Written Assignment 1

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1 Calculator language: Syntax

digits = "0" | "1" | "2" | "3" | "4" | "5" | "6" | "7" | "8" | "9" ;

boolconst = "tt" | "ff" ;

int = ["-"] digits {digits} ;

be = boolconst
  | be "&&" be
  | be "||" be
  | ae "==" ae
  | ae "<" ae ;

ae = int
  | ae "+" ae
  | ae "×" ae
  | "if" be "then" ae "else" ae ;

2 Calculator language: Operational semantics

2.1 Values that do not step (Defining here)

b1, b2, b = "tt" | "ff" ;
i_1, i_2, i = ["-"] digits {digits} ;

2.2 Semantics for be "&&" be

Let b_1, b_2, b $\in$ boolconst
Let be_1, be_1^1, be_2, be_2^1 $\in$ be

$$\frac{b_1 \&\& b_2 = b}{(b_1 \text{``\&\&''} b_2) \to b}$$

$$\frac{be_1 \to b_1}{(be_1 \text{``\&\&''} b_2) \to (b_1 \text{``\&\&''} b_2)}$$

$$\frac{be_1 \to be_1^1}{(be_1 \text{``\&\&''} b_2) \to (be_1^1 \text{``\&\&''} b_2)}$$

$$\frac{be_2 \to b_2}{(b_1 \text{“\&\&”} be_2) \to (b_1 \text{“\&\&”} b_2)}$$

$$\frac{be_2 \to be_2^1}{(b_1 \text{“\&\&”} be_2) \to (b_1 \text{“\&\&”} be_2^1)}$$

$$\frac{be_1 \to b_1, \quad be_2 \to be_2^1}{(be_1 \text{“\&\&”} be_2) \to (b_1 \text{“\&\&”} be_2^1)}$$

$$\frac{be_1 \to be_1^1, \quad be_2 \to b_2}{(be_1 \text{“\&\&”} be_2) \to (be_1^1 \text{“\&\&”} b_2)}$$

$$\frac{be_1 \to b_1, \quad be_2 \to b_2}{(be_1 \text{“\&\&”} be_2) \to (b_1 \text{“\&\&”} b_2)}$$

$$\frac{be_1 \to be_1^1, \quad be_2 \to be_2^1}{(be_1 \text{“\&\&”} be_2) \to (be_1^1 \text{“\&\&”} be_2^1)}$$

2.3 Semantics for be “||” be

Let b_1, b_2, b $\in$ boolconst

Let be_1, be_1^1, be_2, be_2^1 $\in$ be

$$\frac{b_1 \,||\, b_2 = b}{(b_1 \text{“}||\text{”} b_2) \to b}$$

$$\frac{be_1 \to b_1}{(be_1 \text{“}||\text{”} b_2) \to (b_1 \text{“}||\text{”} b_2)}$$

$$\frac{be_1 \to be_1^1}{(be_1 \text{“}||\text{”} b_2) \to (be_1^1 \text{“}||\text{”} b_2)}$$

$$\frac{be_2 \to b_2}{(b_1 \text{“}||\text{”} be_2) \to (b_1 \text{“}||\text{”} b_2)}$$

$$\frac{be_2 \to be_2^1}{(b_1 \text{“}||\text{”} be_2) \to (b_1 \text{“}||\text{”} be_2^1)}$$

$$\frac{be_1 \to b_1, \quad be_2 \to be_2^1}{(be_1 \text{“}||\text{”} be_2) \to (b_1 \text{“}||\text{”} be_2^1)}$$

$$\frac{be_1 \to be_1^1, \quad be_2 \to b_2}{(be_1 \text{“}||\text{”} be_2) \to (be_1^1 \text{“}||\text{”} b_2)}$$

$$\frac{be_1 \to b_1, \quad be_2 \to b_2}{(be_1 \text{“}||\text{”} be_2) \to (b_1 \text{“}||\text{”} b_2)}$$

$$\frac{be_1 \to be_1^1, \quad be_2 \to be_2^1}{(be_1 \text{“}||\text{”} be_2) \to (be_1^1 \text{“}||\text{”} be_2^1)}$$

2.4 Semantics for ae "==" ae

Let i_1, i_2, i $\in$ int

Let b $\in$ boolconst

Let ae_1, ae_1^1, ae_2, ae_2^1 $\in$ ae

$$\frac{(i_1 == i_2) = b}{(i_1 \text{``}==\text{''} i_2) \rightarrow b}$$

$$\frac{ae_1 \rightarrow i_1}{(ae_1 \text{``}==\text{''} i_2) \rightarrow (i_1 \text{``}==\text{''} i_2)}$$

$$\frac{ae_1 \rightarrow ae_1^1}{(ae_1 \text{``}==\text{''} i_2) \rightarrow (ae_1^1 \text{``}==\text{''} i_2)}$$

$$\frac{ae_2 \rightarrow i_2}{(i_1 \text{``}==\text{''} ae_2) \rightarrow (i_1 \text{``}==\text{''} i_2)}$$

$$\frac{ae_2 \rightarrow ae_2^1}{(i_1 \text{``}==\text{''} ae_2) \rightarrow (i_1 \text{``}==\text{''} ae_2^1)}$$

$$\frac{ae_1 \rightarrow i_1, \quad ae_2 \rightarrow ae_2^1}{(ae_1 \text{``}==\text{''} ae_2) \rightarrow (i_1 \text{``}==\text{''} ae_2^1)}$$

$$\frac{ae_1 \rightarrow ae_1^1, \quad ae_2 \rightarrow i_2}{(ae_1 \text{``}==\text{''} ae_2) \rightarrow (ae_1^1 \text{``}==\text{''} i_2)}$$

$$\frac{ae_1 \rightarrow i_1, \quad ae_2 \rightarrow i_2}{(ae_1 \text{``}==\text{''} ae_2) \rightarrow (i_1 \text{``}==\text{''} i_2)}$$

$$\frac{ae_1 \rightarrow ae_1^1, \quad ae_2 \rightarrow ae_2^1}{(ae_1 \text{``}==\text{''} ae_2) \rightarrow (ae_1^1 \text{``}==\text{''} ae_2^1)}$$

2.5 Semantics for ae "<" ae

Let i_1, i_2, i $\in$ int

Let b $\in$ boolconst

Let ae_1, ae_1^1, ae_2, ae_2^1 $\in$ ae

$$\frac{(i_1 < i_2) = b}{(i_1 \text{``}<\text{''} i_2) \rightarrow b}$$

$$\frac{ae_1 \rightarrow i_1}{(ae_1 \text{``}<\text{''} i_2) \rightarrow (i_1 \text{``}<\text{''} i_2)}$$

$$\frac{ae_1 \rightarrow ae_1^1}{(ae_1 \text{``}<\text{''} i_2) \rightarrow (ae_1^1 \text{``}<\text{''} i_2)}$$

$$\frac{ae_2 \rightarrow i_2}{(i_1 \text{``}<\text{''} ae_2) \rightarrow (i_1 \text{``}<\text{''} i_2)}$$

$$\frac{ae_2 \rightarrow ae_2^1}{(i_1 \text{``}<\text{''} ae_2) \rightarrow (i_1 \text{``}<\text{''} ae_2^1)}$$

$$\frac{ae_1 \to i_1, \quad ae_2 \to ae_2^1}{(ae_1 \text{“}<\text{”} ae_2) \to (i_1 \text{“}<\text{”} ae_2^1)}$$

$$\frac{ae_1 \to ae_1^1, \quad ae_2 \to i_2}{(ae_1 \text{“}<\text{”} ae_2) \to (ae_1^1 \text{“}<\text{”} i_2)}$$

$$\frac{ae_1 \to i_1, \quad ae_2 \to i_2}{(ae_1 \text{“}<\text{”} ae_2) \to (i_1 \text{“}<\text{”} i_2)}$$

$$\frac{ae_1 \to ae_1^1, \quad ae_2 \to ae_2^1}{(ae_1 \text{“}<\text{”} ae_2) \to (ae_1^1 \text{“}<\text{”} ae_2^1)}$$

2.6 Semantics for ae “+” ae

Let i_1, i_2, i ∈ int

Let ae_1, ae_1^1, ae_2, ae_2^1 ∈ ae

$$\frac{i_1 + i_2 = i}{(i_1 \text{“}+\text{”} i_2) \to i}$$

$$\frac{ae_1 \to i_1}{(ae_1 \text{“}+\text{”} i_2) \to (i_1 \text{“}+\text{”} i_2)}$$

$$\frac{ae_1 \to ae_1^1}{(ae_1 \text{“}+\text{”} i_2) \to (ae_1^1 \text{“}+\text{”} i_2)}$$

$$\frac{ae_2 \to i_2}{(i_1 \text{“}+\text{”} ae_2) \to (i_1 \text{“}+\text{”} i_2)}$$

$$\frac{ae_2 \to ae_2^1}{(i_1 \text{“}+\text{”} ae_2) \to (i_1 \text{“}+\text{”} ae_2^1)}$$

$$\frac{ae_1 \to i_1, \quad ae_2 \to ae_2^1}{(ae_1 \text{“}+\text{”} ae_2) \to (i_1 \text{“}+\text{”} ae_2^1)}$$

$$\frac{ae_1 \to ae_1^1, \quad ae_2 \to i_2}{(ae_1 \text{“}+\text{”} ae_2) \to (ae_1^1 \text{“}+\text{”} i_2)}$$

$$\frac{ae_1 \to i_1, \quad ae_2 \to i_2}{(ae_1 \text{“}+\text{”} ae_2) \to (i_1 \text{“}+\text{”} i_2)}$$

$$\frac{ae_1 \to ae_1^1, \quad ae_2 \to ae_2^1}{(ae_1 \text{“}+\text{”} ae_2) \to (ae_1^1 \text{“}+\text{”} ae_2^1)}$$

2.7 Semantics for ae "×" ae

Let i_1, i_2, i ∈ int

Let ae_1, ae_1^1, ae_2, ae_2^1 ∈ ae

$$\frac{i_1 \times i_2 = i}{(i_1 \text{“} \times \text{”} i_2) \to i}$$

$$\frac{ae_1 \to i_1}{(ae_1 \text{“} \times \text{”} i_2) \to (i_1 \text{“} \times \text{”} i_2)}$$

$$\frac{ae_1 \to ae_1^1}{(ae_1 \text{“} \times \text{”} i_2) \to (ae_1^1 \text{“} \times \text{”} i_2)}$$

$$\frac{ae_2 \to i_2}{(i_1 \text{“} \times \text{”} ae_2) \to (i_1 \text{“} \times \text{”} i_2)}$$

$$\frac{ae_2 \to ae_2^1}{(i_1 \text{“} \times \text{”} ae_2) \to (i_1 \text{“} \times \text{”} ae_2^1)}$$

$$\frac{ae_1 \to i_1, \quad ae_2 \to ae_2^1}{(ae_1 \text{“} \times \text{”} ae_2) \to (i_1 \text{“} \times \text{”} ae_2^1)}$$

$$\frac{ae_1 \to ae_1^1, \quad ae_2 \to i_2}{(ae_1 \text{“} \times \text{”} ae_2) \to (ae_1^1 \text{“} \times \text{”} i_2)}$$

$$\frac{ae_1 \to i_1, \quad ae_2 \to i_2}{(ae_1 \text{“} \times \text{”} ae_2) \to (i_1 \text{“} \times \text{”} i_2)}$$

$$\frac{ae_1 \to ae_1^1, \quad ae_2 \to ae_2^1}{(ae_1 \text{“} \times \text{”} ae_2) \to (ae_1^1 \text{“} \times \text{”} ae_2^1)}$$

2.8 Semantics for if be then ae else ae : (Lazy-If)

Let i_1, i_2, i ∈ int

Let ae_1, ae_1^1, ae_2, ae_2^1 ∈ ae

Let b_1, b_2, b ∈ boolconst

Let be_1, be_1^1, be_2, be_2^1 ∈ be

$$\frac{i1}{if \;\; \text{“}tt\text{”} \;\; then \;\; i_1 \;\; else \;\; ae_2 \to i_1}$$

$$\frac{i2}{if \;\; \text{“}ff\text{”} \;\; then \;\; ae_1 \;\; else \;\; i_2 \to i_2}$$

$$\frac{ae_1 \to i_1}{if \;\; \text{“}tt\text{”} \;\; then \;\; ae_1 \;\; else \;\; ae_2 \to if \;\; \text{“}tt\text{”} \;\; then \;\; i_1 \;\; else \;\; ae_2}$$

$$\frac{ae_1 \to ae_1^1}{if \;\; \text{“}tt\text{”} \;\; then \;\; ae_1 \;\; else \;\; ae_2 \to if \;\; \text{“}tt\text{”} \;\; then \;\; ae_1^1 \;\; else \;\; ae_2}$$

$$\frac{ae_2 \to i_2}{if \;\; \text{“}ff\text{”} \;\; then \;\; ae_1 \;\; else \;\; ae_2 \to if \;\; \text{“}ff\text{”} \;\; then \;\; ae_1 \;\; else \;\; i_2}$$

$$\frac{ae_2 \to ae_2^1}{if\ \ \text{“}ff\text{”}\ \ then\ \ ae_1\ \ else\ \ ae_2 \to if\ \ \text{“}ff\text{”}\ \ then\ \ ae_1\ \ else\ \ ae_2^1}$$

$$\frac{be_1 \to b_1}{if\ \ be_1\ \ then\ \ ae_1\ \ else\ \ ae_2 \to if\ \ b1\ \ then\ \ ae_1\ \ else\ \ ae_2}$$

$$\frac{be_1 \to be_1^1}{if\ \ be_1\ \ then\ \ ae_1\ \ else\ \ ae_2 \to if\ \ be_1^1\ \ then\ \ ae_1\ \ else\ \ ae_2}$$

2.9 Semantics for if be then ae else ae : (Eager-If)

Let i_1, i_2, i $\in$ int

Let ae_1, ae_1^1, ae_2, ae_2^1 $\in$ ae

Let b_1, b_2, b $\in$ boolconst

Let be_1, be_1^1, be_2, be_2^1 $\in$ be

$$\frac{i1}{if\ \ \text{“}tt\text{”}\ \ then\ \ i_1\ \ else\ \ i_2 \to i_1}$$

$$\frac{i2}{if\ \ \text{“}ff\text{”}\ \ then\ \ i_1\ \ else\ \ i_2 \to i_2}$$

$$\frac{be_1 \to b_1}{if\ \ be_1\ \ then\ \ i_1\ \ else\ \ i_2 \to if\ \ b_1\ \ then\ \ i_1\ \ else\ \ i_2}$$

$$\frac{be_1 \to be_1^1}{if\ \ be_1\ \ then\ \ i_1\ \ else\ \ i_2 \to if\ \ be_1^1\ \ then\ \ i_1\ \ else\ \ i_2}$$

$$\frac{ae_1 \to i_1}{if\ \ be_1\ \ then\ \ ae_1\ \ else\ \ i_2 \to if\ \ be_1\ \ then\ \ i_1\ \ else\ \ i_2}$$

$$\frac{ae_2 \to i_2}{if\ \ be_1\ \ then\ \ i_1\ \ else\ \ ae_2 \to if\ \ be_1\ \ then\ \ i_1\ \ else\ \ i_2}$$

$$\frac{ae_1 \to i_1, ae_2 \to ae_2^1}{if\ \ be_1\ \ then\ \ ae_1\ \ else\ \ ae_2 \to if\ \ be_1\ \ then\ \ i_1\ \ else\ \ ae_2^1}$$

$$\frac{ae_1 \to ae_1^1, ae_2 \to i_2}{if\ \ be_1\ \ then\ \ ae_1\ \ else\ \ ae_2 \to if\ \ be_1\ \ then\ \ ae_1^1\ \ else\ \ i_2}$$

$$\frac{ae_1 \to ae_1^1, ae_2 \to ae_2^1}{if\ \ be_1\ \ then\ \ ae_1\ \ else\ \ ae_2 \to if\ \ be_1\ \ then\ \ ae_1^1\ \ else\ \ ae_2^1}$$

1. Lazy-If is preferred over Eager-If as it ends up computing less to arrive at the same result when compared to Eager-If as in Eager-If all the expressions in body are evaluated irrespective of the guard condition.

2. Also in the case of Eager-If if the arguments doesn't terminate the function call doesn't terminate.

3 Lambda calculus

3.1

$(\lambda f.\ \lambda x.\ f\ x)\ (\lambda x.\ x+1)\ 5$
$\rightarrow (\lambda x.\ (\lambda x.\ x+1)\ x)\ 5$
$\rightarrow (\lambda x.\ x+1)\ 5$
$\rightarrow 6$

3.2

$(\lambda f.\ \lambda x.\ \lambda y.\ f\ y\ x)\ (\lambda x.\ \lambda y.\ x-y)\ 5\ 3$
$\rightarrow ((\lambda x.\ \lambda y.\ (\lambda x.\ \lambda y.\ x-y)\ y\ x)\ 5\ 3)$
$\rightarrow ((\lambda y.\ (\lambda x.\ \lambda y.\ x-y)\ y\ 5)\ 3)$
$\rightarrow ((\lambda x.\ \lambda y.\ x-y)\ 3\ 5)$
$\rightarrow ((\lambda y.\ 3-y)\ 5)$
$\rightarrow$(3 - 5)
$\rightarrow$ -2

3.3

$(\lambda x.\ x\ x)\ (\lambda x.\ x\ x)$
$\rightarrow (\lambda x.\ x\ x)\ (\lambda x.\ x\ x)$
$\rightarrow (\lambda x.\ x\ x)\ (\lambda x.\ x\ x)$
$\rightarrow .$
$\rightarrow .$
$\rightarrow .$
$\rightarrow .$
$\rightarrow .(Neverstops! - Infinitely\ recursive)$

4 Recursion

fibonacci = fix f. $\lambda n.\ if\ n<2\ then\ 1\ else\ (f(n-1)+f(n-2))$

Test n=3:

$\rightarrow [\lambda n.\ if\ n<2\ then\ 1\ else\ ((fix\ f...)(n-1)+(fix\ f...)(n-2)]\ 3$
$\rightarrow if\ 3<2\ then\ 1\ else\ ((fix\ f...)(3-1)+(fix\ f...)(3-2))$
$\rightarrow ((fix\ f...)(2)+(fix\ f...)(1))$
$\rightarrow ((if\ 1<2\ then\ 1\ else\ ...)\ +\ (if\ 0<2\ then\ 1\ else\ ...)\ +\ (if\ 1<2\ then\ 1\ else\ ...)))$
$\rightarrow 1\ +\ 1\ +\ 1$
$\rightarrow 3$