

# Notes on Formal Compiler Construction with the II Framework

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<http://github.com/ChristianoBraga/PiFramework>



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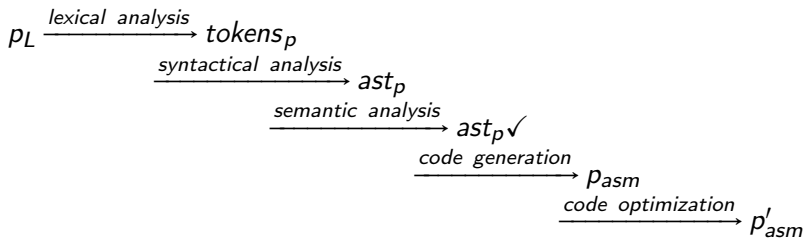
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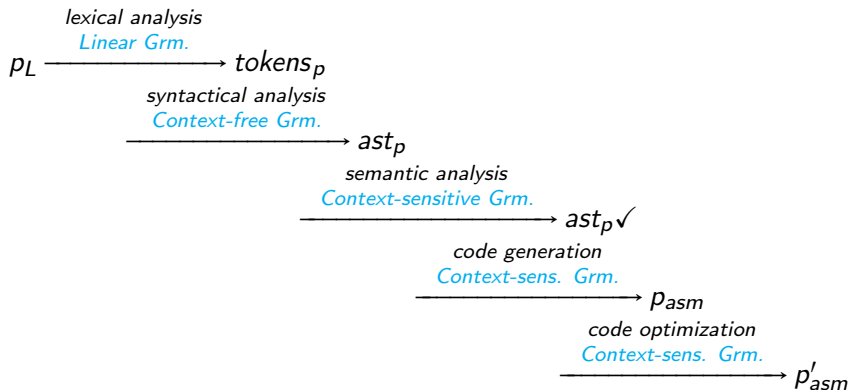
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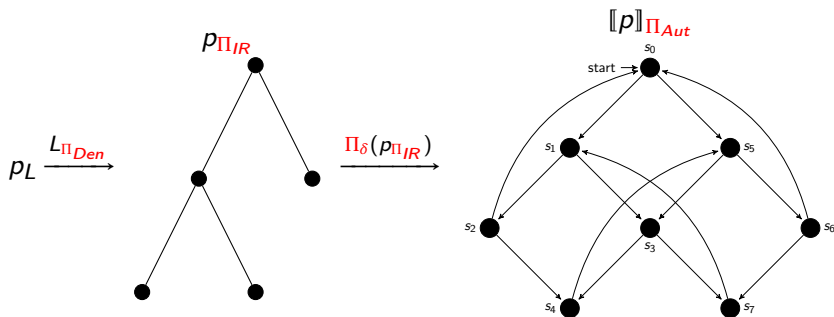
# Compiler pipeline



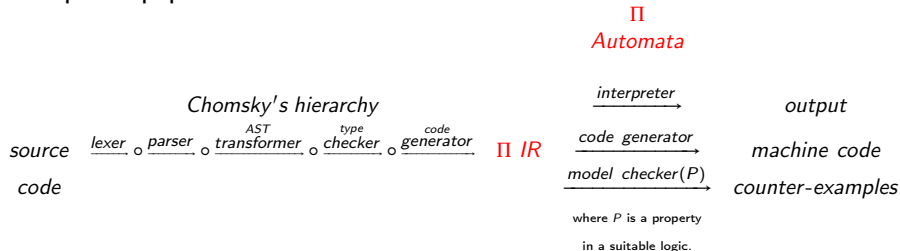
# Compiler pipeline and formal languages



# Compiler pipeline with the $\Pi$ Framework I



# Compiler pipeline with the $\Pi$ Framework II



- $\Pi$  IR defines a set of constructions common to many programming languages.
- $\Pi$  IR constructions have a formal automata-based semantics in  $\Pi$  automata.
- One may execute (or validate) a program in a given language by running its associated  $\Pi$  IR program.

# Compiler pipeline with the $\Pi$ Framework III

- $\Pi$  Framework:  
<http://github.com/ChristianoBraga/PiFramework>
- Notes on Formal Compiler Construction with the  $\Pi$  Framework:  
<https://github.com/ChristianoBraga/PiFramework/blob/master/notes/notes.pdf>.

# A calculator

We wish to compute simple arithmetic expressions such as  $5 * (3 + 2)$ .



# A calculator: Lexer

$\langle \textit{digit} \rangle \quad ::= [0..9]$

$\langle \textit{digits} \rangle \quad ::= \langle \textit{digit} \rangle^+$

$\langle \textit{boolean} \rangle \quad ::= \text{'true'} \mid \text{'false'}$

# A calculator: concrete syntax

$\langle \text{exp} \rangle ::= \langle \text{aexp} \rangle \mid \langle \text{bexp} \rangle$

$\langle \text{aexp} \rangle ::= \langle \text{aexp} \rangle '+' \langle \text{term} \rangle \mid \langle \text{aexp} \rangle '-' \langle \text{term} \rangle \mid \langle \text{term} \rangle$

$\langle \text{term} \rangle ::= \langle \text{term} \rangle '*' \langle \text{factor} \rangle \mid \langle \text{term} \rangle '/' \langle \text{factor} \rangle \mid \langle \text{factor} \rangle$

$\langle \text{factor} \rangle ::= '(' \langle \text{aexp} \rangle ')' \mid \langle \text{digits} \rangle$

$\langle \text{bexp} \rangle ::= \langle \text{boolean} \rangle \mid '\sim' \langle \text{bexp} \rangle \mid \langle \text{bexp} \rangle \langle \text{boolop} \rangle \langle \text{bexp} \rangle$   
 $\mid \langle \text{aexp} \rangle \langle \text{iop} \rangle \langle \text{aexp} \rangle$

$\langle \text{boolop} \rangle ::= '=' \mid '/\backslash' \mid '\backslash/'$

$\langle \text{iop} \rangle ::= '<' \mid '>' \mid '<=' \mid '>='$

# A calculator: abstract syntax

$$\langle exp \rangle ::= \langle digits \rangle \mid \langle boolean \rangle \mid \langle exp \rangle \langle bop \rangle \langle exp \rangle$$
$$\langle bop \rangle ::= '+' \mid '-' \mid '*' \mid '/' \mid '=' \mid '\backslash' \mid '\wedge' \mid '<' \mid '>' \mid '<=' \mid '>='$$

# A calculator: $\Pi$ denotations I

Let  $D$  in  $\langle \text{digits} \rangle$ ,  $B$  in  $\langle \text{boolean} \rangle$  and  $E_1, E_2$  in  $\langle \text{exp} \rangle$ ,

$$\llbracket D \rrbracket_{\Pi} = \text{Num}(D) \quad (1)$$

$$\llbracket B \rrbracket_{\Pi} = \text{Boo}(B) \quad (2)$$

$$\llbracket E_1 + E_2 \rrbracket_{\Pi} = \text{Sum}(\llbracket E_1 \rrbracket_{\Pi}, \llbracket E_2 \rrbracket_{\Pi}) \quad (3)$$

$$\llbracket E_1 - E_2 \rrbracket_{\Pi} = \text{Sub}(\llbracket E_1 \rrbracket_{\Pi}, \llbracket E_2 \rrbracket_{\Pi}) \quad (4)$$

$$\llbracket E_1 * E_2 \rrbracket_{\Pi} = \text{Mul}(\llbracket E_1 \rrbracket_{\Pi}, \llbracket E_2 \rrbracket_{\Pi}) \quad (5)$$

$$\llbracket E_1 / E_2 \rrbracket_{\Pi} = \text{Div}(\llbracket E_1 \rrbracket_{\Pi}, \llbracket E_2 \rrbracket_{\Pi}) \quad (6)$$

$$\llbracket E_1 < E_2 \rrbracket_{\Pi} = \text{Lt}(\llbracket E_1 \rrbracket_{\Pi}, \llbracket E_2 \rrbracket_{\Pi}) \quad (7)$$

$$\llbracket E_1 \leq E_2 \rrbracket_{\Pi} = \text{Le}(\llbracket E_1 \rrbracket_{\Pi}, \llbracket E_2 \rrbracket_{\Pi}) \quad (8)$$

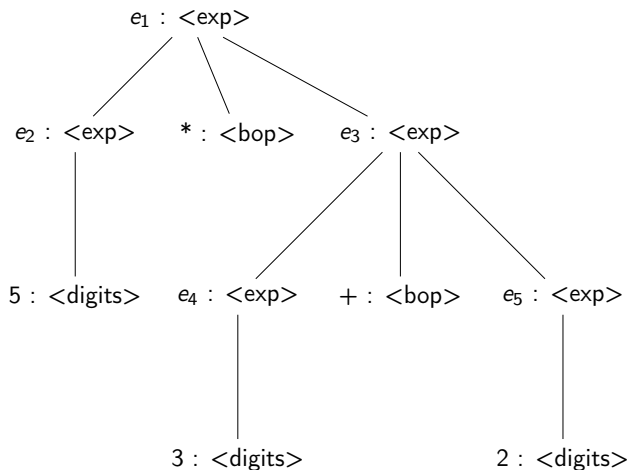
$$\llbracket E_1 > E_2 \rrbracket_{\Pi} = \text{Gt}(\llbracket E_1 \rrbracket_{\Pi}, \llbracket E_2 \rrbracket_{\Pi}) \quad (9)$$

$$\llbracket E_1 \geq E_2 \rrbracket_{\Pi} = \text{Ge}(\llbracket E_1 \rrbracket_{\Pi}, \llbracket E_2 \rrbracket_{\Pi}) \quad (10)$$

# A calculator: $\Pi$ denotations II

- $\Pi$  denotations are *functions*  $\llbracket \cdot \rrbracket_{\Pi} : AST \rightarrow \Pi \text{ IR}$ , where *AST* denotes the *datatype* for the abstract syntax tree and  $\Pi \text{ IR}$  denotes the datatype for  $\Pi \text{ IR}$  programs.
- Note that  $\llbracket \cdot \rrbracket_{\Pi}$  has *trees* as parameters, instances of *AST*. The example expression  $5 * (3 + 2)$  becomes

# A calculator: $\Pi$ denotations III



# A calculator: $\Pi$ denotations IV

$$\llbracket 5 * (3 + 2) \rrbracket_{\Pi} = \text{Mul}(\llbracket 5 \rrbracket_{\Pi}, \llbracket (3 + 2) \rrbracket_{\Pi}) \quad \text{by Equation 5}$$

$$\text{Mul}(\llbracket 5 \rrbracket_{\Pi}, \llbracket (3 + 2) \rrbracket_{\Pi}) = \text{Mul}(\text{Num}(5), \llbracket (3 + 2) \rrbracket_{\Pi}) \quad \text{by Equations 1}$$

$$\text{Mul}(\text{num}(5), \llbracket (3 + 2) \rrbracket_{\Pi}) = \text{Mul}(\text{Num}(5), \text{Sum}(\llbracket 3 \rrbracket_{\Pi}, \llbracket 2 \rrbracket_{\Pi})) \quad \text{by Equation 3}$$

$$\text{Mul}(\text{Num}(5), \text{Sum}(\llbracket 3 \rrbracket_{\Pi}, \llbracket 2 \rrbracket_{\Pi})) = \text{Mul}(\text{Num}(5), \text{Sum}(\text{Num}(3), \llbracket 2 \rrbracket_{\Pi})) \quad \text{by Equation 1}$$

$$\text{Mul}(\text{Num}(5), \text{Sum}(\text{Num}(3), \llbracket 2 \rrbracket_{\Pi})) = \text{Mul}(\text{Num}(5), \text{Sum}(\text{Num}(3), \text{Num}(2))) \quad \text{by Equation 1}$$

# A calculator: executing $\Pi$ IR with $\Pi$ automata

A  $\Pi$  automaton is a 5-tuple  $\mathcal{A} = (G, Q, \delta, q_0, F)$ , where  $G$  is a context-free grammar,  $Q$  is the set of states,  $q_0$  is the initial state,  $F \subseteq Q$  is the set of final states and

$$\delta : L(G)^* \times L(G)^* \times Store \rightarrow Q,$$

where  $L(G)$  is the language generated by  $G$  and  $Store$  represents the memory. (Elements in a set  $S^*$  are represented by terms  $[s_1, s_2, \dots, s_n]$ .)

$$\begin{aligned} \delta([Mul(Num(5), Sum(Num(3), Num(2))), \emptyset, \emptyset) &= \delta([Num(5), Sum(Num(3), Num(2)), \#MUL], \emptyset, \emptyset) \\ \delta([Num(5), Sum(Num(3), Num(2)), \#MUL], \emptyset, \emptyset) &= \delta([Sum(Num(3), Num(2)), \#MUL], [Num(5)], \emptyset) \\ \delta([Sum(Num(3), Num(2)), \#MUL], [Num(5)], \emptyset) &= \delta([Num(3), Num(2), \#SUM, \#MUL], [Num(5)], \emptyset) \\ \delta([Num(3), Num(2), \#SUM, \#MUL], [Num(5)], \emptyset) &= \delta([Num(2), \#SUM, \#MUL], [Num(3), Num(5)], \emptyset) \\ \delta([Num(2), \#SUM, \#MUL], [Num(3), Num(5)], \emptyset) &= \delta([\#SUM, \#MUL], [Num(2), Num(3), Num(5)], \emptyset) \\ \delta([\#SUM, \#MUL], [Num(2), Num(3), Num(5)], \emptyset) &= \delta([\#MUL], [Num(5), Num(5)], \emptyset) \\ \delta([\#MUL], [Num(5), Num(5)], \emptyset) &= \delta(\emptyset, [Num(25)], \emptyset) \\ \delta(\emptyset, [Num(25)], \emptyset) &= Num(25) \end{aligned}$$



Excerpt of  $\Pi$  IR expressions

$\langle Statement \rangle ::= \langle Exp \rangle$

$\langle Exp \rangle ::= \langle ArithExp \rangle \mid \langle BoolExp \rangle \langle Exp \rangle$

$\langle ArithExp \rangle ::= 'Num'(\langle digits \rangle) \mid 'Sum'(\langle Exp \rangle, \langle Exp \rangle) \mid$   
 $'Sub'(\langle Exp \rangle, \langle Exp \rangle) \mid 'Mul'(\langle Exp \rangle, \langle Exp \rangle)$

$\langle BoolExp \rangle ::= 'Eq'(\langle Exp \rangle, \langle Exp \rangle) \mid 'Not'(\langle Exp \rangle)$

# $\Pi$ automaton semantics for $\Pi$ IR expressions I

- Recall that  $\delta : L(G)^* \times L(G)^* \times Store \rightarrow Q$ , and let  $N, N_i \in \mathbb{N}$ ,  $C, V \in L(G)^*$ ,  $S \in Store$ ,

$$\delta(Num(N) :: C, V, S) = \delta(C, Num(N) :: V, S) \quad (11)$$

$$\delta(Sum(E_1, E_2) :: C, V, S) = \delta(E_1 :: E_2 :: \#SUM :: C, V, S) \quad (12)$$

$$\delta(\#SUM :: C, Num(N_1) :: Num(N_2) :: V, S) = \delta(C, (N_1 + N_2) :: V, S) \quad (13)$$

...

$$\delta(Not(E) :: C, V, S) = \delta(E :: \#NOT :: C, V, S) \quad (14)$$

$$\delta(\#NOT :: C, Boo(true) :: V, S) = \delta(C, Boo(false) :: V, S) \quad (15)$$

$$\delta(\#NOT :: C, Boo(false) :: V, S) = \delta(C, Boo(true) :: V, S) \quad (16)$$

- Notation  $h :: ls$  denotes the concatenation of element  $h$  with the list  $ls$ .
- $C$  represents the *control* stack.  $V$  represents the *value* stack.  $S$  denotes the memory store.
- $\delta(\emptyset, V, S)$  denotes an *accepting state*.

## $\Pi$ automaton semantics for $\Pi$ IR expressions II

- On a particular implementation of the  $\Pi$  Framework, as in Python, `<digits>` denote built-in numbers in implementation language, such that all arithmetic operations, such as `+`, are defined. That's why  $N_i$  are in  $\mathbb{N}$ .

## $\Pi$ IR commands

- Commands are language constructions that require a *memory* store to be evaluated.

$\langle \text{Statement} \rangle ::= \langle \text{Cmd} \rangle$

$\langle \text{Exp} \rangle ::= \text{'Id'}(\langle \text{String} \rangle)$

$\langle \text{Cmd} \rangle ::= \text{'Assign'}(\langle \text{Id} \rangle, \langle \text{Exp} \rangle)$   
|  $\text{'Loop'}(\langle \text{BoolExp} \rangle, \langle \text{Cmd} \rangle)$   
|  $\text{'CSeq'}(\langle \text{Cmd} \rangle, \langle \text{Cmd} \rangle)$

# $\Pi$ automaton semantics for $\Pi$ IR commands I

- A location  $l \in Loc$  denotes a memory cell.
- Storable and Bindable sets denote the data that may be mapped to by identifiers and locations on the memory and environment respectively.
- $Store = Id \mapsto Storable$ ,  $Env = Loc \mapsto Bindable$ ,  $Loc \subseteq Store$ ,  $\mathbb{N} \subseteq Loc, Bindable$ .
- Now the transition function is  $\delta : L(G)^* \times L(G)^* \times Env \times Store \rightarrow Q$ , and let  $W \in String$ ,  $C, V \in L(G)^*$ ,  $S \in Store$ ,  $E \in Env$ ,  $B \in Bindable$ ,  $l \in Loc$ ,  $T \in Storable$ ,  $X \in \langle Exp \rangle$ ,  $M, M_1, M_2 \in \langle Cmd \rangle$ , and

# $\Pi$ automaton semantics for $\Pi$ IR commands II

expression  $S' = S/[I \mapsto N]$  means that  $S'$  equals to  $S$  in all indices but  $I$  that is bound to  $N$ ,

$$\delta(Id(W) :: C, V, E, S) = \delta(C, B :: V, E, S), \quad (17)$$

where  $E[W] = I$  and  $S[I] = B$ ,

$$\delta(Assign(W, X) :: C, V, E, S) = \delta(X :: \#ASSIGN :: C, W :: V, E, S'), \quad (18)$$

$$\delta(\#ASSIGN :: C, T :: W :: V, E, S) = \delta(C, V, E, S'), \quad (19)$$

where  $E[W] = I$  and  $S' = S/[I \mapsto T]$ ,

$$\delta(Loop(X, M) :: C, V, E, S) = \delta(X :: \#LOOP :: C, Loop(X, M) :: V, E, S), \quad (20)$$

$$\delta(\#LOOP :: C, Boo(true) :: Loop(X, M) :: V, E, S) = \delta(M :: Loop(X, M) :: C, V, E, S), \quad (21)$$

$$\delta(\#LOOP :: C, Boo(false) :: Loop(X, M) :: V, E, S) = \delta(C, V, E, S), \quad (22)$$

$$\delta(CSeq(M_1, M_2) :: C, V, E, S) = \delta(M_1 :: M_2 :: C, V, E, S). \quad (23)$$

## $\Pi$ IR declarations

- Declarations are statements that create an environment, binding identifiers to (bindable) values.
- In  $\Pi$  IR, a bindable value is either a Boolean value, an integer or a location.
- From a syntactic standpoint, all classes are monotonically extended.

$\langle \text{Statement} \rangle ::= \langle \text{Dec} \rangle$

$\langle \text{Exp} \rangle ::= \text{'Ref'}(\langle \text{Exp} \rangle) \mid \text{'DeRef'}(\langle \text{Id} \rangle) \mid \text{'ValRef'}(\langle \text{Id} \rangle)$

$\langle \text{Dec} \rangle ::= \text{'Bind'}(\langle \text{Id} \rangle, \langle \text{Exp} \rangle) \mid \text{'DSeq'}(\langle \text{Dec} \rangle, \langle \text{Dec} \rangle)$

$\langle \text{Cmd} \rangle ::= \text{'Blk'}(\langle \text{Dec} \rangle, \langle \text{Cmd} \rangle)$

## $\Pi$ automaton semantics for $\Pi$ IR declarations I

Let  $BlockLocs = Set\{Loc\}$ , now the transition function is  $\delta : L(G)^* \times L(G)^* \times Env \times Store \times BlockLocs \rightarrow Q$ , and let  $L, L' \in BlockLocs$ ,  $Loc \subseteq Storable$ , and  $S/L$  means the store  $S$  without the locations in  $L$ ,



$\Pi$  automaton semantics for  $\Pi$  IR declarations II

$$\delta(\text{Ref}(X) :: C, V, E, S, L) = \delta(X :: \#REF :: C, V, E, S, L), \quad (24)$$

$$\delta(\#REF :: C, T :: V, E, S, L) = \delta(C, I :: V, E, S', L'), \text{ where } S' = S \cup [I \mapsto T], I \notin S, L' = L \cup \{I\}, \quad (25)$$

$$\delta(\text{DeRef}(\text{Id}(W)) :: C, V, E, S, L) = \delta(C, I :: V, E, S, L), \text{ where } I = E[W], \quad (26)$$

$$\delta(\text{ValRef}(\text{Id}(W)) :: C, V, E, S, L) = \delta(C, T :: V, E, S, L), \text{ where } T = S[S[E[W]]], \quad (27)$$

$$\delta(\text{Bind}(\text{Id}(W), X) :: C, V, E, S, L) = \delta(X :: \#BIND :: C, W :: V, E, S, L), \quad (28)$$

$$\delta(\#BIND :: C, B :: W :: E' :: V, E, S, L) = \delta(C, ([W \mapsto B] \cup E') :: V, E, S, L), \text{ where } E' \in \text{Env}, \quad (29)$$

$$\delta(\#BIND :: C, B :: W :: H :: V, E, S, L) = \delta(C, [W \mapsto B] :: H :: V, E, S, L), \text{ where } H \notin \text{Env}, \quad (30)$$

$$\delta(\text{DSeq}(D_1, D_2), X) :: C, V, E, S, L) = \delta(D_1 :: D_2 :: C, V, E, S, L), \quad (31)$$

$$\delta(\text{Blk}(D, M) :: C, V, E, S, L) = \delta(D :: \#BLKDEC :: M :: \#BLKCMD :: C, L :: V, E, S, \emptyset), \quad (32)$$

$$\delta(\#BLKDEC :: C, E' :: V, E, S, L) = \delta(C, E :: V, E/E', S, L), \quad (33)$$

$$\delta(\#BLKCMD :: C, E :: L :: V, E', S, L') = \delta(C, V, E, S', L), \text{ where } S' = S/L. \quad (34)$$

## $\Pi$ IR abstractions

- Abstractions extend Bindables by allowing a name to be bound to a list of formal parameters, a list of identifiers, and a block in the environment.
- Such names can be called and applied to actual parameters, a list of expressions.

$\langle Dec \rangle \quad ::= \text{'Bind'}(\langle Id \rangle, \langle Abs \rangle)$

$\langle Abs \rangle \quad ::= \text{'Abs'}(\langle Formals \rangle, \langle Blk \rangle)$

$\langle Formals \rangle \quad ::= \langle Id \rangle^*$

$\langle Cmd \rangle \quad ::= \text{'Call'}(\langle Id \rangle, \langle Actuals \rangle)$

$\langle Actuals \rangle \quad ::= \langle Exp \rangle^*$

# $\Pi$ automaton semantics for $\Pi$ IR abstractions — Closures I

We chose a static binding semantics for abstractions. Therefore, we interpret abstractions as *closures* formed by an abstraction together with its declaration environment which defines the context in which the abstraction will be evaluated.

$$\textit{Closure} : \textit{Formals} \times \textit{Blk} \times \textit{Env} \rightarrow \textit{Bindable}$$

# $\Pi$ automaton semantics for $\Pi$ IR abstractions — Example I

```
1 |m\Pi source code:
2 |# In this example we encapsulate the iterative calculation
3 |# of the factorial within a function call.
4 |let var z = 1
5 |in
6 |    let fn f(x) =
7 |        let var y = x
8 |        in
9 |            while not (y == 0)
10 |            do
11 |                z := z * y
12 |                y := y - 1
13 |in f(10)
```

# $\Pi$ automaton semantics for $\Pi$ IR abstractions — Example II

```
1  $\Pi$  IR AST:  
2 Blk(Bind(Id(z), Ref(Num(1))),  
3   Blk(BindAbs(Id(f), Abs(Id(x),  
4     Blk(Bind(Id(y), Ref(Id(x))),  
5       Loop(Not(Eq(Id(y), Num(0))),  
6         CSeq(Assign(Id(z), Mul(Id(z), Id(y))),  
7           Assign(Id(y), Sub(Id(y), Num(1)))))))))  
8   Call(Id(f), Num(10)))
```

# $\Pi$ automaton semantics for $\Pi$ IR abstractions I

Let  $F \in \text{Formals}$ ,  $B \in \text{Blk}$ ,  $I \in \text{Id}$ ,  $A \in \text{Actuals}$ ,  $V_i \in \text{Value}$ ,  $1 \leq i \leq n$ ,  $n \in \mathbb{N}$ ,

$$\delta(\text{Abs}(F, B) :: C, V, E, S, L) = \delta(C, \text{Closure}(F, B, E) :: V, E, S, L) \quad (35)$$

$$\delta(\text{Call}(I, [X_1, X_2, \dots, X_n])) :: C, V, E, S, L) = \quad (36)$$

$$\delta(X_n :: X_{n-1} :: \dots :: X_1 :: \# \text{CALL}(I, n) :: C, V, E, S, L)$$

$$\delta(\# \text{CALL}(I, n) :: C, [V_1, V_2, \dots, V_n] :: V, [I \mapsto \text{Closure}(F, B, E_1)] E_2, S, L) = \quad (37)$$

$$\delta(B :: \# \text{BLKCMD} :: C, E_2 :: V, (E_1 / \text{match}(F, [V_1, V_2, \dots, V_n])), S, L)$$

$$\text{match} : \text{Id}^* \times \text{Values}^* \rightarrow \text{Env}$$

$$\text{match}(fl, al) = \text{if } |fl| \neq |al| \text{ then } \{\} \text{ else } \_ \text{match}(fl, al, \{\})$$

$$\_ \text{match} : \text{Id}^* \times \text{Values}^* \times \text{Env} \rightarrow \text{Env}$$

$$\_ \text{match}(\[], \[], E) = E$$

$$\_ \text{match}(f, a, E) = \{f \mapsto a\} E$$

$$\_ \text{match}(f :: fl, a :: al, E) = \_ \text{match}(fl), al, \{f \mapsto a\} E$$

## $\Pi$ IR recursive abstractions

- Abstractions can be recursive to allow for the declaration of recursive functions.

$\langle Dec \rangle \quad ::= \text{'Rbnd'}(\langle Id \rangle, \langle Abs \rangle)$

# $\Pi$ automaton semantics for $\Pi$ IR recursive abstractions — Recursive closures I

In the context of static binding semantics for abstractions, in a call to a recursive function, the evaluation of identifiers needs to be reminded about the binding of the function name to a closure.

$$Rec : Formals \times Blk \times Env \times Env \rightarrow Bindable$$
$$unfold : Env \rightarrow Env$$
$$reclose_E : Env \rightarrow Env$$



# $\Pi$ automaton semantics for $\Pi$ IR recursive abstractions — Recursive closures II

$$\text{unfold}(E) = \text{reclose}_E(E) \quad (38)$$

$$\text{reclose}_E(I \mapsto \text{Closure}(F, B, E')) = (I \mapsto \text{Rec}(F, B, E', E)) \quad (39)$$

$$\text{reclose}_E(I \mapsto \text{Rec}(F, B, E', E'')) = (I \mapsto \text{Rec}(F, B, E', E)) \quad (40)$$

$$\text{reclose}_E(I \mapsto v) = (I \mapsto v) \text{ if } v \neq \text{Closure}(F, B, E) \quad (41)$$

$$\text{reclose}_E(E_1 \cup E_2) = \text{reclose}_E(E_1) \cup \text{reclose}_E(E_2) \quad (42)$$

$$\text{reclose}_E(\emptyset) = \emptyset \quad (43)$$

# $\Pi$ automaton semantics for $\Pi$ IR recursive abstractions — Recursive closures I

$$\delta(Rbnd(I, Abs(F, B)) :: C, V, E, S, L) =$$

$$\delta(C, unfold(I \mapsto Closure(F, B, E)) :: V, E, S, L) \quad (44)$$

$$\delta(\#CALL(I, n) :: C, [V_1, V_2, \dots, V_n] :: V, E, S, L) = \quad (45)$$

$$\delta(B :: \#BLKCMD :: C, E :: V, E', S, L)$$

$$\text{where } E = \{I \mapsto Rec(F, B, E_1, E_2)\} \cup E_3 \quad (46)$$

$$E' = E_1 / unfold(E_2) / match(F, [V_1, V_2, \dots, V_n]) \quad (47)$$

# $\Pi$ IR expressions in Python I

```
1 class Statement:
2     def __init__(self, *args):
3         self.opr = args
4     def __str__(self):
5         ret =str(self.__class__.__name__)+ "("
6         for o in self.opr:
7             ret +=str(o)
8         ret +=")"
9         return ret
10 class Exp(Statement): pass
11 class ArithExp(Exp): pass
```

## $\Pi$ IR expressions in Python II

```
1 class Num(ArithExp):
2     def __init__(self, f):
3         assert(isinstance(f, int))
4         ArithExp.__init__(self, f)
5 class Sum(ArithExp):
6     def __init__(self, e1, e2):
7         assert(isinstance(e1, Exp) and isinstance(e2, Exp))
8         ArithExp.__init__(self, e1, e2)
9 ...
```

## $\Pi$ IR expressions in Python III

```
1 class BoolExp(Exp): pass
2 class Eq(BoolExp):
3     def __init__(self, e1, e2):
4         assert(isinstance(e1, Exp) and isinstance(e2, Exp))
5         BoolExp.__init__(self, e1, e2)
6 ...
```

## $\Pi$ IR expressions in Python IV

```
1 exp = Sum(Num(1), Mul(Num(2), Num(4)))
2 print(exp)
3
4 Sum(Num(1)Mul(Num(2)Num(4)))
```

## $\Pi$ IR expressions in Python V

```
1 exp2 =Mul(2, 1)
2 -----
3
4
5
6
7
8
9
10
11
12
13
14
```

AssertionError Traceback (most recent call last)

<ipython-input-7-00fd40a79a54> in <module>()

---->1 exp2 =Mul(2, 1)

<ipython-input-5-42a82e58862f> in \_\_init\_\_(self, e1, e2)

```
28 class Mul(ArithExp):
29 def __init__(self, e1, e2):
30 assert(isinstance(e1, Exp) and isinstance(e2, Exp))
31 ArithExp.__init__(self, e1, e2)
32 class BoolExp(Exp): pass
```

AssertionError:

# $\Pi$ automaton for $\Pi$ IR expressions I

```
1  ## Expressions
2  class ValueStack(list): pass
3  class ControlStack(list): pass
4  class ExpKW:
5      SUM = "#SUM"
6      SUB = "#SUB"
7      MUL = "#MUL"
8      EQ = "#EQ"
9      NOT = "#NOT"
```



## $\Pi$ automaton for $\Pi$ IR expressions II

```
1 class ExpPiAut(dict):
2     def __init__(self):
3         self["val"] = ValueStack()
4         self["cnt"] = ControlStack()
5     def __evalSum(self, e):
6         e1 = e.opr[0]
7         e2 = e.opr[1]
8         self.pushCnt(ExpKW.SUM)
9         self.pushCnt(e1)
10        self.pushCnt(e2)
11    def pushCnt(self, e):
12        cnt = self.cnt()
13        cnt.append(e)
14    ...
```

## $\Pi$ automaton for $\Pi$ IR expressions III

```
1 ea =ExpPiAut()  
2 print(exp)  
3 ea.pushCnt(exp)  
4 while not ea.emptyCnt():  
5     ea.eval()  
6     print(ea)
```

# $\Pi$ automaton for $\Pi$ IR expressions IV

```
1 Sum(Num(1)Mul(Num(2)Num(4)))
2 {'val': [], 'cnt': ['#SUM', <__main__.Num object at 0x111851470>, <
    __main__.Mul object at 0x1118516d8>]}
3 {'val': [], 'cnt': ['#SUM', <__main__.Num object at 0x111851470>, '#MUL',
    <__main__.Num object at 0x111851630>, <__main__.Num object
    at 0x1118516a0>]}
4 {'val': [4], 'cnt': ['#SUM', <__main__.Num object at 0x111851470>, '#MUL',
    <__main__.Num object at 0x111851630>]}
5 {'val': [4, 2], 'cnt': ['#SUM', <__main__.Num object at 0x111851470>, '#MUL',
    <__main__.Num object at 0x111851630>]}
6 {'val': [8], 'cnt': ['#SUM', <__main__.Num object at 0x111851470>]}
7 {'val': [8, 1], 'cnt': ['#SUM']}
8 {'val': [9], 'cnt': []}
```

## $\Pi$ IR commands I

```
1 class Cmd(Statement): pass
2 class Id(Exp):
3     def __init__(self, s):
4         assert(isinstance(s, str))
5         Exp.__init__(self, s)
6 class Assign(Cmd):
7     def __init__(self, i, e):
8         assert(isinstance(i, Id) and isinstance(e, Exp))
9         Cmd.__init__(self, i, e)
10 class Loop(Cmd):
11     def __init__(self, be, c):
12         assert(isinstance(be, BoolExp) and isinstance(c, Cmd))
13         Cmd.__init__(self, be, c)
14 class CSeq(Cmd):
15     def __init__(self, c1, c2):
16         assert(isinstance(c1, Cmd) and isinstance(c2, Cmd))
17         Cmd.__init__(self, c1, c2)
```

## $\Pi$ IR commands II

```
1 cmd =Assign(Id("x"), Num(1))
2 print(type(cmd))
3 print(cmd)
4 <class '__main__.Assign'>
5 Assign(Id(x)Num(1))
```

# $\Pi$ automaton for $\Pi$ IR commands I

Environment, Location, Store and commands opcodes.

```
1 ## Commands
2 class Env(dict): pass
3 class Loc(int): pass
4 class Sto(dict): pass
5 class CmdKW:
6     ASSIGN = "#ASSIGN"
7     LOOP = "#LOOP"
```

## $\Pi$ automaton for $\Pi$ IR commands II

$\Pi$  automaton for commands extends the  $\Pi$  automaton for expressions.

```
1 class CmdPiAut(ExpPiAut):
2     def __init__(self):
3         self["env"] = Env()
4         self["sto"] = Sto()
5         ExpPiAut.__init__(self)
6     def env(self):
7         return self["env"]
8     def getLoc(self, i):
9         en = self.env()
10        return en[i]
11    def sto(self):
12        return self["sto"]
13    def updateStore(self, l, v):
14        st = self.sto()
15        st[l] = v
```

## $\Pi$ automaton for $\Pi$ IR commands III

$\Pi$  semantics for assignment.

$$\begin{aligned}\delta(\text{Assign}(W, X) :: C, V, E, S) &= \delta(X :: \#ASSIGN :: C, W :: V, E, S'), \\ \delta(\#ASSIGN :: C, T :: W :: V, E, S) &= \delta(C, V, E, S'), \\ \text{where } E[W] &= I \text{ and } S' = S/[I \mapsto T].\end{aligned}$$

```
1 def __evalAssign(self, c):
2     i = c.opr[0]
3     e = c.opr[1]
4     self.pushVal(i.opr[0])
5     self.pushCnt(CmdKW.ASSIGN)
6     self.pushCnt(e)
7 def __evalAssignKW(self):
8     v = self.popVal()
9     i = self.popVal()
10    l = self.getLoc(i)
11    self.updateStore(l, v)
```



## $\Pi$ automaton for $\Pi$ IR commands IV

$\Pi$  semantics for identifiers.

$$\delta(Id(W) :: C, V, E, S) = \delta(C, B :: V, E, S),$$

where  $E[W] = I$  and  $S[I] = B$ .

```
1  def __evalId(self, i):  
2      s =self.sto()  
3      l =self.getLoc(i)  
4      self.pushVal(s[l])
```

## $\Pi$ automaton for $\Pi$ IR commands V

$\Pi$  semantics for loop: recursive step.

$$\delta(\text{Loop}(X, M) :: C, V, E, S) = \delta(X :: \#LOOP :: C, \text{Loop}(X, M) :: V, E, S).$$

```
1 def __evalLoop(self, c):
2     be = c.opr[0]
3     bl = c.opr[1]
4     self.pushVal(Loop(be, bl))
5     self.pushVal(bl)
6     self.pushCnt(CmdKW.LOOP)
7     self.pushCnt(be)
```

## $\Pi$ automaton for $\Pi$ IR commands VI

$\Pi$  semantics for loop: basic steps.

$$\delta(\#LOOP :: C, \text{Boo}(\text{true}) :: \text{Loop}(X, M) :: V, E, S) = \delta(M :: \text{Loop}(X, M) :: C, V, E, S),$$
$$\delta(\#LOOP :: C, \text{Boo}(\text{false}) :: \text{Loop}(X, M) :: V, E, S) = \delta(C, V, E, S).$$

```
1 def __evalLoopKW(self):
2     t =self.popVal()
3     if t:
4         c =self.popVal()
5         lo =self.popVal()
6         self.pushCnt(lo)
7         self.pushCnt(c)
8     else:
9         self.popVal()
10        self.popVal()
```

## $\Pi$ automaton for $\Pi$ IR commands VII

$\Pi$  semantics for command composition.

$$\delta(CSeq(M_1, M_2) :: C, V, E, S) = \delta(M_1 :: M_2 :: C, V, E, S).$$

```
1  def __evalCSeq(self, c):  
2      c1 = c.opr[0]  
3      c2 = c.opr[1]  
4      self.pushCnt(c2)  
5      self.pushCnt(c1)
```

## $\Pi$ automaton for $\Pi$ IR commands VIII

Commands are now on the top of the food chain.

```
1 def eval(self):
2     c =self.popCnt()
3     if isinstance(c, Assign):
4         self.__evalAssign(c)
5     elif c ==CmdKW.ASSIGN:
6         self.__evalAssignKW()
7     elif isinstance(c, Id):
8         self.__evalId(c.opr[0])
9     elif isinstance(c, Loop):
10        self.__evalLoop(c)
11    elif c ==CmdKW.LOOP:
12        self.__evalLoopKW()
13    elif isinstance(c, CSeq):
14        self.__evalCSeq(c)
15    else:
16        self.pushCnt(c)
17        ExpPiAut.eval(self)
```

# $\Pi$ IR declarations in Python I

DeRef and ValRef not implemented yet.

```
1 ## Declarations
2 class Dec(Statement): pass
3 class Bind(Dec):
4     def __init__(self, i, e):
5         assert (isinstance(i, Id) and isinstance(e, Exp))
6         Dec.__init__(self, i, e)
7 class Ref(Exp):
8     def __init__(self, e):
9         assert (isinstance(e, Exp))
10        Exp.__init__(self, e)
11 class Blk(Cmd):
12     def __init__(self, d, c):
13         assert (isinstance(d, Dec) and isinstance(c, Cmd))
14        Cmd.__init__(self, d, c)
```

## $\Pi$ IR declarations in Python II

```
1 class DSeq(Dec):  
2     def __init__(self, d1, d2):  
3         assert (isinstance(d1, Dec) and isinstance(d2, Dec))  
4         Dec.__init__(self, d1, d2)
```

## $\Pi$ IR declarations in Python III

```
1  ## Declarations
2  class DecExpKW(ExpKW):
3      REF = "#REF"
4
5  class DecCmdKW(CmdKW):
6      BLKDEC = "#BLKDEC"
7      BLKCMD = "#BLKCMD"
8
9  class DecKW:
10     BIND = "#BIND"
11     DSEQ = "#DSEQ"
12
13 class DecPiAut(CmdPiAut):
14     def __init__(self):
15         self["locs"] = []
16         CmdPiAut.__init__(self)
17
18     def locs(self):
```



## $\Pi$ IR declarations in Python IV

```
19     return self["locs"]
20
21 def pushLoc(self, l):
22     ls =self.locs()
23     ls.append(l)
24
25 def __evalRef(self, e):
26     ex =e.opr[0]
27     self.pushCnt(DecExpKW.REF)
28     self.pushCnt(ex)
29
30 def __newLoc(self):
31     sto =self.sto()
32     if sto:
33         return max(list(sto.keys())) +1
34     else:
35         return 0.0
36
37 def __evalRefKW(self):
```

## $\Pi$ IR declarations in Python V

```
38     v =self.popVal()
39     l =self.__newLoc()
40     self.updateStore(l, v)
41     self.pushLoc(l)
42     self.pushVal(l)
43
44 def __evalBind(self, d):
45     i =d.opr[0]
46     e =d.opr[1]
47     self.pushVal(i)
48     self.pushCnt(DecKW.BIND)
49     self.pushCnt(e)
50
51 def __evalBindKW(self):
52     l =self.popVal()
53     i =self.popVal()
54     x =i.opr[0]
55     self.pushVal({x: l})
56
```

## $\Pi$ IR declarations in Python VI

```
57 def __evalDSeq(self, ds):
58     d1 =ds.opr[0]
59     d2 =ds.opr[1]
60     self.pushCnt(DecKW.DSEQ)
61     self.pushCnt(d2)
62     self.pushCnt(d1)
63
64 def __evalDSeqKW(self):
65     d2 =self.popVal()
66     d1 =self.popVal()
67     d1.update(d2)
68     self.pushVal(d1)
69
70 def __evalBlk(self, d):
71     ld =d.opr[0]
72     c =d.opr[1]
73     l =self.locs()
74     self.pushVal(list(l))
75     self.pushVal(c)
```

## $\Pi$ IR declarations in Python VII

```
76     self.pushCnt(DecCmdKW.BLKDEC)
77     self.pushCnt(1d)
78
79     def __evalBlkDecKW(self):
80         d =self.popVal()
81         c =self.popVal()
82         l =self.locs()
83         self.pushVal(l)
84         en =self.env()
85         ne =en.copy()
86         ne.update(d)
87         self.pushVal(en)
88         self["env"] =ne
89         self.pushCnt(DecCmdKW.BLKCMD)
90         self.pushCnt(c)
91
92     def __evalBlkCmdKW(self):
93         en =self.popVal()
94         ls =self.popVal()
```

## $\Pi$ IR declarations in Python VIII

```
95     self["env"] =en
96     s =self.sto()
97     s ={k: v for k, v in s.items() if k not in ls}
98     self["sto"] =s
99     # del ls
100     ols =self.popVal()
101     self["locs"] =ols
102
103 def eval(self):
104     d =self.popCnt()
105     if isinstance(d, Bind):
106         self.__evalBind(d)
107     elif d ==DecKW.BIND:
108         self.__evalBindKW()
109     elif isinstance(d, DSeq):
110         self.__evalDSeq(d)
111     elif d ==DecKW.DSEQ:
112         self.__evalDSeqKW()
113     elif isinstance(d, Ref):
```

## $\Pi$ IR declarations in Python IX

```
14         self.__evalRef(d)
15     elif d == DecExpKW.REF:
16         self.__evalRefKW()
17     elif isinstance(d, Blk):
18         self.__evalBlk(d)
19     elif d == DecCmdKW.BLKDEC:
20         self.__evalBlkDecKW()
21     elif d == DecCmdKW.BLKCMD:
22         self.__evalBlkCmdKW()
23     else:
24         self.pushCnt(d)
25         CmdPiAut.eval(self)
```

## $\Pi$ IR declarations in Python X

```
1 dc =DecPiAut()
2 fac =Loop(Not(Eq(Id("y"), Num(0))),
3             CSeq(Assign(Id("x"), Mul(Id("x"), Id("y"))),
4                   Assign(Id("y"), Sub(Id("y"), Num(1))))))
5 dec =DSeq(Bind(Id("x"), Ref(Num(1))),
6           Bind(Id("y"), Ref(Num(200))))
7 fac_blk =Blk(dec, fac)
8 dc.pushCnt(fac_blk)
9 while not dc.emptyCnt():
10     aux =dc.copy()
11     dc.eval()
12     if dc.emptyCnt():
13         print(aux)
```

# $\Pi$ IR abstractions in Python I

```
1
2 class Formals(list):
3     def __init__(self, f):
4         if isinstance(f, list):
5             for a in f:
6                 if not isinstance(a, Id):
7                     raise IllFormed(self, a)
8             self.append(f)
9         else:
10            raise IllFormed(self, f)
11
12 class Abs:
13     def __init__(self, f, b):
14         if isinstance(f, list):
15             if isinstance(b, Blk):
16                 self._opr = [f, b]
17             else:
18                 raise IllFormed(self, b)
```



## $\Pi$ IR abstractions in Python II

```
19     else:
20         raise IllFormed(self, f)
21
22     def formals(self):
23         return self._opr[0]
24
25     def blk(self):
26         return self._opr[1]
27
28     def __str__(self):
29         ret =str(self.__class__.__name__) + "("
30         formals =self.formals()
31         ret +=str(formals[0]) # First formal argument
32         for i in range(1, len(formals)):
33             ret +=", "
34             ret +=str(formals[i]) # Remaining formal arguments
35         ret +=", "
36         ret +=str(self.blk()) # Abstraction block
37         ret +=")"
```

## $\Pi$ IR abstractions in Python III

```
38     return ret
39
40 class BindAbs(Bind):
41     '''
42     BindAbs is a form of bind but that receives an Abs instead of an
43     expression.
44     '''
45     def __init__(self, i, p):
46         if isinstance(i, Id):
47             if isinstance(p, Abs):
48                 Dec.__init__(self, i, p)
49             else:
50                 raise IllFormed(self, p)
51         else:
52             raise IllFormed(self, i)
53
54 class Actuals(list):
55     def __init__(self, a):
56         if isinstance(a, list):
```

## $\Pi$ IR abstractions in Python IV

```
57     for e in a:
58         if not isinstance(e, Exp):
59             raise IllFormed(self, e)
60     self.append(a)
61 else:
62     raise IllFormed(self, a)
63
64 class Call(Cmd):
65     def __init__(self, f, actuals):
66         if isinstance(f, Id):
67             if isinstance(actuals, list):
68                 Cmd.__init__(self, f, actuals)
69             else:
70                 raise IllFormed(self, actuals)
71         else:
72             raise IllFormed(self, f)
73
74     def caller(self):
75         return self.operand(0)
```

## $\Pi$ IR abstractions in Python V

```
76
77 def actuals(self):
78     return self.operand(1)
79
80 class Closure(dict):
81     def __init__(self, f, b, e):
82         if isinstance(f, list):
83             if isinstance(b, Blk):
84                 # I wanted to write assert(isinstance(e, Env)) but it
85                                     fails.
86
87                 if isinstance(e, dict):
88                     self["for"] = f # Formal parameters
89                     self["env"] = e # Current environment
90                     self["block"] = b # Procedure block
91                 else:
92                     raise IllFormed(self, e)
93             else:
94                 raise IllFormed(self, b)
95         else:
```

## $\Pi$ IR abstractions in Python VI

```
94         raise IllFormed(self, f)
95
96     def __str__(self):
97         ret =str(self.__class__.__name__) + "("
98         formals =self.formals()
99         fst_formal =formals[0] # First formal argument
100         ret +=str(fst_formal)
101         for i in range(1, len(formals)):
102             ret +=", "
103             formal =formals[i] # Remaining formal arguments
104             ret +=str(formal)
105         ret +=", "
106         ret +=str(self.blk()) # Closure block
107         ret +=")"
108         return ret
109
110     def formals(self):
111         return self['for']
112
```

## $\Pi$ IR abstractions in Python VII

```
13 def env(self):
14     return self['env']
15
16 def blk(self):
17     return self['block']
18
19 class AbsPiAut(DecPiAut):
20     def __evalAbs(self, a):
21         if not isinstance(a, Abs): # p must be an abstraction
22             raise EvaluationError(self, "Function __evalAbs called with
23                                     no abstraction but with "
24                                     , a, " instead.")
25
26         else:
27             f = a.formals() # Formal parameters
28             b = a.blk() # Body
29             e = self.env() # Current environment
30             # Closes the given abs. with the current env
31             c = Closure(f, b, e)
32             # Closure c is pushed to the value stack such that
```

## $\Pi$ IR abstractions in Python VIII

```
30     self.pushVal(c)
```

```
31     # a BIND may create a new binding to a given identifier.
```

```
32  
33 def __match(self, f, a):
```

```
34     '''
```

```
35     Given a list of formal parameters and a list of actual parameters
```

```
36     it returns an environment relating the elements of the former  
37     with the latter.
```

```
38     '''
```

```
39 if isinstance(f, list):
```

```
40     if isinstance(a, list):
```

```
41         if len(f) == 0:
```

```
42             return {}
```

```
43         if len(f) == len(a) and len(f) > 0:
```

```
44             # For some reason, f[0] is a tuple, not an Id.
```

```
45             f0 = f[0]
```

```
46             a0 = a[0]
```

```
             b0 = {f0.id(): a0.num()}
```

## $\Pi$ IR abstractions in Python IX

```
47     if len(f) == 1:
48         return b0
49     else:
50         # For some reason, f[0] is a tuple, not an Id.
51         f1 = f[1]
52         a1 = a[1]
53         b1 = {f1.id(): a1.num()}
54         e = b0.update(b1)
55         for i in range(2, len(f)):
56             fi = f[i][0]
57             ai = a[i][0]
58             e.update({fi.id(): ai.num()})
59         return e
60     else:
61         raise EvaluationError("Call to '__match' on " + str(self) +
                                ": " + "formals and"
                                "actuals differ in"
                                "size.")
62 else:
```



## $\Pi$ IR abstractions in Python X

```
raise EvaluationError("Call to '.__match' on " +str(self) +":  
                        "+" no formals, but with  
                        ", f, " instead.")
```

```
def __evalCall(self, c):  
    '''
```

*Essentially, a call is translated into a block.  
If we were programming  $\pi$  in a symbolic language,  
we could simply create a proper block and push it to the control  
stack.*

*However, the environment is not symbolic: is a dictionary of  
objects.*

*To create a block we would need to "pi-IR-fy" it, that is,  
recreate the*

*pi IR tree from the concrete environment and joint it with  
matches created*

*also at pi IR level. These would be pushed back into the control  
stack and*

## $\Pi$ IR abstractions in Python XI

reobjectified. Thus, to avoid pi-IRfication and reevaluation of the environment we manipulate it at the object level, which is dangerous but seems to be correct.

In this implementation, actual parameters are already evaluated.  
'''

```
if not isinstance(c, Call): # c must be a Call object
    raise EvaluationError("Call to __evalCall with no Call object
                           but with ", c, " instead
                           .")
else:
    # Procedure to be called
    caller = c.caller()
    # Retrieves the current environment.
    e = self.env()
    # Retrieves the closure associated with the caller function.
    clos = e[caller.id()]
```

# $\Pi$ IR abstractions in Python XII

```

89     # Retrieves the actual parameters from the call.
90     a =c.actuals()
91     # Retrieves the formal parameters from the closure.
92     f =clos.formals()
93     # Matches formals and actuals, creating an environment.
94     d =self.__match(f, a)
95     # Retrives the closure's environment.
96     ce =clos.env()
97     # The caller's block must run on the closures environment
98     # overwritten with the matches.
99     d.update(ce)
100    self["env"] =d
101    self.pushVal(self.locs())
102    # Saves the current environment in the value stack.
103    self.pushVal(e)
104    # Pushes the keyword BLKCMD for block completion.
105    self.pushCnt(DecCmdKW.BLKCMD)
106    # Pushes the body of the caller function into the control
        stack.
    
```

## $\Pi$ IR abstractions in Python XIII

```
07         self.pushCnt(clos.blk())
08
09     def eval(self):
10         d =self.popCnt()
11         if isinstance(d, Abs):
12             self.__evalAbs(d)
13         elif isinstance(d, Call):
14             self.__evalCall(d)
15         else:
16             self.pushCnt(d)
17             DecPiAut.eval(self)
```

# Complete Imp grammar in EBNF notation I

```

1 @@grammar::IMP
2 @@eol_comments ::/#.*?/start = @:cmd ;
3
4
5
6 cmd =nop | let | assign | loop | call ;
7
8 call =i:identifier '(' { a:actual }* ')' ;
9
10 actual =e1:expression { ',' e2:expression }* | {} ;
11
12 nop ='nop' ;
13
14 loop =op:'while' ~ e:expression 'do' { c:cmd }+ ;
15
16 assign =id:identifier op:':=' ~ e:expression ;
17
18 let =op:'let' ~ d:dec 'in' { c:cmd }+ ;

```

# Complete Imp grammar in EBNF notation II

```

19
20 dec =var | fn ;
21
22 var =op:'var' ~ id:identifier '=' e:expression ;
23
24 fn =op:'fn' ~ id:identifier '(' f:formal ')' '=' c:cmd ;
25
26 formal =i1:identifier { ',' i2:identifier }* | {} ;
27
28 expression =@:bool_expression ;
29
30 bool_expression =negation | equality | conjunction | disjunction
31                  | lowereq | greatereq | lowerthan | greaterthan
32                  | add_expression ;
33
34 equality =left:add_expression op:"==" ~ right:bool_expression ;
35
36 conjunction =left:add_expression op:"and" ~ right:bool_expression ;
37

```

# Complete Imp grammar in EBNF notation III

```

38 disjunction =left:add_expression op:"or" ~ right:bool_expression ;
39
40 lowereq =left:add_expression op:"<=" ~ right:add_expression ;
41
42 greatereq =left:add_expression op:">=" ~ right:add_expression ;
43
44 lowerthan =left:add_expression op:"<" ~ right:add_expression ;
45
46 greaterthan =left:add_expression op:">" ~ right:add_expression ;
47
48 parenthesisexp ='(' ~ @:bool_expression ')';
49
50 negation =op:'not' ~ b:bool_expression ;
51
52 add_expression =addition | subtraction | @:mult_expression ;
53
54 addition =left:mult_expression op:"+" ~ right:add_expression ;
55
56 subtraction =left:mult_expression op:"- " ~ right:add_expression ;

```

# Complete Imp grammar in EBNF notation IV

```
57  
58 mult_expression = multiplication | division  
59                 | atom  
60                 | parenthesisexp ;  
61  
62 multiplication = left:atom op:"*" ~ right:mult_expression ;  
63  
64 division = left:atom op:"/" ~ right:mult_expression ;  
65  
66 atom = number | truth | identifier ;  
67  
68 number = /\d+/ ;  
69  
70 identifier = /(?!\\d)\\w+/ ;  
71  
72 truth = 'True' | 'False' ;
```



# Example: iterative factorial

```
1 # The classic iterative factorial example
2 let var z = 1
3 in
4   let var y = 10
5   in
6     while not (y == 0)
7     do
8       z := z * y
9       y := y - 1
```

## Example: iterative factorial within a function

```
1 # In this example we encapsulate the iterative calculation
2 # of the factorial within a function call.
3 let var z = 1
4 in
5   let fn f(x) =
6     let var y = x
7     in
8       while not (y == 0)
9       do
10         z := z * y
11         y := y - 1
12   in f(10)
```