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1 a

Suppose an accelerator delivers a luminosity of $100fb^{-1}$ over the past year. In that year, they saw 2500 events which produced and X particle.

Luminosity is defined as:

$$dN = L \times d\sigma \tag{1}$$

The total number of events is given by $\int dN$. Therefore, the total cross-section is:

$$\sigma = \int d\sigma = \int \frac{dN}{L} = \frac{2500}{100fb^{-1}} = 25fb$$

For reference, a femptobarn is $1 \times 10^{-43}m^2$.

2 b

How do we know that $\delta^{(4)}(\sum p_i^\mu - \sum p_j^\mu)$ is lorentz- invariant?

Consider an arbitrary lorentz transformation Λ_μ^ν. Suppose we shift to a different frame, transforming all of the variables in the above expression.

$$\begin{aligned}\delta^{(4)}(\sum p_i^\mu - \sum p_j^\mu) &\rightarrow \delta^{(4)}\left(\sum \Lambda_\mu^\nu p_i^\mu - \sum \Lambda_\mu^\nu p_j^\mu\right) \\ &\rightarrow \delta^{(4)}\left(\Lambda_\mu^\nu\left(\sum p_i^\mu - \sum p_j^\mu\right)\right)\end{aligned}$$

Since the matrix Λ_μ^ν is a lorentz transformation, it is invertible. Therefore $\Lambda_\mu^\nu(\sum p_i^\mu - \sum p_j^\mu) = 0$ implies $\sum p_i^\mu - \sum p_j^\mu = 0$, since Λ_μ^ν has no kernel. Similarly, $\Lambda_\mu^\nu(\sum p_i^\mu - \sum p_j^\mu) \neq 0$ implies $\sum p_i^\mu - \sum p_j^\mu \neq 0$. The functions agree pointwise.

Therefore, since for all lorentz transformations Λ_μ^ν,

$$\delta^{(4)}(\sum p_i^\mu - \sum p_j^\mu) = \delta^{(4)}(\Lambda_\mu^\nu(\sum p_i^\mu - \sum p_j^\mu))$$

$\delta^{(4)}(\sum p_i^\mu - \sum p_j^\mu)$ is lorentz-invariant.

3 c

The differential cross section for $2 \rightarrow n$ particle scattering is given by (Schwartz 5.22):

$$d\sigma = \frac{1}{(2E_1)(2E_2) \mid \vec{v_1} - \vec{v_2} \mid} \mid \mathcal{M} \mid^2 d\Pi_{\text{LIPS}}$$

where $d\Pi_{\text{LIPS}}$ is given by:

$$d\Pi_{\text{LIPS}} := (2\pi)^4 \delta^{(4)}(\Sigma p) \Pi_j \frac{d^3 p_j}{(2\pi)^3} \frac{1}{2E_j}$$

I'm trying to answer the question, what are the units of $\mid \mathcal{M} \mid$? The units of $d\sigma$ should be area, or -2. The units of $\frac{1}{(2E_1)(2E_2)|\vec{v_1}-\vec{v_2}|}$ are -2, since the velocity has energy dimension 0. Therefore, we expect the energy dimension of $\mid \mathcal{M} \mid^2$ to cancel out the energy dimension of $d\Pi_{\text{LIPS}}$.

Using the relation $\delta^4(0) = \frac{TV}{(2\pi)^4}$, we can see that the deltafunction has energy dimension -4. The energy dimension of momentum and energy are both 1, so the energy dimension of $d\Pi_{\text{LIPS}}$ is $2n - 4$, since an energy dimension of 2 is added for each particle in the product.

Therefore, the energy dimension of $\mid \mathcal{M} \mid^2$ is $4 - 2n$. For 2 particles scattering to 2 particles, the scattering amplitude is unitless.