Notes on Formal Compiler Construction with the Π Framework

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http://github.com/ChristianoBraga/PiFramework

Compiler pipeline

$$
\begin{array}{l}
p_L \xrightarrow{lexical\ \ analysis} tokens_p \\
\qquad \xrightarrow{syntactical\ \ analysis} ast_p \\
\qquad\qquad \xrightarrow{semantic\ \ analysis} ast_p\checkmark \\
\qquad\qquad\qquad \xrightarrow{code\ \ generation} p_{asm} \\
\qquad\qquad\qquad\qquad \xrightarrow{code\ \ optimization} p'_{asm}
\end{array}
$$

Compiler pipeline and formal languages

$$p_L \xrightarrow{\substack{\textit{lexical analysis} \\ \textit{Linear Grm.}}} tokens_p$$

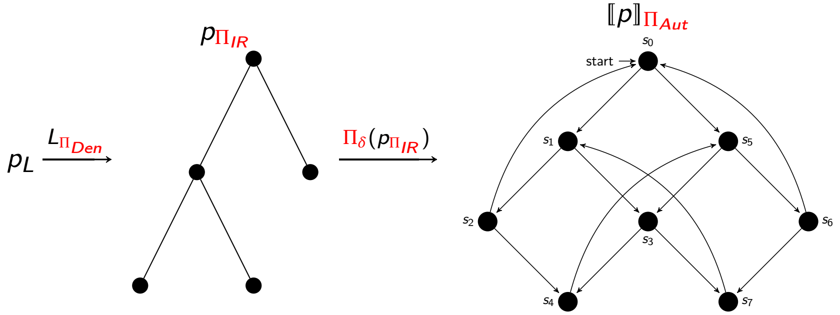
$$\xrightarrow{\substack{\textit{syntactical analysis} \\ \textit{Context-free Grm.}}} ast_p$$

$$\xrightarrow{\substack{\textit{semantic analysis} \\ \textit{Context-sensitive Grm.}}} ast_p\checkmark$$

$$\xrightarrow{\substack{\textit{code generation} \\ \textit{Context-sens. Grm.}}} p_{asm}$$

$$\xrightarrow{\substack{\textit{code optimization} \\ \textit{Context-sens. Grm.}}} p'_{asm}$$

Compiler pipeline with the Π Framework I



Compiler pipeline with the Π Framework II

Π

Automata

Chomsky's hierarchy

$$\textit{source code} \xrightarrow{\textit{lexer}} \circ \xrightarrow{\textit{parser}} \circ \xrightarrow[\textit{transformer}]{\textit{AST}} \circ \xrightarrow[\textit{checker}]{\textit{type}} \circ \xrightarrow[\textit{generator}]{\textit{code}} \Pi\ \textit{IR}$$

$$\Pi\ \textit{IR} \xrightarrow{\textit{interpreter}} \textit{output}$$

$$\Pi\ \textit{IR} \xrightarrow{\textit{code generator}} \textit{machine code}$$

$$\Pi\ \textit{IR} \xrightarrow{\textit{model checker}(P)} \textit{counter-examples}$$

where P is a property

in a suitable logic.

- Π IR defines a set of constructions common to many programming languages.
- Π IR constructions have a formal automata-based semantics in Π automata.
- One may execute (or validate) a program in a given language by running its associated Π IR program.

Compiler pipeline with the Π Framework III

- Π Framework: `http://github.com/ChristianoBraga/PiFramework`
- Notes on Formal Compiler Construction with the Π Framework: `https://github.com/ChristianoBraga/PiFramework/blob/master/notes/notes.pdf`.

A calculator

We wish to compute simple arithmetic expressions such as $5 * (3 + 2)$.

A calculator: Lexer

$$\langle \mathit{digit} \rangle ::= [0..9]$$

$$\langle \mathit{digits} \rangle ::= \langle \mathit{digit} \rangle^{+}$$

$$\langle \mathit{boolean} \rangle ::= \text{‘}\mathtt{true}\text{’} \mid \text{‘}\mathtt{false}\text{’}$$

A calculator: concrete syntax

$$
\begin{array}{lcl}
\langle exp \rangle & ::= & \langle aexp \rangle \mid \langle bexp \rangle \\[1em]
\langle aexp \rangle & ::= & \langle aexp \rangle \text{ ‘+’ } \langle term \rangle \mid \langle aexp \rangle \text{ ‘-’ } \langle term \rangle \mid \langle term \rangle \\[1em]
\langle term \rangle & ::= & \langle term \rangle \text{ ‘}*\text{’ } \langle factor \rangle \mid \langle term \rangle \text{ ‘/’ } \langle factor \rangle \mid \langle factor \rangle \\[1em]
\langle factor \rangle & ::= & \text{‘(’ } \langle aexp \rangle \text{ ‘)’} \mid \langle digits \rangle \\[1em]
\langle bexp \rangle & ::= & \langle boolean \rangle \mid \text{‘}\sim\text{’ } \langle bexp \rangle \mid \langle bexp \rangle \langle boolop \rangle \langle bexp \rangle \\
 & \mid & \langle aexp \rangle \langle iop \rangle \langle aexp \rangle \\[1em]
\langle boolop \rangle & ::= & \text{‘=’} \mid \text{‘/}\backslash\text{’} \mid \text{‘}\backslash\text{/’} \\[1em]
\langle iop \rangle & ::= & \text{‘}<\text{’} \mid \text{‘}>\text{’} \mid \text{‘}<=\text{’} \mid \text{‘}>=\text{’}
\end{array}
$$

A calculator: abstract syntax

$$\langle \mathit{exp} \rangle \quad ::= \quad \langle \mathit{digits} \rangle \mid \langle \mathit{boolean} \rangle \mid \langle \mathit{exp} \rangle \; \langle \mathit{bop} \rangle \; \langle \mathit{exp} \rangle$$

⟨*bop*⟩ ::= ‘+’ | ‘-’ | ‘*’ | ‘/’ | ‘=’ | ‘\/‘ | ‘/\’ | ‘<’ | ‘>’ | ‘<=’ | ‘>=’

A calculator: Π denotations I

Let D in <digits>, B in <boolean> and E_1, E_2 in <exp>,

$$[\![D]\!]_\Pi = Num(D) \tag{1}$$

$$[\![B]\!]_\Pi = Boo(B) \tag{2}$$

$$[\![E_1 + E_2]\!]_\Pi = Sum([\![E_1]\!]_\Pi, [\![E_2]\!]_\Pi) \tag{3}$$

$$[\![E_1 - E_2]\!]_\Pi = Sub([\![E_1]\!]_\Pi, [\![E_2]\!]_\Pi) \tag{4}$$

$$[\![E_1 * E_2]\!]_\Pi = Mul([\![E_1]\!]_\Pi, [\![E_2]\!]_\Pi) \tag{5}$$

$$[\![E_1 / E_2]\!]_\Pi = Div([\![E_1]\!]_\Pi, [\![E_2]\!]_\Pi) \tag{6}$$

$$[\![E_1 < E_2]\!]_\Pi = Lt([\![E_1]\!]_\Pi, [\![E_2]\!]_\Pi) \tag{7}$$

$$[\![E_1 <= E_2]\!]_\Pi = Le([\![E_1]\!]_\Pi, [\![E_2]\!]_\Pi) \tag{8}$$

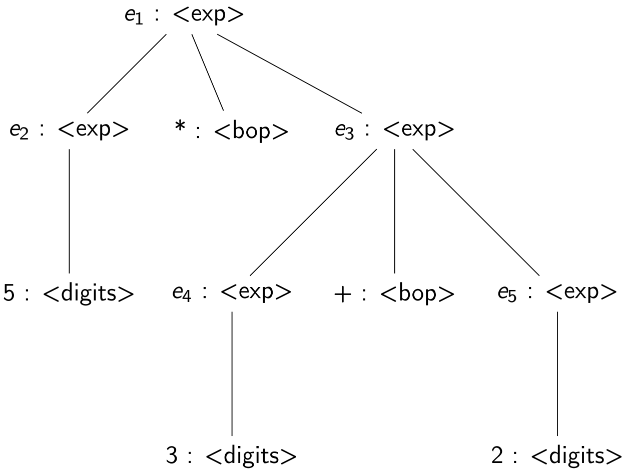
$$[\![E_1 > E_2]\!]_\Pi = Gt([\![E_1]\!]_\Pi, [\![E_2]\!]_\Pi) \tag{9}$$

$$[\![E_1 >= E_2]\!]_\Pi = Ge([\![E_1]\!]_\Pi, [\![E_2]\!]_\Pi) \tag{10}$$

A calculator: Π denotations II

- Π denotations are *functions* $[\![\cdot]\!]_{\Pi} : AST \rightarrow \Pi\ IR$, where AST denotes the *datatype* for the abstract syntax tree and $\Pi\ IR$ denotes the datatype for $\Pi\ IR$ programs.
- Note that $[\![\cdot]\!]_{\Pi}$ has *trees* as parameters, instances of AST. The example expression $5 * (3 + 2)$ becomes

A calculator: Π denotations III



A calculator: Π denotations IV

$$
\begin{array}{rcll}
[\![5 * (3+2)]\!]_\Pi & = & Mul([\![5]\!]_\Pi, [\![(3+2)]\!]_\Pi) & \textit{by Equation } 5 \\
Mul([\![5]\!]_\Pi, [\![(3+2)]\!]_\Pi) & = & Mul(Num(5), [\![(3+2)]\!]_\Pi) & \textit{by Equation } 1 \\
Mul(num(5), [\![(3+2)]\!]_\Pi) & = & Mul(Num(5), Sum([\![3]\!]_\Pi, [\![2]\!]_\Pi) & \textit{by Equation } 3 \\
Mul(Num(5), Sum([\![3]\!]_\Pi, [\![2]\!]_\Pi) & = & Mul(Num(5), Sum(Num(3), [\![2]\!]_\Pi) & \textit{by Equation } 1 \\
Mul(Num(5), Sum(Num(3), [\![2]\!]_\Pi) & = & Mul(Num(5), Sum(Num(3), Num(2)) & \textit{by Equation } 1
\end{array}
$$

A calculator: executing Π IR with Π automata

A Π automaton is a 5-tuple $\mathscr{A} = (G, Q, \delta, q_0, F)$, where G is a context-free grammar, Q is the set of states, q_0 is the initial state, $F \subseteq Q$ is the set of final states and

$$\delta : L(G)^* \times L(G)^* \times Store \to Q,$$

where $L(G)$ is the language generated by G and $Store$ represents the memory. (Elements in a set S^* are represented by terms $[s_1, s_2, \ldots, s_n]$.)

$$\begin{aligned}
\delta([Mul(Num(5), Sum(Num(3), Num(2)], \emptyset, \emptyset) &= \delta([Num(5), Sum(Num(3), Num(2)), \#MUL], \emptyset, \emptyset) \\
\delta([Num(5), Sum(Num(3), Num(2)), \#MUL], \emptyset, \emptyset) &= \delta([Sum(Num(3), Num(2)), \#MUL], [Num(5)], \emptyset) \\
\delta([Sum(Num(3), Num(2)), \#MUL], [Num(5)], \emptyset) &= \delta([Num(3), Num(2), \#SUM, \#MUL], [Num(5)], \emptyset) \\
\delta([Num(3), Num(2), \#SUM, \#MUL], [Num(5)], \emptyset) &= \delta([Num(2), \#SUM, \#MUL], [Num(3), Num(5)], \emptyset) \\
\delta([Num(2), \#SUM, \#MUL], [Num(3), Num(5)], \emptyset) &= \delta([\#SUM, \#MUL], [Num(2), Num(3), Num(5)], \emptyset) \\
\delta([\#SUM, \#MUL], [Num(2), Num(3), Num(5)], \emptyset) &= \delta([\#MUL], [Num(5), Num(5)], \emptyset) \\
\delta([\#MUL], [Num(5), Num(5)], \emptyset) &= \delta(\emptyset, [Num(25)], \emptyset) \\
\delta(\emptyset, [Num(25)], \emptyset) &= Num(25)
\end{aligned}$$

Excerpt of Π IR expressions

⟨*Statement*⟩	::=	<Exp>
⟨*Exp*⟩	::=	<ArithExp> | <BoolExp> <Exp>
⟨*ArithExp*⟩	::=	'Num'(<digits>) | 'Sum'(<Exp> , <Exp>) | 'Sub'(<Exp>, <Exp>) | 'Mul'(<Exp>, <Exp>)
⟨*BoolExp*⟩	::=	'Eq'(<Exp>, <Exp>) | 'Not'(<Exp>)

Π automaton semantics for Π IR expressions I

- Recall that $\delta : L(G)^* \times L(G)^* \times Store \to Q$, and let $N, N_i \in \mathbb{N}$, $C, V \in L(G)^*$, $S \in Store$,

$$\delta(Num(N) :: C, V, S) = \delta(C, Num(N) :: V, S) \tag{11}$$

$$\delta(Sum(E_1, E_2) :: C, V, S) = \delta(E_1 :: E_2 :: \#SUM :: C, V, S) \tag{12}$$

$$\delta(\#SUM :: C, Num(N_1) :: Num(N_2) :: V, S) = \delta(C, (N_1 + N_2) :: V, S) \tag{13}$$

$$\ldots$$

$$\delta(Not(E) :: C, V, S) = \delta(E :: \#NOT :: C, V, S) \tag{14}$$

$$\delta(\#NOT :: C, Boo(true) :: V, S) = \delta(C, Boo(false) :: V, S) \tag{15}$$

$$\delta(\#NOT :: C, Boo(false) :: V, S) = \delta(C, Boo(true) :: V, S) \tag{16}$$

- Notation $h :: ls$ denotes the concatenation of element h with the list ls.
- C represents the *control* stack. V represents the *value* stack. S denotes the memory store.
- $\delta(\emptyset, V, S)$ denotes an *accepting state*.

Π automaton semantics for Π IR expressions II

- On a particular implementation of the Π Framework, as in Python, `<digits>` denote built-in numbers in implementation language, such that all arithmetic operations, such as +, are defined. That's why N_i are in $\mathbb{N}$.

Π IR commands

- Commands are language constructions that require a *memory* store to be evaluated.

$$
\begin{array}{lcl}
\langle \mathit{Statement} \rangle & ::= & <\text{Cmd}> \\
\langle \mathit{Exp} \rangle & := & \text{`Id`}(<\text{String}>) \\
\langle \mathit{Cmd} \rangle & ::= & \text{`Assign'}(<\text{Id}>, <\text{Exp}>) \\
 & \mid & \text{`Loop'}(<\text{BoolExp}>, <\text{Cmd}>) \\
 & \mid & \text{`CSeq'}(<\text{Cmd}>, <\text{Cmd}>)
\end{array}
$$

Π automaton semantics for Π IR commands I

- A location $l \in Loc$ denotes a memory cell.
- Storable and Bindable sets denote the data that may be mapped to by identifiers and locations on the memory and environment respectively.
- $Store = Id \mapsto Storable$, $Env = Loc \mapsto Bindable$, $Loc \subseteq Store$, $\mathbb{N} \subseteq Loc, Bindable$.
- Now the transition function is $\delta : L(G)^* \times L(G)^* \times Env \times Store \to Q$, and let $W \in String$, $C, V \in L(G)^*$, $S \in Store$, $E \in Env$, $B \in Bindable$, $l \in Loc$, $T \in Storable$, $X \in < Exp >$, $M, M_1, M_2 \in < Cmd >$,and

Π automaton semantics for Π IR commands II

expression $S' = S/[I \mapsto N]$ means that S' equals to S in all indices but I that is bound to N,

$$\delta(Id(W) :: C, V, E, S) = \delta(C, B :: V, E, S), \tag{17}$$
$$\text{where } E[W] = I \text{ and } S[I] = B,$$
$$\delta(Assign(W, X) :: C, V, E, S) = \delta(X :: \#ASSIGN :: C, W :: V, E, S'), \tag{18}$$
$$\delta(\#ASSIGN :: C, T :: W :: V, E, S) = \delta(C, V, E, S'), \tag{19}$$
$$\text{where } E[W] = I \text{ and } S' = S/[I \mapsto T],$$
$$\delta(Loop(X, M) :: C, V, E, S) = \delta(X :: \#LOOP :: C, Loop(X, M) :: V, E, S), \tag{20}$$
$$\delta(\#LOOP :: C, Boo(true) :: Loop(X, M) :: V, E, S) = \delta(M :: Loop(X, M) :: C, V, E, S), \tag{21}$$
$$\delta(\#LOOP :: C, Boo(false) :: Loop(X, M) :: V, E, S) = \delta(C, V, E, S), \tag{22}$$
$$\delta(CSeq(M_1, M_2) :: C, V, E, S) = \delta(M_1 :: M_2 :: C, V, E, S). \tag{23}$$

Π IR declarations

- Declarations are statements that create an environment, binding identifiers to (bindable) values.
- In Π IR, a bindable value is either a Boolean value, an integer or a location.
- From a syntactic standpoint, all classes are monotonically extended.

$\langle \mathit{Statement} \rangle$ ::= <Dec>

$\langle \mathit{Exp} \rangle$ ::= 'Ref'(<Exp>)> | 'DeRef'(<Id>) | 'ValRef'(<Id>)

$\langle \mathit{Dec} \rangle$ ::= 'Bind'(<Id>, <Exp>) | 'DSeq'(<Dec>, <Dec>)

$\langle \mathit{Cmd} \rangle$ ::= 'Blk'(<Dec>, <Cmd>)

Π automaton semantics for Π IR declarations I

Let $BlockLocs = \mathscr{P}(Loc)$, now the transition function is $\delta : L(G)^* \times L(G)^* \times Env \times Store \times BlockLocs \rightarrow Q$, and let $L, L' \in BlockLocs$, $Loc \subseteq Storable$, and S/L means the store S without the locations in L,

Π automaton semantics for Π IR declarations II

$$\delta(Ref(X) :: C, V, E, S, L) = \delta(X :: \#REF :: C, V, E, S, L), \tag{24}$$

$$\delta(\#REF :: C, T :: V, E, S, L) = \delta(C, l :: V, E, S', L'), \text{ where } S' = S \cup [l \mapsto T], l \notin S, L' = L \cup \{l\}, \tag{25}$$

$$\delta(DeRef(Id(W)) :: C, V, E, S, L) = \delta(C, l :: V, E, S, L), \text{ where } l = E[W], \tag{26}$$

$$\delta(ValRef(Id(W)) :: C, V, E, S, L) = \delta(C, T :: V, E, S, L), \text{ where } T = S[S[E[W]]], \tag{27}$$

$$\delta(Bind(Id(W), X) :: C, V, E, S, L) = \delta(X :: \#BIND :: C, W :: V, E, S, L), \tag{28}$$

$$\delta(\#BIND :: C, B :: W :: E' :: V, E, S, L) = \delta(C, ([W \mapsto B] \cup E') :: V, E, S, L), \text{ where } E' \in \textbf{\textit{Env}}, \tag{29}$$

$$\delta(\#BIND :: C, B :: W :: H :: V, E, S, L) = \delta(C, [W \mapsto B] :: H :: V, E, S, L), \text{ where } H \notin \textbf{\textit{Env}}, \tag{30}$$

$$\delta(DSeq(D_1, D_2), X) :: C, V, E, S, L) = \delta(D_1 :: D_2 :: C, V, E, S, L), \tag{31}$$

$$\delta(Blk(D, M) :: C, V, E, S, L) = \delta(D :: \#BLKDEC :: M :: \#BLKCMD :: C, L :: V, E, S, \emptyset), \tag{32}$$

$$\delta(\#BLKDEC :: C, E' :: V, E, S, L) = \delta(C, E :: V, E/E', S, L), \tag{33}$$

$$\delta(\#BLKCMD :: C, E :: L :: V, E', S, L') = \delta(C, V, E, S', L), \text{ where } S' = S/L. \tag{34}$$

Π IR abstractions

- Abstractions extend Bindables by allowing a name to be bound to a list of formal parameters, a list of identifiers, and a block in the environment.
- Such names can be called and applied to actual parameters, a list of expressions.

$$
\begin{array}{lcl}
\langle \mathit{Dec} \rangle & ::= & \text{`Bind'(<Id>, <Abs>)} \\
\langle \mathit{Abs} \rangle & ::= & \text{`Abs'(<Formals>, <Blk>)} \\
\langle \mathit{Formals} \rangle & ::= & \text{<Id>}^{*} \\
\langle \mathit{Cmd} \rangle & ::= & \text{`Call`(<Id>, <Actuals>)} \\
\langle \mathit{Actuals} \rangle & ::= & \text{<Exp>}^{*}
\end{array}
$$

Π automaton semantics for Π IR abstractions — Closures I

We chose a static binding semantics for abstractions. Therefore, we interpret abstractions as *closures* formed by an abstraction together with its declaration environment which defines the context in which the abstraction will be evaluated.

$$Closure : Formals \times Blk \times Env \to Bindable$$

Π automaton semantics for Π IR abstractions — Example I

```
ImΠ source code:
# In this example we encapsulate the iterative calculation
# of the factorial within a function call.
let var z = 1
in
    let fn f(x) =
        let var y = x
        in
            while not (y == 0)
            do
                z := z * y
                y := y - 1
    in f(10)
```

Π automaton semantics for Π IR abstractions — Example II

```
Π IR AST:
Blk(Bind(Id(z), Ref(Num(1))),
  Blk(BindAbs(Id(f), Abs(Id(x),
    Blk(Bind(Id(y), Ref(Id(x))),
      Loop(Not(Eq(Id(y), Num(0))),
        CSeq(Assign(Id(z), Mul(Id(z), Id(y))),
            Assign(Id(y), Sub(Id(y), Num(1)))))))),
    Call(Id(f), Num(10))))
```

Π automaton semantics for Π IR abstractions I

Let $F \in Formals$, $B \in Blk$, $I \in Id$, $A \in Actuals$, $V_i \in Value$, $1 \leq i \leq n$, $n \in \mathbb{N}$,

$$\delta(\boldsymbol{Abs}(F,B) :: C, V, E, S, L) = \delta(C, \boldsymbol{Closure}(F,B,E) :: V, E, S, L) \tag{35}$$

$$\begin{aligned} \delta(\boldsymbol{Call}(I,[X_1,X_2,\ldots,X_n])) :: C, V, E, S, L) = \\ \delta(X_n :: X_{n-1} :: \ldots :: X_1 :: \#\boldsymbol{CALL}(I,n) :: C, V, E, S, L) \end{aligned} \tag{36}$$

$$\begin{aligned} \delta(\#\boldsymbol{CALL}(I,n) :: C, [V_1,V_2,\ldots,V_n] :: V, [I \mapsto \boldsymbol{Closure}(F,B,E_1)]E_2, S, L) = \\ \delta(B :: \#BLKCMD :: C, E_2 :: V, (E_1/\boldsymbol{match}(F,[V_1,V_2,\ldots,V_n])), S, L) \end{aligned} \tag{37}$$

$$\begin{aligned} \boldsymbol{match} : \boldsymbol{Id}^* \times \boldsymbol{Values}^* \to \boldsymbol{Env} \\ \boldsymbol{match}(fl, al) = \boldsymbol{if}\ |fl| \neq |al|\ \boldsymbol{than}\ \{\}\ \boldsymbol{else}\ \_\boldsymbol{match}(fl, al, \{\}) \\ \_\boldsymbol{match} : \boldsymbol{Id}^* \times \boldsymbol{Values}^* \times \boldsymbol{Env} \to \boldsymbol{Env} \\ \_\boldsymbol{match}([],[],E) = E \\ \_\boldsymbol{match}(f,a,E) = \{f \mapsto a\}E \\ \_\boldsymbol{match}(f :: fl, a :: al, E) = \_\boldsymbol{match}(fl), al, \{f \mapsto a\}E) \end{aligned}$$

Π IR recursive abstractions

- Abstractions can be recursive to allow for the declaration of recursive functions.

$\langle \mathit{Dec} \rangle$ ::= 'Rbnd'(<Id>, <Abs>)

Π automaton semantics for Π IR recursive abstractions — Recursive closures I

In the context of static binding semantics for abstractions, in a call to a recursive function, the evaluation of identifiers needs to be reminded about the binding of the function name to a closure.

$$
\begin{aligned}
\mathit{Rec} &: \mathit{Formals} \times \mathit{Blk} \times \mathit{Env} \times \mathit{Env} \to \mathit{Bindable} \\
\mathit{unfold} &: \mathit{Env} \to \mathit{Env} \\
\mathit{reclose}_E &: \mathit{Env} \to \mathit{Env}
\end{aligned}
$$

Π automaton semantics for Π IR recursive abstractions — Recursive closures II

$$unfold(E) = reclose_E(E) \tag{38}$$

$$reclose_E(I \mapsto Closure(F, B, E')) = (I \mapsto Rec(F, B, E', E)) \tag{39}$$

$$reclose_E(I \mapsto Rec(F, B, E', E'')) = (I \mapsto Rec(F, B, E', E)) \tag{40}$$

$$reclose_E(I \mapsto v) = (I \mapsto v) \text{ if } v \neq Closure(F, B, E) \tag{41}$$

$$reclose_E(E_1 \cup E_2) = reclose_E(E_1) \cup reclose_E(E_2) \tag{42}$$

$$reclose_E(\emptyset) = \emptyset \tag{43}$$

Π automaton semantics for Π IR recursive abstractions — Recursive closures I

$$
\begin{aligned}
&\delta(Rbnd(I, Abs(F, B)) :: C, V, E, S, L) = \\
&\qquad \delta(C, unfold(I \mapsto Closure(F, B, E)) :: V, E, S, L) && (44) \\
&\delta(\#CALL(I, n) :: C, [V_1, V_2, \ldots, V_n] :: V, E, S, L) = \\
&\qquad \delta(B :: \#BLKCMD :: C, E :: V, E', S, L) && (45) \\
&\text{where } E = \{I \mapsto Rec(F, B, E_1, E_2)\} \cup E_3 \\
&\qquad E' = E / E_1 / unfold(E_2) / match(F, [V_1, V_2, \ldots, V_n])
\end{aligned}
$$

Π IR expressions in Python I

```
class Statement:
    def __init__(self, *args):
        self.opr =args
    def __str__(self):
        ret =str(self.__class__.__name__)+"("
        for o in self.opr:
            ret +=str(o)
        ret +=")"
        return ret
class Exp(Statement): pass
class ArithExp(Exp): pass
```

Π IR expressions in Python II

```
class Num(ArithExp):
    def __init__(self, f):
        assert(isinstance(f, int))
        ArithExp.__init__(self,f)
class Sum(ArithExp):
    def __init__(self, e1, e2):
        assert(isinstance(e1, Exp) and isinstance(e2, Exp))
        ArithExp.__init__(self, e1, e2)
...
```

Π IR expressions in Python III

```
class BoolExp(Exp): pass
class Eq(BoolExp):
    def __init__(self, e1, e2):
        assert(isinstance(e1, Exp) and isinstance(e2, Exp))
        BoolExp.__init__(self, e1, e2)
...
```

Π IR expressions in Python IV

```
exp =Sum(Num(1), Mul(Num(2), Num(4)))
print(exp)

Sum(Num(1)Mul(Num(2)Num(4)))
```

Π IR expressions in Python V

```
exp2 =Mul(2, 1)
---------------------------------------------------------------------------
AssertionError Traceback (most recent call last)
<ipython-input-7-00fd40a79a54> in <module>()
---->1 exp2 =Mul(2, 1)

<ipython-input-5-42a82e58862f> in __init__(self, e1, e2)
     28 class Mul(ArithExp):
     29 def __init__(self, e1, e2):
--->30 assert(isinstance(e1, Exp) and isinstance(e2, Exp))
     31 ArithExp.__init__(self, e1, e2)
     32 class BoolExp(Exp): pass

AssertionError:
```

Π automaton for Π IR expressions I

```
## Expressions
class ValueStack(list): pass
class ControlStack(list): pass
class ExpKW:
    SUM ="#SUM"
    SUB ="#SUB"
    MUL ="#MUL"
    EQ ="#EQ"
    NOT ="#NOT"
```

Π automaton for Π IR expressions II

```
class ExpPiAut(dict):
    def __init__(self):
        self["val"] =ValueStack()
        self["cnt"] =ControlStack()
    def __evalSum(self, e):
        e1 =e.opr[0]
        e2 =e.opr[1]
        self.pushCnt(ExpKW.SUM)
        self.pushCnt(e1)
        self.pushCnt(e2)
    def pushCnt(self, e):
        cnt =self.cnt()
        cnt.append(e)
...
```

Π automaton for Π IR expressions III

```
ea =ExpPiAut()
print(exp)
ea.pushCnt(exp)
while not ea.emptyCnt():
    ea.eval()
    print(ea)
```

Π automaton for Π IR expressions IV

```
Sum(Num(1)Mul(Num(2)Num(4)))
{'val': [], 'cnt': ['#SUM', <__main__.Num object at 0x111851470>, <
        __main__.Mul object at 0x1118516d8>]
        }
{'val': [], 'cnt': ['#SUM', <__main__.Num object at 0x111851470>, '#MUL'
        , <__main__.Num object at
        0x111851630>, <__main__.Num object
        at 0x1118516a0>]}
{'val': [4], 'cnt': ['#SUM', <__main__.Num object at 0x111851470>, '#MUL
        ', <__main__.Num object at
        0x111851630>]}
{'val': [4, 2], 'cnt': ['#SUM', <__main__.Num object at 0x111851470>, '#
        MUL']}
{'val': [8], 'cnt': ['#SUM', <__main__.Num object at 0x111851470>]}
{'val': [8, 1], 'cnt': ['#SUM']}
{'val': [9], 'cnt': []}
```

Π IR commands I

```python
class Cmd(Statement): pass
class Id(Exp):
    def __init__(self, s):
        assert(isinstance(s, str))
        Exp.__init__(self, s)
class Assign(Cmd):
    def __init__(self, i, e):
        assert(isinstance(i, Id) and isinstance(e, Exp))
        Cmd.__init__(self, i, e)
class Loop(Cmd):
    def __init__(self, be, c):
        assert(isinstance(be, BoolExp) and isinstance(c, Cmd))
        Cmd.__init__(self, be, c)
class CSeq(Cmd):
    def __init__(self, c1, c2):
        assert(isinstance(c1, Cmd) and isinstance(c2, Cmd))
        Cmd.__init__(self, c1, c2)
```

Π IR commands II

```
cmd =Assign(Id("x"), Num(1))
print(type(cmd))
print(cmd)
<class '__main__.Assign'>
Assign(Id(x)Num(1))
```

Π automaton for Π IR commands I

Environment, Location, Store and commands opcodes.

```
## Commands
class Env(dict): pass
class Loc(int): pass
class Sto(dict): pass
class CmdKW:
    ASSIGN ="#ASSIGN"
    LOOP ="#LOOP"
```

Π automaton for Π IR commands II

Π automaton for commands extends the Π automaton for expressions.

```
class CmdPiAut(ExpPiAut):
    def __init__(self):
        self["env"] =Env()
        self["sto"] =Sto()
        ExpPiAut.__init__(self)
    def env(self):
        return self["env"]
    def getLoc(self, i):
        en =self.env()
        return en[i]
    def sto(self):
        return self["sto"]
    def updateStore(self, l, v):
        st =self.sto()
        st[l] =v
```

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Π automaton for Π IR commands III

Π semantics for assignment.

$$\delta(Assign(W,X) :: C, V, E, S) = \delta(X :: \#ASSIGN :: C, W :: V, E, S'),$$
$$\delta(\#ASSIGN :: C, T :: W :: V, E, S) = \delta(C, V, E, S'),$$
$$\text{where } E[W] = l \text{ and } S' = S/[l \mapsto T].$$

```python
    def __evalAssign(self, c):
        i =c.opr[0]
        e =c.opr[1]
        self.pushVal(i.opr[0])
        self.pushCnt(CmdKW.ASSIGN)
        self.pushCnt(e)
    def __evalAssignKW(self):
        v =self.popVal()
        i =self.popVal()
        l =self.getLoc(i)
        self.updateStore(l, v)
```

Π automaton for Π IR commands IV

Π semantics for identifiers.

$$\delta(Id(W) :: C, V, E, S) = \delta(C, B :: V, E, S),$$
$$\text{where } E[W] = l \text{ and } S[l] = B.$$

```
    def __evalId(self, i):
        s =self.sto()
        l =self.getLoc(i)
        self.pushVal(s[l])
```

Π automaton for Π IR commands V

Π semantics for loop: recursive step.

$$\delta(Loop(X,M) :: C, V, E, S) = \delta(X :: \#LOOP :: C, Loop(X,M) :: V, E, S).$$

```
    def __evalLoop(self, c):
        be =c.opr[0]
        bl =c.opr[1]
        self.pushVal(Loop(be, bl))
        self.pushVal(bl)
        self.pushCnt(CmdKW.LOOP)
        self.pushCnt(be)
```

Π automaton for Π IR commands VI

Π semantics for loop: basic steps.

$$\delta(\#LOOP :: C, Boo(true) :: Loop(X,M) :: V, E, S) = \delta(M :: Loop(X,M) :: C, V, E, S),$$
$$\delta(\#LOOP :: C, Boo(false) :: Loop(X,M) :: V, E, S) = \delta(C, V, E, S).$$

```
    def __evalLoopKW(self):
        t =self.popVal()
        if t:
            c =self.popVal()
            lo =self.popVal()
            self.pushCnt(lo)
            self.pushCnt(c)
        else:
            self.popVal()
            self.popVal()
```

Π automaton for Π IR commands VII

Π semantics for command composition.

$$\delta(CSeq(M_1, M_2) :: C, V, E, S) = \delta(M_1 :: M_2 :: C, V, E, S).$$

```
    def __evalCSeq(self, c):
        c1 =c.opr[0]
        c2 =c.opr[1]
        self.pushCnt(c2)
        self.pushCnt(c1)
```

Π automaton for Π IR commands VIII

Commands are now on the top of the food chain.

```
    def eval(self):
        c =self.popCnt()
        if isinstance(c, Assign):
            self.__evalAssign(c)
        elif c ==CmdKW.ASSIGN:
            self.__evalAssignKW()
        elif isinstance(c, Id):
            self.__evalId(c.opr[0])
        elif isinstance(c, Loop):
            self.__evalLoop(c)
        elif c ==CmdKW.LOOP:
            self.__evalLoopKW()
        elif isinstance(c, CSeq):
            self.__evalCSeq(c)
        else:
            self.pushCnt(c)
            ExpPiAut.eval(self)
```

Π IR declarations in Python I

DeRef and ValRef not implemented yet.

```
## Declarations
class Dec(Statement): pass
class Bind(Dec):
   def __init__(self, i, e):
       assert (isinstance(i, Id) and isinstance(e, Exp))
       Dec.__init__(self, i, e)
class Ref(Exp):
   def __init__(self, e):
       assert (isinstance(e, Exp))
       Exp.__init__(self, e)
class Blk(Cmd):
   def __init__(self, d, c):
       assert (isinstance(d, Dec) and isinstance(c, Cmd))
       Cmd.__init__(self, d, c)
```

Π IR declarations in Python II

```
class DSeq(Dec):
    def __init__(self, d1, d2):
        assert (isinstance(d1, Dec) and isinstance(d2, Dec))
        Dec.__init__(self, d1, d2)
```

Π IR declarations in Python III

```
## Declarations
class DecExpKW(ExpKW):
    REF ="#REF"

class DecCmdKW(CmdKW):
    BLKDEC ="#BLKDEC"
    BLKCMD ="#BLKCMD"

class DecKW:
    BIND ="#BIND"
    DSEQ ="#DSEQ"

class DecPiAut(CmdPiAut):
    def __init__(self):
        self["locs"] =[]
        CmdPiAut.__init__(self)

    def locs(self):
```

Π IR declarations in Python IV

```
        return self["locs"]

    def pushLoc(self, l):
        ls =self.locs()
        ls.append(l)

    def __evalRef(self, e):
        ex =e.opr[0]
        self.pushCnt(DecExpKW.REF)
        self.pushCnt(ex)

    def __newLoc(self):
        sto =self.sto()
        if sto:
            return max(list(sto.keys())) +1
        else:
            return 0.0

    def __evalRefKW(self):
```

Π IR declarations in Python V

```
        v = self.popVal()
        l = self.__newLoc()
        self.updateStore(l, v)
        self.pushLoc(l)
        self.pushVal(l)

    def __evalBind(self, d):
        i = d.opr[0]
        e = d.opr[1]
        self.pushVal(i)
        self.pushCnt(DecKW.BIND)
        self.pushCnt(e)

    def __evalBindKW(self):
        l = self.popVal()
        i = self.popVal()
        x = i.opr[0]
        self.pushVal({x: l})

```

Π IR declarations in Python VI

```
    def __evalDSeq(self, ds):
        d1 =ds.opr[0]
        d2 =ds.opr[1]
        self.pushCnt(DecKW.DSEQ)
        self.pushCnt(d2)
        self.pushCnt(d1)

    def __evalDSeqKW(self):
        d2 =self.popVal()
        d1 =self.popVal()
        d1.update(d2)
        self.pushVal(d1)

    def __evalBlk(self, d):
        ld =d.opr[0]
        c =d.opr[1]
        l =self.locs()
        self.pushVal(list(l))
        self.pushVal(c)
```

Π IR declarations in Python VII

```
        self.pushCnt(DecCmdKW.BLKDEC)
        self.pushCnt(ld)

    def __evalBlkDecKW(self):
        d =self.popVal()
        c =self.popVal()
        l =self.locs()
        self.pushVal(l)
        en =self.env()
        ne =en.copy()
        ne.update(d)
        self.pushVal(en)
        self["env"] =ne
        self.pushCnt(DecCmdKW.BLKCMD)
        self.pushCnt(c)

    def __evalBlkCmdKW(self):
        en =self.popVal()
        ls =self.popVal()
```

Π IR declarations in Python VIII

```
        self["env"] =en
        s =self.sto()
        s ={k: v for k, v in s.items() if k not in ls}
        self["sto"] =s
        # del ls
        ols =self.popVal()
        self["locs"] =ols

    def eval(self):
        d =self.popCnt()
        if isinstance(d, Bind):
            self.__evalBind(d)
        elif d ==DecKW.BIND:
            self.__evalBindKW()
        elif isinstance(d, DSeq):
            self.__evalDSeq(d)
        elif d ==DecKW.DSEQ:
            self.__evalDSeqKW()
        elif isinstance(d, Ref):
```

Π IR declarations in Python IX

```
        self.__evalRef(d)
    elif d ==DecExpKW.REF:
        self.__evalRefKW()
    elif isinstance(d, Blk):
        self.__evalBlk(d)
    elif d ==DecCmdKW.BLKDEC:
        self.__evalBlkDecKW()
    elif d ==DecCmdKW.BLKCMD:
        self.__evalBlkCmdKW()
    else:
        self.pushCnt(d)
        CmdPiAut.eval(self)
```

Π IR declarations in Python X

```
dc =DecPiAut()
fac =Loop(Not(Eq(Id("y"), Num(0))),
         CSeq(Assign(Id("x"), Mul(Id("x"), Id("y"))),
              Assign(Id("y"), Sub(Id("y"), Num(1)))))
dec =DSeq(Bind(Id("x"), Ref(Num(1))),
         Bind(Id("y"), Ref(Num(200))))
fac_blk =Blk(dec, fac)
dc.pushCnt(fac_blk)
while not dc.emptyCnt():
   aux =dc.copy()
   dc.eval()
   if dc.emptyCnt():
      print(aux)
```

Π IR abstractions in Python I

```
class Formals(list):
    def __init__(self, f):
        if isinstance(f, list):
            for a in f:
                if not isinstance(a, Id):
                    raise IllFormed(self, a)
            self.append(f)
        else:
            raise IllFormed(self, f)

class Abs:
    def __init__(self, f, b):
        if isinstance(f, list):
            if isinstance(b, Blk):
                self._opr =[f, b]
            else:
                raise IllFormed(self, b)
```

Π IR abstractions in Python II

```
        else:
            raise IllFormed(self, f)

    def formals(self):
        return self._opr[0]

    def blk(self):
        return self._opr[1]

    def __str__(self):
        ret =str(self.__class__.__name__) +"("
        formals =self.formals()
        ret +=str(formals[0]) # First formal argument
        for i in range(1, len(formals)):
            ret +=", "
            ret +=str(formals[i]) # Remaining formal arguments
        ret +=", "
        ret +=str(self.blk()) # Abstraction block
        ret +=")"
```

Π IR abstractions in Python III

```
        return ret

class BindAbs(Bind):
    '''
    BindAbs is a form of bind but that receives an Abs instead of an
    expression.
    '''
    def __init__(self, i, p):
        if isinstance(i, Id):
            if isinstance(p, Abs):
                Dec.__init__(self, i, p)
            else:
                raise IllFormed(self, p)
        else:
            raise IllFormed(self, i)

class Actuals(list):
    def __init__(self, a):
        if isinstance(a, list):
```

Π IR abstractions in Python IV

```
            for e in a:
                if not isinstance(e, Exp):
                    raise IllFormed(self, e)
            self.append(a)
        else:
            raise IllFormed(self, a)

class Call(Cmd):
    def __init__(self, f, actuals):
        if isinstance(f, Id):
            if isinstance(actuals, list):
                Cmd.__init__(self, f, actuals)
            else:
                raise IllFormed(self, actuals)
        else:
            raise IllFormed(self, f)

    def caller(self):
        return self.operand(0)
```

Π IR abstractions in Python V

```python

    def actuals(self):
        return self.operand(1)

class Closure(dict):
    def __init__(self, f, b, e):
        if isinstance(f, list):
            if isinstance(b, Blk):
                # I wanted to write assert(isinstance(e, Env)) but it
                                                      fails.
                if isinstance(e, dict):
                    self["for"] =f # Formal parameters
                    self["env"] =e # Current environment
                    self["block"] =b # Procedure block
                else:
                    raise IllFormed(self, e)
            else:
                raise IllFormed(self, b)
        else:
```

Π IR abstractions in Python VI

```
            raise IllFormed(self, f)

    def __str__(self):
        ret =str(self.__class__.__name__) +"("
        formals =self.formals()
        fst_formal =formals[0] # First formal argument
        ret +=str(fst_formal)
        for i in range(1, len(formals)):
            ret +=", "
            formal =formals[i] # Remaining formal arguments
            ret +=str(formal)
        ret +=", "
        ret +=str(self.blk()) # Closure block
        ret +=")"
        return ret

    def formals(self):
        return self['for']

```

Π IR abstractions in Python VII

```
    def env(self):
        return self['env']

    def blk(self):
        return self['block']

class AbsPiAut(DecPiAut):
    def __evalAbs(self, a):
        if not isinstance(a, Abs): # p must be an abstraction
            raise EvaluationError(self, "Function __evalAbs called with
                                        no abstraction but with "
                                        , a, " instead.")
        else:
            f =a.formals() # Formal parameters
            b =a.blk() # Body
            e =self.env() # Current environment
            # Closes the given abs. with the current env
            c =Closure(f, b, e)
            # Closure c is pushed to the value stack such that
```

Π IR abstractions in Python VIII

```
            self.pushVal(c)
            # a BIND may create a new binding to a given identifier.

    def __match(self, f, a):
        '''
        Given a list of formal parameters and a list of actual parameters,
        it returns an environment relating the elements of the former with the latter.
        '''
        if isinstance(f, list):
            if isinstance(a, list):
                if len(f) ==0:
                    return {}
                if len(f) ==len(a) and len(f) >0:
                # For some reason, f[0] is a tuple, not an Id.
                    f0 =f[0]
                    a0 =a[0]
                    b0 ={f0.id(): a0.num()}
```

Π IR abstractions in Python IX

```
            if len(f) ==1:
                return b0
            else:
                # For some reason, f[0] is a tuple, not an Id.
                f1 =f[1]
                a1 =a[1]
                b1 ={f1.id(): a1.num()}
                e =b0.update(b1)
                for i in range(2, len(f)):
                    fi =f[i][0]
                    ai =a[i][0]
                    e.update({fi.id(): ai.num()})
                return e
        else:
            raise EvaluationError("Call to '__match' on " +str(self) +
                                  ": " +"formals and
                                  actuals differ in
                                  size.")
    else:
```

Π IR abstractions in Python X

```
            raise EvaluationError("Call to '__match' on " +str(self) +":
                                                " +" no formals, but with
                                                 ", f, " instead.")

    def __evalCall(self, c):
        '''
        Essentially, a call is translated into a block.
        If we were progrmming pi in a symbolic language,
        we could simply crete a proper block and push it to the control
                                    stack.
        However, the environment is not symbolic: is a dictionary of
                                    objects.
        To create a block we would need to "pi-IR-fy" it, that is,
                                    recreate the
        pi IR tree from the concrete environmnet and joint it with
                                    matches created
        also at pi IR level. These would be pushed back into the control
                                    stack and
```

Π IR abstractions in Python XI

```
    reobjectifyed. Thus, to avoid pi-IRfication and reevaluatuation
                                          of the
    environment we manipulate it at the object level, which is
                                          dangerous but
    seems to be correct.

    In this implementation, actual parameters are already evaluated.
    '''
    if not isinstance(c, Call): # c must be a Call object
        raise EvaluationError("Call to __evalCall with no Call object
                                              but with ", c, " instead
                                              .")
    else:
        # Procedure to be called
        caller =c.caller()
        # Retrieves the current environment.
        e =self.env()
        # Retrieves the closure associated with the caller function.
        clos =e[caller.id()]
```

Π IR abstractions in Python XII

```
        # Retrieves the actual parameters from the call.
        a =c.actuals()
        # Retrieves the formal parameters from the closure.
        f =clos.formals()
        # Matches formals and actuals, creating an environment.
        d =self.__match(f, a)
        # Retrives the closure's environment.
        ce =clos.env()
        # The caller's block must run on the closures environment
        # overwritten with the matches.
        d.update(ce)
        self["env"] =d
        self.pushVal(self.locs())
        # Saves the current environment in the value stack.
        self.pushVal(e)
        # Pushes the keyword BLKCMD for block completion.
        self.pushCnt(DecCmdKW.BLKCMD)
        # Pushes the body of the caller function into the control
                                                  stack.
```

Π IR abstractions in Python XIII

```python
                self.pushCnt(clos.blk())

    def eval(self):
        d =self.popCnt()
        if isinstance(d, Abs):
            self.__evalAbs(d)
        elif isinstance(d, Call):
            self.__evalCall(d)
        else:
            self.pushCnt(d)
            DecPiAut.eval(self)
```

Complete Imp grammar in EBNF notation I

```
@@grammar::IMP
@@eol_comments ::/#.*?/start = @ : cmd ;
cmd =nop | let | assign | loop | call ;
call =i:identifier '(' { a:actual }* ')' ;
actual =e1:expression { ',' e2:expression }* | {} ;
nop ='nop' ;
loop =op:'while' ~ e:expression 'do' { c:cmd }+ ;
assign =id:identifier op:':=' ~ e:expression ;
let =op:'let' ~ d:dec 'in' { c:cmd }+ ;
```

Complete Imp grammar in EBNF notation II

```
dec =var | fn ;

var =op:'var' ~ id:identifier '=' e:expression ;

fn =op:'fn' ~ id:identifier '(' f:formal ')' '=' c:cmd ;

formal =i1:identifier { ',' i2:identifier }* | {} ;

expression =@:bool_expression ;

bool_expression =negation | equality | conjunction | disjunction
               | lowereq | greatereq | lowerthan | greaterthan
               | add_expression ;

equality =left:add_expression op:"==" ~ right:bool_expression ;

conjunction =left:add_expression op:"and" ~ right:bool_expression ;
```

Complete Imp grammar in EBNF notation III

```
disjunction =left:add_expression op:"or" ~ right:bool_expression ;

lowereq =left:add_expression op:"<=" ~ right:add_expression ;

greatereq =left:add_expression op:">=" ~ right:add_expression ;

lowerthan =left:add_expression op:"<" ~ right:add_expression ;

greaterthan =left:add_expression op:">" ~ right:add_expression ;

parentesisexp ='(' ~ @:bool_expression ')' ;

negation =op:'not' ~ b:bool_expression ;

add_expression =addition | subtraction | @:mult_expression ;

addition =left:mult_expression op:"+" ~ right:add_expression ;

subtraction =left:mult_expression op:"-" ~ right:add_expression ;
```

Complete Imp grammar in EBNF notation IV

```

mult_expression =multiplication | division
                | atom
                | parentesisexp ;

multiplication =left:atom op:"*" ~ right:mult_expression ;

division =left:atom op:"/" ~ right:mult_expression ;

atom =number | truth | identifier ;

number =/\d+/ ;

identifier =/(?!\d)\w+/ ;

truth ='True' | 'False' ;
```

Example: iterative factorial

```
# The classic iterative factorial example
let var z = 1
in
    let var y = 10
    in
        while not (y == 0)
        do
            z := z * y
            y := y - 1
```

Example: iterative factorial within a function

```
# In this example we encapsulate the iterative calculation
# of the factorial within a function call.
let var z = 1
in
    let fn f(x) =
        let var y = x
        in
            while not (y == 0)
            do
                z := z * y
                y := y - 1
    in f(10)
```