The IMP Programming Language

Christiano Braga cbraga@ic.uff.br

Instituto de Computação Universidade Federal Fluminense



Abstract. IMP is a simple imperative programming language created to illustrate the use of the Basic Programming Languages Constructs library (BPLC). It is implemented in the Maude language.

1 IMP syntax

```
⟨program⟩ ::= 'module' ⟨ident⟩ ⟨clauses⟩ 'end'
\langle clauses \rangle ::= \langle var \rangle^? \langle const \rangle^? \langle init \rangle^? \langle proc \rangle^+
\langle var \rangle := \text{`var'} \langle ident \rangle^+
\langle const \rangle := 'const' \langle ident \rangle^+
\langle init \rangle := 'init' \langle ini \rangle^+
\langle ini \rangle := \langle ident \rangle '=' \langle exp \rangle
\langle proc \rangle := \text{`proc'} \langle ident \rangle \text{`('} \langle ident \rangle^* \text{`)'} \langle block \rangle
\langle block \rangle := `\{' \langle cmd \rangle^+ `\}'
\langle cmd \rangle := \langle ident \rangle `:=' \langle exp \rangle
    |\langle if \rangle| \langle while \rangle| \langle print \rangle| \langle exit \rangle
    |\langle call \rangle |\langle seq \rangle |\langle choice \rangle
\langle if \rangle := \text{`if'} \langle boolexp \rangle \langle cmd \rangle \text{`else'} \langle cmd \rangle
     | 'if' \langle boolexp \rangle \langle cmd \rangle 'else' \langle block \rangle
         'if' \langle boolexp \langle \langle block \rangle 'else' \langle cmd \rangle
          'if' \langle boolexp \langle \langle block \rangle 'else' \langle block \rangle
\langle while \rangle := \text{`while'} \langle boolexp \rangle \langle block \rangle
⟨print⟩ := 'print' '(' ⟨exp⟩ ')'
\langle exit \rangle := \text{`exit'`('} \langle exp \rangle ')'
```

```
\langle call \rangle := \langle ident \rangle \, (' \langle exp \rangle^* \, ')'
\langle seq \rangle := \langle cmd \rangle \, ';' \langle cmd \rangle
\langle choice \rangle := \langle cmd \rangle \, '|' \langle cmd \rangle
\langle exp \rangle := \langle ident \rangle
| \langle ident \rangle \langle arithop \rangle \langle ident \rangle
| \langle boolexp \rangle
\langle arithop \rangle := '+' | '-' | '*' | '|' | '/'
\langle boolexp \rangle := \langle ident \rangle \langle boolop \rangle \langle ident \rangle
\langle boolop \rangle := '\sim' | '==' | '<' | '<=' | '>' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>=' | '>='
```

2 IMP semantics

- 2.1 Genralized SMC Machines & BPLC
- 2.2 GSMC Rules for IMP
- 3 Maude implementation

3.1 A Maude primer

Syntax

- Modules
- Signatures
- Equations
- Rules

Semantics. Modules, transition systems and Kripke structures.

Metalevel

3.2 Parser

Module TOKEN specifies what Tokens, TokenLists and NeTokenList are. They are constructed with operators token, bubble and neTokenList, where attribute Exclude specify which terms are not to be constructed by the given constructor.

```
⟨token-module⟩≡
  fmod TOKEN is
  pr QID-LIST .
  sorts Token Bubble TokenList NeTokenList .
  subsort Token < TokenList .

op _,_ : TokenList TokenList -> TokenList [assoc prec 5] .
```

```
op token : Qid -> Token
    [special
                             (1 \ 1)
      (id-hook Bubble
                             (<Qids> : ~> Qid)
      op-hook qidSymbol
       id-hook Exclude
                             ( true false nop print ))] .
op bubble : QidList -> Bubble
    [special
      (id-hook Bubble
                             (1 - 1)
       op-hook qidListSymbol (__ : QidList QidList ~> QidList)
       op-hook qidSymbol
                             (<Qids> : ~> Qid)
       id-hook Exclude
                             ( | if then else end { }
                               while ; := = nop )) ] .
op neTokenList : QidList -> NeTokenList
    [special
      (id-hook Bubble
                             (1 - 1)
      op-hook qidListSymbol (__ : QidList QidList ~> QidList)
       op-hook qidSymbol
                             (<Qids> : ~> Qid)
       id-hook Exclude
                              ( . ))] .
endfm
```

Module IMP-GRAMMAR implements the grammar of Section 1 in Maude. Each sort implements a non-terminal of the grammar in Section 1 whereas constants, tokens, as described in module TOKEN, or operators implement terminals. For instance, the following rule in the grammar of Section 1

```
\langle proc \rangle := \text{`proc'} \langle ident \rangle \text{`('} \langle ident \rangle^* \text{`)'} \langle block \rangle
```

is implemented in Module IMP-GRAMMAR by the operation declarations

```
op proc__: Token BlockCommandDecl -> ProcDecl [prec 50].
op proc_'(_')_: Token TokenList BlockCommandDecl -> ProcDecl [prec 50].
together with the declarations for sorts ProcDecl and BlockCommandDecl.
\langle imp-grammar-module\rangle \equiv
  fmod IMP-GRAMMAR is
   pr TOKEN .
   pr RAT .
   inc PREDICATE-DECL .
   inc COMMAND-DECL .
   sorts VariablesDecl ConstantsDecl OperationsDecl ProcDeclList
         ProcDecl FormalsDecl BlockCommandDecl ExpressionDecl
         InitDecl InitDeclList InitDecls ClausesDecl
         ModuleDecl Expression .
   subsort InitDecl < InitDeclList .</pre>
   subsort VariablesDecl ConstantsDecl ProcDeclList
            InitDecls < ClausesDecl .</pre>
   subsort BlockCommandDecl < CommandDecl .</pre>
   subsort ProcDecl < ProcDeclList .</pre>
   subsort PredicateDecl < ExpressionDecl .</pre>
   *** Boolean expressions
   op _==_ : Token Token -> PredicateDecl [prec 30] .
   op _==_ : Token ExpressionDecl -> PredicateDecl [prec 30] .
   op _==_ : ExpressionDecl Token -> PredicateDecl [prec 30]
   op _==_ : ExpressionDecl ExpressionDecl -> PredicateDecl
              [prec 30] .
   op _/\_ : PredicateDecl PredicateDecl -> PredicateDecl
              [assoc comm prec 35] .
      ~ : PredicateDecl -> PredicateDecl .
   *** Arithmetic expressions
   op _+_ : Token Token -> ExpressionDecl [gather(e E) prec 15] .
   op _+_ : Token ExpressionDecl -> ExpressionDecl [gather(e E) prec 15] .
   op _+_ : ExpressionDecl Token -> ExpressionDecl [gather(e E) prec 15] .
   op _+_ : ExpressionDecl ExpressionDecl -> ExpressionDecl
```

[gather(e E) prec 15] .

```
op _-_ : Token Token -> ExpressionDecl [gather(e E) prec 15] .
op _-_ : Token ExpressionDecl -> ExpressionDecl [gather(e E) prec 15] .
op _-_ : ExpressionDecl Token -> ExpressionDecl [gather(e E) prec 15] .
op _-_ : ExpressionDecl ExpressionDecl -> ExpressionDecl
         [gather(e E) prec 15] .
op _*_ : Token Token -> ExpressionDecl [gather(e E) prec 10] .
op _*_ : Token ExpressionDecl -> ExpressionDecl [gather(e E) prec 10] .
op _*_ : ExpressionDecl Token -> ExpressionDecl [gather(e E) prec 10] .
op _*_ : ExpressionDecl ExpressionDecl -> ExpressionDecl
         [gather(e E) prec 10] .
op _/_ : Token Token -> ExpressionDecl [prec 20] .
op _/_ : Token ExpressionDecl -> ExpressionDecl [prec 20] .
op _/_ : ExpressionDecl Token -> ExpressionDecl [prec 20] .
op _/_ : ExpressionDecl ExpressionDecl -> ExpressionDecl [prec 20] .
*** Commands
op _:=_ : Token Token -> CommandDecl [prec 40] .
op _:=_ : Token ExpressionDecl -> CommandDecl [prec 40] .
op _() : Token -> CommandDecl .
op _(_) : Token Bubble -> CommandDecl .
op print : Token -> CommandDecl .
op print : ExpressionDecl -> CommandDecl .
op _;_ : CommandDecl CommandDecl -> CommandDecl [assoc prec 50] .
op _|_ : CommandDecl CommandDecl -> CommandDecl [assoc prec 50] .
op while_do_ : PredicateDecl BlockCommandDecl -> CommandDecl [prec 40] .
op {_} : CommandDecl -> BlockCommandDecl [prec 35] .
op if__else_ : PredicateDecl CommandDecl CommandDecl -> CommandDecl
   [prec 40] .
*** Declarations
op module_end : Token ClausesDecl -> ModuleDecl [prec 80] .
op __ : ClausesDecl ClausesDecl -> ClausesDecl [assoc comm prec 70] .
op var_ : TokenList -> VariablesDecl [prec 60] .
op const_ : TokenList -> ConstantsDecl [prec 60] .
op init_ : InitDecl -> InitDecls [prec 60] .
op init_ : InitDeclList -> InitDecls [prec 60] .
op _=_ : Token Token -> InitDecl [prec 40] .
op _=_ : Token ExpressionDecl -> InitDecl [prec 40] .
op _,_ : InitDeclList InitDeclList -> InitDeclList [assoc prec 50] .
op proc__ : Token BlockCommandDecl -> ProcDecl [prec 50] .
op proc_'(_')_ : Token TokenList BlockCommandDecl -> ProcDecl
   [prec 50] .
```

```
op __ : ProcDeclList ProcDeclList -> ProcDeclList
  [assoc comm prec 60] .
endfm
```

3.3 Compiling to BPLC

Compilation from IMP to BPLC is quite trivial as there exists a one-to-one correspondence between IMP constructions and BPLC. Essentially, an IMP module gives rise to a BPLC dec. IMP var and const are declarations and so is a proc declaration that gives rise to a prc declaration in BPLC.

```
⟨compile-imp-to-bplc⟩≡
  mod COMPILE-IMP-TO-BPLC is
  inc PREDICATE-DECL .
  inc COMMAND-DECL .
  pr BPLC .
  pr META-LEVEL .

⟨Compiling IMP expressions to BPLC Exp⟩
 ⟨Compiling IMP commands to BPLC Cmd⟩
 ⟨Compiling IMP declarations to BPLC Dec⟩
```

The compilation from IMP to BPLC exp relates IMP tokens to BPLC Id, IMP arithmetic and boolean expressions to BPLC Exp. In particular, the compilation of a token has to check if the token is a primitive type, either Rat (for Rational numbers) or Bool (for Boolean values), or an identifier. Since Rat and Bool are tokenized and we need Maude metalevel descent function downTerm to help us parse them into proper constants

```
\langle Compiling\ tokens \rangle \equiv
   op compileId : Qid -> Id .
  eq compileId(I:Qid) = idn(downTerm(I:Qid, 'Qid)) .
  op compileId : Term -> Id .
   eq compileId('token[I:Qid]) =
      if (metaParse(upModule('RAT, false),
           downTerm(I:Qid, 'Qid), 'Rat) :: ResultPair)
          then rat(downTerm(getTerm(metaParse(upModule('RAT, false),
                downTerm(I:Qid, 'Qid), 'Rat)), 1/2))
      else
       if (metaParse(upModule('BOOL, false),
            downTerm(I:Qid, 'Qid), 'Bool) :: ResultPair)
       then boo(downTerm(getTerm(metaParse(upModule('BOOL, false),
                      downTerm(I:Qid, 'Qid), 'Bool)), true))
       else idn(downTerm(I:Qid, 'Qid))
       fi
      fi.
```

Arithmetic expressions are mapped to their prefixed counterpart in BPLC, e.g, an IMP expression a + b is compiled to add(compileExp(a), compileExp(b)).

```
⟨Compiling arithmetic expressions⟩≡
  op compileExp : Term → Exp .
  ceq compileExp(I:Qid) = compileId(I:Qid)
  if not(I:Qid :: Constant) .
  eq compileExp('token[I:Qid]) = compileId('token[I:Qid]) .
  eq compileExp('_+_[T1:Term, T2:Term]) =
     add(compileExp(T1:Term), compileExp(T2:Term)) .
  eq compileExp('_-_[T1:Term, T2:Term]) =
     sub(compileExp(T1:Term), compileExp(T2:Term)) .
  eq compileExp('_*_[T1:Term, T2:Term]) =
     mul(compileExp(T1:Term), compileExp(T2:Term)) .
  eq compileExp('_/_[T1:Term, T2:Term]) =
     div(compileExp(T1:Term), compileExp(T2:Term)) .
```

A treatment similar to arithmetic expressions is given to Boolean expressions, with a particular care with constants, as we did in «Compiling tokens». Here, however, we do not require a call to downTerm as the constants for PredicateDecl are already typed.

```
\langle Compiling\ boolean\ expressions \rangle \equiv
   eq compileExp('true.PredicateDecl) = boo(true) .
   eq compileExp('false.PredicateDecl) = boo(false) .
  eq compileExp('~[T:Term]) = neg(compileExp(T:Term)) .
  eq compileExp('_/\_[T1:Term,T2:Term]) =
      and(compileExp(T1:Term), compileExp(T2:Term)) .
  eq compileExp('_\/_[T1:Term,T2:Term]) =
      or(compileExp(T1:Term), compileExp(T2:Term)) .
  eq compileExp('_==_[T1:Term,T2:Term]) =
      eq(compileExp(T1:Term), compileExp(T2:Term)) .
  eq compileExp('_>_[T1:Term,T2:Term]) =
      gt(compileExp(T1:Term), compileExp(T2:Term)) .
  eq compileExp('_>=_[T1:Term,T2:Term]) =
      ge(compileExp(T1:Term), compileExp(T2:Term)) .
   eq compileExp('_<_[T1:Term,T2:Term]) =
      lt(compileExp(T1:Term), compileExp(T2:Term)) .
   eq compileExp('_<=_[T1:Term,T2:Term]) =
      le(compileExp(T1:Term), compileExp(T2:Term)) .
```

```
\langle Compiling\ IMP\ commands\ to\ BPLC\ Cmd \rangle \equiv
  op compileCmd : Term -> Cmd .
  eq compileCmd('nop.CommandDecl) = nop .
  eq compileCmd('_:=_['token[I:Qid], T:Term]) =
      assign(compileId('token[I:Qid]), compileExp(T:Term)) .
  eq compileCmd(''{_'}[T:Term]) = blk(compileCmd(T:Term)) .
  eq compileCmd('_;_[T1:Term, T2:Term]) =
      seq(compileCmd(T1:Term), compileCmd(T2:Term)) .
  eq compileCmd('_|_[T1:Term, T2:Term]) =
      choice(compileCmd(T1:Term), compileCmd(T2:Term)) .
  eq compileCmd('if__else_[T1:Term, T2:Term, T3:Term]) =
      if(compileExp(T1:Term), compileCmd(T2:Term),
         compileCmd(T3:Term)) .
  eq compileCmd('while_do_[T1:Term, T2:Term]) =
      loop(compileExp(T1:Term), compileCmd(T2:Term)) .
  eq compileCmd('print[T:Term]) = print(compileId(T:Term)) .
  *** Compiling IMP commands to BPLC Cmd:
  *** Procedure calls deserve more attention...
  eq compileCmd('_'(')['token[I:Qid]]) =
      cal(compileId('token[I:Qid])) .
  eq compileCmd('_'(_')['token[I:Qid], 'bubble[Q:Qid]]) =
      cal(compileId('token[I:Qid]), compileActuals(Q:Qid)) .
  eq compileCmd('_'(_')['token[I:Qid],
                 'bubble['__[TL:TermList]]]) =
      cal(compileId('token[I:Qid]), compileActuals(TL:TermList)) .
  op compileActuals : TermList -> Actuals .
  eq compileActuals(Q:Qid) = compileId('token[Q:Qid]) .
  eq compileActuals((TL1:TermList, "',.Qid , TL2:TermList)) =
      act(makeExp(TL1:TermList) , compileActuals(TL2:TermList)) .
  eq compileActuals(TL:TermList) = makeExp(TL:TermList) [owise] .
  op makeExp : TermList -> Exp .
  eq makeExp(Q:Qid) = compileId(Q:Qid) .
  ceq makeExp(TL:TermList) =
       compileExp(
        getTerm(
         metaParse(upModule('IMP-GRAMMAR, false),
          makeExprDeclQidList(TL:TermList), 'ExpressionDecl)))
    if (metaParse(upModule('IMP-GRAMMAR, false),
               makeExprDeclQidList(TL:TermList), 'ExpressionDecl) ::
              ResultPair) .
```

```
op makeExprDeclQidList : TermList -> QidList .
eq makeExprDeclQidList((empty).TermList) = (nil).QidList .
ceq makeExprDeclQidList(Q:Qid) = downTerm(Q:Qid, 'error)
if downTerm(Q:Qid, 'error) =/= 'error .
eq makeExprDeclQidList((Q:Qid , TL:TermList)) =
    makeExprDeclQidList(Q:Qid) makeExprDeclQidList(TL:TermList) .
```

```
\langle Compiling\ IMP\ declarations\ to\ BPLC\ Dec \rangle \equiv
  *** Compiling variables. The initialization clause is
  *** necessary to properly declare variables according to BPLC
  *** ref construct.
  op compileVar : Term Term -> Dec .
  op $compileVar : Term Term -> Dec .
  eq compileVar('var_[TL1:TermList], 'init_[TL2:TermList]) =
      $compileVar(TL1:TermList, TL2:TermList) .
  eq $compileVar('token[I:Qid], '_=_['token[I:Qid], T:Term]) =
      ref(compileId('token[I:Qid]), compileExp(T:Term)) .
  eq $compileVar('token[I:Qid],
      ('_',_['_=_['token[I:Qid], T:Term], TL:TermList])) =
     ref(compileId('token[I:Qid]), compileExp(T:Term)) .
  ceq $compileVar('token[I1:Qid],
       ('_',_['_=_['token[I2:Qid], T:Term], TL:TermList])) =
       $compileVar('token[I1:Qid], TL:TermList)
    if I1:Qid =/= I2:Qid .
  eq $compileVar('_',_['token[I:Qid], IS:TermList],
                   '_=_['token[I:Qid], T:Term]) =
      ref(compileId('token[I:Qid]), compileExp(T:Term)) .
  eq $compileVar('_',_['token[I:Qid], IS:TermList],
                  '_',_[TL:TermList]) =
      dec($compileVar('token[I:Qid], '_',_[TL:TermList]),
       $compileVar(IS:TermList, '_',_[TL:TermList])) .
  *** Compiling constants.
  op compileConst : Term Term -> Dec .
  op $compileConst : Term Term -> Dec .
  eq compileConst('const_[T1:Term], 'init_[T2:Term]) =
      $compileConst(T1:Term, T2:Term) .
  eq $compileConst('token[I:Qid],
                    '_=_['token[I:Qid], T:Term]) =
      cns(compileId('token[I:Qid]), compileExp(T:Term)) .
  eq $compileConst('token[I:Qid],
                   ('_',_['_=_['token[I:Qid], T:Term], TL:TermList])) =
      cns(compileId('token[I:Qid]), compileExp(T:Term)) .
  ceq $compileConst('token[I1:Qid],
                   ('_',_['_=_['token[I2:Qid], T:Term], TL:TermList])) =
      $compileConst('token[I1:Qid], TL:TermList)
  if I1:Qid =/= I2:Qid .
  eq $compileConst('_',_['token[I:Qid], IS:TermList],
                    '_=_['token[I:Qid], T:Term]) =
      cns(compileId('token[I:Qid]), compileExp(T:Term)) .
  eq $compileConst(',',_['token[I:Qid], IS:TermList], ',',_[TL:TermList]) =
```

```
dec($compileConst('token[I:Qid], '_',_[TL:TermList]),
    $compileConst(IS:TermList, '_',_[TL:TermList])) .
*** Compiling procedure declarations.
op compileProc : TermList -> Dec .
op compileToFormals : TermList -> Formals .
eq compileProc('__[TL1:TermList, TL2:TermList]) =
   dec(compileProc(TL1:TermList), compileProc(TL2:TermList)) .
eq compileProc('proc__['token[0:Qid], TL:TermList, T:Term]) =
   {\tt prc(compileId('token[0:Qid]),\ compileCmd(T:Term))}\ .
eq compileProc('proc_'(_')_['token[0:Qid], TL:TermList, T:Term]) =
   prc(compileId('token[0:Qid]),
    compileToFormals(TL:TermList), compileCmd(T:Term)) .
eq compileToFormals('token[0:Qid]) = par(compileId('token[0:Qid])) .
eq compileToFormals('_',_[TL1:TermList, TL2:TermList]) =
   for(compileToFormals(TL1:TermList), compileToFormals(TL2:TermList)) .
*** Compiling modules.
op compileMod : Term -> Dec .
eq compileMod('module__end['token[I:Qid],
              '__['var_[T1:Term],
              '__['const_[T2:Term],
              '__['init_[T3:Term],
                               T:Term]]]]) =
   dec(compileVar('var_[T1:Term],'init_[T3:Term]),
    dec(compileConst('const_[T2:Term], 'init_[T3:Term]),
     compileProc(T:Term))) .
eq compileMod('module__end['token[I:Qid],
              '__['var_[T1:Term],
              '__['init_[T3:Term],
              T:Term]]]) =
   dec(compileVar('var_[T1:Term],'init_[T3:Term]),
        compileProc(T:Term)) .
eq compileMod('module_end['token[I:Qid],
              '__['var_[T1:Term],
              '__['const_[T2:Term],
              '__['init_[T3:Term],
              '__[TL:TermList]]]]) =
   dec(compileVar('var_[T1:Term],'init_[T3:Term]),
    dec(compileConst('const_[T2:Term], 'init_[T3:Term]),
         compileProc('__[TL:TermList]))) .
eq compileMod('module__end['token[I:Qid],
              '__['var_[T1:Term],
              '__['init_[T3:Term],
              '__[TL:TermList]]]) =
```

3.4 Pretty-printing IMP programs, their execution and model checking results

```
\langle imp\text{-}pretty\text{-}printing \rangle \equiv
 mod IMP-PRETTY-PRINTING is
  pr BPLC-MODEL-CHECKER .
  pr COMPILE-TO-PYTHON .
  pr INT .
  op printTermList : TermList -> QidList .
  eq printTermList(empty) = (nil).QidList .
  eq printTermList((Q:Qid,TL:TermList)) =
      downTerm(Q:Qid, 'Qid) printTermList(TL:TermList) .
  op printTokens : Term -> QidList .
   eq printTokens(Q:Qid) = Q:Qid .
   eq printTokens('token[Q:Qid]) = downTerm(Q:Qid, 'Qid) .
   eq printTokens('_',_[T1:Term, T2:Term]) =
      printTokens(T1:Term) printToken('',) '\s printTokens(T2:Term) .
   *** Pretty-printing IMP Expressions
  op printExp : Term -> QidList .
  eq printExp('true.PredicateDecl) = 'true .
   eq printExp('false.PredicateDecl) = 'false .
  eq printExp('bubble['__[TL:TermList]]) = printTermList(TL:TermList) .
   eq printExp('token[Q:Qid]) = downTerm(Q:Qid, 'Qid) .
  eq printExp('_==_[T1:Term, T2:Term]) =
      printExp(T1:Term) printToken('==) printExp(T2:Term) .
  eq printExp('_\/_[T1:Term, T2:Term]) =
      printExp(T1:Term) printToken('\/) printExp(T2:Term) .
  eq printExp('_/\_[T1:Term, T2:Term]) =
      printExp(T1:Term) printToken('\/) printExp(T2:Term) .
  eq printExp('~[T:Term]) =
      printToken('~) printExp(T:Term) .
  eq printExp(',<_[T1:Term, T2:Term]) =</pre>
      printExp(T1:Term) printToken('<) printExp(T2:Term) .</pre>
  eq printExp('_<=_[T1:Term, T2:Term]) =</pre>
      printExp(T1:Term) printToken('<=) printExp(T2:Term) .</pre>
  eq printExp('_>_[T1:Term, T2:Term]) =
      printExp(T1:Term) printToken('>) printExp(T2:Term) .
  eq printExp('_>=_[T1:Term, T2:Term]) =
      printExp(T1:Term) printToken('>=) printExp(T2:Term) .
   eq printExp('_+_[T1:Term, T2:Term]) =
      printExp(T1:Term) printToken('+) printExp(T2:Term) .
  eq printExp('_*_[T1:Term, T2:Term]) =
      printExp(T1:Term) printToken('*) printExp(T2:Term) .
   eq printExp('_-_[T1:Term, T2:Term]) =
```

```
printExp(T1:Term) printToken('-) printExp(T2:Term) .
eq printExp('_/_[T1:Term, T2:Term]) =
   printExp(T1:Term) printToken('/) printExp(T2:Term) .
eq printExp(T:Term) =
   metaPrettyPrint(upModule('IMP-GRAMMAR, false), T:Term) [owise] .
*** Pretty-printing IMP Commands
op printCmd : Term Nat -> QidList .
eq printCmd('nop.CommandDecl, N:Nat) = 'nop .
eq printCmd('print[T:Term], N:Nat) =
   printToken('print) printToken('() printExp(T:Term) printToken('()) .
eq printCmd('_'(_')[T1:Term, T2:Term], N:Nat) =
   printExp(T1:Term) printToken(''() printExp(T2:Term) printToken('')) .
eq printCmd('_|_[T1:Term, '_;_[T2:TermList]], N:Nat) =
   printSpaces(N:Nat)
   printCmd(T1:Term, N:Nat) printToken('|) '\n
   printSpaces(N:Nat) printToken('()
   printCmd('_;_[T2:TermList], N:Nat) printToken('')) .
ceq printCmd('_|_[T:TermList, F:Qid[TL:TermList]], N:Nat) =
   printSpaces(N:Nat) printCmd(T:TermList, N:Nat) printToken('|)
   printCmd(F:Qid[TL:TermList], N:Nat)
if F:Qid =/= '_;_ .
ceq printCmd('_;_[F1:Qid[TL1:TermList], F2:Qid[TL2:TermList]], N:Nat) =
   printCmd(F1:Qid[TL1:TermList], N:Nat) '\s printToken(';)
   printCmd(F2:Qid[TL2:TermList], N:Nat)
if F1:Qid =/= '_;_ and F2:Qid =/= '_;_ .
ceq printCmd('_;_[F1:Qid[TL1:TermList], F2:Qid[TL2:TermList]], N:Nat) =
   printSpaces(N:Nat)
   printCmd(F1:Qid[TL1:TermList], N:Nat) '\s printToken(';) '\n
   printSpaces(N:Nat)
   printCmd(F1:Qid[TL1:TermList], N:Nat)
if F1:Qid == '_;_ or F2:Qid == '_;_ .
eq printCmd('_:=_[T1:Term, T2:Term], N:Nat) =
   printExp(T1:Term) printToken(':=) printExp(T2:Term) .
eq printCmd('while_do_[T1:Term, T2:Term], N:Nat) =
   printToken('while) ' printToken('() printExp(T1:Term) printToken('()) '
   printToken('do)
   printCmd(T2:Term, N:Nat) .
eq printCmd('if__else_[T1:Term, T2:Term, T3:Term], N:Nat) =
   printToken('if) ' printToken('(') printExp(T1:Term) printToken('')) '
   printCmd(T2:Term, N:Nat)
    '\s printToken('else) printCmd(T3:Term, N:Nat) .
eq printCmd(''{_'}[C:Constant], N:Nat) =
    '\s printToken(''{})
   printCmd(C:Constant, 0)
```

```
printToken(',') .
eq printCmd(''{_'}['_;_[TL:TermList]], N:Nat) =
    '\s printToken(''{{}} '\n
   printSpaces(N:Nat + incr)
   printCmd('_;_[TL:TermList], N:Nat + incr) '\n
   printSpaces(N:Nat) printToken('') .
eq printCmd(','{_'}[',_|_[TL:TermList]], N:Nat) =
    '\s printToken('`{}) '\n
   printCmd('_|_[TL:TermList], N:Nat + incr) '\n
   printSpaces(N:Nat) printToken('') .
ceq printCmd(''{_'}[F:Qid[TL:TermList]], N:Nat) =
    '\s printToken(''{{}} '\n
   printSpaces(N:Nat + incr)
   printCmd(F:Qid[TL:TermList], N:Nat + incr) '\n
   printSpaces(N:Nat) printToken('')
if F:Qid =/= '_;_ /\ F:Qid =/= '_|_ .
eq printCmd(T:Term, N:Nat) =
   printSpaces(N:Nat)
   metaPrettyPrint(upModule('IMP-GRAMMAR, false), T:Term) [owise] .
*** Pretty-printing IMP Declarations
op printModule : Term -> QidList .
eq printModule('module__end[T1:Term, T2:Term]) =
   printToken('module) printTokens(T1:Term)
       printClauses(T2:Term)
        '\n printToken('end) .
op printClauses : Term -> QidList .
eq printClauses('proc__[T1:Term, T2:Term]) =
    '\n printSpaces(level)
   printToken('proc) printTokens(T1:Term)
   printCmd(T2:Term, level) .
eq printClauses('__['proc__[T1:Term, T2:Term], T3:Term]) =
    '\n printSpaces(level) printToken('proc) printTokens(T1:Term)
   printCmd(T2:Term, level)
   printClauses(T3:Term) .
eq printClauses('proc_'(_')_[T1:Term, T2:Term, T3:Term]) =
    '\n printSpaces(level)
   printToken('proc) printTokens(T1:Term)
   printToken('() printTokens(T2:Term) printToken('())
   printCmd(T3:Term, level) .
eq printClauses('__['proc_'(_')_[T1:Term, T2:Term, T3:Term], T4:Term]) =
    '\n printSpaces(level) printToken('proc)
   printTokens(T1:Term) printToken(''() printTokens(T2:Term) printToken(''))
```

```
printCmd(T3:Term, level)
   printClauses(T4:Term) .
eq printClauses('__['init_[T1:Term], T2:Term]) =
    '\n printSpaces(level) printToken('init)
   printInit(T1:Term) printClauses(T2:Term) .
eq printClauses('__['const_[T1:Term], T2:Term]) =
    '\n printSpaces(level) printToken('const)
   printTokens(T1:Term) printClauses(T2:Term) .
 eq printClauses('__['var_[T1:Term], T2:Term]) =
        '\n printSpaces(level) printToken('var)
   printTokens(T1:Term) printClauses(T2:Term) .
op printInit : Term -> QidList .
eq printInit('_=_[T1:Term, T2:Term]) =
   printTokens(T1:Term) printToken('=) printExp(T2:Term) .
eq printInit(',_',_[T1:Term, T2:Term]) =
   printInit(T1:Term) printToken('',) '\s printInit(T2:Term) .
*** Pretty-printing parse error
op printQidList : QidList -> QidList .
eq printQidList(nil) = (nil).QidList .
eq printQidList(Q:Qid QL:QidList) = printToken(Q:Qid) printQidList(QL:QidList) .
op printToken : Qid -> Qid .
eq printToken(Q:Qid) = '\b '\! Q:Qid '\o .
op printInputWithError : QidList Nat -> QidList .
op $printInputWithError : QidList Int QidList -> QidList .
eq printInputWithError(QL:QidList, N:Nat) =
   $printInputWithError(QL:QidList, N:Nat, (nil).QidList) .
eq $printInputWithError(nil, I:Int, QL:QidList) = QL:QidList .
eq $printInputWithError(QL:QidList, 0, (nil).QidList) = (nil).QidList .
ceq $printInputWithError(QL1:QidList, 0, (QL2:QidList Q:Qid)) =
   QL2:QidList '\r '\u '>> Q:Qid '«< 'HERE '\o
   printQidList(QL1:QidList)
if QL2:QidList =/= nil .
eq $printInputWithError(Q:Qid QL1:QidList, I:Int, QL2:QidList) =
   $printInputWithError(QL1:QidList, (I:Int + (- 1)), QL2:QidList
       printToken(Q:Qid)) .
op printParseError : QidList ResultPair? -> Qid .
eq printParseError('module Q:Qid QL:QidList , noParse(N:Nat)) =
    'IMP: 'Error 'at 'position
   metaPrettyPrint(upModule('NAT, false), upTerm(N:Nat))
        'while 'parsing 'Module Q:Qid ': '\n
```

```
printInputWithError(('module Q:Qid QL:QidList), N:Nat) .
eq printParseError(QL:QidList, ambiguity(T1:ResultPair, T2:ResultPair)) =
    'IMP: '\r 'Ambiguous 'parse '\o '\n
   metaPrettyPrint(upModule('IMP-GRAMMAR, false), getTerm(T1:ResultPair)) '\n
   '\r 'vs. '\o '\n
   metaPrettyPrint(upModule('IMP-GRAMMAR, false), getTerm(T2:ResultPair)) .
*** Pretty-print exec and mc output
op printState : Env Store -> QidList .
eq printState(noEnv, S:Store) = (nil).QidList .
eq printState((I:Id |-> bind(L:Loc)) , ((L:Loc |-> store(R:Rat)), S:Store)) =
   printId(I:Id) '= metaPrettyPrint(upModule('RAT, false), upTerm(R:Rat)) .
ceq printState(((I:Id |-> bind(L:Loc)), E:Env) ,
   ((L:Loc |-> store(R:Rat)), S:Store)) =
   printId(I:Id) '=
   metaPrettyPrint(upModule('RAT, false), upTerm(R:Rat))
   printState(E:Env , (L:Loc |-> store(R:Rat), S:Store))
if E:Env =/= noEnv .
eq printState((I:Id |-> bind(L:Loc)), (L:Loc |-> store(B:Bool), S:Store)) =
   printId(I:Id) '= metaPrettyPrint(upModule('BOOL, false), upTerm(B:Bool)) .
ceq printState(((I:Id |-> bind(L:Loc)), E:Env) ,
    ((L:Loc |-> store(B:Bool)), S:Store)) =
       printId(I:Id) '= metaPrettyPrint(upModule('RAT, false), upTerm(B:Bool)) '',
       printState(E:Env , (L:Loc |-> store(B:Bool), S:Store))
if E:Env =/= noEnv .
eq printState(((I:Id |-> B:Bindable), E:Env) , S:Store) =
   printState(E:Env , S:Store) [owise] .
op printId : Id -> Qid .
eq printId(idn(Q:Qid)) = (Q:Qid).Qid .
op printCnt : ControlStack ValueStack -> QidList .
eq printCnt((LOOP C:ControlStack),
             (V:Value val(loop(E:Exp, K:Cmd)) VS:ValueStack)) =
   printToken('while) printToken('() printExp(E:Exp) printToken('()) '...
eq printCnt((choice(K1:Cmd, K2:Cmd) C:ControlStack), VS:ValueStack) =
   printCmd(K1:Cmd) printToken('|) printCmd(K2:Cmd) .
eq printCnt(C:ControlStack, VS:ValueStack) =
    '\n 'Constrol 'stack:
   metaPrettyPrint(upModule('BPLC+META-LEVEL, false),
    upTerm(C:ControlStack))
    '\n 'Value 'stack:
    metaPrettyPrint(upModule('BPLC+META-LEVEL, false),
    upTerm(VS:ValueStack)) .
```

```
op printExp : Exp -> QidList .
eq printExp(rat(R:Rat)) = '\g metaPrettyPrint(upModule('RAT, false),
    upTerm(R:Rat)) '\o .
eq printExp(boo(B:Bool)) = '\g metaPrettyPrint(upModule('BOOL, false),
    upTerm(B:Bool)) '\o .
eq printExp(neg(E:Exp)) = printToken('~') printExp(E:Exp) .
eq printExp(and(E1:Exp, E2:Exp)) =
   printExp(E1:Exp) printToken('/\) printExp(E2:Exp) .
eq printExp(eq(E1:Exp, E2:Exp)) =
   printExp(E1:Exp) printToken('==) printExp(E2:Exp) .
eq printExp(idn(Q:Qid)) = printTokens(Q:Qid) .
eq printExp(E:Exp) = metaPrettyPrint(upModule('BPLC, false), upTerm(E:Exp)) .
op printCmd : Cmd -> QidList .
eq printCmd(seq(C1:Cmd, C2:Cmd)) =
   printCmd(C1:Cmd) printToken(';) printCmd(C2:Cmd) .
eq printCmd(assign(I:Id, E:Exp)) =
   printExp(I:Id) printToken(':=) printExp(E:Exp) .
eq printCmd(if(E:Exp, C1:Cmd, C2:Cmd)) =
   printToken('if) printExp(E:Exp) '...
eq printCmd(choice(C1:Cmd, C2:Cmd)) = printCmd(C1:Cmd)
   printToken('|) printCmd(C2:Cmd) .
eq printCmd(C:Cmd) = metaPrettyPrint(upModule('BPLC, false), upTerm(C:Cmd)) .
op printConf : Conf -> QidList .
eq printConf(< env : E:Env , sto : S:Store , exc : X:Exc ,
               cnt : C:ControlStack, val : V:ValueStack, ...:Set{SemComp} > ) =
   if X:Exc == CNT
   then printCnt(C:ControlStack, V:ValueStack) ', [ printState(E:Env, S:Store) ', ]
       else 'IMP: 'Internal 'error 'while 'printing 'configuration.
   fi.
op printValueStack : ValueStack -> QidList .
eq printValueStack(evs) = '\b 'No 'output. '\o .
eq printValueStack(val(R:Rat) evs) =
   metaPrettyPrint(upModule('RAT, false), upTerm(R:Rat)) .
eq printValueStack(val(B:Bool) evs) =
   metaPrettyPrint(upModule('BOOL, false), upTerm(B:Bool)) .
eq printValueStack(V:Value VS:ValueStack) =
   printValueStack(V:Value) printValueStack(VS:ValueStack) .
op printOut : Conf -> QidList .
eq printOut(< out : 0:ValueStack , ...:Set{SemComp} > ) =
   printValueStack(0:ValueStack) .
```

```
op printExec : Term ~> QidList .
ceq printExec(T:Term) = printOut(downTerm(T:Term, < noSemComp >))
 if downTerm(T:Term, < noSemComp >) =/= < noSemComp > .
op printTraceStep : TraceStep -> QidList .
 eq printTraceStep({T:Term, Y:Type, R:Rule}) =
    printConf(downTerm(T:Term, < noSemComp >)) .
 op printTrace : Trace Nat -> QidList .
 eq printTrace(nil, N:Nat) = (nil).QidList .
 eq printTrace(TS:TraceStep, N:Nat) =
    '\b 'State
   metaPrettyPrint(upModule('NAT, false),
    upTerm(N:Nat)) ': '\o '\n
    printTraceStep(TS:TraceStep) .
ceq printTrace(TS:TraceStep T:Trace, N:Nat) =
    '\b 'State
   metaPrettyPrint(upModule('NAT, false),
    upTerm(N:Nat)) ': '\o '\n
    printTraceStep(TS:TraceStep) '\n printTrace(T:Trace, (N:Nat + 1))
 if T:Trace =/= nil.
op printTransitionList : TransitionList -> QidList .
 eq printTransitionList(nil) = (nil).QidList .
eq printTransitionList({C:Conf, R:RuleName}) = printConf(C:Conf) .
ceq printTransitionList({C:Conf, R:RuleName} TL:TransitionList) =
    printConf(C:Conf) '\b '-> '\o printTransitionList(TL:TransitionList)
if TL:TransitionList =/= nil .
op printModelCheckResult : ModelCheckResult QidList -> QidList .
eq printModelCheckResult(B:Bool, QL:QidList) =
    'IMP: '\b 'Model 'check 'result 'to 'command
    '\c QL:QidList '\o '\b ': '\o '\n
        metaPrettyPrint(upModule('BOOL, false), upTerm(B:Bool)) .
 eq printModelCheckResult(
    counterexample(TL1:TransitionList, TL2:TransitionList), QL:QidList) =
    'IMP: '\b 'Model 'check 'counter 'example 'to 'command
    '\c QL:QidList '\o '\b ': '\o '\n
    '\b 'Path 'from 'the 'initial 'state: '\o
    '\n printTransitionList(TL1:TransitionList)
    '\n '\b 'Loop: '\o
    '\n printTransitionList(TL2:TransitionList) .
endm
```

3.5 IMP command line interface

```
\langle imp\text{-}interface \rangle \equiv
 mod IMP-INTERFACE is
  pr LOOP-MODE * (sort State to LoopState).
  pr COMPILE-IMP-TO-BPLC .
  pr IMP-PRETTY-PRINTING .
  sorts Dec? MetaIMPModule Command IMPState .
  subsort Term < MetaIMPModule .</pre>
   subsort Dec < Dec? .
   subsort IMPState < LoopState .</pre>
  op noDec : -> Dec? .
  op noModule : -> MetaIMPModule .
  op idle : -> Command .
  op <_;_;_> : MetaIMPModule Dec? QidList -> IMPState .
  op init : -> System .
  op init'IMP'no'banner : -> System .
  op banner : -> QidList .
  eq banner = '\g 'IMP 'Prototype '\o '
               ''( '\y '\! 'March '2018 '\o '') .
  eq init = [nil, < noModule ; noDec ; nil >, banner] .
  eq init'IMP'no'banner = [nil, < noModule ; noDec ; nil >, nil] .
  vars QIL QIL' QIL' QIL1 QIL2 : QidList .
  rl [version] : ['version, L:LoopState, QIL] =>
                   [nil, L:LoopState, banner] .
   *** Loading a module.
   crl [in] : ['module Q:Qid QIL,
               < M:MetaIMPModule ; D:Dec? ; QIL' >, QIL'] =>
    if (T:ResultPair? :: ResultPair)
    then [nil, < getTerm(T:ResultPair?);</pre>
                 compileMod(getTerm(T:ResultPair?));
                  'IMP: '\b 'Module Q:Qid 'loaded. '\o >, QIL'']
    else [nil, < noModule ; noDec ; nil >,
          printParseError('module Q:Qid QIL, T:ResultPair?)]
    fi
   if T:ResultPair? :=
      metaParse(upModule('IMP-GRAMMAR, false),
       'module Q:Qid QIL, 'ModuleDecl) .
   *** Viewing a module.
```

```
crl [view] : ['view , < M:MetaIMPModule ; D:Dec ; QIL' >, QIL''] =>
             [nil, < M:MetaIMPModule ; D:Dec ; QIL' >,
                          printModule(M:MetaIMPModule)]
 if M:MetaIMPModule =/= noModule .
*** Viewing a module.
crl [python] : ['py , < M:MetaIMPModule ; D:Dec ; QIL' >, QIL''] =>
               [nil, < M:MetaIMPModule ; D:Dec ; QIL' >,
                     dec2Dec1(D:Dec, D:Dec, 0)]
 if M:MetaIMPModule =/= noModule .
*** Executing a command.
crl [exec-unbounded] : [('exec QIL),
                        < M:MetaIMPModule ; D:Dec ; QIL' >, QIL'] =>
 if P:ResultPair? :: ResultPair
 then [nil, < M:MetaIMPModule; D:Dec; QIL' > ,
       if compileCmd(getTerm(P:ResultPair?)) :: Cmd
       then
         'IMP: '\b 'Execution 'result 'for '\c QIL '\o '\b ': '\o
         printExec(getTerm(metaRewrite(upModule('BPLC+META-LEVEL, false),
          'run[upTerm(compileCmd(getTerm(P:ResultPair?))),
               upTerm(D:Dec)], unbounded)))
       else 'IMP: '\r 'Internal 'Error 'while 'compiling '\s QIL
 else [nil, < M:MetaIMPModule ; D:Dec ; QIL' >,
       'IMP: '\r 'Error 'while 'processing 'command '\o 'exec '\s QIL ]
 fi
 if P:ResultPair? :=
   metaParse(upModule('IMP-GRAMMAR, false), QIL, 'CommandDecl) .
*** Model check command
crl [mc] : [('mc QIL1 '|= QIL2),
            < 'module__end[T1:Term, T2:Term] ; D:Dec ; QIL' >, QIL''] =>
 if ((P1:ResultPair? :: ResultPair) and (P2:ResultPair? :: ResultPair))
 then [nil, < 'module_end[T1:Term, T2:Term]; D:Dec; QIL' >,
       if compileCmd(getTerm(P1:ResultPair?)) :: Cmd
       then
       printModelCheckResult(downTerm(
         getTerm(metaReduce(
          makeMCModule(printTokens(T1:Term), D:Dec),
           '_',_|=?_[upTerm(D:Dec),
          upTerm(compileCmd(getTerm(P1:ResultPair?))),
           getTerm(P2:ResultPair?)])),true), QIL1 '\s '|= '\s QIL2)
       else 'IMP: '\r 'Internal 'Error 'while
            'processing 'command '\o 'mc QIL1 ' |= QIL2
```

```
fi ]
  else [nil, < 'module__end[T1:Term, T2:Term] ; D:Dec ; QIL' >,
               'IMP: '\r 'Error 'while 'processing 'command '\o
               'mc QIL1 '|= QIL2]
 fi
 if P1:ResultPair? :=
       metaParse(upModule('IMP-GRAMMAR, false), QIL1, 'CommandDecl) /\
   P2:ResultPair? :=
      metaParse(makeMCModule(printTokens(T1:Term), D:Dec), QIL2, 'Formula) .
*** Output
crl [out] : [nil, < M:MetaIMPModule ; D:Dec? ; QIL' > , QIL'] =>
             [nil, < M:MetaIMPModule ; D:Dec? ; nil > , QIL']
 if QIL' =/= \min .
*** Command error
 crl [com-error] : [Q:Qid QIL, < M:MetaIMPModule ; D:Dec ; QIL' > , QIL'] =>
     [nil, < M:MetaIMPModule ; D:Dec ; QIL' > ,
      'IMP: '\r 'Error 'no 'such 'command '\o Q:Qid QIL]
  if (Q:Qid =/= 'module) /\ (Q:Qid =/= 'view) /\
     (Q:Qid =/= 'exec) /\ (Q:Qid =/= 'mc) /\ (Q:Qid =/= 'py) .
{\tt endm}
loop init .
```