Integrating Differential Equations- HW Problems

Solve the following differential equations.

1. $\frac{dy}{dx} = \frac{2}{x^3}$; $\quad y(1) = 3$
2. $\frac{dy}{dx} = xe^{(x^2)}$; $\quad y(0) = 2.$

In problems 3 and 4 find the position function $x(t)$ given the acceleration function $a(t)$, initial velocity $v_0 = v(0)$, and initial position, $x_0 = x(0)$.

3. $a(t) = -2, \quad v_0 = 10, \quad x_0 = -4$
4. $a(t) = 6t + 1, \quad v_0 = -3, \quad x_0 = 2.$

5. At time $t = 0$ the brakes of a car are applied to a car travelling at $22.2m/sec$ ($\approx$ 80km/hr). The car decelerates at a constant rate of $20m/sec^2$. Calculate how far the car travels before it comes to a stop.

1.) $\frac{dy}{dx} = \frac{2}{x^3}$; $y(1) = 3$

$$y = \int \frac{2}{x^3}\, dy$$

$$y = 2\int \frac{1}{x^3}\, dy$$

$$2\int x^{-3}\, dy$$

$$\boxed{y = 2\left(\frac{x^{-2}}{-2}\right) + C}$$ general solution

$y = 2\left(\frac{x^{-2}}{-2}\right) + 4$ $y(1) = 2\left(\frac{1^{-2}}{-2}\right) + C = 3$

$$\boxed{y = -x^{-2} + 4}$$

particular solution

$y(1) = -1 + C = 3$

$+1 \quad +1$

$$\boxed{C = 4}$$

2)

$$\frac{dy}{dx} = xe^{(x^2)}; \quad y(0) = 2.$$

$$\int \frac{\cancel{x}e^{(u)}}{2\cancel{x}}\,du \qquad u = x^2$$

$$\frac{du}{2x} = dx$$

$$\frac{1}{2}\int e^{(u)}\,du$$

$$\boxed{y = \frac{1}{2}\left(e^{x^2}\right) + C}$$ general solution

$$y(0) = \frac{1}{2} + C = 2$$

$$-\frac{1}{2} \qquad -\frac{1}{2}$$

$$C = 1.5$$

$$\boxed{y = \frac{1}{2}\left(e^{x^2}\right) + 1.5}$$

particular solution

3.) $a(t) = -2$

$v(t) = \int -2\,dt$

$v(t) = -2t + C$

$v(0) = -2(0) + C = 10$

$C = 10$

$v(t) = -2t + 10$

$x(t) = \int -2t + 10\,dt$

$x(t) = -2\frac{t^2}{2} + 10t + C$

$x(0) = -2\frac{(0)^2}{2} + 10(0) + C = -4$

$\boxed{x(t) = -2\frac{t^2}{2} + 10t - 4}$

position function

4.)

$a(t) = 6t + 1$

$v(t) = \int 6t + 1\,dt$

$v(t) = 6\frac{t^2}{2} + 1t + C$

$v(0) = 6\frac{(0)^2}{2} + 1(0) + C = -3$

$v(t) = 3t^2 + 1t - 3$

$x(t) = \int 3t^2 + 1t - 3\,dt$

$x(t) = \frac{3t^3}{3} + \frac{t^2}{2} - 3t + C$

$x(0) = C = 2$

position function

$\boxed{x(t) = t^3 + \frac{t^2}{2} - 3t + 2}$

Distance traveled before stopping

Given:

5.) $V_0 = 80\,Km/hr \approx 22.2\,m/sec$

$a = -20\,m/s^2$

$V = 0\,m/s$

$v^2 = v_0^2 + 2a\Delta x$

$0^2 = (22.2)^2 + 2(-20)\Delta x$

$0 = (22.2)^2 - 40\Delta x$

$+40\Delta x \qquad +40\Delta x$

$\frac{40\Delta x}{40} = \frac{(22.2)^2}{40}$

$\Delta x = 12.321\,m$

Separable Differential Equations- HW Problems

In problems 1-4 find the general solution to the differential equations.

1. $\frac{dy}{dx} = \frac{x^2}{y^2}$

2. $\frac{dy}{dx} = 3y$

3. $\frac{dy}{dx} - \sqrt{4xy} = 0$

4. $\frac{dy}{dx} - (1 + x)(1 + y) = 0.$

In problems 5-8 find the particular solution to the differential equation.

5. $\frac{dy}{dx} = \frac{2x+3x^2}{3y^2}$; $y(1) = 2$

6. $\frac{dy}{dx} - 2e^{(x+2y)} = 0$; $y(0) = 1$

7. $\frac{dy}{dx} + 3x^2y^2 + 2xy^2 = 0$; $y(1) = 3$

8. $\frac{dy}{dx} = 3x^2(y^2 + 1)$; $y(0) = 1$

9. A population of bacteria grows at a rate proportional to the size of the existing population. Suppose that the population grows to 10 times its original size in 24 hours. How long did it take for the population to double?

10. Suppose you invest $\$10{,}000$ in an account that grows at a rate of 4% per year compounded continuously.

a. How long does it take for the investment to double its original amount?

b. How much money will be in the account after 10 years?

11. A turkey is heated in an oven to $200°F$. At time $t = 0$ it is taken out of the oven and put on a counter in a room whose temperature is $70°F$. After 20 minutes the turkey's temperature is $150°F$. At what time will the turkey's temperature be $125°F$?

1.) $\frac{dy}{dx} = \frac{x^2}{y^2}$

$(dx)\ y^2 \frac{dy}{dx} = x^2\ (dx)$

$y^2\,dy = x^2\,dx$

$\int y^2\,dy = \int x^2\,dx$

$3\left(\frac{y^3}{3}\right) = \left(\frac{x^3}{3} + c\right)3$

$\sqrt[3]{y^3} = \sqrt[3]{x^3 + 3c}$

$\boxed{y = \sqrt[3]{x^3 + 3c}}$

general solution

2.)

$$\left(\frac{1}{y}\right)\frac{dy}{dx} = 3y\left(\frac{1}{y}\right)$$

$$(dx)\left(\frac{1}{y}\right)\frac{dy}{dx} = 3(dx)$$

$$\frac{1}{y}dy = 3dx$$

$$\int\frac{1}{y}dy = \int 3dx$$

$$e^{(\ln|y|)} = e^{(3x+c)}$$

$$\boxed{y = e^{3x+c}}$$

general Solution

3.)

$$\left(\frac{dy}{dx}\right) = \left(\sqrt{4xy}\right)$$

$$\left(\frac{1}{y^{\frac{1}{2}}}\right)\frac{dy}{dx} = (4xy)^{\frac{1}{2}}\left(\frac{1}{y^{\frac{1}{2}}}\right)$$

$$(dx)\left(\frac{1}{\sqrt{y}}\right)\frac{dy}{dx} = (4x)^{\frac{1}{2}}(dx)$$

$$\int\frac{1}{\sqrt{y}}dy = \int\sqrt{4x}\,dx$$

General Solution: $y = \left(\frac{8x^{\frac{3}{2}}}{3} + \frac{c}{2}\right)^2$

$u = y$

$$\int\frac{1}{\sqrt{u}}dy = 2\int\sqrt{x}\,dx$$

$$\int u^{-\frac{1}{2}}du = 2\left(\frac{2(x)^{\frac{3}{2}}}{3}\right) + c$$

$$2u^{\frac{1}{2}} = \frac{4x^{\frac{3}{2}}}{3} + c$$

$$\frac{\cancel{2}(y)^{\frac{1}{2}}}{\cancel{2}} = \frac{\frac{4x^{\frac{3}{2}}}{3} + c}{2}$$

$$\left(\sqrt{y}\right)^2 = \left(\frac{8x^{\frac{3}{2}}}{3} + \frac{c}{2}\right)^2$$

4.)

$$\left(\frac{1}{1+y}\right)\frac{dy}{dx} = (1+x)\cancel{(1+y)}\left(\frac{1}{(1+y)}\right)$$

$$(dx)\left(\frac{1}{1+y}\right)\frac{dy}{dx} = (1+x)(dx)$$

$$\int \frac{1}{1+y}\,dy = \int 1+x\,dx$$

$$e^{(\ln|1+y|)} = e^{\left(\frac{x^2}{2}+x+c\right)}$$

$$1+y = e^{\left(\frac{x^2}{2}+x+c\right)}$$

$$-1 \qquad -1$$

$$\boxed{y = e^{\left(\frac{x^2}{2}+x+c\right)} - 1}$$

general Solution

5)

$$(3y^2)\frac{dy}{dx} = \frac{2x+3x^2}{\cancel{3y^2}}(3y^2)$$

$$(dx)\,3y^2\frac{dy}{dx} = 2x+3x^2\,(dx)$$

$$\int 3y^2\,dy = \int 2x+3x^2\,dx$$

$$\cancel{3}\left(\frac{y^3}{\cancel{3}}\right) = \cancel{2}\left(\frac{x^2}{\cancel{2}}\right) + \cancel{3}\frac{x^3}{\cancel{3}} + C$$

$$\sqrt[3]{y^3} = \sqrt[3]{x^2+x^3+C}$$

$$y = \sqrt[3]{x^2+x^3+C}$$

$$y(1) = \left(\sqrt[3]{2+C}\right)^3 = (2)^3$$

$$2+C = 8$$

$$-2 \qquad -2$$

$$C = 6$$

$$\boxed{y = \sqrt[3]{x^2+x^3+6}}$$

particular solution

6)

$$\frac{dy}{dx} = -2e^{(x+2y)}$$

$$\frac{dy}{dx} = -2(e^x * e^{2y})$$

$$\left(\frac{1}{-2e^{2y}}\right)\frac{dy}{dx} = -2e^x \times \cancel{-2e^{2y}\left(\frac{1}{-2e^{2y}}\right)}$$

$$(dx)\frac{1}{-2e^{2y}}\left(\frac{dy}{dx}\right) = -2e^x\,(dx)$$

$$\int \frac{1}{-2e^{2y}}\,dy = \int -2e^x\,dx$$

$$-\frac{1}{2}\int e^{-2y}\,dy = -2\int e^x\,dx$$

$u = -2y\,dy$ $\quad \left(\frac{1}{4}(e^{-2y})\right)4 = (-2(e^x) + C)4$

$\frac{u}{-2} = dy$

$$\ln(e^{-2y}) = \ln(-8(e^x) + 4C)$$

$$\frac{\cancel{-2}y}{\cancel{-2}} = \frac{\ln(-8(e^x) + 4C)}{-2}$$

$$\boxed{y = \frac{\ln(-8(e^x) + 4C)}{-2}}$$

genral solution

$$y(1) = \frac{\ln(-8(e^1) + 4C)}{-2} = 2$$

$$(-2)\frac{\ln(-8(e^1) + 4C}{\cancel{-2}} = 2(-2)$$

$$\ln(-8e^1 + 4C) = (-4)$$

$$e \qquad\qquad e$$

$$-8e^1 + 4C = e^{-4}$$

$$+8e^1 \qquad\qquad +8e^1$$

$$\frac{4C}{4} = \frac{e^{-4} + 8e^1}{4}$$

$$C = \left(\frac{e^{-4} + 8e^1}{4}\right)$$

$$y = \ln\left(-8(e^x) + \cancel{4}\left(\frac{e^{-4} + 8e^1}{\cancel{4}}\right)\right)$$

$$\boxed{y = \ln(-8e^x + e^{-4} + 8e^1)}$$

particular Solution

7.) $\frac{dy}{dx} = -3x^2y^2 - 2xy^2$

$\left(\frac{1}{y^2}\right)\frac{dy}{dx} = \cancel{y^2}(-3x^2-2x)\cancel{\left(\frac{1}{y^2}\right)}$

$\cancel{(dx)}\left(\frac{1}{y^2}\right)\frac{dy}{\cancel{dx}} = (-3x^2-2x)(dx)$

$\int \frac{1}{y^2}\,dy = \int -3x^2-2x\,dx$

$\int y^{-2}\,dy = \int -3x^2-2x\,dx$

$\frac{y^{-1}}{-1} = -3\frac{x^3}{3} - 2\frac{x^2}{2} + C$

$(y) - \frac{1}{y} = \left(-\cancel{3}\frac{x^3}{\cancel{3}} - \cancel{2}\frac{x^2}{\cancel{2}} + C\right)y$

$\frac{-1}{(-x^3-x^2+C)} = \frac{(-x^3-x^2+C)\,y}{(-x^3-x^2+C)}$

$\boxed{\frac{-1}{(-x^3-x^2+C)} = y}$

general solution

$y(1) = \frac{-1}{-2+C} = 3$

$\cancel{(-2+C)}\frac{-1}{\cancel{-2+C}} = 3(-2+C)$

$1 = \frac{5}{3} \Leftarrow -\frac{1}{3} = -2 + C$
$\quad +2 \qquad +2$

particular solution

$\boxed{y = \frac{1}{\left(-x^3-x^2+\frac{5}{3}\right)}}$

8) $\left(\frac{1}{y^2+1}\right)\frac{dy}{dx} = 3x^2\cancel{(y^2+1)}\left(\frac{1}{y^2+1}\right)$

$$\cancel{(dx)}\left(\frac{1}{y^2+1}\right)\left(\frac{dy}{\cancel{dx}}\right) = 3x^2\,(dx)$$

$$\int\left(\frac{1}{y^2+1}\right)dy = \int 3x^2\,(dx)$$

$$\tan\left(\tan^{-1}(y)\right) = \underset{\tan}{\left(\cancel{3}\left(\frac{x^3}{\cancel{3}}\right)+C\right)}$$

$$\boxed{y = \tan(x^3+C)}$$

general solution

+5

$$y(0) = \underset{\tan^{-1}}{\left(\tan(0^3+C)\right)} = \underset{\tan^{-1}}{(1)}$$

$$0^3 + C = \tan^{-1}(1)$$

particular solution

$$\boxed{y = \tan\left(x^3 + \tan^{-1}(1)\right)}$$

growth is in 24hrs

10x in 24hrs $\quad p(t) = Ae^{\left(\frac{\ln\left(\frac{10A}{A}\right)}{.00273972 6}\right)(t)}$

$2A = Ae^{\left(\frac{\ln(10^{1})}{.002739726}\right)(t)} \Rightarrow$

next page

9)

$\frac{dp}{dt} = kp$

$\int \frac{1}{p}\,dp = \int k\,dt$

$\ln p = kt + c$

$e^{\ln p} = e^{(kt+c)}$

$p(t) = Ae^{kt}$ rate, time

$p(t) = Ae^{\left(\frac{1095\ln(10)}{3}\right)(t)}$

Ex: $p\left(\frac{3}{1095}\right) = 5e^{\left(\frac{1095\ln(10)}{3}\right)\left(\frac{3}{1095}\right)}$

$p\left(\frac{3}{1095}\right) = 50$

$24\text{hr} \times \frac{1\text{ year}}{8760\text{hr}} = \boxed{.002739726\text{yrs}}$

$\frac{24}{8760} = \boxed{\frac{3}{1095}}$

$\frac{10(8)}{8} = \boxed{\frac{8e^{(k)(.002739726)}}{8}}$

$\ln\left(\frac{10(8)}{8}\right) = \ln\left(e^{(k)(.002739726)}\right)$

$\frac{\ln\left(\frac{10(8)}{8}\right)}{.002739726} = \frac{k(.002739726)}{.002739726}$

$\frac{\ln\left(\frac{10(8)}{8}\right)}{\left(\frac{3}{1095}\right)} = k$

$$\cancel{2A} = \cancel{A}e^{\left(\frac{\ln(10)}{.002739726}\right)(t)}$$

$$2 = e^{\left(\frac{\ln(10)}{.002739726}\right)(t)}$$

$$\left(\frac{\ln(2)}{\frac{1095\ln(10)}{3}}\right) = \frac{\left(\frac{\ln(10)}{.002739726}\right)(t)}{\frac{\ln(10)}{.002739726}} \Rightarrow$$

Ex:

$$P\left(\frac{\ln(2)}{\left(\frac{1095\ln(10)}{3}\right)}\right) = 25e^{\left(\frac{1095\ln(10)}{3}\right)\left(\frac{\ln(2)}{\frac{1095\ln(10)}{3}}\right)}$$

$$P\left(\frac{\ln(2)}{\left(\frac{1095\ln(10)}{3}\right)}\right) = 50$$

$$t = \left(\frac{\ln(2)}{\left(\frac{1095\ln(10)}{3}\right)}\right) \text{ yrs}$$

$t = 7.225$ hrs

(Find time pop to double)

10)

$$I(t) = 10{,}000e^{(.04)(t)}$$

a)

$$\frac{20{,}000}{10{,}000} = \frac{10{,}000e^{(.04)(t)}}{10{,}000}$$

$$2 = e^{(.04)(t)}$$

$$\ln(2) = \ln\left(e^{(.04)(t)}\right)$$

$$\frac{\ln(2)}{.04} = \frac{(.04)(t)}{.04}$$

b)

$$I(10) = 10{,}000e^{(.04)(10)}$$

$I(10) = \$14{,}918.24698$

$$\boxed{t = \frac{\ln(2)}{.04} \text{ yrs}}$$

1D

$$\frac{dT}{dt} = k(200-T)$$

$$\frac{1}{200-T}\,dt = k\,dt$$

$$\int \frac{1}{200-T}\,dt = \int k\,dt$$

$$-\ln(200-T) = kt + c$$

$$\ln(200-T) = -kt - c$$

$$e^{(\ln(200-T))} = e^{(-kt-c)}$$

$$200-t = Ae^{-kt}$$

General Solution $T(t) = 200 - Ae^{-kt}$

$$T(0) = 200 - Ae^{0} = 70$$

$$-70 \qquad 70$$

$$\boxed{130 = A}$$

$$T(t) = 200 - 130e^{-kt}$$ Particular solution

$$T(20) = 200 - 130e^{-k(20)} = 150$$

$$-200 \qquad -200$$

$$\frac{-50}{-130} = \frac{-130e^{-k20}}{-130}$$

$$\frac{\ln\left(\frac{50}{130}\right)}{20} = \frac{-k20}{20}$$

$$-k = \left(\frac{-\ln\left(\frac{50}{130}\right)}{20}\right)$$

$$125 = 200 - 130e^{\left(\frac{-\ln\left(\frac{50}{130}\right)}{20}\right)(t)}$$

$$-200 \qquad -200$$

$$\frac{-75}{-130} = \frac{-130e^{\left(\frac{\ln\left(\frac{50}{130}\right)}{20}\right)(t)}}{-130}$$

$$\frac{\ln\left(\frac{75}{130}\right)}{\left(\frac{\ln\left(\frac{50}{130}\right)}{20}\right)} = \frac{\left(\frac{\ln\left(\frac{50}{130}\right)}{20}\right)(t)}{\left(\frac{\ln\left(\frac{50}{130}\right)}{20}\right)}$$

$$\boxed{\frac{\ln\left(\frac{75}{130}\right)}{\left(\frac{\ln\left(\frac{50}{130}\right)}{20}\right)} = t \text{ minutes}}$$

First Order Linear Differential Equations- HW Problems

In problems 1 and 2 find the general solution to the differential equations.

1. $(x^2+1)\frac{dy}{dx}+2xy-3x=0$
2. $xy'+y=4\sqrt{x}\,;\quad x>0.$

In problems 3-7 solve the initial value problems.

3. $\frac{dy}{dx}=-2xy+2xe^{(-x^2)};\quad y(0)=2$
4. $xy'=2y+x^3\sin(x)\,;\quad y(1)=0$
5. $(x^2+1)y'+5xy=x;\quad y(0)=2$
6. $xy'+4y=\frac{e^{-x}}{x^3}\,;\quad y(1)=0,\ \ x>0$
7. $(1+x)y'+y=\cos(x)\,;\quad y(0)=1,\ \ 1+x>0.$

1)

$$\frac{\cancel{(x^2+1)}\frac{dy}{dx} + 2xy = 3x}{\cancel{(x^2+1)}}$$

$$\frac{dy}{dx} + \left(\frac{2x}{x^2+1}\right)y = 3x$$

$$e^{\int P_x dx} = e^{\int \frac{2x}{x^2+1} dx} = e^{\ln|x^2+1|} = x^2+1$$

$$\underbrace{(x^2+1)\frac{dy}{dx} + 2xy}_{} = 3x^3 + 3x$$

$$\frac{d}{dx}\left[(x^2+1)y\right] = 3x^3+3x$$

$$\int \frac{d}{dx}\left[(x^2+1)y\right]dx = \int 3x^3+3x\,dx$$

$$\frac{\cancel{(x^2+1)}y}{\cancel{x^2+1}} = \left(\frac{3\frac{x^4}{4} + 3\frac{x^2}{2} + C}{x^2+1}\right)$$

$$\boxed{y = \left(\frac{\frac{3x^4}{4} + 3\frac{x^2}{2} + C}{x^2+1}\right)}$$

2)

$$x\frac{dy}{dx} + y = 4\sqrt{x} \; ; \; x>0$$

$$\frac{x\frac{dy}{dx} + y}{x} = \frac{4\sqrt{x}}{x}$$

$$\frac{dy}{dx} + \underbrace{\frac{1}{x}}_{P} y = \underbrace{\frac{4\sqrt{x}}{x}}_{Q}$$

$$e^{\int \frac{1}{x}dx} = e^{\ln|x|} = x \Leftarrow I$$

$$\int \frac{d}{dx}\left[x \cdot y\right]dx = \int 4\sqrt{x}\,dx$$

$$xy = 4\int x^{\frac{1}{2}}dx$$

$$\frac{xy}{x} = \frac{4\left(\frac{2x^{\frac{3}{2}}}{3}\right) + C}{x}$$

$$\boxed{y = \left|\frac{\frac{8x^{\frac{3}{2}}}{3}}{x}\right| + \frac{C}{|x|}}$$

3)

$$\frac{dy}{dx} + 2xy = 2xe^{(-x^2)} \; ; \; y(0) = 2$$

$$I = e^{\int 2x\,dx} = e^{x^2}$$

$$\frac{d}{dx}\left[e^{x^2} * y\right] = 2xe^{(-x^2+x^2)}$$

$$e^{x^2} y = \int 2x\,dx$$

$$\frac{e^{x^2} y}{e^{x^2}} = \frac{x^2 + C}{e^{x^2}}$$

(General solution)

$$\left[y = \frac{x^2}{e^{x^2}} + \frac{C}{e^{x^2}}\right]$$

$$y(0) = C = 2$$

(Particular solution)

$$\left[y = \frac{x^2}{e^{x^2}} + \frac{2}{e^{x^2}}\right]$$

4.) $x\frac{dy}{dx} = 2y + x^3\sin x;\ y(1)=0$

$$\frac{x\frac{dy}{dx} - 2y}{x} = \frac{x^3\sin x}{x}$$

$$\frac{dy}{dx} - \frac{2y}{x} = \frac{x^3\sin x}{x}$$

$$I = e^{\int -\frac{2}{x}dx} = e^{-2\ln|x|} = \left(e^{\ln|x|}\right)^{-2} = x^{-2}$$

$$\int \frac{d}{dx}\left[x^{-2} * y\right]dx = \int x^2\sin x\,dx$$

$$(x^{-2})*(y) = \int \underset{u}{x^2}\,\underset{dv}{\sin x\,dx} = x^2(-\cos x) - \int(-\cos x)2x\,dx$$

$$+2\int \underset{dv}{\cos x}\,\underset{u}{x}\,dx$$

$du = 2x\,dx \quad v = -\cos x$

$v = \sin x$

$du = dx$

$$x^2(-\cos x) + 2\left(x\sin x - \int \sin x\,dx\right)$$

$$\frac{x^{-2}(y)}{x^{-2}} = \frac{x^2(-\cos x) + 2(x\sin x + \cos x) + C}{x^{-2}}$$

General Solution

$$\boxed{y = \frac{x^2(-\cos x) + 2(x\sin x + \cos x) + C}{x^{-2}}}$$

$$y(1) = \frac{1^2(-\cos(1)) + 2((1\sin(1) + \cos(1)) + C}{1^{-2}}$$

$$y(1) = C = -1.034752508$$

Particular solution

$$\boxed{y = \frac{x^2(-\cos x) + 2(x\sin x + \cos x)}{x^{-2}} - 1.034752508}$$

5) $$\frac{(x^2+1)\frac{dy}{dx} + 5xy}{(x^2+1)} = \frac{x}{(x^2+1)}; \quad y(0)=2$$

$$\frac{dy}{dx} + \left(\frac{5x}{x^2+1}\right)y = \frac{x}{x^2+1}$$

$$I = e^{\int \frac{5x}{x^2+1} d} = e^{\frac{5}{2}\ln|x^2+1|} = (x^2+1)^{\frac{5}{2}}$$

$$(x^2+1)^{\frac{5}{2}} * (y) = \int x(x^2+1)^{\frac{5}{2}}\,dx$$

$u = x^2+1$

$\frac{du}{2x} = dx \qquad \int \frac{\cancel{x}(u)^{\frac{5}{2}}}{2\cancel{x}}\,du$

$$\frac{1}{2}\int u^{\frac{5}{2}}\,du$$

$$\frac{(x^2+1)^{\frac{5}{2}}(y)}{(x^2+1)^{\left(\frac{5}{2}\right)}} = \frac{\frac{2}{14}(x^2+1)^{\frac{7}{2}} + c}{(x^2+1)^{\left(\frac{5}{2}\right)}}$$

particular solution

$$\boxed{y = \frac{1}{7}(x^2+1) + \frac{\left(\frac{13}{7}\right)}{(x^2+1)^{\frac{5}{2}}}}$$

$y(0) = \frac{1}{7} + C = 2 \quad \Longleftarrow \quad \boxed{y = \frac{1}{7}(x^2+1) + \frac{C}{(x^2+1)^{\frac{5}{2}}}}$

$C = \left(\frac{13}{7}\right)$

general solution

6) $\dfrac{x\frac{dy}{dx} + 4y}{x} = \dfrac{\frac{e^{-x}}{x^3}}{x}$; $y(1)=0,\ x>0$

$$\frac{dy}{dx} + \left(\frac{4}{x}\right)y = \frac{e^{-x}}{x^2}$$

$$I = e^{\int \frac{4}{x}dx} = e^{4\ln|x|} = x^4$$

$$x^4 * y = \int \underset{u}{x^2}\,\underset{dv}{e^{-x}dx} = x^2(-e^{-x}) - \int(-e^{-x})x\,dx$$

$du = x\,dx$

$v = -e^{-x}$

$$x^2(-e^{-x}) + \int \underset{dv}{e^{-x}}\,\underset{u}{x}\,dx$$

$du = dx$

$v = -e^{-x}$

$$((x)(-e^{-x}) - \int(-e^{-x})\,dx$$

$$(x)(-e^{-x}) + \int e^{-x}dx$$

$$(-e^{-x})$$

$y(1) = |1(-e^{-1}) - (1(-e^{-1}) + (-e^{-1}))| + C = 0$

$y(1) = C = .367879441$

Particular Solution

$$y = \frac{|x^2(-e^{-x}) - (x(-e^{-x}) + (-e^{-x}))| - .367879441}{|x^4|}$$

$$\frac{x^4 * y}{x^4} = \frac{x^2(-e^{-x}) - (x(-e^{-x}) + (-e^{-x})) + C}{x^4}$$

$$y = \frac{|x^2(-e^{-x}) - (x(-e^{-x}) + (-e^{-x}))| + C}{|x^4|}$$

general solution

7.)

$$\frac{dy}{dx} + \left(\frac{1}{1+x}\right)y = \frac{\cos(x)}{1+x} \; ; \; y(0)=1, \; 1+x>0$$

$$I = e^{\int \frac{1}{1+x}dx} = e^{\ln|1+x|} = 1+x$$

$$y(1+x) = \int \cos(x)\,dx$$

$$\frac{y(1+x)}{(1+x)} = \frac{\sin x + c}{(1+x)}$$

$$\boxed{y = \frac{\sin x + c}{|(1+x)|}}$$ general solution

$\Downarrow$

$$y(0) = c = 1$$

particular solution

$$\boxed{y = \frac{\sin x + 1}{|(1+x)|}}$$