Sec. 8.3 Ex. 7

Let $X \sim N(\mu, \sigma^2)$

a) We want to test

$$H_0: \sigma^2 = \overbrace{30}^{\sigma_0^2} \quad \text{vs} \quad H_1: \sigma^2 = \overbrace{80}^{\sigma_1^2}$$

at significance level $\alpha = 0.05$ using a sample of size $n = 19$

The test statistic for this hypothesis test is

$$Q = \frac{(n-1)S^2}{\sigma_0^2} = \frac{18\,S^2}{30} \sim \chi^2(18)$$

The critical region for this test is

$$q \geq \chi^2_\alpha(n-1) = \chi^2_{0.05}(18) = 28.87$$

where q is the observed value of the test statistic Q.

Equivalently, we reject H_0 if the observed value of the sample variance is such that

$$s^2 \geq \frac{\sigma_0^2 \chi^2_\alpha(n-1)}{n-1} = \frac{(30)(28.87)}{18} \approx 48.12$$

b) $\beta = P(\text{Error Type II})$

$= P(\text{not reject } H_0 \mid H_1 \text{ true})$

$= P(S^2 < 48.12 \mid \sigma^2 = 80)$

Under $H_1: \sigma^2 = 80$, we have that

$$Q = \frac{(n-1)S^2}{\sigma_1^2} = \frac{18\,S^2}{80} \sim \chi^2(18)$$

and so

$$\beta = P\left(\frac{18 \cdot S^2}{80} < \frac{(18)(48.12)}{80}\right)$$

$$= P\left(Q < 10.83\right) \simeq 0.10$$

slightly under 0.10 actually