

Find the Inverse Laplace Transform of the given function.
Use partial Fractions when appropriate (Now Inverse Laplace)

Based on Q6 of H.W. 20

$$F(s) = \frac{5s^2 - 3s - 4}{s^3 + 5s^2 + 6s}$$

Apply partial Fractions (do Algebra)

$$\mathcal{L}^{-1} \left[\frac{-4}{6s} + \frac{22}{5(s+2)} + \frac{50}{3(s+3)} \right]$$

$$\frac{1}{6} \mathcal{L}^{-1} \left[\frac{-4}{s} \right] + \frac{22}{5} \mathcal{L}^{-1} \left[\frac{1}{s+2} \right] + \frac{50}{3} \mathcal{L}^{-1} \left[\frac{1}{s+3} \right]$$

$$\frac{1}{6}(-4) + \frac{22}{5}(e^{-2t}) + \frac{50}{3}(e^{-3t})$$

↑ Answer

$$\frac{5s^2 - 3s - 4}{s(s^2 + 5s + 6)} = \frac{5s^2 - 3s - 4}{(s)(s+2)(s+3)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$5s^2 - 3s - 4 = A(s+2)(s+3) + B(s+3)(s) + C(s+2)(s)$$

put $s=0$

$$-4 = A(2)(3)$$

$$\frac{-4}{6} = \frac{A \cdot 6}{6}$$

$$A = -\frac{4}{6}$$

put $s=-2$

$$20 + 6 - 4 = B(5)$$

$$\frac{22}{5} = \frac{B(5)}{5}$$

$$B = \frac{22}{5}$$

put $s=-3$

$$45 + 9 - 4 = 3C$$

$$\frac{50}{3} = \frac{3C}{3}$$

$$C = \frac{50}{3}$$