

# 1 Homogenous Transformation

Two frames  $o_0x_0y_0z_0$  and  $o_1x_1y_1z_1$  are related by the homogenous transformation, which sends frame 0 to frame 1:

$$H = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

A particle has velocity  $v_1(t) = [3, 1, 0]^T$  relative to the frame  $o_1x_1y_1z_1$ . Since the velocity and the frame are both constant, to find the velocity in frame zero, it suffices to express the vector  $v_1$  in frame 0. The inverse transformation is:

$$H^{-1} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since  $v_1$  is a vector, the rotation is all that we need to consider. Therefore,  $v_0$  is:

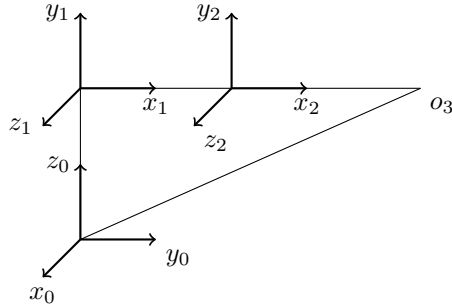
$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}$$

# 2 Jacobian for 3-link Elbow Manipulator

Compute the jacobian  $J_{11}$  for the 3-link elbow manipulator of Example 4.9 and show that it agrees with Equation (4.98).

From Eq. 4.58, if there are only revolute joints on a manipulator,

$$J_{v_i} = z_{i-1} \times (o_n - o_{i-1}) \quad (2)$$



Show that the determinant agrees with Equation (4.99).