Sec. 8.4 Ex. 5

Let $Y \sim b(n,p)$ with $n = 209$

We test $H_0 : p = \overset{p_0}{\overbrace{0.07}}$ vs $H_1 : p > 0.07$

Since $Y \sim b(n,p)$ and $n = 192$ is large, the Central Limit Theorem guarantees that:

$$\frac{Y - np}{\sqrt{np(1-p)}} \text{ is approximately } N(0,1)$$

Under $H_0 : p = p_0$ $(p_0 = 0.07)$, we have:

$$Z = \frac{Y - np_0}{\sqrt{np_0(1-p_0)}} \text{ is approximately } N(0,1)$$

We reject $H_0$ at significance level $\alpha$ if the observed value $z$ of the test statistic $Z$ is such that

$$z \geq z_\alpha$$

Since we are given that $y = 23$, we have that

$$z = \frac{y - np_0}{\sqrt{np_0(1-p_0)}} = \frac{23 - 192(0.07)}{\sqrt{192 \cdot (0.07) \cdot (0.93)}} \approx 2.27$$

a) At significance level $\alpha = 0.05$, we have

$$z_{0.05} = 1.645$$

and so $\quad z = 2.27 > 1.645 = z_{0.05}$

Thus, we reject $H_0$ at significance level 0.05.

b) At significance level $\alpha = 0.01$, we have

$$z_{0.01} = 2.326$$

and so $\quad z = 2.27 < 2.326 = z_{0.01}$

Thus, we do not reject $H_0$ at significance level 0.01.

c) The p-value is

$$\begin{aligned}
\text{p-value} &= P(Z > z \mid H_0 \text{ true}) \\
&= P(Z > 2.27) \\
&= 1 - \phi(2.27) \\
&\simeq 1 - 0.9884 \\
&\simeq 0.0116
\end{aligned}$$