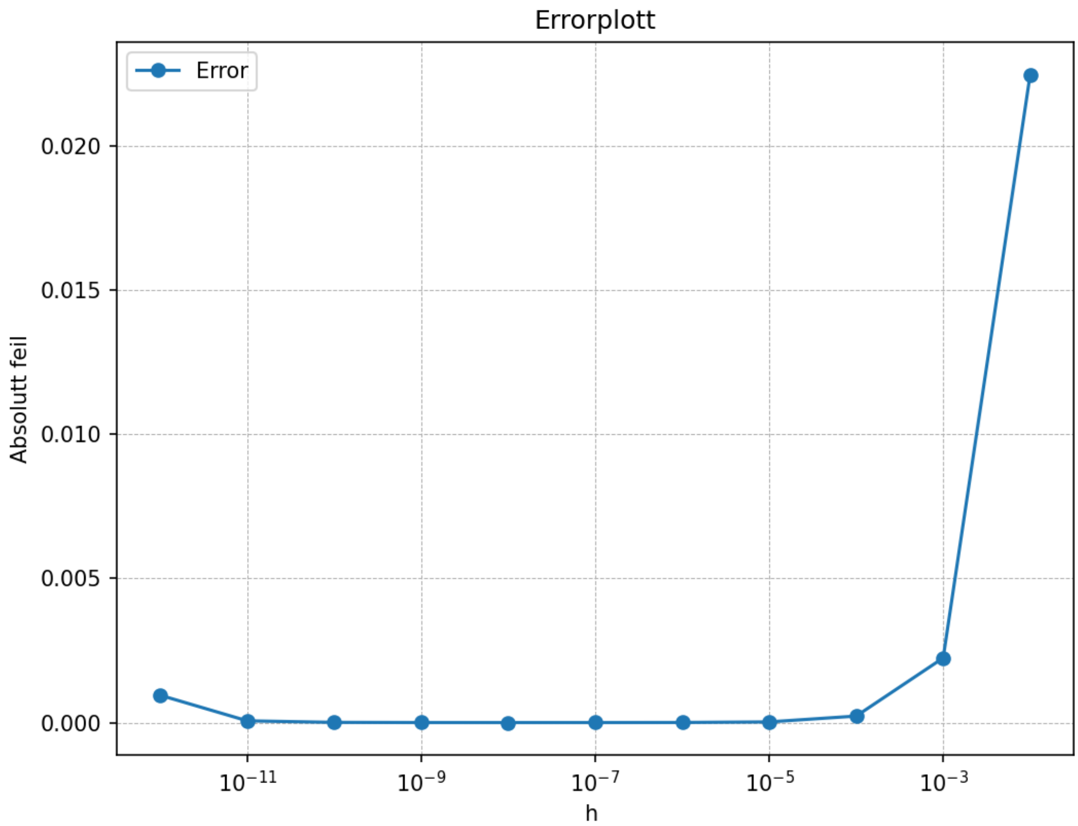
Standardprosjekt

1) For å løse denne oppgaven lagde jeg et pythonprogram som finner feilen for ulike h og plotter den.

```python
import numpy as np
import matplotlib.pyplot as plt

h_values=[0.01,0.001,0.0001,0.00001,0.000001,0.0000001,0.00000001,0.000000001,0.0000000001,0.00000000001,0.000000000001]

def f(x):
    return np.exp(x)

def f_prime_analytic(x):
    return np.exp(x)

def f_prime_numerical(x,h):
    return (f(x+h)-f(x))/h

calc_list=[]

for h in h_values:
    calc_list.append(f_prime_numerical(1.5,h))


for i in range(len(calc_list)-1):
    print("h: ",h_values[i]," Error: ",abs(calc_list[i]-f_prime_analytic(1.5)))

plt.figure(figsize=(8,6))
plt.plot(h_values,abs(calc_list-f_prime_analytic(1.5)),'-o',label='Error')
plt.xscale('log')

plt.xlabel("h ")
plt.ylabel("Absolutt feil")
plt.title("Errorplott")



plt.legend()
plt.grid(True, linestyle="--", linewidth=0.5)
plt.show()
```



Her ser man at for store h blir feilen veldig stor, for veldig små h vil feilen begynne å øke. Så svaret er da at for h mindre enn 1E-11 vil det oppstå problemer.

2)

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \frac{h^3}{6}f'''(x) + O(h^4)$$

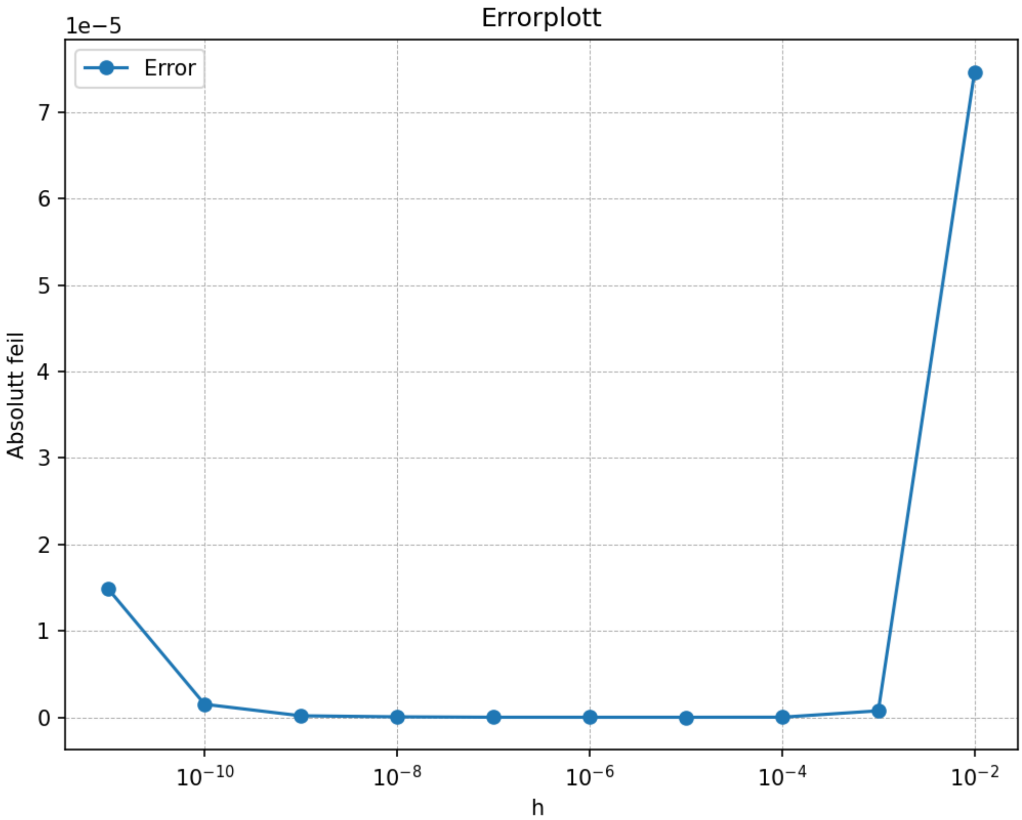
$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2}f''(x) - \frac{h^3}{6}f'''(x) + O(h^4)$$

$$f(x+h) - f(x-h) = 2hf'(x) + \frac{2h^3}{6}f'''(x)$$

$$\frac{f(x+h) - f(x-h)}{2h} = f'(x) + \frac{h^2}{6}f''(x)$$

$$\text{feil} \approx \frac{h^2}{6}e^x$$

Her har jeg beskrevet taylorutviklingen for f(x+h) og f(x-h) og delt på 2h for å få en taylorutvikling for gitt formel. Feilen blir da det siste leddet. Antakelsen er at for store h vil feilen så klart bli veldig stor, og bli lavere for mindre h, helt til h blir veldig liten og feilen blir større.

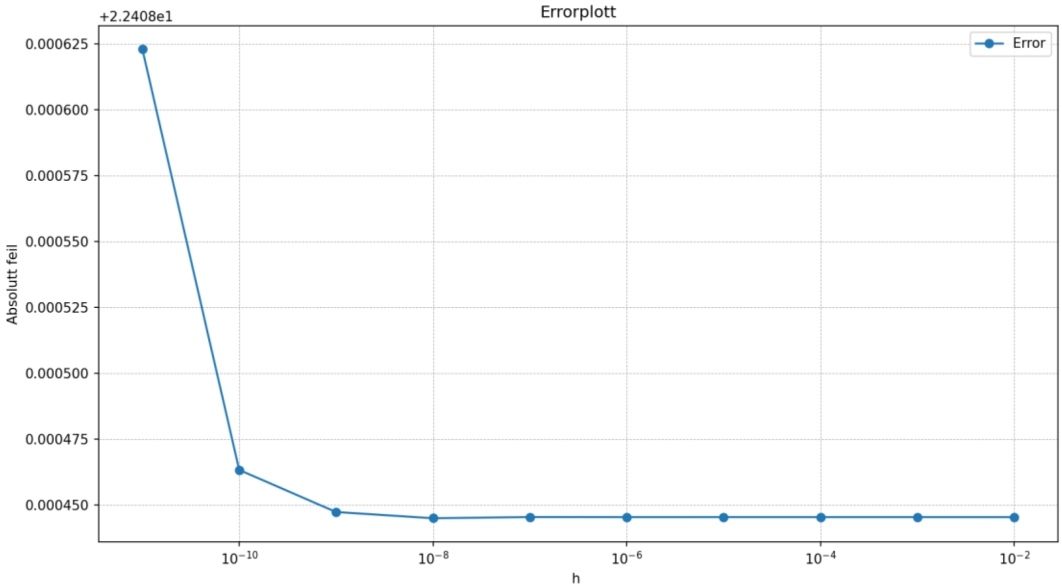


Dette er plottet for formelen over. Her ser man at antakelsen stemmer. Feilen er veldig stor for h>1E-2 og blir større igjen for h<1E-10.

Koden er så si identisk som forrige.

```python
import numpy as np
import matplotlib.pyplot as plt

h_values=[0.01,0.001,0.0001,0.00001,0.000001,0.0000001,0.00000001,0.000000001,0.0000000001,0.00000000001]

def f(x):
    return np.exp(x)

def f_prime_analytic(x):
    return np.exp(x)

def f_prime_numerical(x,h):
    return (f(x+h)-f(x-h))/(2*h)



calc_list=[]

for h in h_values:
    calc_list.append(f_prime_numerical(1.5,h))


for i in range(len(calc_list)):
    print("h: ",h_values[i]," Error: ",abs(calc_list[i]-f_prime_analytic(1.5)))

plt.figure(figsize=(8,6))
plt.plot(h_values,abs(calc_list-f_prime_analytic(1.5)),'-o',label='Error')
plt.xscale('log')

plt.xlabel("h ")
plt.ylabel("Absolutt feil")
plt.title("Errorplott")



plt.legend()
plt.grid(True, linestyle="--", linewidth=0.5)
plt.show()
```

3) Samme prosedyre, så si samme kode her også.

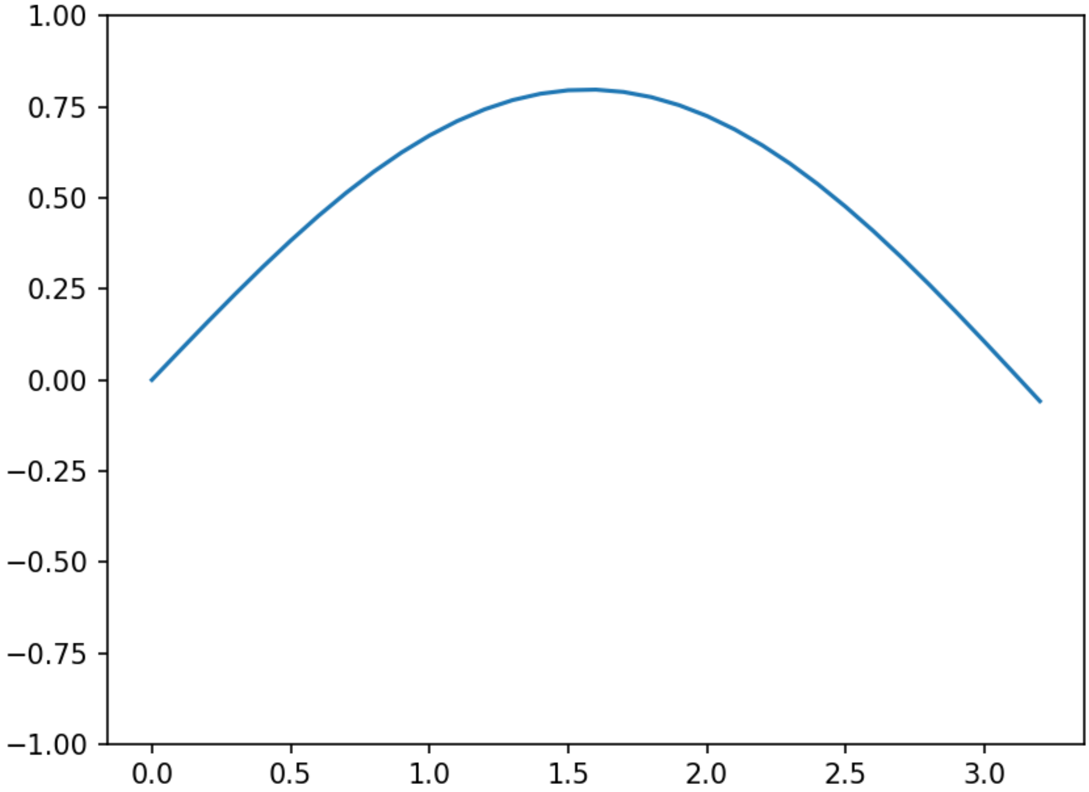


Feilen vokser for ekstremt små h, her begynner feilen å vokse for h<1E-8.

4)

```python
1  import numpy as np
2  import matplotlib.pyplot as plt
3  import matplotlib.animation as animation
4
5  def solve_heat_equation(h, k, T, L):
6      x = np.arange(0, L+h, h)
7      t = np.arange(0, T+k, k)
8
9      u = np.sin(x)
10     U = [u.copy()]
11
12     lambd = k / h**2
13     if lambd > 0.5:
14         print(f"Ustabil for lambda={lambd:.2f}, velg annen h og/eller k")
15         return None
16
17     for _ in t[1:]:
18         u_new = u.copy()
19         for j in range(1, len(x)-1):
20             u_new[j] = u[j] + lambd * (u[j+1] - 2*u[j] + u[j-1])
21         u = u_new
22         U.append(u.copy())
23
24     return x, t, U
25
26 def animate_solution(x, t, U):
27     fig, ax = plt.subplots()
28     line, = ax.plot(x, U[0])
29     ax.set_ylim(-1, 1)
30
31     def update(n):
32         line.set_ydata(U[n])
33         return line,
34
35     ani = animation.FuncAnimation(fig, update, frames=len(t), interval=50)
36     plt.show()
37
38
39 h_values = [1000,0.1, 0.05,0.01]
40 k_values = [0.01,0.01, 0.001,0.01]
41 T, L = 1, np.pi
42
43 for h in h_values:
44     for k in k_values:
45         print(f"Kjører for h={h}, k={k}")
46         result = solve_heat_equation(h, k, T, L)
47         if result:
48             x, t, U = result
49             animate_solution(x, t, U)
50
```

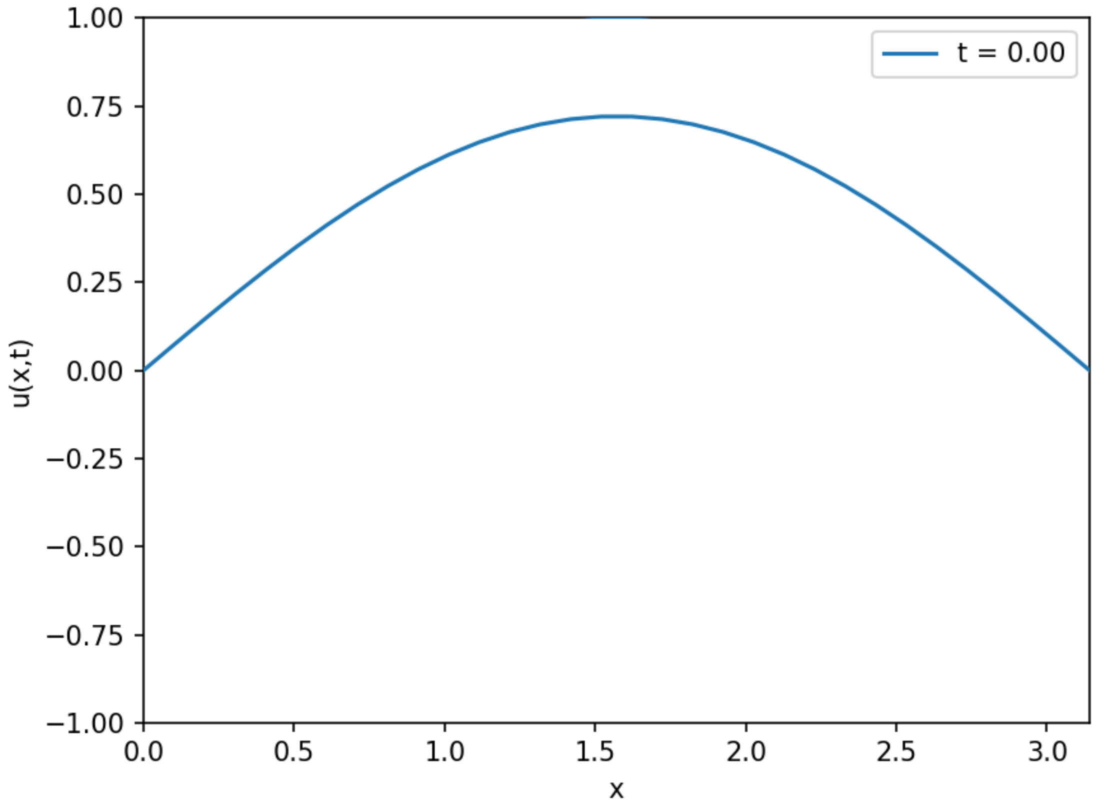
Her har jeg definert variabelen k/h**2. Linje 39 og 40 er to lister for ulike verdier for h og k. Her itererer jeg gjennom begge listene og animerer resultatene.




```
Kjører for h=0.1, k=0.01
Ustabil for lambda=1.00, velg annen h og/eller k
Kjører for h=0.05, k=0.01
Ustabil for lambda=4.00, velg annen h og/eller k
Kjører for h=0.05, k=0.01
Ustabil for lambda=4.00, velg annen h og/eller k
Kjører for h=0.05, k=0.001
Kjører for h=0.05, k=0.01
Ustabil for lambda=4.00, velg annen h og/eller k
Kjører for h=0.01, k=0.01
Ustabil for lambda=100.00, velg annen h og/eller k
Kjører for h=0.01, k=0.01
Ustabil for lambda=100.00, velg annen h og/eller k
Kjører for h=0.01, k=0.001
Ustabil for lambda=10.00, velg annen h og/eller k
Kjører for h=0.01, k=0.01
Ustabil for lambda=100.00, velg annen h og/eller k
```

5)

```python
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.animation as animation

def solve_heat_eq(h, k, L, T):
    Nx = int(L / h)
    Nt = int(T / k)
    x = np.linspace(0, L, Nx+1)
    t = np.linspace(0, T, Nt+1)
    r = k / h**2

    U = np.zeros((Nx+1, Nt+1))
    U[:, 0] = np.sin(x)

    A = np.zeros((Nx-1, Nx-1))
    for i in range(Nx-1):
        A[i, i] = 1 + 2 * r
        if i > 0:
            A[i, i-1] = -r
        if i < Nx-2:
            A[i, i+1] = -r

    for j in range(Nt):
        b = U[1:Nx, j]
        U[1:Nx, j+1] = np.linalg.solve(A, b)

    return x, t, U

def animate(x, t, U):
    fig, ax = plt.subplots()
    line, = ax.plot(x, U[:, 0], label="t = 0")
    ax.set_xlim([x[0], x[-1]])
    ax.set_ylim([-1, 1])
    ax.set_xlabel("x")
    ax.set_ylabel("u(x,t)")
    ax.legend()

    def update(frame):
        line.set_ydata(U[:, frame])
        ax.legend([f"t = {t[frame]:.2f}"])
        return line,

    ani = animation.FuncAnimation(fig, update, frames=len(t), interval=50, blit=True)
    plt.show()

def main():
    h_values=[0.1,0.001,0.0001]
    k_values=[0.01,0.001,0.000001]
    for h in h_values:
        for k in k_values:
            print(f"Kjører for h={h},k={k}")
            L,T=np.pi,1
            x,t,U=solve_heat_eq(h,k,L,T)
            animate(x,t,U)

if __name__ == "__main__":
    main()
```



6)

```python
def solve_heat_eq(h, k, L, T):
    Nx = int(L / h)
    Nt = int(T / k)
    x = np.linspace(0, L, Nx+1)
    t = np.linspace(0, T, Nt+1)
    r = k / h**2
    U = np.zeros((Nx+1, Nt+1))
    U[:, 0] = np.sin(x)

    A = np.zeros((Nx-1, Nx-1))
    B=np.zeros((Nx-1,Nx-1))
    for i in range(Nx-1):
        A[i, i] = 1 + 2 * r
        B[i,i]=1-2*r
        if i > 0:
            A[i, i-1] = -r
            B[i,i-1]=r
        if i < Nx-2:
            A[i, i+1] = -r
            B[i,i+1]=r

    for j in range(Nt):
        b = B.dot(U[1:Nx,j])
        U[1:Nx, j+1] = np.linalg.solve(A, b)

    return x, t, U

def animate(x, t, U):
    fig, ax = plt.subplots()
    line, = ax.plot(x, U[:, 0], label="t = 0")
    ax.set_xlim([x[0], x[-1]])
    ax.set_ylim([-1, 1])
    ax.set_xlabel("x")
    ax.set_ylabel("u(x,t)")
    ax.legend()

    def update(frame):
        line.set_ydata(U[:, frame])
        ax.legend([f"t = {t[frame]:.2f}"])
        return line,

    ani = animation.FuncAnimation(fig, update, frames=len(t), interval=50, blit=True)
    plt.show()

def main():
    h_values=[0.1,0.001,0.0001]
    k_values=[0.01,0.001,0.000001]
    for h in h_values:
        for k in k_values:
            print(f"Kjører for h={h},k={k}")
            L,T=np.pi,1
            x,t,U=solve_heat_eq(h,k,L,T)
            animate(x,t,U)

if __name__ == "__main__":
    main()
```

Sammenlikningen forteller at for store h og store k foretrekkes Crank Nickolson og implisitt metode. Dette er på grunn av at $\lambda \leq 0.5$ for stabilitet. Ellers vil feilen bli for stor. Problemet med Crank Nickolson og implisitt er at de er ganske tunge beregningsmessig sett. Der Crank Nickolson er den mest tunge. Eksplisitt er bedre egnet for ganske enkle beregninger gitt $\lambda \leq 0.5$.