Sec. 7.2 Ex. 11

We have two independent random samples $\left\{ \begin{array}{ll} X_1, \ldots, X_{n_x} , & n_x = 60 \\ Y_1, \ldots, Y_{n_y} , & n_y = 60 \end{array} \right\}$, both with unknown mean and variance

Since both sample sizes are large, we may use that

$\dfrac{\bar{X} - \mu_x}{S_x/\sqrt{n_x}}$ and $\dfrac{\bar{Y} - \mu_y}{S_y/\sqrt{n_y}}$ are approximately $N(0,1)$ and so

$$Z = \frac{\bar{X} - \bar{Y} - (\mu_x - \mu_y)}{\sqrt{\dfrac{S_x^2}{n_x} + \dfrac{S_y^2}{n_y}}}$$ is approximately $N(0,1)$

Since

$$P\left(Z \le z_\alpha \right) \approx 1 - \alpha ,$$

we get that

$$P\left(\bar{X} - \bar{Y} - z_\alpha \sqrt{\frac{S_x^2}{n_x} + \frac{S_y^2}{n_y}} \le \mu_x - \mu_y \right) \approx 1 - \alpha$$

Setting $\bar{X} = \bar{x} = 671$, $\bar{Y} = \bar{y} = 480$, $s_x = 129$, $s_y = 93$, $m_x = 60$, $m_y = 60$,
observing that $\alpha = 0.05$ (because we want $1 - \alpha = 0.95$) and checking on the standard normal table for the value $z_\alpha = z_{0.05} = 1.645$, we obtain the 95% confidence approximate lower bound for $\mu_x - \mu_y$ as

$$\bar{x} - \bar{y} - z_\alpha \sqrt{\frac{s_x^2}{m_x} + \frac{s_y^2}{m_y}} = 671 - 480 - 1.645 \sqrt{\frac{(129)^2}{60} + \frac{(93)^2}{60}} = 157.227$$

The corresponding (one-sided) 95% approximate confidence interval is

$$[157.227, \infty)$$