Sec 8.6 Ex 8

$X_1, \dots, X_n \sim N(\mu, 140^2)$ [sample size $n = 25$]

Test $H_0: \mu = 715$ vs $H_1: \mu < 715$ using critical region $C = \{\bar{x} : \bar{x} < 668.94\}$

Solution:

a) $K(\mu) = P(\text{Rej } H_0 \mid H_1 \text{ true}) = P(\bar{X} < 668.94 \mid \mu < 715) = P\left(\frac{\bar{X} - \mu}{140/5} < \frac{668.94 - \mu}{140/5} \right)$

Note that under $H_1: \mu < 715$, we have that $Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{\bar{X} - \mu}{140/5} \sim N(0,1)$ and so

$K(\mu) = P\left(\frac{\bar{X} - \mu}{140/5} < \frac{668.94 - \mu}{140/5} \right) = \Phi\left(\frac{668.94 - \mu}{140/5} \right)$ where Φ is the cdf of $N(0,1)$

b) The significance level is $\alpha = P(\text{Rej } H_0 \mid H_0 \text{ true}) = P(\bar{X} < 668.94 \mid \mu = 715)$

$= K(715) = \Phi\left(\frac{668.94 - 715}{140/5} \right) = \Phi(-1.645) = 0.05$

because $z_{0.05} = 1.645$ (table)

c) $K(663.94) = \Phi\left(\frac{663.94 - 663.94}{140/5}\right) = \Phi(0) = 0.5$

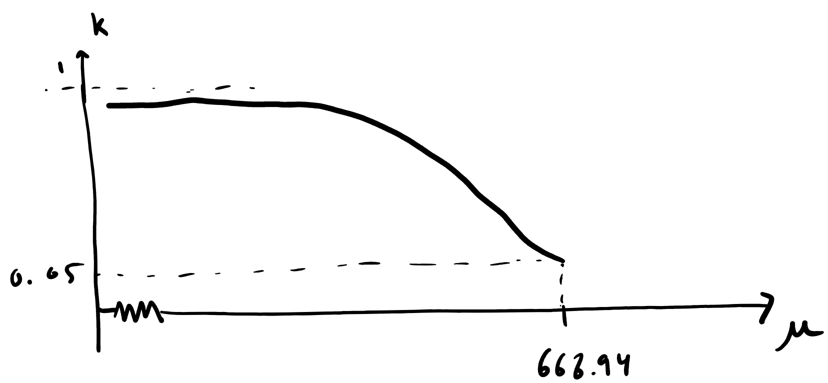
item a)

$K(622.33) = \Phi\left(\frac{668.94 - 622.33}{140/5}\right) = \Phi(1.645) = 0.95$

item a)

because $z_{0.05} = 1.645$ (table)

d)



e) Computing the sample mean of the 25 observations $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{425 + 710 + 661 + \dots + 975}{25} = 667.92$

Since $667.92 < 663.94$, we reject H_0.

f) $\text{p-value} = P(\bar{x} \leq 667.92 \mid \mu = 715) = P\left(\frac{\bar{x} - 715}{140/5} \leq \frac{667.92 - 715}{140/5} \right) =$

$$= \Phi\left(\underbrace{\frac{667.92 - 715}{140/5}}_{\approx -1.68} \right) \approx \Phi(-1.68) \underbrace{=}_{\text{symmetry of } N(0,1)} 1 - \Phi(1.68) = 1 - 0.9535 = 0.0465$$