based on Q9, $y=e^{5x}$ is one Solution to $y^{(3)}-7y''+20y'-50y=0$ find the General solution.

(1) $\underline{y=e^{rx}}$, $\underline{y'=re^{rx}}$, $\underline{y''=r^2e^{rx}}$, $\underline{y^{(3)}=r^3e^{rx}}$

(2) $e^{rx}(r^3-7r^2+20r-50)=0$

$$r^3-7r^2+20r-50=0$$

(3)

$$\begin{array}{c|cccc} 5 & 1 & -7 & 20 & -50 \\ & \downarrow & 5 & -10 & 50 \\ \hline & 1 & -2 & 10 & 0 \end{array}$$

$\pm1 \pm2 \pm25 \pm10 \pm5$

Substitute in 5, for cubic function & you'll get 0, $r=5$

reducing cubic function with synthetic division

result

$(r^2-2r+10)=0$

(4) Use Quadratic Formula

$$\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(50)}}{2(1)} = 1\pm\frac{\sqrt{196}}{2}i$$

$$r=1\pm7i$$

(5) plug r-values into our General Sol (specifically 5, $\pm$7 from our Conjugate)

$$\boxed{y=C_1^{5x}+C_2\cos(7x)+C_3\sin(7x)}$$

(General Solution)