2) $x^2y'' + (6x + x^2)y' + xy = 0$

$$y'' + \frac{(6x + x^2)}{x^2}y' + \frac{xy}{x^2} = 0$$

$$p(x) = x\left(\frac{6x + x^2}{x^2}\right) \quad q(x) = x^2\left(\frac{x}{x^2}\right)$$

$$p(x) = \frac{6x + x^2}{x} \quad q(x) = x$$

find their r-values at 0

$$p(0) = \frac{0}{0} \quad q(0) = 0$$

The point 0 is not Analytic

So, we can't proceed to set up our indicial equation, or find its R-values.
Therefore, there is no Frobenius Solution.

3) $x^2y'' - xy' + y = 0$

$y'' - \frac{x}{x^2}y' + \frac{y}{x^2} = 0$

$p(x) = x\left(-\frac{x}{x^2}\right) \quad q(x) = x^2\left(\frac{1}{x^2}\right)$

$p(0) = -1 \quad q(0) = 1$

$r(r+1) + r + 1$

$r^2 + r + r + 1$

$r^2 + 2r + 1$

$(r+1)(r+1)$

$r_1 = r_2 = -1$

There is only one Frobenius Solution due to $r_1 = r_2$

$y = y_1(\ln x) + \sum_{n=0}^{\infty} b_n x^n$

$y' = y_1\left(\frac{1}{x}\right) + y_1'(\ln x) + \sum_{n=1}^{\infty} n b_n x^{n-1}$

$y'' = y_1\left(-\frac{1}{x^2}\right) + y_1'\left(\frac{1}{x}\right) + y_1'(\ln x) + \sum_{n=2}^{\infty} n(n-1) b_n x^{n-2}$

$x^2\left[y_1\left(-\frac{1}{x^2}\right) + y_1'\left(\frac{1}{x}\right) + y_1'(\ln x) + \sum_{n=2} n(n-1) b_n x^{n-2}\right] - x\left[y_1\left(\frac{1}{x}\right) + y_1'(\ln x) + \sum_{n=1} n b_n x^{n-1}\right] + \left[y_1(\ln x) + \sum_{n=0} b\right.$

$\underline{-y_1} + y_1' x + y_1' x^2 \ln x + \sum_{n=2}^{\infty} n(n-1) b_n x^n \underline{- y_1} + y_1' x \ln x + \sum_{n=1} n b_n x^{n+1}$
$+ y_1(\ln x) + \sum_{n=0} b_n x^n$

$\ln x\left[y_1' x^2 + y_1' x + y_1\right] - 2y_1 + \sum_{n=2}^{\infty} n(n-1) b_n x^n + \sum_{n=1}^{\infty} n b_n x^{n+1}$
$+ \sum_{n=0} b_n x^n$

$-2\left(\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}\right) + \sum_{n=2}^{\infty} n(n-1) b_n x^n + \sum_{n=1}^{\infty} n b_n x^n + \sum_{n=0}^{\infty} b_n x^{n+1} = 0$

$-2\left(\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}\right) + b_1 x + 4b_2 x^2 + \sum_{n=1}^{\infty} n b_n x^n + \sum_{n=0}^{\infty} b_n x^{n+1} = 0$

$\sum_{n=1}^{\infty} \left[b_{n-1} + n b_n\right] x^n \qquad \sum_{n=1}^{\infty} b_{n-1} x^n = 0$

$-2\left(\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}(n!)^2}\right) + b_{n-1} + b_n = 0$

$b_n = 2\left(\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}(n!)^2}\right) + b_{n-1} \qquad n \geq 1$

continued

$n = 1, \; b_1 = -\frac{2}{4} + b_0$

$n = 2, \; b_3 = \frac{2}{64} + b_1 = \frac{2}{64} + \left(-\frac{2}{4} + b_0\right)$

$-\frac{33}{64} + b_0$

$= 0$