ENERGY

Pre-class Assignment: Energy

Instruction: Complete each statement by filling in the blanks from the given *list of words/phrases.

1. A woman holds a bowling ball in a fixed position. The work she does on the ball is <u>zero</u>.
2. A man pushes a very heavy load across a horizontal floor. The work done by gravity on the load is <u>zero</u>.
3. When you do <u>positive</u> work on a particle, its kinetic energy increases.
4. Friction is a <u>nonconservative</u> force.
5. The SI unit of <u>power</u> is watt.

* kinetic energy, positive, work, zero, not zero, conservative, energy, negative, gravitational potential energy, nonconservative, power, spring potential energy

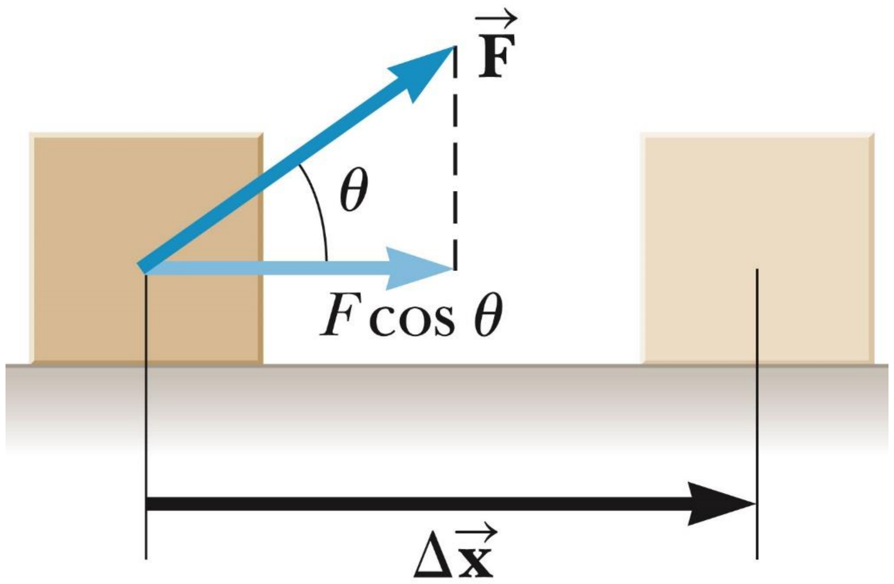
A. Work done by a Constant Force

The work done on an object by a constant force F_x is

$$W = F_x \Delta x = (F \cos \theta) \Delta x$$

where θ is the angle between the applied force $\vec{F}$ and displacement $\Delta\vec{x}$

SI unit of work: joule (J), 1 J = 1 N·m

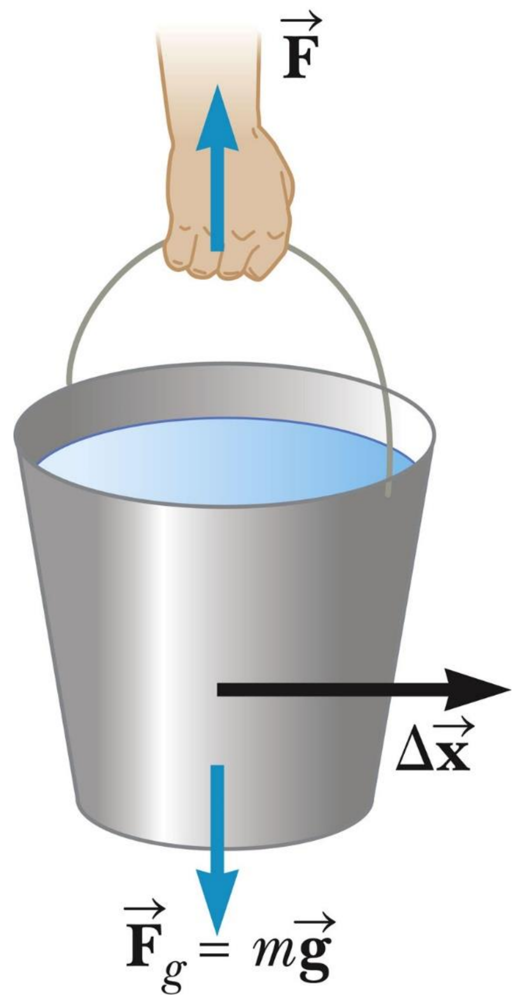


Important points about work:

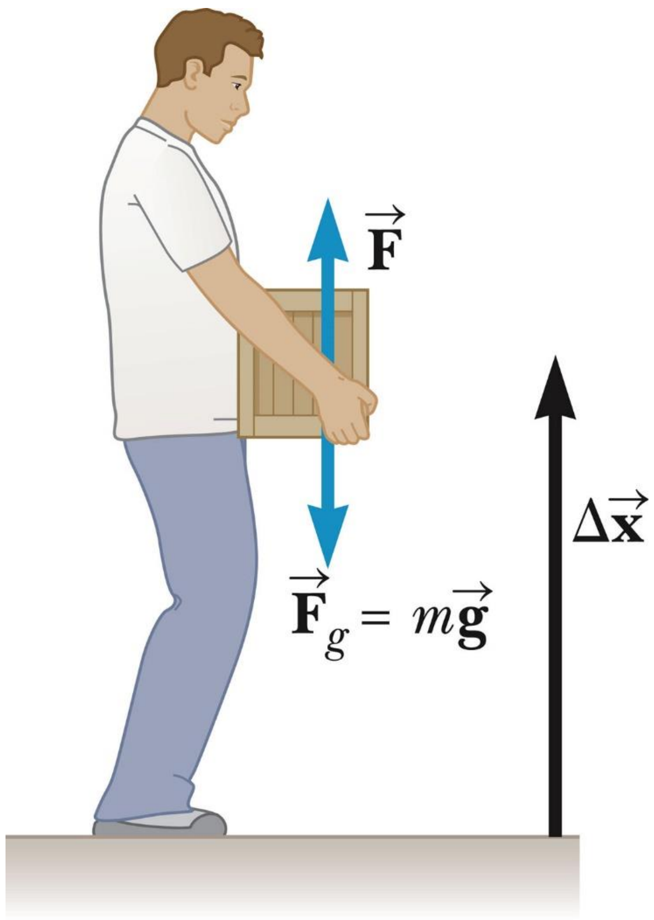
1. Work is a scalar quantity.

2. If the magnitude of displacement is zero, work is zero, even if a force is applied.

3. If the entire force is perpendicular to the displacement, there is no work done by the force at all.



4. Work can be either positive or negative, depending on whether a component of the force points in the same direction as the displacement or in the opposite direction.

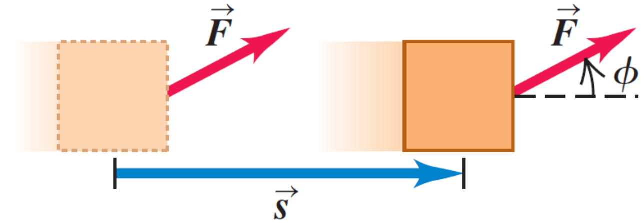
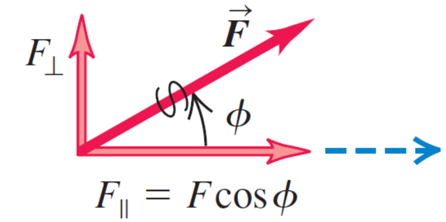
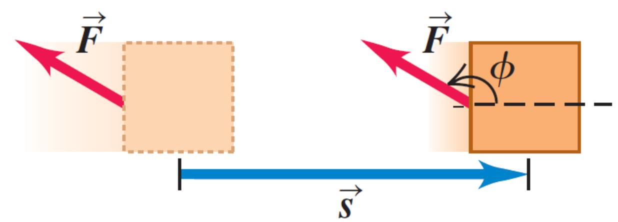
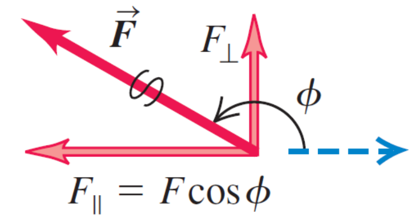
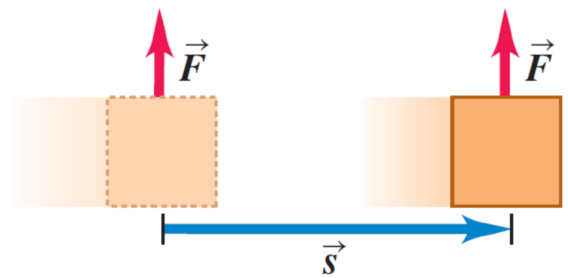
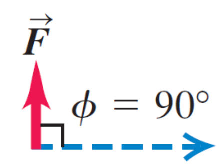


The net or total work done on an object can also be computed using one of the following equations:

$$W_{net} = \left(\sum F_x\right)\Delta x = \sum W_n$$

where $\sum F_x$ is the net external force and W_n is the individual work done by a number n = 1, 2, 3, ... of external forces acting on the object.

6.4 A constant force $\vec{F}$ can do positive, negative, or zero work depending on the angle between $\vec{F}$ and the displacement $\vec{s}$.

Direction of Force (or Force Component)	Situation	Force Diagram
(a) **Force $\vec{F}$ has a component in direction of displacement:** $W = F_{\parallel}s = (F\cos\phi)s$ Work is *positive*.		
(b) **Force $\vec{F}$ has a component opposite to direction of displacement:** $W = F_{\parallel}s = (F\cos\phi)s$ Work is *negative* (because $F\cos\phi$ is negative for $90° < \phi < 180°$).		
(c) **Force $\vec{F}$ (or force component $F_{\perp}$) is perpendicular to direction of displacement:** The force (or force component) does *no* work on the object.		

B. Kinetic Energy and Work-Energy Theorem

Kinetic energy (SI unit: J) is energy of motion. It depends on the object's speed but not its location.

$$KE = \frac{1}{2}mv^2$$

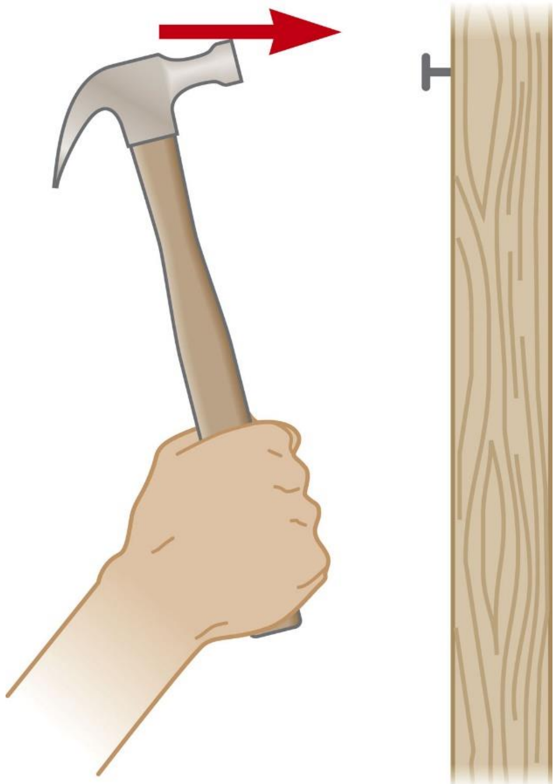
where m is the mass of the object and v is the speed of the object.

When a net force performs work on an object, the result is a change in the kinetic energy of the object.

$$W_{net} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \Delta KE$$

This is known as the Work-Energy Theorem.

The work-energy theorem deals with the work done by the net external force. It does not apply to the work done by an individual force, unless that force happens to be the only one present. It is also valid if the work is done by a varying force.



Note that energy has the same SI unit as work, the joule. Energy, like work, is a scalar quantity.

Checkpoint Questions:

1. Work is negative if the force and displacement are in the opposite direction. True or False?
2. In the figure below, a block moves to the right in the positive x-direction through the displacement $\Delta\vec{x}$ while under the influence of a force with the same magnitude F. Which of the following is the correct order of the amount of work done by the force F, from the most positive to most negative?

a) d, c, a, b

b) c, a, b, d

c) c, a, d, b

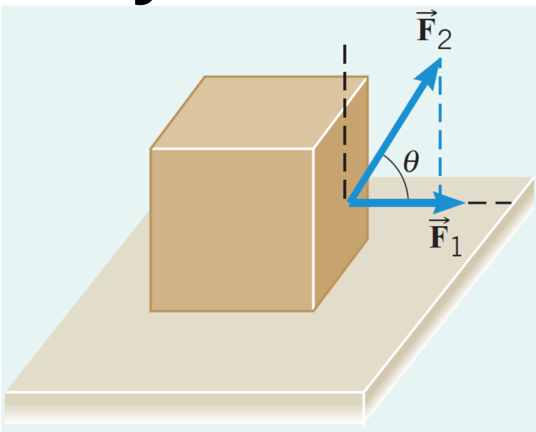
3. Two forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ are acting on the box shown below, causing the box to move across the floor. The two force vectors are drawn to scale. Which one of the following statements is correct?

a) $\overrightarrow{F_2}$ does more work than $\overrightarrow{F_1}$ does.

b) $\overrightarrow{F_1}$ does more work than $\overrightarrow{F_2}$ does.

c) Both forces do the same amount of work.

d) Neither force does any work.



4. A sailboat is moving at a constant velocity. Is work being done by a net external force acting on the boat? [Yes or No]
5. A net external force acts on a particle that is moving along a straight line. This net force is not zero. Which one of the following statements is correct?
   a) The velocity, but not the kinetic energy, of the particle is changing.
   b) The kinetic energy, but not the velocity, of the particle is changing.
   c) Both the velocity and the kinetic energy of the particle are changing.

C. Gravitational Potential Energy and Spring Potential Energy

Potential energy (SI unit: J) refers to energy that is stored in a system.

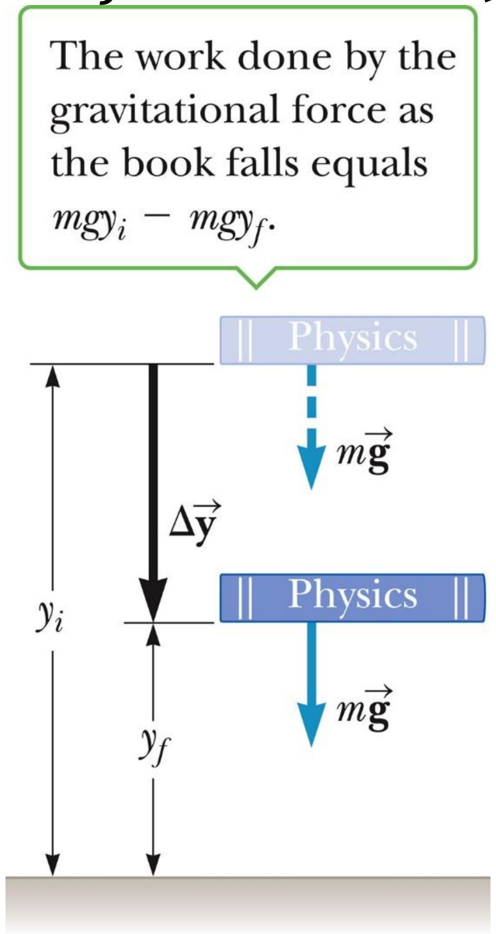
1. The gravitational potential energy is the energy that an object has by virtue of its location relative to the surface of the earth. The reference position for the heights can be taken anywhere.

$$PE_g = mgy$$

where m is mass of the object, g is the acceleration due to gravity, and y is the position of the object.

The work done on any object by the gravitational force is:

$$W_g = -(mgy_f - mgy_i) = -\Delta PE_g$$



2. The force exerted by a spring is determined by Hooke’s law:

$$F_s = -kx$$

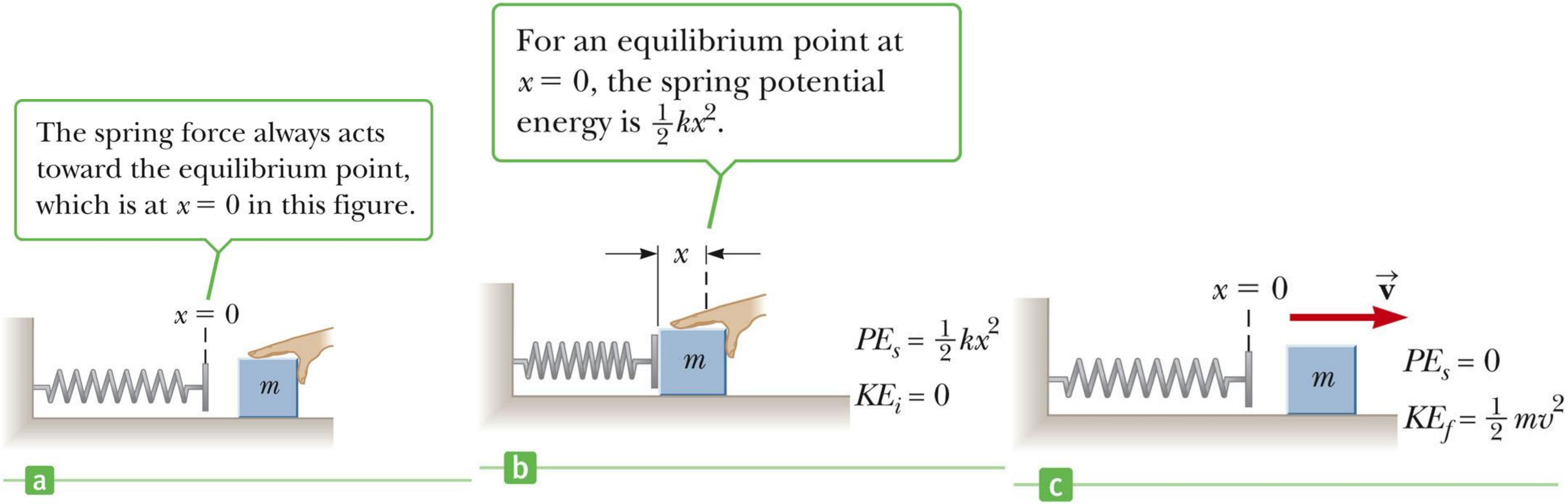
where k is the spring or force constant and x is the magnitude of displacement of the spring from its original length.

This spring force is associated with the spring (elastic) potential energy of the system. It can be thought of as the energy stored in the deformed spring (one that is either compressed or stretched from its equilibrium position). The reference position is taken at the equilibrium position of the spring.

$$PE_s = \frac{1}{2}kx^2$$

The work done on any object by the spring force is:

$$W_s = -\left(\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2\right) = -\Delta PE_s$$



D. Systems and Energy Conservation

1. Conservative and Nonconservative Forces

   a. Conservative force (gravitational force, spring force):

      i. It can be expressed as the difference between the initial and final values of a potential-energy function.

      ii. It is reversible.

      iii. It is independent of the path of the object and depends only on the starting and ending points.

iv. When the starting and ending points are the same, the total work is zero.

KE	PE	E = KE + PE
0 J	600 000 J	600 000 J
200 000 J	400 000 J	600 000 J
400 000 J	200 000 J	600 000 J
600 000 J	0 J	600 000 J

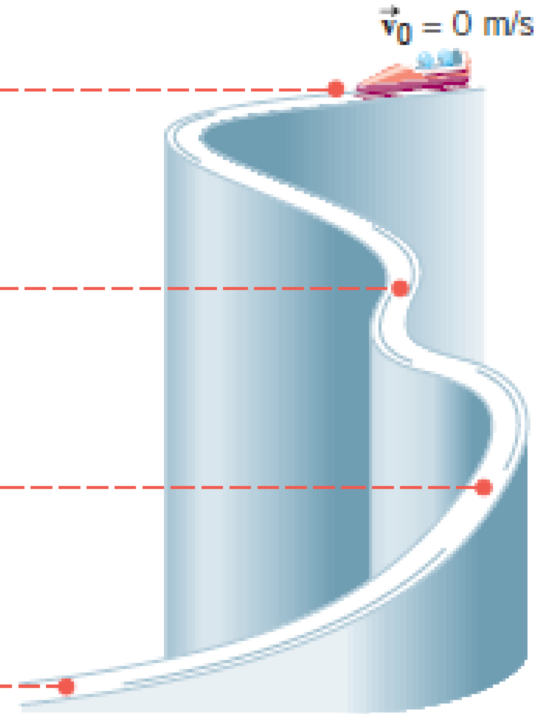


Figure 6.15 If friction and wind resistance are ignored, a bobsled run illustrates how kinetic and potential energy can be interconverted, while the total mechanical energy remains constant. The total mechanical energy is 600 000 J, being all potential energy at the top and all kinetic energy at the bottom.

b. Nonconservative forces (friction, drag force) are forces which do not have properties of conservative forces as stated above. These forces generally dissipate energy.

https://phet.colorado.edu/en/simulations/energy-skate-park-basics

2. Principle of Conservation of Energy

The work–energy theorem can be rewritten in terms of the work done by conservative forces and the work done by nonconservative forces:

$$W_{nc} + W_c = \Delta KE$$

Note that the sum of kinetic and potential energy is called *E*, the total mechanical energy of the system. The work done by nonconservative forces is also related to internal energy, which is associated with the change in state of materials (such as temperature)

$$W_{nc} = -\Delta E_{int}$$

Thus the principle of Conservation of Energy in general form is

$$\Delta KE + \Delta PE + \Delta E_{int} = 0$$

In words:

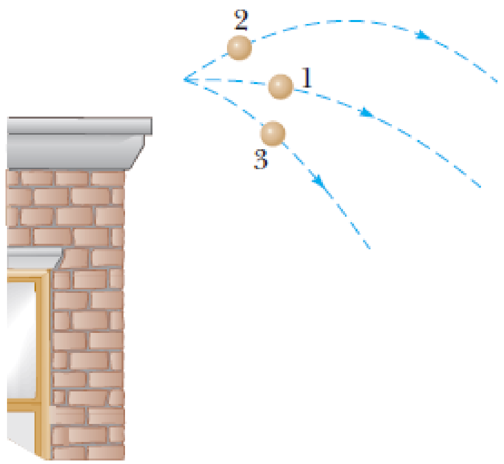
"Energy is never created or destroyed. It only changes form."

Checkpoint Questions:

1. In a simulation on earth, an astronaut in his space suit climbs up a vertical ladder. On the moon, the same astronaut makes the exact same climb. Which one of the following statements correctly describes how the gravitational potential energy of the astronaut changes during the climb?
   a) It changes by a greater amount on the earth.
   b) It changes by a greater amount on the moon.
   c) The change is the same in both cases.

2. A mass attached to a vertical spring causes the spring to stretch, and the mass to move downwards. As this happens, what can you say about the spring's potential energy (PE_s) and the gravitational potential energy (PE_g) of the mass-Earth system?
   a) both PE_s and PE_g decrease
   b) PE_s increases and PE_g decreases
   c) both PE_s and PE_g increase
   d) PE_s decreases and PE_g increases
   e) PE_s increases and PE_g is constant

3. Three identical balls are thrown from the top of a building, all with the same initial speed. The first ball is thrown horizontally, the second at some angle above the horizontal, and the third at some angle below the horizontal, as shown in the figure. Neglecting air resistance, rank the speeds of the balls as they reach the ground, from fastest to slowest.



a) 1, 2, 3
b) 2, 1, 3
c) 3, 1, 2
d) All three balls strike the ground at the same speed.

4. Bob, of mass *m*, drops from a tree limb at the same time that Esther, also of mass *m*, begins her descent down a frictionless slide. If they both start at the same height above the ground, which of the following is true about their kinetic energies as they reach the ground?
   a) Bob’s kinetic energy is greater than Esther’s.
   b) Esther’s kinetic energy is greater than Bob’s.
   c) They have the same kinetic energy.
   d) The answer depends on the shape of the slide.

5. A net external nonconservative force does positive work on a particle. Based solely on this information, you are justified in reaching only one of the following conclusions. Which one is it?
   a) The kinetic and potential energies of the particle both decrease.
   b) The kinetic and potential energies of the particle both increase.
   c) Neither the kinetic nor the potential energy of the particle changes.

d) The total mechanical energy of the particle decreases.
e) The total mechanical energy of the particle increases.

E. Power

Power is the rate of energy transfer with time.

SI unit: watt (1 W = 1 J/s)

Average power is defined as work done divided by the time interval:

$$\bar{P} = \frac{W}{\Delta t} = F\bar{v}$$

or the product of a constant force with magnitude F and average velocity with magnitude $\bar{v}$. This force F is the component of force $\vec{F}$ in the direction of the average velocity $\vec{v}_{av}$.

Instantaneous power:

$$P = Fv$$

where both the force $\vec{F}$ and velocity $\vec{v}$ must be parallel, but can change with time.

Note that in electric power generation, one kilowatt-hour (kWh) is the energy transferred in 1 h at the constant rate of 1 kW = 1 000 J/s.

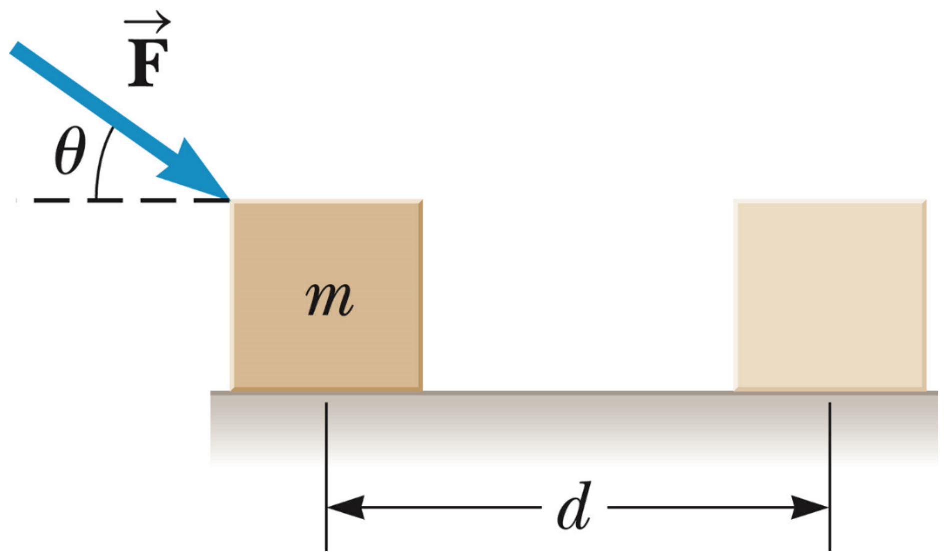
Checkpoint Questions:

1. Engine A has a greater power rating than engine B. Which one of the following statements correctly describes the abilities of these engines to do work?
   a) Engines A and B can do the same amount of work, but engine A can do it more quickly.
   b) Engines A and B can do the same amount of work in the same amount of time.
   c) In the same amount of time, engine B can do more work than engine A.

2. Is it correct to conclude that one engine is doing twice the work that another is doing just because it is generating twice the power? [Yes or No]

* A weight lifter lifts a 350-N set of weights from ground level to a position over his head, a vertical distance of 2.00 m. How much work does the weight lifter do, assuming he moves the weights at constant speed?

* A block of mass m = 2.50 kg is pushed a distance d = 2.20 m along a frictionless horizontal table by a constant applied force of magnitude F = 16.0 N directed at an angle θ = 25.0º below the horizontal as shown in the figure. Determine the work done by (a) the applied force, (b) the normal force exerted by the table, (c) the force of gravity, and (d) the net force on the block.



* Starting from rest, a 5.00-kg block slides 2.50 m down a rough 30.0^{o} incline. The coefficient of kinetic friction between the block and the incline is μ_k = 0.436. Determine (a) the work done by the force of gravity, (b) the work done by the friction force between block and incline, and (c) the work done by the normal force.

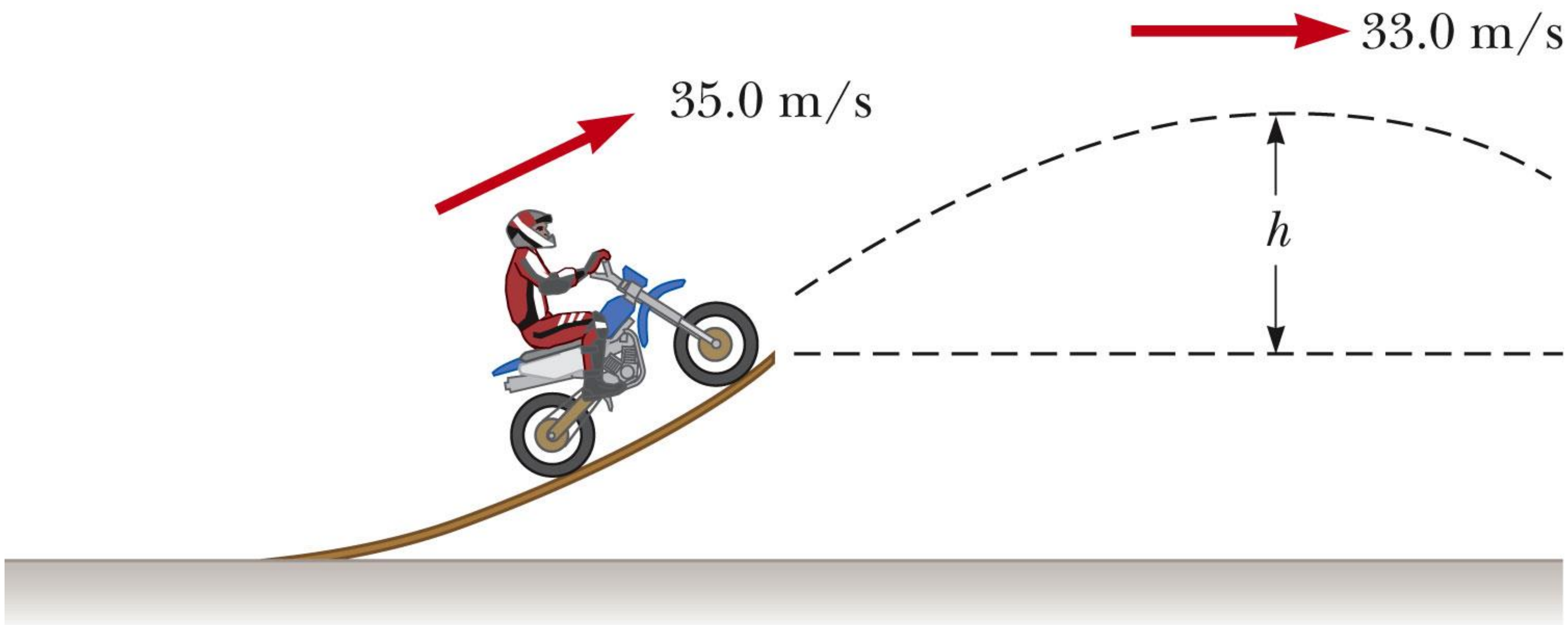
* While running, a person dissipates about 0.60 J of mechanical energy per step per kilogram of body mass. If a 60-kg person develops a power of 70 W during a race, how fast is the person running? (Assume a running step is 1.5 m long.)

* A 0.60-kg particle has a speed of 2.0 m/s at point *A* and a kinetic energy of 7.5 J at point *B*. What is (a) its kinetic energy at *A*? (b) Its speed at point *B*? (c) The total work done on the particle as it moves from *A* to *B*.

* A 0.20-kg stone is held 1.3 m above the top edge of a water well and then dropped into it. The well has a depth of 5.0 m. Taking y = 0 at the top edge of the well, what is the gravitational potential energy of the stone-Earth system (a) before the stone is released and (b) when it reaches the bottom of the well. (c) What is the change in gravitational potential energy of the system from release to reaching the bottom of the well?

* When a 2.50-kg object is hung vertically on a certain light spring described by Hooke's law, the spring stretches 2.76 cm. (a) What is the force constant of the spring? (b) If the 2.50-kg object is removed, how far will the spring stretch if a 1.25-kg block is hung on it? (c) How much work must an external agent do to stretch the same spring 8.00 cm from its unstretched position?

* A daredevil on a motorcycle leaves the end of a ramp with a speed of 35.0 m/s as in the figure below. If his speed is 33.0 m/s when he reaches the peak of the path, what is the maximum height that he reaches? Ignore friction and air resistance.



* $W = F\cos\theta\,\Delta y = (350)(\cos 0^\circ)(2.00)$

$W \approx \boxed{700\,J}$

* (a) $W_F = (F\cos\theta)\Delta x = (16\cos 25^\circ)\,2.2$

$W_F \approx \boxed{31.9\,J}$

(b) $W_n = (n\cos 90^\circ)\Delta x \approx \boxed{0}$

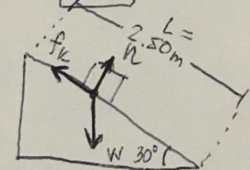
(c) $W_g = (mg\cos 90^\circ)\Delta x \approx \boxed{0}$

(d) $W_{net} = W_F + W_n + W_g$

$= 31.9\,J + 0 + 0$

$\approx \boxed{31.9\,J}$

* (a)



(c) $W_n = (n\cos 90^\circ)L$

$W_n = \boxed{0}$

$w = mg = (5)9.8 \approx 49.0\,N$

$W_g = -\Delta PE_g = -mg(y_f - y_i) = mg(y_i - y_f)$

$W_g = mg\,L\sin 30^\circ$

$= (49)(2.50)\sin 30^\circ \approx \boxed{61.3\,J}$

(b) $n = mg\cos 30^\circ$

$f_k = \mu_k n = (0.436)(49)\cos 30^\circ = 18.5\,N$

$W_f = (f_k\cos\theta)L = [(18.5)\cos 180^\circ]\,2.50 = \boxed{-46.3\,J}$

* Power $= \dfrac{\text{work done}}{\Delta t}$

$= \dfrac{(\text{work per step per unit mass})(\text{mass})(\#\text{ steps})}{\Delta t}$

or $70W = \left(0.60\,\dfrac{J/\text{step}}{kg}\right)(60\,kg)\left(\dfrac{\Delta N}{\Delta t}\right)$

where $\dfrac{\Delta N}{\Delta t} = 1.9\,\dfrac{\text{steps}}{s}$

$V_{av} = \dfrac{\Delta x}{\Delta t} = \left(\dfrac{\Delta N}{\Delta t}\right)(\text{distance traveled per step})$

$= \left(1.9\,\dfrac{\text{step}}{s}\right)\left(1.5\,\dfrac{m}{\text{step}}\right) \approx \boxed{2.9\,m/s}$

* (a) $KE_A = \frac{1}{2}mv_A^2 = \frac{1}{2}(0.60)(2.0)^2$

$KE_A \approx \boxed{1.2\,J}$

(b) $KE_B = \frac{1}{2}mv_B^2$

$v_B = \sqrt{\dfrac{2KE_B}{m}} = \sqrt{\dfrac{2(7.5\,J)}{0.60\,kg}}$

$v_B \approx \boxed{5.0\,m/s}$

(c) $W_{net} = \Delta KE = KE_B - KE_A$

$= 7.5\,J - 1.2\,J$

$\approx \boxed{6.3\,J}$

* (a) $PE_i = mgy_i = (0.20)(9.8)(1.3)$

$PE_i \simeq \boxed{2.5\,J}$

(b) $PE_f = mgy_f = (0.20)(9.8)(-5.0)$

$PE_f \simeq \boxed{-9.8\,J}$

(c) $\Delta PE = PE_f - PE_i = -9.8\,J - 2.5\,J$

$\Delta PE \simeq \boxed{-12\,J}$

* (a) $k = \dfrac{|F_g|}{|\Delta x|} = \dfrac{mg}{|\Delta x|}$

$= \dfrac{(2.50)(9.8)}{2.76 \times 10^{-2}}$

$k \simeq \boxed{8.88 \times 10^{2}\ N/m}$

(b) $(\Delta x)_2 = \frac{1}{2}(\Delta x)_1 = \frac{1}{2}(2.76\,cm) \simeq \boxed{1.38\,cm}$

(c) $W_{\text{done on spring}} = (PE_s)_f - (PE_s)_i$

$= \frac{1}{2}Kx_f^2 - 0$

$= \frac{1}{2}(8.88 \times 10^{2}\,N/m)\,(8.00 \times 10^{-2}\,m)^2$

$\simeq \boxed{2.84\,J}$

By conservation of energy

$$* \quad \frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

$$h = y_f - y_i = \frac{v_i^2 - v_f^2}{2g}$$

$$= \frac{(35)^2 - (33)^2}{2(9.8)} \simeq \boxed{6.94\,m}$$