Sec. 6.4 Ex. 13

$X_1, X_2, \dots, X_n$ random sample from Uniform distribution on $(\theta-1, \theta+1)$, $\theta \in \mathbb{R}$, that is, the pdf of each X_i, $i=1,\dots,n$, is given by

$$f(x) = \begin{cases} \dfrac{1}{(\theta+1-(\theta-1))}, & \text{if } x \in (\theta-1,\theta+1) \\ 0, & \text{otherwise} \end{cases} = \begin{cases} \frac{1}{2}, & \text{if } x \in (\theta-1,\theta+1) \\ 0, & \text{otherwise} \end{cases}$$

a) Note that $E[X] = \dfrac{(\theta+1)+(\theta-1)}{2} = \theta$ and so the method of moments estimator for θ is determined by the condition

$$E[X] = \frac{1}{n}\sum_{i=1}^{n} X_i$$

that is, $\tilde{\theta} = \overline{X}$

b) Note that

$$E[\tilde{\theta}] = E[\bar{X}] = E\left[\frac{1}{m}\sum_{i=1}^{m} X_i\right] = \frac{1}{m}\sum_{i=1}^{m} \overbrace{E[X_i]}^{\theta} = \frac{1}{m}\sum_{i=1}^{m}\theta = \frac{1}{m}\cdot m\theta = \theta$$

linearity of expected value

We conclude that $\tilde{\theta} = \bar{X}$ is an unbiased estimator of θ

c) A point estimate for θ using the method of moments estimator $\tilde{\theta} = \bar{X}$ is

$$\tilde{\theta} = \frac{6.61 + 7.70 + 6.98 + 8.36 + 7.26}{5} = 7.382$$

d) The point estimate for θ using the given MLE of

$$\hat{\theta} = \frac{\min(X_i) + \max(X_i)}{2}$$

is

$$\hat{\theta} = \frac{6.61 + 8.36}{2} = 7.485$$