MA678 Final Project

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2022-11-18

```
library(tidyverse)
## — Attaching packages
                                                                 tidyverse
1.3.2 -
## √ ggplot2 3.4.0
                        ✓ purrr
                                   0.3.5
## √ tibble 3.1.8
                        √ dplyr
                                   1.0.10
## √ tidyr
             1.2.1

√ stringr 1.4.1

## √ readr
            2.1.3

√ forcats 0.5.2

## Warning: package 'ggplot2' was built under R version 4.2.2
## Warning: package 'tidyr' was built under R version 4.2.2
## Warning: package 'readr' was built under R version 4.2.2
## Warning: package 'purrr' was built under R version 4.2.2
## — Conflicts —
tidyverse_conflicts() —
## X dplyr::filter() masks stats::filter()
## X dplyr::lag()
                    masks stats::lag()
library(dplyr)
library(ggplot2)
library(grid)
library(gridExtra)
## Attaching package: 'gridExtra'
## The following object is masked from 'package:dplyr':
##
##
       combine
#library(rstanarm)
library(lmerTest)
## Warning: package 'lmerTest' was built under R version 4.2.2
## Loading required package: lme4
## Warning: package 'lme4' was built under R version 4.2.2
## Loading required package: Matrix
```

```
## Warning: package 'Matrix' was built under R version 4.2.2
##
## Attaching package: 'Matrix'
##
## The following objects are masked from 'package:tidyr':
##
       expand, pack, unpack
##
##
##
## Attaching package: 'lmerTest'
##
## The following object is masked from 'package:lme4':
##
##
       lmer
##
## The following object is masked from 'package:stats':
##
##
       step
library(lme4)
library(merTools)
## Warning: package 'merTools' was built under R version 4.2.2
## Loading required package: arm
## Loading required package: MASS
##
## Attaching package: 'MASS'
## The following object is masked from 'package:dplyr':
##
##
       select
##
##
## arm (Version 1.13-1, built: 2022-8-25)
##
## Working directory is C:/Users/Jiahao Liu/Desktop
library(correlation)
## Warning: package 'correlation' was built under R version 4.2.2
##
## Attaching package: 'correlation'
##
## The following object is masked from 'package:arm':
##
##
       display
library(corrplot)
```

```
## corrplot 0.92 loaded
##
## Attaching package: 'corrplot'
## The following object is masked from 'package:arm':
##
##
       corrplot
library("PerformanceAnalytics")
## Loading required package: xts
## Warning: package 'xts' was built under R version 4.2.2
## Loading required package: zoo
## Warning: package 'zoo' was built under R version 4.2.2
## Attaching package: 'zoo'
## The following objects are masked from 'package:base':
##
##
       as.Date, as.Date.numeric
##
##
## Attaching package: 'xts'
## The following objects are masked from 'package:dplyr':
##
       first, last
##
##
##
## Attaching package: 'PerformanceAnalytics'
## The following object is masked from 'package:graphics':
##
##
       legend
```

Data Processing

```
##
     song_name
                       song_popularity song_duration_ms
                                                           acousticness
## Length:13070
                            : 0.00
                       Min.
                                        Min.
                                               : 12000
                                                          Min.
                                                                 :0.000001
## Class :character
                       1st Qu.: 37.00
                                        1st Qu.: 182967
                                                          1st Ou.:0.025000
## Mode :character
                       Median : 51.00
                                        Median : 211486
                                                          Median :0.147000
##
                              : 48.49
                                               : 218627
                                                          Mean
                       Mean
                                        Mean
                                                                 :0.277770
##
                       3rd Qu.: 63.00
                                        3rd Qu.: 244506
                                                          3rd Qu.:0.479750
##
                              :100.00
                                               :1799346
                                                                 :0.996000
                       Max.
                                        Max.
                                                          Max.
##
     danceability
                                       instrumentalness
                                                               key
                         energy
## Min.
           :0.0000
                     Min.
                            :0.00107
                                       Min.
                                              :0.000000
                                                          Min.
                                                                 : 0.00
   1st Qu.:0.5242
                     1st Qu.:0.49000
                                       1st Qu.:0.000000
                                                          1st Qu.: 2.00
```

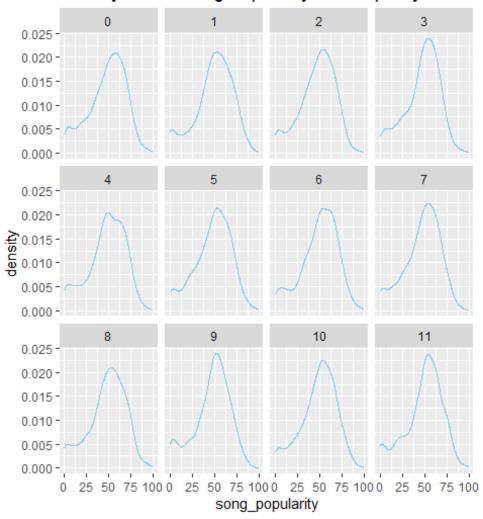
```
Median :0.6370
                    Median :0.66700
                                     Median :0.000022
                                                       Median: 5.00
##
        :0.6250
                    Mean :0.63572
                                     Mean :0.096672
                                                       Mean : 5.32
   Mean
   3rd Qu.:0.7410
                    3rd Qu.:0.81500
##
                                     3rd Qu.:0.005910
                                                       3rd Qu.: 8.00
##
   Max.
          :0.9870
                    Max. :0.99900
                                     Max. :0.997000
                                                       Max. :11.00
##
      liveness
                      loudness
                                       audio_mode
                                                     speechiness
##
   Min.
          :0.0119
                          :-38.768
                                           :0.000
                                                    Min.
                    Min.
                                     Min.
                                                           :0.0000
   1st Ou.:0.0933
                    1st Ou.: -9.537
                                     1st Ou.:0.000
                                                    1st Ou.:0.0373
##
   Median :0.1210
                    Median : -6.859
                                     Median :1.000
                                                    Median :0.0544
##
   Mean
         :0.1805
                    Mean : -7.790
                                     Mean :0.633
                                                    Mean :0.1008
                    3rd Qu.: -5.041
##
   3rd Qu.:0.2230
                                     3rd Qu.:1.000
                                                    3rd Qu.:0.1150
##
   Max. :0.9860
                    Max. : 1.585
                                     Max. :1.000
                                                    Max. :0.9410
##
     tempo
                    time signature
                                  audio valence
## Min. : 0.00
                    Min. :0.000
                                   Min. :0.0000
   1st Qu.: 98.07
                    1st Qu.:4.000
                                   1st Qu.:0.3340
## Median :120.03
                    Median :4.000
                                   Median :0.5290
## Mean :121.15
                    Mean :3.952
                                   Mean :0.5286
## 3rd Qu.:139.96
                    3rd Qu.:4.000
                                   3rd Qu.:0.7300
## Max. :242.32
                   Max. :5.000
                                   Max. :0.9840
```

Variable Selection

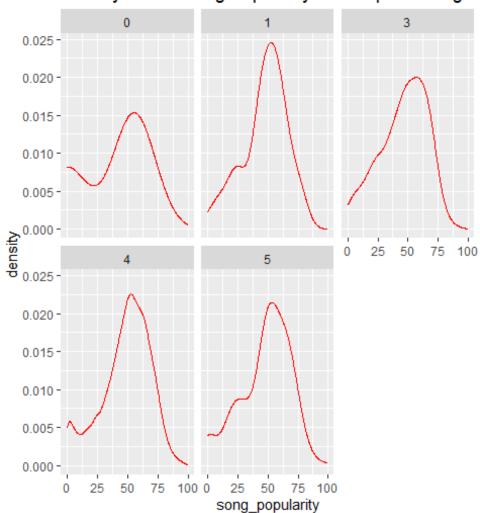
```
## # A tibble: 12 × 2
##
       key
             n
##
      <dbl> <int>
##
         0 1484
  1
## 2
         1 1404
##
  3
         2 1202
## 4
         3
            394
## 5
         4
            954
## 6
         5 1113
## 7
         6
            911
## 8
         7 1457
## 9
            915
         8
## 10
         9
           1252
## 11
            922
        10
## 12
        11 1062
## # A tibble: 5 × 2
## time_signature
##
             <dbl> <int>
## 1
                 0
                       3
## 2
                 1
                      57
                     626
## 3
                 3
## 4
                 4 12208
## 5
                 5
                     176
## # A tibble: 2 × 2
    audio mode n
##
         <dbl> <int>
```

1 0 4797 ## 2 1 8273

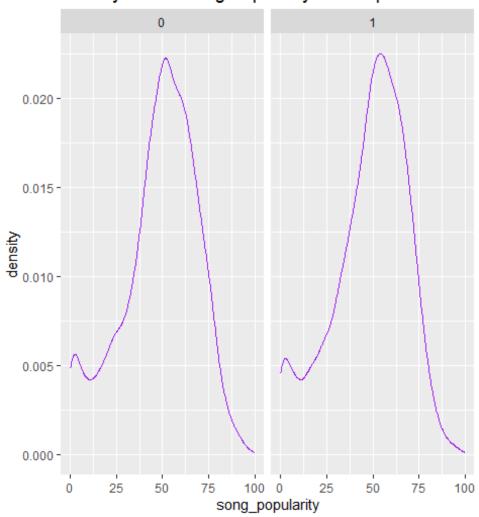
Density Plot of Song Popularity in Group Key



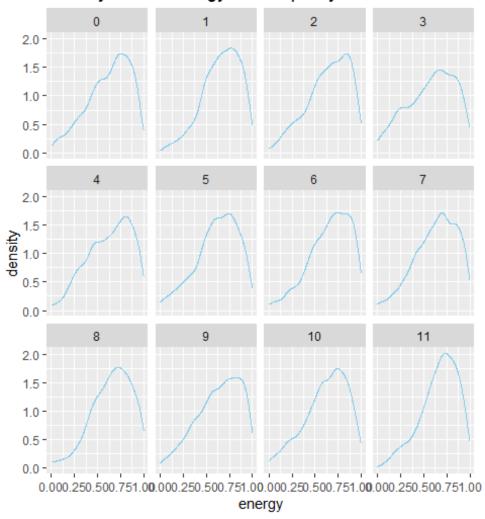
Density Plot of Song Popularity in Group Time Signa



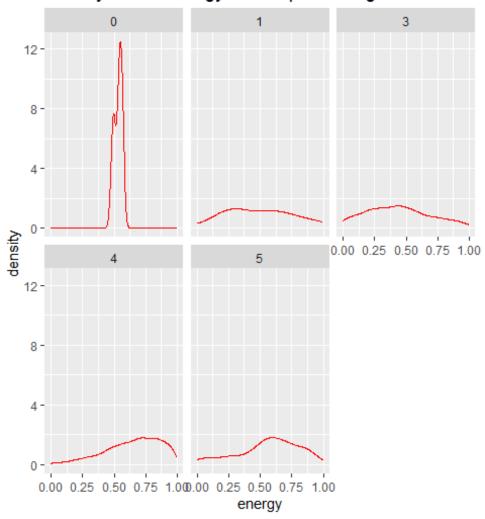
Density Plot of Song Popularity in Group Audio Mod€



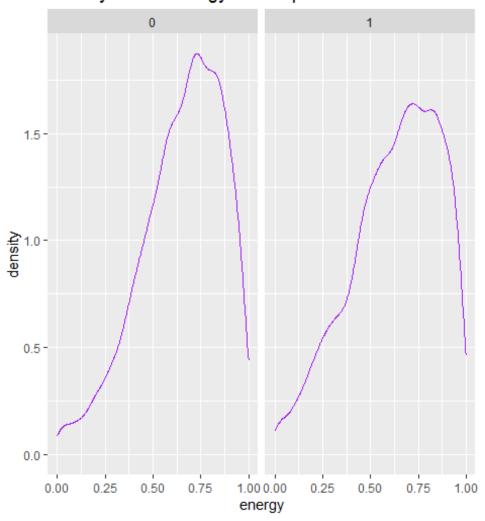
Density Plot of Energy in Group Key



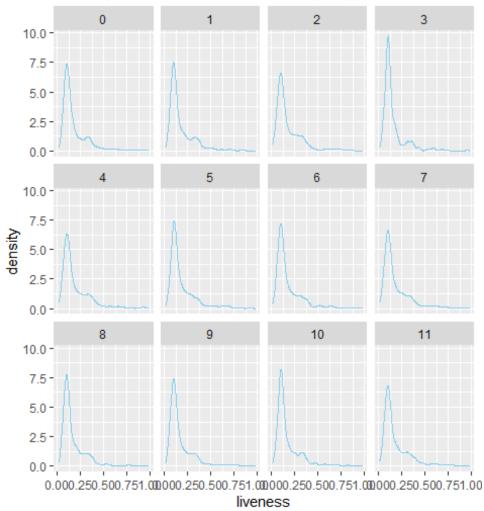
Density Plot of Energy in Group Time Signature



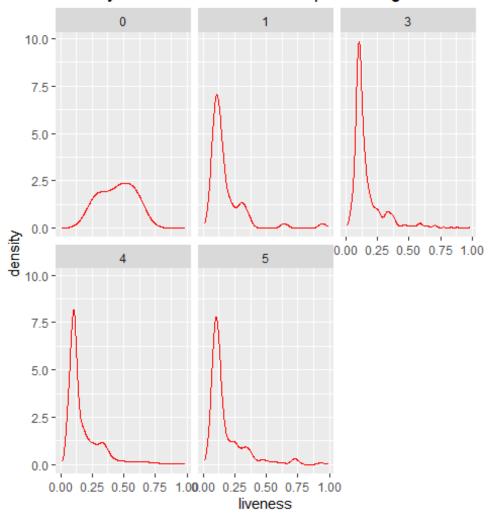
Density Plot of Energy in Group Audio Mode



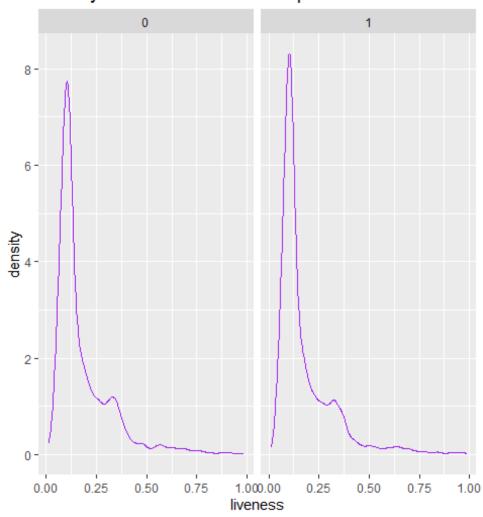
Density Plot of Liveness in Group Key



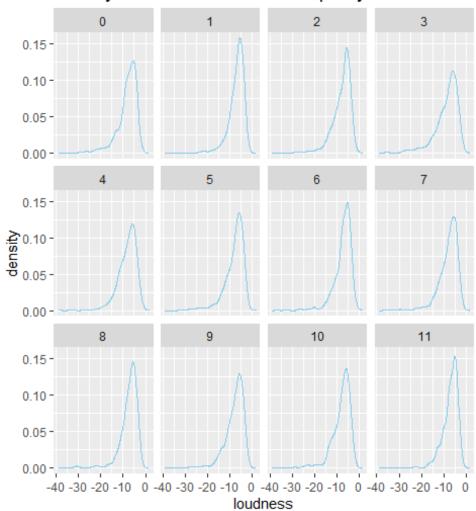
Density Plot of Liveness in Group Time Signature



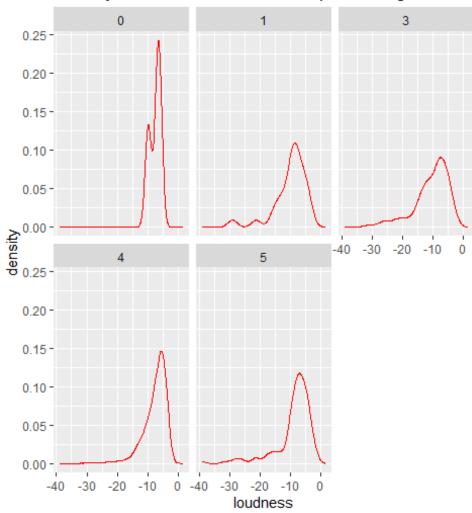
Density Plot of Liveness in Group Audio Mode



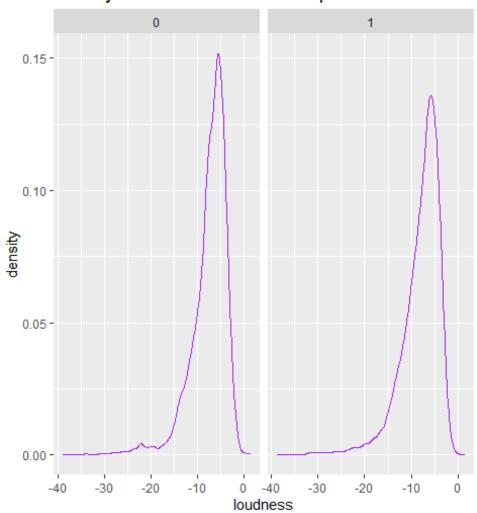
Density Plot of Loudness in Group Key



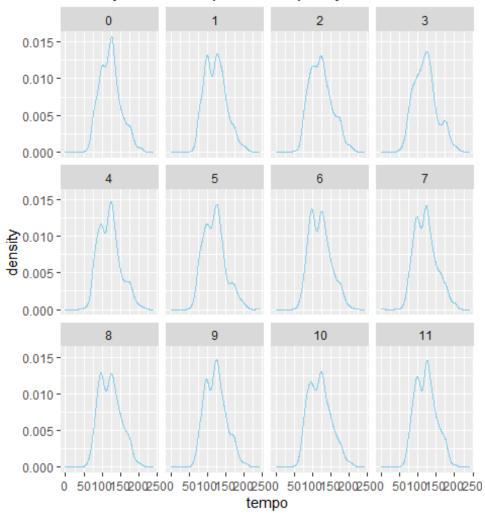
Density Plot of Loudness in Group Time Signature



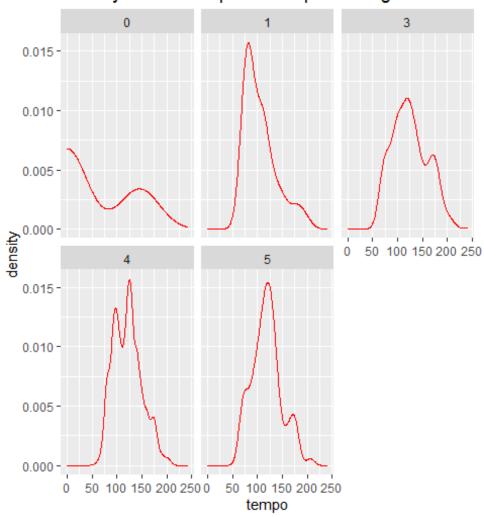
Density Plot of Loudness in Group Audio Mode



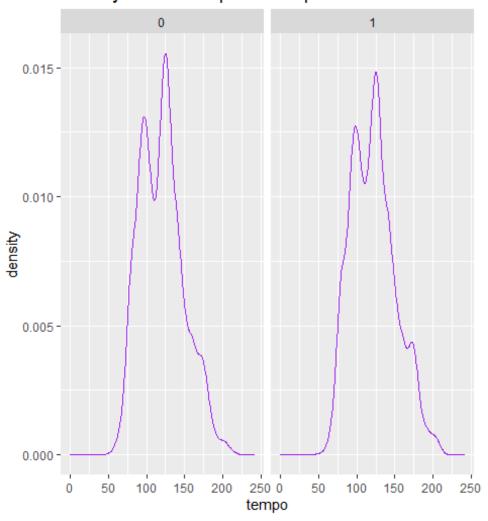
Density Plot of tempo in Group Key



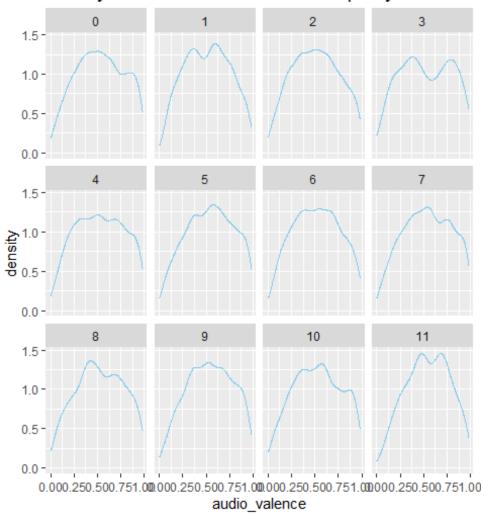
Density Plot of Tempo in Group Time Signature



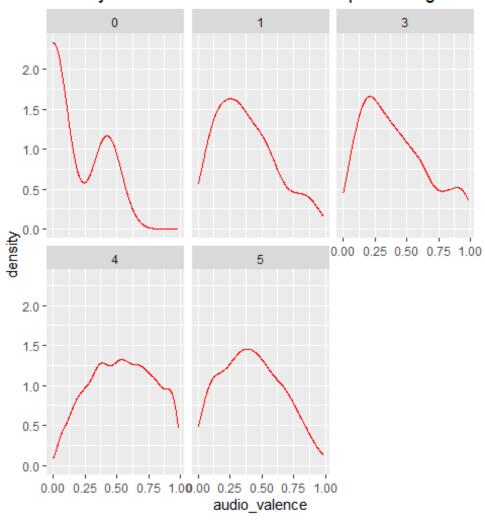
Density Plot of Tempo in Group Audio Mode



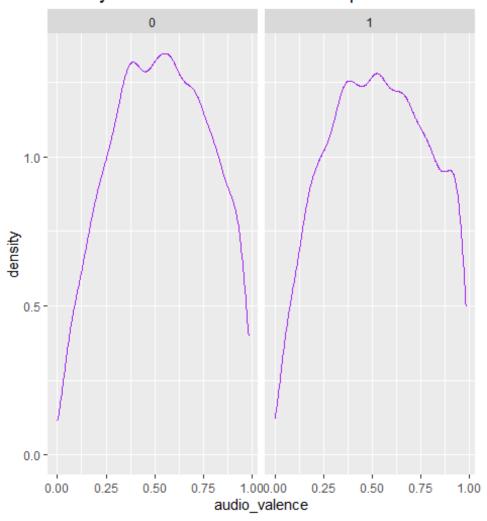
Density Plot of Audio Valence in Group Key



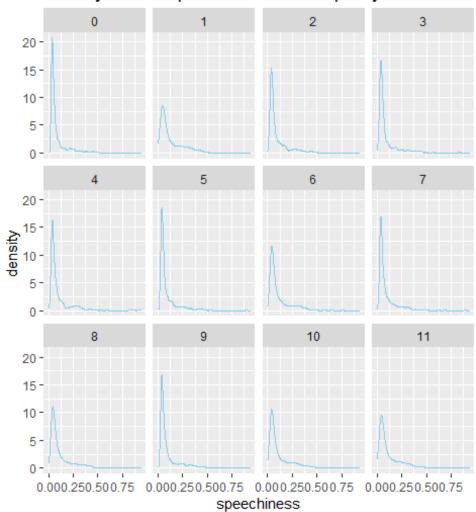
Density Plot of Audio Valence in Group Time Signature



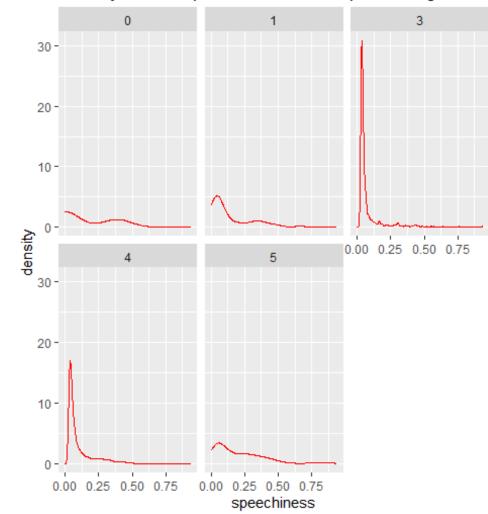
Density Plot of Audio Valence in Group Audio Mode



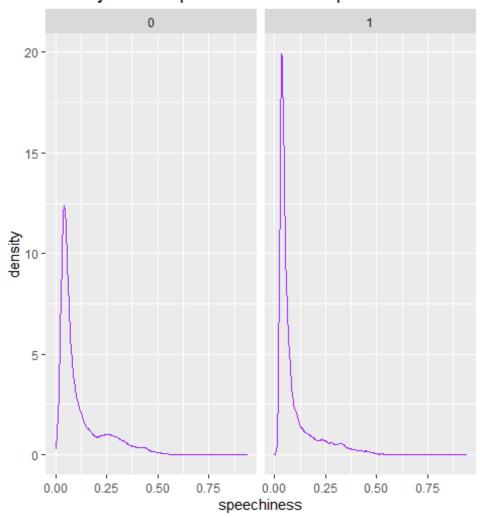
Density Plot of Speechiness in Group Key



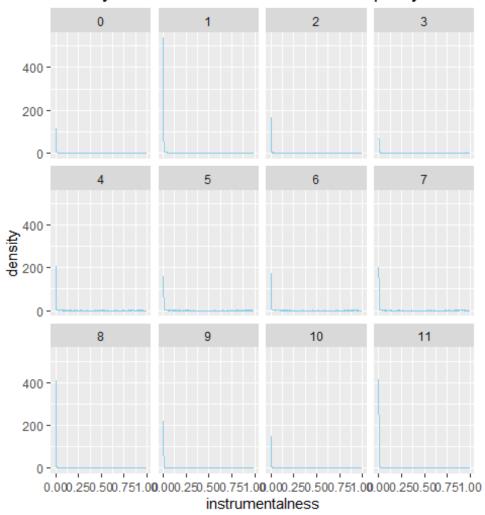
Density Plot of Speechiness in Group Time Signature



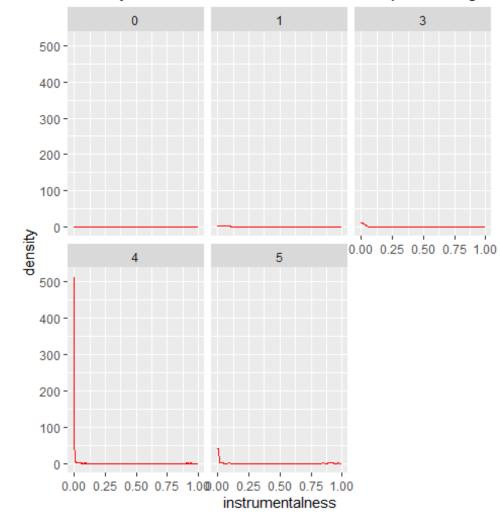
Density Plot of Speechiness in Group Audio Mode



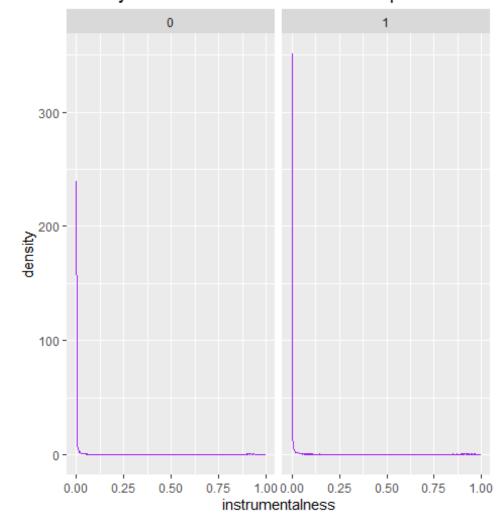
Density Plot of Instrumentalness in Group Key



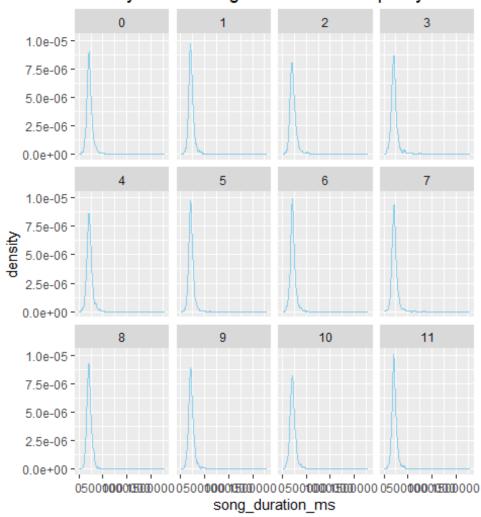
Density Plot of Instrumentalness in Group Time Signa



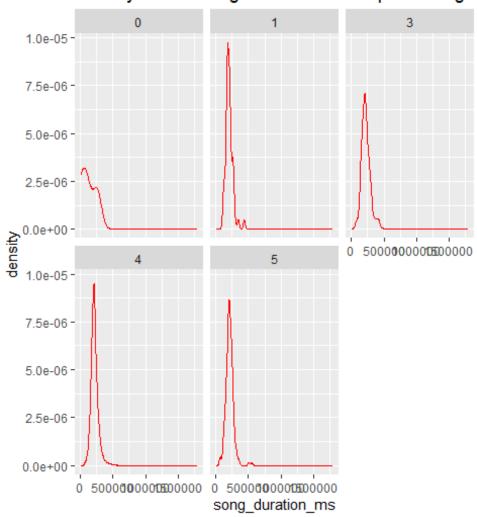
Density Plot of Instrumentalness in Group Audio Mode



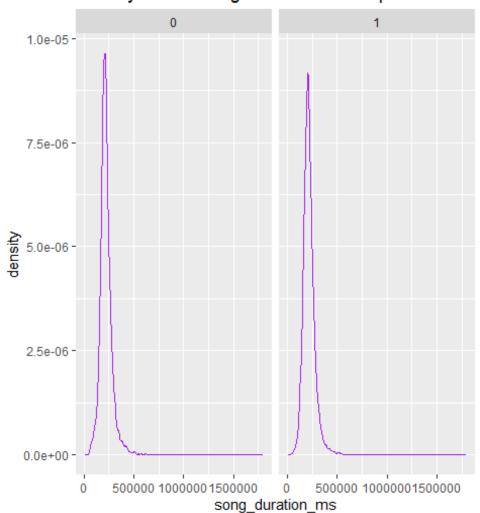
Density Plot of Song Duration in Group Key



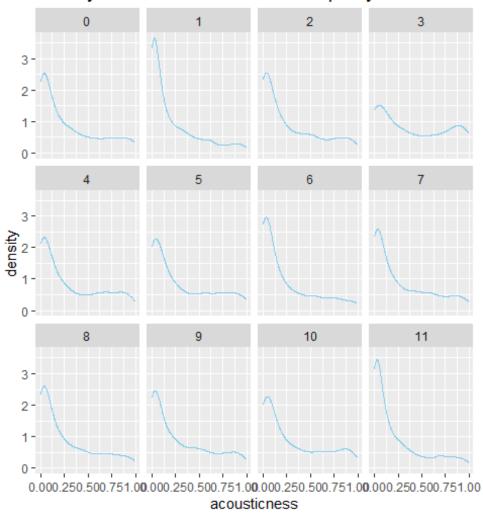
Density Plot of Song Duration in Group Time Signa



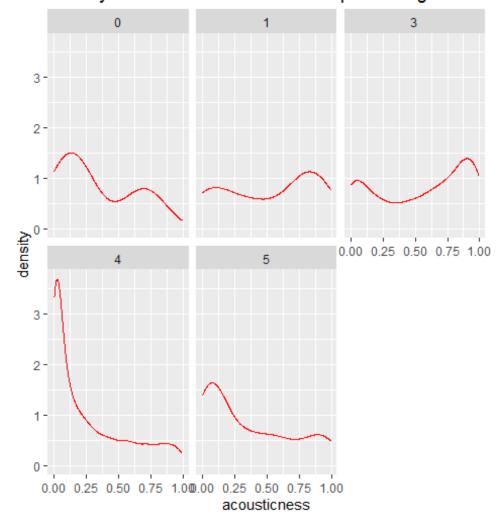
Density Plot of Song Duration in Group Audio Mode



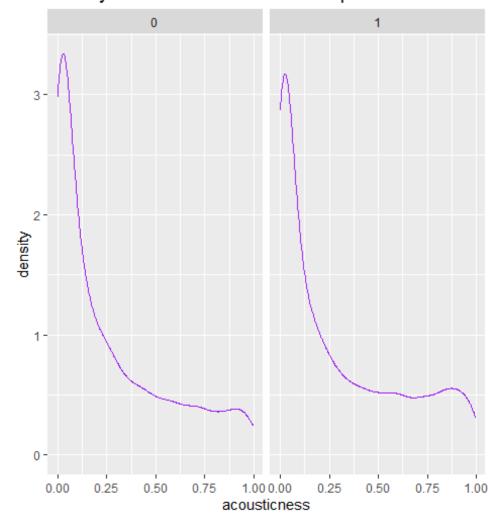
Density Plot of Acousticness in Group Key



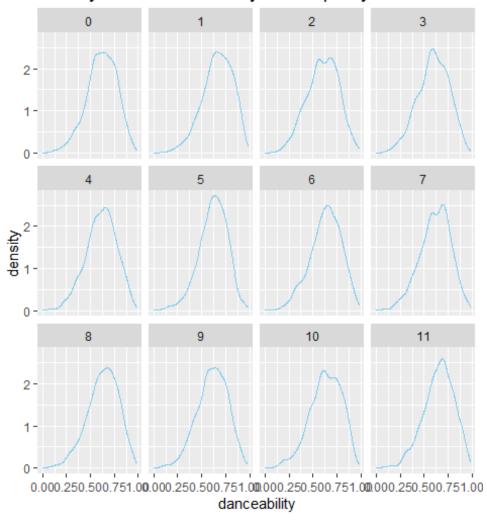
Density Plot of Acousticness in Group Time Signature



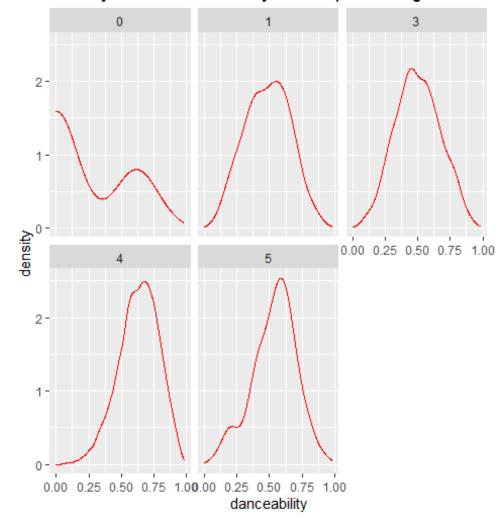
Density Plot of Acousticness in Group Audio Mode



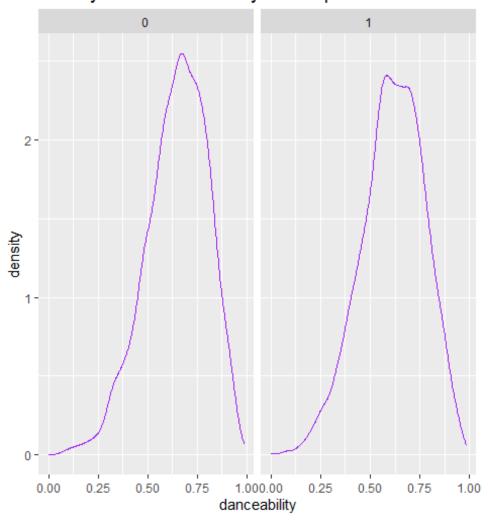
Density Plot of Danceability in Group Key



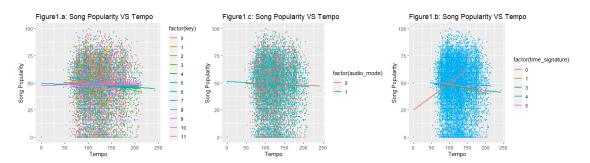
Density Plot of Danceability in Group Time Signature



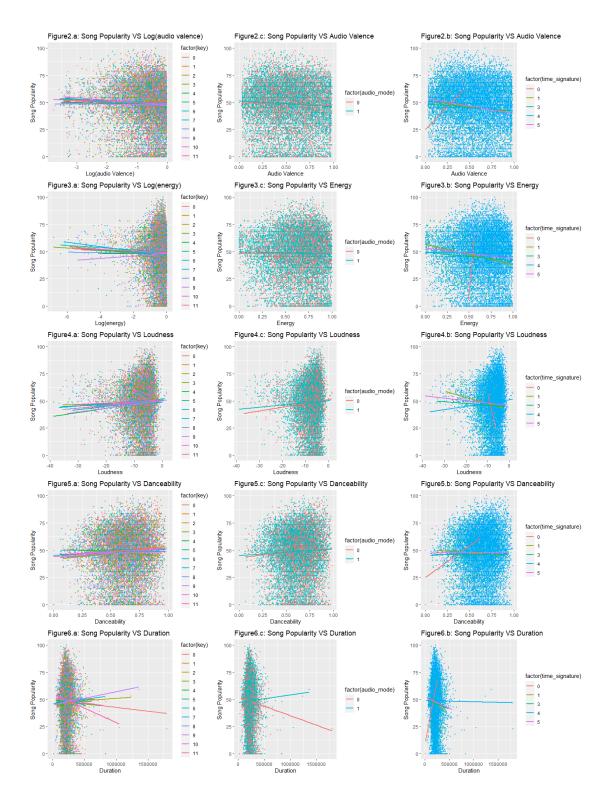
Density Plot of Danceability in Group Audio Mode

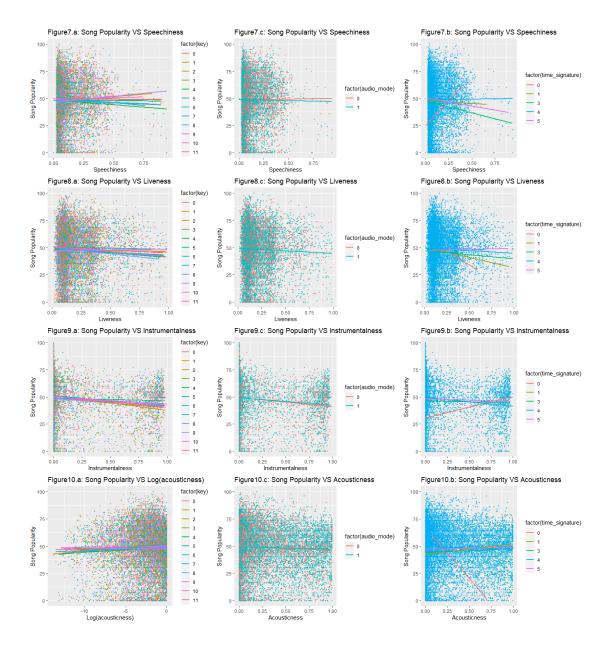


Variable Selection



Warning: Removed 2 rows containing non-finite values (`stat_smooth()`).





Correlation

```
< .001***
                             energy | 0.11 | [ 0.09, 0.12] |
                                                               12.15
## song duration ms |
< .001***
## song duration ms | instrumentalness | -0.03 | [-0.04, -0.01] |
                                                                -3.08
0.012*
                           liveness | 0.02 | [ 0.01,
## song_duration_ms |
                                                     0.041
                                                                2.69
0.028*
                           loudness | 0.04 | [ 0.02,
## song_duration_ms |
                                                     0.05] |
                                                                4.05 l
< .001***
## song duration ms |
                         speechiness | -0.08 | [-0.10, -0.06] |
                                                                -9.27
< .001***
                              tempo | 0.02 | [ 0.00, 0.03] |
## song_duration_ms |
                                                                1.95
0.152
## song duration ms |
                       audio_valence | -0.07 | [-0.09, -0.05] |
                                                                -7.84
< .001***
                       danceability | -0.18 | [-0.20, -0.16] |
## acousticness
                                                               -20.88
< .001***
## acousticness
                             energy | -0.68 | [-0.69, -0.67] |
                                                              -106.22
< .001***
## acousticness
                  21.68
< .001***
                           liveness | -0.09 | [-0.11, -0.07] |
## acousticness
                                                               -10.32
< .001***
## acousticness
                           loudness | -0.57 | [-0.58, -0.56] |
                                                               -79.81
< .001***
                         speechiness | -0.09 | [-0.11, -0.07] |
## acousticness
                                                               -10.51
< .001***
                              tempo | -0.15 | [-0.16, -0.13] |
                                                               -17.00
## acousticness
< .001***
                       audio valence | -0.12 | [-0.14, -0.11] |
## acousticness
                                                               -14.14
< .001***
## danceability
                             energy | 0.06 | [ 0.05, 0.08] |
                                                                7.31
< .001***
## danceability
                  -14.65
< .001***
## danceability
                           liveness | -0.09 | [-0.11, -0.08] |
                                                               -10.83
< .001***
## danceability
                           loudness | 0.18 | [ 0.17, 0.20] |
                                                               21.12
< .001***
## danceability
                         speechiness | 0.20 | [ 0.19,
                                                     0.22]
                                                               23.81
< .001***
## danceability
                              tempo | -0.13 | [-0.14, -0.11] |
                                                               -14.61
< .001***
                       audio_valence | 0.35 | [ 0.34, 0.37] |
                                                               42.95
## danceability
< .001***
## energy
                  -27.14
< .001***
## energy
                           liveness | 0.18 | [ 0.16, 0.19] |
                                                               20.63
< .001***
                           loudness | 0.77 | [ 0.76, 0.78] |
## energy
                                                              138.08
```

```
< .001***
                           speechiness |
                                          0.08 | [ 0.06,
                                                          0.09]
                                                                      8.91
## energy
< .001***
## energy
                                 tempo |
                                          0.18 | [ 0.16,
                                                          0.201 |
                                                                     21.08 |
< .001***
## energy
                         audio_valence | 0.31 | [ 0.30, 0.33] |
                                                                     37.57
< .001***
                              liveness | -0.04 | [-0.06, -0.03] |
                                                                     -5.02
## instrumentalness |
< .001***
                              loudness | -0.41 | [-0.42, -0.39] |
## instrumentalness |
                                                                    -50.68
< .001***
## instrumentalness |
                           speechiness | -0.08 | [-0.10, -0.07] |
                                                                     -9.45
< .001***
                                                                     -5.22
## instrumentalness |
                                 tempo | -0.05 | [-0.06, -0.03] |
< .001***
## instrumentalness |
                         audio_valence | -0.20 | [-0.22, -0.19] |
                                                                    -23.56
< .001***
## liveness
                              loudness | 0.12 | [ 0.10,
                                                         0.13] |
                                                                     13.29
< .001***
## liveness
                           speechiness
                                          0.09 | [ 0.07,
                                                          0.11] |
                                                                     10.26
< .001***
## liveness
                                 tempo |
                                          0.03 | [ 0.01,
                                                          0.04] |
                                                                      2.97
0.015*
## liveness
                         audio valence
                                          0.01 \mid [0.00]
                                                          0.03] |
                                                                      1.63
0.206
## loudness
                                         0.08 | [ 0.07,
                           speechiness |
                                                          0.10] |
                                                                      9.59
< .001***
## loudness
                                          0.14 | [ 0.13,
                                                          0.16]
                                 tempo |
                                                                     16.43
< .001***
## loudness
                         audio valence |
                                         0.21 | [ 0.20,
                                                          0.23]
                                                                     25.01
< .001***
## speechiness
                                 tempo |
                                          0.05 | [ 0.04,
                                                          0.07] |
                                                                      6.06
< .001***
## speechiness
                         audio valence
                                          0.01 \mid [-0.01]
                                                          0.03] |
                                                                      1.33
0.206
                         audio valence
                                          0.05 | [ 0.03,
## tempo
                                                          0.071
                                                                      5.53
< .001***
##
## p-value adjustment method: Holm (1979)
## Observations: 13070
corr_music <- cor(music_data_cor)</pre>
corrplot(corr_music, method = "number", diag = FALSE, type = "lower")
```

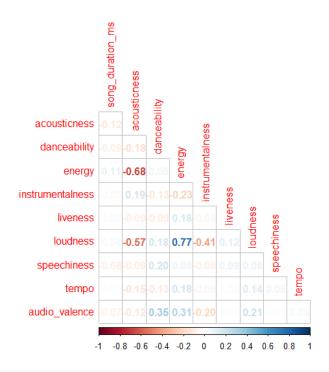


chart.Correlation(music_data_cor, histogram=TRUE, pch=19)

Warning in par(usr): argument 1 does not name a graphical parameter

Warning in par(usr): argument 1 does not name a graphical parameter

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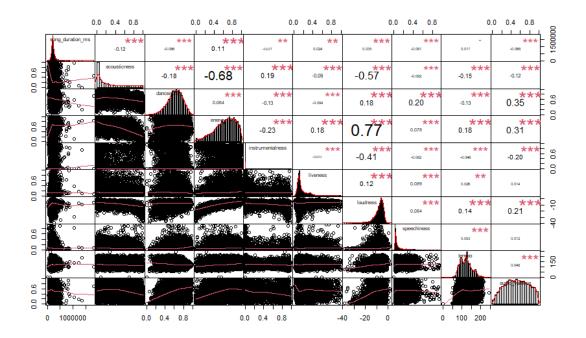
Warning in par(usr): argument 1 does not name a graphical parameter

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```



Fit Model

```
model <- lmer(song_popularity ~ tempo + danceability + audio_valence +
liveness + loudness + (1 + tempo + danceability|key) + (1|time_signature) +
(1|audio_mode), data = music_data)

## boundary (singular) fit: see help('isSingular')

## Warning: Model failed to converge with 4 negative eigenvalues: -2.0e+00 -
9.1e+00

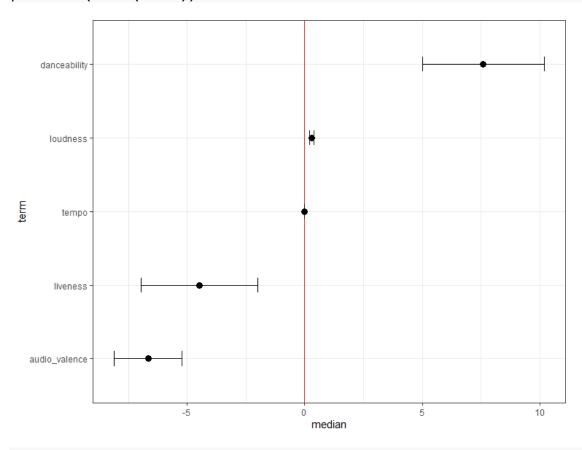
## -3.4e+02 -1.2e+03

summary(model)</pre>
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: song_popularity ~ tempo + danceability + audio_valence + liveness
       loudness + (1 + tempo + danceability | key) + (1 | time_signature) +
##
##
       (1 | audio_mode)
##
      Data: music_data
## REML criterion at convergence: 115396.8
##
## Scaled residuals:
##
      Min
               1Q Median
                               3Q
                                      Max
## -2.7740 -0.5528 0.1424 0.7216 2.4867
##
## Random effects:
## Groups
                               Variance Std.Dev. Corr
                  Name
## key
                   (Intercept) 1.603e+00
                                         1.265996
##
                  tempo
                               1.507e-05 0.003882 -1.00
                   danceability 1.869e+00 1.367055 -0.76 0.76
##
## time signature (Intercept) 2.530e+02 15.904768
                               3.940e+02 19.850453
## audio mode
                   (Intercept)
## Residual
                               3.988e+02 19.969965
## Number of obs: 13070, groups: key, 12; time_signature, 5; audio_mode, 2
##
## Fixed effects:
##
                  Estimate Std. Error
                                              df t value Pr(>|t|)
                                                   3.164 0.00156 **
## (Intercept)
                  5.036e+01 1.592e+01
                                       1.294e+04
## tempo
                 -1.627e-02 6.278e-03 6.830e+02 -2.591 0.00977 **
## danceability
                 7.505e+00 1.299e+00 3.775e+01
                                                   5.778 1.17e-06 ***
## audio valence -6.840e+00 7.664e-01 1.267e+04 -8.925 < 2e-16 ***
                                       1.306e+04 -3.910 9.27e-05 ***
## liveness
                 -4.771e+00
                           1.220e+00
## loudness
                 3.043e-01 4.520e-02 1.286e+04 6.732 1.74e-11 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Correlation of Fixed Effects:
##
               (Intr) tempo dncblt ad vln livnss
               -0.054
## tempo
## danceabilty -0.048 0.201
## audio_valnc -0.006 -0.073 -0.312
## liveness
               -0.024 0.009 0.116 -0.029
## loudness
               0.041 -0.154 -0.117 -0.141 -0.125
## optimizer (nloptwrap) convergence code: 0 (OK)
## boundary (singular) fit: see help('isSingular')
ranova(model)
## boundary (singular) fit: see help('isSingular')
## Warning: Model failed to converge with 1 negative eigenvalue: -2.3e+02
```

```
## boundary (singular) fit: see help('isSingular')
## Warning: Model failed to converge with 3 negative eigenvalues: -1.5e+00 -
5.2e+00
## -8.0e+02
## boundary (singular) fit: see help('isSingular')
## Warning: Model failed to converge with 2 negative eigenvalues: -1.5e+00 -
1.1e+01
## boundary (singular) fit: see help('isSingular')
## Warning: Model failed to converge with 4 negative eigenvalues: -6.1e+00 -
1.2e+01
## -1.3e+03 -4.7e+03
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## song popularity ~ tempo + danceability + audio_valence + liveness +
loudness + (1 + tempo + danceability | key) + (1 | time signature) + (1 |
audio mode)
##
                                                    npar logLik
                                                                   AIC
LRT Df
                                                      15 -57698 115427
## <none>
## tempo in (1 + tempo + danceability | key)
                                                      12 -57689 115402 -
## danceability in (1 + tempo + danceability | key)
                                                      12 -57701 115426
5.1712 3
## (1 | time signature)
                                                      14 -57697 115421
3.4402 1
                                                      14 -57698 115424 -
## (1 | audio_mode)
0.8801 1
##
                                                    Pr(>Chisq)
## <none>
## tempo in (1 + tempo + danceability | key)
                                                        1.0000
## danceability in (1 + tempo + danceability | key)
                                                        0.1597
## (1 | time signature)
                                                        1.0000
## (1 | audio mode)
                                                        1.0000
VarCorr(model)
   Groups
                   Name
                                Std.Dev.
                                           Corr
                                 1.2659959
## key
                   (Intercept)
##
                   tempo
                                 0.0038822 -1.000
                   danceability 1.3670545 -0.761 0.761
##
## time signature (Intercept) 15.9047685
## audio_mode
                   (Intercept) 19.8504527
## Residual
                                19,9699651
```

```
# Check the confidence interval to random effects
confint.merMod(model, method = 'Wald')
                        2.5 %
                                    97.5 %
##
## .sig01
                           NA
                                        NA
## .sig02
                           NA
                                        NA
## .sig03
                           NA
                                        NA
## .sig04
                                        NA
                           NA
## .sig05
                                        NA
                           NA
## .sig06
                           NA
                                        NA
## .sig07
                           NA
                                        NA
## .sig08
                           NA
                                        NA
## .sigma
                           NA
                                        NA
## (Intercept)
                 19.16256998 81.553999815
## tempo
                  -0.02856893 -0.003961337
## danceability
                  4.95938199 10.050901362
## audio valence -8.34266364 -5.338253987
## liveness
                 -7.16307452 -2.379751467
## loudness
                  0.21569048 0.392860235
# Fixed Effect
plotFEsim(FEsim(model))
```

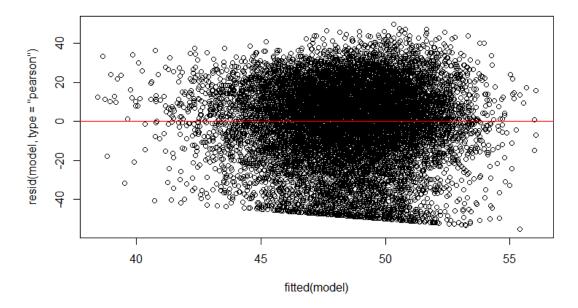


```
# Random Effect
round(ranef(model)$key, digits = 4)
```

```
(Intercept) tempo danceability
## 0
          -0.4415 0.0014
                                0.1306
## 1
           0.2288 -0.0007
                                0.8507
## 2
          -0.8040 0.0025
                                0.2677
## 3
           0.2578 -0.0008
                               -0.6062
## 4
           0.1129 -0.0003
                               -0.4721
## 5
           0.1656 -0.0005
                                0.0153
## 6
           1.3714 -0.0042
                               -0.4203
## 7
          -0.1107 0.0003
                               -0.5004
## 8
          -0.3975 0.0012
                               -0.0023
          -0.3423 0.0010
## 9
                               -0.3820
## 10
          0.2703 -0.0008
                                0.2386
## 11
          -0.3110 0.0010
                                0.8806
round(ranef(model)$time_signature, digits = 4)
##
     (Intercept)
## 0
         -5.7595
## 1
          1.4134
## 3
          0.5875
## 4
          2.2786
## 5
          1.4799
round(ranef(model)$audio_mode, digits = 4)
##
     (Intercept)
## 0
         -0.2717
## 1
          0.2717
```

Model Check

```
# Residuals VS Fitted value
plot(fitted(model), resid(model, type = "pearson"))
abline(0,0, col="red")
```



```
# Q-Q PLot
qqnorm(resid(model))
qqline(resid(model), col = "red")
```

Normal Q-Q Plot

