1 Connection between spin and statistics

The spin-statistics theorem connects the spins of particles, and the result of interchanging the particles in a state. One of the primary results is that particles with integer spin are symmetric under interchange of the particles, while particles with half-integer spin are antisymmetric. Among other things, this shows that no two fermions can be in the same state.

Spin is connected to statistics by many factors: the S-matrix is only lorentz-invariant if the correct statistics are applied. The assumption that the total energy of the system remain bounded from below also implies the spin-statistics theorem. But the main idea is that spin affects how a state transforms under spatial translations and rotations. The half-integer representations mandate antisymmetry under exchange, while the integer representations mandate symmetry.

2 Arguments for Spin-Statistics connection

- For a field of spin s, a rotation of angle ϕ induces a phase of $e^{i\phi s}$. To interchange two identical particles, one way of doing this is to rotate the world by π around the midpoint of two particles. Therefore, for integer-spin particles, the phase is $+1$ and for half-integer spin particles the phase is -1. This is not a proof. It merely gives me a good idea of what to try to prove. If the exchange of particles is not path-dependant, then the spin-statistics connection has been determined. To show that there is no path-dependance, some topologists showed that a relative rotation is characterized up to homeomorphism by a single angle ϕ, that which that angles rotated around each other. In three dimensions, if the particles rotate around each other by 2π, one making a loop inside the other's larger loop, then their loops can be unentangled in the third spatial dimension. So the loops are only defined up to a phase of π. Therefore, every exchange of particles is homemorphic to every other, and thus the above argument works.

  This argument appeals to my intuition the best, although there's a bit of topology magic.

- Lorentz invariance of the S-matrix. In this case, the commutation relations within the time-ordered product are specified, and the lorentz-invariance is checked. This is convincing but offers no intuition.

- Stability I'm confused why the free complex scalar field is discussed before a real scalar field. The argument is clear here. I don't want this to be the method I use to prove things, though, because I would not expect every theory to have a positive definite energy density.

- Causality. This argument shows that the commutation relations must be correct, otherwise the operators of observables would commute (or anticommute) outside of the light cone.

3 Confusing

The arguments about proving the spin-statistics theorem with causality were definitely the most confusing in the chapter. It may just be because of glossing over the definitions of D and D_1.