Calendar Effects in Monthly Time Series: Detection by Spectrum Analysis and Graphical Methods

WILLIAM S. CLEVELAND and SUSAN J. DEVLIN*

Many time series, particularly national business and economic series, which are reported monthly and represent a total of the series for each month, contain calendar effects due to changing month length, weekly periodicities, and holidays. It is important to *detect* and *remove* this spurious calendar variation to allow a better appreciation of the variation in the series due to important factors. This article discusses detection. Two sets of diagnostic methods for detecting calendar effects in monthly time series, spectrum analyses and time domain graphical displays, are described. These methods can be used in an initial analysis to decide if calendar adjustment is necessary and can be used on an adjusted series to determine if the adjustment has properly removed all of the calendar effects.

KEY WORDS: Calendar adjustment; Trading-day adjustment; Seasonal adjustment; Spectrum analysis; Graphics.

1. INTRODUCTION

1.1 Calendar Effects

Many time series, particularly national business and economic series, are reported monthly and represent a total, or time aggregation, of the series for each month. Because the month length is not the same for all months and because many of these series have strong weekly periodicities (i.e., the series for a particular day depends on the day of the week), the aggregated monthly series will vary, in part, due to the changing month length, the effect of the weekly periodicity, and the effect of holidays. We shall refer to this variation as calendar effects in the monthly series. Examples of economic series with strong calendar effects are manufacturing shipments, barrels of imported petroleum products, money supply, housing starts, telephone messages and revenues, and retail sales. To allow an effective interpretation of aggregated monthly series it is important to remove from the series that variation which is due to calendar effects. We shall refer to this as calendar adjustment.

1.2 The Investigation of Calendar Effects

Relatively little research on calendar effects has been carried out in the past, which contrasts with the large amount of work devoted to seasonal adjustment (e.g., Zellner 1979). Young (1965) and Eisenpress (1956) discuss calendar effects and present methods for carrying out an adjustment. The X-11 package (Shiskin, Young, and Musgrave 1967) contains a calendar adjustment option that is referred to as a "trading-day adjustment." Granger (1963) has studied series whose calendar effects depend on the number of work days in a month. The imbalance in research on calendar adjustment and seasonal adjustment would seem to be inappropriate since for many series the calendar variation is at least as important as the yearly seasonal variation not due to calendar effects.

In this article we present two sets of methods for detecting calendar effects in a monthly time series. These methods focus only on that part of the calendar variation due to weekly periodicity and changing month length, and do not explicitly consider holiday effects. The first set, which is described in Section 2, consists of spectrum analyses in which peaks in the spectrum at certain "calendar frequencies" indicate the presence of calendar effects. The second set, which is described in Section 3, consists of graphical displays in which the presence of certain patterns indicates the presence of calendar effects. In Sections 2 and 3 the techniques are illustrated by their application to an artificial "weekday" series, Bell System revenues from toll calls, an international airline passenger series, and manufacturers' shipments.

1.3 Detection and Modeling

There are two stages in the overall analysis of calendar effects in which these graphical and spectrum diagnostic techniques can be used. The first stage is one in which the techniques would be used to decide if calendar effects are present and are sufficiently important to warrant modeling and adjustment. (Techniques for such modeling and adjustment are discussed elsewhere (Cleveland and Devlin 1980).) The second stage occurs after an adjustment has been carried out. Here the techniques can be applied to check the adequacy of the adjustment by checking for the presence of remaining calendar effects in the adjusted series. Thus the use of the detection procedures is analogous to the use of the usual summed, lagged product estimates of autocorrelation in time series modeling (Box and Jenkins 1976). The autocorrelation function, which is relatively simple to estimate, is used initially to determine if autocorrelation is present. If so, a model is used to account for it, an adjusted series (residuals) is computed, and the auto-

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© Journal of the American Statistical Association
September 1980, Volume 75, Number 371
Applications Section

correlation function of the adjusted series is studied to determine if there is any residual autocorrelation.

2. SPECTRUM ANALYSIS FOR DETECTING CALENDAR EFFECTS

2.1 Overview

Suppose $c(t)$ is a periodic series with period l so that $c(t + l) = c(t)$. For l even, $c(t)$ can be written in the form

$$c(t) = \mu + \sum_{j=1}^{l/2} \left\{\alpha_j \cos\left(2\pi t \frac{j}{l}\right) + \beta_j \sin\left(2\pi t \frac{j}{l}\right)\right\} \quad (2.1)$$

where

$$\alpha_j = \frac{2}{l} \sum_{t=1}^{l} c(t) \cos\left(2\pi t \frac{j}{l}\right),$$

$$\beta_j = \frac{2}{l} \sum_{t=1}^{l} c(t) \sin\left(2\pi t \frac{j}{l}\right),$$

for $j = 1, \ldots, (l/2) - 1,$

$$\mu = \frac{1}{l} \sum_{t=1}^{l} c(t) ,$$

$$\alpha_{l/2} = \frac{1}{l} \sum_{t=1}^{l} c(t) \cos(\pi t) ,$$

and

$$\beta_{l/2} = 0 .$$

(A similar formula holds when l is odd.) The spectrum $c(t)$ at frequency $f_j = j/l$, for $j = 1, \ldots, l/2$, is the squared amplitude, $\alpha_j^2 + \beta_j^2$, of the harmonic component in (2.1) at frequency f_j. The spectrum at other frequencies is 0.

Let $x(t)$ be an aggregated monthly series and let $x_m(y)$ be the mth monthly subseries of $x(t)$ so that for the yth year,

$$x_m(y) = x(12(y - 1) + m) .$$

Calendar effects result in periodic components of the form (2.1) in $x(t)$ and $x_m(t)$ at certain known frequencies, which will be referred to as "calendar frequencies" and will be discussed further in Sections 2.2.1 and 2.2.2. The first spectrum procedure for identifying calendar effects is to estimate the spectrum of $x(t)$ and look for peaks at the calendar frequencies. The second spectrum procedure is to estimate the spectrum of $x_m(y)$ for each month (excluding February, which behaves differently from the other months), average the spectrum estimates across the 11 months for each frequency, and look for peaks at the calendar frequencies. For example, in Figures C through E the peaks in the spectrum estimates at the calendar frequencies shown by the vertical dotted lines are indications of calendar effects. The principal reason for introducing a second spectrum estimate is to have a corroborative procedure. There are times, however, when one procedure will yield a very clear indication of calendar effects and the other will not.

2.2 The Spectrum of Calendar Effects

2.2.1 The Calendar Frequencies. We shall think of the unaggregated series as a continuous parameter time series, $X(T)$, where the units of the parameter T are days. Let T_0 be the beginning of the first month and let T_t be the time at the end of the tth month. Then the aggregated monthly series is

$$x(t) = \int_{T_{t-1}}^{T_t} X(T)dT ,$$

for $t = 1, 2, \ldots$. Let $C(T)$ be a weekly periodicity in $X(T)$ (i.e., $C(T + 7) = C(T)$) whose integral over a period of 7 days is 0, and suppose $X(T) = B + C(T) + R(T)$, where B is a constant and $R(T)$ is the remaining variation in $X(T)$. Then $x(t) = b(t) + c(t) + r(t)$, where b, c, and r are the aggregates of B, C, and R, respectively. The calendar component in $x(t)$ induced through the aggregation process is $b(t) + c(t)$. The term $b(t)$ is B times the number of days in month t. As we shall see, $c(t)$ has a more complicated pattern.

All 30-day months that start on a particular day of the week, say Friday, will have the same value of $B + C(T)$ and therefore the same value of $b(t) + c(t)$. A similar statement holds for 31-, 29-, and 28-day months. If we neglect the fact that leap year is omitted every 400 years, the calendar has a period of 28 years. Thus $b(t) + c(t)$ has a period of 28 years = 336 months. Let $c_m(y)$ be the mth monthly subseries of $c(t)$ so that for the yth year $c_m(y) = c(12(y - 1) + m)$. Define $b_m(y)$ in a similar fashion. Then $b_m(y) + c_m(y)$ has a period of 28 years. Thus from (2.1), the spectrum of $b_m(y) + c_m(y)$ is concentrated at frequencies $j/28$, for $j = 1, \ldots, 14$, and the spectrum of $b(t) + c(t)$ is concentrated at the frequencies $j/336$, for $j = 1, \ldots, 168$. But knowledge of these "calendar frequencies" is not very informative since there are so many. It is necessary to know which of these frequencies are typically important for series in practice. In the next sections we shall use a line of reasoning that will lead to a small number of "important" calendar frequencies and then demonstrate the validity of this reasoning in practice by several examples.

2.2.2 The Important Calendar Frequencies. Since $B + C(T)$ is a periodic function with period equal to 7 days, we can write

$$B + C(T) = \sum_{k=0}^{\infty} \gamma_k \cos\left(2\pi \frac{kT}{7} + \phi_k\right),$$

where γ_k is the amplitude of the cosine at frequency $k/7$ cycles/day and ϕ_k is the phase. Thus for the aggregated calendar effects

$$b(t) + c(t) = \sum_{k=0}^{\infty} \gamma_k h_k(t) \quad (2.2)$$

where

$$h_k(t) = \int_{T_{t-1}}^{T_t} \cos\left(2\pi \frac{kT}{7} + \phi_k\right) dT . \quad (2.3)$$

1. Values of Spectrum of $h_k(t)$ Greater Than .1

Frequency	Spectrum	k
.083	.100	0
.220	.151	2
.250	.210	0
.304	.157	2
.333	.161	0
.348	2.649	1
.402	.119	1
.416	.653	0
.432	.473	1
.500	.210	0

For two reasons the contributions in (2.2) for small k are the most important ones and those for larger k have a negligible effect. The first is that the spectrum of $h_k(t)$, which is derived in Section A.1 of the Appendix, becomes small for large k. Table 1 shows the important calendar frequencies for $b(t) + c(t)$, which are defined to be frequencies at which the spectrum of some $h_k(t)$ is greater than .1. Only values of k less than or equal to 2 appear. The second reason is an empirical result; for most weekly patterns, γ_k, which depends on the shape of the weekly pattern, will be small except for small values of k. For example, suppose $B + C(T)$ is a weekday indicator series, which means $B + C(T)$ equals 1 on weekdays and equals 0 on weekends. If $T = 0$ is taken to be Wednesday at noon, then $B + C(T)$ is symmetric about 0, so that $\phi_k = 0$,

$$\gamma_0 = \frac{1}{7}\int_{-3.5}^{3.5} (B + C(T))dT = \frac{5}{7},$$

and, for $k \geq 1$,

$$\gamma_k = \frac{2}{7}\int_{-3.5}^{3.5} (B + C(T)) \cos\left(2\pi\frac{k}{7}T\right) dT$$
$$= 2\,\frac{\sin\left(2\pi\frac{k}{7}2.5\right)}{\pi k}.$$

Thus γ_k decreases rapidly for the weekday indicator series.

Let $h_{km}(y)$ be the mth monthly subseries of $h_k(t)$ so that $h_{km}(y) = h_k(12(y - 1) + m)$ and $b_m(y) = c_m(y) = \sum_{k=0}^{\infty} \gamma_k h_{km}(y)$. For the same reasons given in the previous paragraph, the spectrum of $b_m(y) + c_m(y)$ typically will be determined by $h_{km}(y)$ for small k. (The derivation of the spectra of $h_{km}(y)$ is given in Section A.2 of the Appendix.) For a fixed k the spectra of the $h_{km}(y)$ for m corresponding to 30-day months are the same, and the spectra for m corresponding to 31-day months are the same. Furthermore, the values of these two sets of spectra are similar. February, however, has a spectrum that is quite different from the other two. Thus the procedure in practice will be to estimate 11 monthly subseries spectra (excluding February) and look at their average. The important calendar frequencies for the average are shown in Table 2. These are defined to be the frequencies at which, for some k, the average of the 11 spectra of $h_{km}(y)$ is greater than .075.

A heuristic explanation can be given for the importance of the calendar frequencies .348 cycles/month and .179 cycles/year, which have the largest spectrum values in Tables 1 and 2, respectively. Suppose the lengths of all months were equal to

$$\frac{365.25}{12}\,days = 30.4375\,days\ .$$

Suppose a cosine with a period of 7 days is sampled every month. Then the sampled series has a frequency of

$$\frac{cycle}{7\ days} = \frac{30.4375/7\ cycles}{month} = 4.348\ cycles/month$$

and the alias of this frequency is .348 cycles/month. If the sampled series is now further sampled once per year, the resulting series has a frequency of

$$\frac{.348\ cycles}{month} = \frac{4.179\ cycles}{year},$$

which aliases to .179 cycles/year.

2.3 Estimating the Spectrum of $x(t)$ and $x_m(y)$

2.3.1 Removing Trend and Seasonal. Often the aggregated monthly time series, $x(t)$, will contain strong trend and yearly seasonal components. It is generally desirable to remove these components from $x(t)$ before estimating the spectra, since the components tend to have a substantial influence on the estimates and can obscure the effects at the calendar frequencies. One procedure for removing the trend and seasonal is to decompose $x(t)$ into trend plus seasonal plus irregular by using a procedure such as SABL (Cleveland, Dunn, and Terpenning 1978 and 1979) or Census X–11 (Shiskin, Young, and Musgrave 1967) and then to estimate the spectra by using the irregular. A second procedure is to difference the series; if U is the shift operator $U_l x(t) = x(t - l)$, this means computing spectrum estimates from $(1 - U_1)^{d_1}(1 - U_{12})^{d_2}x(t)$ where d_1 and d_2 are chosen to remove the trend and seasonal, respectively (see Box and Jenkins 1976). In the examples in Sections 2 and 3 the trend and seasonal removal is carried out by using SABL with the length of the trend smooth equal to 7 and the length of the seasonal smooth equal to 11.

2. Values of Average Spectrum of $h_{km}(y)$ Greater Than .075

Frequency	Spectrum	k
.071	.124	1
.107	.078	2
.179	3.854	1
.286	.080	3
.357	.446	2
.429	.079	1
.464	.137	3

2.3.2 Transforming the Data. In many cases the trend, seasonal, and irregular are nonadditive, and the seasonal component has oscillations whose amplitudes increase as the trend increases. In the case in which $x(t) > 0$, this nonadditivity can be removed by a power transformation of the form

$$\begin{aligned} u^{(p)} &= u^p \quad \text{for} \quad p > 0 \\ &= \log u \quad \text{for} \quad p = 0 \\ &= -u^p \quad \text{for} \quad p < 0 \ . \end{aligned}$$

A procedure for selecting the value of p is given in Cleveland, Dunn, and Terpenning (1978 and 1979). Transforming $x(t)$ to make the seasonal oscillations more nearly stable is desirable because it not only facilitates the trend and seasonal removal but also generally leads to simpler models to explain the variation, both calendar and otherwise, in the series. In particular the transformation tends to result in the additivity of the calendar component as assumed in Section 2.2.1.

2.3.3 Clipping Outliers. Outliers in a series whose spectrum is to be estimated can have a substantially distorting effect on the estimate (Kleiner, Martin, and Thomson 1979). When a series with outliers is decomposed into trend, seasonal, and irregular by a routine such as SABL, which robustly estimates the trend and the seasonal, the outliers become part of the irregular. To guard against the distorting effect of outliers in a spectrum estimate from an irregular component, the irregular will be modified by the procedure described in the next paragraph, called "clipping."

As we have discussed in Section 2.2.1, for the 30-day months there is one of seven values that $b(t) + c(t)$ can take, which is determined by the day of the week on which the month starts, and similarly for the 31-day months there are seven values that $b(t) + c(t)$ can take. Furthermore, the two sets of seven values will typically be very similar. Thus the irregular will be divided into eight groups. The first seven groups are formed from the irregular, with February excluded, according to the starting day of the month. The eighth group consists of the February values. Now for a set of numbers the midmean is defined to be the average of all values between the quartiles, the upper (lower) semimidmean is defined to be the midmean of all values above (below) the median, and (Tukey 1977) one step = 1.5 (upper semimidmean minus lower semimidmean). For each group the semimidmeans and one step are computed. Any value in a group that is greater than the quantity, upper semimidmean + 1.5 steps, is set equal to this quantity; any value in the group that is less than the quantity, lower semimidmean − 1.5 steps, is set equal to this quantity.

2.3.4 Computing the Spectrum Estimate. Let $z(v)$, for $v = 1, \ldots, n$, be a series whose spectrum is to be estimated. One method of estimation begins by first subtracting the mean, $\bar{z}$, from $z(v)$ and then multiplying the result by a data window (Bloomfield 1976), $w(v)$, such as

$$\begin{aligned} w(v) &= .5 - .5 \cos(\pi v/(r+1)) \ , \quad v = 1, \ldots, r \\ &= 1 \ , \quad v = r+1, \ldots, n-r \\ &= .5 - .5 \cos(\pi(n+1-v)/(r+1)) \ , \\ &\qquad\qquad v = n-r+1, \ldots, n \ . \end{aligned}$$

(For the spectrum estimates in the examples of this article, r will be taken to be 20 percent of n.) Multiplying by $w(v)$ reduces the chance of a strong peak at one frequency causing spurious peaks at other frequencies. The spectrum estimate at frequency f is the squared modulus of the Fourier transform of the tapered data,

$$s(f) = \frac{1}{w} \left| \sum_{v=1}^{n} (z(v) - \bar{z}) w(v) \exp(2\pi i f v) \right|^2$$

where $w = \sum_{v=1}^{n} w^2(v)$. Since we are looking for very narrow-band peaks in the spectrum at particular frequencies, it is generally not necessary to smooth $s(f)$ as is frequently done in spectrum estimation. $s(f)$ does not estimate the spectrum as defined in Section 2.1, but rather estimates a constant times the spectrum. However, since the constant does not depend on frequency and therefore does not interfere with the identification of peaks, we shall not adjust the spectrum estimates in the examples in Section 3.

2.4 Peak Picking

The procedure for identifying calendar effects in the data is to plot both the spectrum estimate from $x(t)$ and the average spectrum estimate from $x_m(y)$ and check for peaks at the calendar frequencies described in Section 2.2.2. Let us first consider the "peak picking" procedure for the spectrum estimate from $x(t)$ suggested by the values in Table 1. Each of the frequencies in the table with $k = 0$ is one of the yearly seasonal frequencies of the form $j/12$ cycles/month, for $j = 1, \ldots, 6$. However, peaks in the spectrum estimate at these seasonal frequencies would not often provide us with diagnostic information about the existence of calendar effects, for often the series will have yearly seasonal behavior due to causes other than the calendar effects. The removal of the yearly seasonal from $x(t)$, which removes the peaks in the spectrum estimate at the seasonal frequencies, does not reduce the effectiveness of the estimate at other calendar frequencies for detecting calendar effects. The largest spectrum value in Table 1 occurs for $k = 1$ at frequency .348 cycles/month. A peak in an estimate of the spectrum of $x(t)$ at this frequency is an almost certain indication of effects. (Some other cause is, of course, possible, but is highly unlikely). The next largest nonseasonal peak occurs at .432 cycles/month. These two frequencies, as our examples will illustrate, are the most important for diagnosing calendar effects in the spectrum estimate from $x(t)$.

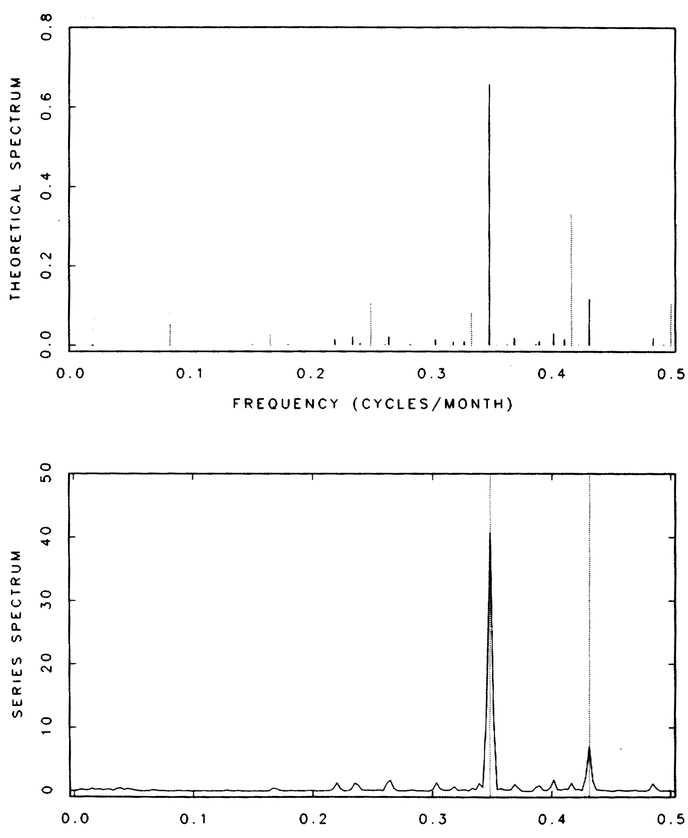
For the average spectrum estimate from $x_m(y)$, Table 2 shows that the two important frequencies for diagnosing calendar effects are .179 and .357 cycles/year. Thus peaks in the average spectrum estimate from $x_m(y)$ at these frequencies are an indication of a calendar effect.

2.5 Examples

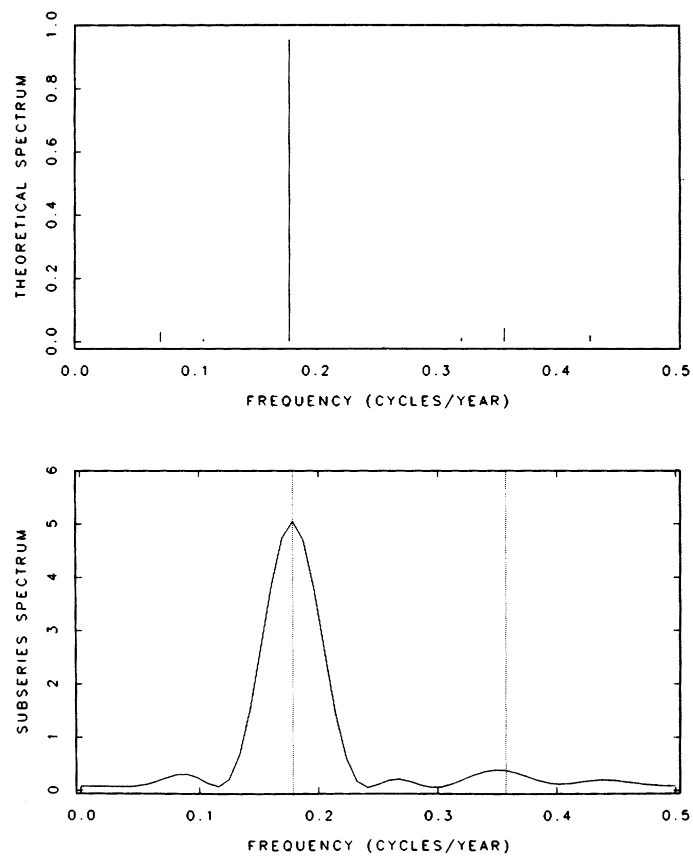
2.5.1 The Weekday Series. To illustrate the procedures of the previous sections, suppose $X(T)$ is the weekday indicator defined in Section 2.2.2. Thus $X(T)$ is a pure calendar series so that $X(T) = B + C(T)$ and $x(t)$ is simply the number of weekdays in a month. The spectrum of $x(t)$ is shown in the top panel of Figure A; the values of the spectrum at the yearly seasonal frequencies, $j/12$ cycles/month for $j = 1, \ldots, 6$, are shown by dotted lines. The average of the spectra of the 11 monthly subseries is shown in the top panel of Figure B.

For January 1958 to December 1977, $x(t)$ was processed by the methods described in Section 2.3. First, $x(t)$ was decomposed into trend, seasonal, and irregular using the SABL decomposition procedure described in Section 2.3.1. (No transformation or clipping was used in this case.) The spectra were then estimated by the

A. Spectrum of the Weekday Series (The top panel shows the theoretical spectrum of the series. The dotted lines are used for the values at the yearly seasonal frequencies. The bottom panel shows the spectrum estimate from the clipped irregular. The dotted vertical lines indicate the two important calendar frequencies.)



B. Spectrum of the Weekday Subseries (The top panel shows the average of the theoretical spectra. The bottom panel shows the estimate from the clipped irregular. The dotted vertical lines indicate the two important calendar frequencies.)

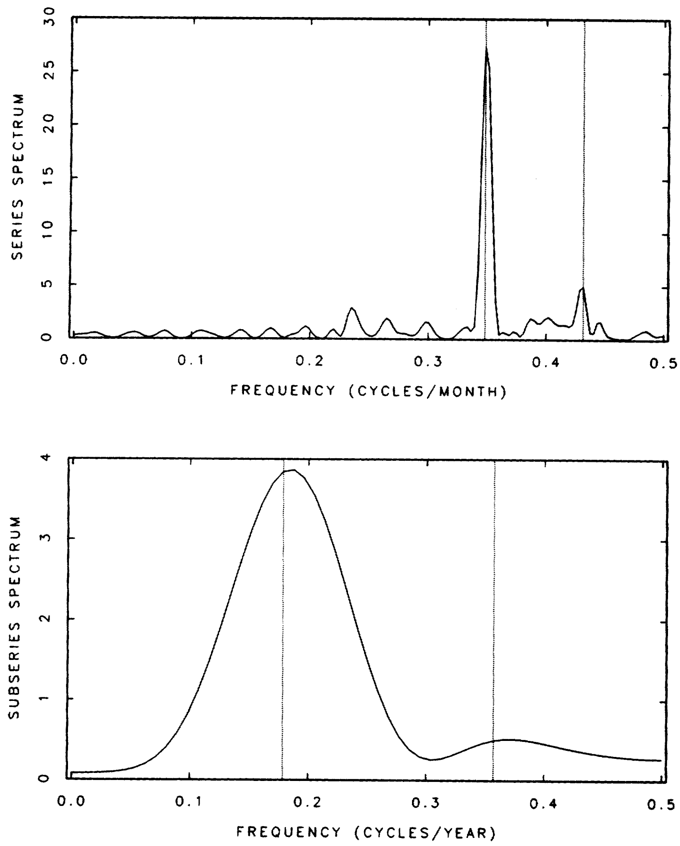


method described in Section 2.3.4 and plotted in the bottom panels of Figures A and B. Vertical lines have been drawn at the most important calendar frequencies (.348 and .432 cycles/month in Figure A; and .179 and .357 cycles/year in Figure B).

For the estimate from $x(t)$, peaks occur at the two important calendar frequencies. Were this a real series with an unknown structure it is clear that the estimate would serve to identify the calendar effects. The peaks at the seasonal frequencies are absent because the seasonal component has been removed. The estimate from $x_m(y)$ also clearly demonstrates the calendar effects since peaks occur at the two important calendar frequencies. This estimate, however, is not substantially affected by the seasonal component removal since a (nearly) periodic seasonal component in $x(t)$ contributes power in the individual subseries spectra at (low) zero frequency, which is removed by the subtraction of the mean.

2.5.2 Bell System Toll Revenues. The investigation of calendar effects in Bell System revenues from toll calls from January 1964 to October 1973 was begun by applying the transformation procedure described in Section 2.3.2. The value of p that most nearly stablizes the seasonal oscillations is .25. The transformed series was then decomposed into trend, seasonal, and irregular by using SABL. Figure C shows the spectrum estimates of

C. Spectrum Estimates From Clipped Irregular of Fourth Root Toll Revenues (The top panel shows the spectrum estimate from the series. The bottom panel shows the average spectrum estimate from the monthly subseries.)



the clipped irregular and the monthly subseries of the clipped irregular. Very large peaks occur at calendar frequencies, which indicates that a large fraction of the variation in the irregular is due to calendar effects. In fact, behavior at the calendar frequencies is quite similar to that of the spectrum estimates for the weekday series in Figures A and B.

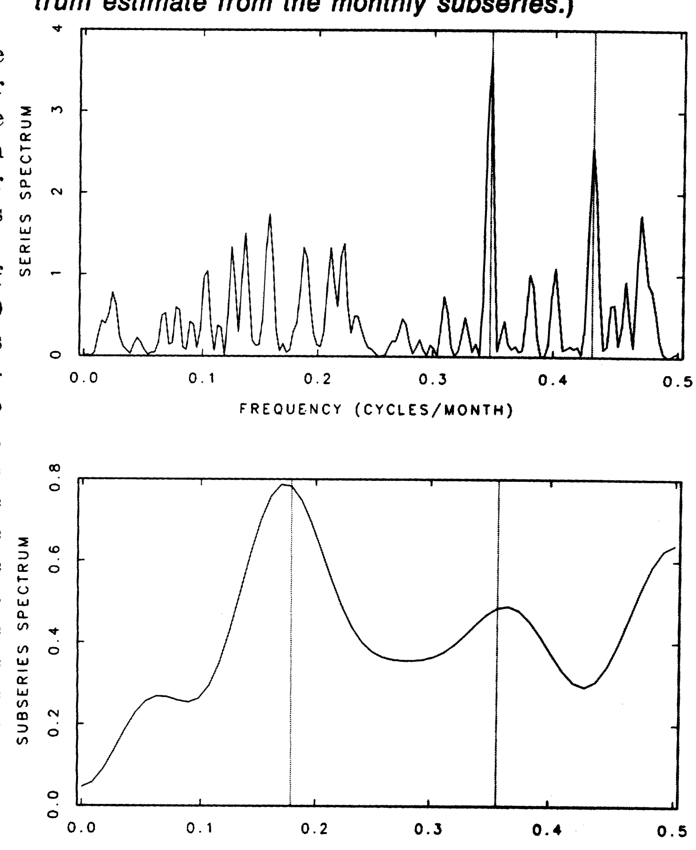
2.5.3 International Airline Series. The number of passengers on international flights from January 1949 to December 1960 has been used by Box and Jenkins (1976) and Brown (1962) to illustrate the use of forecasting methodology. The data were transformed to stabilize the seasonal oscillations by taking logarithms, and SABL was used to remove the seasonal and the trend. In Figure D the spectrum estimates from the clipped irregular and the 11 monthly subseries of the clipped irregular show calendar effects in the airline series, since peaks occur at all four of the important calendar frequencies. Thus the variation of the airline series is, in part, due to calendar effects, so that if these effects were incorporated into a model, they could be used to enhance forecasts because current and future values of the calendar effects are known.

2.5.4 Manufacturers' Shipments. The values of all shipments of manufactured durable goods in the United States are reported in Bureau of Economic Analysis (1976) and are calendar adjusted using X-11. Figure E shows the spectrum estimates of the logarithms of the calendar adjusted data from January 1960 to December 1974 using the clipped irregular from SABL. The peak at .348 cycles/month in the estimate of the top panel and the peaks at .179 and .357 cycles/year in the estimate of the bottom panel indicate there are residual calendar effects still in the series. Thus the X-11 calendar adjustment procedure does not appear to have successfully removed all of the calendar effects from the data.

3. TIME DOMAIN GRAPHICAL METHODS

Various time domain graphical displays of $x(t)$ and $x_m(y)$ can also be used for detection of calendar effects. The basic approach has been to design displays that allow the detection of the periodic behavior due to calendar effects, which we discussed in Section 2. The time domain displays are, in fact, not as powerful as the spectrum analyses for detecting the periodic behavior, but they are somewhat simpler to apply, and outliers and peculiar behavior at specific points in time can be detected. As with the spectrum analysis, we will generally not want to work with the aggregated monthly data but rather with the data after preanalysis by transformation and trend-seasonal removal.

D. Spectrum Estimates From Clipped Irregular of the Logarithms of International Airline Passengers (The top panel shows the spectrum estimate from the series. The bottom panel shows the average spectrum estimate from the monthly subseries.)



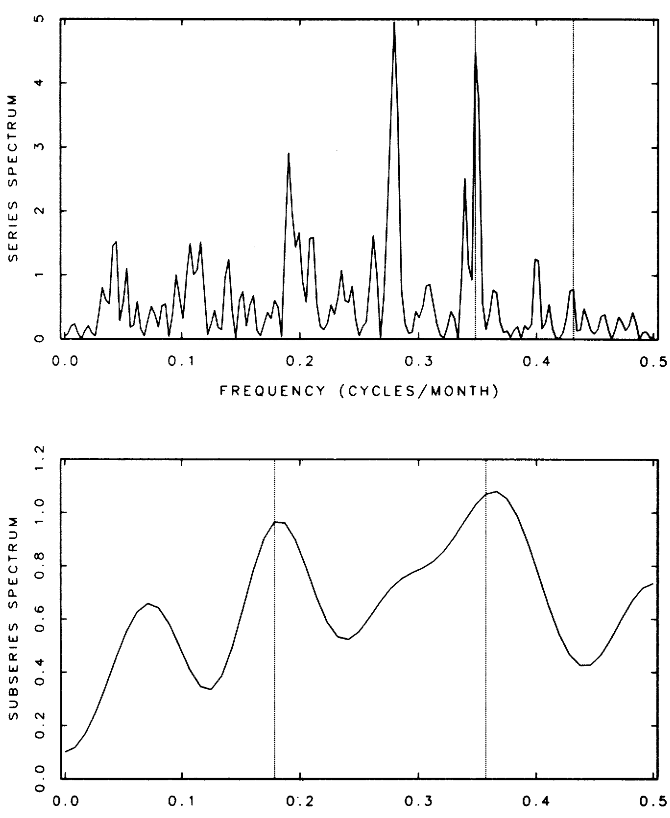
As discussed in Section 2.3.3, when a series with outliers is decomposed into trend, seasonal, and irregular by a routine such as SABL, which robustly estimates the trend and seasonal, the outliers become part of the irregular. Thus when an irregular component is displayed we do not want the outliers to cause a large increase in the scale of the plot and thereby substantially reduce the resolution of the calendar pattern. To prevent this, we shall modify the irregular component, before plotting, by the clipping procedure described in Section 2.3.3.

3.1 Seasonal Adjustment Plots

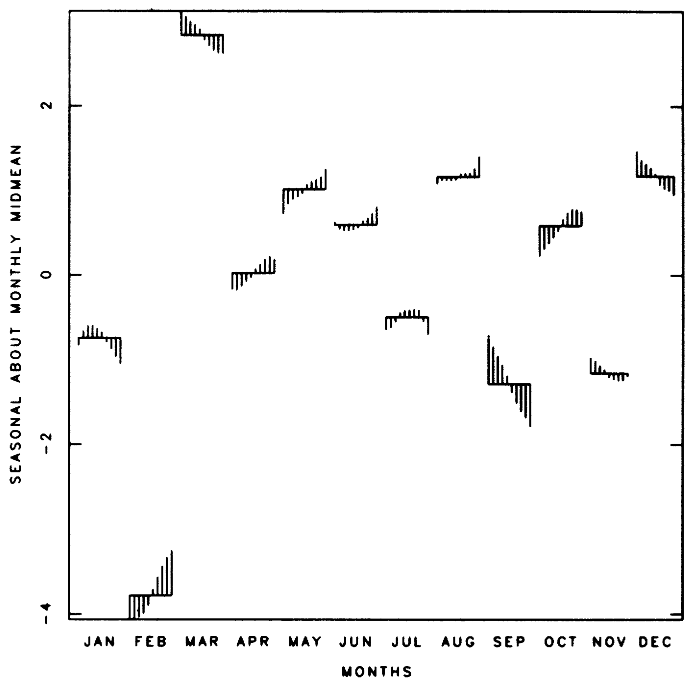
For situations in which the monthly data are decomposed into trend, seasonal, and irregular, certain plots that have already been proposed for enhancing seasonal adjustment procedures (Cleveland, Dunn, and Terpenning 1978 and 1979) can also serve to assist in detecting calendar effects. In this section we shall describe two of these plots.

3.1.1 Plots of the 12 Monthly Subseries of the Irregular. In Section 2.2.2 it was shown that a calendar effect typically results in oscillations with a period of 5.6 years in each of the monthly subseries of the irregular, excluding February. Thus plots of the monthly subseries of the clipped irregular can reveal the calendar effect if the effect accounts for a major part of the variation in the irregular. For less pronounced effects, however, the calendar pattern in the irregular is more clearly revealed in the graphical display that will be described in Section 3.2.

E. Spectrum Estimates From Clipped Irregular of the Logarithms of Manufacturers' Shipments (The top panel shows the spectrum estimate from the series. The bottom panel shows the average spectrum estimate from the monthly subseries.)



F. Seasonal-by-Month Plot for Fourth Root Toll Revenues



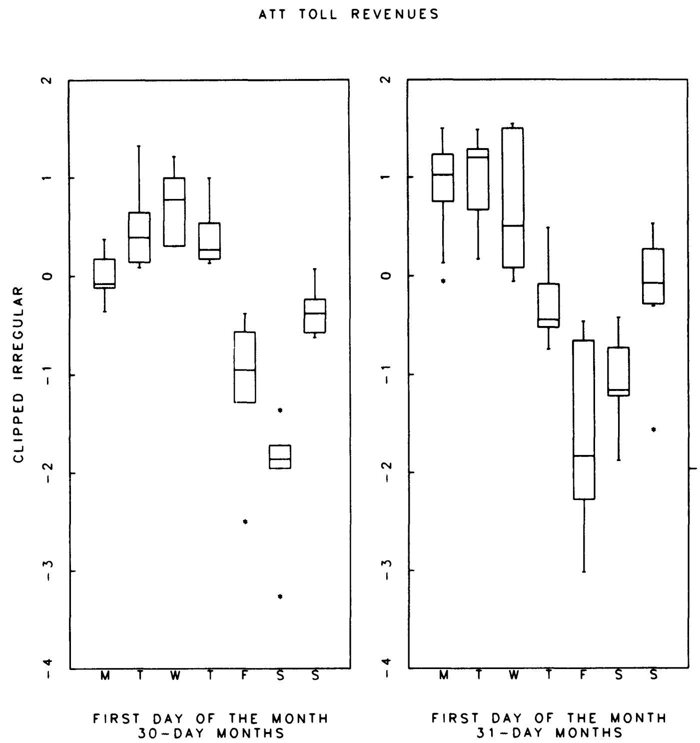
3.1.2 Seasonal-by-Month Plots. A second useful plot, called the seasonal-by-month plot, is one in which the seasonal for each month is plotted against year. An example is given in Figure F where the seasonal of the fourth root of Bell System toll revenues is displayed. The midmean for each month is portrayed by a horizontal line, and the seasonal component for that month is portrayed by vertical lines emanating from the horizontal line.

In Section 2.2.1 it was shown that the effect of aggregating the constant term, B, in $X(T)$ is to produce a component, $b(t)$, in $x(t)$ where $b(t)$ is the number of days in the month times B. Almost all of the variation in $b(t)$ is concentrated at the yearly seasonal frequencies, and therefore almost all of $b(t)$ is contained in the seasonal. The seasonal component will contain, of course, the yearly seasonal behavior not due to calendar causes; however, calendar effects can be detected in the seasonal-by-month plot by detecting a correlation between the level of each monthly subseries (as portrayed by the midmean) and the number of days in the month.

For example, in Figure F it is clear that the midmeans are highly correlated with month length so that almost all of the seasonal component of toll revenues consists of calendar effects. We have already seen in Section 2.5.2 that this is also true of the irregular. Thus apart from the overall trend the major factor in the variation of toll revenues is the calendar effect.

The task of detecting calendar effects is but one of many tasks that the seasonal-by-month plot allows one

G. Starting-Day-of-the-Month Plot for the Clipped Irregular of Fourth Root Toll Revenues



to carry out. (For example, evolution of the seasonal through time can be assessed.) If we were designing this display just for calendar detection, we would then quite likely use a different method (e.g., plotting midmeans versus monthly length). But in practice, because resources are finite, it is desirable to have multipurpose displays like the seasonal-by-month plot, which, while perhaps not the optimal display for detecting calendar effects in the seasonal, nevertheless appears to perform quite adequately for the examples on which we have tested it.

3.2 A Highly Tailored Calendar Plot

The special structure of the calendar effect can be exploited to design a plot that is quite powerful for identifying a calendar effect. As we have seen in Section 2.2.1, for the 30-day months there are seven values that $b(t) + c(t)$ can take, which are determined by the day of the week on which the month starts, and for the 31-day months there are seven values that $b(t) + c(t)$ can take. Thus a calendar effect can be detected by plotting, for each month (except February), the value of $x(t)$ against the day of the week on which the month starts. A dependence of the values on the day of the week indicates a calendar effect.

Since the patterns in the 30-day months are the same, the four plots can be combined into one. Similarly, the seven 31-day months can be combined into one. This is done in Figure G for the clipped irregular of the fourth roots of Bell System toll revenues. For each starting day of the month a schematic plot (Tukey 1977) is used to summarize the distribution of the values. The line inside the rectangle of the schematic plot is the midmean and the upper and lower edges of the rectangle are the upper semimidmean and the lower semimidmean, respectively. The top of the vertical line extending from the top of the box is the largest data point that is within one step of the upper semimidmean. All points beyond one step are plotted individually. A similar procedure is followed for the bottom of the diagram. (The midmean, semimidmeans, and step are defined in Section 2.3.3.) In Figure G it is quite clear that the distributions are very different for different days of the week, which indicates a strong calendar effect.

4. SUMMARY

Spectrum analyses and time domain graphical methods have been used to detect calendar effects in monthly aggregated time series. First, these methods can be used in an initial analysis to decide if calendar adjustment is necessary. For example, it was shown that revenues from toll calls and the number of international airline passengers have significant calendar effects. The methods also can be used on an adjusted series to determine if the adjustment has properly removed all of the calendar effects. For example, it was shown that the manufacturers' shipments series, which was adjusted by the X–11 procedure, has residual calendar effects.

Let $x(t)$ be the aggregated monthly subseries and let $x_m(y)$, for $m = 1, \ldots, 12$, be the 12 monthly subseries of $x(t)$. Two types of spectrum estimates are computed in the following manner:

1. Transform $x(t)$.
2. Remove the trend and seasonal from the transformed $x(t)$.
3. Clip outliers.
4. Compute the spectrum estimate of the resulting series in (3).
5. Compute the average spectrum estimate of the 11 monthly subseries (excluding February) of the resulting series in (3).

A peak in the spectrum estimate from (4) at .348 or .432 cycles/month, or a peak in the spectrum estimate from (5) at .179 or .357 cycles/month, indicates the presence of calendar effects.

Two time domain graphical methods are also useful for detecting effects. One is to plot each of the 12 monthly subseries of the seasonal component of $x(t)$ and look for a correlation between the levels of the subseries and the lengths of the months. The second graphical method is to plot each value of the clipped irregular (with February omitted) against the day of the week on which the month starts and look for a pattern in the locations of the seven sets of values. The time domain graphics are not so sensitive a tool for detecting calendar effects as the spectrum estimates, but the graphics, when they reveal the effects, do give information about the nature of the calendar component and are somewhat simpler to apply.

APPENDIX: SPECTRUM OF $h_k(t)$ AND $h_{km}(t)$

A.1 The Spectrum of $h_k(t)$

Let $T_0 = 0$ and let $d(t)$ be the number of days in the tth month so that

$$T_t = \sum_{j=1}^{t} d(j) \ .$$

Let

$$\bar{d} = \frac{1}{48} \sum_{t=1}^{48} d(t) = 30.4375$$

be the average number of days per month. Since $d(t)$ has period 48 months (i.e., $d(t) = d(t+48)$),

$$r_t = T_t - \bar{d}t$$

has period 48 months.

For $k = 0$, we have from (2.3) that $h_0(t) = d(t)$. Since $d(t)$ has period 48 the spectrum is given by (2.1). For $k \geq 1$, we have from (2.3)

$$h_k(t) = \frac{\sin(\omega_k T_t + \phi_k) - \sin(\omega_k T_{t-1} + \phi_k)}{\omega_k} \qquad \text{(A.1)}$$

where

$$\omega_k = \frac{2\pi k}{7} \cdot$$

Let

$$g_k(t) = \exp(i\omega_k \bar{d}t)[\exp(i\omega_k r_t) - \exp(i\omega_k(r_{t-1} - \bar{d}))]$$

then

$$h_k(t) = (2\omega_k i)^{-1}[g_k(t)\exp(i\phi_k) - \bar{g}_k(t)\exp(-i\phi_k)] \ .$$

Since r_t has period 48 we can write

$$\exp(i\omega_k r_t) = \sum_{j=0}^{47} \xi_{kj} \exp(i\theta_j t)$$

where

$$\theta_j = \frac{2\pi j}{48}$$

and

$$\xi_{kj} = \frac{1}{48} \sum_{t=0}^{47} \exp(i\omega_k r_t)\exp(-i\theta_j t) \ .$$

Thus

$$g_k(t) = \sum_{j=0}^{47} \zeta_{kj} \exp(2\pi i f_k(j)t)$$

where $f_k(j)$ is the alias of

$$\frac{\omega_k \bar{d} + \theta_j}{2\pi}$$

and

$$\zeta_{kj} = \xi_{kj}(1 - \exp(-2\pi i f_k(j))) \ .$$

Thus

$$h_k(t) = \frac{1}{\omega_k} \sum_{j=0}^{47} im(\zeta_{kj} \exp(i\phi_k)) \cos 2\pi f_k(j)t + \mathrm{re}(\zeta_{kj} \exp(i\phi_k)) \sin 2\pi f_k(j)t \ .$$

Thus the spectrum of $h_k(t)$ at frequency $f_k(j)$ is

$$\frac{|\zeta_{kj}|^2}{\omega_k^2} \ .$$

A.2 The Spectrum of $h_{km}(y)$

For $k = 0$, $h_{0m}(y)$ is the number of days in the mth month of the yth year. Thus, except for February, $h_{0m}(y)$ is constant (as a function of y) so that the spectrum is concentrated at zero frequency. For February, $h_{0m}(y)$ is periodic with period 4.

Suppose $k \geq 1$. Let $b_m(y)$ be the time (in units of days) at the end of the mth month in the yth year and let $a_m(y)$ be the time at the beginning of the month. For $k \geq 1$, we have from (A.1)

$$h_{km}(y) = \frac{\sin(\omega_k b_m(y) + \phi_k) - \sin(\omega_k a_m(y) + \phi_k)}{\omega_k} \ . \qquad \text{(A.2)}$$

Let $b'_m(y)$ be $b_m(y)$ reduced modulo 7 to one of the integers 0, ..., 6. Define $a'_m(y)$ similarly. Then $h_{km}(y)$ may be written as in (A.2) but with $a_m(y)$ and $b_m(y)$ replaced by $a'_m(y)$ and $b'_m(y)$, respectively. Since leap year occurs every fourth year, $a'_m(y)$ and $b'_m(y)$ consist of the integers (mod 7) with every fourth value removed. For example, if the month is March, if $b'_m(1) = 1$, and if year 4 is a leap year, then the sequence begins with 1, 2, 3, 5, 6, 0, 1, 3, 4.... Thus $a'_m(y)$ and $b'_m(y)$ are periodic with period 28 and, consequently, so is $h_{km}(y)$. If the month has 31 days, then $b'_m(y) = a'_m(y) + 3$ (mod 7), whereas if the month has 30 days, $b'_m(y) = a'_m(y) + 2$ (mod 7). Thus the spectra for all the 31-day monthly subseries are equal and the spectra for all the 30-day montly subseries are equal. February stands alone.

Since, for $k \geq 0$, $h_{km}(y)$ has period 28, we have from (2.1) that the spectrum is concentrated at the frequencies $j/28$, $j = 1, \ldots, 14$. The procedure of averaging the 11 spectrum estimates from the monthly subseries of $x(t)$, excluding February, is justifiable because the difference between the 30-day and 31-day theoretical spectra is not large. February has been excluded since its values are substantially different.

[*Received May 1979. Revised March 1980.*]

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