

MIDTERM Review

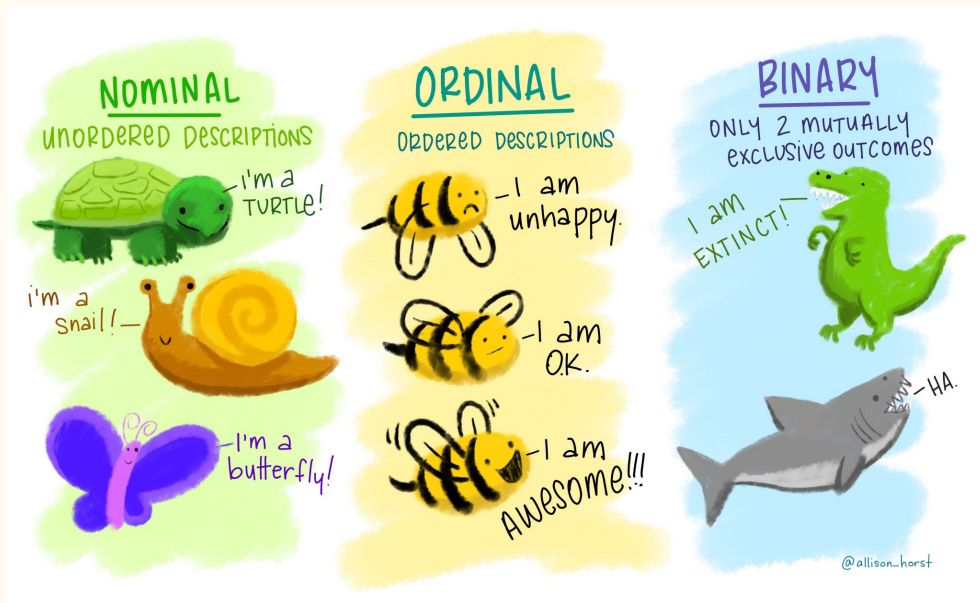
STA130F24

Data Types

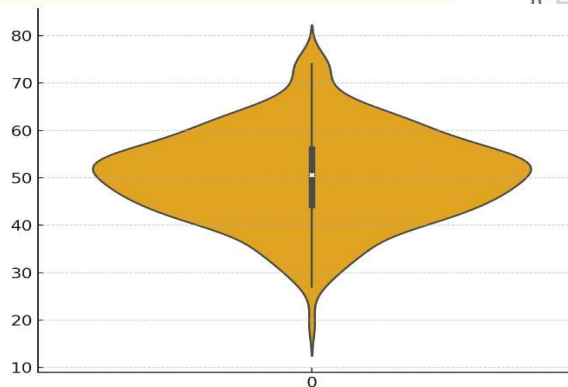
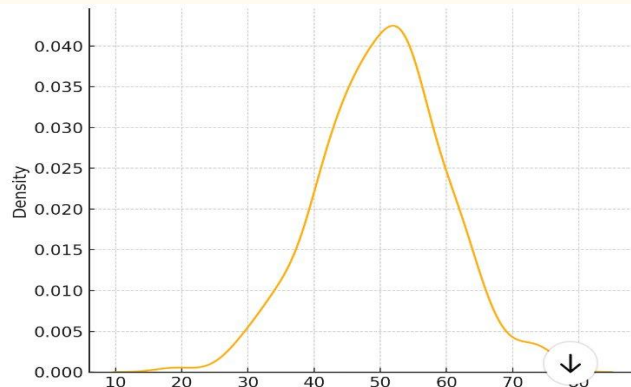
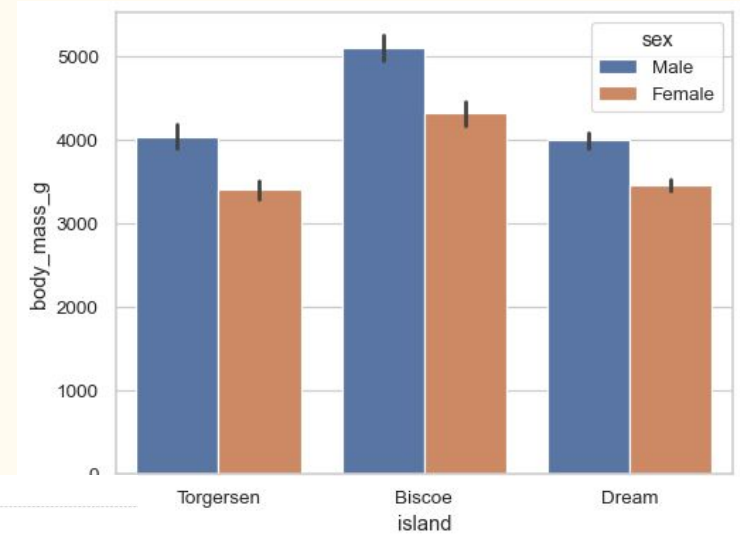
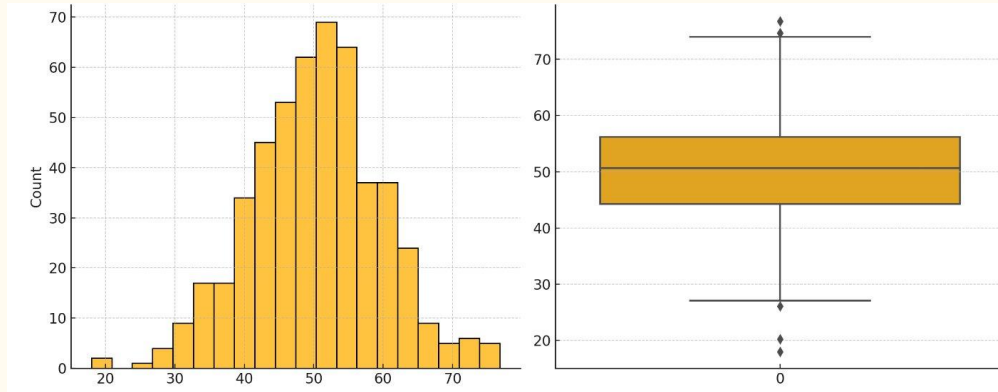
Quantitative variables



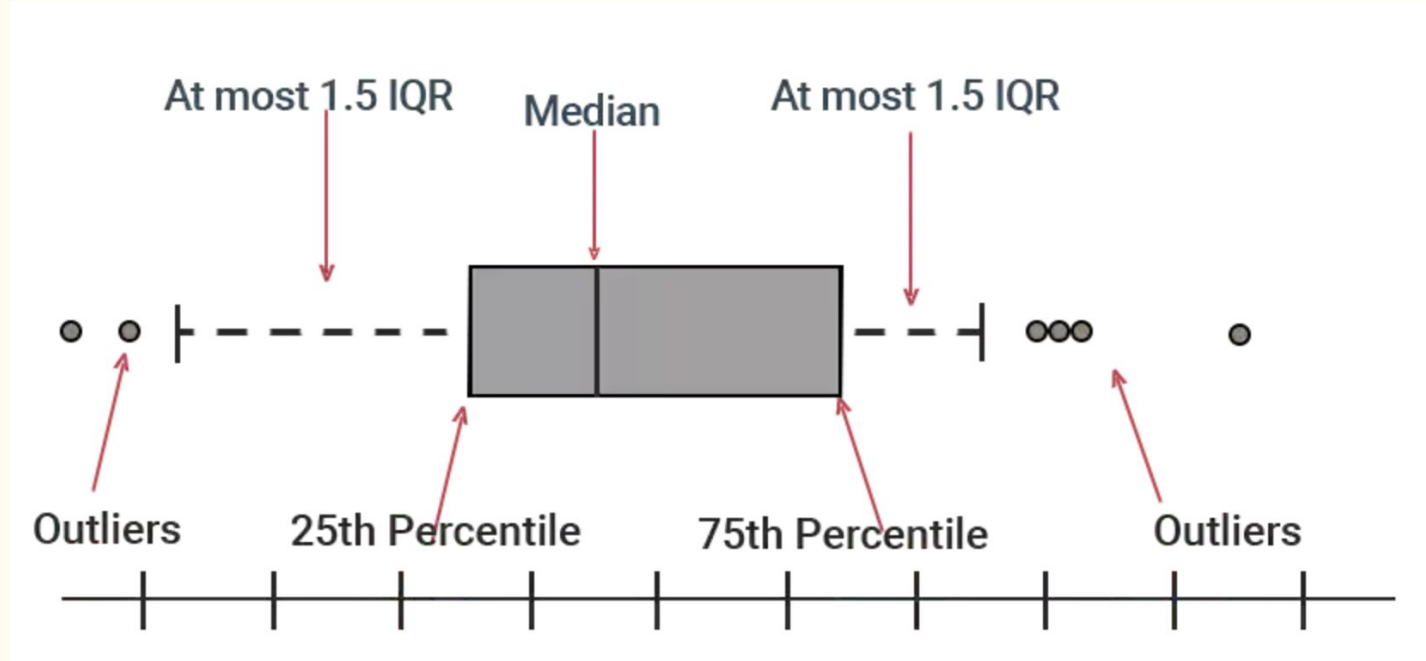
Qualitative variables



How to Visualize your Data?

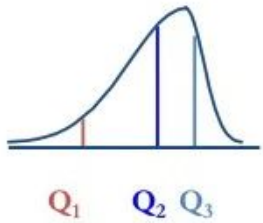


Median and Potential Outliers

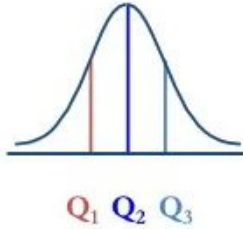


Skewness and Modality

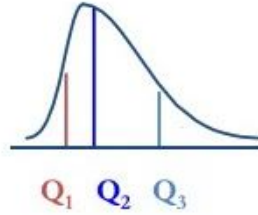
Left-Skewed



Symmetric



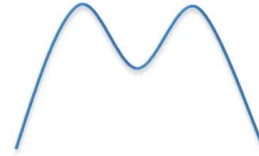
Right-Skewed



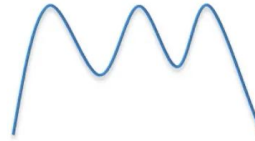
Unimodal



Bimodal

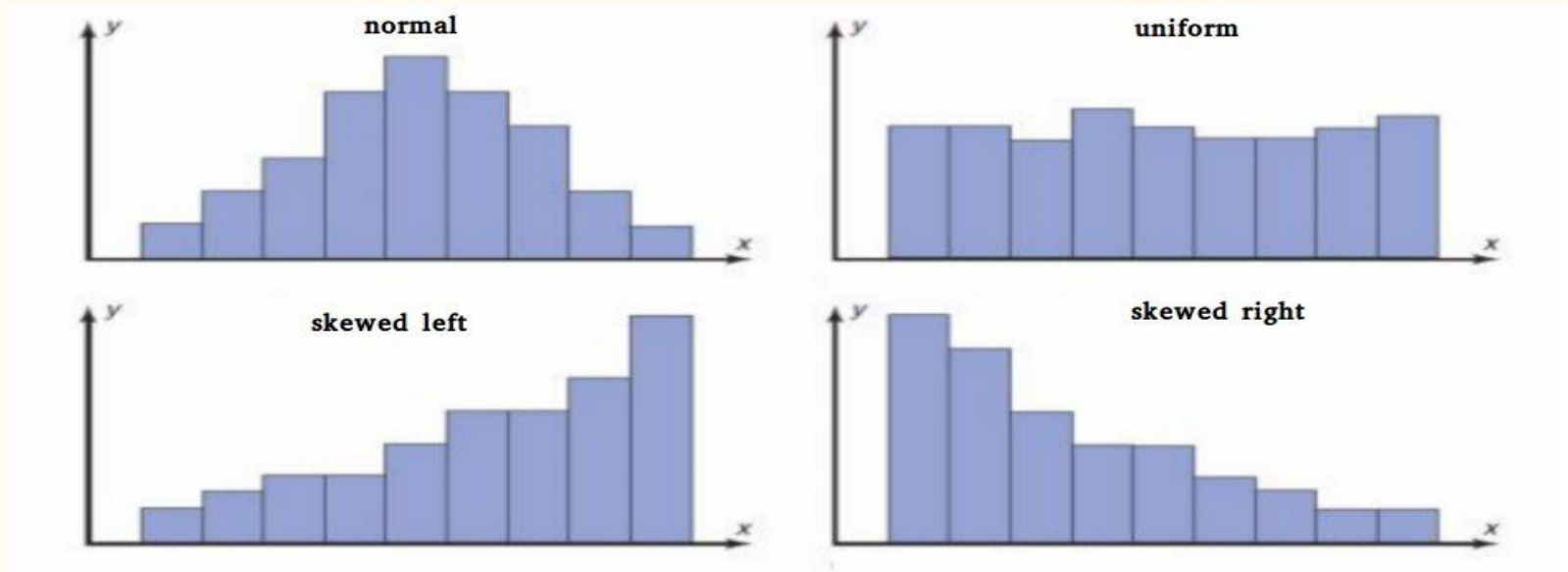


Multimodal

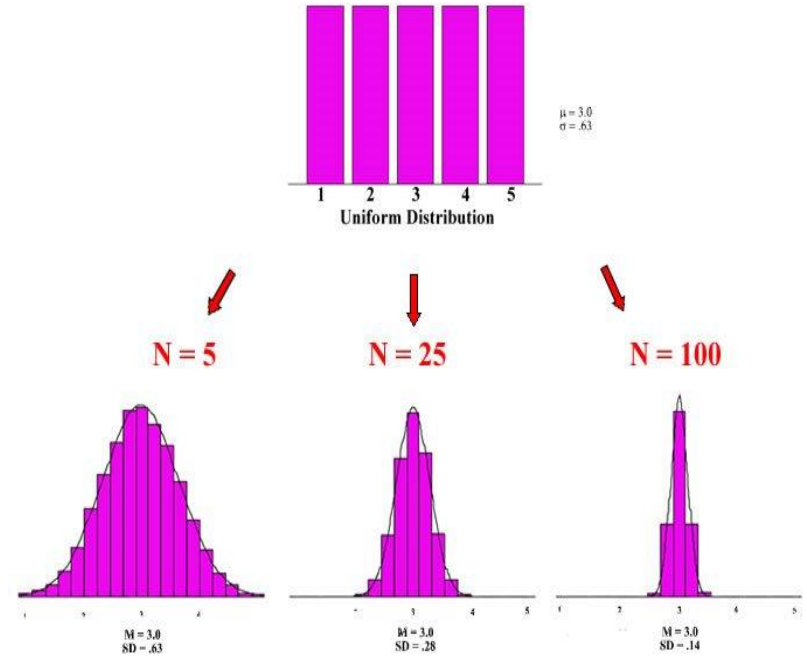
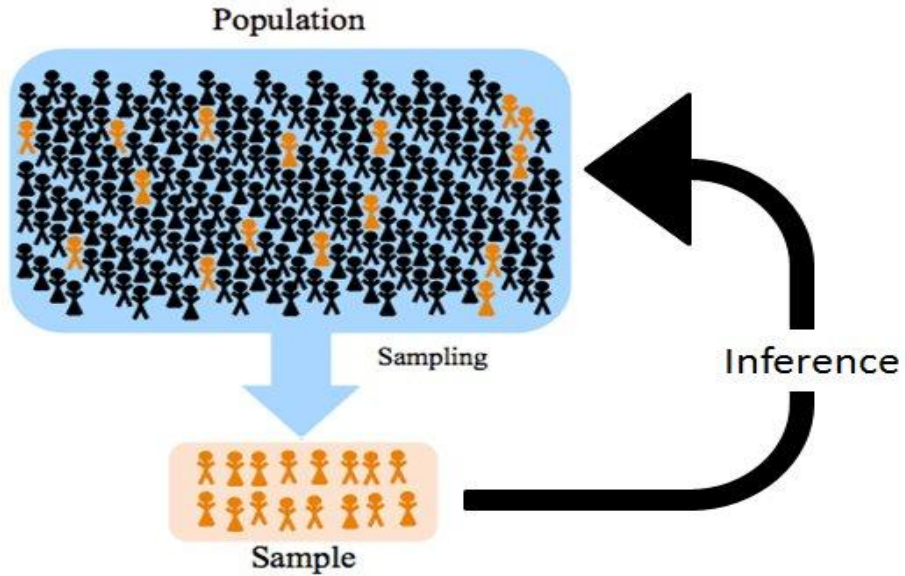


What is Distribution?

The way in which the values of a random variable are spread or allocated.



Sampling Distribution



Standard Deviation vs Standard Error

Standard deviation: the amount of variation or dispersion of individual data points within a single sample. It shows how spread out the data points are around the sample mean.

Standard error: how much variability there is in the sample mean as an estimate of the population mean

$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1}}$$

- x_i = each individual data point.
- \bar{x} = sample mean.
- n = sample size.

$$SE = \frac{s}{\sqrt{n}}$$

How number of samples (n) drives SE?

The standard error of the mean *decreases as the sample size increases*, indicating more "precise" estimates with larger samples.

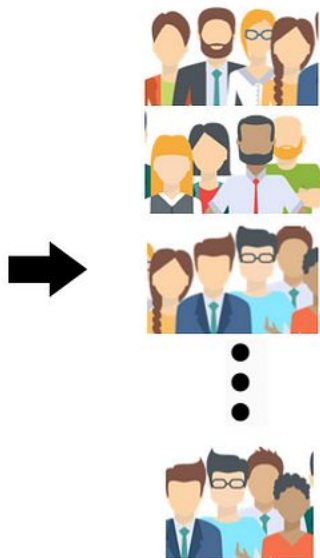
$$SE = \frac{s}{\sqrt{n}}$$

Bootstrapping

Sample Size N



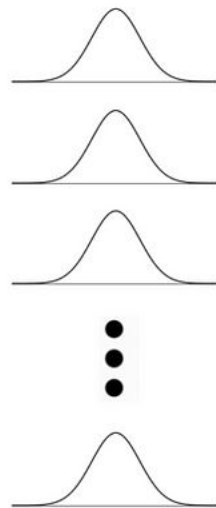
B Bootstrap
Samples, each size n



Estimate θ

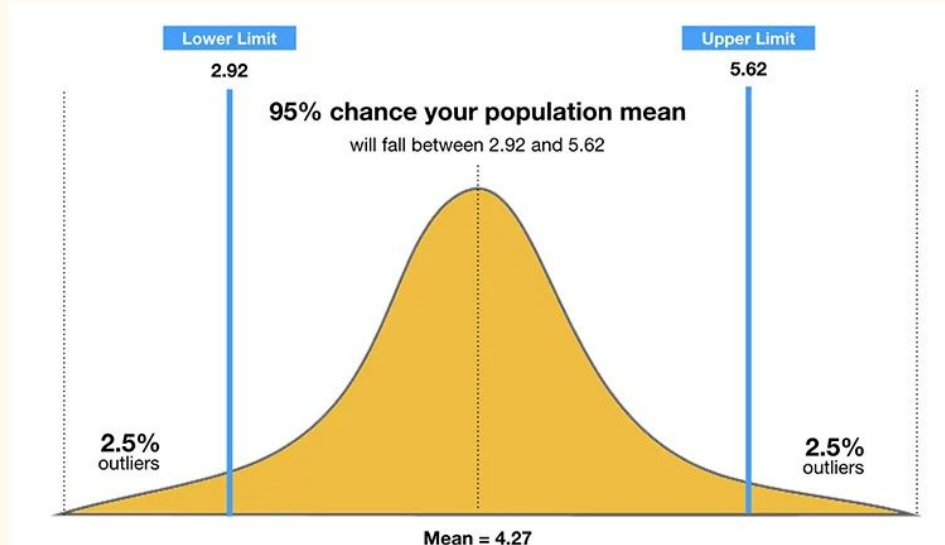
θ
 θ
 θ
 \vdots
 θ

Inference



Confidence Interval

- A plausible range of values for a population parameter.
- The most common choice for the confidence level is **95%**, meaning that the chance that an independent sample will result in the construction of a confidence interval which "captures" the actual true value of the population parameter is 95%.



Hypothesis Testing

- Hypothesis test give a decision about whether a hypothesis is consistent with the observed data.
 - **Null hypothesis** (H_0)
 - The working theory until proven otherwise
 - **Alternative hypothesis** (H_a)
 - The opposing theory of the null
 - **Test statistic**
 - Evidence from the data to help make the decision about H_0
 - **P-value**
 - Probability the test statistic supports H_0 assuming its true
 - **Significance level** (α)
 - Threshold to determine if we reject H_0 based on the p-value

Null Hypothesis vs Alternative Hypothesis

- **Null hypothesis** (H_0)
 - The working theory until proven otherwise
- **Alternative hypothesis** (H_a)
 - The opposing theory of the null

The **null hypothesis** usually simply states the "no effect" (on average) assumption

H_0 : The vaccine has no effect (**on average**) on patient health

H_1 : H_0 is false

To emphasize that "**(on average)**" refers to the population parameter μ (the average effect), it is helpful to more formally (and concisely) express this equivalently as

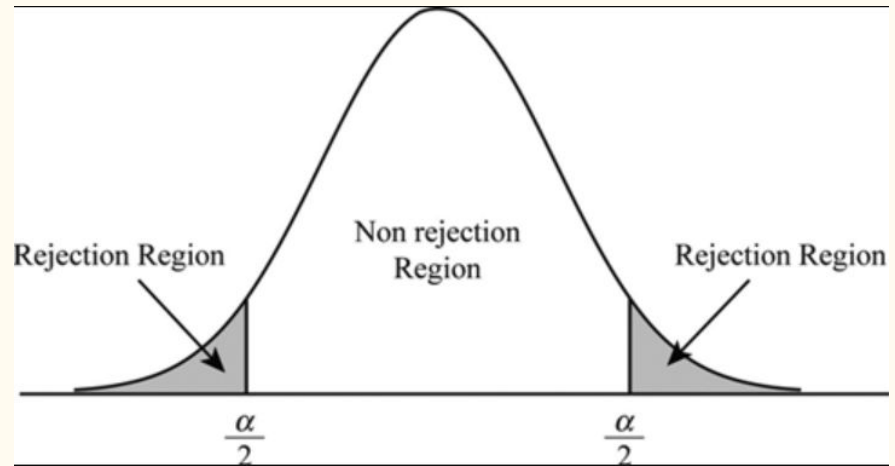
$$H_0 : \mu = 0 \quad \text{and} \quad H_A : H_0 \text{ is false}$$

Test Statistics (Observed vs Simulated)

- **Test statistic:** mean, median, mode, variance, etc.
- **Observed test statistic:** calculated based on sample data.
- **Simulated test statistic:** calculated based on the simulation under the assumption that null hypothesis is true.

P-value & Significance level

- **P-value:** The probability that a test statistic (simulated test statistic) is as or more extreme than the observed test statistic under the **assumption that the null hypothesis is true**.
- **Significance level / Rejection level:** threshold to determine if we reject the null hypothesis based on the p-value.



P-value and the strength of the evidence

p-value	Evidence
$p > 0.1$	No evidence against the null hypothesis
$0.1 \geq p > 0.05$	Weak evidence against the null hypothesis
$0.05 \geq p > 0.01$	Moderate evidence against the null hypothesis
$0.01 \geq p > 0.001$	Strong evidence against the null hypothesis
$0.001 \geq p$	Very strong evidence against the null hypothesis

Type I and Type II Errors

Decision	Null Hypothesis is True	Null Hypothesis is False
Reject Null	Type I Error (with chance α)	Correct Decision
Fail to Reject	Correct Decision	Type II Error (with chance β)

- **Type I.** If a medical test incorrectly shows that a patient has a disease when they actually don't.
- **Type II.** If a medical test incorrectly shows that a patient does not have a disease when they actually do.

One-sided vs Two-sided Hypothesis Test

One-Tailed Test (Left Tail)	Two-Tailed Test	One-Tailed Test (Right Tail)
$H_0 : \mu_X = \mu_0$ $H_1 : \mu_X < \mu_0$	$H_0 : \mu_X = \mu_0$ $H_1 : \mu_X \neq \mu_0$	$H_0 : \mu_X = \mu_0$ $H_1 : \mu_X > \mu_0$
Rejection = $(1 - \alpha)$	Rejection = $(1 - \alpha) / 2$	
