MONASH University

MONASH BUSINESS SCHOOL

ETF3231/5231: Business forecasting

Week 4: The forecasters' toolbox

https://bf.numbat.space/

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Outline

1. A tidy forecasting workflow
2. Some simple forecasting methods
3. Residual diagnostics
4. Distributional forecasts and prediction intervals
5. Forecasting with transformations
6. Forecasting and decomposition
7. Evaluating forecast accuracy
8. Time series cross-validation

Outline

A tidy forecasting workflow

you may iterate this a few times

Specify

Tidy → Visualise → Specify → Estimate → Evaluate → Visualise

Estimate → Forecast

R: Demo GDP example

Outline

Some simple forecasting methods - Benchmarks

- `MEAN(y)`: Average method
- `NAIVE(y)`: Naïve method
- `SNAIVE(y ~ lag(m))`: Seasonal naïve method
- `RW(y ~ drift())`: Drift method

Note: distinguish between a method and a model

Method	Forecasts	Implicit model $\varepsilon_t \overset{iid}{\sim} N(0,\sigma^2)$
MEAN( )	$\hat{y}_{T+h\|T} = \bar{y} = (y_1 + \dots + y_T)/T$	
NAIVE( )	$\hat{y}_{T+h\|T} = y_T$	
SNAIVE( )	$\hat{y}_{T+h\|T} = y_{T+h-m(k+1)}$	
DRIFT( )	$\hat{y}_{T+h\|T} = y_T + \frac{h}{(T-1)} \sum_{t=2}^{T} (y_t - y_{t-1})$ $= y_T + \frac{h}{T-1}(y_T - y_1)$	

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$$= y_T + \frac{h}{T-1}(y_T - y_1)$$

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Model fitting

- The `model()` function trains models to data.
- The `forecast()` function generates forecasts.

`SNAIVE(y ~ lag(m))`: Seasonal naïve method

- Forecasts equal to last value from same season.
- Forecasts: $\hat{y}_{T+h|T} = y_{T+h-m(k+1)}$, where m = seasonal period and k is the integer part of $(h-1)/m$.

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Example : quarterly data ($m=4$)

$h=2$: $(h-1)/m = (2-1)/4 = 0.25 \Rightarrow k=0$ (integer part)

$$\hat{y}_{T+2|T} = \hat{y}_{T+2-m(0+1)} = y_{T+2-4} = y_{T-2}$$

Q_2	Q_3	Q_4	Q_1	Q_2
$T-2$	$T-1$	T	$T+1$	$T+2$

`SNAIVE(y ~ lag(m))`: Seasonal naïve method

- Forecasts equal to last value from same season.
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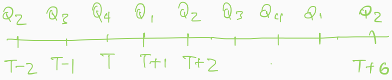
$$\hat{y}_{T+2|T} = \hat{y}_{T+2-m(0+1)} = y_{T+2-4} = y_{T-2}$$

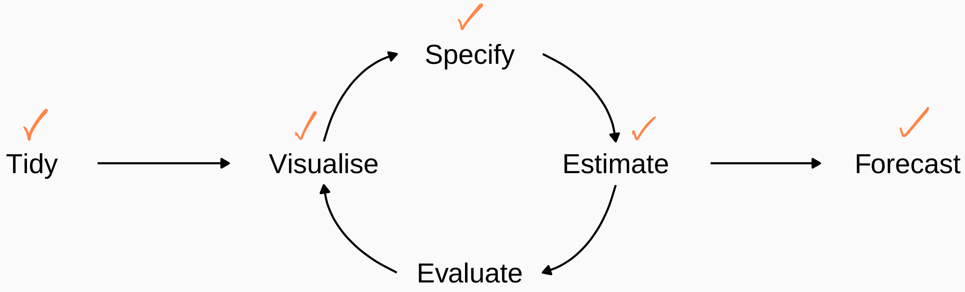
$h=6$: $(h-1)/m = (6-1)/4 = 1.25 \Rightarrow k=1$

$$\hat{y}_{T+6|T} = y_{T+6-4(1+1)} = y_{T-2}$$



A tidy forecasting workflow - Recap

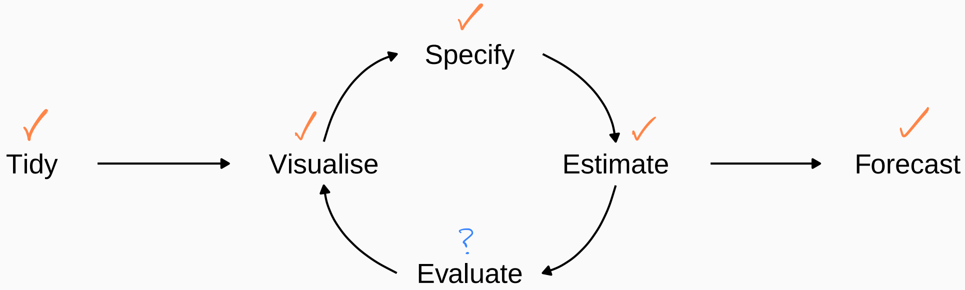
Complete Seminar Activities 1-3 before going through the slide



- Introduced four simple methods / benchmarks (mean, naïve, seas naïve, drift)

A tidy forecasting workflow - Recap

Complete Seminar Activities 1-2 before going through the slide

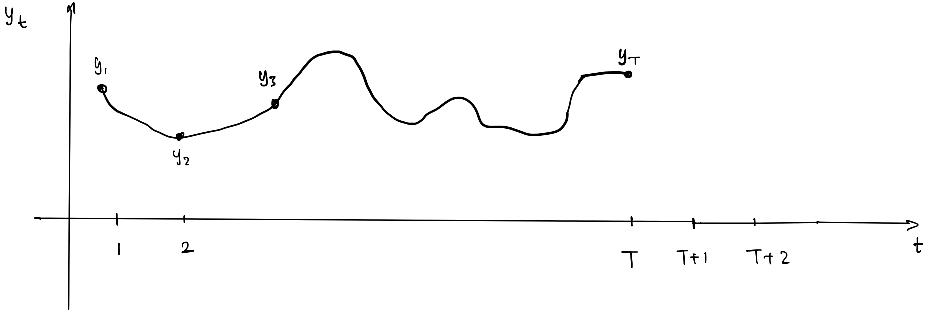


- Introduced four simple methods / benchmarks (mean, naïve, seas naïve, drift)
- Evaluate : 1. How well do these fit the data
  2. How well do they actually forecast

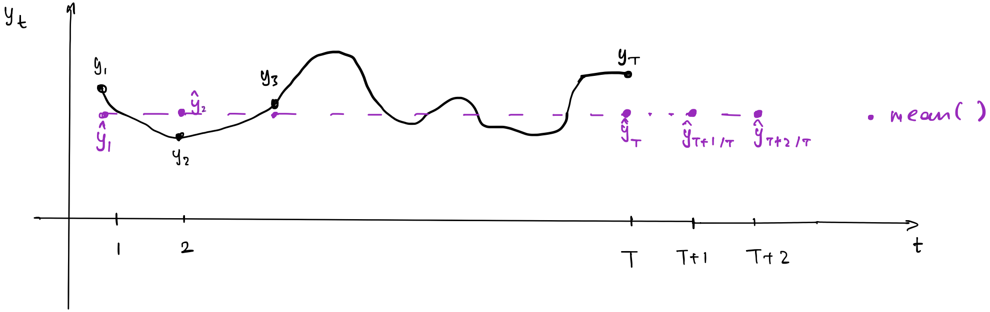
Outline

Forecasting residuals

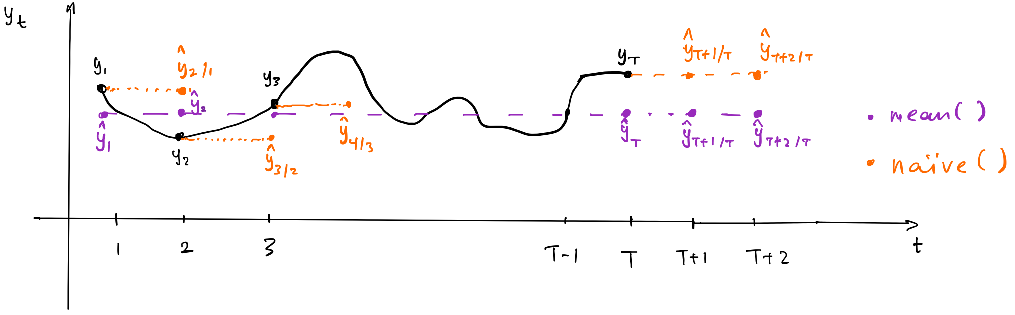
Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.



y_t
y_1
y_2
y_3
y_T
1
2
T
T+1
T+2
t



y_t
y_1
$\hat{y}_2$
y_3
y_T
$\hat{y}_1$
y_2
$\hat{y}_T$
$\hat{y}_{T+1/T}$
$\hat{y}_{T+2/T}$
• mean( )
1
2
T
T+1
T+2
t


y_t
y_1
$\hat{y}_{2|1}$
$\hat{y}_2$
y_3
$\hat{y}_1$
y_2
$\hat{y}_{3|2}$
$\hat{y}_{4|3}$
y_T
$\hat{y}_{T+1|T}$
$\hat{y}_{T+2|T}$
$\hat{y}_T$
$\hat{y}_{T+1|T}$
$\hat{y}_{T+2|T}$
mean( )
naïve( )
1
2
3
T-1
T
T+1
T+2
t

Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

Assumptions

1. $\{e_t\}$ uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
2. $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

Useful properties (for distributions & prediction intervals)

3. $\{e_t\}$ have constant variance.
4. $\{e_t\}$ are normally distributed.

ACF of residuals

- We assume that the residuals are white noise (uncorrelated, mean zero, constant variance). If they aren't, then there is information left in the residuals that should be used in computing forecasts.
- So a standard residual diagnostic is to check the ACF of the residuals of a forecasting method.

$H_0: \rho_\ell = 0$

- We expect these to look like white noise.

Portmanteau tests

Consider a whole set of r_k values, and develop a test to see whether the set is significantly different from a zero set.

Ljung-Box test

$H_0: \rho_1 = \rho_2 = \dots = \rho_\ell = 0 \sim WN$

$$Q^* = T(T+2)\sum_{k=1}^{\ell}(T-k)^{-1}r_k^2 \sim \chi^2_{\ell-k}$$

$k =$ no. of parameters in the model

$\ell > k$

where ℓ is max lag being considered and T is number of observations.

- My preferences: $\ell = 10$ for non-seasonal data, $\ell = 2m$ for seasonal data.
- Better performance, especially in small samples.

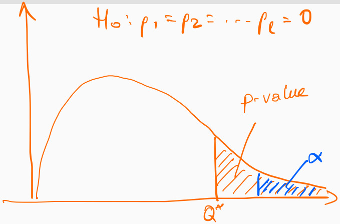
Portmanteau tests

- If data are WN, Q^* has χ^2 distribution with $(\ell - K)$ degrees of freedom where K = no. parameters in model.
- When applied to raw data, set $K = 0$.
- `lag` = ℓ, `dof` = K

```
augment(fit) %>%
  features(.resid, ljung_box, lag=10, dof=0)
```

$H_0: \rho_1 = \rho_2 = \cdots \rho_\ell = 0$

```
                                         Q*       p-value
## # A tibble: 1 x 4
##   Symbol .model        lb_stat lb_pvalue
##   <chr>  <chr>           <dbl>     <dbl>
## 1 FB     NAIVE(Close)     12.1     0.276
```



Seminar Activities 3-4

Cannot reject H_0 if $Q^* > \alpha$, ie WN

Outline

Forecast distributions

Assuming residuals: have zero mean, are uncorrelated, normal, with variance = $\hat{\sigma}^2$:

Mean:	$y_{T+h\|T} \sim N(\bar{y}, (1 + 1/T)\hat{\sigma}^2)$
Naïve:	$y_{T+h\|T} \sim N(y_T, h\hat{\sigma}^2)$
Seasonal naïve:	$y_{T+h\|T} \sim N(y_{T+h-m(k+1)}, (k + 1)\hat{\sigma}^2)$
Drift:	$y_{T+h\|T} \sim N(y_T + \frac{h}{T-1}(y_T - y_1), h\frac{T+h}{T}\hat{\sigma}^2)$

where k is the integer part of $(h - 1)/m$.

Note that when $h = 1$ and T is large, these all give the same approximate forecast variance: $\hat{\sigma}^2$.

So for h=1 in-sample variance can be used (no need to do anything more)

NAÏVE METHOD : $\hat{y}_{T+h|T} = y_T$

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RANDOM WALK MODEL : $y_t = y_{t-1} + \varepsilon_t \quad , \varepsilon_t \sim iid\, N(0, \sigma^2)$

$t = 1, \ldots, T$

NAÏVE METHOD : $\hat{y}_{T+h|T} = y_T$

RANDOM WALK MODEL : $y_t = y_{t-1} + \varepsilon_t$, $\varepsilon_t \sim iid\ N(0, \sigma^2)$

$t = 1, \ldots, T$

Distinguish between residuals $e_t = y_t - \hat{y}_{t|t-1}$ & model errors ε_t

we estimate σ^2 by $\hat{\sigma}^2$ var of residuals

NAÏVE METHOD : $\hat{y}_{T+h/T} = y_T$

RANDOM WALK MODEL : $y_t = y_{t-1} + \varepsilon_t$, $\varepsilon_t \sim \text{iid } N(0, \sigma^2)$

$t = 1, \dots, T$

Distinguish between residuals $e_t = y_t - \hat{y}_{t|t-1}$ & model errors ε_t

we estimate σ^2 by $\hat{\sigma}^2$ var of residuals

Question: What would the RW model look like at T+h? Let's do it sequentially.

$y_t = y_{t-1} + \varepsilon_t \qquad t = 1, \ldots, T. \qquad \varepsilon_t \sim iid\, N(0, \sigma^2)$

Let's go out-of-sample/future $t = T + \underline{1}$.

$y_t = y_{t-1} + \varepsilon_t \qquad t = 1, \ldots, T. \qquad \varepsilon_t \sim iid\, N(0, \sigma^2)$

Let's go out-of-sample/future $t = T+1$.

$$y_{T+1} = y_T + \varepsilon_{T+1}$$

$$y_{T+2} = y_{T+1} + \varepsilon_{T+2} = y_T + \varepsilon_{T+1} + \varepsilon_{T+2}$$

$$y_{T+3} = y_{T+2} + \varepsilon_{T+3} = y_T + \varepsilon_{T+1} + \varepsilon_{T+2} + \varepsilon_{T+3}$$

$$y_{T+h} = y_T + \sum_{i=0}^{h} \varepsilon_{T+h-i}$$

So y_T + all errors between y_T + y_{T+h}

mean

$$E(y_{T+h}) = E(y_T) + \sum_{i=0}^{h-1} E(\varepsilon_{T+h-i}) = y_T$$

$$\Rightarrow \quad \hat{y}_{T+h/T} = y_T$$

a sensible model for the naive method.

variance

$$var(y_{T+h}) = var(y_T) + \sum_{i=0}^{h-1} var(\varepsilon_{T+h-i})$$

$$= 0 + \underbrace{(\sigma^2 + \sigma^2 + \dots + \sigma^2)}_{h} + cov(\quad)$$

$i = 0 \quad 1 \quad \dots \quad h-1$

$T+h-i = T+h \quad T+h-1 \quad T+\cancel{h}-\cancel{h}+1$

$$\Rightarrow \quad var(\hat{y}_{T+h|T}) = h\sigma^2$$

(notice this depends on h)

HOMEWORK: derive mean & variance for the mean model

$y_t = \mu + \varepsilon_t \qquad \varepsilon_t \sim \text{iid } N(0, \sigma^2)$

$\boxed{\begin{array}{l}\text{RECALL mean method} \\ \hat{y}_{T+h} = \bar{y}\end{array}}$

At T+h $\quad y_{T+h} = \mu + \varepsilon_{T+h}$

mean $\quad E(y_{T+h}) = \mu + \cancel{E(\varepsilon_{T+h})}^{0} = \mu \implies \hat{y}_{T+h|T} = \mu$

which we estimate using the sample mean: $\quad \hat{\mu} = \frac{1}{T}\sum_{t=1}^{T} y_t = \bar{y}$ $\quad$ a sensible model for the mean method

variance $\quad \text{var}(y_{T+h}) = \text{var}(\mu) + \text{var}(\varepsilon_{T+h}) = \text{var}(\bar{y}) + \text{var}(\varepsilon_{T+h})$

$$\text{var}(\bar{y}) = \frac{1}{T^2}\sum_{t=1}^{T}\text{var}(y_t) = \frac{1}{T^2}\sum_{t=1}^{T}\text{var}(\mu+\varepsilon_t) = \frac{1}{T^2}\sum_{t=1}^{T}\text{var}(\varepsilon_t) = \frac{1}{T^2}T\sigma^2 = \frac{\sigma^2}{T}$$

$\implies \text{var}(y_{T+h}) = \frac{\sigma^2}{T} + \sigma^2 = \sigma^2\left(1 + 1/T\right)$ $\quad$ (this does not depend on h)

Prediction intervals

- Assuming forecast errors are normally distributed, then a 95% PI is

$$\hat{y}_{T+h|T} \pm 1.96\hat{\sigma}_h$$

where $\hat{\sigma}_h$ is the st dev of the h-step distribution.

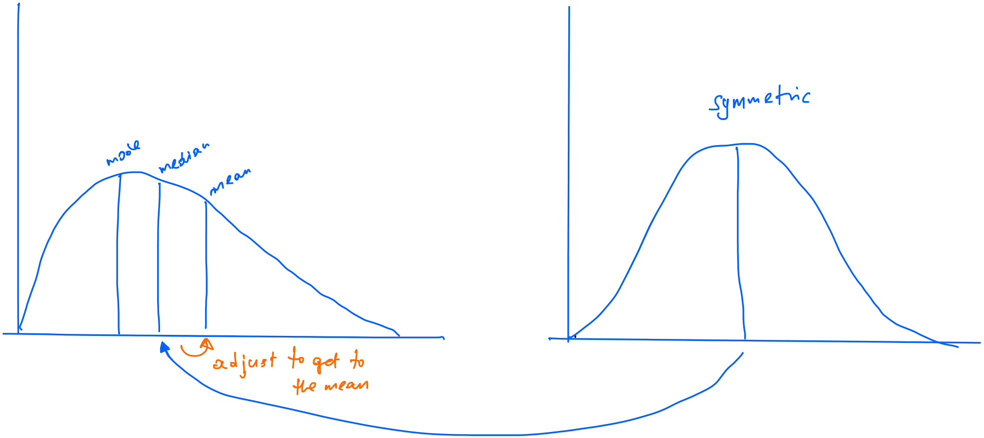
- When $h = 1$, $\hat{\sigma}_h$ can be estimated from the residuals.
- Point forecasts often useless without a measure of uncertainty (such as prediction intervals).
- Prediction intervals require a stochastic model (with random errors, etc).
- Usually too narrow due to unaccounted uncertainty.

Sources of uncertainty:

1. model errors
2. estimation uncertainty for both model & parameters
3. model choice

Seminar Activity 5

Outline


mode
median
mean
symmetric
adjust to get to
the mean

Outline

Decomposition models

`decomposition_model()` creates a decomposition model $\hat{A}_t$

- You must provide a method for forecasting the `season_adjust` series. $\hat{S}_t$
- A seasonal naive method is used by default for the `seasonal` components.
- The variances from both the seasonally adjusted and seasonal forecasts are combined.

Seminar Activity 6

Outline

SUMMARY

Train : residual diagnostics

- in-sample
- one-step
- fitted values (or forecasts)

Test : forecast evaluation

- hold-out sample (out-of-sample)
- multi-step
- forecasts

SUMMARY

Train: residual diagnostics

- in-sample
- one-step
- fitted values (or forecasts)

→

- evaluation versus model assumptions
  - uncorrelated
  - constant variance
  - mean zero
  - normal

  $\sim$ iid $N(0, \sigma^2)$

Test: forecast evaluation

- hold-out sample (out-of-sample)
- multi-step
- forecasts

SUMMARY

Train : residual diagnostics

- in-sample
- one-step
- fitted values (or forecasts)

Test : forecast evaluation

- hold-out sample (out-of-sample)
- multi-step
- forecasts

- ME (bias)
- MAE , MSE → RMSE
- MAPE
- RMSSE , MASE

Seminar Activity 7-8

Outline

Creating the rolling training sets

There are three main rolling types which can be used.

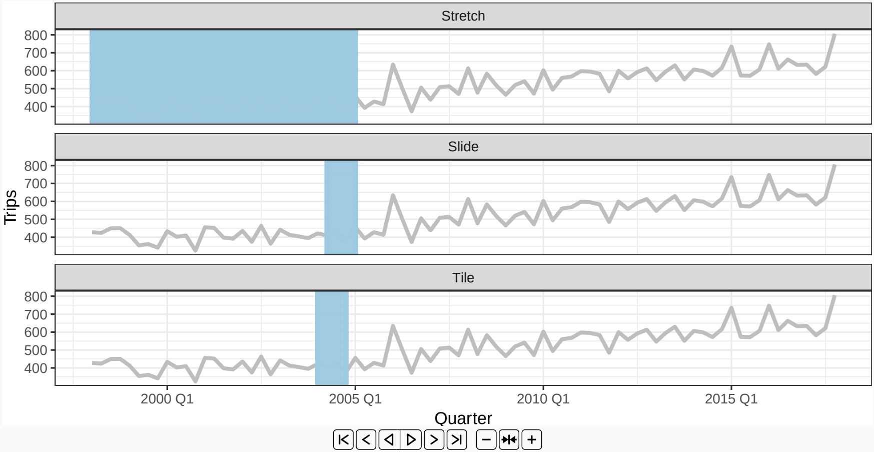
- Stretch: extends a growing length window with new data.
- Slide: shifts a fixed length window through the data.
- Tile: moves a fixed length window without overlap.

Three functions to roll a tsibble: `stretch_tsibble()`, `slide_tsibble()`, and `tile_tsibble()`.

For time series cross-validation, stretching windows are most commonly used.

Disadvantage: computationally intensive / time committment

Creating the rolling training sets



Seminar Activity 9