The conditions $f([-1,2]) = [-.515, -.1]$

and

$\max\{|f'(x)| : -1 \leq x \leq 2\} = .35$

Satisfies fixed point iteration because $[-.515, -.1]$ indicates the function mapping onto itself with $x_1 = f(x_0)$ being contained within $[-1,2]$. In addition, $\max\{|f'(x)| : -1 \leq x \leq 2\}$ shows that with each iteration the distance between the input & output $\left(.35|x_n - x_{n-1}|\right)$ is shrunk by a factor of .35. If the factor was ≥ 1 then the condition would be invalid.

Find $f([-1,2])$

$f(x) = \frac{x^3}{20} - \frac{x}{4} - .3$

Step 1: find critical points

$f'(x) = \frac{3x^2}{20} - \frac{1}{4}$

Set $= 0$, for critical points

$\frac{3x^2}{20} - \frac{1}{4} = 0 \Rightarrow \frac{3x^2}{20} = \frac{1}{4}$

Step 2: Eval Boundary points & other critical points

$\left(\frac{20}{3}\right)\frac{3x^2}{20} = \frac{1}{4}\left(\frac{20}{3}\right)$

$x^2 = \frac{20}{12}$

$x^2 = \frac{10}{6}$

$x^2 = \frac{5}{3} \Rightarrow x = \pm\sqrt{\frac{5}{3}}$

$x = -1$

$x = 2$

positive value lies in interval → $x = \sqrt{\frac{10}{4}}$

$f(-1) = -.1$

$f(2) = -.4$

$f\left(\sqrt{\frac{5}{3}}\right) = -.515166$

global max

Answer: $[-.1, -.515166]$

global min

find max $\{|f'(x)| : -1 \leq x \leq 2\}$

(step 1) find critical points

$$f(x) = \frac{x^3}{20} - \frac{x}{4} - .3$$

$$\boxed{\begin{array}{l} x=-1 \\ x=2 \end{array}}$$

$$f''(x) = \frac{6x}{20} = \frac{3x}{10}$$

$$\left(\frac{10}{3}\right)\frac{3x}{10} = 0\left(\frac{10}{3}\right)$$

$$\boxed{x=0}$$

(step 2) Eval with $f'(x)$

$$f'(x) = \frac{3x^2}{20} - \frac{1}{4}$$

$$f'(0) = -\frac{1}{4}$$

$$f'(-1) = -\frac{1}{10}$$

$$f'(2) = \frac{7}{20} = .35$$

$$\boxed{\text{max} = .35}$$

Answer