Eliminating Seasonal Movements from Series of Economic Data

By publishing this study, prepared by the staff of its Department of Research and Statistics, the Bank deutscher Länder hopes to encourage and promote the work that is now being conducted by various institutions with a view to eliminating seasonal variation from series of economic data. Where it seems appropriate the Bank will until further notice make use in its Reports of the series adjusted according to the procedure here described; of course it will also be glad to make use of series "adjusted" by other methods, if the institutions which are mainly engaged on such studies should consider other methods to be more expeditious.

In a very large number of cases, both in the narrower monetary sphere and in the other fields with which our Reports currently deal, diagnosing economic developments involves weighing the question whether and to what extent short-term changes, which are from time to time observed in individual items, can be regarded as indications of a change in general trends. Many changes of direction which the statistical series exhibit in the course of individual months, quarters and years are in fact no more than periodically recurring phenomena, so that it is not possible without further qualification to draw from them conclusions as to the trend over a longer period. Other changes may be purely "random" in the sense that they are due to a combination of numerous causes which cannot be determined in detail at all. And then there are changes which are due to ascertainable causes of a passing nature. In their case one of course still has to decide for purposes of analysis whether one can expect the previous trend to continue after the disappearance of these temporary causes, or whether one must expect certain secondary effects to last for a considerable time after disappearance of the primary cause.

Narrow limits are of course set to any attempt to base so far as possible exclusively on safe and objective evidence such judgments regarding the indicative significance of short-term changes. Hence in the diagnosing of given situations importance will always attach to the weighing of factors which escape direct statistical ascertainment. Nevertheless one will have to use all the possibilities, especially those offered by a comparison with similar changes in earlier periods, in order to reach a proper judgment as to the significance of short-term fluctuations.

This applies in the first place to the question how far short-term changes in economic data are to be attributed to the natural seasonal recurrence of summer and winter, sowing and harvest, and to the similar seasonal rhythm of many social phenomena such as the bunching of holiday periods, Christmas and spring buying, end-of-season sales, tax payment dates, and periodical payments of wages, salaries, pensions, rent and interest. Here too an absolutely unequivocal judgment is never possible. In the first place the very seasonal variations of temperature and weather differ quite appreciably from year to year, while it is not possible to ascertain precisely how far the divergencies in the movement of the particular economic series in question in the course of individual years are due to just these differences in the weather. Besides this the "social habits" — and this applies especially to those which give rise to seasonal changes in production and turnover — are subject to a certain variation, as in the amount of Christmas bonuses or the extent of winter building.

And yet it is no use because of these difficulties to cast doubt from the outset on every attempt at quantifying, in the sense of splitting the observed short-term changes into a component which is rather to be ascribed to the seasonal trend, and a residual component which may possibly be relevant to diagnosis of the economic trend in general; in fact there is simply no choice as to whether it is at all possible, before interpreting actual changes, to ascertain and make appropriate allowance for the changes which are called seasonal. On the contrary, almost every statement as to whether certain general tendencies are growing stronger or weaker already implies an assumption as to the movement to be expected on seasonal grounds, since one can describe as a change in the trend

only something that goes beyond movements of this kind recurring at regular intervals. All that remains for choice is therefore whether one's idea of what is to be called seasonal shall be based on purely haphazard recollection, or simply on what happened in the previous year, or whether it might not perhaps be better to form that idea on the basis of systematic analysis of a number of previous years.

Inadequate Methods of Eliminating Seasonal Movements

A method which has become widely generalised in economic reporting for avoiding the difficulties which periodically recurring variations place in the way of interpreting economic statistics is to compare the figure for any given part of a year, quarter or month, not with the immediately preceding similar period, but with the corresponding part of the previous year, quarter or month. Frequently for instance the increase or decrease in monthly industrial output is stated by comparison with output in the same month of the previous year. Similarly, because many tax payments are due quarterly, tax revenues are usually compared with those received in the corresponding month of the previous quarter as reckoned for tax purposes. This procedure has the advantage of simplicity, and indeed is usually adequate where the trend is steady; but once in a while it may result in very seriously misleading conclusions.

If for example the monthly figures in a given year are above those for the corresponding months in the previous year, and if between one month and the next this difference between the monthly figures increases, then one is prone to conclude from this that the general trend is upward; and yet in reality that increase in the difference may quite possibly be due to the fact that the figure for the month a year earlier corresponding to the second month was particularly low. The change in such growth-ratios indicates only to what extent the current year's trend differs from that in the previous year; but this does not at all show whether, and if so to what extent, the overall trend has altered. Suppose that towards the end of one year there was a very large rise, which then gave way to a stationary phase at the beginning of the next year. After such a movement it may well happen that during the second half of the current year the growth-ratios will decrease, even though the trend has already changed to a fresh rise. In such a case it is enough for the rate of the fresh rise to be slower than a year before to produce further declining growth-ratios, even though a fresh upward movement has in reality been proceeding for some time.

The case is quite similar where for instance one compares the actual change between one month and the next with the change between the two corresponding months in the preceding year or in still earlier ones. From such comparisons too all that one learns is to what extent the overall movement in the current year differs from that in the years taken as a basis for comparison. But one does not learn whether the trend, regarded absolutely, has a rising or a falling tendency. This is because the month-to-month change in each of the years taken as a basis of comparison is determined not only by the seasonal, which means periodic, change, but also by the overall trend then prevailing.

In order to overcome these difficulties, and to eliminate false conclusions which may easily be drawn from them, attempts have frequently been made to base analyses on a certain "normal course" of the seasonal variations, formed from the average of a number of previous years, in the shape of what are called "seasonal normals". No matter by which of the numerous methods such "seasonal normals" are found, they do usually give for each individual calendar month (January, February and so forth) a single figure which indicates either by how much *per cent* or by what *absolute* amount the values for that calendar month in the relative time-series (for industrial production, bookings of orders or the like) have, on the average of the years taken as basis for comparison, differed from the yearly average or from the overall trend. This procedure already offers substantial advantages as compared with the simple year-to-year comparisons mentioned above.

Without now entering into these advantages in detail, and without on the other hand setting forth at this point the difficulties which not only the last-named but also all similarly constructed procedures necessarily encounter, let

us at least say that all those procedures which operate with a normal-seasonal-movement table consisting of a single series of twelve figures for the individual calendar months suffer from two special disadvantages, which can be avoided by a certain refinement. In the first place it is necessary with these procedures to decide in advance whether one is going to regard as "normal" seasonal deviations which from year to year remain constant in their *absolute* amount, or whether one shall call the movement "normal" when the seasonal deviations reckoned as *percentages* show constancy. Secondly, with these procedures, after establishment of the "seasonal normal" it is usually not verified to what extent the empirically found movement differed during the period taken as basis of comparison from the seasonal movement assumed as "normal"; it is no longer to be seen how far the individual seasonal figures, found as averages, in fact represent the seasonal trend in each period taken for comparison.

A New Procedure for Eliminating Seasonal Movements

Since as already mentioned the simple year-to-year comparison frequently yields no results appropriate to the purpose, and since moreover certain weaknesses are inherent in adjustment for seasonal movements with the aid of simple figures of normal seasonal movement, we have worked out and tested a procedure, which is in many respects of a new kind, for eliminating seasonal movements. A detailed account will now be given of the individual steps in this procedure. The most important indications for practical use of the diagrams which were obtained under this procedure are stated and explained at the close of this study (see Steps 9 and 10 on pages 46/47).

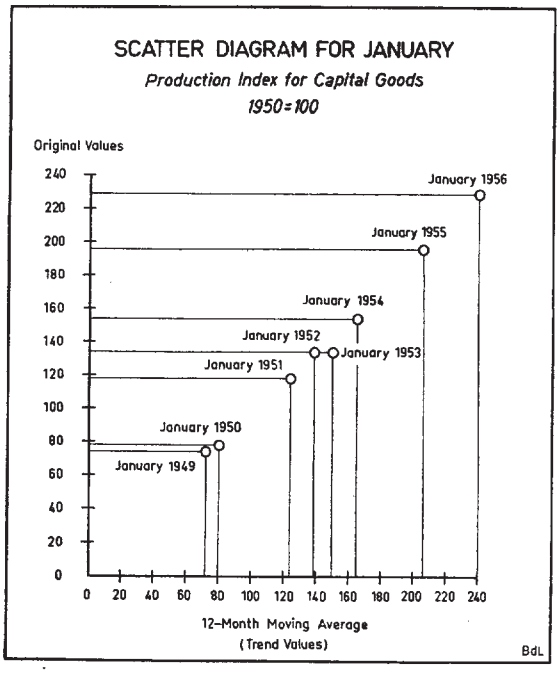
Step 1

For all months in the past period taken for comparison one establishes in addition to the original statistical figure — the so-called *original, or unadjusted, value* — the "moving *twelve-month average*" (as the appropriate "trend value"): that is to say, for example, one establishes for January 1954 the figure which results if one adds together all the monthly figures from August 1953 to June 1954 inclusive, plus half the monthly figures for July 1953 and July 1954, and then divides their total by twelve[1]).

Step 2

For every one of the twelve calendar months there is constructed a *scatter diagram*, such for example as is shown for January in Graph 1. In this scatter diagram there is plotted for each individual year a *point*, the height of which above the abscissa corresponds to the actual January value in the year in question, and the distance of which from the ordinate is determined by the twelve-month average appropriate to that value[2]).

Graph 1



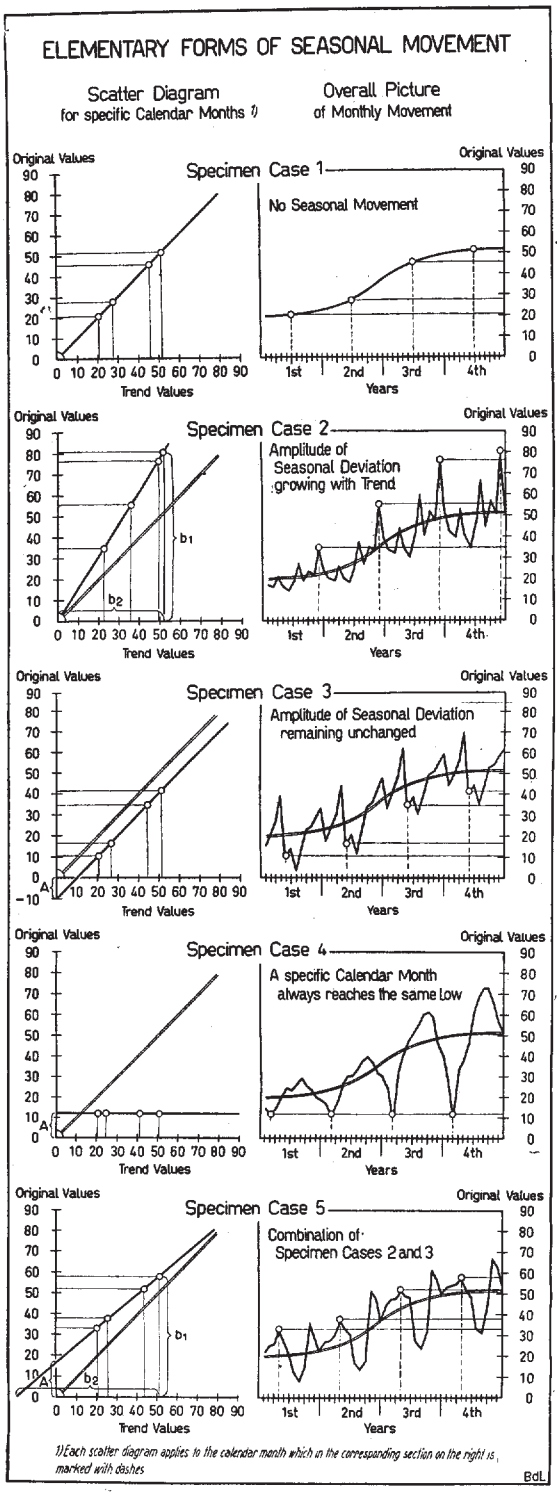
This scatter diagram gives an impression of whether the divergence of the individual January values from the twelve-month average in each case shows a certain regularity. Thus, if for example the January values always happened to

[1]) As can be seen from step 6, owing to the subsequent systematic correction of the trend it makes almost no difference what trend values are initially taken as basis. That is why it is possible in many cases to take a short cut to the "final" trend by immediately selecting a trend by the freehand method, subject to subsequent systematic adjustment.

[2]) In cases where only monthly figures are available allowance can be made for *variation in the dates of public holidays and end-of-season sales* by constructing separate scatter diagrams, for instance, for all the months of April in which the weeks before Easter represented a major part of the month, and for all the months of April in which this was not so. Where data for relatively short periods are available scatter diagrams can for instance be constructed in each case for the "last three weeks before Easter", and appropriate account can be taken of them alongside the diagrams for the calendar months.

correspond precisely with the appropriate twelve-month average — and this would point to the fact that the January values showed no seasonal deviation at all — then all the individual points in the said scatter diagram would lie on the thickly drawn diagonal (Graph 2, Specimen Case 1). If however the monthly figures were for example always just 55 per cent higher than the corresponding twelve-month average, then the individual points would fall on the line representing the "multiplicative seasonal component" which is shown in Graph 2, Specimen Case 2, running somewhat steeper than the diagonal. If, to take a third specimen case, the monthly values in all years fell short of the twelve-month average not by a constant *percentage*, but always just by an equal *absolute amount*, then this would be reflected in the scatter diagram in the fact that the line joining the said points would run parallel to the diagonal, as illustrated in Specimen Case 3 of Graph 2 ("additive seasonal component"). If however as in Specimen Case 4 the points lie with fair accuracy on a horizontal line, this would mean that the monthly values in question did not, as in the cases already explained, differ from the twelve-month average by a constant percentage (Specimen Case 2) or by an equal absolute amount in each case (Specimen Case 3); it would mean that, in relative independence of the overall trend and therefore also comparatively independent of the appropriate twelve-month average, they reached a fairly constant absolute value in the calendar month in question in all the observed years. (This is a specimen case which becomes somewhat important above all in connection with the figures for building activity and unemployment, because during the winter months the time-series for both of these reach certain extreme values, the amount of which shows almost no connection with the overall trend.) Of course there are also combinations between the elementary forms of Specimen Cases 1 to 4 which solely for the sake of simplicity were first described; these combinations indeed play an important part in practical analysis. Suppose that the points lie roughly on a line as shown in the scatter diagram for Specimen Case 5. This would mean that the amount by

Graph 2



which the original values for the month in question in the observed period differed from the twelve-month average consisted of a constant percentage and an unchanging absolute amount. This is a combination of the "multi-

plicative" seasonal component in Specimen Case 2 and the "additive" seasonal component in Specimen Case 3. In all cases the component for seasonal deviation (A) which has to be measured in terms of absolute amount can be read off at the intersection of the line in question with the ordinate; and the component (B) to be measured in terms of a percentage can be seen as the percentage amount by which the distances designated b_1 in the diagram differ from those designated b_2.

In reality of course the points which one obtains by plotting individual empirical number-series never lie precisely on one straight line. The straight lines are in all cases no more than auxiliary lines by which the general location of the points is to be roughly indicated in the scatter diagram.

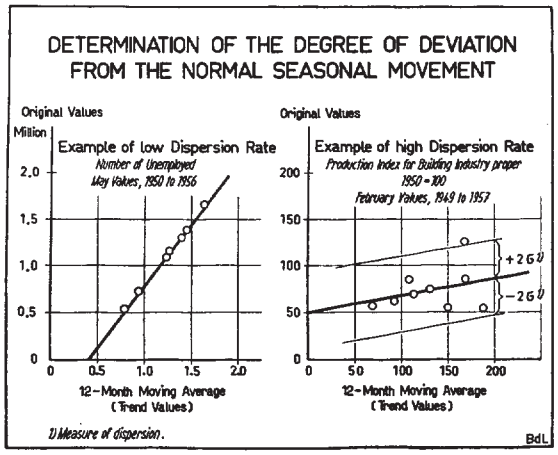
Step 3

As soon as the position of the points in the scatter diagram can in fact be indicated by a straight line, even if only as a rough approximation, such a straight line is drawn in the scatter diagram (Graph 1) as what is called a *regression line;* in accordance with the usual statistical methods this is done in such a way that the squares of the vertically measured distances from the individual points to this straight line remain on the average as small as possible[1]). If by chance the various points from the outset lie fairly close to a straight line, then if the auxiliary line is in the optimum position the remaining said average of the square of all the deviations will of course be comparatively *small* (see Graph 3). The less the location of the points from the outset resembles a straight line, on the other hand, the greater this mean-square deviation will be.

Thus through the scatter diagram one not only obtains a visual idea of whether the deviations between the values for a specified calendar month and the twelve-month average in each of the preceding years showed any regularity *at all;* one also obtains at the same time an indication of the *nature* of this regularity — whether for example the January values were usually below

[1]) In many considered cases where only small "additive" components were to be observed, straight lines through the origin of the system of coordinates were preferred. As to this see No. 3 of the Notes as to the Theory on page 47.

Graph 3



the twelve-month average by a definite percentage or a definite absolute amount; and one also obtains a measure of whether the ascertained regularity was *more strictly or less strictly* realised. In the present conditions of relatively rapid change in many economic phenomena, and in view of the fact that at the moment the period open for observation in the German Federal Republic comprises only the few years since Currency Reform, precisely the existence of a measure of how closely the observed regularity was in fact realised is of great importance for practical analysis.

Step 4

In a *further graph,* which differs from the scatter diagrams previously mentioned in comprising all the monthly results for the whole of the observed period as usual in calendar order, both the statistical unadjusted data and also the above-mentioned moving twelve-month averages are entered (Graph 4). As will be seen, the line for the moving twelve-month average already presents *a certain picture of the current overall trend.*

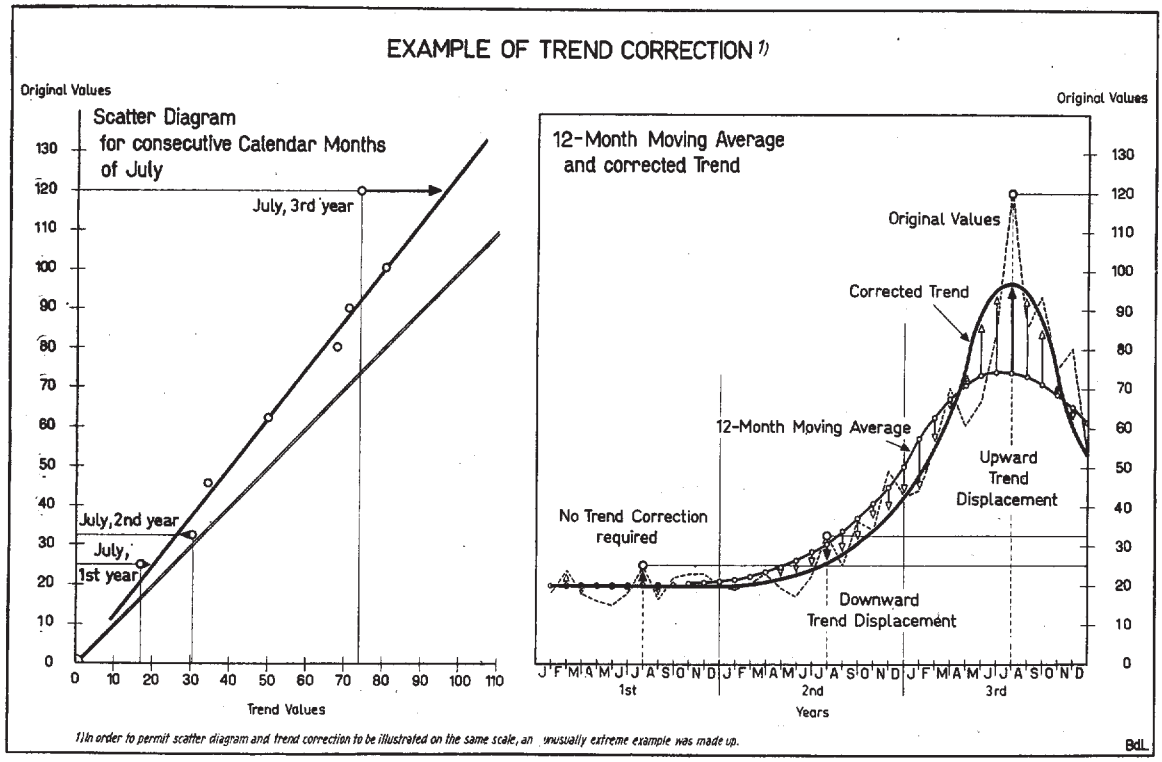
Step 5

The *picture of the overall trend,* as it first emerges from the twelve-month averages, *is then systematically corrected* with the help of the information obtained from the scatter diagrams previously mentioned. For this purpose, in the first place the horizontal distance, designated by arrows in Graph 4, from the "regression line" is ascertained for every individual point in the scatter diagrams. These distances indicated

by arrows are transferred for every month out of the scatter diagram into the above-mentioned graph, reckoning from the moving twelve-month average which is to indicate the overall trend as a first approximation — counting upwards if the original value in the scatter diagram lay to the left of the regression line, and otherwise downwards. Any such arrow in Graph 4 indicates in which direction the "trend value" would have to be moved for the month in question, so that in that month the remaining proportion between the original value and the relevant new trend value should accurately correspond to the proportion which was shown by the regression line for the relative calendar month, which means that it was considered to be the "normal seasonal movement" in the first approximation.

The idea underlying this necessarily somewhat complicated operation is the following. It might well be that in the scatter diagrams first formed the possibly quite large distance of a point from the plotted regression line is not due at all — as it first appears to be — to the fact that the monthly figure in question for *random* reasons differed by a particularly large margin from the average seasonal deviation as reflected in that regression line, but is on the contrary due to the fact that in the section of the curve in question the moving twelve-month average first selected as basis in the scatter diagrams shows the overall trend only imperfectly. Whether that is in fact the case, and if so in what section of the curve, is very clearly shown by the arrows that were put in. If in fact the arrows for a number of successive months point in different directions, so that it is impossible to find any smooth line by which the overall trend could be better characterised, then that is an indication that the deviation of the points in the scatter diagram from the regression line characterising the average seasonal movement is in fact merely "random", and was due to particular events of a merely passing kind in the months in question. But if the arrows for a number of successive months mostly point in the same direction, that indicates that this part of the curve reflects the overall trend only very badly. In these cases therefore the curve which is to describe the overall trend is shifted; the moving twelve-month average first selected is replaced by a curve which in some measure corresponds to the location of the arrow-points. For this purpose

Graph 4



it has been shown, by the subsequent checks which the procedure renders possible, that the simple freehand procedure is perfectly practicable and adequate. It needs only to be borne in mind that, on an overall view, any upward and any downward shifts should roughly balance each other out.

Step 6

On the basis of this corrected line for the overall trend *Steps 2 and 3 are now repeated.* That is to say, one plots in twelve new scatter diagrams, in the manner previously described, points the height of which above the abscissa again corresponds to the original monthly value given by the statistics, while the distance from the ordinate is now given by the value which the newly obtained curve for the overall trend (Step 5) reaches in the month in question. In most of the cases dealt with the majority of the points in this second process already lay from the outset fairly close to a straight line. This indicates that the greater deviations found on the first run over the course were indeed mainly due to the fact that the moving twelve-month average gave only a poor idea of the overall trend, and that the correction has now resulted in a quite plausible apportioning of the whole unsteady movement in the original series between a component rather attributable to the overall trend and another component attributable rather to seasonal phenomena in each of the years.

While all points are taken into account in forming the first preparatory scatter diagrams, in each of the last scatter diagrams individual points lying in very extreme positions are sometimes disregarded when ascertaining the optimum location of the straight lines. In fact, it is very likely that those values which after all corrections still lie a very long way from the regression line are due to "singular" causes. Where such a cause is in fact established, the extreme value in question is left out of account in determining the location of the regression line, since after all the location of that line is intended simply to afford a measure of the percentage and absolute amounts by which the individual months "usually", i. e. for seasonal reasons, differed from the values determined by the overall trend.

In case of need the procedure described as "Step 6" can of course be repeated a number of times until the location of the arrow-points in Graph 4 shows that it is not possible further to improve the curve describing the overall trend. To that extent the process resembles that of gradual approximation through repeated corrections which has been applied with great success in other branches of science.

Remarkably enough, where the first provisional line for the overall trend is arbitrarily selected, such repeated application of the procedure described for correcting the location of the overall trend-line always leads to results which resemble each other more and more closely, and in most cases already differ from each other only insignificantly even at the second correction, always provided that in every case one requires the final shape of that line to fulfil the same formal requirements. One of these formal requirements is for example that at least some months shall lie between any two flex points at which the direction of this overall trend-line changes. Otherwise, if for instance one were to change the direction appreciably at still shorter intervals, this line would lose its character as evidence of the overall — i. e., at least medium-term — trend; and in the extreme case, if monthly changes of unlimited extent were admitted, it would ultimately coincide with the seasonally adjusted series itself. Yet the object of the procedure here described is precisely to obtain better information on the medium-term trend, beyond the monthly variations due to seasonal and random causes.

On the other hand however it would not be correct to take the fact that even on differing initial assumptions as to the overall trend repeated application of the procedure described leads to results which resemble each other more and more closely as already affording evidence that thereby one had arrived at anything like the "true" or "only correct" apportionment of the movement as between a random component, one due more to seasonal causes, and one attributable rather to the overall trend. In actual fact, the position here is like that in other branches of statistics, for instance in the calculation of price indices, or in other working up of individual statistical data so as to present an overall con-

clusion as to the general situation. There are of course some results which are definitely "wrong" inasmuch as they are based on incorrect original series or on inappropriate methods. But in the case of mass phenomena which always have to be regarded from a number of points of view, and hence also in the apportioning of numerous individual figures in a time-series as between an essential component on the one hand and on the other hand unessential subsidiary phenomena which are temporary (seasonal) or completely "random", there is no "really correct" value which one can surmise by intuition or gradually ascertain with constantly increasing care and precision. On the contrary, even in connection with overall statistical conclusions which are based on unimpeachable data and reached by unassailable methods there is always some margin for arbitrary decision. That margin can no doubt be reduced in individual cases according to the actual question asked by the user. But even then room is still left for results diverging to a certain extent; and objective considerations show that in principle among these results, even where further progress is made in research on the subject, not one can be singled out as the only result that is correct.

Not least because of the "range of dispersion" resulting from the mere fact that in principle there are always several statistical methods that in themselves are equally justified, it seems appropriate, even as regards the purely outward presentation, so far as possible to present in such a way the results of applying methods of this kind as not to give an impression of having reached a supposed degree of computational precision which in principle does not exist in the sphere of statistical mass phenomena. There is the further point that in the present case of separating the changes "normally" to be expected according to the seasons from those changes which may possibly indicate a change in the trend it is of special interest already to know, at the time when the procedure is applied, to what extent the changes in the observed years conformed to what is here assumed to be "normal". In fact, the wider the range of dispersion, that is to say, the less closely the changes in an individual calendar month by comparison with the immediately preceding and the immediately following month conformed in past years to what has here been found to be the "normal" monthly seasonal deviation, the less occasion there will be to suppose that there is a change in the overall trend as soon as new monthly figures come in which differ appreciably from the result that is to be expected in the light of the ascertained "normal".

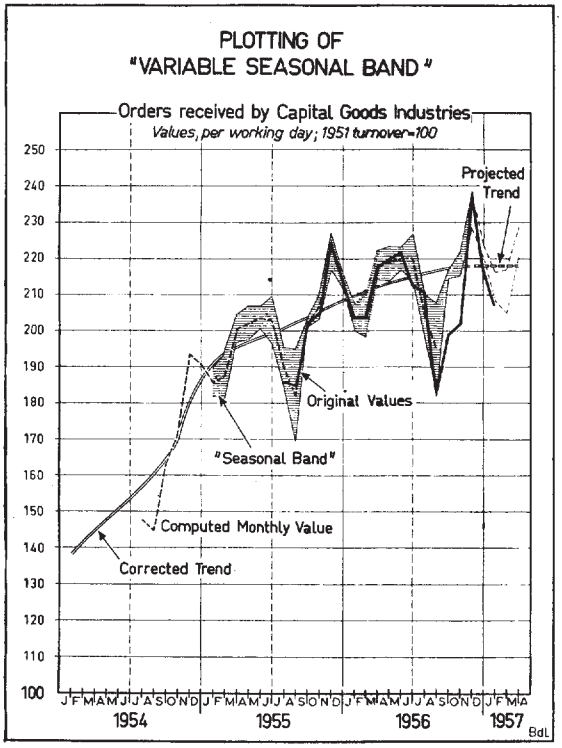
Step 7

In order to obtain an initial measure of the zone of dispersion, on the basis of the scatter diagrams established on the last trend correction the *mean-square deviation* of the observed monthly values from the "normal values" represented by the regression line is determined for each calendar month, as indicated in Step 3 on page 42.

Step 8

All the obtained results are amalgamated to form a *final graph* in the following manner. First the corrected line obtained according to Step 5 is plotted for the general trend. Starting from this line one plots the "computed monthly value" (Graph 5) for each current month on the basis of the figures for the "normal" percentage and absolute deviation of the individual monthly values, as this can be read according to Step 2 from the location of the regression line in the scatter diagram. The line joining these points shows what the course would have been if the actual movement had precisely corresponded with the rule now ascertained. Starting from this line, which reflects the hypothetical course corresponding to the law that has been found, one now marks out the observed range of dispersion for each individual value, upward or downward, by a distance which is determined by twice the root of the mean-square deviation mentioned in Steps 3 and 7. As can be seen in the example depicted, the range of deviation usually differs a good deal as between individual months. While in the example shown the September values during the period taken for comparison followed fairly closely the rule which was ascertained by the procedure, this is so in the case of the August values only to a smaller extent. The line joining the ends of the distances so plotted gives what

Graph 5



is called the *Variable Seasonal Band*[1]). Finally one also plots the original values in the same graph. The closer these original values lie to the centre of the seasonal band, the nearer the result for the month in question corresponded to the regular trend now covered by the overall trend-line and the seasonal components. If an earlier original value lies outside the "seasonal band", this means that the result for the month in question is regarded as one which for random reasons is particularly high or low.

Step 9

The most important *practical application* is of course *when new monthly figures come in*, because, as mentioned at the outset, the prime object of the entire procedure is to obtain the most objective possible evidence as to whether the changes observed most recently can be regarded as pointing to an alteration of the overall trend. So as to make it easier to decide whether they do, the overall trend is ascertained, on the basis of the above-described procedure, say up to the last month but three before the month under report: in the case of the present Report for March, for example, as far as last November inclusive. (In ascertaining the trend for the last months it is of course no longer possible to start from the moving twelve-month average. Instead, for the last part of the period one has to rely entirely on the freehand procedure mentioned in Step 5, with subsequent systematic correction.) Up till then the overall trend is drawn thickly in the graph. For the next four months after that, and purely as an aid to diagnosis, one draws a further trend-line which corresponds precisely to the prolongation of the trend observed up till the last basis month: that is, in the present example, up till November. From this auxiliary line the previously described "seasonal band" is plotted on the basis of the regularly recurring seasonal movement which was observed in the preceding years and which has been ascertained through the procedure described, this being also done only in order to make the work of diagnosis easier and on no account with the object of making a forecast. If now for several months the figures as they become freshly known generally fall above or below the "seasonal band" which has been drawn in, this indicates that it is possible to say with a high degree of probability that there has been a fresh change in the overall trend.

Step 10

The picture becomes still clearer if from the outset the seasonal deviations which can be described as "normal" in the light of observations in the basis years are deducted from the original figures, and if only the residual value is shown, which then contains no more than the amount to be ascribed to the overall trend plus what is due to "random influences". In this way one obtains a line for the "*values adjusted for seasonal variation*" (cf. the graph on page 23 of this Report, showing the movement in the amount of orders received). If one proceeds in a corresponding manner with the other component elements of Graph 5, then in place of the "seasonal band" one obtains a "*band adjusted*

[1]) The idea of showing the range of dispersion in the form of a "seasonal band" originates from O. *Anderson* jun. (see "Eine neue Variante der Saisonberechnung von statistischen Zeitreihen" — A New Variant for Seasonal Calculation of Statistical Time-Series — in the Mitteilungsblatt für mathematische Statistik — Bulletin for Mathematical Statistics — Volume II, pages 50 to 56). While however Anderson selected a band of constant width, the procedure quoted has here been further developed, in that the range of dispersion for each calendar month has been separately ascertained, and from this a "variable" seasonal band has been formed.

for seasonal variation": that is to say, as the graph on page 23 shows, a band which now directly follows the overall trend. If in this graph a monthly value "adjusted for seasonal variation" lies in *past months* outside the band, this means that the amount remaining after the "normal" seasonal change has been deducted from the original value differs more from the plotted trend-line than was to be expected as probable in the light of the experience of the years taken as basis. If a number of *new* monthly values coming in lie uniformly above or below the plotted "seasonally adjusted band" then, as in Graph 5, this means that the overall trend has probably changed as compared with the trend observable up till the last basis month.

Notes as to the Theory

1) In the procedure described the seasonal deviations are split into an additive and a multiplicative component, that is to say into a component which is independent of and one which is dependent on the trend. Here a warning must be given in regard to the component dependent on the trend. Even though in the German Federal Republic in all years since the war a very clear proportionality can be discerned as between the overall growth of certain figures for production and turnover and the amplitude of the seasonal deviations, it is not at the moment yet possible to decide whether this amplitude in fact depends on the absolute level of the figures for the underlying processes of production and turnover. It is not impossible — and this must always be borne in mind when the procedure is applied — that such connections between the level of those figures and the amplitude of the seasonal deviation represent a merely apparent connection, that is to say what is called an illusory or nonsense correlation. If for example it has been observed that during the period since Currency Reform, when most of the economic quantities showed a fairly steady rise from year to year, a seasonal peak has become constantly greater in absolute amount, this alone will not mean that the extent of the seasonal peaks is generally growing as the absolute level becomes higher. On the contrary it is quite possible that this seasonal peak may have an inherent tendency to become greater simply because of a continuous change in certain social habits. The appearance as though this increase in the seasonal deviation were connected with the rise in the absolute level would then be due only to the "random" circumstance that during the years when social habits were continuously changing in the direction of an increase in the seasonal deviation the absolute figures for production, turnover and so forth were also increasing.

A *separation* between *trend-conditioned* and merely *time-conditioned* changes in the amplitude of seasonal deviations is for example made in the procedure used by the Federal Reserve Board (cf. "Adjustment for Seasonal Variation" by H. C. *Barton* jun. in the Federal Reserve Bulletin for June 1941). In principle the procedure which we now propose also lends itself to separate treatment of these two components. (The place of the two-dimensional regression diagrams would in that event be taken by three-dimensional constructions, showing the time-dimension as a third axis in addition to the original values and the trend values.) For practical purposes however this extension yields no usable results in present German conditions, because the apportionment which then by a kind of multiple correlation is obtained mathematically between "time-conditioned" and "trend-conditioned" changes in the amplitude of the seasonal variations, with due regard for fresh data, changes abruptly owing to the high degree of correlation between "time" and "trend", and so affords no reliable evidence. On the other hand the procedure which we here describe always affords, also in its present form, the possibility of verifying to what extent there is a connection between lapse of time and seasonal deviation. (For this purpose one needs only to compare with each other in the scatter diagram the inclination of the straight lines joining the origin to the various "annual points".) Over and above this there is also the possibility, as already described, of allowing for additive components and of carefully demonstrating the range of dispersion in the individual case, which is not immediately possible in the above-mentioned procedure of the Federal Reserve Board.

2) Since, when one "forms averages" by means of regression lines, the two points furthest from each other in the system of coordinates exercise a certain "leverage", and so acquire relatively great weight, the resulting distortions are *corrected* in extreme cases by omission of one of the two points.

3) Where the lines selected as regression lines in the procedure which we here describe are not just any lines, but are exclusively straight lines from the origin, this corresponds almost precisely to the *traditional procedures which use a table of "multiplicative" normal seasonal movements*. Thus our proposed method includes these procedures as a possible and at the same time highly important special case, which will be used where the original values in fact almost exclusively comprise proportional seasonal components. In this special case, apart from demonstration of the range of dispersion, a difference remains only in two points. First, in the procedures mentioned it is usual to select as average the arithmetic mean, whereas the least-squares method chosen by us produces another average. Secondly, in the traditional procedures the seasonal factor is often determined in the first place as a proportionality constant between the monthly values (remaining after random fluctuations have been eliminated) and their respective *trend values*, but in the subsequent application it is treated as a proportion between the original value and the trend adjusted for seasonal variation (i. e. *trend plus random component*), whereas in the procedure which we here describe the multiplicative component is in all cases related to the trend and never to the "trend plus random component".

4) In so far as only lines parallel to the diagonal are admitted as regression lines in the procedure which we here describe, this procedure corresponds to the traditional methods which operate only with an additive seasonal component. Hence these methods also are contained as a special case in the procedure described by us, subject of course to the qualification that in them the average is determined not as an arithmetic (or geometric) mean but according to the least-squares method.