Sec. 8.3 Ex. 3

Suppose $X \sim N(\mu, \sigma^2)$

a) We want to test

$$H_0: \sigma^2 = \sigma_0^2 \quad vs \quad H_1: \sigma^2 > \sigma_0^2$$

where $\sigma_0 = 140$ at significance level $\alpha = 0.05$

using a sample of size $n = 25$

The test statistic for this hypothesis test would be

$$Q = \frac{(n-1) S^2}{\sigma_0^2} = \frac{24 S^2}{140^2} \sim \chi^2(24)$$

The critical region for this test is 

$$q \geq \chi^2_\alpha (n-1) = \chi^2_{0.05}(24) = 36.42$$

where q is the observed value of the test statistic Q.

b) We need to evaluate the sample variance first

(using the given data $x_1, \ldots, x_{25}$)

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 = \frac{1}{n-1} \left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2 \right) \approx 23{,}827$$

and then

$$q = \frac{(n-1)s^2}{\sigma_0^2} = \frac{24 \cdot (23{,}827)}{(140)^2} \approx 29.18$$

Since $q < \chi^2_{0.05}(24) = 36.42$, we do not reject H_0.