1 Lagrange Interpolating Polynomial

We are trying to write a polynomial which, if we are considering the points $x_0, \ldots, x_n$, is equal to one at x_i, and equal to zero for all x_j, with $j \neq i$. We exhibit this polynomial below.

$$p_i(x) = \prod_{j \neq i} \frac{x - x_j}{x_i - x_j} \tag{1}$$

Note that $p_i(x_i) = \prod_{j \neq i} \frac{x_i - x_j}{x_i - x_j}$, and since all of the numerators and denominators are equal, we can see that $p_i(x_i) = 1$. For $p_i(x_k) = \prod_{j \neq i} \frac{x_k - x_j}{x_i - x_j}$, if $k \neq i$, since j ranges over all indices except i, one of the j's will be equal to k. That will make the numerator zero, and thus the whole product will be zero. Therefore, $p_i(x_k) = \delta_{ik}$.

If we want our polynomial to have the value $f(x_i)$ at each point x_i, we can sum several of these polynomials, so that the resultant polynomial has the characteristics which we desire:

$$p(x) = \sum_i f(x_i) p_i(x) \tag{2}$$

$$= \sum_i f(x_i) \left(\prod_{j \neq i} \frac{x - x_j}{x_i - x_j} \right) \tag{3}$$

This polynomial has the required values at each point.

2 Differentiating a Lagrange Interpolating Polynomial