Sec. 6.5 Ex. 3

Let $x_1, \dots, x_m$ represent the $m = 10$ given midterm scores and

$y_1, \dots, y_m$ the respective final exam scores.

a) Using the values in the table, we find that

$$\sum_{i=1}^{m} x_i = 745, \quad \sum_{i=1}^{m} y_i = 868, \quad \sum_{i=1}^{m} x_i^2 = 55917, \quad \sum_{i=1}^{m} x_i y_i = 65087$$

We obtain that

$$\bar{x} = \frac{1}{m} \sum_{i=1}^{m} x_i = 74.5$$

$$\hat{\alpha} = \frac{1}{m} \sum_{i=1}^{m} y_i = 86.8$$

$$\hat{\beta} = \frac{\sum_{i=1}^{m} x_i y_i - \frac{1}{m}\left(\sum_{i=1}^{m} x_i\right) \cdot \left(\sum_{i=1}^{m} y_i\right)}{\sum_{i=1}^{m} x_i^2 - \frac{1}{m}\left(\sum x_i\right)^2} = \frac{65{,}087 - \frac{1}{10} \cdot (745) \cdot (868)}{55917 - \frac{1}{10}(745)^2} = \frac{842}{829}$$

The least squares regression line $\left(y = \hat{\alpha} + \hat{\beta}\,(x - \bar{x}) \right)$ is then

$$y = 86.8 + \frac{842}{829}\,(x - 74.5)$$

c) To evaluate $\widehat{\sigma^2}$, we start by evaluating

$$\hat{y}_i = 86.8 + \frac{842}{829}\,(x_i - 74.5)$$ for each of the 10 values $x_1, \dots, x_{10}$

We then evaluate $(y_i - \hat{y}_i)^2$ for each $i = 1, 2, \dots, 10$ to get

$$\widehat{\sigma^2} = \frac{1}{n} \underbrace{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}_{179.9981} \approx \frac{1}{10} \cdot 179.998 \approx 17.9998$$