Nested Pseudo Likelihood (NPL)

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Fixed Point in different spaces

Integrated value function space

$$V_\sigma = \Gamma(V_\sigma) = \int \max_a \left\{ u(x,a) + \epsilon(a) + \beta \sum_{x'} V_\sigma(x') f(x'|x,a) \right\} g(\epsilon|x')$$

Idea: Keep on updating integrated value function, V_σ, until convergence to solve the model

Choice probability space / Policy function space

$$P = \Psi(P) = \Lambda(\psi(P))$$

Idea: Keep on updating the choice probabilities, P, until convergence to solve the model.

Fixed point in policy function / choice probability space

Fixed Point:

$$P = \Psi(P) = \Lambda(\psi(P))$$

where

$$V_\sigma = \psi(P) = [I - \beta F^U(P)]^{-1} \sum_a \{P(a) * (u(a) + E[\epsilon(a)|P])\}$$

$$P(a|x) = \Lambda(V_\sigma) = \frac{1}{1 + \sum_{j \neq a} \exp\{(v(x,j) - v(x,a))/\sigma\}}$$

- $F^U(P)$ is the unconditional state transition matrix,
- $v(x,a)$ is the choice specific value function,
- $E[\epsilon(a)|P] = \gamma - \ln(P(a))$
- γ is the Euler-Mascheroni constant ≈ 0.5772

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From Bertel's slides

Nested Pseudo Likelihood Algorithm

Initialization

- Let $\hat{\theta}_f$ be an estimate of θ_f .
- Start with an initial guess for the conditional choice probabilities, $P^0 \in [0,1]^{MJ}$.

At iteration $K \geq 1$, apply the following steps:

- Step 1: Obtain a new pseudo-likelihood estimate of α, α^K, as

$$\alpha^K = \arg\max_{\alpha \in \Theta} \sum_{i=1}^{n} \ln \Psi_{\alpha,\hat{\theta}_f}(P^{K-1})(a_i|x_i) \tag{3}$$

  where $\Psi_\theta(P)(a|x)$ is the (a,x)'s element of $\Psi_\theta(P)$.
- Step 2: Update P using the arg max from step 1, i.e.

$$P^K = \Psi_{(\alpha^K,\hat{\theta}_f)}(P^{K-1}) \tag{4}$$

- Iterate in K until convergence in P (and α) is reached.