

Sec 1.1 Ex 16

p_m , $m=0,1,2,\dots$ is probability that a policy holder will file m claims (in a five-year period)

Told that $p_{m+1} = \frac{1}{4} p_m$, $m=0,1,2,\dots$ so that $p_0 \rightarrow p_1 = \frac{1}{4} p_0 \rightarrow p_2 = \left(\frac{1}{4}\right)^2 p_0 \Rightarrow \dots \Rightarrow p_m = \left(\frac{1}{4}\right)^m p_0$

Define the event A : "policy holder files two or more claims"

Then $P(A) = 1 - P(\bar{A})$ where \bar{A} : "policy holder files less than 2 claims"

$$= 1 - \underbrace{(p_0 + p_1)}_{P(\bar{A})} = 1 - p_0 - p_1 = 1 - p_0 - \frac{1}{4} p_0$$

All that remains to be done is to find p_0 :

We know that $\sum_{m=0}^{\infty} p_m = 1$

and

$$\sum_{m=0}^{\infty} \left(\frac{1}{4}\right)^m p_0 = 1 \Leftrightarrow p_0 \sum_{m=0}^{\infty} \left(\frac{1}{4}\right)^m = 1$$

$$\text{Hence } P(A) = 1 - p_0 - \frac{1}{4} p_0 = 1 - \frac{3}{4} - \frac{1}{4} \cdot \frac{3}{4} = \frac{1}{16}$$

$$\Leftrightarrow p_0 \frac{1}{1 - \frac{1}{4}} = 1 \Leftrightarrow p_0 = \frac{3}{4}$$