MOTION IN TWO DIMENSIONS

Pre-class Assignment: Motion in Two Dimensions

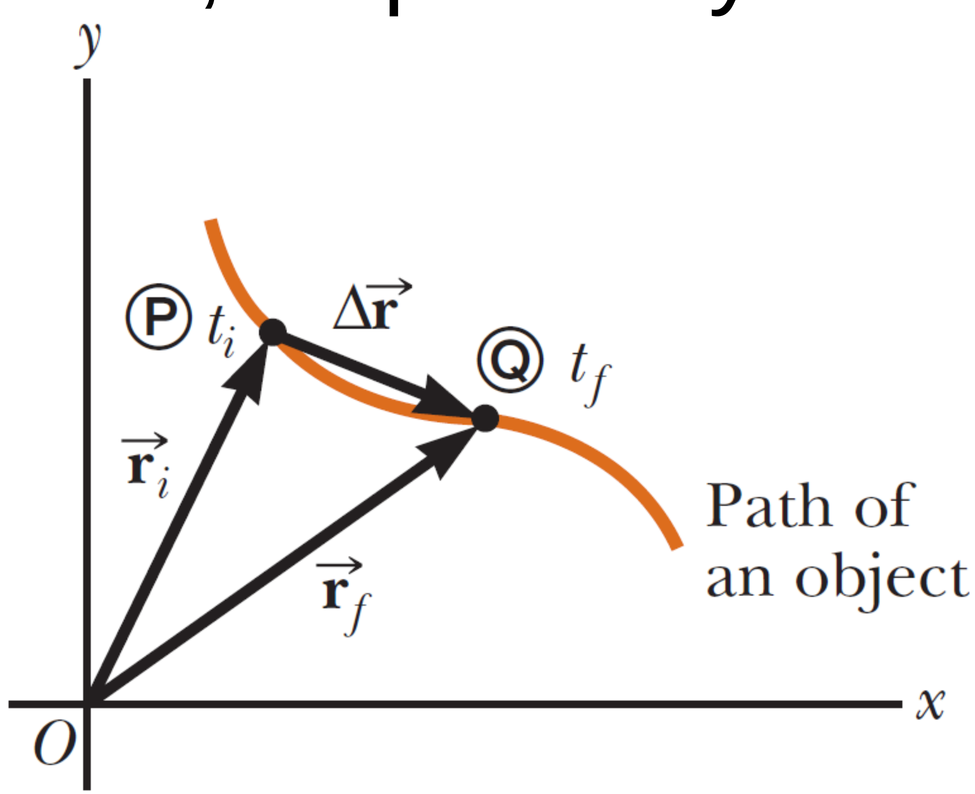
1. A car moving around a circular track with constant speed has an acceleration.
2. The horizontal component of acceleration in projectile motion is equal to 0 m/s^2.
3. The range is the horizontal distance travelled between launching and landing, assuming the projectile returns to the same vertical level at which it was fired.
4. You are in a car that is moving to the right at a constant velocity. You shoot a ball straight upward. In the absence of air resistance, the ball would land in the car.

5. Two metal balls are the same size but one weighs twice as much as the other. The two balls roll off a horizontal table with the same speed. In this situation, both projectiles hit the floor at <u>the same</u> horizontal distance from the base of the table.

* in the car, in front of the car, constant, the same, maximum height, behind the car, range, has no, different, has an, 0, -9.8

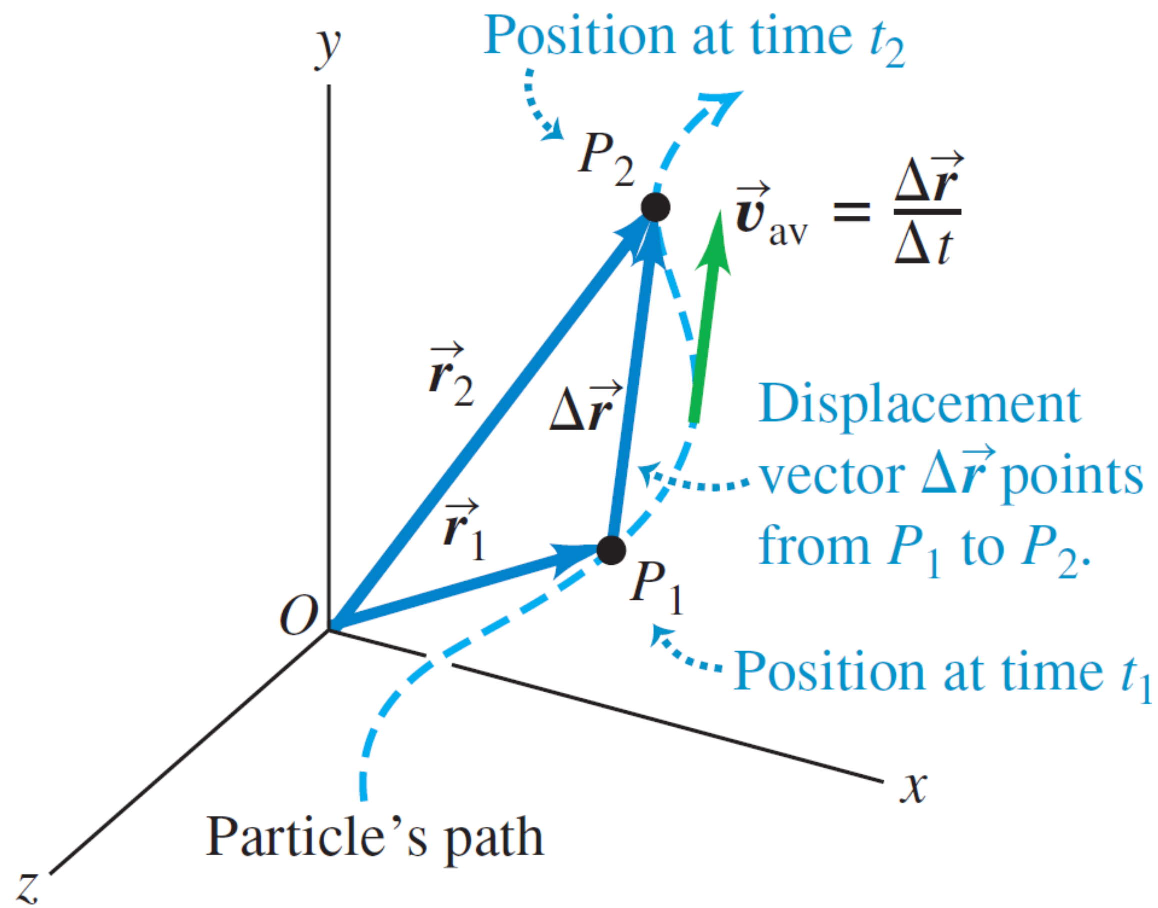
A. Displacement, Velocity, and Acceleration in Two Dimensions

1. Displacement (SI unit: m) is defined as $\Delta\vec{r} \equiv \vec{r}_f - \vec{r}_i$ where $\vec{r}_f$ and $\vec{r}_i$ are the final and initial position vectors, respectively.

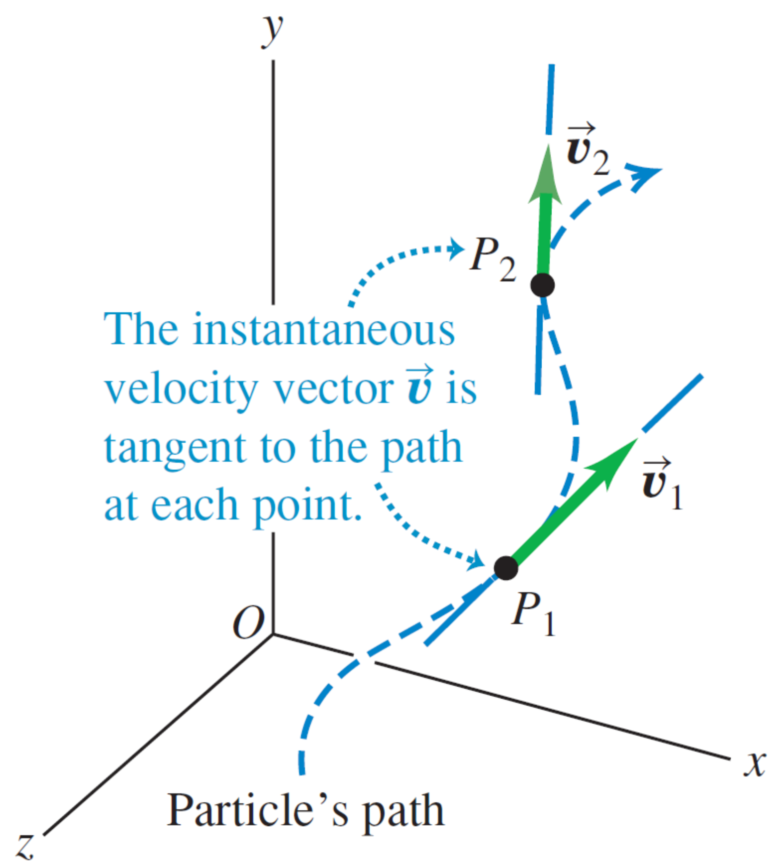


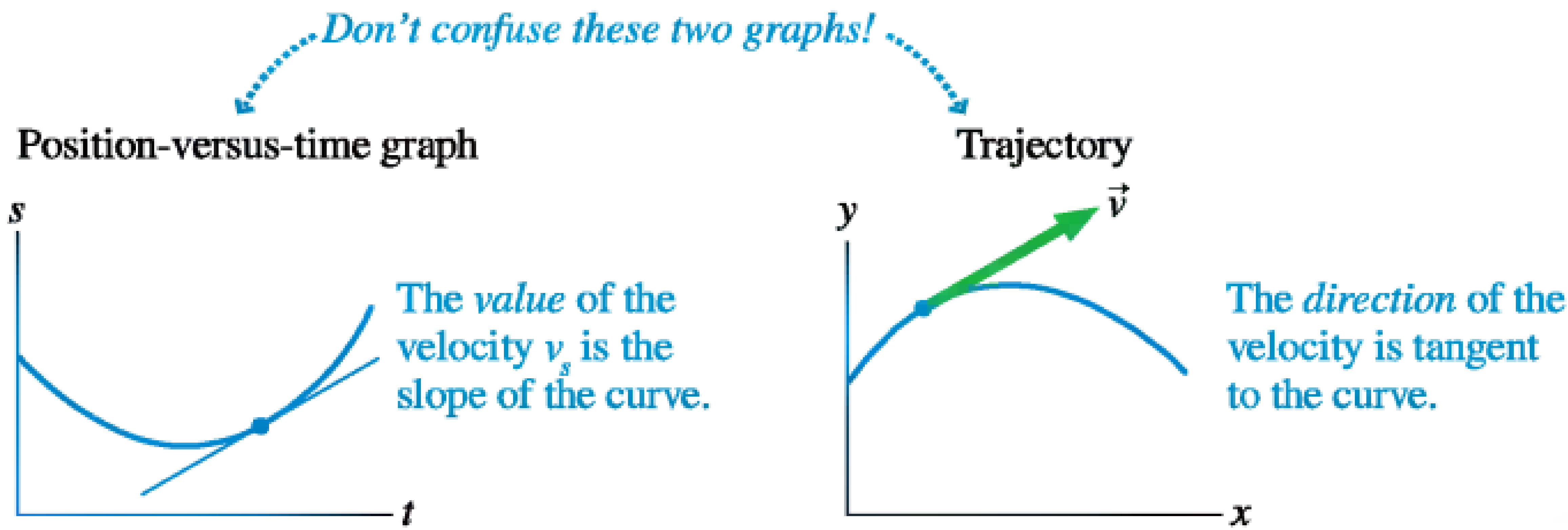
2. Velocity (SI unit: m/s)

   a. Average velocity is defined as $\vec{v}_{av} \equiv \frac{\Delta\vec{r}}{\Delta t}$. Its direction is the same as the displacement vector $\Delta\vec{r}$.



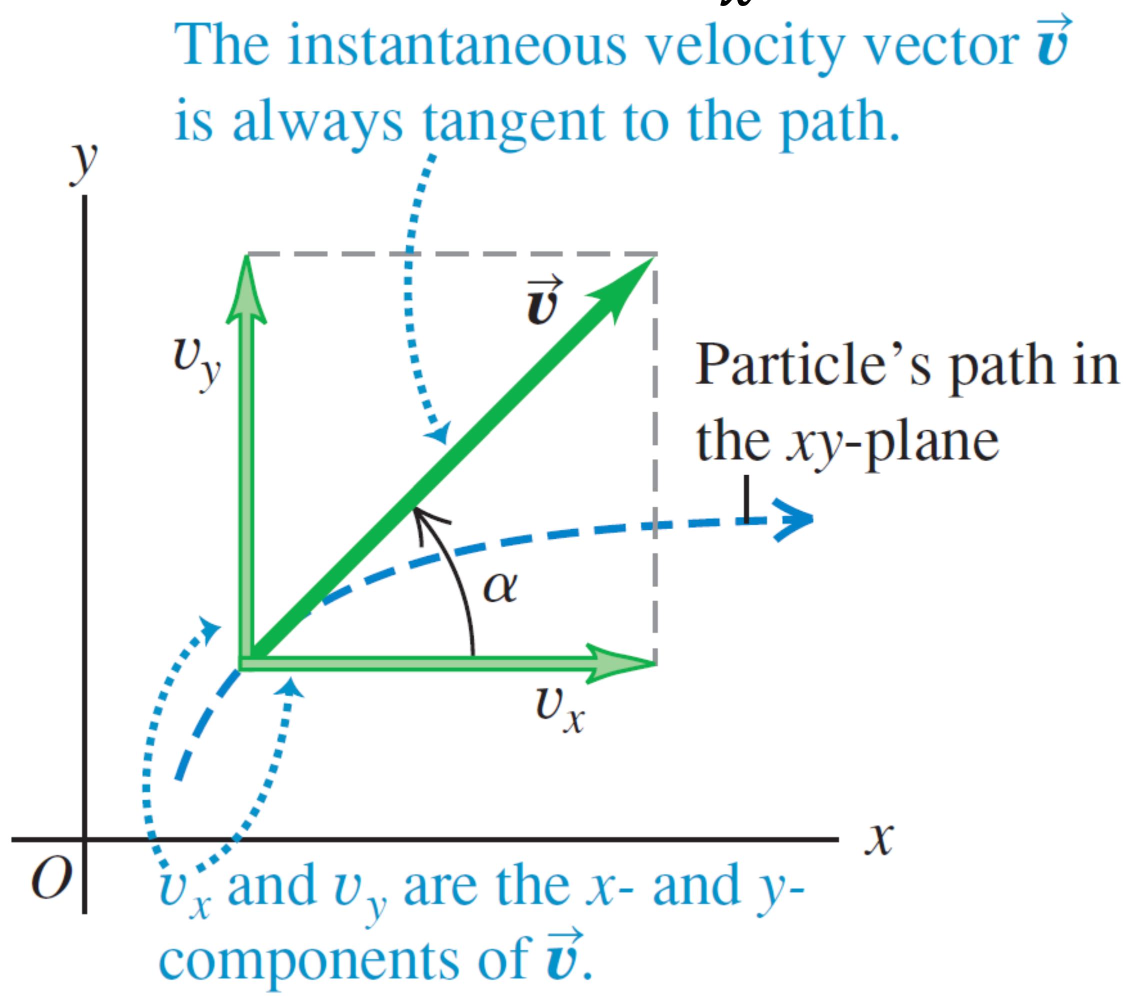
b. Instantaneous velocity is defined as $\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$. Its direction is along a line that is tangent to the object's path and in the direction of its motion.




Don't confuse these two graphs!
Position-versus-time graph
s
t
The *value* of the velocity v_s is the slope of the curve.
Trajectory
y
x
$\vec{v}$
The *direction* of the velocity is tangent to the curve.

i. Magnitude: $v = \sqrt{v_x^2 + v_y^2}$

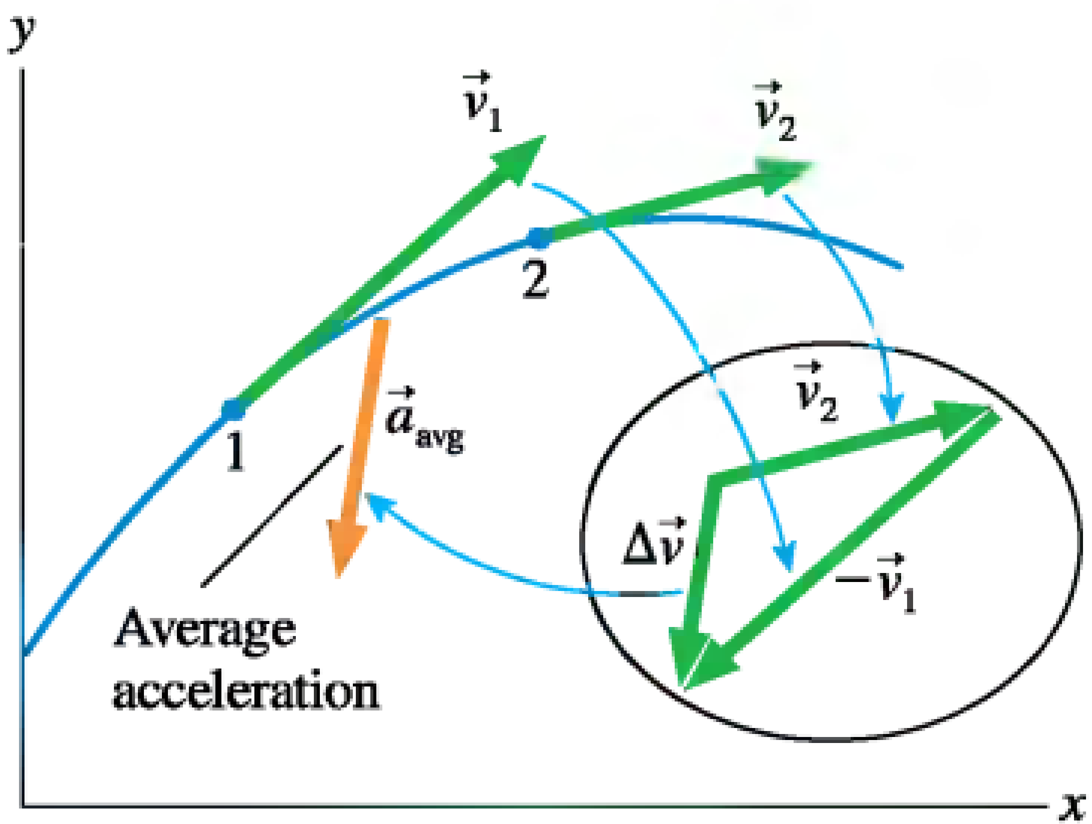
ii. Direction: $\alpha = \tan^{-1}\frac{v_y}{v_x}$

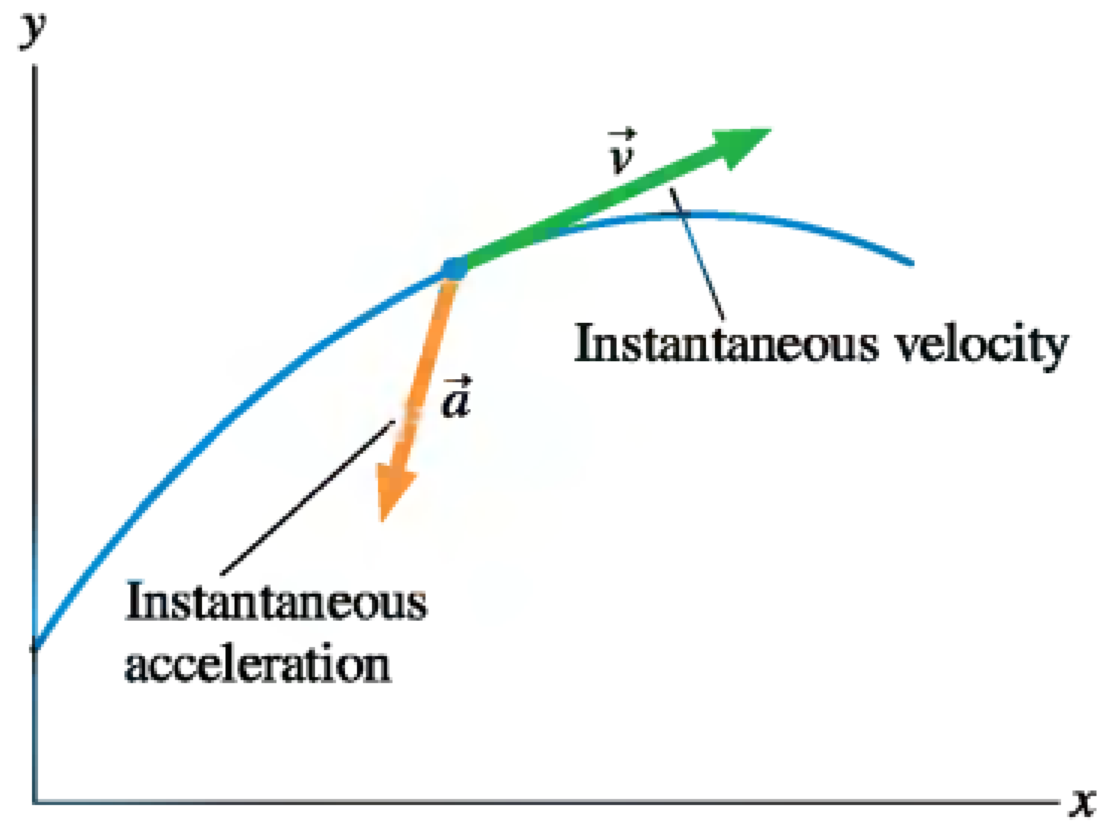


3. Acceleration (SI unit: m/s^2)

a. Average acceleration is defined as $\vec{a}_{av} \equiv \frac{\Delta \vec{v}}{\Delta t}$.

b. Instantaneous acceleration is defined as $\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$.

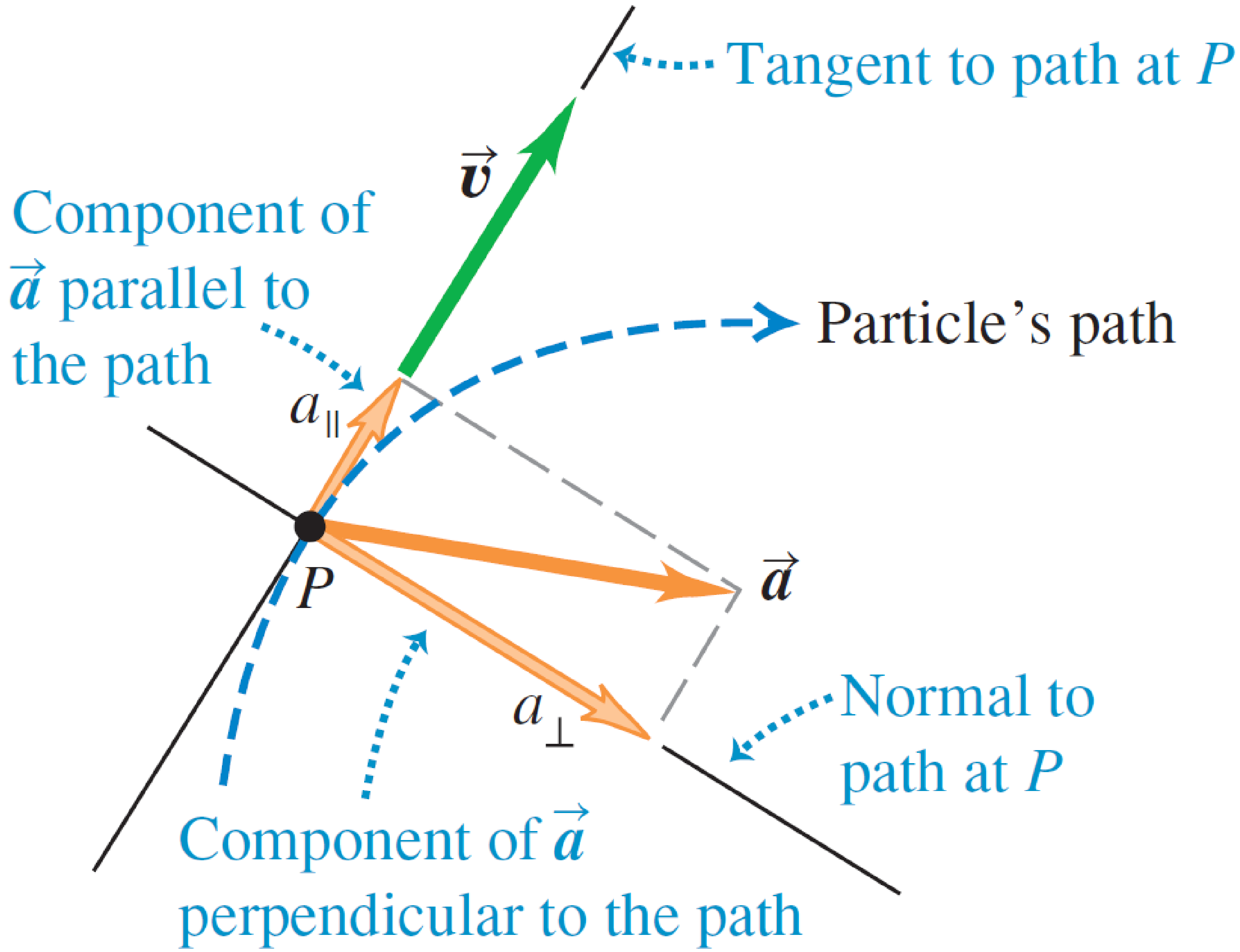


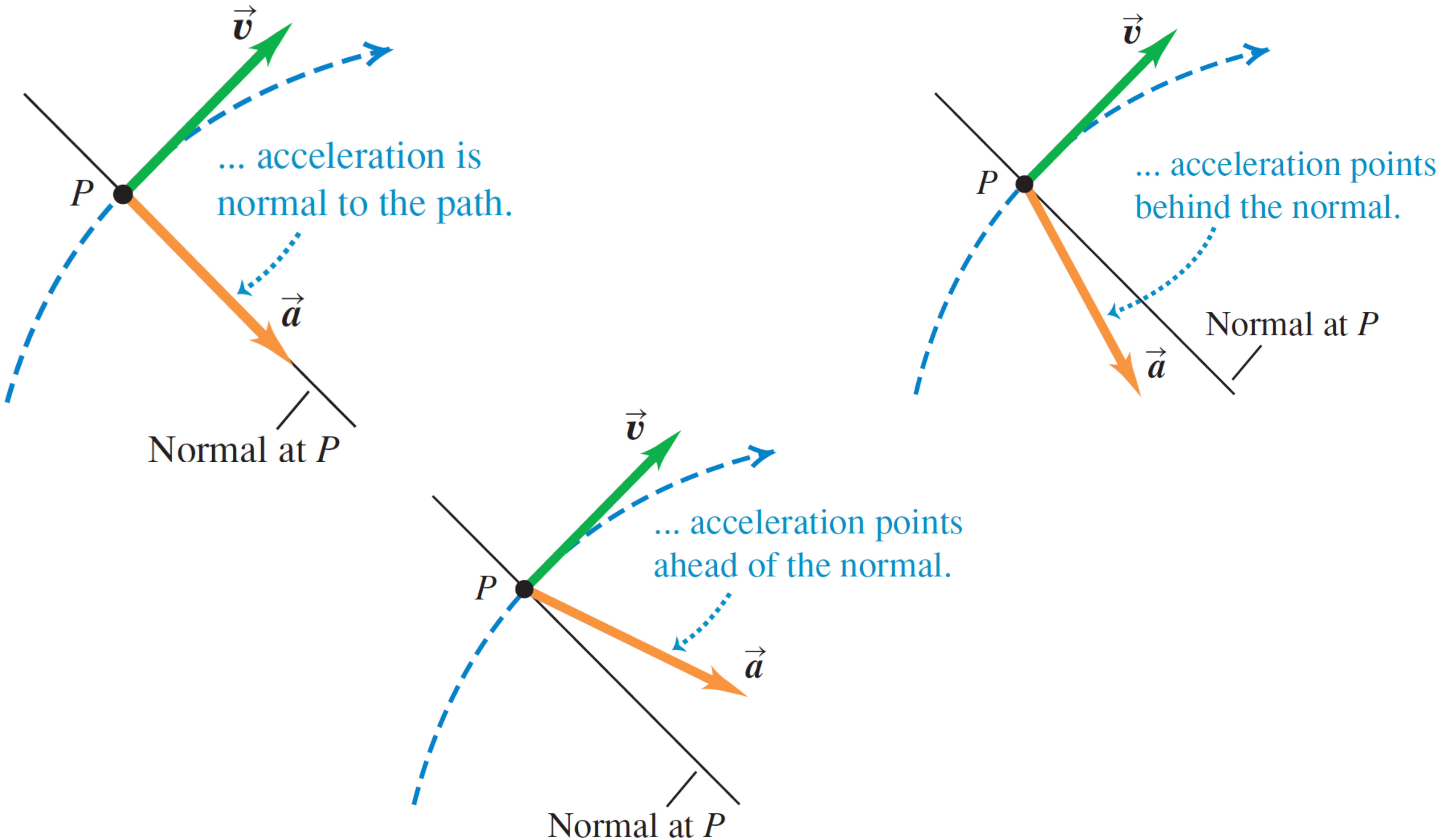


Components of Acceleration:

i. Acceleration $a_{\parallel}$ parallel (tangent to the path) to velocity changes the magnitude of velocity (speed).

ii. Acceleration $a_{\perp}$ perpendicular (normal to the path) to velocity changes the direction of velocity.


Tangent to path at P
$\vec{v}$
Component of
$\vec{a}$ parallel to
the path
Particle's path
$a_{\parallel}$
P
$\vec{a}$
$a_{\perp}$
Normal to
path at P
Component of $\vec{a}$
perpendicular to the path



$\vec{v}$
... acceleration is normal to the path.
P
$\vec{a}$
Normal at P
$\vec{v}$
... acceleration points behind the normal.
P
Normal at P
$\vec{a}$
$\vec{v}$
... acceleration points ahead of the normal.
P
$\vec{a}$
Normal at P

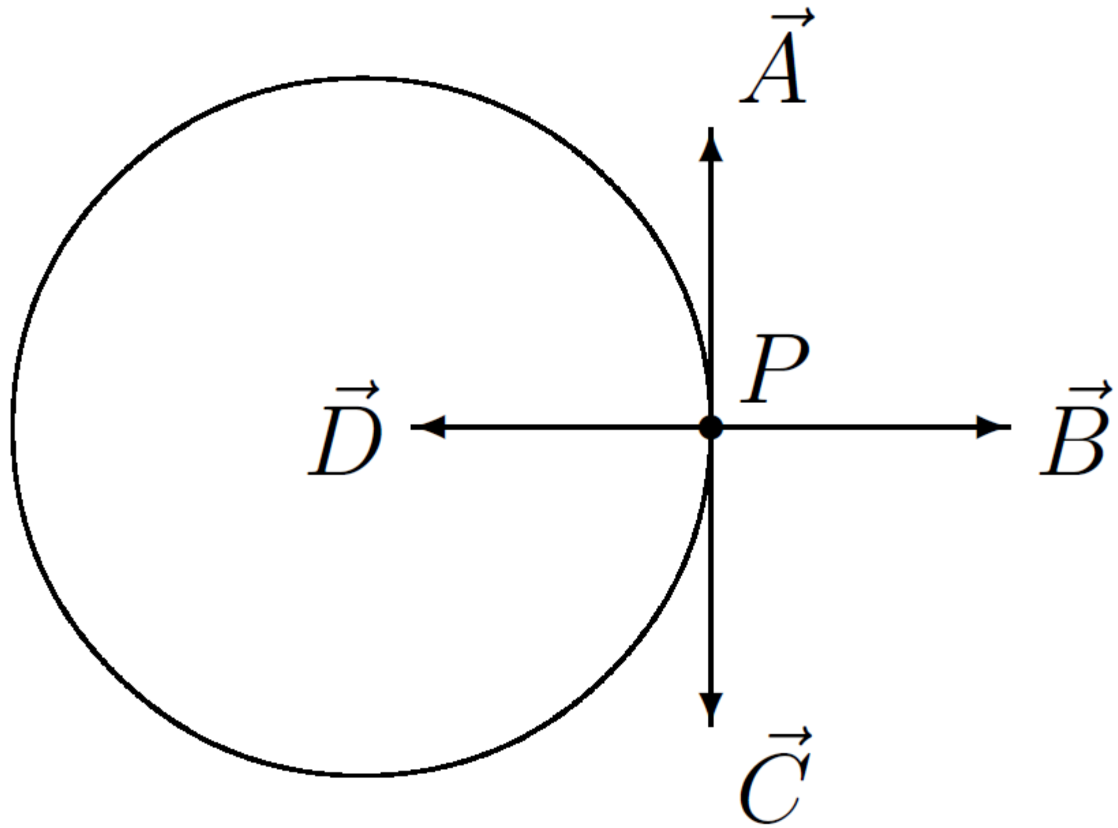
Checkpoint Questions:

1. An object moves in a circle and returns to its initial position. The total distance covered is equal to ______ while the magnitude of displacement is equal to ______.
2. Which of the following objects can’t be accelerating? (a) An object moving with a constant speed; (b) an object moving with a constant velocity; (c) an object moving along a curve.
3. The average acceleration vector has the same direction with ______.

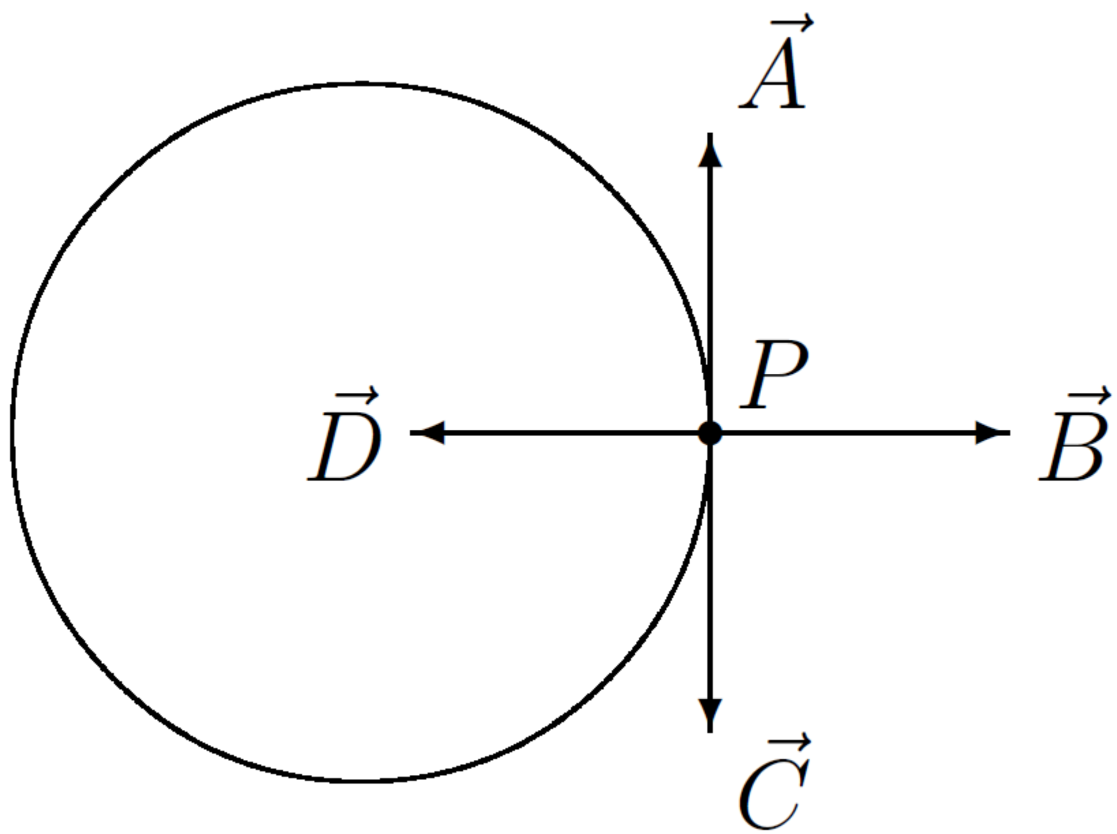
4. The acceleration $a_\perp$ perpendicular to velocity changes the ______ of velocity.

5. The acceleration in the figure will cause the particle to ______ (speed up or slow down) and ______ (curve upward or curve downward)?

6. A car is driving at a constant speed in a clockwise direction around a circular track. Which of the labelled vectors is the velocity vector at point P?



7. A car is driving at a constant speed in a clockwise direction around a circular track. Which of the labelled vectors is the acceleration vector at point P?



B. Motion in Two Dimensions

1. Motion in Two Dimensions

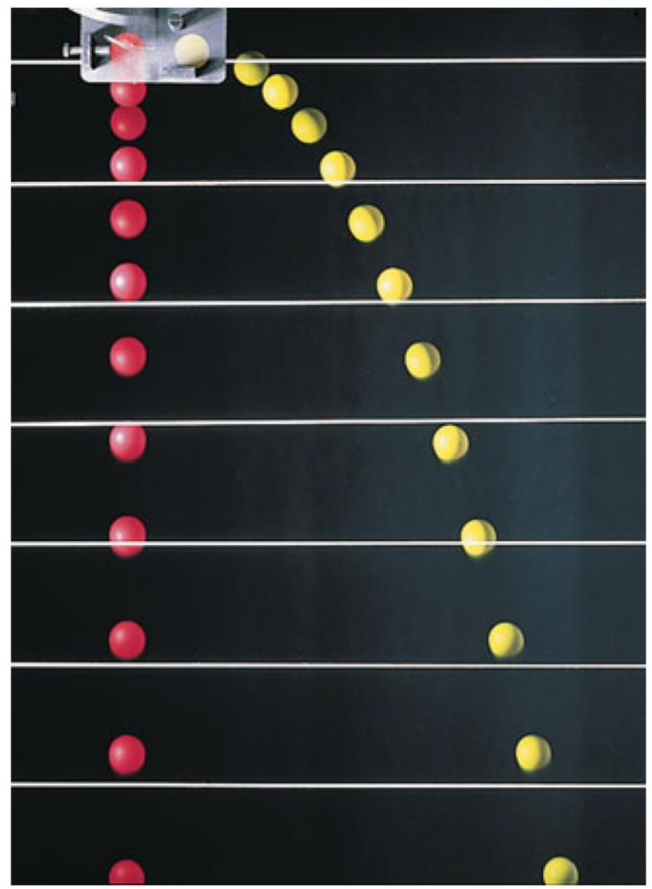
x-direction	*y*-direction
$v_x = v_{ox} + a_x t$	$v_y = v_{oy} + a_y t$
$x - x_o = v_{ox} t + \frac{1}{2} a_x t^2$	$y - y_o = v_{oy} t + \frac{1}{2} a_y t^2$
$v_x^2 = v_{ox}^2 + 2a_x(x - x_o)$	$v_y^2 = v_{oy}^2 + 2a_y(y - y_o)$
Note: The x and y motions are independent of each other. Components $x_o, y_o, x, y, v_{ox}, v_{oy}, v_x, v_y, a_x, a_y$ may be positive (+) or negative (-) depending on the direction of the vectors.	

2. Example of Two-Dimensional Motion: Projectile Motion

   Projectile motion occurs if the object moves under the influence of gravity only. The motion follows a parabolic path called a trajectory.

Important points about projectile motion:

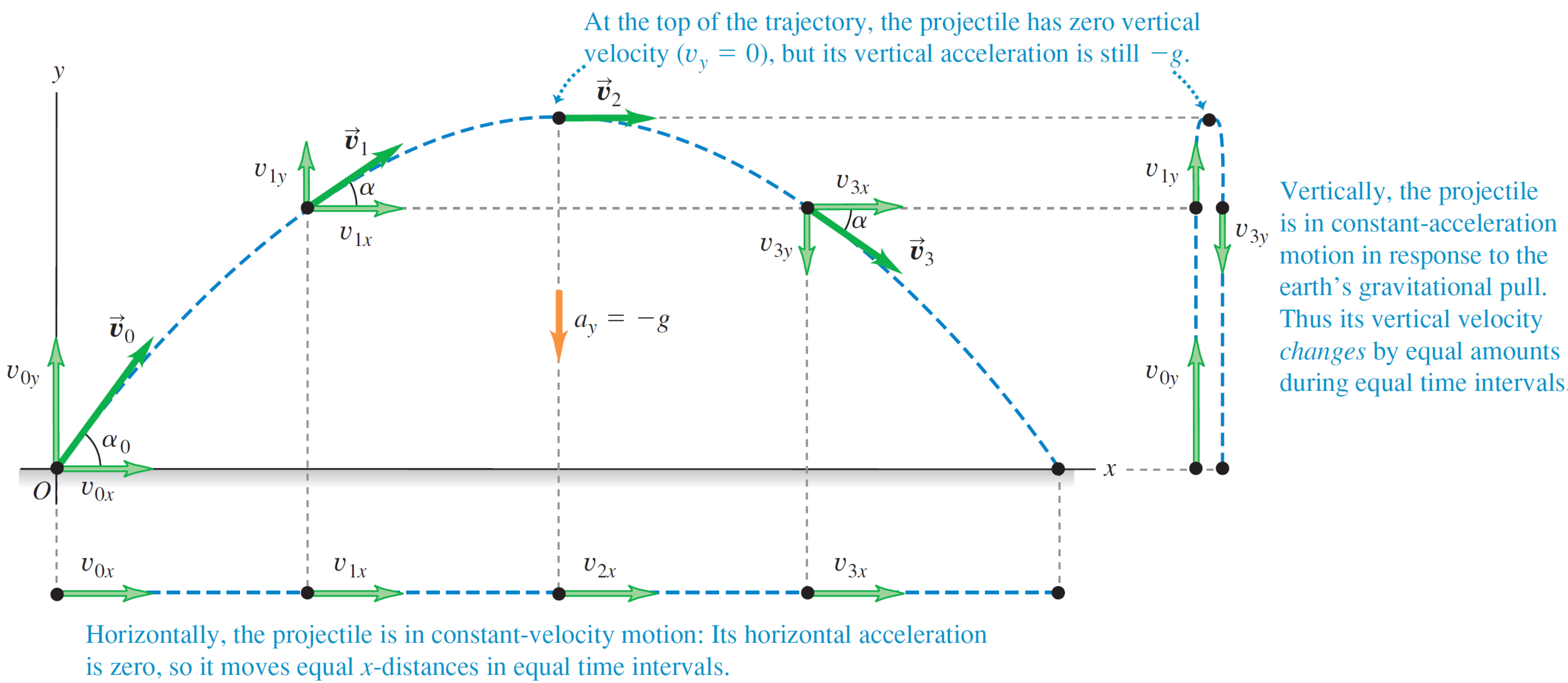
a. A projectile launched horizontally falls in the same time as a freely falling object that is released from rest at the same height where the projectile was launched.



b. Projectile motion is made up of two independent motions:

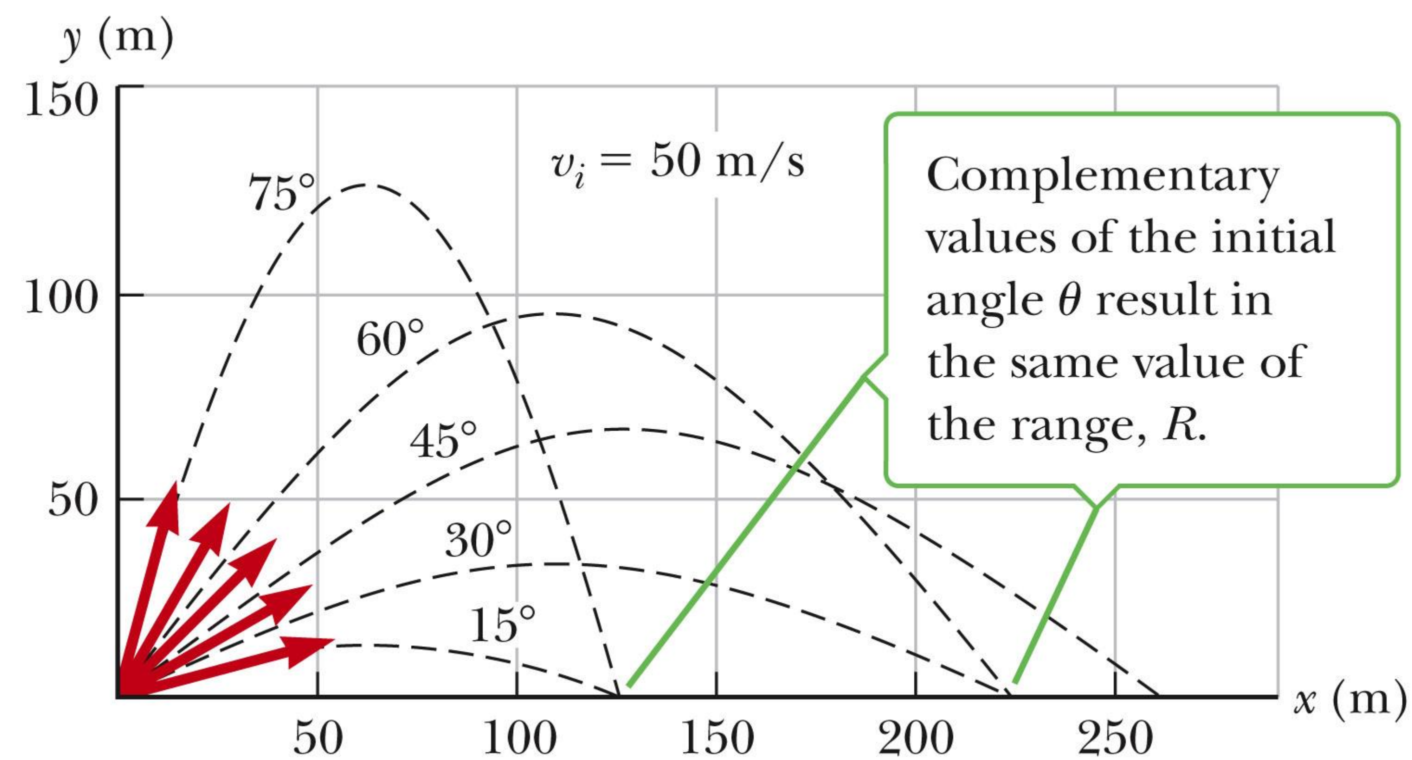
i. Constant-velocity motion in the horizontal direction.

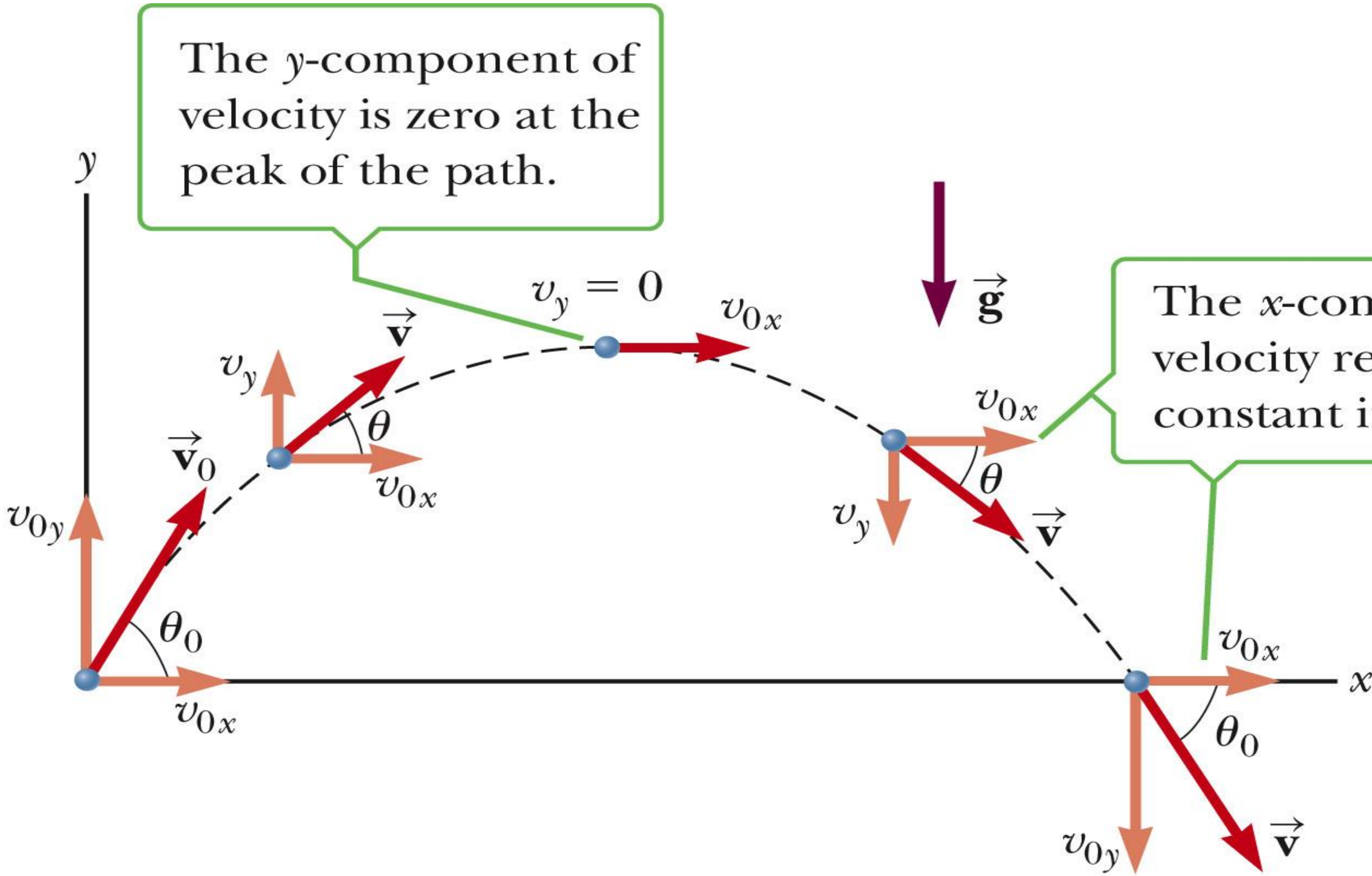
ii. Free-fall motion in the vertical direction.


At the top of the trajectory, the projectile has zero vertical velocity ($v_y = 0$), but its vertical acceleration is still $-g$.
y
$\vec{v}_2$
$\vec{v}_1$
v_{1y}
α
v_{1x}
v_{3x}
α
v_{3y}
$\vec{v}_3$
v_{1y}
v_{3y}
v_{0y}
$\vec{v}_0$
v_{0y}
$a_y = -g$
α_0
O
v_{0x}
x
Vertically, the projectile is in constant-acceleration motion in response to the earth's gravitational pull. Thus its vertical velocity *changes* by equal amounts during equal time intervals.
v_{0x}
v_{1x}
v_{2x}
v_{3x}
Horizontally, the projectile is in constant-velocity motion: Its horizontal acceleration is zero, so it moves equal x-distances in equal time intervals.

c. At the highest point of a projectile's trajectory. Only the y-component of the velocity is zero. Acceleration (a_x=0, a_y= - 9.8m/s^2) due to gravity is constant in projectile motion.
d. The x and y kinematic equations have the same value for t.

e. If the projectile is launched at an angle θ_o with the horizontal, then $v_{ox} = v_o \cos\theta_o$ and $v_{oy} = v_o \sin\theta_o$. Complementary initial angles θ_o give the same horizontal range. The projectile has the maximum horizontal range when it is launched at a 45° angle.




The y-component of velocity is zero at the peak of the path.
The x-component of velocity remains constant in time.
$v_y = 0$
v_{0x}
$\vec{\mathbf{g}}$
$\vec{\mathbf{v}}$
v_y
θ
$\vec{\mathbf{v}}_0$
v_{0y}
θ_0
y
x

Checkpoint Questions:

1. The x part of motion occurs exactly as it would if the y part did not occur at all. Similarly, the y part of the motion occurs exactly as it would if the x part of the motion did not exist. True or False?
2. There is no velocity at the peak of the projectile's trajectory. True or False?
3. ______ initial angles give the same horizontal range.
4. A projectile launched at a ______ angle gives the maximum horizontal range.