| Find the Inverse Laplace of function f(s) |

$$f(s) = \frac{\left(\frac{s\left(e^{1}-e^{2}\right)}{s^{2}+1}\right)+\left(\frac{4!\left(e^{e^{1}}-e^{e^{6}}\right)}{s^{5}}\right)+\left(\frac{5\left(e^{2\pi}-e^{8\pi}\right)}{s}\right)}{2s^{2}+8}$$

$$\mathcal{L}^{-1}[F(s)] = (e^1 - e^2)\mathcal{L}^{-1}\left[\frac{\frac{s}{s^2+1}}{2s^2+8}\right] + 24(e^{e^1} - e^{e^5})\mathcal{L}^{-1}\left[\frac{\frac{1}{s^5}}{2s^2+8}\right] + (e^{2\pi} - e^{8\pi})\mathcal{L}^{-1}\left[\frac{\left(\frac{5}{s}\right)}{2s^2+8}\right]$$

What I did was rewrote the terms inside the inverse laplace also some linearity

$$\left[\frac{s}{(s^2+1)(2s^2+8)}\right] \qquad \left[\frac{1}{(s^5)(2s^2+8)}\right] \qquad \left[\frac{5}{(s)(2s^2+8)}\right]$$

use Partial fractions

$$\frac{s}{(s^2+1)(2s^2+8)} = \frac{As+B}{(s^2+1)} + \frac{Cs+D}{(2s^2+8)}$$

$s = As + B(2s^2+8) + Cs + D(s^2+1)$

$s = 2As^3 + 8As + B2s^2 + 8B + Cs^3 + Cs + Ds^2 + D$

$C: 0 = 8B + D$

$s: 1 = C + 8A$

$s^2: 0 = D + 2B$

$s^3: 0 = 2A + C$

$1 = -2A + 8A$

$\frac{1}{6} = \frac{6A}{6}$ $\boxed{A = \frac{1}{6}}$

$\boxed{-\frac{2}{6} = C}$

$D = 0 \quad B = 0$

$$\boxed{\frac{1}{6}\left[\frac{s}{s^2+1}\right] + \frac{-2}{6}\left[\frac{s}{2s^2+8}\right]}$$

$$\frac{1}{(s^5)(2s^2+8)} = \frac{1}{2s^5(s^2+4)} = \frac{As^4+Bs^3+Cs^2+Ds+E}{2s^5} + \frac{Fs+G}{s^2+4}$$

$= s^2+4[As^4+Bs^3+Cs^2+Ds+E] + s^5[Fs+G]$

$1 = (1s^6A + 1s^5B + C1s^4 + 1s^3D + 1s^2E)$

$+ (4As^4 + 4Bs^3 + 4Cs^2 + 4Ds + 4E)$

$+ (2s^6F + 2s^5G)$

$s^1: 0 = 4D$ $\quad s^4: 0 = C + 4A$

$s^2: 0 = 4C + E$ $\quad s^5: 0 = 2G + B$

$s^3: 0 = D + 4B$ $\quad s^6: 0 = 2F + A$

$C: 1 = 4E$

$E = \frac{1}{4}, D = 0, C = -\frac{1}{16}, B = 0, G = 0,$

$\frac{1}{64} = A$ $\quad -\frac{1}{128} = F$

$$\boxed{\frac{\left(\frac{1}{64}\right)s^4 - \frac{1}{16}s^2 + \frac{1}{4}}{2s^5} - \frac{1}{128}\left(\frac{s}{s^2+4}\right)}$$

$$\frac{5}{(s)(2s^2+8)} = \frac{A}{s} + \frac{Bs+C}{2s^2+8}$$

$5 = A(2s^2+8) + Bs + C(s)$

$5 = 2s^2A + 8A + Bs^2 + Cs$

$s^2: 0 = 2A + B$

$s: 0 = C$

$C: 5 = 8A$

$\frac{5}{8} = A, B = -\frac{10}{8}, C = 0$

$$\boxed{\frac{5}{8}\left(\frac{1}{s}\right) - \frac{10}{8}\left(\frac{s}{2s^2+8}\right)}$$

↓ ↓ ↓ ↓

Continued

$$\mathcal{L}^{-1}[F(s)] = (e^{1}-e^{2})\mathcal{L}^{-1}\left[\frac{\frac{s}{s^2+1}}{2s^2+8}\right] + 24(e^{1}-e^{6})\mathcal{L}^{-1}\left[\frac{\frac{1}{s^5}}{2s^2+8}\right] + (e^{2\pi}-e^{8\pi})\mathcal{L}^{-1}\left[\frac{\left(\frac{5}{s}\right)}{2s^2+8}\right]$$

Evaluating $\mathcal{L}^{-1}[\ ]$

$$\mathcal{L}^{-1}[F(s)] = \underbrace{(e^{1}-e^{2})\mathcal{L}^{-1}\left[\frac{1}{6}\left[\frac{s}{s^2+1}\right] + \frac{-2}{12}\left[\frac{s}{s^2+4}\right]\right]} + \underbrace{24(e^{1}-e^{6})\mathcal{L}^{-1}\left[\frac{1}{128s} - \frac{1}{32s^3} + \frac{1}{8s^5} - \frac{1}{128}\left(\frac{s}{s^2+4}\right)\right]}$$

$$+ \underbrace{(e^{2\pi}-e^{8\pi})\mathcal{L}^{-1}\left[\frac{5}{8}\left(\frac{1}{s}\right) - \frac{10}{16}\left(\frac{s}{s+4}\right)\right]}$$

Further Evaluation

$$(e^{1}-e^{2})\left[\frac{1}{6}\cos(t) - \frac{2}{12}\cos(2t)\right] + 24(e^{1}-e^{6})\left[\frac{1}{128}u(t) - \frac{1}{64}\mathcal{L}^{-1}\left[\frac{2}{s^3}\right] + \frac{1}{192}\mathcal{L}^{-1}\left[\frac{24}{s^5}\right] - \frac{1}{128}\cos 2t\right]$$

$$+ (e^{2\pi}-e^{8\pi})\left[\frac{5}{8}u(t) - \frac{5}{8}\cos 2t\right]$$

t^2 $\quad t^n \overset{\mathcal{L}}{\Rightarrow} \frac{n!}{s^{n+1}}$ $\quad t^4$

Full Answer

$$F(t) = (e^{1}-e^{2})\left[\frac{1}{6}\cos(t) - \frac{2}{12}\cos(2t)\right] + 24(e^{1}-e^{6})\left[\frac{1}{128}u(t) - \frac{1}{64}t^2 + \frac{1}{192}t^4 - \frac{1}{128}\cos(2t)\right]$$

$$+ (e^{2\pi}-e^{8\pi})\left[\frac{5}{8}u(t) - \frac{5}{8}\cos(2t)\right]$$