The Boys H.W. 6 Scenario

Suppose that a motorboat is moving at 100 ft/s when its motor suddenly quits after hitting a whale on the coast and that 10 s later the boat has slowed to 3 ft/s.

(1) Assume that the resistance it encounters while ramming into a whale along the coast is proportional to its velocity. How far will the boat travel?

(2) Assuming that the resistance is proportional to the square of the velocity, how far does the motorboat travel in the first minute after its motor quits?

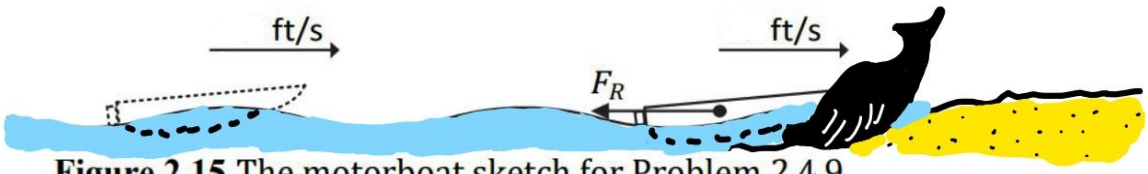

Figure 2.15 The motorboat sketch for Problem 2.4.9.

(1)

$$\frac{dv}{dt} = -kv$$

$$\left(\frac{dt}{v}\right)\frac{dv}{dt} = -kv\left(\frac{dt}{v}\right)$$

$$\int \frac{1}{v}\,dv = \int -k\,dt$$

$$\ln|v| = -kt_0 + C$$

$$e^{(\ln|V|)} = e^{(-kt_0 + C)}$$

$$V = Ae^{(-kt_0)}$$

Remember: $V(0) = 100\ ft/s$ & $V(10) = 3\ ft/s$

$100 = Ae^{(-k \cdot 0)}$ ⟹ $A = 100$

$3 = 100e^{(-k \cdot 10)}$

$\ln|3/100| = (-10k)$

$K = \frac{\ln|3/100|}{-10}$

Therefore, $V(t) = 100\ e^{\left(-\left(\frac{\ln|3/100|}{-10}\right)t\right)}$

Then, Separate variables for definition of Velocity & Solve for x, which represents your distance.

Def: $\frac{dx}{dt} = V$

$(dt)\ \frac{dx}{dt} = V(dt)$

$\int_c^x 1\,dx = \int_0^{t_0} 100e^{-kt}\,dt$

$x = 100\int_0^t \frac{e^u}{-k}\,du$ $\quad u = -kt$, $\frac{du}{-k} = dt$

$x = \frac{100}{-k}(e^{-kt})\Big|_0^t$

$x = \frac{100}{-k}(e^{-kt} - 1) =$

When $t \to \infty$

$x = \lim_{t\to\infty} \frac{100}{-k}(e^{-kt} - 1) \approx 285\ ft$

(2)

$$\frac{(dt)}{v^2}\frac{dv}{dt} = -kv^2\frac{(dt)}{v^2}$$

$$\int \frac{1}{v^2}dv = \int -k\,dt$$

$$-v^{-1} = -tk + C$$

$$(-1)\ -\frac{1}{v} = -tk + C\ (-1)$$

$$(v)\ \frac{1}{v} = tk - C\ (v)$$

$$\frac{1}{tk-C} = v$$

$V(0) = 100,\ V(10) = 3$

$$(-)\frac{1}{100} = \frac{1}{-C}(-C) \qquad \frac{1}{10k+\frac{1}{100}}(10k+.01) = 3$$

$$\frac{(-)100}{100} = \frac{1}{100} \qquad \frac{(10k+.01)3}{3} = \frac{1}{3}$$

$$(-1) - C = \frac{1}{100}(-1) \qquad 10k + .01 = \frac{1}{3}$$

$$\boxed{C = -\frac{1}{100}} \qquad -.01 \quad -.01$$

$$\frac{10k}{10} = \frac{\left(\frac{97}{300}\right)}{10}$$

$$\boxed{k = \frac{\left(\frac{97}{300}\right)}{10}}$$

$$\frac{dx}{dt} = V(t) = \frac{1}{t\left(\frac{\frac{97}{300}}{10}\right) + .01}$$

$$\int \frac{dx}{dt}dt = \int \frac{1}{t\left(\frac{\frac{97}{300}}{10}\right) + .01}dt$$