1 Problem 1

I want to use linear programming to optimally plan time-investment into all of my classes.

1.1 Decision Variables

The decision variables for an optimal study plan are the number of hours per week spent studying for each class. The number of hours per week will not vary each week. On weeks with an exam instead of a problem set, the time will be used to study for an exam. I have six classes, listed below. I am a math major. Some of the classes are math classes.

Class Name	Major?
QFT	no
Math Modeling	yes
Topology	yes
Combinatorics	yes
Robotic Manipulation	no
French	no

The decision variables are restricted by physical constraints. For example, each one must be positive. I'm assuming also that I do not have to spend an integer number of hours per week on each class.

There is also a constraint on the total number of hours spent per week. It's more helpful to think about constraints per day:

Constraint	amount hrs / day
Sleep	6
Time with Spouse	3
Lecture	3
Transit Time	1
Meal Time	1

This leaves 10 hours per day available to study. On weekends, lecture is absorbed into spouse time. Therefore, we conclude that there are 70 available total hours.

I am not placing any constraints on the total time spent on one class.

The decision space is therefore:

$$D = \left\{ h \in \mathbb{R}^6 \mid \text{for } 1 \leq i \leq 6, h_i > 0; \sum_{i=1}^{6} h_i < 70 \right\}$$

1.2 Optimization Goal

I am trying to minimize the total time spent studying overall. If $\vec{h} \in D$ is a set of decision variables, the total time per week is given by $\sum_{i=1}^{6} h_i = \vec{h} \cdot \vec{1}$ ($\vec{1} = (1,1,1,1,1,1)$). I am not trying to minimize time spent studying for one class over the other, I am treating each class equally.

1.3 GPA Constraints

I have two GPA constraints. My GPA in all of my classes, the average of the grades in each class, must be above a 3.0. My GPA in all of my major classes, again meaning the average, must be above a 3.5.

I need to determine how the number of hours spent on each class per week determines the grade in that class. I'm going to assume that the number of hours spent per week is averaged out over the whole semester.

Since I want to use linear optimization, I am restricted to a linear relationship between the overall class grade and the number of hours spent studying for it. I am going to try and determine two values for each class: the estimated grade (on a four point scale) that I will recieve if I do no work at all outside of lecture, and the estimated amount of hours per week to perfectly complete the problem set. I am going to assume that the number of hours per week to complete the problem set perfectly is the same as the number of hours per week of studying required for the exam to get a perfect score. This will simplify the analysis.

If a class has a no-effort grade g_0, and a requires h_p hours of work per week for a perfect score, then the grade is given by the formula:

$$g(h) = g_0 + (4 - g_0)\left(\frac{h}{h_p}\right)$$

This places an implicit constraint on the number of hours per week I can spend. Since my grade in each class cannot be above 4.0 (if I have this kind of an attitude, I'm not going to get any A+'s), under the assumptions of my model I cannot spend more time in each class than the h_p for that class. We should revise the state space:

$$D = \left\{ h \in \mathbb{R}^6 \mid \text{for } 1 \leq i \leq 6, 0 < h_i < h_{p_i}; \sum_{i=1}^{6} h_i < 70 \right\}$$

1.3.1 Quantum Field Theory

This class is a graduate class, so the grades will be higher. However this class has regular reading assignments in addition to the problem set. Each one of these reading assignments takes 2 hours, and there are two per week. In addition, the homework is long and difficult, taking an average of eight hours.

The final is only 25 percent of the grade, in-class work is 10 percent, and the rest is homework. This class is also very difficult for me, with new material. I estimate that I will recieve a 1.0 with no work, and that I will need 12 hours of work per week to recieve a 4.0.

1.3.2 Math Modeling

This class is a 3000-level math class, so the grades will not be as inflated as in graduate classes, but the grades will not be like those of weed-out classes. This class has regular homework but gives significant weight to a project which I will count as ”during lecture”. If I do alright on the tests, I think that I can get a 2.0 in the class with no work spent on the problem sets. The problem set last week took about 4 hours, so being conservative, I estimate that I can get a 4.0 in this class with 6 hours per week.

1.3.3 Topology

This class is a 4000-level math class, so grades will be relatively inflated. In addition, the class time is spent doing the reading assignments. The homeworks are weekly and relatively time consuming proof assignments. Since there are tests and reading assignments that can be done in class, I estimate that I can recieve a 2.0 in this class with no work done outside of class. I estimate that I can get a 4.0 in this class with 7 hours of work outside of class per week.

1.3.4 Combinatorics

This class is a 4000-level math class, so I expect the grades to be relatively inflated. The class time is spent on doing proofs, so if I only copy the statement of the proof at the end while using the class time to work on problem sets. Through this technique, I think that I can achieve a grade of 1.5 in this class with no outside work, and complege a problem set in 5 hours.

1.3.5 Robotic Manipulation

This class is a 4000-level crosslisted CS/MechE class. Most of the graded assignments are group projects, so I can get my group members to do most of the work through devious emailing and group member choice. I therefore estimate that I can get a 2.5 without doing any of the work outside of class. The work will take me 4 hours per week to get a 4.0, because I can start the projects early.

1.3.6 French

French is a 2000-level class I'm taking for the language requirement. The reading is discussed in class, so I don't have to do it if I make general comments responding to others. The class has online homework assignments that need to be complete. I speak french reasonable well, so I think that I can get a 2.5 in this class with no out-of-class work, and a 4.0 spending 1 hour per week.

I can summarize the parameters in a table:

g_0 (no-work grade)	h_p (hours per week for 4.0)
1.0	12
2.0	6
2.0	7
1.5	5
2.5	4
2.5	1

1.4 Summary of Constraints

1.4.1 Explicit Constraints

1. I can't work more than the total number of available hours:

$$h_1 + \cdots + h_6 \leq 70$$

2. The average grade must be above 3.0.

$$\frac{\sum_{i=1}^{6} b_i + 4.0\left(\frac{h_i}{(h_p)_i}\right)}{6} \geq 3.0$$

3. The average grade in major classes must be above 3.5

$$\frac{\sum_{i=\{2,3,4\}} b_i + 4.0\left(\frac{h_i}{(h_p)_i}\right)}{3} \geq 3.5$$

1.4.2 Implicit Constraints

1. I can't work fewer than 0 hours per week. For each $1 \leq i \leq 6$, $h_i \geq 0$.

2. I can get a maximum of 4.0 in each class, so I cannot work longer than the number of hours required for a perfect. For each $1 \leq i \leq 6$, $h_i \leq h_p$.

1.5 Results

When I solve this linear constrained optimization problem with matlab, I find the following schedule:

Class	Hours per Week
QFT	0.0
Math Modeling	1.5
Topology	0.0
Combinatorics	5.0
Robotic Manipulation	0.0
French	0.375

2 Problem 2: Robustness and Model Assumptions

I want to find out how robust my predictions are. Specifically, if the estimates of my parameters are flawed, how much can I expect this to change the results? Will it make me study more than optimal? How will it affect my GPA?

2.1 Robustness Testing through Random Noise

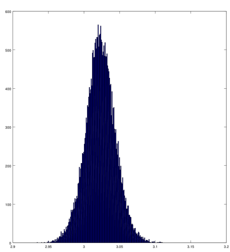

Figure 1: Total GPA's, gaussian noise added with $\sigma = 0.1$

I can do robustness testing through adding random noise to the parameters, and evaulating how much it violates my explicit constraints. I will have to make sure that the implicit constraints are still satisfied. Perturbations of the b_i no-work grade parameter must leave the parameter between 0.0 and 4.0.

Perturbations of the h_p hours required for pset only need to remain positive. We see that if we add gaussian noise to the parameters, the distribution of possible gps's is approximately gaussian. We can try and plot the ways that the mean and standard deviation of the GPA is related to the standard deviation of the added noise.

2.2 Other Neglected Aspects

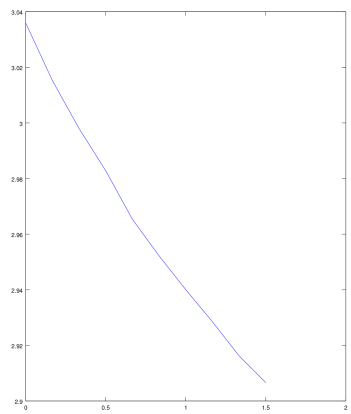

Figure 2: Mean Overall Grade, as σ of noise on hours studying required is varied

2.3 Other Neglected Aspects

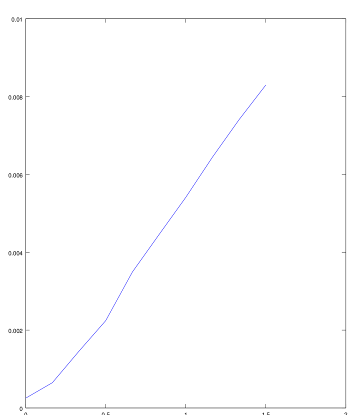

Figure 3: Standard Div of Overall Grade, as σ of noise on hours studying required is varied

Obviously, success in a class is determined by more than just the time spent studying for it. It can be determined by how the study time is distributed over the week, and what types of studying is needed. I haven't taken into account variances accross the semester in study time. There could be additional parameters, representing studying vs. work on homework. I could have several parameters for each class representing different study techniques.

Even if we restrict success in a class to be a function of only the time spent, there's very little reason to expect the relationship to be linear. Studying also has significant nonlinear effects. A 1-minute studying session is almost certainly not 1/100th of a 100 minute study session. I don't know whether it's more effective or less effective. I definitely expect that the function $g(h)$ will saturate on the high end.

The linear objective function is pretty accurate. It could be generalized by weighting different classes differently, to express a preference in studying for one class over another. For example, suppose I strongly dislike french, 5x more than I dislike studying for other classes. Instead of optimizing to minimize $(1,1,1,1,1,1) \cdot \vec{h}$, I can try to minimize $(1,1,1,1,1,5) \cdot \vec{h}$. The results are summarized below.

Class	Hours per Week (Original)	Hours per Week (new)
QFT	0.0	0.0
Math Modeling	6.0	6.0
Topology	1.75	1.75
Combinatorics	5.0	5.0
Robotic Manipulation	0.0	4.0
French	0.375	0

3 Problem 3

3.1 "Spread" and "Relative Spread"

The "spread" is the difference between the maximum final value, and the minimum final value. The "relative spread" attempts to normalize this value, by dividing by the expected value of the population in the continuum limit. So, the relative spread compares the difference in highest and lowest values to the expected value of the population. This is good at making sure that differences in the spread which are extremely small compared to the actual values of the population can be distinguished from differences in spread which correspond to a significiant portion of the population.

Since the spread and relative spread use the maximum and minimum values exclusively, I expect that running more samples would get more and more unlikely behavior, represented as strings of good or bad luck reproducing. Therefore the spread and relative spread should increase when more trials are run.

For example, on the lecture script, I get a relative spread of 0.646 when I run with 80 random sequences, and 0.531 when I run with 40.

The spread is good for determining deterministic and stochastic models of growth because it represents how much of a deviation from the expected value we might expect when trying to make decisions based on the model.

3.2 Different Initial Population Sizes

How does varying the initial population size affect the relative spread?

initial population	relative spread after 20 iterations
2	1.3065
3	0.9528
4	0.7884
5	0.9000
10	06211
20	0.4424
40	0.2984
80	0.1793
160	0.1050

The relative spread is a good way of determining how well the deterministic model reflects the stochastic model. The lower the relative spread, the more accurate I can expect the deterministic model to be.

3.3 Different Iteration Times

How does adding more iterations affect the relative spread?

number of iterations	relative spread
20	0.65233
25	0.58804
30	0.60614

It seems like adding more iterations decreases the relative spread, which seems like a restatement of the so-called "law of large numbers", that the larger these sequences get, the less "remarkable" they become.