In Problems 1-4 find the Inverse Laplace transform of the given function.

1.) $F(s) = s^{-4} e^{-s}$

Rule: $\mathcal{L}^{-1}\left(e^{-as}(G(s))\right) = u(t-a)\,g(t-a)$

$\mathcal{L}^{-1}(F(s)) = u(t-4)\frac{1}{6}((t-4)^3)$

$\frac{1}{6}\mathcal{L}^{-1}\left(\frac{6}{s^4}\right) = \frac{1}{6}(t^3)$

$x(t) = u(t-1)\frac{(t-1)^3}{6}$

answer

2.) $F(s) = \frac{e^{-3s}}{s-3}$

$\mathcal{L}^{-1}(F(s)) = u(t-3)\,e^{3(t-3)}$

$x(t) = u(t-3)\,e^{3t-9}$

answer

$g(s) = \frac{1}{s-3}$

$\mathcal{L}^{-1}(g(s))\quad e^{3t}$

3.) $F(s) = \frac{e^{-2s}}{s^2+4}$

$\mathcal{L}^{-1}(F(s)) = u(t-2)\frac{1}{2}\sin 2(t-2)$

$g(s) = \frac{1}{s^2+4}$

$\frac{1}{2}\mathcal{L}^{-1}\left(\frac{2}{s^2+4}\right) = \frac{1}{2}(\sin 2t)$

$x(t) = u(t-2)\frac{\sin(2t-4)}{2}$

answer

4.) $F(s) = \frac{s e^{-s}}{s^2+9}$

$\mathcal{L}^{-1}(F(s)) = u(t-1)\cos 3(t-1)$

$g(s) = \frac{s}{s^2+9} = \cos 3t$

Answer: $u(t-1)\cos(3t-3)$

In Problems 5-7 find the Laplace transform of the given function.

5.) $\begin{cases} f(t)=3 & 0\le t\le 4 \\ =0 & t>4 \end{cases}$

$f(t)=3[u(t)-u(t-4)]+0[\cancel{u(t-4)}]$

$f(t)=3u(t)-3u(t-4)$

$\mathcal{L}[f(t)]=\frac{3}{s}-3e^{-4s}\mathcal{L}[1]$

$=\frac{3}{s}-e^{-4s}\left[\frac{1}{s}\right]$

6.) $\begin{cases} f(t)=t & 0\le t\le 2\pi \\ =0 & t>2\pi \end{cases}$

$f(t)=t[u(t)-u(t-2\pi)]+0[\cancel{u(t-2\pi)}]$

$f(t)=tu(t)-tu(t-2\pi)$

$\mathcal{L}[f(t)]=e^{-s}\mathcal{L}[t]-e^{-s}\mathcal{L}[t-2\pi]$

$=e^{-s}\cdot\frac{1}{s^2}-e^{-s}\left[\frac{1}{s^2}-\frac{2\pi}{s}\right]$

7.) $\begin{cases} f(t)=\sin(2t) & 0\le t\le 2\pi \\ =0 & t>2\pi \end{cases}$ $=$ $f(t)=\sin(2t)[u(t)-u(t-2\pi)]+0[\cancel{u(t-2\pi)}]$

$f(t)=\sin(2t)u(t)-\sin(2t)u(t-2\pi)$

$\mathcal{L}[f(t)]=e^{0}\cdot\frac{2}{s^2+2}-e^{-2\pi s}\cdot\frac{2}{s^2+2}$

In problems 8-10 Solve the initial Value Problems

must be a typo because there is only 8-9 problems

8.) $x'' + 9x = f(t)$, where
$f(t) = 1 \quad 0 \le t < 2$
$= 0 \quad t \ge 2$
and $x(0) = x'(0) = 0$

$x = f(t)$

$\mathcal{L}[f(t)] = \underline{f(s)}$

$\mathcal{L}[f''(t)] = \underline{s^2 f(s) - s f(0) - f'(0)}$

$$[s^2 f(s) - s f(0) - f'(0)] + 9[f(s)] = \mathcal{L}[f(t)]$$

$$\mathcal{L}[f(t)] = \mathcal{L}[(1)u(t) - u(t-2) + 0(\cancel{u}(t-2))]$$

$$\mathcal{L}[u(t) - u(t-2)]$$

$$= \underline{\frac{1}{s} - e^{-2s}\frac{1}{s}}$$

$$s^2 f(s) + 9f(s) = \frac{1}{s} - \frac{e^{-2s}}{s}$$

$$s^2 + 9(f(s)) = \frac{1}{s} - \frac{e^{-2s}}{s}$$

$$\mathcal{L}^{-1}\left[f(s) = \frac{1}{s(s^2+9)} - \frac{e^{-2s}}{s(s^2+9)}\right]$$

↓ Continued

9.) $x'' + 4x = f(t)$, where
$f(t) = \sin t \quad 0 \le t \le 2\pi$
$= 0 \quad t > 2\pi$
and $x(0) = x'(0) = 0$

$$[s^2 f(s) - s f(0) - f'(0)] + 4[f(s)] = \mathcal{L}[f(t)]$$

$$\mathcal{L}[f(t)] = \mathcal{L}[\sin t(u(t) - u(t-2\pi)) + 0(u(\cancel{t}-2\pi))]$$

$$= e^{0} \cdot \frac{1}{s^2+1} - e^{2\pi s} \cdot \frac{1}{s^2+1} = \underline{\frac{1}{s^2+1} - \frac{e^{2\pi s}}{s^2+1}}$$

$$s^2 f(s) + 4f(s) = \frac{1}{s^2+1} - \frac{e^{2\pi s}}{s^2+1}$$

$$(s^2+4) f(s) = \frac{1}{s^2+1} - \frac{e^{2\pi s}}{s^2+1}$$

$$\mathcal{L}^{-1}\left[f(s) = \frac{1}{s^2+1(s^2+4)} - \frac{e^{2\pi s}}{s^2+1(s^2+4)}\right]$$

↓ Continued

In problems 8-10 Solve the initial Value Problems

must be a typo because there is only 8-9 problems

8.) Continued

$$\mathcal{L}^{-1}\left[f(s)=\frac{1}{s(s^2+9)}-\frac{e^{-2s}}{s(s^2+9)}\right]$$

$$\frac{1}{s(s^2+9)}=\frac{A}{s}+\frac{Bs+D}{s^2+9}$$

$$1=A(s^2+9)+Bs+D(s)$$

$$1=s^2A+9A+Bs^2+Ds$$

$$\frac{1}{9}=\frac{9A}{9} \quad \boxed{A=\frac{1}{9}} \quad 0=A+B$$

$$\boxed{B=-\frac{1}{9}} \quad \boxed{0=D}$$

$$f(t)=\frac{1}{9}\mathcal{L}^{-1}\left[\frac{1}{s}\right]-\frac{1}{9}\mathcal{L}^{-1}\left[\frac{s}{s^2+9}\right]-\mathcal{L}^{-1}\left[\frac{e^{-2s}}{s(s^2+9)}\right]$$

$$\boxed{f(t)=\frac{1}{9}[u(t)]-\frac{1}{9}[\cos 3t] -u(t-2)\left[\frac{1}{9}[u(t-2)]-\frac{1}{9}[\cos 3(t-2)]\right]}$$

Answer

9.) Continued

$$\mathcal{L}^{-1}\left[f(s)=\frac{1}{s^2+1(s^2+4)}-\frac{e^{2\pi s}}{s^2+1(s^2+4)}\right]$$

$$\frac{1}{s^2+1(s^2+4)}=\frac{As+B}{s^2+1}+\frac{Cs+D}{s^2+4}$$

$$1=As+B(s^2+4)+Cs+D(s^2+1)$$

$$1=As^3+4As+Bs^2+4B+Cs^3+Cs+Ds^2+D$$

$$C: 1=4B+D$$

$$s^3: 0=A+C$$

$$s^2: 0=B+D$$

$$s: 0=4A+C$$

$$\frac{1}{3}=\frac{3B}{3} \quad \boxed{B=\frac{1}{3}} \quad \boxed{D=-\frac{1}{3}}$$

$$\boxed{\begin{matrix}A=0\\C=0\end{matrix}}$$

$$f(t)=\frac{\frac{1}{3}}{s^2+1}+\frac{-\frac{1}{3}}{s^2+4}-\mathcal{L}^{-1}\left[\frac{e^{2\pi s}}{s^2+1(s^2+4)}\right]$$

$$\boxed{f(t)=\frac{1}{3}[\sin t]-\frac{1}{6}[\sin 2t] -u(t+2\pi)\left[\frac{1}{3}[\sin t+2\pi]-\frac{1}{6}[\sin 2(t+2\pi)]\right]}$$

Answer