APM 466 Assignment 2

Mathematical Theory of Finance

Due April 2, 2018

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Requirement:

Calculate the 1,3 and 5-year probability of default of Loblaw's under each of the following assumptions:

- ✓ A Markov chain model with two states: solvency and default, calibrated to one of its bond prices.
- ✓ A Merton model, using company's assets, company's liabilities and asset volatilities implied by their stock volatility.

The assignment must contain: all explanations of the work done, all assumptions made, and the results with probabilities in the table and chart formats.

Part 1

We first need to give an insight into Markov chain model.

"When the default process follows a Markov Chain the probabilities of default/solvency for period (t_i, t_{i+1}) are given by the matrix": [1]

	Solvency	Default
Solvency	q_i	$1 - q_i$
Default	0	1

"And the Markov Chain for the constant credit Spread $h_i = h = -\ln(q_i)$" [2]

Here are some assumption for Markov chain model.

We can also use p_i to represent the probability of default, where $p_i = 1 - q_i$; We conclude that, $p_i + q_i = 1$.

Moreover, assume the identical and independent of the probability for each stage.

We use the formula $Value = clean\ price + AI = \sum p_i e^{-r_i t_i} q_i$

In the nth period, the table of Markov Chain model will then be:

	Solvency	Default
Solvency	$q^n\ or\ (1-p)^n$	$1 - q^n or\ 1 - (1-p)^n$
Default	0	1

[1] *MAT1856S/APM466S, Luis A. Seco, Lecture Slides, Portfolio Credit Risk – Part 1*
[2] *MAT1856S/APM466S, Luis A. Seco, Lecture Slides, Portfolio Credit Risk – Part 1*

Now, we can calculate using the data obtained. [3]

Loblaw CoLtd	3.748	2019-Mar-12	101.49
Canada	0.250	2018-May-01	99.91
Canada	0.750	2019-May-01	99.00

Today is March 22th,2018, round to April 1st, 2018. (Assume the same yield rate less than one year)

For the Canada bond maturity on May 1st, 2018, its last payment date was November 1st, 2017, which is 5 months ago.

Dirty Price:

$$Dirty\ Price = Accrued\ Interest + Clean\ Price = \sum_i p_i e^{-r(t_i)t_i}$$

Accrued Interest

$$Accrued\ Interest = \frac{\#days\ since\ last\ payment}{365} * Annual\ coupon\ rate$$

$$Dirty\ Price = 99.91 + \frac{5}{12} * 0.250 = 100.01417 = \left(\frac{0.250}{2} + 100\right) e^{-r_1 * \frac{1}{12}}$$

$$100.01417 = 100.125 e^{-r_1 * \frac{1}{12}}$$

$$e^{-r_1 * \frac{1}{12}} = \frac{100.01417}{100.125} = 0.9989$$

$$r_1 * \frac{1}{12} = 0.00111; r_1 = 0.01329$$

Loblaw:

$$Value = Clean\ Price + AI = \sum_i p_i e^{-r(t_i)t_i} q_i$$

Maturity of Loblaw is Mar 12th, 2019, we round it to Mar 1st, 2019.

There are two more cash flows, one is on its maturity, and the other one is on Sep 1st, 2018.

Sep 1st, 2018 is 5 months away from now; and Mar 1st, 2019 is 11 months away from now. Last payment was 1 month ago.

Let q be the monthly probability of solvency.

$$101.49 + \frac{1}{12} * 3.748 = \frac{3.748}{2} * e^{-r_1 * \frac{5}{12}} * q^5 + \left(\frac{3.748}{2} + 100\right) e^{-r_1 * \frac{11}{12}} * q^{11}$$

$$101.8023 = 1.86365 * q^5 + 100.64045 * q^{11}$$

$$q = 0.999369$$

Then, by Markov chain model,

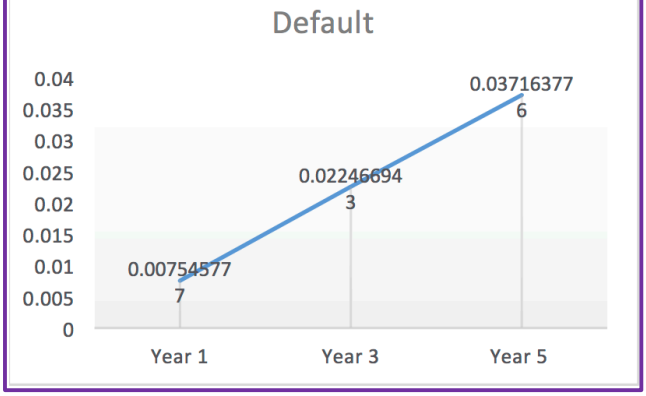
One-year solvency is q^{12}; default: $1 - q^{12}$

Three-year solvency is q^{36}; default: $1 - q^{36}$

Five-year solvency is q^{60}; default: $1 - q^{60}$

Finally, use Excel to obtain the table and graph shown on the right.

0.999369	Year 1	Year 3	Year 5
Solvency	0.992454	0.97753306	0.96283622
Default	0.007546	0.022467	0.037164



[3] "Canadian Fixed Income", http://www.pfin.ca/canadianfixedincome/Default.aspx

Part 2

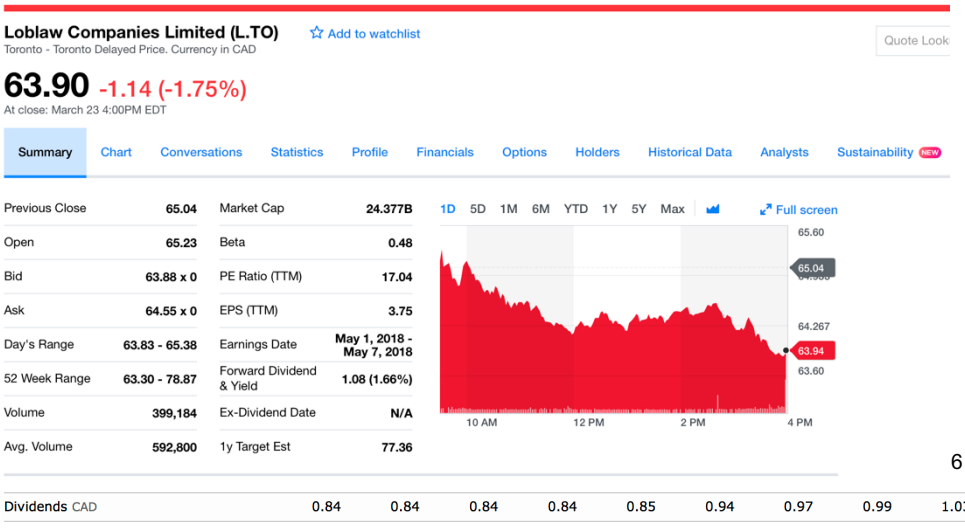
According to slides, a Merton model,

- "Introduced the view that equity value is a call option on the value of the assets of the firm, with a strike price equal to the firm's debt.
- In particular the stock price embodies a forecast of the firms default probabilities, in the same way that an option embodies an implied forecast of the option being exercised".[4]
- Firm value: V_T
- Debt: K
- If $V_t > k$: $\begin{cases} Debt\ holder: k \\ Share\ holder: V_t - k \end{cases}$
- If $V_t < k$: $\begin{cases} Debt\ holder: V_t \\ Share\ holder: 0 \end{cases}$
- $S_T = \max(V_T - K, 0)$
- $B_T = V_t - \max(V_t - K, 0) = \min(V_T, k)$

Pricing equity, assumptions:[5]

1. "Assume the firm's value follows a geometric Brownian motion process
2. Assume no transaction costs (including bankruptcy costs)
3. Stock price is the value of the option on the firm's assets, can price it with the Black-Scholes methodology. Tradeable assets."

Now we can search the values of Loblaw.



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Dividends CAD	0.84	0.84	0.84	0.84	0.85	0.94	0.97	0.99	1.03	1.07	1.07

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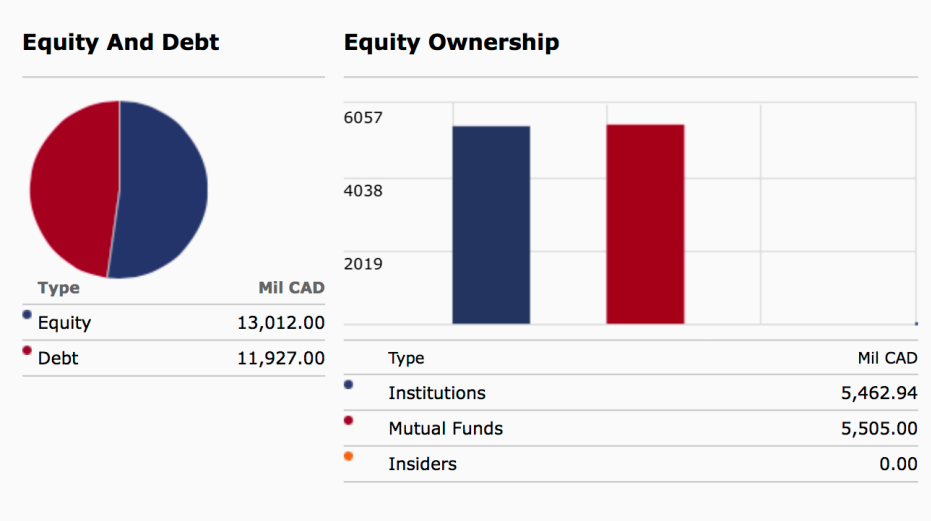
[4] *MAT1856S/APM466S, Luis A. Seco, Lecture Slides, Portfolio Credit Risk 3*
[5] *MAT1856S/APM466S, Luis A. Seco, Lecture Slides, Portfolio Credit Risk 3*
[6] *Yahoo Finance, Loblaw, https://ca.finance.yahoo.com/quote/L.TO?p=L.TO*
[7] *Morningstar, Loblaw, http://quote.morningstar.ca/Quicktakes/stock/keyratios.aspx?t=L&region=CAN&culture=en-CA&ops=clear*



[8]

We download the Max period historical data from Yahoo Finance[9] and open in Excel.

In a new column, let $\text{return} = \frac{Price_{close} - Price_{open}}{Price_{open}}$;

$ie. for\ 2008-04-15; price\ open\ is\ 30.549999; price\ close\ is\ 31.76$; thus; return is $\frac{31.76-30.549999}{30.549999} = 0.039607;$

We then generate return of every daily prices. After Deleting all the null values, we are able to obtain the daily standard deviation by entering STDEV.S to all the returns, which is 0.012923549. To get yearly standard deviation, multiply the daily value by the square root of number of business days, which is around 250-260, we use 255 days in our calculation. Therefore, $0.012923549 * sqrt(255) = 0.20637$ is the yearly standard deviation.

Thus,
$\text{Price} = \$63.90$
$\text{Dividends: } 1.07$
$\text{Equity: } 13{,}012.00 \text{ Mil CAD}$
$\text{Debt: } 11{,}927.00 \text{ Mil CAD}$
$\text{Risk free rate: } 2.677\%$ (use options calculator; set days to expiration as 365)[10]
$k = De^{rt}$
$k_1 = 11{,}927.00 * e^{2.677\% * 1} = 12{,}250.59$
$k_3 = 11{,}927.00 * e^{2.677\% * 3} = 12{,}924.37$
$k_5 = 11{,}927.00 * e^{2.677\% * 5} = 13{,}635.20$

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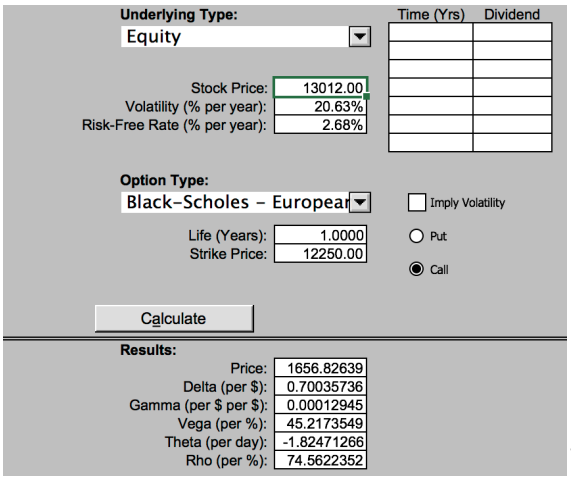
[8] *Morningstar, Loblaw,* http://quote.morningstar.ca/Quicktakes/stock/keyratios.aspx?t=L&region=CAN&culture=en-CA&ops=clear

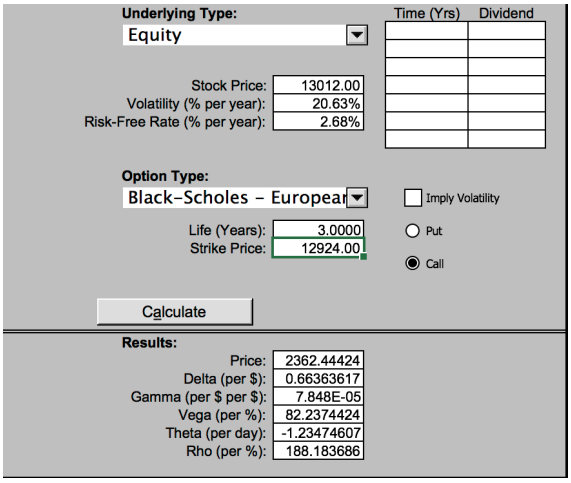
[9] *Yahoo Finance,* https://ca.finance.yahoo.com/quote/L.TO/history?period1=789886800&period2=1521950400&interval=1d&filter=history&frequency=1d

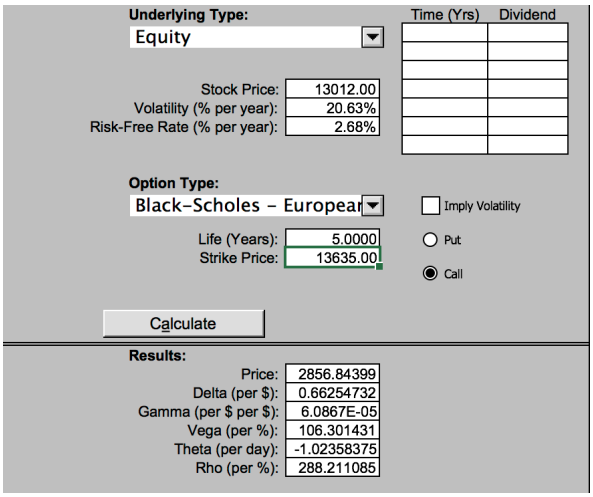
[10] *Option Calculator,* http://www.cboe.com/framed/IVolframed.aspx?content=https%3a%2f%2fcboe.ivolatility.com%2fcalc%2findex.j%3fcontract%3d2660B4FB-47DF-41C4-BA33-C0E1D5F4B131&sectionName=SEC_TRADING_TOOLS&title=CBOE%20-%20IVolatility%20Services



Year-1
Stock Price: 13,012.00
Volatility (SDyear) = 0.20637
Risk-Free rate: 2.677%
Life: 1 year
Strike Price(k_1) = 12,250
Get:
Price: 1656.83
Delta: 0.70036

Year-3
Stock Price: 13,012.00
Volatility (SDyear) = 0.20637
Risk-Free rate: 2.677%
Life: 3 years
Strike Price(k_1) = 12,924
Get:
Price: 2362.44
Delta: 0.6637





Year-5
Stock Price: 13,012.00
Volatility (SDyear) = 0.20637
Risk-Free rate: 2.677%
Life: 5 years
Strike Price(k_1) = 13,635
Get:
Price: 2856.84
Delta: 0.6625

[11] RMFI, https://www-2.rotman.utoronto.ca/~hull/RMFI%20Software/RMFISoftwareindex.htm

Now, $\sigma_{asset} = \sigma_s * \frac{s}{v}\frac{dv}{ds}, where\ \frac{dv}{ds}\ is\ delta$

$\text{Asset} = \text{Equity} + \text{Debt} = 13{,}012.00 + 11{,}927.00 = 24{,}939.00$

When T=1, $\sigma_{asset} = 0.20637 * \frac{1656.83}{24{,}939} * 0.70036 = 0.00960$

When T=3, $\sigma_{asset} = 0.20637 * \frac{2362.44}{24{,}939} * 0.6637 = 0.01297$

When T=5, $\sigma_{asset} = 0.20637 * \frac{2856.84}{24{,}939} * 0.6625 = 0.01566$

Probability of default:

$$distance\ to\ default_t = \frac{\ln\left(\frac{V_A}{L_t}\right) - \frac{\sigma_a^2}{2}T}{\sigma_a\sqrt{T}}$$

$$\text{T} = 1: distance\ to\ default_t = \frac{\ln\left(\frac{V_A}{L_t}\right) - \frac{\sigma_a^2}{2}T}{\sigma_a\sqrt{T}} = \frac{\ln\left(\frac{13012+11927}{11927}\right) - \frac{0.00960^2}{2}*1}{0.00960*1} = 76.83$$

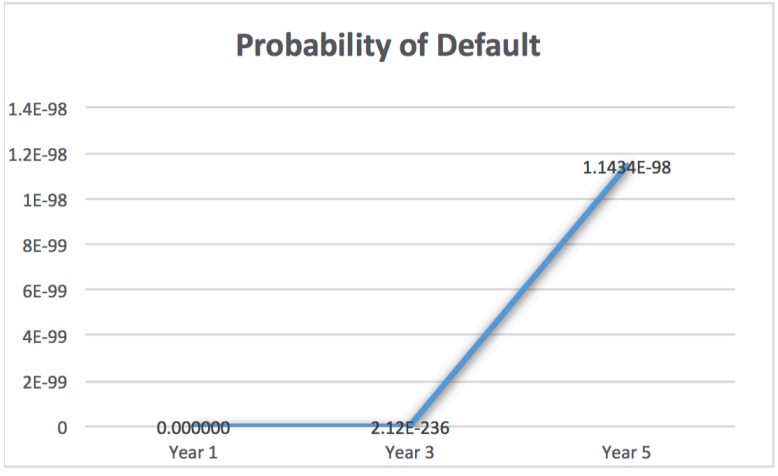
$$\text{T} = 3: distance\ to\ default_t = \frac{\ln\left(\frac{V_A}{L_t}\right) - \frac{\sigma_a^2}{2}T}{\sigma_a\sqrt{T}} = \frac{\ln\left(\frac{13012+11927}{11927}\right) - \frac{0.01297^2}{2}*3}{0.01297*\sqrt{3}} = 32.81$$

$$\text{T} = 5: distance\ to\ default_t = \frac{\ln\left(\frac{V_A}{L_t}\right) - \frac{\sigma_a^2}{2}T}{\sigma_a\sqrt{T}} = \frac{\ln\left(\frac{13012+11927}{11927}\right) - \frac{0.01566^2}{2}*5}{0.01566*\sqrt{5}} = 21.05$$

Then, use Excel to generate the graph. Set N(-d1) as norm.dist(-d1,0,1,True), which is the probability of default, then we get probability of default approximate to 0. This result implies the little probability that Loblaw will default.

T	1	3	5
d1	76.83	32.81	21.05
N(-d1)	0.000000	2.12E-236	1.1434E-98

YEAR	Year 1	Year 3	Year 5
Prob Default	0.000000	2.12E-236	1.1434E-98



Reference

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4. Yahoo Finance, Loblaw, *https://ca.finance.yahoo.com/quote/L.TO?p=L.TO*

5. Morningstar, Loblaw, *http://quote.morningstar.ca/Quicktakes/stock/keyratios.aspx?t=L&region=CAN&culture=en-CA&ops=clear*

6. Option Calculator, *http://www.cboe.com/framed/IVolframed.aspx?content=https%3a%2f%2fcboe.ivolatility.com%2fcalc%2findex.j%3fcontract%3d2660B4FB-47DF-41C4-BA33-C0E1D5F4B131&sectionName=SEC_TRADING_TOOLS&title=CBOE%20-%20IVolatility%20Services*

7. RMFI, *https://www-2.rotman.utoronto.ca/~hull/RMFI%20Software/RMFISoftwareindex.htm*