In problems 2-5 a mass is attached to a spring, with spring constant k, and a dashpot, with damping constant c. The initial velocity of the mass is v_0 and the initial position is x_0. ① Find the position function of the mass, $x(t)$, and identify if the motion is overdamped, critically damped, or underdamped. If the motion is underdamped, write the position in the form $x(t) = Ce^{-pt}(\cos(\omega t - \alpha))$. ② Also find the undamped position function (ie where $c = 0$), $u(t) = C\cos(\omega t - \alpha)$.

) $m=5$, $k=46$, $c=16$, $x_0=-8$, $v_0=-2$

check if motion is overdamped

$c^2-4km = 16^2-4(46)(5) = -664 < 0$

There fore motion is underdamped

(position function of underdamped)

$$\boxed{x(t)=\sqrt{\frac{8050}{83}}\,e^{-\frac{8}{5}t}\left(\cos\left(\sqrt{\frac{46}{5}}\,t - 3.7642\right)\right)}$$

c, w_0, α

Based on newton laws of free motion $\rightarrow mx''+cx'+kx=0$

$5x''+16x'+46x=0$, $x=e^{rx}$

$e^{rx}(5r^2+16r+46)=0$ $\quad x'=re^{rx}$

$x''=r^2e^{rx}$

$$r=\frac{-(16)\pm\sqrt{(16)^2-4(5)(46)}}{2(5)}$$

$$r=\left(-\frac{8}{5}\pm\frac{\sqrt{664}}{10}i\right)$$

$$\boxed{x(t)=e^{-\frac{8}{5}t}\left(A\cos\frac{\sqrt{664}}{10}(t)+B\sin\frac{\sqrt{664}}{10}(t)\right)}$$

$$x'(t)=e^{-\frac{8}{5}t}\left(-\frac{\sqrt{664}}{10}A\sin\frac{\sqrt{664}}{10}(t)+\frac{\sqrt{664}}{10}B\cos\frac{\sqrt{664}}{10}(t)\right)$$
$$-\frac{8}{5}e^{-\frac{8}{5}t}\left(A\cos\frac{\sqrt{664}}{10}(t)+B\sin\frac{\sqrt{664}}{10}(t)\right)$$

$-8=x(0)=A$

$-2=x'(0)=\frac{\sqrt{664}}{10}B-\frac{8}{5}A$

$w_0=\sqrt{\frac{k}{m}}$

$\boxed{w_0=\sqrt{\frac{46}{5}}}$

$-2=\frac{\sqrt{664}}{10}B-\frac{8}{5}(-8)$

$-2=\frac{\sqrt{664}}{10}B+\frac{64}{5}$

$-\frac{64}{5} \qquad -\frac{64}{5}$

$-\frac{74}{5}=\frac{\sqrt{664}}{10}B$

$\boxed{c=\frac{8050}{83}}$ $\quad$ $\boxed{A=-8}$ $\quad$ $\boxed{B=-\frac{740}{5\sqrt{664}}}$

$$c=\sqrt{(-8)^2+\left(-\frac{740}{5\sqrt{664}}\right)^2}=\sqrt{\frac{8050}{83}}$$

$A<0$ (2nd & 3rd Quadrant) $\boxed{\alpha=\pi+\tan^{-1}\frac{B}{A}=3.7642}$

Undamped Case: $c=0$

$x(t) = mx'' + Kx = 0$, $x = e^{rx}$ $x'' = r^2 e^{rx}$

$5x'' + 46x = 0$

$e^{rx}(5r^2 + 46) = 0$

$\frac{5r^2}{5} = \frac{-46}{5}$

$r = \pm\sqrt{\frac{-46}{5}}$

$r = \pm\sqrt{\frac{46}{5}}\, i$

$$\boxed{u(t) = A\cos\left(\sqrt{\tfrac{46}{5}}(t)\right) + B\sin\left(\sqrt{\tfrac{46}{5}}t\right)}$$

$$\boxed{u'(t) = -\sqrt{\tfrac{46}{5}}A\sin\left(\sqrt{\tfrac{46}{5}}\right)(t) + \sqrt{\tfrac{46}{5}}B\cos\left(\sqrt{\tfrac{46}{5}}(t)\right)}$$

$-8 = u(0) = A$

$-2 = u'(0) = \sqrt{\frac{46}{5}}B$

$A = -8$	$C = \sqrt{\frac{1482}{23}}$
$B = -\frac{2}{\sqrt{46/5}}$	$\alpha = 3.2242$

$$\boxed{u(t) = \sqrt{\tfrac{1482}{23}}\cos\left(\sqrt{\tfrac{46}{5}}t - 3.2242\right)}$$

(undampened position function)

$C = \sqrt{A^2 + b^2} = \sqrt{(-8)^2 + \left(\frac{-2}{\sqrt{46/5}}\right)^2} = \sqrt{\frac{1482}{23}}$

$\alpha = \pi + \tan\left(\frac{b}{a}\right) = 3.2242$

$A < 0$

$$\boxed{w_0 = \sqrt{\tfrac{K}{m}} = \sqrt{\tfrac{46}{5}}}$$