$$\int_0^{\pi/2} u\left(u^6 (1-u^4)\right) du$$

$$\int_0^{\pi/2} u\left(u^6 - u^{10}\right) du$$

$$\int_0^{\pi/2} \left(u^7 - u^{11}\right) du$$

$$\frac{\sin\theta^8}{8} - \frac{\sin\theta^{12}}{12} \qquad \frac{u^8}{8} - \frac{u^{12}}{12}$$

$$\int_0^{\pi/2} \sin^7\theta \cos^5\theta \, d\theta$$

$$\int_0^{\pi/2} (\sin^6\theta)(\sin\theta)\cos^5\theta \, d\theta$$

$$\int_0^{\pi/2} (\sin^2\theta)^3(\sin\theta)\cos^5\theta \, d\theta$$

$$\int_0^{\pi/2} (1-\cos^2\theta)^3(\sin\theta)\cos^5\theta \, d\theta$$

$u = \cos\theta$

$$\int_0^{\pi/2} \frac{(1-\cos^2\theta)^3(\cancel{\sin\theta})\cos^5\theta \, du}{\cancel{\sin\theta}}$$

$$\frac{du}{\sin\theta} = \frac{\sin\theta \, d\theta}{\sin\theta}$$

$$\int_0^{\pi/2} (1-u^2)^3(u5)$$

$$\int_0^{\pi/2} (u^5 - u^{10})^3$$

$$\frac{\cos\theta^{16}}{16} - \frac{\cos\theta^{31}}{31}$$

$$u^{15} - u^{30}$$

$$\frac{u^{16}}{16} - \frac{u^{31}}{31}$$

$$(1-u^2)(1-u^2)(1-u^2)$$

$$(1-u^2-u^2+u^4)$$

$$(1-u^2)(1-2u^2+u^4)$$

$$(-u^5)(1-2u^2+u^4-u^2+2u^4-u^6)$$

$$-u^5+2u^7-u^9+u^7-2u^9+u^{11}$$

$$\boxed{-u^5+3u^7-3u^9+u^{11}}$$

$$\int_0^{\frac{\pi}{2}} (\sin^7\theta)(\cos^4\theta)(\cos\theta)\,d\theta$$

$$\int_0^{\frac{\pi}{2}} (\sin^7\theta)(1-\sin^2\theta)^2(\cos\theta)\,d\theta$$

$$\int_0^{\frac{\pi}{2}} \frac{(\sin^7\theta)(1-\sin^2\theta)^2\cancel{(\cos\theta)}\,du}{\cancel{\cos\theta}}$$

$u = \cos\theta\,d\theta$

$$\int_0^{\frac{\pi}{2}} (\sin^7\theta)(1-\sin^2\theta)^2\,du$$

$u = \sin\theta\,d\theta$

$\frac{du}{\cos\theta} = \frac{\cos\theta\,d\theta}{\cos\theta}$

$$\int_0^{\frac{\pi}{2}} (u^7)(1-u^2)^2\,du$$

$1-2u^2+u^4$

$(u^7-2u^9+u^{11})$

$(u^7)(1-u^2+u^4)$

$(1-u^2)(1-u^2)$

$1-u^2-u^2+u^4$

$\frac{u^8}{8}-2\frac{u^{10}}{10}+\frac{u^{12}}{12}$

Suppose f and g are continuous that w/ $f(x) \geq g(x) \geq 0$ for all $x > a$

(i) If $\int_a^\infty f(x)$ converges, so does

$$\int_1^\infty g(x)\,dx$$

(ii)

If $\int_a^\infty g(x)\,dx$ diverges, so does

$$\int_1^\infty f(x)\,dx$$

ex: Show that $\int_1^\infty e^{-x^2}\,dx$ is convergent

If $x \geq 1$, then $x^2 \geq x$

$\Rightarrow -x^2 \leq -x$

$$\int_1^\infty e^{-x}\,dx = \lim_{t \to \infty} \int_1^t e^{-x}\,dx$$

$$\lim_{t \to \infty} \left(-e^{-x} \Big|_1^t = \lim_{t \to \infty} \left(-e^{-t} + e^{-1}\right)\right.$$

$$= e^{-1}$$

ie taking $f(x) = e^{-x}$
and $g(x)$ $g(x) = e^{-x^2}$
in the comparison theorem
shows if $\int_1^\infty e^{-x} dx$
converges so does $\int_1^\infty e^{-x^2} dx$

ex: shows $\int_1^\infty \frac{1+e^{-x}}{x} dx$ is divergent

$$\frac{1+e^{-x}}{x} > \frac{1}{x} \quad \forall x \geq t$$

for all

$$\int_1^\infty \frac{1}{x} dx = \lim_{t \to \infty} \int_1^t \frac{1}{x} dx =$$

$$\lim_{t \to \infty} \ln|x| \Big|_1^t$$

show a smaller thing diverges

$$= \lim_{t \to \infty} (\ln|t|)$$

$$= \infty$$

Comparison theorem
$\int_1^\infty \frac{1+e^{-x}}{x} dx$ must
also diverge

def ~~A~~ sequence can be thought of a ordered list of values

ex: Sequence can be defined recursively like the fibonacci seq is the seq $\{f_1, f_2, f_3$ where $f_1 = f_2 = 1$ and

$f_n = f_{n-1} + f_{n-2}$ for $n \geq 2$

$\{f_n\}_{n=1}^{\infty} \{1, 1, 2, 3, 5, 8 \ldots\}$

ex: We can also defined seq whose nth term

is given by $f(n) = n^2 - 1$

this seq would b

$\{0, 3, 8, 15, 24 \ldots\}$

Arithmetic Sequence

★ common dif $\{a_n\}$

def: An arithmetic sequence between consecutive terms is constant $(a_{n+1} - a_n)$

ex $\{17, 24, 31, 38, \ldots\}$ $\{7n + 17\}_{n=0}^{\infty}$

$7n + 17$

common dif — first term

ex: $\{1, 3, 5, 7, 9, \ldots\}$ arithmetic seq

$$\{2n + 1\}_{n=0}^{\infty}$$

thm: Let $\{a_n\} = \{a_1, a_2, a_3, \ldots\}$ be an arithmetic sequence with common difference d. Then, $\{a_n\} = \{nd + a_1\}$

II. Geometric Sequences

def: If the ratio of 2 consecutive terms is constant then it a geometric series, the ratio is called a common ratio

ex: $\{360, 180, 90, 45, \ldots\}$

Common ratio $\frac{180}{360}$

Notice seq can be written

$$\{360(\tfrac{1}{2})^n\}_{n=0}^{\infty}$$

common ratio

ex: $\{5, 15, 45, 135 \ldots\}$

$$= \{5(3)^n\}_{n=0}^{\infty}$$

ex: $\{a_n\} = \{3, 2, \frac{4}{3}, \frac{8}{9}\}$

$$\{3(\tfrac{2}{3})^n\}_{n=0}^{\infty}$$

$a_1\left(\frac{x}{y}\right)^n$

Thm: Let $\{a_n\} = \{a_0, a_1, a_2, \dots\}$

be a geometric sequence of

common ratio r

Then $\{a_n\} = \{a_0 r^n\}$

III Alternate Sequences

def: If the terms of a seq alternate between positive and negative values, we call this an alternating sequence

ex: $\{-2, 2, -2, 2, -2, 2, \dots\}$

$\frac{2}{3}\left(-\frac{1}{3}\right)^n$ $= \{(-1)^n \cdot 2\}_{n=1}^{\infty}$

could al

$\frac{2}{3}\left(\frac{1}{3}\right)^n\}$

thm: Let $\{a_n\} = \{a_0, a_1, a_2, \dots\}$

be a geometric sequence of

common ratio r

Then $\{a_n\} = \{a_0 r^n\}$

III Alternate Sequences

def: If the terms of a seq

alternate between positive and

negative values, we call this an

alternating sequence

ex: $\{-2, 2, -2, 2, -2, 2, \dots\}$

$\frac{2}{3}\left(-\frac{1}{3}\right)^n \quad = \{(-1)^n \cdot 2\}_{n \geq 1}^{\infty}$

could al

$\left\{\frac{2}{3}\left(\frac{1}{3}\right)^n\right\}$

def: A sequence $\{a_n\}$ has limit L if we can make the terms a_n as close to L as we want by making n sufficiently large. In this case, we write $\lim_{n\to\infty} a_n = L$ and say the sequence converges if L exists. Otherwise, we say the sequence diverges

ex: $\{3n+4\} = \{a_n\}$

$\lim_{n\to\infty} \boxed{a_n = \infty}$ "this diverges" to infinity

ex: $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$

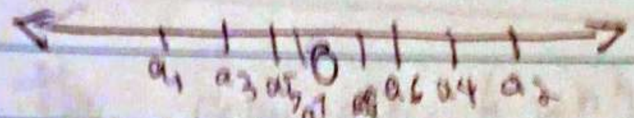
$\lim_{n\to\infty} \boxed{a_n = 0}$

§11.1 Sequences

Thm: Let $\{a_n\}$ be a sequence

I $\lim_{n \to \infty} |a_n| = 0$,

Then $\lim_{n \to \infty} a_n = 0$



1

1

{

ex. $\{a_n\} = \{1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \dots\}$

$\lim_{n \to \infty} |a_n| = 0$

$\lim_{n \to \infty} a_n = 0$

divergent

Q: For what values of r is $\{r^n\}_{n=0}^{\infty}$ convergent?

Consid $|r| > 1$, then $\lim_{n \to \infty} r^n = \infty$

Consider $r = 1$, then $\lim_{n \to \infty} r^n = 1$

Consider $r = -1$, then $\lim_{n\to\infty} r^n$

$$\{1, -1, 1, -1, 1, \ldots\}$$

Consider $|r| < 1$

$$\lim_{n\to\infty} r^n = 0$$

The sequence $\{r^n\}_{n=0}^{\infty}$

Converges only when $-1 < r \leq 1$

$$\lim_{n\to\infty} r^n = \begin{cases} 1 & \text{if } r = 1 \\ 0 & \text{if } |r| < 1 \end{cases}$$

(Diverges otherwise)

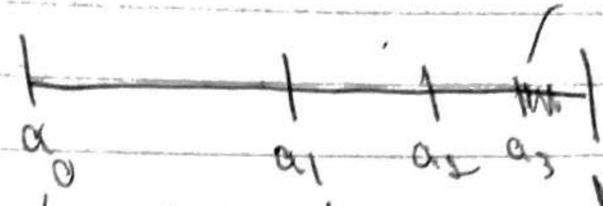
def: A sequence $\{a_n\}$

is called increasing if

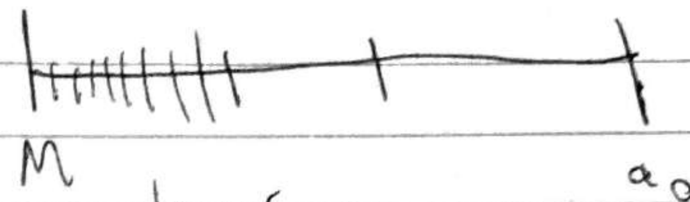
$a_n < a_{n+1}$ $\forall n$ and is
called decreasing if

$a_n > a_{n+1}$ $\forall n$ A sequence which increasing or decreasing is said to be monotonic

must converg if upper



bounded by lower term



thm: A bounded Monotonic sequence is Convergent.

§ 11.2 Series

def: A Series is an ordered Summands (terms)

in a finite series finite number of terms $\sum_{n=1}^{n} a_j = a_1 + a_2 + \dots + a_j + a_n$

$\sum_{n=1}^{\infty} a_j = a_1 + a_2 + a_3 + \dots + a_j$ In an infinite series, there are an infinite number of summands

A Series is an ordered Sum

The Sum of the series is the Value of the sum, if it exists

24.68

ex: 24.68

$2(10)^1 + 4(10)^0 + 6(10)^{-1} + 8(10)^{-2}$

This is a finite series.

ex: $\pi = 3.14159265\ldots$

$\pi = 3(10)^0 + 1(10)^{-1} + 4(10)^{-2}$

This is an infinite series

def: If the terms ~~app~~ of a sum form a arithmetic (geometric, or alternating) sequence, we call it an arithmetic (geometric, alternating) series

ex: $\sum_{i=1}^{4} (2i+1) = 3+5+7+9 = 24$ ← finite arithmetic series ex $\sum_{i=1}^{4}$ =

$$\sum_{i=1}^{4} 2^{i} = 2+4+8+16$$

finite geometric series

$= 30$

$$\sum_{i=1}^{4} 2^{i}$$

Properties of series

(i) $\sum_{n=i}^{j} c a_n$ — you can factor c out

$$= c \sum_{n=i}^{j} a_n \text{, where } c \in \mathbb{R}$$

(ii) $\sum_{n=i}^{k} a_n = \sum_{n=i}^{j-1} a_n + \sum_{n=j}^{k} a_n$ where $i \le j \le k$

Sums of Arithmetic Series:

ex: $\sum_{i=2}^{15}(3i+2) = 8 + 11 + \dots + 44 + 47$

$15-2+1$ $\qquad \sum_{i=2}^{15}(3i+2) = 47 + 44 + \dots + 11 + 8$

← number of terms

$(j-i)+1$ number of terms

$$2\sum_{i=2}^{15}(3i+2) = 55+55 \dots 55+55$$

$$2\sum_{i=2}^{15}(3i+2) = 14(55)$$

$$\sum_{i=2}^{15}(3i+2) = \frac{14(55)}{2}$$

ex: Find the sum of the first n positive integers

$$\sum_{i=1}^{n} i = 1 + 2 + \dots + (n-1) + n$$

$$\sum_{i=1}^{n} i = n + (n-1) + \dots + 2 + 1$$

$$2\sum_{i=1}^{n} i = (n+1) + (n+1) + \dots + (n+1) + (n+1)$$

$$2\sum_{i=1}^{n} i = n(n+1) \Rightarrow \sum_{i=1}^{n} i$$

Partial Sum?

$$\frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n = \frac{1}{2^2} + \frac{1}{2^3}$$

$$\frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n = \frac{1}{2}$$

first $\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n = 1$

$\frac{1}{2}$

A Geometric Series looks lik

$$\sum_{n=0}^{\infty} ar^n = a + ar + ar^3 + ar^3 + \ldots$$

Suppose $r \neq 1$, $\sum_{n=0}^{\infty} ar^n = a + ar + ar + ar^2 +$

$$r\sum_{n=0}^{\infty} ar^n = ar + ar^2 + ar^3$$

$$\sum_{n=0}^{\infty} ar^n - r\sum_{n=0}^{\infty} ar^n = a$$

$$\sum_{n=0}^{\infty} ar^n(1-r) = a$$

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

first term

$$\frac{\frac{1}{2}}{1-\frac{1}{2}}$$

common Ration

$1-(-\frac{1}{2}) = 2$

$S_n = a + ar + \dots$

$rS_n = ar + ar^2 \dots\dots ar^{n-1} + ar^n$

$S_n - rS_n = a -$

Consider $|r| < 1$

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

$|r| >$ diverges

The Geometric Series

$$\sum_{n=1}^{\infty} ar^{n-1}$$ is convergent

limit of partial sum is sum of series

common ratio

if $|r| < 1$ and its sum is

$$\sum_{n=1}^{\infty} = \frac{a}{1-r}$$

when common ratio is less than 1

$$\sum_{-}^{\infty} 5\left(-\frac{2}{3}\right)^n$$

Since $|r| = |\frac{2}{3}| < 1$ converges at

$\dots =$

ex: $0.\overline{3}$ is a geometric series

$$.\overline{3} = .3 + .03 + .003$$

$$= \frac{3}{10} + \frac{3}{10^2} + \frac{3}{10^3} + \dots$$

$\Rightarrow$ Common ratio is $\frac{1}{10}$ and $\left|\frac{1}{10}\right| < 1$

$\Rightarrow$ the series converges to

$$\frac{.3}{1 - \frac{1}{10}}$$

$$\Rightarrow .\overline{3} = \frac{1}{3}$$

$S_1 = -1$

$S_2 = 0$

$S_3 = -1$

$S_4 = 0$

$\lim_{n \to \infty} S_n$ does not exist

You can find

first n term finite by adding

first 1

first 2

first 3

(telescoping sum) ex: $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

$$\frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$$

$$= \left(1-\cancel{\tfrac{1}{2}}\right) + \left(\cancel{\tfrac{1}{2}}-\cancel{\tfrac{1}{3}}\right) + \left(\tfrac{1}{3}-\tfrac{1}{4}\right)\dots$$

$$1 = A(n+1) + B(n)$$

just left with first term

$n=0 \Rightarrow A=1$

$n=-1 \Rightarrow B=-1$

$$S_1 = 1-\frac{1}{2} = \frac{1}{2}$$

$$S_2 = \left(1-\frac{1}{2}\right) + \left(\frac{1}{2}-\frac{1}{3}\right) + \left(\frac{1}{3}-\frac{1}{4}\right) = 1-\frac{1}{4} = \frac{3}{4}$$

$$S_n = \frac{n}{n+1}$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(\frac{n}{n+1}\right)\frac{\frac{1}{n}}{\frac{1}{n}}$$

$$\frac{1}{1+\frac{1}{n}} = 1$$

Arithmetic Series diverges

ar^n Smaller less than 1 to power

Test for Divergence :
(nth term test)

If $\lim_{n \to \infty} a_n \neq 0$

then $\sum a_n$ divergens

0 may diverge or converge

Contra positive

$p \Rightarrow q$

$\neg q \Rightarrow \neg p$

Show the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges

$$\sum_{n=1}^{\infty} \frac{1}{n} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots$$

$S_1 = 1$

$S_2 = 1.5$

$S_4 = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} > 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 2$

$S_8 = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{8} > 1 + \frac{1}{2} + \frac{1}{4}$
$+ \frac{1}{4} + \frac{1}{8} + \frac{1}{8}$
$+ \frac{1}{8} + \frac{1}{8}$
$= 2.5$

$S_{2^n} \geq \frac{1}{2} n + 1$

$\Rightarrow$ going to infinity

ex: $\sum_{n=1}^{\infty} \frac{2n^2+7}{3n^2+n+1}$

$\lim_{n\to\infty} \frac{2n^2+7}{3n^2+n+1} = \frac{2}{3} \neq 0$

$\Rightarrow \sum_{n=1}^{\infty} \frac{2n^2+7}{3n^2+n+1}$ diverges

by the nth term test

(Test for Divergence)

ex $\sum_{n=1}^{\infty} \frac{3}{n(n+1)} + \sum_{n=1}^{\infty} \frac{1}{2^n}$

$= 3\sum_{n=1}^{\infty} \frac{1}{n(n+1)} + \sum_{n=1}^{\infty} \frac{1}{2^n}$

$3(1) + \frac{\frac{1}{2}}{1-\frac{1}{2}} = 3+1 = \boxed{4}$

Convergent

(1)

Def: A linear Transformation is a Mapping of a function between two modules that preserves the operations of addition and Scalar Multiplication

(2)

2a) $T: \mathbb{R}^2 \to \mathbb{R}$

$$T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 8x_1 - 3x_2$$

$$T\begin{bmatrix} 1 \\ 0 \end{bmatrix} = 8(1) - 3(0) = 8$$

$$T\begin{bmatrix} 0 \\ 1 \end{bmatrix} = 8(0) - 3(1) = -3$$

$$T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 8 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

2b) $T: \mathbb{R}^2 \to \mathbb{R}^2$

$$T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_1 \end{bmatrix}$$

Solve:

$$T\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$T\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

2c) $T: \mathbb{R}^2 \to \mathbb{R}^2$

$$T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_1 - x_2 \end{bmatrix}$$

Solve

$$T\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+0 \\ 1-0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$T\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0+1 \\ 0-1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

2d) $T: \mathbb{R}^3 \to \mathbb{R}^3$

$$T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$T\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$T\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$T\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

2e)

$$T : \mathbb{R}^3 \to \mathbb{R}^3$$

$$T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 + x_3 \\ 0 \end{bmatrix}$$

$$T\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0+0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$T\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1+0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$T\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0+1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

(3) Counter-Example

3a) $T : \mathbb{R}^2 \to \mathbb{R}$

$$T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 8x_1 - 3x_2$$

$$T\begin{bmatrix} 1 \\ 0 \end{bmatrix} = 8(1)$$

$$T\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

(3) Counter-Example

3b) $$T\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right) = \sqrt{a_1^2 + a_1^2}$$

$$T\left(\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}\right) = \sqrt{b_1^2 + b_2^2}$$

$$T\left(\begin{bmatrix} a_1+b_1 \\ a_2+b_2 \end{bmatrix}\right) = \sqrt{(a_1+b_1)^2 + (a_2+b_2)^2}$$

$$\sqrt{a_1^2 + a_2^2} + \sqrt{b_1^2 + b_2^2} \neq \sqrt{(a_1+b_1)^2 + (a_2+b_2)^2}$$

3a) $$T\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right) = (a_1)^2 + (a_2)^2$$

$$T\left(\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}\right) = (b_1)^2 + (b_2)^2$$

$$T\left(\begin{bmatrix} a_1+b_1 \\ a_2+b_2 \end{bmatrix}\right) = (a_1+b_1)^2 + (a_2+b_2)^2$$

$$\left((a_1)^2 + (a_2)^2\right) + \left((b_1)^2 + (b_2)^2\right) \neq (a_1+b_1)^2 + (a_2+b_2)^2$$

3c)

$$T\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right) = \begin{bmatrix} a_1 + 5 \\ a_2 \end{bmatrix}$$

$$T\begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \end{bmatrix} = \begin{bmatrix} a_1 + b_1 + 5 \\ a_2 + b_2 \end{bmatrix}$$

$$T\left(\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}\right) = \begin{bmatrix} b_1 + 5 \\ b_2 \end{bmatrix}$$

$$\begin{bmatrix} a_1 + 5 \\ a_2 \end{bmatrix} + \begin{bmatrix} b_1 + 5 \\ b_2 \end{bmatrix} \neq \begin{bmatrix} a_1 + b_1 + 5 \\ a_2 + b_2 \end{bmatrix}$$

3d)

$$T\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right) = \begin{bmatrix} \frac{a_1}{\sqrt{a_1^2 + a_2^2}} \end{bmatrix}$$

$$T\left(\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}\right) = \begin{bmatrix} \frac{b_1}{\sqrt{b_1^2 + b_2^2}} \end{bmatrix}$$

$$T\left(\begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \end{bmatrix}\right) = \begin{bmatrix} \frac{a_1 + b_1}{\sqrt{(a_1 + b_1)^2 + (a_2 + b_2)^2}} \end{bmatrix}$$

$$\begin{bmatrix} \frac{a_1}{\sqrt{a_1^2 + a_2^2}} \end{bmatrix} + \begin{bmatrix} \frac{b_1}{\sqrt{b_1^2 + b_2^2}} \end{bmatrix} \neq \begin{bmatrix} \frac{a_1 + b_1}{\sqrt{(a_1 + b_1)^2 + (a_2 + b_2)^2}} \end{bmatrix}$$

(1)

a) $\begin{bmatrix} 8 & -3 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

e) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

---

(2)

$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 5 \\ -6 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ 2 & 5 \\ 3 & -6 \end{bmatrix}$

(3)

$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 1 \\ 3 \\ 1 \end{bmatrix}$

$T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -5 \\ 2 \\ 0 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} 3 & -5 \\ 1 & 2 \\ 3 & 0 \\ 1 & 0 \end{bmatrix}$

(4) $\mathbb{R}^4 \to \mathbb{R}^3$

$$T(x) = \begin{bmatrix} 4 & 5 & 9 & -2 \\ 6 & 5 & 1 & 12 \\ 3 & 4 & 8 & -3 \end{bmatrix} x$$

a) After Gaussian

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -5 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 = 7\alpha + 4x_3$

$x_2 = -6\alpha - 5x_3$

$$\left\{ \alpha \begin{pmatrix} 7 \\ -6 \end{pmatrix} + x_3 \begin{pmatrix} 4 \\ -5 \end{pmatrix} \right\}$$

kurnal $T(x)$

$$\text{Range}(T) = \begin{bmatrix} 4 \\ 6 \\ 3 \end{bmatrix} \begin{bmatrix} 5 \\ 5 \\ 4 \end{bmatrix}$$

b) Hint

$\ker(T) = \text{Null}(A)$

$\text{Range}(T) = \text{Col}(A)$

(5)

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad S(T(\bar{x}))$$

$$S\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad = S\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$S \circ T = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(b)

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_2 \\ x_3 \\ x_1 \end{bmatrix}$$

$$T\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$T\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \\ x_1 \end{bmatrix}$$

$$T\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \leftrightarrow R_1$

$R_1 \leftrightarrow R_3$ $$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right]$$

$$T^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$X > 1$ converges

integral test: continuous positive decreasing

Integral Test: f cont, decreasing, positive on $[1, \infty)$ w/

$f(n) = a_n$

$\sum_{n=1}^{\infty} a_n$ converges $\Longleftrightarrow$ $\int_1^{\infty} f(x)\,dx$ converges

Most Powerful Test

recall $\int_1^\infty \frac{1}{x^p}\,dx$

converges whenever $p > 1$

and diverges whenever $p \leq 1$

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges for $p > 1$

and diverges for $p \leq 1$

p-series

ex: $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$ diverges: p-series w/ $p = \frac{1}{5} \leq 1$

ex: $\sum_{n=1}^{\infty} \frac{1}{n^\pi}$ converges: p-series w/ $p = \pi > 1$

ex: $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ $\quad \int_1^\infty \frac{x}{x^2+1}\,dx = \lim_{t\to\infty} \frac{1}{2} \int u^{-1}\,du$

By the integral test the series also diverges $\quad = \lim_{t\to\infty} \frac{1}{2} \ln|u| \Big|_{x=1}^{x=t}$

ex: $\sum_{n=1}^{\infty} \frac{\sqrt{n}+4}{n^2}$

$$\int_1^{\infty} \frac{\sqrt{x}+4}{x^2}\,dx$$

$$= \sum_{n=1}^{\infty} \left(\frac{\sqrt{n}}{n^2} + \frac{4}{n^2} \right)$$

$$= \sum_{n=1}^{\infty} \left(\frac{n^{1/2}}{n^2} + \frac{4}{n^2} \right)$$

$$= \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} + \sum_{n=1}^{\infty} \frac{4}{n^2}$$

$$= \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} + 4\sum_{n=1}^{\infty} \frac{1}{n^2}$$

These are both p-series w/ $p>1$ hence both converge.

So $\sum \frac{\sqrt{n}+4}{n^2}$ converge

Mines would

ex: $\sum_{n=1}^{\infty} \frac{1}{2^n+1}$ of got you no where bigger than diverge diverge

$\frac{1}{2^n+1} < \frac{1}{2^n}$ Smaller than converge convergent

$\sum_{n=1}^{\infty} \frac{1}{2^n}$ is a geometric Series

w/ $|r| = \frac{1}{2} < 1$, By comparison test convergent

$\sum \frac{\ln n}{n}$

$\frac{\ln n}{n} > \frac{1}{n}, n \geq 3$

Since $\sum \frac{1}{3}$ diverges, so most $\sum \frac{\ln n}{n}$ by the comparison test

Limit Comparison Test

ex: $\sum \frac{n}{n^2+1}$

$$\lim_{n\to\infty} \frac{\frac{n}{n^2+1}\cdot\frac{n}{n}}{\frac{1}{n}\cdot\frac{n}{n}} = \lim_{n\to\infty} \frac{n^2}{n^2+1}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$= \lim_{n\to\infty} \frac{1}{1+\frac{1}{n^2}} = 1 < \infty$$

$\Rightarrow \sum \frac{n}{n^2+1}$ has same convergence/divergence as $\sum \frac{1}{n}$ since we know $\sum \frac{1}{n}$ diverges, so must $\sum \frac{n}{n^2+1}$ by the limit comparison test

(1)

Define: inner product
(dot product)

Define (1) Let $\bar{u} = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$ and $\begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$ be two vectors in $\mathbb{R}^n$

The inner product of $\bar{u}$ and $\bar{v}$ is defined as $\bar{u} \cdot \bar{v} = \bar{u}^T \bar{v}$.

$$= \begin{bmatrix} u_1 & u_2 & \cdots & u_n \end{bmatrix}_{1 \times n} \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}_{n \times 1}$$

$$= \underbrace{u_1 v_1 + u_2 v_2 + \cdots u_n v_n}_{\text{Real Number like 4 or 2}}$$

(1)

Define: inner product
(dot product)

Define (1) Let $\bar{u} = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$ and $\begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$ be two

vectors in $\mathbb{R}^n$

The inner product of $\bar{u}$ and $\bar{v}$ is defined as $\bar{u} \cdot \bar{v} = \bar{u}^T \bar{v}$

$$= \begin{bmatrix} u_1 & u_2 & \cdots & u_n \end{bmatrix}_{1\times n} \begin{bmatrix} v_1 \\ \vdots \\ \vdots \\ v_n \end{bmatrix}_{n\times 1}$$

$$= \underbrace{u_1 v_1 + u_2 v_2 + \cdots u_n v_n}_{\text{Real Numer like 4 or 2}}$$

Def: ~~length of $\overline{v}$ (a vector) is defined~~ as

$\|v\| = \sqrt{\overline{v} \circ \overline{v}} = \sqrt{v_1^2 + v_2^2 + \dots v_n^2}$

norm of v

☆ v dot v

not times

real number

Def: Distance between vectors $\overline{u}$ and $\overline{v}$ is $\| \overline{u} - \overline{v} \| = \sqrt{(u_1 - v_1)^2 + (u_2 - u_2)^2 + \dots (u_n - v_n)^2}$

real number

Def: Unit Vector is any vector with the length of 1

Def: Orthogonal Set of $\overline{u}$ and $\overline{v}$ vectors if $\overline{u} \circ \overline{v} = 0$

$$\overline{u} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \quad \overline{v} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

$$\overline{u} \circ \overline{v} = 3(-1) + 1(2) + 1(1) = 0$$

Def: Orthogonal Complement is when all vectors orthogonal to S

it is defined as $S^{\perp}$ S "perp"

Def: Orthonormal set is when colums of a matrix $A^T = A^{-1}$

(2)

let $v = \begin{bmatrix}3\\-1\\5\end{bmatrix}$ and $v = \begin{bmatrix}6\\-2\\3\end{bmatrix}$

Find $u \circ v$

$$3(6) + -1(-2) + 5(3) = 35$$

Find $v \circ u \, \|u\|$

$$\|u\| = \sqrt{3^2 + -1^2 + 5^2}$$

$$= \sqrt{35}$$

$$v \circ u \sqrt{35}$$

$$6(3(\sqrt{35})) + -2(-1(\sqrt{35})) + 3(5(\sqrt{35}))$$

$$= 207.06$$

Find $\|V\|$

$$\|v\| = \sqrt{6^2 + -2^2 + 3^2}$$

$$= \sqrt{36 + 4 + 9}$$

$$= \sqrt{49}$$

$$= \boxed{7}$$

Find $\frac{\bar{u}}{\|u\|}$

(Both Unit Vectors)

$$\frac{u}{\|u\|} = 1$$

Find $\frac{\bar{v}}{\|v\|} = 1$

(3)

$\bar{v} = \begin{bmatrix} -6 \\ 4 \\ -3 \end{bmatrix}$

$$\|\bar{v}\| = \sqrt{-6^2 + 4^2 + -3^2}$$

$$= \sqrt{36 + 16 + 9}$$

unit vector

$$= \sqrt{61}$$

$$\left\| \frac{\bar{v}}{\|\bar{v}\|} \right\| = \sqrt{\left(\frac{-6}{\sqrt{61}}\right)^2 + \left(\frac{4}{\sqrt{61}}\right)^2 + \left(\frac{-3}{\sqrt{61}}\right)^2}$$

$$= 1$$

$\|u - v\|$ (4) Find distance

$u = \begin{bmatrix} 0 \\ -5 \\ 2 \end{bmatrix} \quad v = \begin{bmatrix} -4 \\ -1 \\ 8 \end{bmatrix}$

$$\|u - v\| = \sqrt{(0-(-4))^2 + (-5-(-1))^2 + (2-8)^2}$$

$$= \sqrt{(4)^2 + (-4)^2 + (-6)^2}$$

$$= \sqrt{16 + 16 + 36}$$

$$= \sqrt{68}$$

(5)

Proof: $\bar{u}$ and $\bar{v}$ are orthogonal

vectors $\bar{u} \circ \bar{v} = \bar{0}$

$$\|u+v\|^2 = (\bar{u}+\bar{v}) \circ (\bar{u}+\bar{v})$$

$$= \bar{u} \circ \bar{u} + \underbrace{\bar{u} \circ \bar{v}}_{=0} + \underbrace{\bar{v} \circ \bar{u}}_{=0} + \bar{v} \circ \bar{v}$$

$$= \|u\|^2 + \|v\|^2$$

(6) Show $S^\perp$ is a subspace of $\mathbb{R}^n$

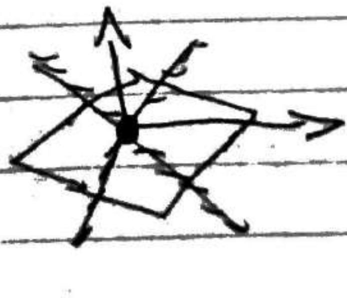
If $S^\perp$ is a subspace of $\mathbb{R}^n$

$\bar{V}$

S

Planes and lines must pass through orign

Therefore use Closure properties



$c(\bar{v} \circ \bar{w})$

$= c(0)$

$= 0$ forall $\bar{w} \in S^\perp$

(7)

a)

$(1)(0) + (-2)(1) + (1)(2)$

$0 + -2 + 2 = 0$

are orthogonal vectors

b)

$-1(3) + 4(-4) + 3(-7)$

$-3 + -16 + -21$

are Not orthogonal vectors

c)

$(2)(4)(0) + (5)(-2)(0) + (-3)(6)(0) = 0$

are orthogonal vectors

d)

$(3)(-1)(3) + (-2)(3)(8) + (1)(-3)(7) + (3)(4)(0)$

$-9 \quad -48 \quad -21 \quad 0$

are Not orthogonal

(1)

$$\frac{\bar{v} \circ b_1}{b_1 \circ b_1} = \frac{9(2) + 7(6)}{9^2 + 7^2} = \frac{18 + 42}{81 + 49} = \frac{60}{130} \quad \frac{3}{3}$$

$$\frac{\bar{v} \circ b_2}{b_2 \circ b_2} = 9(\qquad \frac{1}{2}$$

(1)

a) $$\frac{\bar{V} \circ b_1}{b_1 \circ b_1} = \frac{9(2) + 7(-3)}{2^2 + -3^2} = \frac{18 + (-21)}{4+9} = \frac{-3}{13}$$

$$\frac{\bar{V} \circ b_2}{b_2 \circ b_2} = \frac{9(6) + 7(4)}{6^2 + 4^2} = \frac{54 + 28}{36+16} = \frac{82}{52}$$

$$\boxed{\bar{V} = -\frac{3}{13} b_1 + \frac{82}{52} b_2}$$

b) $$\frac{\bar{V} \circ b_1}{b_1 \circ b_1} = \frac{8(1) - 4(0) - 3(1)}{1^2 + 0^2 + 1^2} = \frac{5}{2}$$

$$\frac{\bar{V} \circ b_2}{b_2 \circ b_2} = \frac{\overset{-8}{8(-1)} - \overset{-16}{4(4)} - \overset{-3}{3(1)}}{-1^2 + 4^2 + 1^2} = -\frac{27}{18}$$

$$\frac{V \circ b_3}{b_3 \circ b_3} = \frac{\overset{16}{8(2)} - \overset{4}{4(1)} - \overset{+6}{3(-2)}}{2^2 + 1^2 + -2^2} = \frac{18}{9}$$

$$\boxed{\bar{V} = \frac{5}{2} b_1 - \frac{27}{18} b_2 + \frac{18}{9} b_3}$$

(2) Find the Orthogonal projection

a) $\frac{\bar{V} \circ b_1}{b_1 \circ b_1} = \frac{1(1) + 3(1) + 5(0)}{1^2 + 1^2 + 0^2} = \frac{4}{2}$

$\frac{\bar{V} \circ b_2}{b_2 \circ b_2} = \frac{1(-1) + 3(1) + 5(0)}{-1^2 + 1^2 + 0^2} = \frac{4}{2}$

$\boxed{\bar{V} = \frac{4}{2} b_1 + \frac{4}{2} b_2}$

$\boxed{\bar{V} = \frac{6+4z}{14} b_1 + \frac{-16-2z}{6} b_2}$

b) $\left\{ \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} \right\}$ and let $v = \begin{bmatrix} -1 \\ 3 \\ 6 \end{bmatrix}$

$\frac{\bar{V} \circ b_1}{b_1 \circ b_1} = \frac{\overset{(-3+3z) + (-3-z) + (12+2z)}{(-1+z)(3) + (3+z)(-1) + (6+z)(2)}}{3^2 + -1^2 + 2^2} = \frac{6+4z}{14}$

$\bar{V} \cdot \begin{bmatrix} -1+z \\ 3+z \\ 6+z \end{bmatrix}$

$\frac{\bar{V} \circ b_2}{b_2 \circ b_2} = \frac{\overset{(-1+z) \quad (-3-z) \quad (-12-2z)}{(-1+z)(1) + (3+z)(-1) + (6+z)(-2)}}{1^2 + -1^2 + -2^2} = \frac{-16-2z}{6}$

Next week monday takehome

14th final exam

§ 11.6 Absolute Convergence and the Ratio and the Ratio and Root Test

def: We say the series $\sum a_n$ is absolutely convergent if $\sum |a_n|$ converges

prop: If $\sum a_n$ is absolutely convergent, then it is convergent.

proof: $-|a_n| \leq a_n \leq |a_n| \; \forall n$

$$\sum -|a_n| \leq \sum a_n \leq \sum |a_n|$$

$$-\sum |a_n| \leq \sum a_n \leq \sum |a_n|$$

ex: $\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2} = \sum$

def: If a series is convergent but not absolutely convergent but not absolutely convergent, we say it is conditionaly convergent.

ex: $\sum_{n=0}^{\infty} \frac{(-1)^n}{n}$

This is convergent conditionally

$$\sum_{n=0}^{\infty} \left| \frac{(-1)^n}{n} \right| = \sum_{n=0}^{\infty} \frac{1}{n}$$

which is the harmonic series which diverges

ie $\sum_{n=0}^{\infty} \frac{(-1)^n}{n}$ is an conditional converge series

~~thm~~

The following for absolute convergence:

The Ratio Test:

If $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = L < 1$, then $\sum a_n$ absolutely converges

If $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = L > 1$, then $\sum a_n$ diverges

note: ratio Test is inconclusive

if $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1$

ex: $\sum_{n=1}^{\infty} \frac{n}{5^n}$

$5^{n+1} = 5^n \cdot 5$

$$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \frac{\frac{n+1}{5^{n+1}}}{\frac{n}{5^n}}$$

$$= \lim_{n \to \infty} \frac{n+1}{5n} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \frac{1}{5} <$$ converges

ex: Test $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ for convergence

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{\frac{(n+1)^{n+1}}{(n+1)n^n}}{\frac{n^n}{n!}}$$

$$= \left(\frac{n+1}{n}\right)^n = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e > 1$$

$$\frac{n!}{n+1!} = \frac{1}{(n+1)}$$

Recall: The convergence of a geometric series is completely dependent on the common ratio

if $|r|<1$, the series converges

if $|r|\geq 1$, the series diverges

$$\sum_{n=0}^{\infty} r^n = 1 + r + r^2 + r^3 + \dots + r^n \dots$$

Notice the nth root of the absolute value of nth term is

$\sqrt[n]{|r^n|} = |r|$

if less than 1
converge

If $|r| \geq 1$, the series diverges

The root test:

(i) : if $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L < 1$, then $\sum a_n$ absolutely converges

(ii) If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L > 1$, then $\sum a_n$ diverges

Note: If $\sqrt[n]{|a_n|} = 1$, the test is inconclusive

lim comparison - (limit must exist as finite non-zero number) everything raised to same power

ex: $\sum \frac{5^n}{(3+n)^n}$

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\frac{5^n}{(3+n)^n}}$$

$$= \lim_{n\to\infty} \frac{5}{3+n} = 0 < 1$$

$\Rightarrow \sum_{n=0}^{\infty} \frac{5^n}{(3+n)^n}$ converges absolutely by the Root Test.

Series is an Ordered Sum

Sum is difference from conditional convergence

§ 11.8 Power Series:

def: A power series is a series of the form

$$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + \dots$$

where c_n are coefficients and x is a variable.

ex: for what values x does the series

$\sum_{n=0}^{\infty} x^n$ converge

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots$$

the power series is geometric series in x with common ratio x

$\Rightarrow$ It will converge when $|x| < 1$

i.e. when $-1 < x < 1$

we also know this series diverges for $|x| \geq 1$ it converges when $x \in (-1, 1)$

The values of x for which a power series converges is the domain of the power series
(also known as interval convergence)

ex: Find the interval of convergence for $\sum_{n=1}^{\infty} \frac{1}{n} x^n$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\frac{1}{n+1} x^{n+1}}{\frac{1}{n} x^n}\right| = \lim_{n\to\infty} \frac{\frac{1}{n+1}|x^{n+1}|}{\frac{1}{n}|x^n|}$$

$$= \lim_{n\to\infty} \frac{n|x^{n+1}|}{(n+1)|x^n|} = \lim_{n\to\infty} \frac{n|x|}{n+1} =$$

$$|x| \lim_{n\to\infty} \frac{n}{n+1} = |x|$$

Ratio Test tells us $\sum \frac{1}{n} x^n$ converges when $|x|<1$

when $-1<x<1$

Ratio Test also tells us

$\sum \frac{1}{n} x^n$ diverges when $|x| > 1$

check $x = 1$: $\sum_{n=1}^{\infty} \frac{1}{n}$ which diverges

when $|x| > 1$

check $x = 1$: $\sum_{n=1}^{\infty} \frac{1}{n}$ which diverges

check $x = -1$ $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$) which converges

interval $x \in (-1, 1)$

converges

ex: $\sum_{n=0}^{\infty} 2^n x^n$

$$\lim_{n\to\infty} \sqrt[n]{2^n x^n}$$

$$\lim_{n\to\infty} 2|x|$$

$$|x| \lim_{n\to\infty} 2 = 2|x|$$

converge when $2|x|<1$ ie $|x|<\frac{1}{2}$

ie $-\frac{1}{2}<x<\frac{1}{2}$

Diverge when $2|x|>1$ ie when ie $x>\frac{1}{2}$

or $x<-\frac{1}{2}$

check $x=\frac{1}{2}$: $\sum_{n=1} 1$ diverges

Check $x=-\frac{1}{2}$ $\sum 2^n(-\frac{1}{2})^n = \sum (-1)^n 2^n (\frac{1}{2})^n$

$\sum (-1)^n$ diverges by nth

$$\lim_{n\to\infty}\left|\frac{(n+1)!\cdot x^{n+1}}{n!\cdot x^n}\right| = \lim_{n\to\infty}|(n+1)x|$$

$$\sum_{n=0}^{\infty} n!\cdot x^n \quad = |x|\lim_{n\to\infty}(n+1)$$

This only converges when

$$\boxed{x=0}$$

def: A more general power series is a power series centered at some number a

$$\sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \dots$$

recall: A more general power series is

$$f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$$ power series centered/about a

Taylor series of $f(x)$ at a is

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

A zero Maclarin

ex: write the M

derivative is function e^x itself always 1

$$f^{(n)}(x) = e^x \quad \forall n$$

centered anywhere is taylor

$$f^{(n)}(0) = 1 \quad \forall n$$

$0! = 1$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

when $x=1$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e = \sum_{n=0}^{\infty} = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$$

ex: Find Taylor series for $f(x) = e^x$ about $a = 2$

$$f^{(n)}(x) = e^x$$

$$f^{(n)}(2) = e^2$$

$$e^x = \sum_{n=0}^{\infty} \frac{e^2}{n!}(x-2)^n$$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$ for what value x converge

By the Ratio Test $\left| \frac{x^{n+1}}{n+1!} \cdot \frac{n!}{x^n} \right|$

$\frac{n!}{(n+1)!} = \frac{1}{n+1}$

$$= \lim_{n \to \infty} \frac{|x|}{n+1}$$

$$|x| \lim_{n \to \infty} \frac{1}{n+1}$$

hence converges $\forall x \in \mathbb{R}$

$$= 0 < 1$$

So $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \; \forall x \in \mathbb{R}$

Ex: $\int e^{x^2} dx$

$$e^{u} = \sum_{n=0}^{\infty} \frac{u^n}{n!}$$

$$\Rightarrow e^{x^2} = \sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$$

$$\text{So } \int e^{x^2} dx = \int \sum_{n=0}^{\infty} \frac{x^{2n}}{n!} dx$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!} \int x^{2n} dx$$

$$\boxed{= \sum_{n=0}^{\infty} \frac{1}{n!} \frac{x^{2n+1}}{2n+1} + C}$$

Maclarin ex: Find Maclarin of $f(x) = \sin x$ and the interval of convergence

$$\left.\begin{aligned} &f(x) = \sin x,\ f(0) = \sin 0 = 0 \\ &f'(x) = \cos x,\ f'(0) = \cos 0 = 1 \\ &f''(x) = -\sin x,\ f''(0) = -\sin 0 = 0 \\ &f'''(x) = -\cos x,\ f'''(0) = -\cos 0 = -1 \end{aligned}\right\} \text{repeat}$$

$$f(x) = \sin x = \sum_{n=0}^{\infty} \frac{f^{(n)}0}{n!} x^n$$

$$= 0 + \frac{1}{1!}x + 0 - \frac{1}{3!}x^3 + 0 + \frac{1}{5!}x^5 + 0$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$$

$$\boxed{\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}}$$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{x^{2(n+1)+1}}{2(n+1)+1!}\right| \left(\frac{(2n+)}{x^{2(n+1)}}\right)$$

$$= \lim_{n\to\infty} \frac{|x^{2n+3}|}{(2n+3)!} \cdot \frac{(2n+1)!}{|x^{2n+1}|}$$

$$\frac{(2n+1)!}{(2n+3)!} = \frac{\cancel{(2n+1)}(2n)}{(2n+3)(2n+1)\cancel{(2n+1)}}$$

$$\left|\frac{x^{2n+3}}{x^{2n+1}}\right| = \left|x^{2n+3-(2n+1)}\right.$$

$$= x^2 \lim_{x \to \infty} \frac{1}{(2n+3)(2n+2)} = 0 < 1$$

Remainder Theorem Alternating is Easier

Find Macclarin Series for Cosine: ($g(x) = \cos x$

$$\frac{d}{dx}(\sin x) = \frac{d}{dx}\left(\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}\right)$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)!} \cdot \frac{d}{dx}\left(x^{2n+1}\right)$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{2n+1}{(2n+1)!} x^{2n}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

$$\ln(1+x)$$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$$\frac{d}{dx}(\ln(1+x)) = \frac{1}{1+x} = \frac{1}{1-(-x)}$$

$$f(x) = S_n(x) + R_n(x)$$
$$R_n(x) = f(x) - S_n(x)$$

Suppose $\sum a_n$ is convergent

with sum S. $\quad$ Can a

This means $\lim_{n \to \infty} S_n = S$

where S_n the n^{th} partial sum

T6

when $x = 1$ $\quad$ $\underbrace{e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}}$

maclarin dumbass $\longrightarrow$ $e = \sum_{n=0}^{\infty} \frac{1}{n!}$

$$e = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots$$

$x = 1$

partial sums

Remainders

$S_1 = 1$

$R_1 = e - 1 \approx 1.718$

$S_2 = 1 + 1 = 2$

$R_2 = e - 2 \approx .718$

$S_3 = 1 + 1 + \frac{1}{2}$

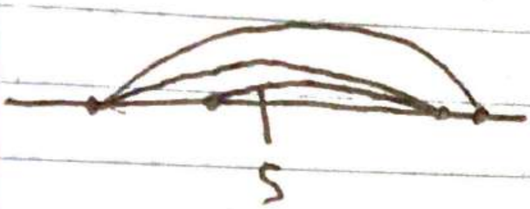
$R_3 = e - 2.5 \approx 0.218$ $\quad$ $S_4 = 1 + 1 + \frac{1}{2} + \frac{1}{6}$

$S_5 =$

thm: Alternating Series Remainder Theorem:

If $s = \sum a_i$ is the sum of an alternating series is the sum of an alternating series i.e. $\{|a_i|\}$ is decreasing and $\lim_{n \to \infty} |a_i| = 0$ then

$$|R_n| = |s - s_n| \leq |a_{n+1}|$$



bouncing back and forth

ex: Find the sum $\sum_{n=1}^{\infty} \frac{-(1)^n}{n^2}$ to within .1

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} = -1 + \frac{1}{4} - \frac{1}{9} + \frac{1}{16} - \frac{1}{25} + \frac{1}{36} \cdots$$

The remainder of adding the first n terms

if used 1

$$|R_1| \leq \frac{1}{4}$$

$$|R_2| \leq \frac{1}{9}$$

$$|R_3| \leq \frac{1}{16} < 0.1$$

stop the most you're off by — first 3 terms within a tenth the sum aprx $-1 + \frac{1}{4} - \frac{1}{9}$ ma tenth thm: Alternating Series Remainder Theorem:

If $s = \sum a_i$ is the sum of an alternating series is the sum of an alternating series i.e. $\{|a_i|\}$ is decreasing and $\lim_{n \to \infty} |a_i| = 0$ then

$$|R_n| = |s - s_n| \leq |a_{n+1}|$$

bouncing back and forth

ex: Find the sum $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ to within .1

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} = -1 + \frac{1}{4} - \frac{1}{9} + \frac{1}{16} - \frac{1}{25} + \frac{1}{36} \dots$$

The remainder of adding the first n terms

if used 1

$$|R_1| \leq \frac{1}{4}$$

$$|R_2| \leq \frac{1}{9}$$

$$|R_3| \leq \frac{1}{16} < 0.1$$

stop the most you're off by

first 3 terms within a tenth the sum apx $-1 + \frac{1}{4} - \frac{1}{9}$ in a tenth

$$\left| \frac{\frac{(-1)^{(n+1)^2} (n+1)^2}{2(n+1)^3+1}}{\frac{(-1)^{n^2} n^2}{2n^3+1}} \right| < 1$$

$(n+1)(n+1)$

$n^2+n+n+1$

$(n+1)(n^2+2n+1)$

n^3+2n^2+n

n^2+2n+1

$$\frac{\cancel{(-1)}^{(n+1)^2}(n+1)}{2(n+1)^3+1} \times \frac{2n^3+1}{\cancel{(-1)}^{n^2} n^2} < 1$$

$$\frac{(-1)^{(1)^2}(n+1)}{2(n+1)^3+1} \times \frac{2n^3+1}{n^2} < 1$$

$2(n^3+3n^2+3n+1)$

$2n^3+6n^2+6n+2$

$$\left| \frac{(-1)^1(n+1)}{\cancel{2n^3}+6n^2+6n+3} \times \frac{\cancel{2n^3}+1}{n^2} \right| < 1$$

$$\lim_{n\to\infty} \left| \frac{(-1)^1(n+1)}{6n^2+6n+3} \times \frac{1}{n^2} \right| = 0 < 1$$

less than 0

is absolute convergent

if it wasn't 2.1 it wouldn't alternate

ex: let $f(x) = \sum \frac{5^n}{n!}(x-3)^n$

Find $f(2.9)$ to within .0001

neglect $|R_0|$ $\cancel{0}$

$f(2.9) = \sum_{n=0}^{\infty} \frac{5^n}{n!}(-.1)^n$

following number needs to be less than .0001

$= \sum_{n=0}^{\infty} (-1)^n \frac{5^n}{10^n n!}$

$= \sum_{n=0}^{\infty} (-1)^n \frac{5^n}{5^n \cdot 2^n n!}$

$= \sum_{n=0}^{\infty} (-1)^n \frac{1}{2^n n!}$

$f(2.9) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n n!}$

$= 1 - \frac{1}{2} + \frac{1}{8} - \frac{1}{48} + \frac{1}{384} - \frac{1}{3840}$

1 term $|R_1| \leq \frac{1}{2}$

2 term $|R_2| \leq \frac{1}{8}$

3 term $|R_3| \leq \frac{1}{48}$

4 terms $|R_4| \leq \frac{1}{384}$

5 terms $|R_5| \leq \frac{1}{3840} = .000260417 < .0001$

$.6065$

$$e = \sum_{n=0}^{\infty} \frac{1}{n!}$$

$T_0(1) = S_0 = 1$ $\qquad R_0 = e-1 \approx 1.7$

$T_1(1) = S_1 = 2$ $\qquad R_1 = e-2 \approx .7$

$T_2(1) = S_2 = 2.5$ $\qquad R_2 = e-2.5 \approx 0.2$

$T_3(1) = S_3 = 2.\overline{6}$ $\qquad R_3 = e-2.6 \approx 0.05$

$T_4(1) = S_4 = 2.7\overline{08}$ $\qquad R_4 = e-2.7083 \approx .001$

---

in general: we let $R_n(x) = f(x) - T_n(x)$

named partial sum

taylor polynomial when x is involved

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = \underbrace{1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}}_{\text{taylor series degree 4}}$$

$\equiv$

$$f(x) = T_n(x) + R_n(x)$$

and call $R_n(x)$ the remainder of the nth degree Taylor polynomial

$$\sum \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Taylor remainder theorem:

$$|R_n(x)| \leq \left| \frac{f^{(n+1)}(z)}{(n+1)} (x-a)^{n+1} \right|$$

where z is a number between a and x which ~~makes~~ expression maximal

want to bound above

$a \to x \quad a \quad x$

$$e = \sum_{n=0}^{\infty} \frac{1}{n!}$$

$T_0(1) = S_0 = 1$ $\quad R_0 = e - 1 \approx 1.7$

$T_1(1) = S_1 = 2$ $\quad R_1 = e - 2 \approx .7$

$T_2(1) = S_2 = 2.5$ $\quad R_2 = e - 2.5 \approx 0.2$

$T_3(1) = S_3 = 2.\overline{6}$ $\quad R_3 = e - 2.6 \approx .05$

$T_4(1) = S_4 = 2.7\overline{08}$ $\quad R_4 = e - 2.7083 \approx .009$

in general: we let $R_n(x) = f(x) - T_n(x)$

named partial sum

taylor polynomial when x is involved

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = \underbrace{1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}}_{\text{taylor series degree 4}}$$

$$\underline{\underline{\ }}\quad f(x) = T_n(x) + R_n(x)$$

and call $R_n(x)$ the remainder of the nth degree Taylor polynomial

$$\sum \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Taylor remainder theorem!

$$|R_n(x)| \le \left| \frac{f^{(n+1)}(z)}{(n+1)} (x-a)^{n+1} \right|$$

where z is a number between a and x which ~~makes~~ expression maximal

want to

$a \quad x \quad a \quad x$

[redacted]

$$R_1(x) \le \frac{f''(z)}{2!} x^2$$

$$\Rightarrow R_1(1) \le \frac{f''(z)}{2}$$

maximal between [0,1]

$$R_1(x) \le \frac{e^{z}}{2!} x^2 = \frac{e}{2!} x^2$$

$x = 1$

$a = 1$

$\boxed{\text{z = will be an endpoint on the test}}$

Maximal on [0,1]

when $z = 1$

$$= R_1(1) \le \approx 1.3591$$

$$R_2(x) \le \frac{f'''(z)}{3!} x^3 = \frac{e^z}{3!} x^3 = \frac{e}{3!} x^3$$

$$R_2(1) \le \frac{e}{6} \approx .453$$

$$R_3(x) \le \frac{e}{4!} x^4$$

ex: Approximate $f(x) = \sqrt{x}$ a Taylor polynomial of degree 2 at value $a = 4$. Then estimate the value
centere
approximation $f(x) \approx T_2(x)$ when $x \in [4, 42]$
when squared

$f(x) = x^{\frac{1}{2}}$ $\qquad f(4) = 2$

$f'(x) = \frac{1}{2} x^{-\frac{1}{2}}$ $\qquad f'(4) = \frac{1}{4}$

$f''(x) = -\frac{1}{4} x^{-\frac{3}{2}}$ $\qquad \boxed{f''(4) = -\frac{1}{32}}$ Second degree

$f'''(4)$

$f'''(x) = \frac{3}{8} x^{-\frac{3}{2}} = \frac{3}{8\sqrt{x^5}}$ - decreas left end point max decre

$f(x) = f(4) + \frac{f'(4)}{1!} ($

$4 \leq x \leq 42$

$\boxed{0 \leq x-4 \leq 2}$ $T_2(x) = 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2$

$$R_2(x) \leq \frac{f'''(z)}{3!}(x-a)^3$$

$$|R_2(x)| \leq \left| \frac{\frac{3}{256}}{6!} \times (x-4)^3 \right| = \frac{f'''(4)}{6}(x-4)^3$$

... Oct 17, 2022 Virtual

§ 5.2: Eigenvalues and eigenvectors

Define: An eigenvector of an $n \times n$ matrix A is a non-zero vector $\bar{x}$ such that $A\bar{x} = \lambda \bar{x}$ for some scalar λ. We call λ the eigenvalue corresponding to the eigenvector $\bar{x}$

$$A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix} \qquad \bar{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$A\bar{x} = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$A \quad \bar{x} \quad = 2 \bar{x}$$

$$\lambda = 2 \quad \bar{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

We call $\lambda = 2$ the eigenvalue and $\bar{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ the eigenvector

$$A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix} \qquad \bar{x} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$A\bar{x} = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ -1 \end{bmatrix}$$

Methods for finding eigenvectors given a specific eigenvalue.

We have to solve the matrix equation

$$A\bar{x} = \lambda \bar{x}$$

$$A\bar{x} - \lambda \bar{x} = \bar{0}$$

$$(A - \lambda I_n)\bar{x} = \bar{0}$$

Eigenvalues

Method for finding eigen values

Method for finding eigenvalues of a Matrix

how did we find eigenvectors?

$$A\bar{x} = \lambda \bar{x}$$

rather

$$A\bar{x} - \lambda \bar{x} = \bar{0}$$

Uatin

$$(A - \lambda I)\bar{x} = 0$$

smoking weed

smoking weed

$$\left[\begin{array}{ccc|ccc} - & - & - & 1 & 0 & 0 \\ - & - & - & 0 & 1 & 0 \\ - & - & - & 0 & 0 & 1 \end{array}\right]$$

$= (2-\lambda)(-6-\lambda) - (3)(3)$

$-[12 - 6\lambda + 2\lambda - \lambda^2] - 9$

turn to identity

$-12 + 4\lambda - 21 =$

A is invertible $\Longleftrightarrow$ det(

A is not invertible $\Longleftrightarrow$ det(A)

Find eigenvalues

$$A = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ -1 & 0 & 2 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} (4-\lambda) & 0 & 1 \\ 2 & (3-\lambda) & 2 \\ -1 & 0 & (2-\lambda) \end{bmatrix}$$

$$\det(A - \lambda I) = (4-\lambda)\left[(3-\lambda)(2-\lambda) - 0(2)\right]$$

$\lambda = 6$

$\lambda = -1$

$6 - 3\lambda - 2\lambda + \lambda^2 = 0$

$\lambda^2 - 5\lambda + 6$

Simplify

$\lambda^2 - 5\lambda + 6 + 3 - \lambda + 0$

$\lambda^2 - 6\lambda + 9 = 0$

$(\lambda - 3)(\lambda - 3)$

$= (0)\left[2(2-\lambda) - (-1)(2)\right]$

$= 0$

$1\left[(0)(2) - (-1)(3-\lambda)\right]$

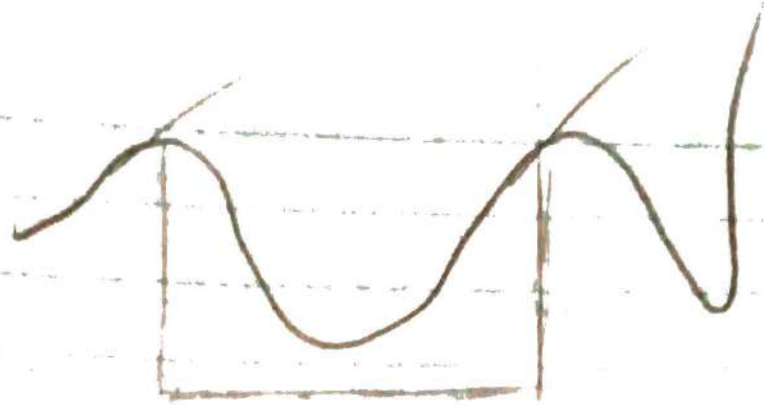
$= 0 - (-3 + \lambda)$

$= 3 - \lambda$

$\boxed{\lambda = 3 \quad \lambda = 3}$

Answer

$$\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$$



find peaks with matching interval

$$\lim_{n\to\infty} \left| \frac{\frac{x^{2(n+1)}}{(2(n+1))!}}{\frac{x^{2n}}{(2n)!}} \right| < 1$$

$$\lim_{n\to\infty} \left| \frac{x^{2n+2}}{2n+2} \times \frac{2n!}{x^{2n}} \right| < 1$$

$$\lim_{n\to\infty} \left| \frac{\cancel{x^{2n}}\, x^2}{2n+2} \times \frac{2n!}{\cancel{x^{2n}}} \right| < 1$$

$$\lim_{n\to\infty} \left| \frac{x^2\, 2n!}{2n+2} \right|$$

$\boxed{\text{Divergent}}$

antipodes are poles

any 2 point far as possible from eachother on a sphere.

Center of a sphere, use midpoint on an antipodes

$\left(\frac{a+b}{2}\right)$ - for every coordinate

were going to figure out how to get tangent planes

how the equation drawn relates to what is put in

Vectors in $\mathbb{R}^3$

~~[illegible]~~

terminal - init

$\vec{v} > \langle x, y, z \rangle$

everything carries from $\mathbb{R}^2$

Standard unit vectors: $i = \langle 1,0,0\rangle$, $j \langle 0,1,0\rangle$, $k \langle 0,0,1\rangle$

2.3: The Dot Product

$\langle 2,3,0\rangle + \langle 1,-1,1\rangle = \langle 3,2,1\rangle$

$\langle 2,3,0\rangle \cdot \langle 1,-1,1\rangle$ Undefined

Definition: the dot product of $u = \langle u_1, u_2, u_3\rangle$ and $\vec{v} \langle v_1, v_2, v_3\rangle$ is

$$\vec{u} + \vec{v} = u_1v_1 + u_2v_2 + u_3v_3$$

number and scalar is interchangeable

Ex: $\langle 2,3,0\rangle \cdot \langle 1,-1,1\rangle = -1$

You got a scalar with a dot product

Theorem: let U, V, and W be vectors and c be a real number

1) $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$

2) $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$

3) $c(\vec{u} \cdot \vec{v}) = (c\vec{u}) \cdot \vec{v} = \vec{u} \cdot (c\vec{v})$

$\vec{v} \cdot \vec{v} = \|\vec{v}\|^2$

$\vec{v}$ onto $\vec{a}$

real numbers squared
cant be negative

Corollary: If $\vec{u} \neq 0$ and $\vec{v} \neq 0$,
then $\vec{u} \cdot \vec{v} = 0$ if and only if

Cant square a vector, no multiplication for of a vector

only dot symbol dot product

$\theta = \pm \frac{\pi}{2}$, ie, $\vec{u}$ and

In terms of
unit length
to get proper
projection

Direction angle
between 3 axis

Section 2.4 The Cross Product

(this section only concerns $\mathbb{R}^3$)

problem: Given nonzero vectors u and v such that $u \neq kv$ for any real number k, can we find a vector $\vec{w}$ orthogonal to $\vec{u}$ and $\vec{v}$?