

# ETF3231/5231: Business forecasting

Ch5. The forecasters' toolbox

<https://bf.numbat.space/>



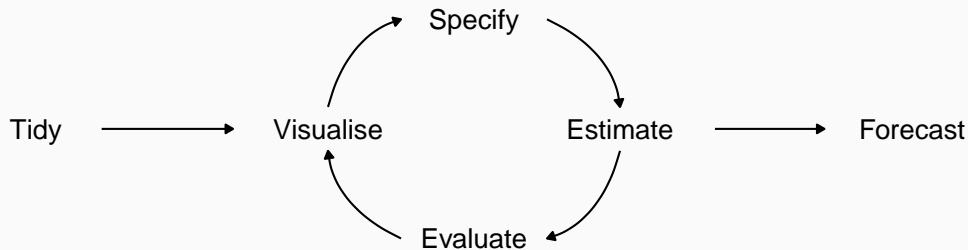
# Outline

- 1 A tidy forecasting workflow
- 2 Some simple forecasting methods
- 3 Residual diagnostics
- 4 Distributional forecasts and prediction intervals
- 5 Forecasting with transformations
- 6 Forecasting and decomposition
- 7 Evaluating forecast accuracy
- 8 Time series cross validation

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# A tidy forecasting workflow



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# Some simple forecasting methods - Benchmarks

- $\text{MEAN}(y)$ : Average method
- $\text{NAIVE}(y)$ : Naïve method
- $\text{SNAIVE}(y \sim \text{lag}(m))$ : Seasonal naïve method
- $\text{RW}(y \sim \text{drift}())$ : Drift method

Note: distinguish between a method and a model

# Model fitting

- The `model()` function trains models to data.
- The `forecast()` function generates forecasts.

## SNAIVE( $y \sim \text{lag}(m)$ ): Seasonal naïve method

- Forecasts equal to last value from same season.
- Forecasts:  $\hat{y}_{T+h|T} = y_{T+h-m(k+1)}$ , where  $m$  = seasonal period and  $k$  is the integer part of  $(h - 1)/m$ .



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# Forecasting residuals

**Residuals in forecasting:** difference between observed value and its fitted value:  $e_t = y_t - \hat{y}_{t|t-1}$ .

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## Assumptions

- 1  $\{e_t\}$  uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
- 2  $\{e_t\}$  have mean zero. If they don't, then forecasts are biased.

## Useful properties (for distributions & prediction intervals)

- 3  $\{e_t\}$  have constant variance.
- 4  $\{e_t\}$  are normally distributed.

## ACF of residuals

- We assume that the residuals are **white noise** (uncorrelated, mean zero, constant variance). If they aren't, then there is information left in the residuals that should be used in computing forecasts.
- So a standard residual diagnostic is to check the ACF of the residuals of a forecasting method.
- We **expect** these to look like white noise.

# Portmanteau tests

Consider a **whole set of**  $r_k$  values, and develop a test to see whether the set is significantly different from a zero set.

## Ljung-Box test

$$Q^* = T(T+2) \sum_{k=1}^{\ell} (T-k)^{-1} r_k^2$$

where  $\ell$  is max lag being considered and  $T$  is number of observations.

- My preferences:  $\ell = 10$  for non-seasonal data,  $\ell = 2m$  for seasonal data.
- Better performance, especially in small samples.

# Portmanteau tests

- If data are WN,  $Q^*$  has  $\chi^2$  distribution with  $(\ell - K)$  degrees of freedom where  $K$  = no. parameters in model.
- When applied to raw data, set  $K = 0$ .
- $\text{lag} = \ell$ ,  $\text{dof} = K$

```
augment(fit) %>%  
  features(.resid, lbjung_box, lag=10, dof=0)
```

```
## # A tibble: 1 x 4  
##   Symbol .model      lb_stat lb_pvalue  
##   <chr>   <chr>      <dbl>    <dbl>  
## 1 FB     NAIVE(Close)    12.1     0.276
```

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# Forecast distributions

Assuming residuals: have zero mean, are uncorrelated, normal, with variance =  $\hat{\sigma}^2$ :

**Mean:**  $y_{T+h|T} \sim N(\bar{y}, (1 + 1/T)\hat{\sigma}^2)$

**Naïve:**  $y_{T+h|T} \sim N(y_T, h\hat{\sigma}^2)$

**Seasonal naïve:**  $y_{T+h|T} \sim N(y_{T+h-m(k+1)}, (k+1)\hat{\sigma}^2)$

**Drift:**  $y_{T+h|T} \sim N(y_T + \frac{h}{T-1}(y_T - y_1), h\frac{T+h}{T}\hat{\sigma}^2)$

where  $k$  is the integer part of  $(h - 1)/m$ .

Note that when  $h = 1$  and  $T$  is large, these all give the same approximate forecast variance:  $\hat{\sigma}^2$ .



# Prediction intervals

- Assuming forecast errors are normally distributed, then a 95% PI is

$$\hat{y}_{T+h|T} \pm 1.96\hat{\sigma}_h$$

where  $\hat{\sigma}_h$  is the st dev of the  $h$ -step distribution.

- When  $h = 1$ ,  $\hat{\sigma}_h$  can be estimated from the residuals.
- Point forecasts often useless without a measure of uncertainty (such as prediction intervals).
- Prediction intervals require a stochastic model (with random errors, etc).
- Usually too narrow due to unaccounted uncertainty.

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# Decomposition models

`decomposition_model()` creates a decomposition model

- You must provide a method for forecasting the `season_adjust` series.
- A seasonal naive method is used by default for the `seasonal` components.
- The variances from both the seasonally adjusted and seasonal forecasts are combined.

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# Creating the rolling training sets

There are three main rolling types which can be used.

- Stretch: extends a growing length window with new data.
- Slide: shifts a fixed length window through the data.
- Tile: moves a fixed length window without overlap.

Three functions to roll a tsibble: `stretch_tsibble()`, `slide_tsibble()`, and `tile_tsibble()`.

For time series cross-validation, stretching windows are most commonly used.