MONASH University

MONASH BUSINESS SCHOOL

ETF3231/5231: Business forecasting

Week 3: Time series decomposition

`https://bf.numbat.space/`

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Outline

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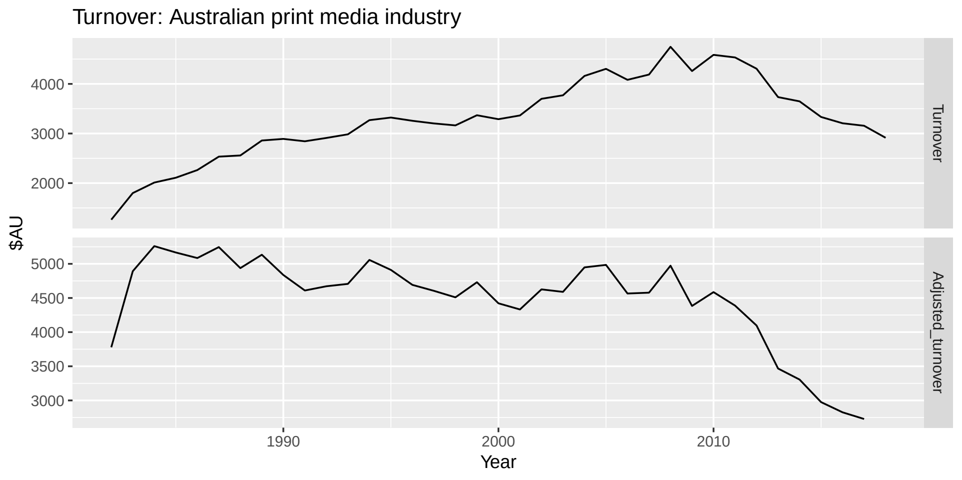
1. Transformations and adjustments
2. Time series components
3. Moving averages
4. Classical decomposition
5. History of time series decomposition
6. STL decomposition

Inflation adjustments



Mathematical transformations

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Denote original observations as $y_1, \dots, y_T$ and transformed observations as $w_1, \dots, w_T$.

Mathematical transformations for stabilizing variation

Square root	$w_t = \sqrt{y_t}$	$\downarrow$
Cube root	$w_t = \sqrt[3]{y_t}$	Increasing
Logarithm	$w_t = \log(y_t)$	strength

Logarithms, in particular, are useful because they are more interpretable: changes in a log value are relative (percent) changes on the original scale.

Box-Cox transformations

Each of these transformations is close to a member of the family of Box-Cox transformations:

$$w_t = \begin{cases} \log(y_t), & \lambda = 0; \\ (\text{sign}(y_t)|y_t|^\lambda - 1)/\lambda, & \lambda \neq 0. \end{cases}$$

- Actually the Bickel-Doksum transformation (allowing for $y_t < 0$)
- $\lambda = 1$: (No substantive transformation)
- $\lambda = \frac{1}{2}$: (Square root plus linear transformation)
- $\lambda = 0$: (Natural logarithm)
- $\lambda = -1$: (Inverse plus 1)

Transformations

- Often no transformation needed.
- Simple transformations are easier to explain and work well enough.
- Transformations can have very large effect on PI.
- If some data are zero or negative, then use $\lambda > 0$.
- `log1p()` can also be useful for data with zeros.
- Choosing logs is a simple way to force forecasts to be positive
- Transformations must be reversed to obtain forecasts on the original scale. (Handled automatically by `fable`.)

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Time series patterns

Recall

Trend pattern exists when there is a long-term increase or decrease in the data.

Cyclic pattern exists when data exhibit rises and falls that are *not of fixed period* (duration usually of at least 2 years).

Seasonal pattern exists when a series is influenced by seasonal factors (e.g., the quarter of the year, the month, or day of the week).

Time series decomposition

$$y_t = f(S_t, T_t, R_t)$$

where	y_t	=	data at period t
	T_t	=	trend-cycle component at period t
	S_t	=	seasonal component at period t
	R_t	=	remainder component at period t

Additive decomposition: $y_t = S_t + T_t + R_t$.

Multiplicative decomposition: $y_t = S_t \times T_t \times R_t$.

Time series decomposition

- Additive model appropriate if magnitude of seasonal fluctuations does not vary with level.
- If seasonal are proportional to level of series, then multiplicative model appropriate.
- Multiplicative decomposition more prevalent with economic series
- Alternative: use a Box-Cox transformation, and then use additive decomposition.
- Logs turn multiplicative relationship into an additive relationship:

$$y_t = S_t \times T_t \times R_t \quad \Rightarrow \quad \log y_t = \log S_t + \log T_t + \log R_t.$$

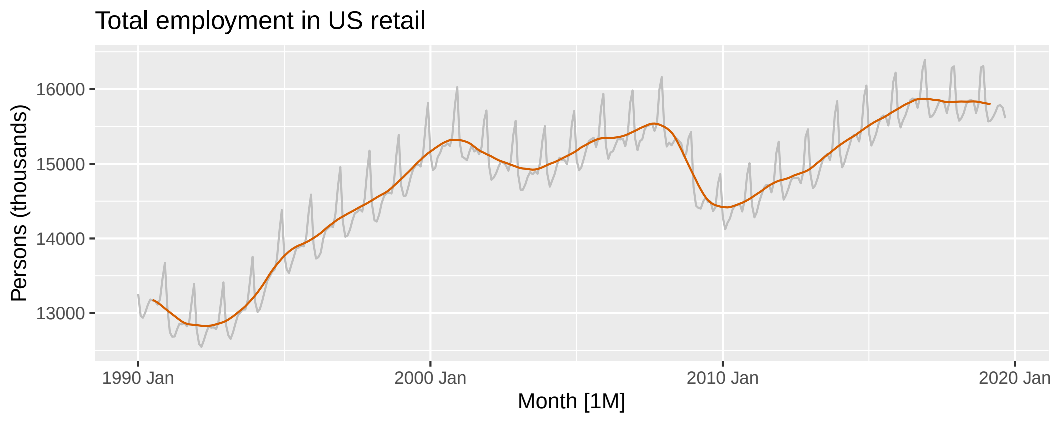
US Retail Employment

```
us_retail_employment <- us_employment |>
  filter(year(Month) >= 1990, Title == "Retail Trade") |>
  select(-Series_ID)
us_retail_employment
```

```
## # A tsibble: 357 x 3 [1M]
##        Month Title        Employed
##        <mth> <chr>           <dbl>
##  1 1990 Jan Retail Trade   13256.
##  2 1990 Feb Retail Trade   12966.
##  3 1990 Mar Retail Trade   12938.
##  4 1990 Apr Retail Trade   13012.
##  5 1990 May Retail Trade   13108.
##  6 1990 Jun Retail Trade   13183.
##  7 1990 Jul Retail Trade   13170.
##  8 1990 Aug Retail Trade   13160.
##  9 1990 Sep Retail Trade   13113.
## 10 1990 Oct Retail Trade   13185.
```

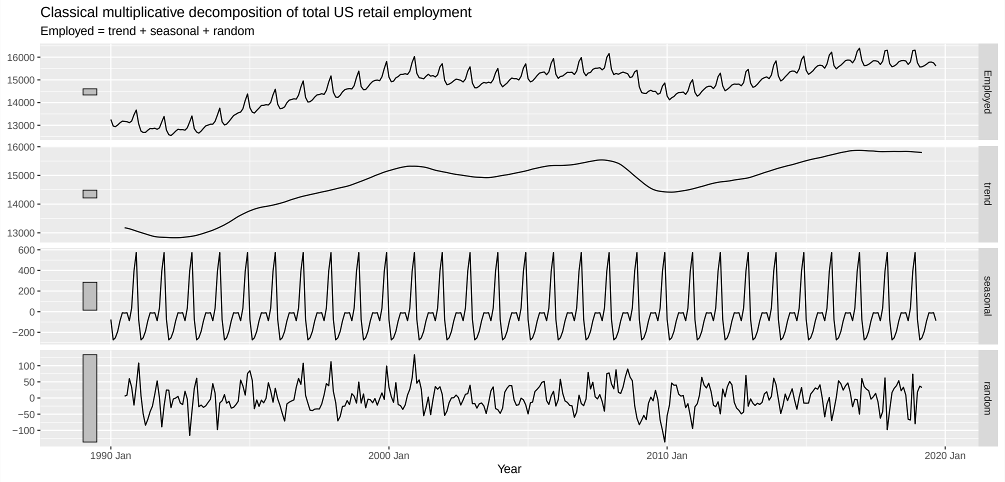
Outline

Moving average trend-cycle

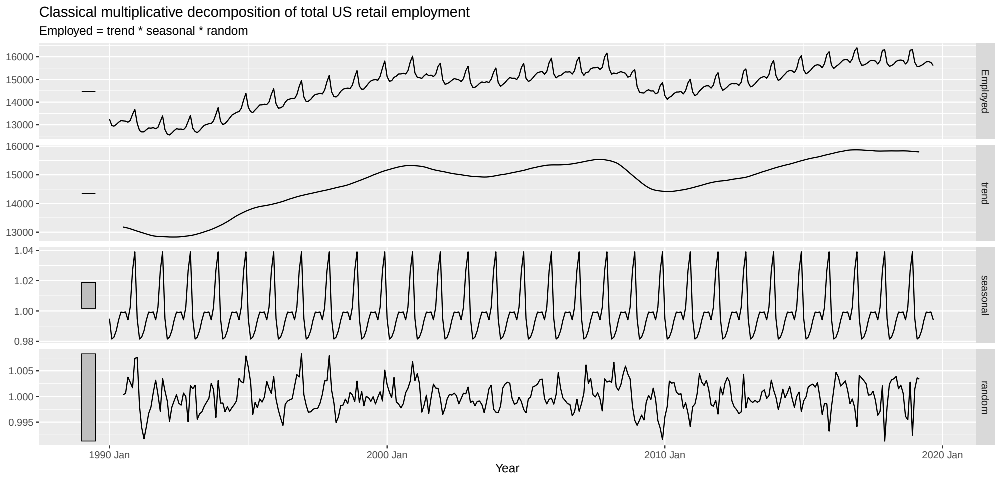


Outline

Additive classical decomposition



Multiplicative classical decomposition



Comments on classical decomposition

- Estimate of trend is unavailable for first few and last few observations.
- Seasonal component repeats from year to year. May not be realistic.
- Not robust to outliers.
- Newer methods designed to overcome these problems.

Outline

History of time series decomposition

- Classical method originated in 1920s.
- Census II method introduced in 1957. Basis for X-11 method and variants (including X-12-ARIMA, X-13-ARIMA)
- STL method introduced in 1983
- TRAMO/SEATS introduced in 1990s.

National Statistics Offices

- ABS uses X-12-ARIMA
- US Census Bureau uses X-13ARIMA-SEATS
- Statistics Canada uses X-12-ARIMA
- ONS (UK) uses X-12-ARIMA
- EuroStat use X-13ARIMA-SEATS

Outline

STL decomposition

- STL: “Seasonal and Trend decomposition using Loess”
- Very versatile and robust.
- Unlike X-12-ARIMA, STL will handle any type of seasonality.
- Seasonal component allowed to change over time, and rate of change controlled by user.
- Smoothness of trend-cycle also controlled by user.
- Robust to outliers
- No trading day or calendar adjustments.
- Only additive.
- Take logs to get multiplicative decomposition.
- Use Box-Cox transformations to get other decompositions.

STL decomposition

```
us_retail_employment |>
  model(STL(Employed)) |>
  components()
```

- `trend(window = ?)` controls wiggliness of trend component.
- `season(window = ?)` controls variation on seasonal component.
- `season(window = 'periodic')` is equivalent to an infinite window.

Default setting

- Season `window = 13`
- Trend `window = nextodd(`
  `ceiling((1.5*period)/(1-(1.5/s.window)))`
- Robust `robust=FALSE`