# Math 3501 - Probability and Statistics I

Preliminary material $\longrightarrow$ useful if not enrolled on MATH 2001 yet

# Equality and inequality symbols

$a = b$ is read *a* is equal to *b*. The symbol $=$ is called the equal sign.

$a \neq b$ is read *a* is not equal to *b*.

$a < b$ is read *a* is less than *b*.

$a > b$ is read *a* is greater than *b*

$a \leq b$ is read *a* is less than or equal to *b*.

$a \geq b$ is read *a* is greater than or equal to *b*.

**Example**

$1 = 1$

$0 \neq 1$

$0 < 1$ and $1 > 0$

$0 \leq 1$, but also $2 \leq 2$

$1 \geq 0$, but also $3 \geq 3$

$\geq \quad \leq$

# Elements of sets

Given a set $A$:

$x \in A$ means that $x$ is an element of $A$ and is read:

- $x$ belongs to $A$
- $x$ is in $A$
- $x$ lies in $A$

$x \notin A$ means that $x$ is not an element of $A$ and is read:

- $x$ does not belong to $A$
- $x$ is not in $A$

**Example**

Given $A = \{2, 3, 4\}$, we have that:

$$3 \in A \qquad \text{but} \qquad 1 \notin A\ .$$

## Operations on sets

Given two sets $A$ and $B$, we define:

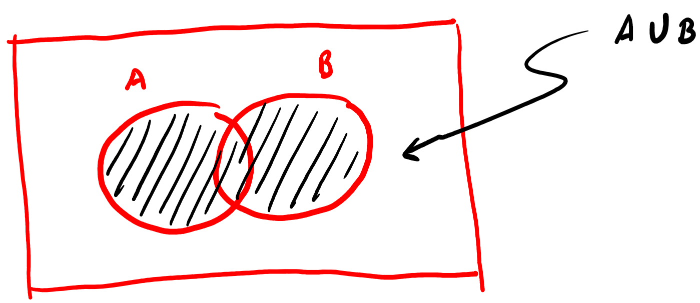
- the *union* of $A$ and $B$, denoted $A \cup B$, as the set of all elements that belong to $A$ or $B$ (or both).
- the *intersection* of $A$ and $B$, denoted $A \cap B$, as the set of all elements that belong to both $A$ and $B$.

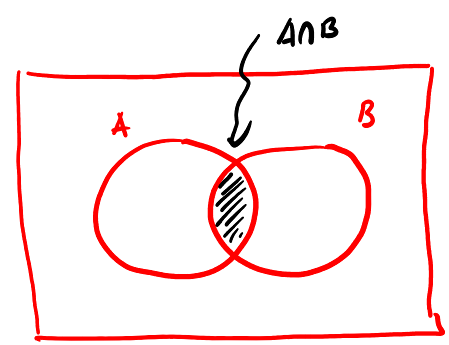
$\{4\} \neq 4$

**Example**

Given $A = \{2, 3, 4, 5\}$ and $B = \{4, 6, 7\}$, we have that:

$$A \cup B = \{2, 3, 4, 5, 6, 7\} \quad \text{and} \quad A \cap B = \{4\} \ .$$



## Some important sets

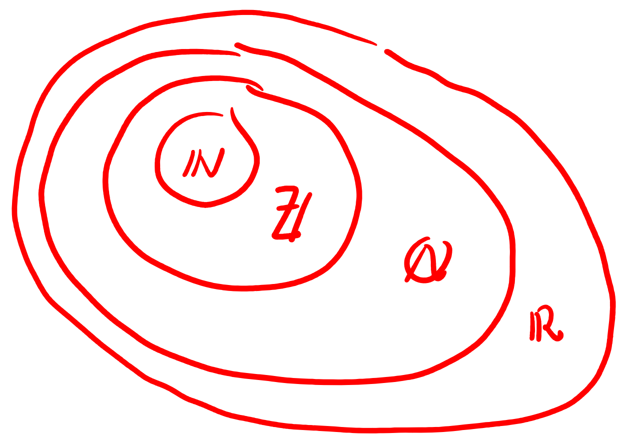
$\emptyset$ denotes the empty set $\{ \}$

$\mathbb{N}$ denotes the set of natural numbers: $\mathbb{N} = \{1, 2, 3, 4, \ldots\}$ $\mathbb{N}$

$\mathbb{Z}$ $\mathbb{Z}$ denotes the set of integer numbers: $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

$\mathbb{Q}$ $\mathbb{Q}$ denotes the set of rational numbers: $\mathbb{Q} = \left\{ \frac{p}{q} : p, q \in \mathbb{Z} \text{ and } q \neq 0 \right\}$

$\mathbb{R}$ $\mathbb{R}$ denotes the set of real numbers



## Set-builder notation

Convenient way of defining sets:

$$\{\text{variable} \mid \text{condition}\}$$

or

$$\{\text{variable} : \text{condition}\}$$

is read as *the set of all "variables" satisfying a given "condition"*.

**Example**

$\{x \in \mathbb{N} : x \leq 5\}$ is the set of all natural numbers less than or equal to 5. $\{1, 2, 3, 4, 5\}$

$\{x \in \mathbb{R} : x \leq 5\}$ is the set of all real numbers less than or equal to 5.

$(-\infty, 5]$

# Interval notation

$x \in \mathbb{R}$

| Inequality | Interval Notation | Graph on Number Line | Description |
|---|---|---|---|
| $a < x < b$ | $(a, b)$ |  | *x* is strictly between *a* and *b* |
| $a \leq x < b$ | $[a, b)$ |  | *x* is between *a* and *b*, to include *a* |
| $a < x \leq b$ | $(a, b]$ |  | *x* is between *a* and *b*, to include *b* |
| $a \leq x \leq b$ | $[a, b]$ |  | *x* is between *a* and *b*, to include *a* and *b* |

# Interval notation

$x \in \mathbb{R}$

| Inequality | Interval Notation | Graph on Number Line | Description |
|---|---|---|---|
| $x > a$ | $(a, \infty)$ |   | $x$ is greater than $a$ |
| $x < a$ | $(-\infty, a)$ |   | $x$ is less than $a$ |
| $x \geq a$ | $[a, \infty)$ |   | $x$ is greater than or equal to $a$ |
| $x \leq a$ | $(-\infty, a]$ |   | $x$ is less than or equal to $a$ |

# Math 3501 - Probability and Statistics I

Applications:
→ Natural sciences (Physics, Biology, Chemistry, ...)
→ Actuarial Science (Insurance)
→ Finance
→ Machine Learning
→ Data Science

## 1.1 - Properties of Probability

branch of mathematics devoted to the study of random or uncertain behavior

Probability theory provides the theoretical framework to do statistics

branch of mathematics devoted to the collection and organization of data, and to the inference of additional information from such data

# Random experiments

A *random experiment* is an experiment for which the outcome cannot be predicted with certainty.

heads or tails

1/2 1/2

## Example

- tossing a fair coin
- rolling a fair six-sided die
- drawing a card from a standard deck of 52 playing cards

**Note**: Even though the specific outcome of a random experiment cannot be predicted with certainty before the experiment is performed, the collection of all possible outcomes is known.

# Outcome space

The collection of all possible outcomes of a random experiment is called the *outcome space* or *sample space*.

In this course, we will often use the letter $S$ to denote outcome spaces

## Example

heads tails

"1" → "face 1 up"

"2" → "face 2 up"

S finite

- tossing a fair coin: $S = \{H, T\}$
- rolling a fair six-sided die: $S = \{1, 2, 3, 4, 5, 6\}$
- number of coin tosses until $H$ occurs: $S = \{1, 2, 3, 4, 5, \ldots\}$ $\mathbb{N}$
- lifespan (in seconds) of a lamp: $S = [0, \infty)$

S is countable

S is uncountable

# Event

subset $A$ of sample space $S$

Given a random experiment with outcome space $S$, an *event* is a subset $A$ of $S$.

When a random experiment is performed and the outcome of the experiment is in $A$, we say that the *event A has occurred*.

$x \in A$

## Example

- tossing a fair coin: $S = \{H, T\}$

  The event "*Heads is observed*" corresponds to the subset $A = \{H\}$ of $S$

- lifespan (in years) of a lamp: $S = [0, \infty)$

  The event "*lamp lasts longer than 2 years*" corresponds to the subset $A = (2, \infty)$ of $S$

- number of coin tosses until H is observed $S = \mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$

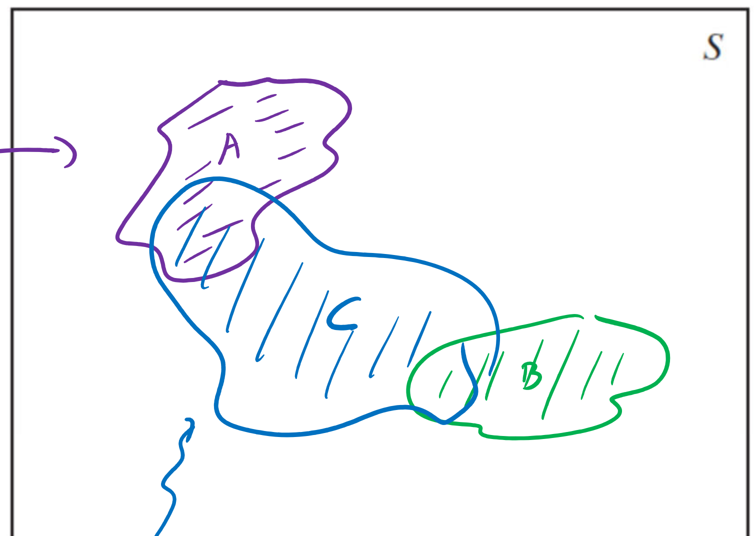
  The event "Heads is observed in at most 4 tosses" corresponds to the subset $A = \{1, 2, 3, 4\}$

# Algebra of sets

*Venn diagram*: represent the outcome space $S$ as a large rectangle and events $A$ as subsets of such rectangle.

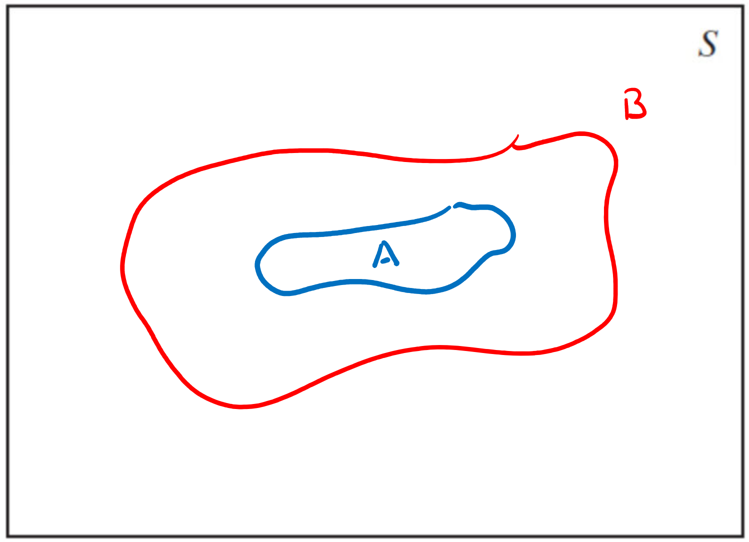
outcome space
sample space

Event A

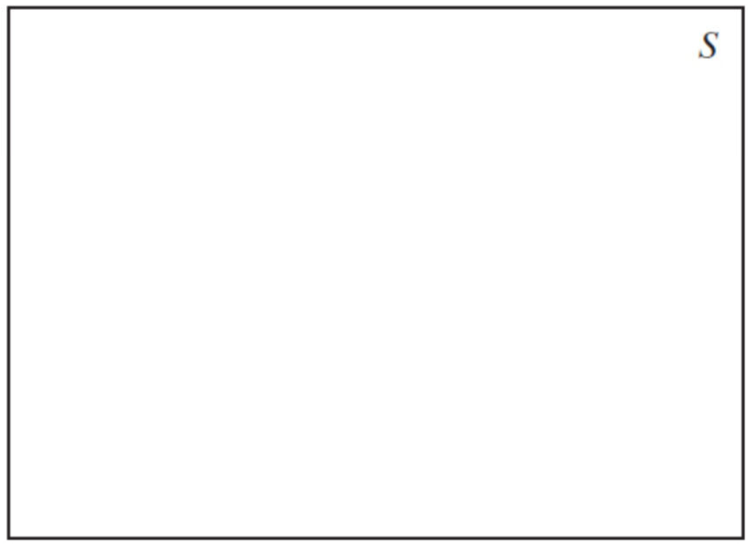



event C

$A \subset B$ denotes that $A$ is a subset of $B$.

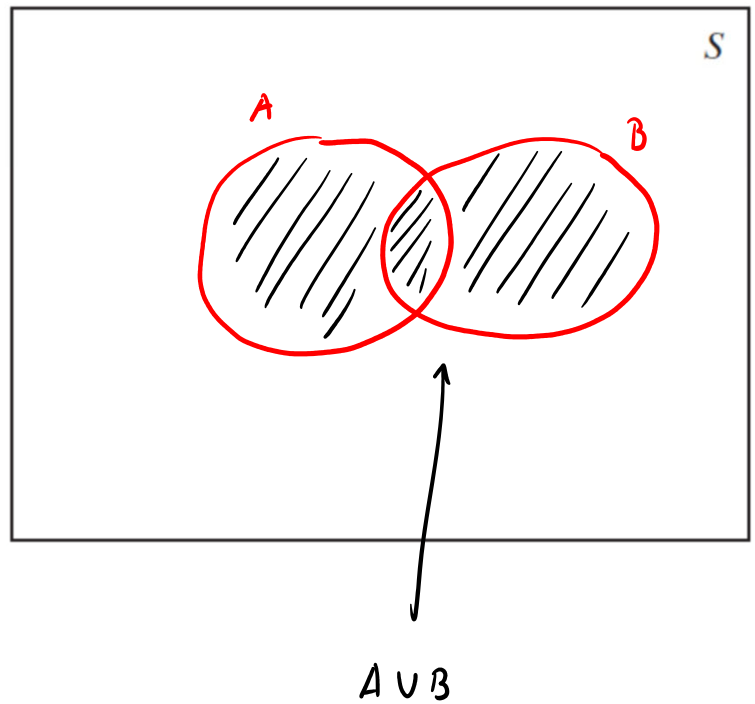


$\emptyset$ denotes the null or empty set.

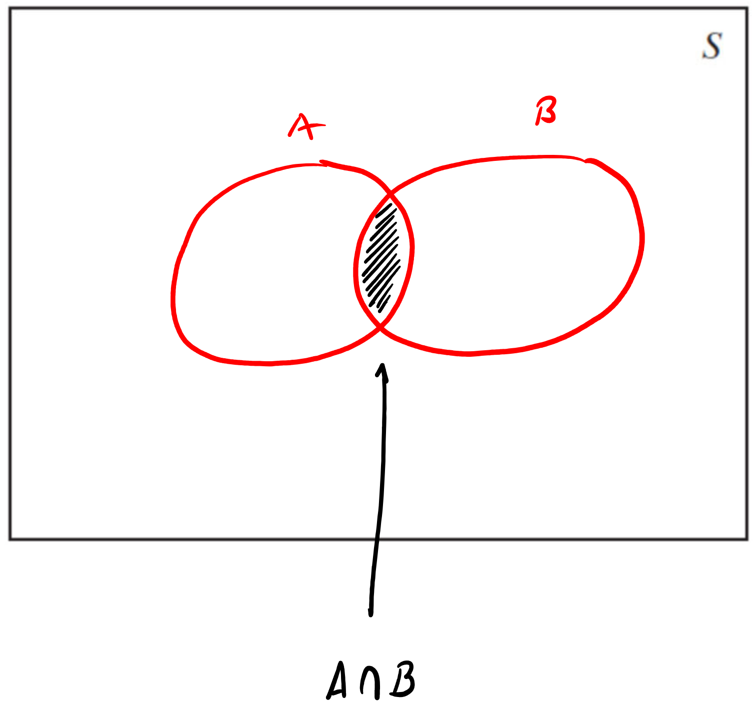



$\phi$ is always a subset of any set!

$A \cup B$ is the union of $A$ and $B$.

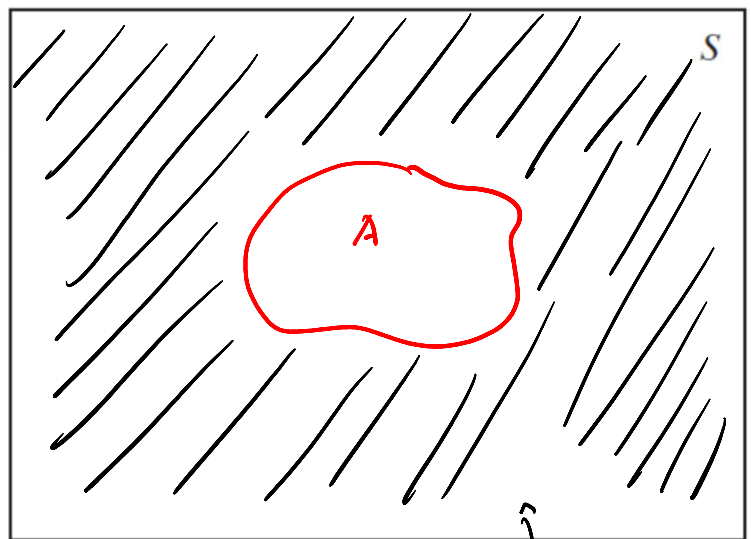


$A \cap B$ is the intersection of $A$ and $B$.



$A'$ (or $\bar{A}$) is the complement of $A$ (in $S$): all elements in $S$ that are not in $A$.

or $A^c$




$\bar{A}$ occurs is equivalent to $A$ does not occur!

$\bar{A}$ is shaded region

every element of $S$ not in $A$