

Written Assignment 4

CS 538, Spring 2020

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1 While-language: basics (15)

- (a) $(x \leftarrow x + 1, s_0)$
 $\rightarrow (x \leftarrow x + 1, [0, 0])$
 $\rightarrow (\mathbf{skip}, [1, 0])$
- (b) $(x \leftarrow 3; y \leftarrow y + x, s_0)$
 $\rightarrow (x \leftarrow 3; y \leftarrow y + x, [0, 0])$
 $\rightarrow (\mathbf{skip}; y \leftarrow y + x, [3, 0])$
 $\rightarrow (y \leftarrow y + x, [3, 0])$
 $\rightarrow (\mathbf{skip}, [3, 3])$
- (c) $(\mathbf{if } x < y \mathbf{ then } x \leftarrow 2 \mathbf{ else } x \leftarrow -2, s_0)$
 $\rightarrow (\mathbf{if } x < y \mathbf{ then } x \leftarrow 2 \mathbf{ else } x \leftarrow -2, [0, 0])$
 $\rightarrow (x \leftarrow -2, [0, 0])$
 $\rightarrow (\mathbf{skip}, [-2, 0])$
- (d) $(x \leftarrow -1; \mathbf{if } x < y \mathbf{ then } x \leftarrow 2 \mathbf{ else } x \leftarrow -2, s_0)$
 $\rightarrow (x \leftarrow -1; \mathbf{if } x < y \mathbf{ then } x \leftarrow 2 \mathbf{ else } x \leftarrow -2, [0, 0])$
 $\rightarrow (\mathbf{skip}; \mathbf{if } x < y \mathbf{ then } x \leftarrow 2 \mathbf{ else } x \leftarrow -2, [-1, 0])$
 $\rightarrow (\mathbf{if } x < y \mathbf{ then } x \leftarrow 2 \mathbf{ else } x \leftarrow -2, [-1, 0])$
 $\rightarrow (x \leftarrow 2, [-1, 0])$
 $\rightarrow (\mathbf{skip}, [2, 0])$
- (e) $(x \leftarrow 2; \mathbf{while } y < x \mathbf{ do } y \leftarrow y + 1, s_0)$
 $\rightarrow (x \leftarrow 2; \mathbf{while } y < x \mathbf{ do } y \leftarrow y + 1, [0, 0])$

$\rightarrow (\text{skip}; \text{while } y < x \text{ do } y \leftarrow y + 1, [2, 0])$
 $\rightarrow (\text{while } y < x \text{ do } y \leftarrow y + 1, [2, 0])$
 $\rightarrow (y \leftarrow y + 1; \text{while } y < x \text{ do } y \leftarrow y + 1, [2, 0])$
 $\rightarrow (\text{skip}; \text{while } y < x \text{ do } y \leftarrow y + 1, [2, 1])$
 $\rightarrow (\text{while } y < x \text{ do } y \leftarrow y + 1, [2, 1])$
 $\rightarrow (y \leftarrow y + 1; \text{while } y < x \text{ do } y \leftarrow y + 1, [2, 1])$
 $\rightarrow (\text{skip}; \text{while } y < x \text{ do } y \leftarrow y + 1, [2, 2])$
 $\rightarrow (\text{while } y < x \text{ do } y \leftarrow y + 1, [2, 2])$
 $\rightarrow (\text{skip}, [2, 2])$

2 While-language: conditionals (10)

$$\begin{array}{c}
\frac{s(b) = \text{true}}{(\text{cond } \{b \Rightarrow c, - \Rightarrow c'\}, s) \rightarrow (c, s)} \qquad \frac{s(b) = \text{false}}{(\text{cond } \{b \Rightarrow c, - \Rightarrow c'\}, s) \rightarrow (c', s)} \\
\\
\frac{s(b_1) = \text{true}}{(\text{cond } \{b_1 \Rightarrow c_1, b_2 \Rightarrow c_2, - \Rightarrow c_3\}, s) \rightarrow (c_1, s)} \qquad \frac{s(b_1) = \text{false and } s(b_2) = \text{true}}{(\text{cond } \{b_1 \Rightarrow c_1, b_2 \Rightarrow c_2, - \Rightarrow c_3\}, s) \rightarrow (c_2, s)} \\
\\
\frac{s(b_1) = \text{false and } s(b_2) = \text{false}}{(\text{cond } \{b_1 \Rightarrow c_1, b_2 \Rightarrow c_2, - \Rightarrow c_3\}, s) \rightarrow (c_3, s)}
\end{array}$$

3 While-language: do-until (5)

$$\overline{(\text{do } c \text{ until } b, s) \rightarrow (c; \text{if } b \text{ then skip else } (\text{do } c \text{ until } b), s)}$$