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NANOGrav Advanced Pulsar Noise Models

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ABSTRACT

A quick run down of the various noise models available in `enterprise` and `enterprise_extensions`. This is an excerpt from an upcoming paper on noise model selection.

1. NOISE MODELS

We describe all stochastic processes in PTA datasets as random Gaussian processes, applied both to achromatic noise (timing perturbations independent of observed-pulse radio frequency) and chromatic noise (timing perturbations dependent on radio frequency according to IPM and ISM physics). Gaussian processes treat an N-dimensional data vector as a single random draw from an equivalent-dimensional Gaussian distribution, whose statistical properties are completely encapsulated by its first and second moments (mean and variance, respectively).

In the following, we describe the use of two different bases with which to model temporal (and radio-band) correlations in pulsar-timing data:

$$\mathbf{s} = \mathbf{F}\,\mathbf{a}, \tag{1}$$

where $\mathbf{s}$ is an n-vector of time-series data, $\mathbf{F}$ is $n \times m$ matrix with m basis functions evaluated at n time stamps, and $\mathbf{a}$ are the weights of each basis function.

The weights of each basis function are given zero-mean Gaussian priors whose variance, ϕ, we model in a variety of different ways:

$$\langle \mathbf{a} \rangle = 0, \quad \langle \mathbf{a}\mathbf{a}^T \rangle = \phi. \tag{2}$$

These variances have parameters that we explicitly search over in a Markov-chain Monte Carlo sampling scheme. The translation between basis weight variances and the full-rank time-domain covariance is given by

$$\langle \mathbf{s}\mathbf{s}^T \rangle = \mathbf{F}\langle \mathbf{a}\mathbf{a}^T \rangle \mathbf{F}^T = \mathbf{F}\phi\mathbf{F}^T. \tag{3}$$

1.1. *Fourier-domain basis*

The simplest estimate of temporal structure in a time-series can be achieved by computing its periodogram, or equivalently by performing a Fourier analysis. The time-series is decomposed onto a basis of sine and cosine functions for each of a range of frequencies, typically varying from a fundamental frequency, $1/T$, in increments of this fundamental frequency up to the Nyquist limit, $2N/T$ (although this does not formally apply for irregularly sampled data). The basis matrix, $\mathbf{F}$, is then a sequence of concatenated sine and cosine column vectors, such that Equation 3 corresponds to the discrete matrix form of the Wiener-Khinchin theorem.

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Power-law spectrum (PLS): The most basic model of long-timescale correlations in PTA data is that of a process whose power spectral density is a power law:

$$S(f_k) = \frac{A_{\rm red}^2}{12\pi^2}\left(\frac{f_k}{\rm yr^{-1}}\right)^{-\gamma_{\rm red}} {\rm yr}^3. \tag{4}$$

This ubiquitous model is used both for modeling the stochastic GW background (where $\gamma = 13/3$ for a population of circular, inspiraling SMBHBs), intrinsic spin instabilities ($\gamma = 4.0???$), and for a Kolmogorov-turbulent ISM ($\gamma = 4.0???$)

Broken–power-law spectrum (BPLS): For additional low-frequency flexibility, we can introduce an inflected spectrum that deviates from a power-law by transitioning to a different slope at low frequencies:

$$S(f_k) = \frac{A_{\rm red}^2}{12\pi^2}\frac{(f_k/{\rm yr}^{-1})^{-\gamma_{\rm red}}}{1+(f_{\rm bend}/f_k)^{\kappa}}\,{\rm yr}^3. \tag{5}$$

Sampson et al. (2015) introduced this model for GWB characterization, where a slope transition indicates that binaries at low frequencies (wide orbital separations) remain coupled to their galactic environments, either through stellar-scattering events or viscous disk interactions. However, as a two-part power-law, the model has more general use in our noise characterization.

Free spectrum (FS): For the most flexible characterization of long-timescale processes as a function of frequency, we adopt independent variance priors for the power-spectrum amplitudes of each sine–cosine pair of red-process Fourier components (see Sec.), corresponding to frequencies k/T, with $k = 1, \ldots, N$, where T is the longest timespan in the dataset, and N is the number of Fourier component pairs. We then derive a joint posterior for all amplitudes.

$$S(f_k) = 10^{\rho_k}, \quad \text{for } k \in [1, N] \tag{6}$$

t-process spectrum (TPS): This model generalizes the Gaussian process prior on the Fourier coefficients to a multivariate Student's T distribution. We apply the conjugate prior to a Gaussian distribution (which is the inverse gamma function) to model this t-process.

$$\begin{aligned} S(f_k) &= \delta_k \times \tfrac{A_{\rm red}^2}{12\pi^2} \left(\tfrac{f_k}{\rm yr^{-1}}\right)^{-\gamma_{\rm red}} \,{\rm yr}^3, \\ p(\delta_k) &= f(\delta_k; \alpha = df/2, \beta = df/2), \end{aligned} \tag{7}$$

where $f(\delta_k; \alpha, \beta)$ is an inverse Gamma function prior (with shape parameters α, β) on coefficients, δ_k, that permit deviations of the spectrum from a strict power-law. This spectrum is sometimes referred to as a "fuzzy" power-law.

1.2. *Time-domain basis*

Some processes have temporal correlations that are more straightforward to model directly in the time domain, e.g. non-stationary processes. To do so, we define a course-grained time-domain basis with nodes that are separated by X!!!! days. Processes defined in this course-grained basis are transformed to the full-rank time series through a linear interpolation.

We use this time-domain basis mostly for describing DM variations and other chromatic ISM effects. As such, the basis matrix, $\mathbf{F}$, will have an additional multiplicative factor of $\nu^{-\alpha}$ for each element, where ν is the radio-frequency at which the TOA is observed, and α is a chromatic index that takes different values depending on the particular ISM mechanism causing the arrival time variation. For interstellar dispersion $\alpha = 2$.

Quasi-periodic (QP): Some DM variations can exhibit periodicity that evolves and changes over time [CITE]. The quasi-periodic covariance function (or "kernel") is the combination of a squared exponential kernel (capturing long timescale variation) and a periodic kernel:

$$\begin{aligned} k_{\rm QP}(t_1, t_2) &= \left(\tfrac{\sigma}{500\,{\rm sec}}\right)^2 + k_{\rm se}(t_1, t_2) \times k_{\rm p}(t_1, t_2) \\ &= \left(\tfrac{\sigma}{500\,{\rm sec}}\right)^2 + \sigma^2 \exp\left(-\tfrac{|t_1 - t_2|^2}{2l^2}\right) \times \\ &\qquad \exp\left(-\Gamma_p \sin^2\left(\tfrac{\pi|t_1 - t_2|}{p}\right)\right), \end{aligned} \tag{8}$$

where σ is an overall variance, l is a length scale of variation, p is the peridicity of variation, Γ_p controls the relative weighting of periodicity versus long timescale variation, and a constant variance is also added as a regularizing term. This model of temporal correlation is locally periodic, allowing the shape of the repeating part of the process to change over time.

Table 1.

Noise process	Chromatic dependence	
	Achromatic	Chromatic
Stochastic		
PLS	✓	✓
BPLS	✓	✓
FS	✓	
TPS	✓	✓
QP		✓
QPRF		✓
Deterministic		
AV		✓
TISM		✓

Quasi-periodic radio-frequency-dependent (QPRF): ISM-induced timing variations may exhibit correlations not only in time, but also in observed radio frequency. To this end, we implement a generalized model of "band noise" to allow for DM variations in different radio bands to be partially independent. We model this using a quasi-periodic kernel for the temporal correlations and a rational-quadratic kernel for the radio-band correlations:

$$\begin{aligned} k_{\rm QPRF}(t_1, t_2; \nu_1, \nu_2) &= \left(\tfrac{\sigma}{500\,{\rm sec}}\right)^2 + k_{\rm se}(t_1, t_2) \times \\ &\qquad k_{\rm p}(t_1, t_2) \times k_{\rm rq}(\nu_1, \nu_2), \end{aligned} \tag{9}$$

where

$$k_{\rm rq}(\nu_1, t\nu_2) = \left(1 + \frac{|\nu_1 - \nu_2|^2}{2\alpha l_{\rm rq}^2}\right)^{-\alpha}. \tag{10}$$

The rational-quadratic kernel is equivalent to adding together many squared exponential kernels with different length-scales, where α controls the relative weighting between short and long term correlations.

1.3. *Deterministic time-domain ISM features*

Annual variation (AV):

Transient ISM under/over-densities (TISM):

REFERENCES

Sampson, L., Cornish, N. J., & McWilliams, S. T. 2015, PhRvD, 91, 084055