Find the Inverse Laplace Transform of the given function.
Use partial Fractions when Appropriate

(Now Inverse laplace)

Based on Q.6 of H.W. 20

$$F(s) = \frac{5s^2-3s-4}{s^3+5s^2+6s}$$

Apply partial fractions (do Algebra)

$$\frac{5s^2-3s-4}{s(s^2+5s+6)} = \frac{5s^2-3s-4}{(s)(s+2)(s+3)} = \frac{A}{(s)} + \frac{B}{(s+2)} + \frac{C}{(s+3)}$$

$$5s^2-3s-4 = A(s+2)(s+3) + B(s+3)(s) + C(s+2)(s)$$

put $s=0$

$$-4 = A(2)(3)$$

$$\frac{-4}{6} = \frac{A6}{6}$$

$$\boxed{A = \frac{-4}{6}}$$

put $s=-2$

$$20+6-4 = B(5)$$

$$\frac{22}{5} = \frac{B(5)}{5}$$

$$\boxed{B = \frac{22}{5}}$$

put $s=-3$

$$45+9-4 = 3C$$

$$\frac{50}{3} = \frac{3C}{3}$$

$$\boxed{C = \frac{50}{3}}$$

$$\mathcal{L}^{-1}\left[\frac{-4}{6s} + \frac{22}{5(s+2)} + \frac{50}{3(s+3)}\right]$$

$$\frac{1}{6}\mathcal{L}^{-1}\left[\frac{-4}{s}\right] + \frac{22}{5}\mathcal{L}^{-1}\left[\frac{1}{s+2}\right] + \frac{50}{3}\mathcal{L}^{-1}\left[\frac{1}{s+3}\right]$$

$$\boxed{\frac{1}{6}(-4) + \frac{22}{5}\left(e^{-2t}\right) + \frac{50}{3}\left(e^{-3t}\right)}$$

Answer