To be handed in before 12 noon, Wednesday May 10, 2017

FMN011 Exam — Computational Part 2

Heat transfer in a thermally stratified chemical reactor

April 2017

Please read the instructions on how to present your report on the course web page.

1 Description of the Problem

Figure 1.1: A chemical reactor tank.

A chemical reactor is thermally stratified. The colder, denser liquid sits on the bottom of the tank, and the warmer liquid rises to the surface. The tank can be idealized as two zones separated by a strong temperature gradient or *thermocline*. The location of the thermocline is defined as the inflection point of the temperature curve as a function of the depth, that is, the point at which $\mathrm{d}^2T/\mathrm{d}z^2 = 0$. At this point, the absolute value of the derivative or gradient, $\mathrm{d}T/\mathrm{d}z$ is a maximum.

The temperature gradient can be used to determine the heat flux across the thermocline, from the surface layer to the bottom layer. Fourier's law gives

$$J = -k\frac{\mathrm{d}T}{\mathrm{d}z}$$

where J is the heat flux in $\mathrm{cal}/(\mathrm{cm}^2 \cdot \mathrm{s})$, $k = 0.01\,\mathrm{cal}/(\mathrm{cm} \cdot \mathrm{s} \cdot^\circ \mathrm{C})$, T is the temperature in °C, and z is the depth in cm.

The following table gives the temperatures in the tank at a particular time:

Depth, m	0	0.5	1	1.5	2	2.5	3
Temperature, °C	70	70	55	22	13	10	10

2 Tasks

Task 1. Use a clamped cubic spline with zero end derivatives to interpolate the given data.

Task 2. Determine the thermocline depth.

Task 3. Compute the flux across this interface.

Task 4. Plot the data together with the curve, T vs z, and also the first derivative and the second derivative.

Task 5. Approximate the temperature at a depth of 1.7 m., and the depth at which the temperature is 50 °C.

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