MAE 6230 – Computational Fluid Dynamics

Spring 2017

Homework # 2

Due date: Tuesday February 28th, 2017 in class or electronically.

Concepts: Automatic generation of finite difference schemes, order of accuracy, boundary conditions.

Teamwork: Please turn in a single document per team of two, with your names on it. Provide a few sentences explaining in details the contribution of each team member to the overall homework.

We are interested in numerically computing the first derivative of a function on a one-dimensional mesh represented by the locations x_i, for $i = 0, \ldots, n$. We consider the general case, where the mesh is *not* uniformly distributed. In order to compute the derivative of a function f at location x_i, i.e., $\frac{\partial f}{\partial x}(x_i)$, we propose to use a computational stencil that extends from x_a to x_b, where a and b are two unspecified integers (probably related to i). We are not interested in Padé-type schemes, i.e., we only allow ourselves to use $f_a = f(x_a), \ldots, f_b = f(x_b)$.

1. Write the Lagrange polynomial that interpolates f on that stencil, i.e., the polynomial that returns the values of f at locations $x_a, \ldots, x_b$.

2. What is the first derivative of that polynomial?

3. Using the previous result, write a code that generates a finite difference scheme to compute the derivative of a function at an arbitrary location, using an arbitrary computational stencil. Provide your code. It should use two main inputs:

   – the computational stencil, in the form of an array of locations $(x_a, \ldots, x_b)$,

   – the location where the derivative should be computed by the schemes (note it could be any location).

   Your code should then return a finite difference coefficient for each stencil point.

4. What is the formal accuracy of the finite difference scheme you have generated?

5. Consider the mesh given by

$$x_i = \frac{i}{n} + \frac{1}{10}\sin\left(\frac{2\pi i}{n}\right), \tag{1}$$

   where n will be increased to study the accuracy of our finite difference scheme. Using that mesh and your finite difference scheme generation code, we want to compute the first derivative of the following function,

$$f(x) = \tanh(10x - 5). \tag{2}$$

   Use a centered 5-point computational stencil (i.e., use $f_{i-2}, \ldots, f_{i+2}$ to compute the derivative at x_i). Show how the error in your solution varies as you refine the mesh by graphing it in a log-log plot. Does it match your expectations? What can you say and do about the boundaries of your mesh?