Bairstow's Method

MATH 4701 Numerical Analysis

Synthetic Division

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1. We form a table with **three** rows and $n + 1$ columns.

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1. We form a table with **three** rows and $n + 1$ columns.
2. We write the coefficients of $P(x)$ sorted by degree in the first row.

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5. Then we multiply the number in the third row we just found by a and we write the product in the second row of the next column.

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5. Then we multiply the number in the third row we just found by a and we write the product in the second row of the next column.
6. When we fill the entry in the third row of the last column, this entry is the remainder of division of $P(x)$ by $x - a$.

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6. When we fill the entry in the third row of the last column, this entry is the remainder of division of $P(x)$ by $x - a$.
7. The rest of the numbers in the third row in the order they appear are the coefficients of the quotient of division of $P(x)$ by $x - a$.

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Example: Find the quotient and remainder of division of $P(x) = x^5 - 4x^4 + 3x^3 + 2x^2 - 5x - 3$ by $x - 2$.

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We form a table with three rows and $n + 1 = 6$ columns.

$a = 2$

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We write the coefficients of $P(x)$ sorted by degree in the first row.

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1	-4	3	2	-5	-3

Example: Find the quotient and remainder of division of $P(x) = x^5 - 4x^4 + 3x^3 + 2x^2 - 5x - 3$ by $x - 2$.

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0					

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0					
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1	-2				

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1	-4	3	2	-5	-3
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1	-4	3	2	-5	-3
0	2	-4	-2	0	-10
1	-2	-1	0	-5	-13

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Remainder$= -13$

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Quotient$= x^4 - 2x^3 - x^2 - 5$

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Notice that since $P(x) = Q(x)(x - a) + R$, $P(a) = Q(a)(a - a) + R = R$.

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Notice that since $P(x) = Q(x)(x - a) + R$, $P(a) = Q(a)(a - a) + R = R$. Therefore, the remainder of division of $P(x)$ by $x - a$ calculates $P(a)$. In this example $P(2) = -13$.

Synthetic Division by a Quadratic Polynomial

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1. We form a table with **four** rows and $n + 1$ columns.

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6. We multiply the number in the fourth row of the previous column by q and we write the product in the second row of the next column.
7. When we fill the entry in the fourth row of the last column, the last two entries a and b in the fourth row determine the remainder of division of $P(x)$ by $x^2 - px - q$ as $ax + b$.

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8. The rest of the numbers in the fourth row in the order they appear are the coefficients of the quotient of division of $P(x)$ by $x^2 - px - q$.

Example: Find the quotient and remainder of division of $P(x) = x^4 - 2x^3 + 4x^2 - x + 7$ by $x^2 - 2x + 5$.

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We form a table with **four** rows and $n + 1 = 5$ columns.

$p = 2 \qquad q = -5$

Example: Find the quotient and remainder of division of $P(x) = x^4 - 2x^3 + 4x^2 - x + 7$ by $x^2 - 2x + 5$.

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0	0			
0				0

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1	-2	4	-1	7
0	0			
0				0
1				

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Example: Find the quotient and remainder of division of $P(x) = x^4 - 2x^3 + 4x^2 - x + 7$ by $x^2 - 2x + 5$.

In each step we calculate the sum of the entries in the first three rows of the current column and write the sum in the fourth row under the same column.

Then we multiply the number in the fourth row of the current column by p and we write the product in the third row of the next column.

We multiply the number in the fourth row of the previous column by q and we write the product in the second row of the next column.

$p = 2 \qquad q = -5$

1	-2	4	-1	7
0	0	-5	0	
0	2	0	-2	0
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When we fill the entry in the fourth row of the last column, the last two entries a and b in the fourth row determine the remainder of division of $P(x)$ by $x^2 - px - q$ as $ax + b$.

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Remainder$= -3x + 12$

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For every complex number $\omega = a + b\mathbf{i}$, its conjugate $\bar{\omega} = a - b\mathbf{i}$ satisfies $\omega\bar{\omega} = |\omega|^2 = a^2 + b^2$ and $\omega + \bar{\omega} = 2a$ which are both real numbers.

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In the above example

$x^2 - 2x + 5 = 0 \Leftrightarrow x^2 - 2x + 1 + 4 = 0$

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Therefore, to calculate $P(1 + 2\mathbf{i})$ we can evaluate $R(1 + 2\mathbf{i}) = -3(1 + 2\mathbf{i}) + 12 = 9 - 6\mathbf{i}$.

Recursive Relations for Division by a Quadratic Polynomial

When we divide polynomial $P(x) = x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$ by a quadratic polynomial $x^2 - px - q$ to get the quotient $Q(x) = x^{n-2} + b_{n-3}x^{n-3} + ... + b_1 x + b_0$ and remainder $R(x) = ux + v$ we have

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from these relations we can define the following recursive (reverse time) system

$$b_i = a_{i+2} + pb_{i+1} + qb_{i+2}$$

starting with initial conditions $b_{n-2} = 1,\ b_{n-1} = b_n = 0$.

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Synthetic division is a way of implementing this recursive system using a table.

If $P(x)$ is a polynomial with real coefficients and $\omega = a + b\mathbf{i}$ is a complex root of $P(x)$, then $\bar{\omega} = a - b\mathbf{i}$ is also a root of $P(x)$.

Bairstow's Method

If $P(x)$ is a polynomial with real coefficients and $\omega = a + b\mathbf{i}$ is a complex root of $P(x)$, then $\bar{\omega} = a - b\mathbf{i}$ is also a root of $P(x)$. In particular, $(x - \omega)(x - \bar{\omega})$ is a factor of $P(x)$.

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In **Bairstow's method** we write the remainder $u(p, q)x + v(p, q)$ of division of $P(x)$ by $x^2 - px - q$ as a function of p and q. The goal is to find values of p and q for which both $u(p, q) = 0$ and $v(p, q) = 0$. Starting with initial guess (p_0, q_0) corresponding with an initial guess of a quadratic factor $x^2 - p_0x - q_0$ of $P(x)$ we use Newton-Raphson iterations on the system of two equations $u(p, q) = 0$ and $v(p, q) = 0$ in two variables p and q to find a sequence (p_n, q_n) corresponding with quadratic polynomials $x^2 - p_nx - q_n$ which are estimates of a quadratic factor of $P(x)$.

Example

Use Bairstow's method to estimate a quadratic factor and its corresponding pair of complex roots for the polynomial $P(x) = x^4 - 5x^3 + x^2 + 12x + 15$. Use initial guess $(p_0, q_0) = (-2, -1)$.

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q	0	0	q		
p	0	p	$p^2 - 5p$		0
	1	$p - 5$			

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Use Bairstow's method to estimate a quadratic factor and its corresponding pair of complex roots for the polynomial $P(x) = x^4 - 5x^3 + x^2 + 12x + 15$. Use initial guess $(p_0, q_0) = (-2, -1)$.

	1	-5	1	12	15
q	0	0	q		
p	0	p	$p^2 - 5p$		0
	1	$p - 5$	$p^2 - 5p + q + 1$		

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p	0	p	$p^2 - 5p$	$p^3 - 5p^2 + qp + p$	0
	1	$p - 5$	$p^2 - 5p + q + 1$	$p^3 - 5p^2 + 2qp + p - 5q + 12$	

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Now we use Newton-Raphson's iteration for the system of non-linear equations

$$\begin{cases} U(p,q) = p^3 - 5p^2 + 2qp + p - 5q + 12 = 0 \\ V(p,q) = qp^2 - 5qp + q^2 + q + 15 = 0 \end{cases}$$

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$p_0 = -2$ and $q_0 = -1$

Example

Use Bairstow's method to estimate a quadratic factor and its corresponding pair of complex roots for the polynomial $P(x) = x^4 - 5x^3 + x^2 + 12x + 15$. Use initial guess $(p_0, q_0) = (-2, -1)$.

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$p_0 = -2$ and $q_0 = -1$

$p_1 = -1.776859504132231 4050$ and $q_1 = -1.2314049586776859 5040$

Example

Use Bairstow's method to estimate a quadratic factor and its corresponding pair of complex roots for the polynomial $P(x) = x^4 - 5x^3 + x^2 + 12x + 15$. Use initial guess $(p_0, q_0) = (-2, -1)$.

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$p_0 = -2$ and $q_0 = -1$
$p_1 = -1.7768595041322314050$ and $q_1 = -1.2314049586776859504 0$
$p_2 = -1.76897527126986$ and $q_2 = -1.28245105573983$

Example

Use Bairstow's method to estimate a quadratic factor and its corresponding pair of complex roots for the polynomial $P(x) = x^4 - 5x^3 + x^2 + 12x + 15$. Use initial guess $(p_0, q_0) = (-2, -1)$.

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$p_3 = -1.76907525986458$ and $q_3 = -1.28291926483910$

Example

Use Bairstow's method to estimate a quadratic factor and its corresponding pair of complex roots for the polynomial $P(x) = x^4 - 5x^3 + x^2 + 12x + 15$. Use initial guess $(p_0, q_0) = (-2, -1)$.

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$p_0 = -2$ and $q_0 = -1$
$p_1 = -1.776859504132231405 0$ and $q_1 = -1.231404958677685950 40$
$p_2 = -1.76897527126986$ and $q_2 = -1.28245105573983$
After three steps of Newton-Raphson iteration, we announce $x^2 + 1.76907525986458x + 1.28291926483910$ as a quadratic factor of $P(x)$.

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Use Bairstow's method to estimate a quadratic factor and its corresponding pair of complex roots for the polynomial $P(x) = x^4 - 5x^3 + x^2 + 12x + 15$. Use initial guess $(p_0, q_0) = (-2, -1)$.

After three steps of Newton-Raphson iteration, we announce $x^2 + 1.76907525986458x + 1.28291926483910$ as a quadratic factor of $P(x)$.

This gives us $a = -\frac{1.76907525986458}{2} = -0.8845376265$ and $b = \pm\sqrt{1.28291926483910 - a^2} = 0.7074690370$ which give us the two conjugate roots $\omega = -0.8845376265 + 0.7074690370\mathbf{i}$ and $\bar{\omega} = -0.8845376265 - 0.7074690370\mathbf{i}$.

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Use Bairstow's method to estimate a quadratic factor and its corresponding pair of complex roots for the polynomial $P(x) = x^4 - 5x^3 + x^2 + 12x + 15$. Use initial guess $(p_0, q_0) = (-2, -1)$.

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In this example since $P(x)$ has degree four and the polynomial $q(x) = x^2 + 1.76907525986458x + 1.28291926483910$ is a factor of $P(x)$, the quotient of division of $P(x)$ by $q(x)$ is also a quadratic factor of $P(x)$.

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Use Bairstow's method to estimate a quadratic factor and its corresponding pair of complex roots for the polynomial $P(x) = x^4 - 5x^3 + x^2 + 12x + 15$. Use initial guess $(p_0, q_0) = (-2, -1)$.

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In the synthetic division table earlier on we found the quotient to be $x^2 + (p-5)x + p^2 - 5p + q + 1$.

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Use Bairstow's method to estimate a quadratic factor and its corresponding pair of complex roots for the polynomial $P(x) = x^4 - 5x^3 + x^2 + 12x + 15$. Use initial guess $(p_0, q_0) = (-2, -1)$.

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Using the values of p_3 and q_3 for p and q we get the quotient to be approximated as $x^2 - 6.769075260 * x + 11.69208432$ and the other two conjugate roots of $P(x)$ to be
$3.384537630 - 0.4868155206\mathbf{i}, 3.384537630 + 0.4868155206\mathbf{i}$.