Parcellations of Vector Fields in Computational Anatomy

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Stanford Statistics

Joint work with Susan Holmes, Xavier Pennec, and Nicolas Bronsard

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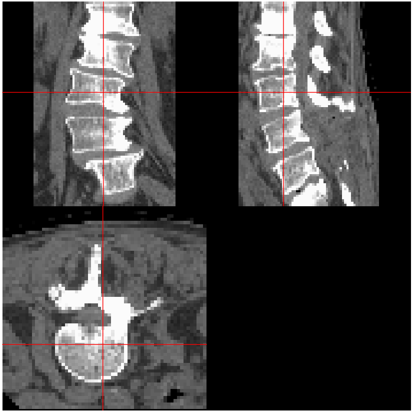
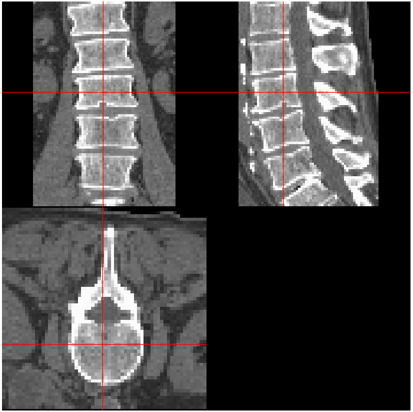
Quantification of Geometric Differences?



Lumbar Back Pain



Control

Computational Anatomy

Analyze geometric difference through deformation.

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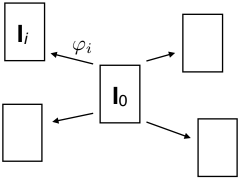
Find deformations φ_i from template $\mathbf{I}_0$ to subject images $\mathbf{I}_i$.



Parcellations in Computational Anatomy

Parcels are the right unit of comparison (not voxels).
Parcels should be spatially contiguous.

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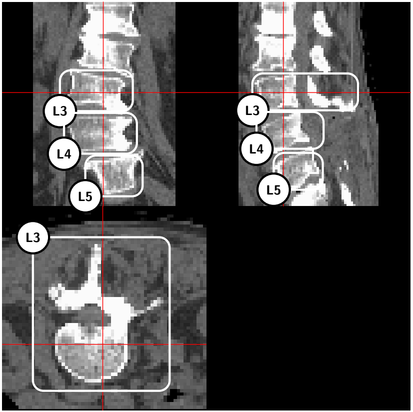
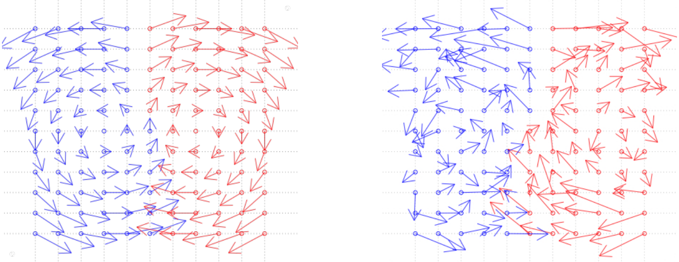


Problem Statement

Data: Deformation fields encoded as velocity fields.



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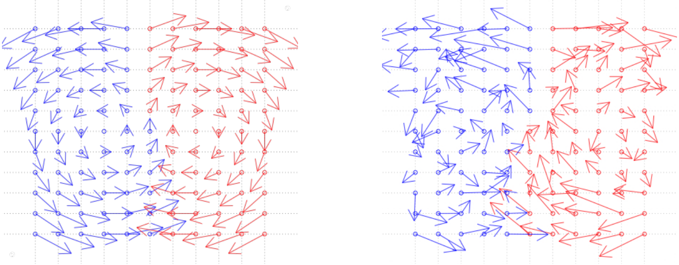

Given: Deformations are composed of locally linear affine transformations.

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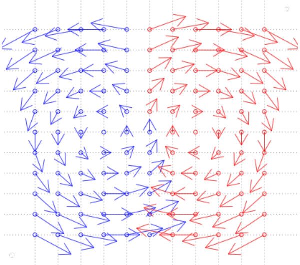 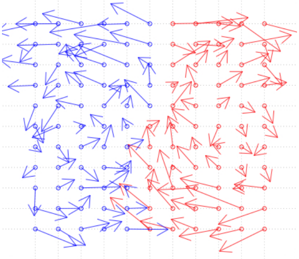

Given: Deformations are composed of locally linear affine transformations.
Infer: Number and shape of parcels.

Problem Statement: More Formal

We observe the velocity field

$$v(x) = \sum_{i=1}^{k} w_i(x) \begin{bmatrix} L_i & q_i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix} + \varepsilon(x)$$

for each subject.

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We want to infer both the #parcels k and shape $w_i(x)$.

Assumption: $w_i(x)$ are non-overlapping binary weight images.

Bayesian Nonparametemetric Model

$$\begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} = \mathbf{W} \begin{bmatrix} \text{Vectorize}(L_1) \\ q_1 \\ \vdots \\ \text{Vectorize}(L_k) \\ q_k \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

matrix $\mathbf{W}$ assigns n voxels to k parcels.

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Distance dependent Chinese restaurant process prior on parcellations:

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Gaussian prior on tranformations parameters L_i and q_i.

Affine Transformations

$$A_i x + b_i = \exp\left(\begin{bmatrix} L_i & q_i \\ 0 & 0 \end{bmatrix}\right)\begin{bmatrix} x \\ 1 \end{bmatrix}$$

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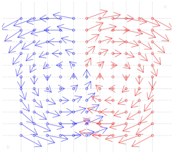
Jordan/Schur decomposition

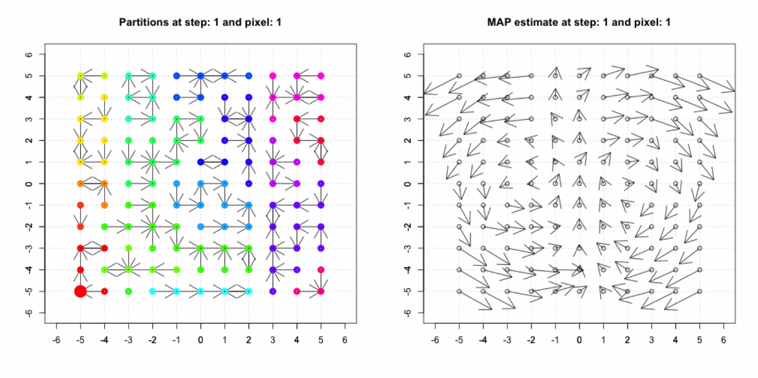
$$L_i = \frac{1}{2}(L_i - L_i^\mathsf{T}) + \frac{1}{2}(L_i + L_i^\mathsf{T})$$

$$L_i = \text{rotation} + \text{scaling} = \theta \begin{bmatrix} 0 & -r_3 & r_2 \\ r_3 & 0 & -r_1 \\ -r_2 & r_1 & 0 \end{bmatrix} + \text{diag}(s)$$

with rotation axis $\begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}$ and rotation angle θ.

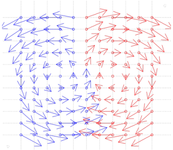
Example: Step 1, Pixel 1

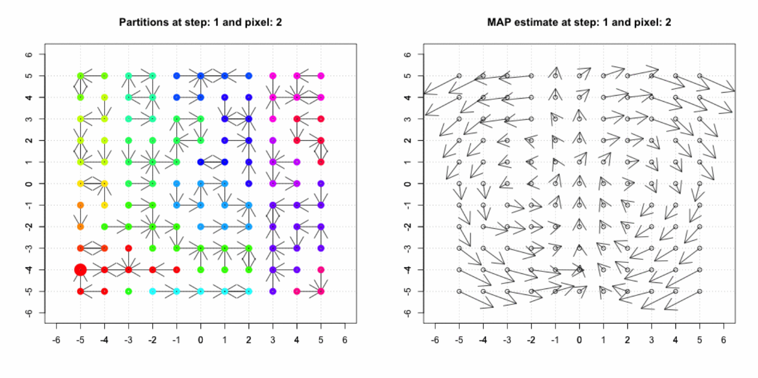




Posterior inference using Gibbs sampler.

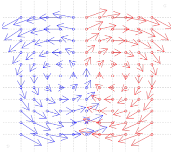
Example: Step 1, Pixel 2

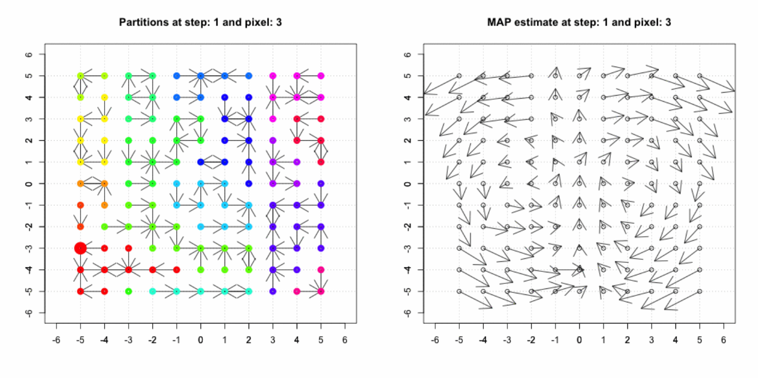




Posterior inference using Gibbs sampler.

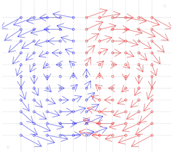
Example: Step 1, Pixel 3

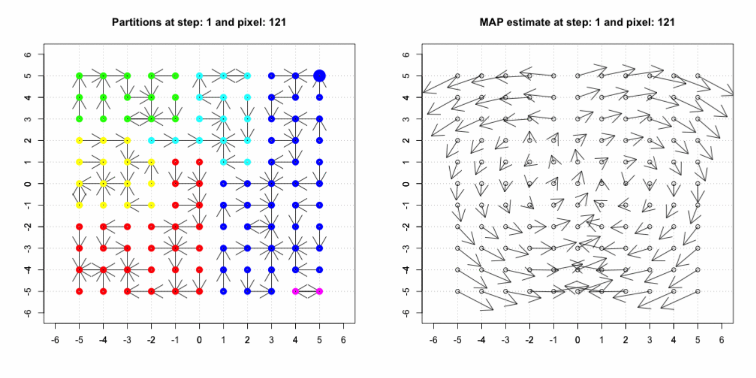




Posterior inference using Gibbs sampler.

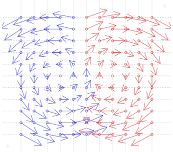
Example: Step 1, Pixel 121

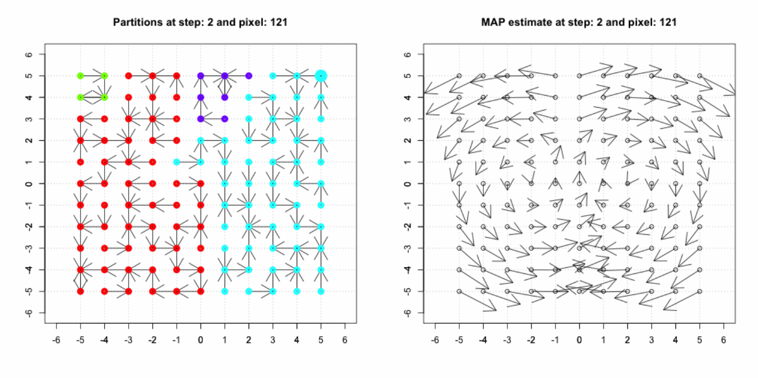




Posterior inference using Gibbs sampler.

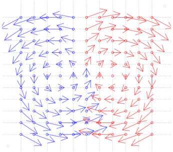
Example: Step 2, Pixel 121

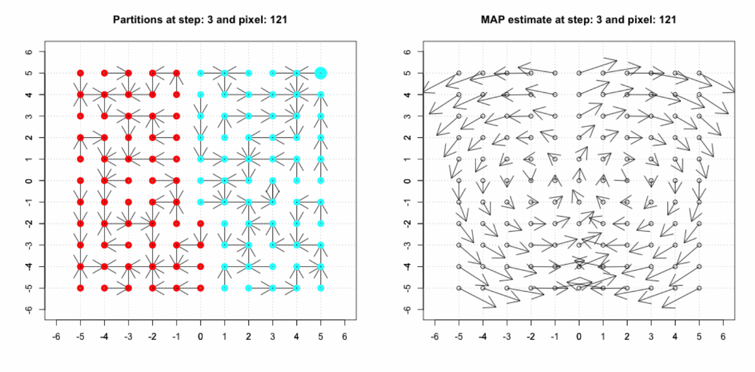




Posterior inference using Gibbs sampler.

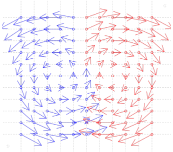
Example: Step 3, Pixel 121

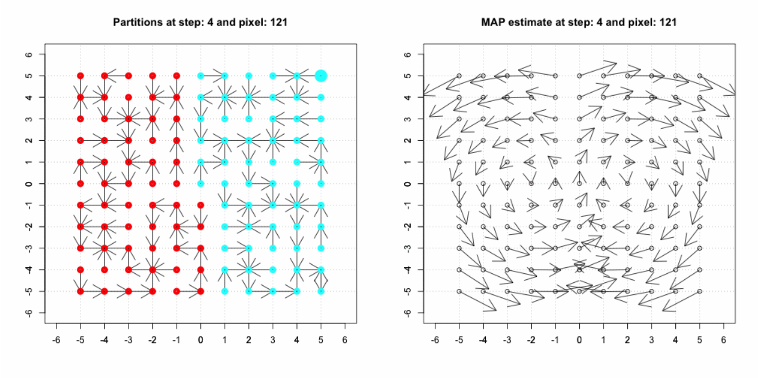




Posterior inference using Gibbs sampler.

Example: Step 4, Pixel 121





Posterior inference using Gibbs sampler.

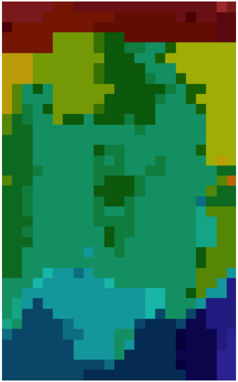
Preliminary Experiments on Spines

Goal: Geometry differences via parcellation: back pain vs control.

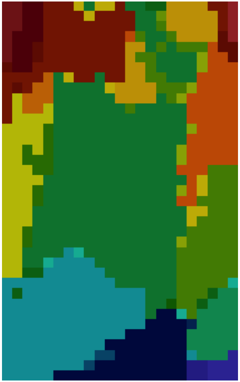
Data: 40 velocity fields computed using non-linear registration algorithm (Log-Demons algorithm).

Gibs Step 10

Template



Back Pain

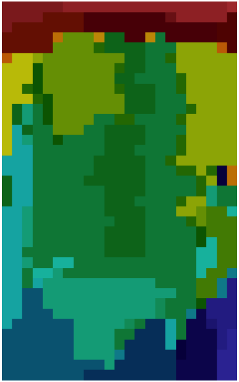


Control

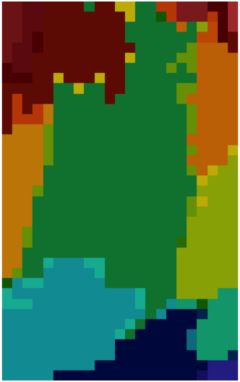
Posterior inference using Gibbs sampler.

Gibs Step 20

Template



Back Pain

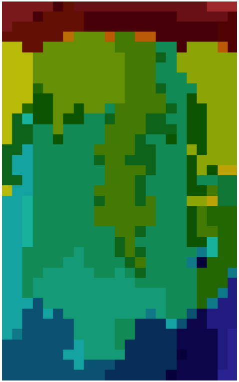


Control

Posterior inference using Gibbs sampler.

Gibs Step 30

Template



Back Pain

Control

Posterior inference using Gibbs sampler.

Conclusion

Code available:
`https://github.com/ChristofSeiler/BayesianNonparametrics`

Are these data-driven parcellations clinically relevant?

Acknowledgements

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Thanks for your attention!

References

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