Sec. 8.1 Ex. 7

Let $X \sim N(\mu, \sigma^2)$

We want to test

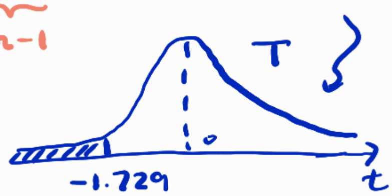
$$H_0: \mu = 47 \quad \text{vs} \quad H_1: \mu < 47$$

and we are told that a random sample of size n is to be collected (yielding a sample mean $\bar{x}$ and sample standard deviation s)

a) Since the distribution variance is not known, under $H_0: \mu = \underbrace{47}_{\mu_0}$, we have that

$$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{\bar{X} - 47}{S/\sqrt{20}} \sim t(\underbrace{19}_{n-1=19})$$

Thus, at a significance level $\alpha = 0.05$ we reject H_0 if the value t of the test statistic T is such that $t \leq -t_\alpha(n-1) = -t_{\underbrace{0.05}_{\alpha}}(\underbrace{19}_{n-1}) = -1.729$



Equivalently, we reject H_0 if

$$\frac{\bar{x} - \mu_0}{s/\sqrt{n}} \leq -t_{0.05}(n-1), \text{ that is, } \bar{x} \leq \mu_0 - t_\alpha(n-1) \cdot \frac{s}{\sqrt{n}}$$

b) Since $\bar{x} = 46.94$, $S = 0.15$, and $n = 20$, we have that $t = \dfrac{\bar{x} - \mu_0}{S/\sqrt{n}} = \dfrac{46.94 - 47}{0.15/\sqrt{20}} \approx -1.79$

Since $t = -1.79 < -1.729 = t_{\underbrace{0.05}_{\alpha}}(\underbrace{19}_{n-1})$, we reject Ho at the significance level $\alpha = 0.05$.

c) The p-value is the probability, under Ho, of observing a value more extreme than the one yielded by the collected sample, that is:

$$\text{p-value} = P(\bar{X} < \bar{x} \mid H_0 \text{ true})$$

$$= P(\bar{X} < 46.94 \mid \mu_0 = 47)$$

under Ho: $T = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim t(n-1)$

$$= P\left(\underbrace{\frac{\bar{X} - 47}{S/\sqrt{n}}}_{T} < \underbrace{\frac{46.94 - 47}{0.15/\sqrt{20}}}_{t \text{ from item b)}} \right)$$

$$= P(T < -1.79) = P(T > 1.79)$$

$$= 1 - \underbrace{P(T \leq 1.79)}_{\text{between } 0.95 \text{ and } 0.975} = \leftarrow \text{between } 0.025 \text{ and } 0.05$$

$\Rightarrow p \in (0.025, 0.05)$

Interpretation: p-value indicates that we do not reject Ho at significance level 0.025 but we do reject Ho at significance level 0.05