Numerical Methods and Their Conditions

- **Newton Method**: An iterative root-finding technique using the formula x_{n+1} = x_n - f(x_n)/f'(x_n). Conditions: $f(x)$ must be differentiable near the root, and $f'(x) \neq 0$.
- **14-bit Representation**: A binary floating-point format using 1 sign bit, several exponent bits, and remaining mantissa bits. The representable range depends on bit allocation and bias.
- **Bisection Method**: A bracketing method for finding roots by repeatedly halving the interval [a,b]. Conditions: $f(x)$ must be continuous, and $f(a)$ and $f(b)$ must have opposite signs.
- **Modified Newton Method**: A variation of Newton’s method that reuses f'(x) to reduce computation. Conditions: f and f' continuous, and $f'(x) \neq 0$ near the root.
- **Method of False Position (Regula Falsi)**: A root-finding method similar to bisection but uses a secant line. Conditions: $f(x)$ continuous on $[a,b]$ and $f(a) \cdot f(b) < 0$.
- **Steffensen Method**: An accelerated fixed-point method using Aitken’s Δ^2 process. Conditions: $g(x)$ must be continuously differentiable and $|g'(x)| < 1$ near the root.
- **Fixed Point Iteration**: Finds a root by solving $x = g(x)$. Conditions: $g(x)$ continuous and differentiable, with $|g'(x)| < 1$ for convergence.
- **Bernoulli Method**: Used for finding polynomial roots by recurrence relations. Conditions: coefficients should not lead to division by zero and the leading term must be non-zero.
- **Root Squaring**: A polynomial transformation method (e.g., Graeffe’s method) that amplifies differences between root magnitudes. Conditions: polynomial coefficients must be real.
- **Two-variable Newton Method**: An extension of Newton’s method for systems of two nonlinear equations using Jacobian matrices. Conditions: differentiable f■(x,y), f■(x,y), and non-singular Jacobian.
- **Linear Difference Equations**: Equations of the form a_n u_{k+n} + ... + a■ u_k = 0. Conditions: coefficients constant for homogeneous equations, and initial terms specified.
- **Linear Difference with Repeating Roots**: When the characteristic polynomial has repeated roots, the solution includes terms like k·r^k. Conditions: must detect multiplicity of roots correctly.
- **Secant Method**: A derivative-free method using two initial guesses: x_{n+1} = x_n - f(x_n)(x_n - x_{n-1})/(f(x_n) - f(x_{n-1})). Conditions: $f(x)$ continuous and initial points near the root.