

Sec 8.6 Ex 7

X_1, \dots, X_m random sample from Bernoulli(p) [sample size $m = 25$]

Test $H_0: p = 0.4$ vs $H_1: p > 0.4$

$Y = \sum_{i=1}^m X_i$ is the number of successes

Test critical region is $C = \{y : y \geq 14\}$

Solution:

$$a) K(p) = P(\underbrace{\text{Rej } H_0}_{Y \geq 14} \mid \underbrace{H_1 \text{ true}}_{p > 0.4}) = P(Y \geq 14 \mid p > 0.4)$$

Note that under $H_1: p > 0.4$, we have that $Y = \sum_{i=1}^m X_i \sim \text{bi}(m, p)$ and so

$$K(p) = P(Y \geq 14 \mid p > 0.4) = \sum_{y=14}^{25} \binom{25}{y} \cdot p^y (1-p)^{25-y}$$

$$b) \text{ The significance level is } \alpha = K(0.4) = P(Y \geq 14 \mid p = 0.4) = \sum_{y=14}^{25} \binom{25}{y} (0.4)^y (0.6)^{25-y}$$

from table! to use the table!

$$= 1 - P(Y < 14 \mid p = 0.4) = 1 - \sum_{y=0}^{13} \binom{25}{y} (0.4)^y (0.6)^{25-y} = 1 - 0.9222 = 0.0778$$

$$c) \quad K(0.45) = P(Y \geq 14 | p=0.45) = 1 - P(Y < 14 | p=0.45) = 1 - \sum_{y=0}^{13} \binom{25}{y} (0.45)^y (0.55)^{25-y} = 1 - 0.8173 = 0.1827$$

$$K(0.5) = P(Y \geq 14 | p=0.5) = 1 - P(Y < 14 | p=0.5) = 1 - \sum_{y=0}^{13} \binom{25}{y} (0.5)^y (0.5)^{25-y} = 1 - 0.6550 = 0.3450$$

$$K(0.6) = P(Y \geq 14 | p=0.6) = P(25 - Y \leq 11 | p=0.6) = \sum_{y=0}^{11} \binom{25}{y} (0.4)^y (0.6)^{25-y} = 0.7323$$

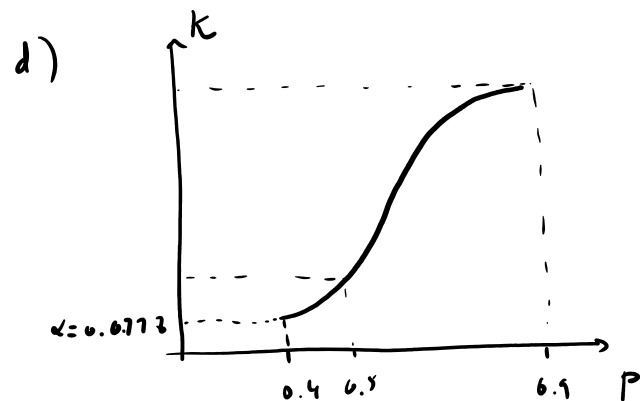
0.7
0.8
0.9

need to rewrite problem in terms of the number of failures $n - Y \sim \text{bi}(n, 1 - p)$
because the table for binomial only gives values for probabilities when $p \leq 0.5$

$$K(0.7) = P(Y \geq 14 | p=0.7) = P(25 - Y \leq 11 | p=0.7) = \sum_{y=0}^{11} \binom{25}{y} (0.3)^y (0.7)^{25-y} = 0.9553$$

$$K(0.8) = P(Y \geq 14 | p=0.8) = P(25 - Y \leq 11 | p=0.8) = \sum_{y=0}^{11} \binom{25}{y} (0.2)^y (0.8)^{25-y} = 0.9935$$

$$K(0.9) = P(Y \geq 14 | p=0.9) = P(25 - Y \leq 11 | p=0.9) = \sum_{y=0}^{11} \binom{25}{y} (0.1)^y (0.9)^{25-y} = 1$$



e) If $y=15$, H_0 would be rejected since $15 \geq 14$ and x_1 lies in the critical region C .

$$\begin{aligned}
 \text{d) } p\text{-value} &= P(Y \geq 15 \mid H_0 \text{ true}) = P(Y \geq 15 \mid p=0.4) = 1 - P(Y < 15 \mid p=0.4) \\
 &= 1 - \sum_{y=0}^{14} \binom{25}{y} (0.4)^y (0.6)^{25-y} = 1 - 0.9656 = 0.0344
 \end{aligned}$$