Bayesian Learning (732A91) Lab3 Report

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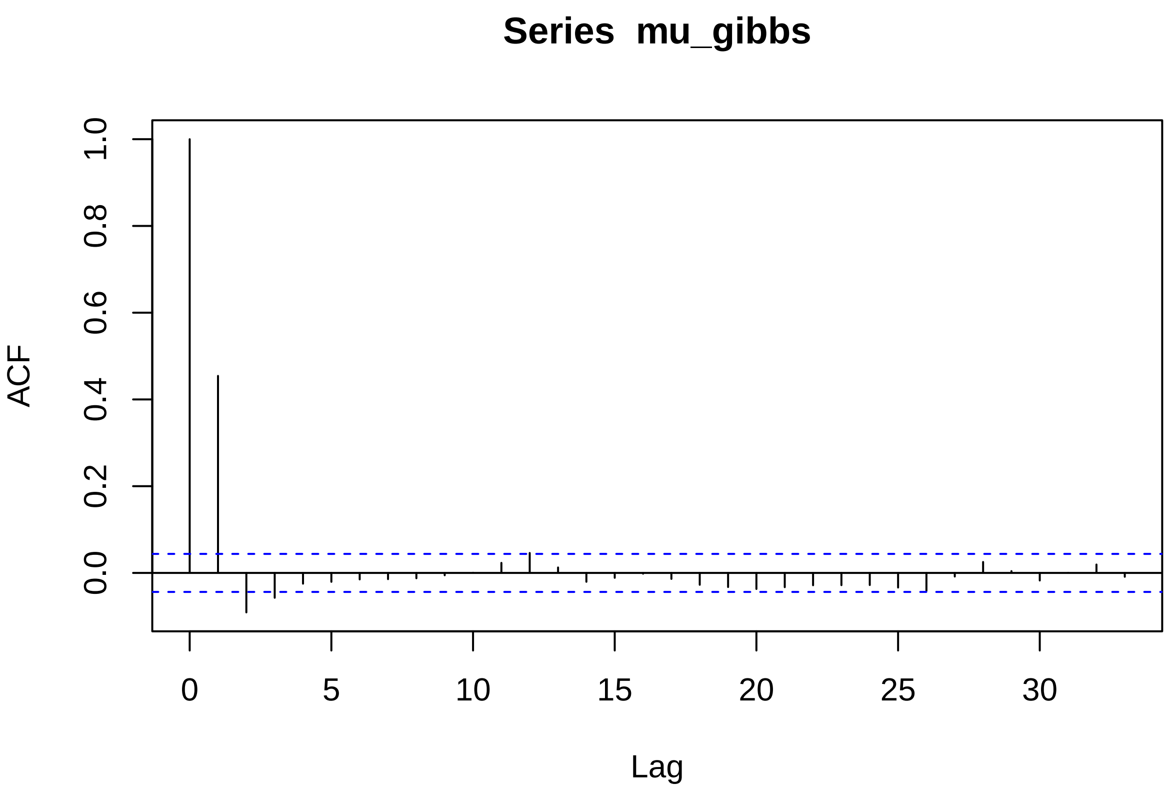
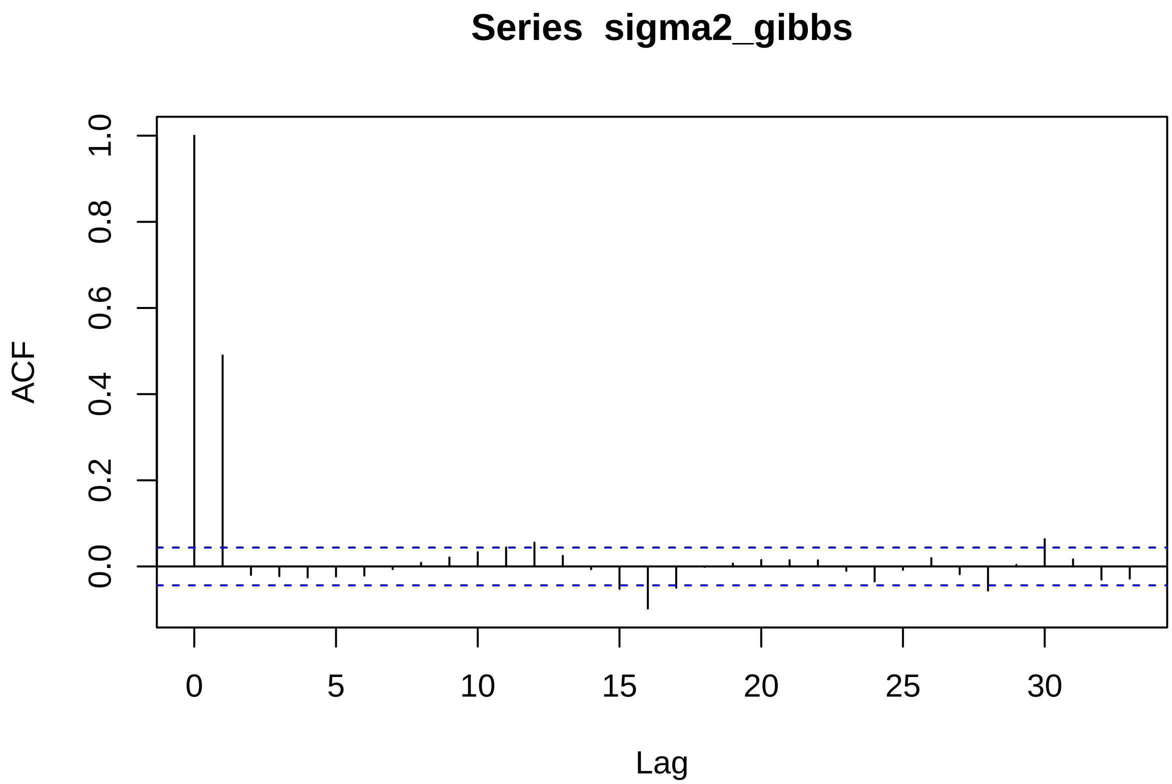
Assignment 1 *Gibbs sampler for a normal model*

Task 1a

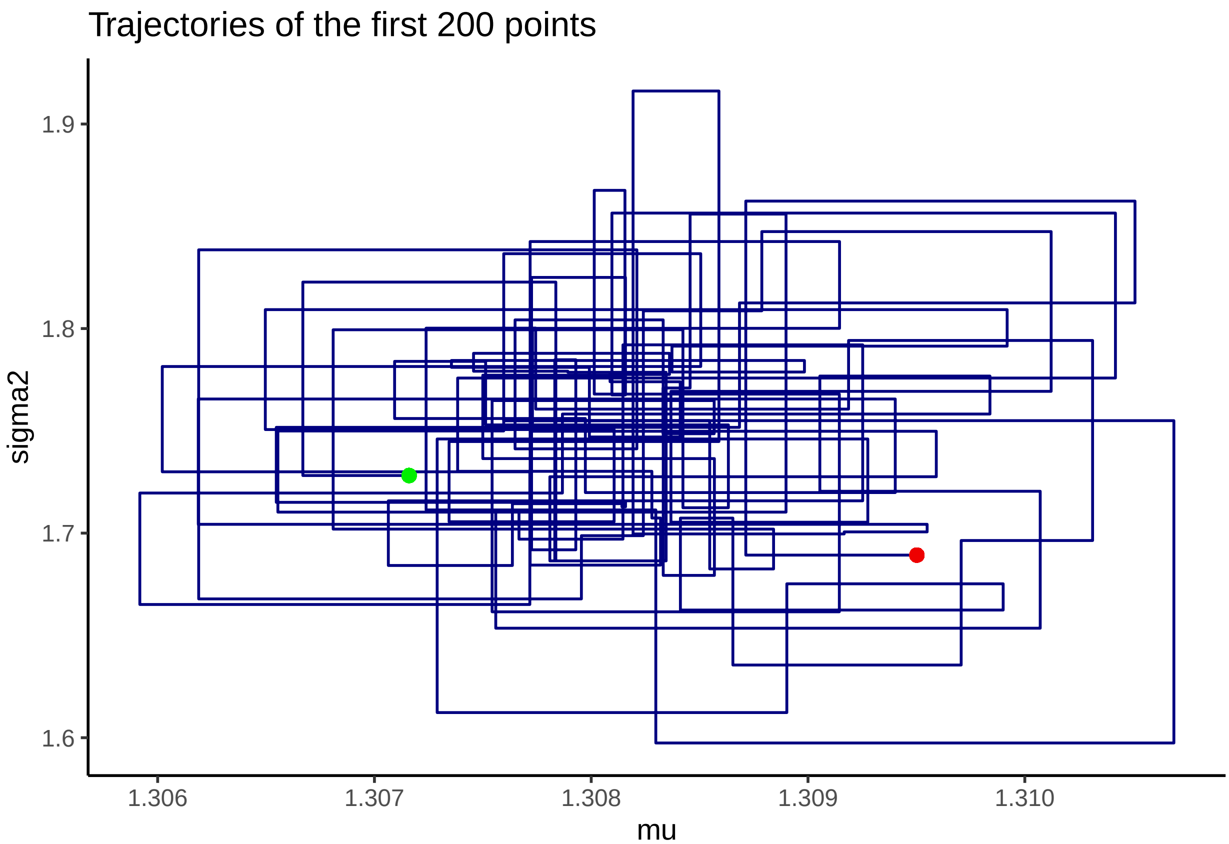
The full conditional posteriors are:

$$\mu|\sigma^2, x \sim N(\mu_n, \tau_n^2)$$
$$\sigma^2|\mu, x \sim Inv - \chi^2\Big(\nu_n, \frac{\nu_0\sigma_0^2 + \sum_{i=1}^{n}(x_i - \mu)^2}{n + \nu_0}\Big)$$



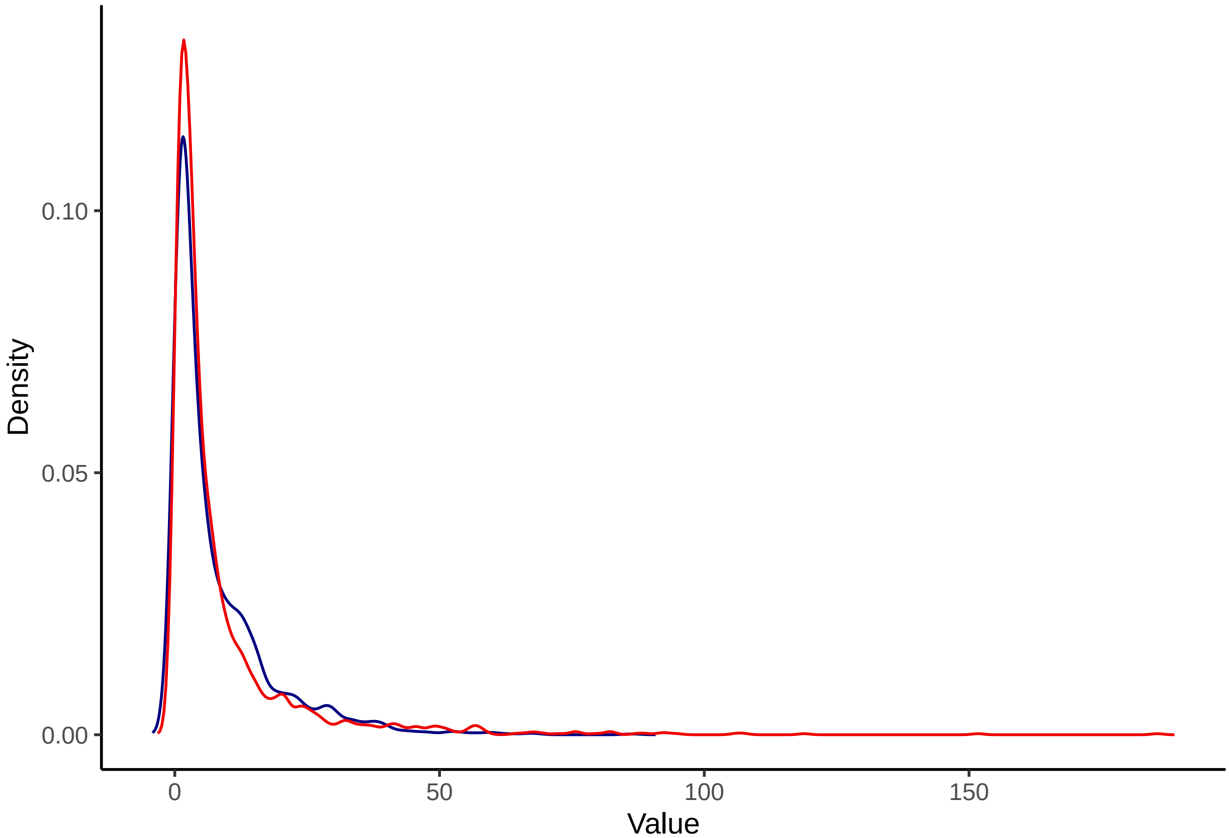


	mu	sigma2
Inefficiency Factors	0.9468888	1.635177



Trajectories of the first 200 points
1.9
1.8
1.7
1.6
sigma2
1.306
1.307
1.308
1.309
1.310
mu

Task b



Assignment 2 *Metropolis Random Walk for Poisson regression*

Task a

```
##
## Call:
## glm(formula = nBids ~ 0 + ., family = poisson, data = ebay)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -3.5800  -0.7222  -0.0441   0.5269   2.4605
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## Const        1.07244    0.03077  34.848  < 2e-16 ***
## PowerSeller -0.02054    0.03678  -0.558   0.5765
## VerifyID    -0.39452    0.09243  -4.268 1.97e-05 ***
## Sealed       0.44384    0.05056   8.778  < 2e-16 ***
## Minblem     -0.05220    0.06020  -0.867   0.3859
## MajBlem     -0.22087    0.09144  -2.416   0.0157 *
## LargNeg      0.07067    0.05633   1.255   0.2096
## LogBook     -0.12068    0.02896  -4.166 3.09e-05 ***
## MinBidShare -1.89410    0.07124 -26.588  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 6264.01  on 1000  degrees of freedom
## Residual deviance:  867.47  on  991  degrees of freedom
## AIC: 3610.3
##
## Number of Fisher Scoring iterations: 5
```

Task b

Poisson regression model: $y_i|beta \sim Poisson[exp(x_i^T\beta)]$, $i = 1, 2, ..., n$

The likelihood function of the Poisson regression model is:

$$L(\theta|X, Y) = \prod_i^n \frac{e^{Y_i\theta^T X_i} e^{-\theta^T X_i}}{Y_i!}$$

The log-likelihood of the Poisson regression model is:

$$l(\theta|X, Y) = log\Big(L(\theta|X, Y)\Big) = \sum_i^n \Big(Y_i\theta^T X_i - e^{\theta^T Xi} - log(Y_i)\Big)$$

In the above formula, θ appears in the first two terms; therefore, given that we are only interested in the finding of the best value of θ, we drop the $log(Y_i)$.

The final log-likelihood of the Poisson regression model is:

$$l(\theta|X, Y) = \sum_i^n \Big(Y_i\theta^T X_i - e^{\theta^T Xi}\Big)$$

https://en.wikipedia.org/wiki/Poisson_regression

The below table illustrates the $J_y^{-1}(\widetilde{\beta})$.

Constant	PowerSeller	VerifyID	Sealed	MinBlem	MajBlem	LargNeg	LogBook	MinBidShare
0.0009455	-0.0007139	-0.0002742	-0.0002709	-0.0004455	-0.0002772	-0.0005128	0.0000644	0.0011099
-0.0007139	0.0013531	0.0000402	-0.0002949	0.0001143	-0.0002083	0.0002802	0.0001182	-0.0005686
-0.0002742	0.0000402	0.0085154	-0.0007825	-0.0001014	0.0002283	0.0003314	-0.0003192	-0.0004293
-0.0002709	-0.0002949	-0.0007825	0.0025578	0.0003577	0.0004532	0.0003376	-0.0001311	-0.0000576
-0.0004455	0.0001143	-0.0001014	0.0003577	0.0036246	0.0003492	0.0000584	0.0000585	-0.0000644
-0.0002772	-0.0002083	0.0002283	0.0004532	0.0003492	0.0083651	0.0004049	-0.0000898	0.0002622
-0.0005128	0.0002802	0.0003314	0.0003376	0.0000584	0.0004049	0.0031751	-0.0002542	-0.0001063
0.0000644	0.0001182	-0.0003192	-0.0001311	0.0000585	-0.0000898	-0.0002542	0.0008385	0.0010374

Constant	PowerSeller	VerifyID	Sealed	MinBlem	MajBlem	LargNeg	LogBook	MinBidShare
0.0011099	-0.0005686	-0.0004293	-0.0000576	-0.0000644	0.0002622	-0.0001063	0.0010374	0.0050548

The below table illustrates the posterior mode values of every feature of the dataset.

	Value
Constant	1.0698412
PowerSeller	-0.0205125
VerifyID	-0.3930060
Sealed	0.4435555
MinBlem	-0.0524663
MajBlem	-0.2212384
LargNeg	0.0706968
LogBook	-0.1202177
MinBidShare	-1.8919850

The below table illustrates the approximate posterior standard deviation values of every feature of the dataset.

	Value
Constant	0.0307484
PowerSeller	0.0367842
VerifyID	0.0922787
Sealed	0.0505745
MinBlem	0.0602047
MajBlem	0.0914607
LargNeg	0.0563477
LogBook	0.0289564
MinBidShare	0.0710968

Task c

For the Random walk Metropolis algorithm:

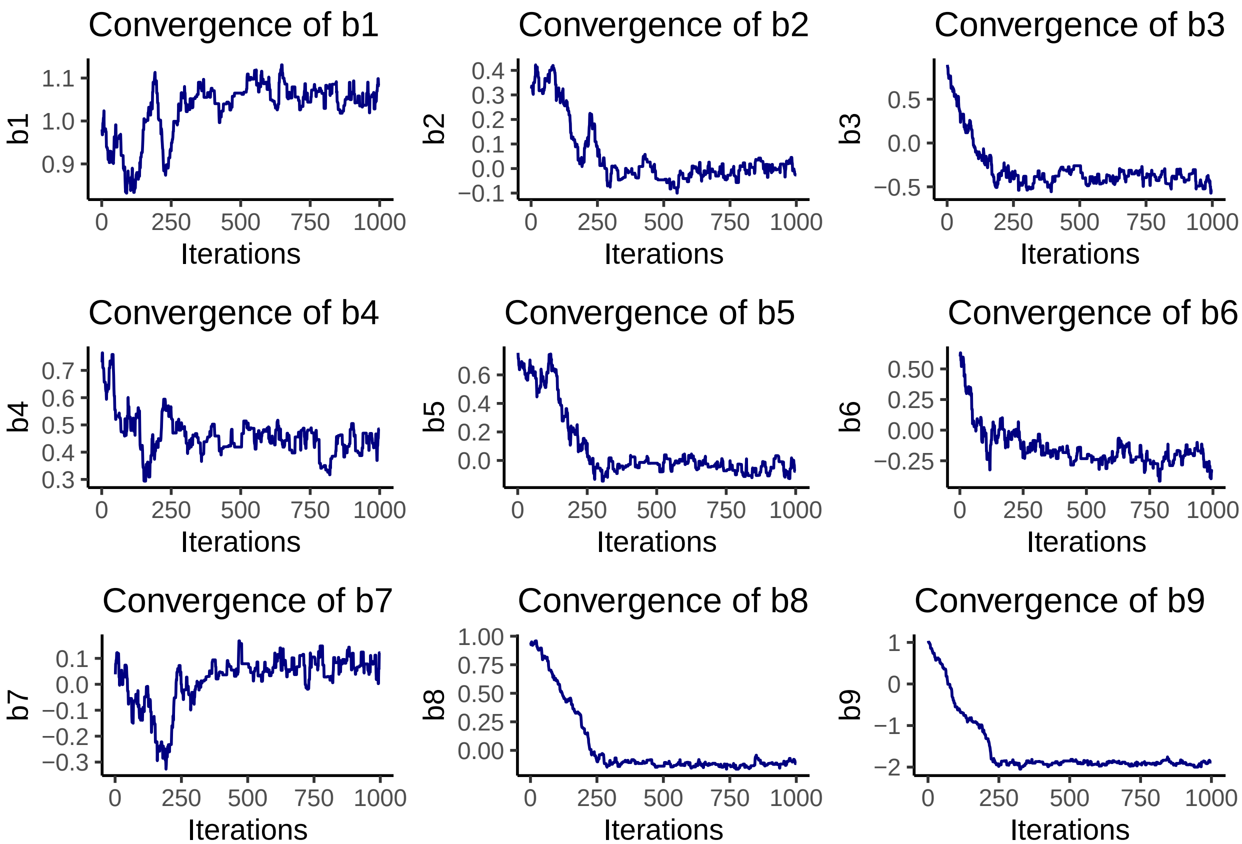
1) Initialize θ^0 and iterate for i = 1, 2, ...

2) Sample proposal: $\theta_p|\theta^{(i-1)} \sim N(\theta^{(i-1)}, c \cdot \Sigma)$, where $\Sigma = J^{-1}_{\hat{\theta},y}$

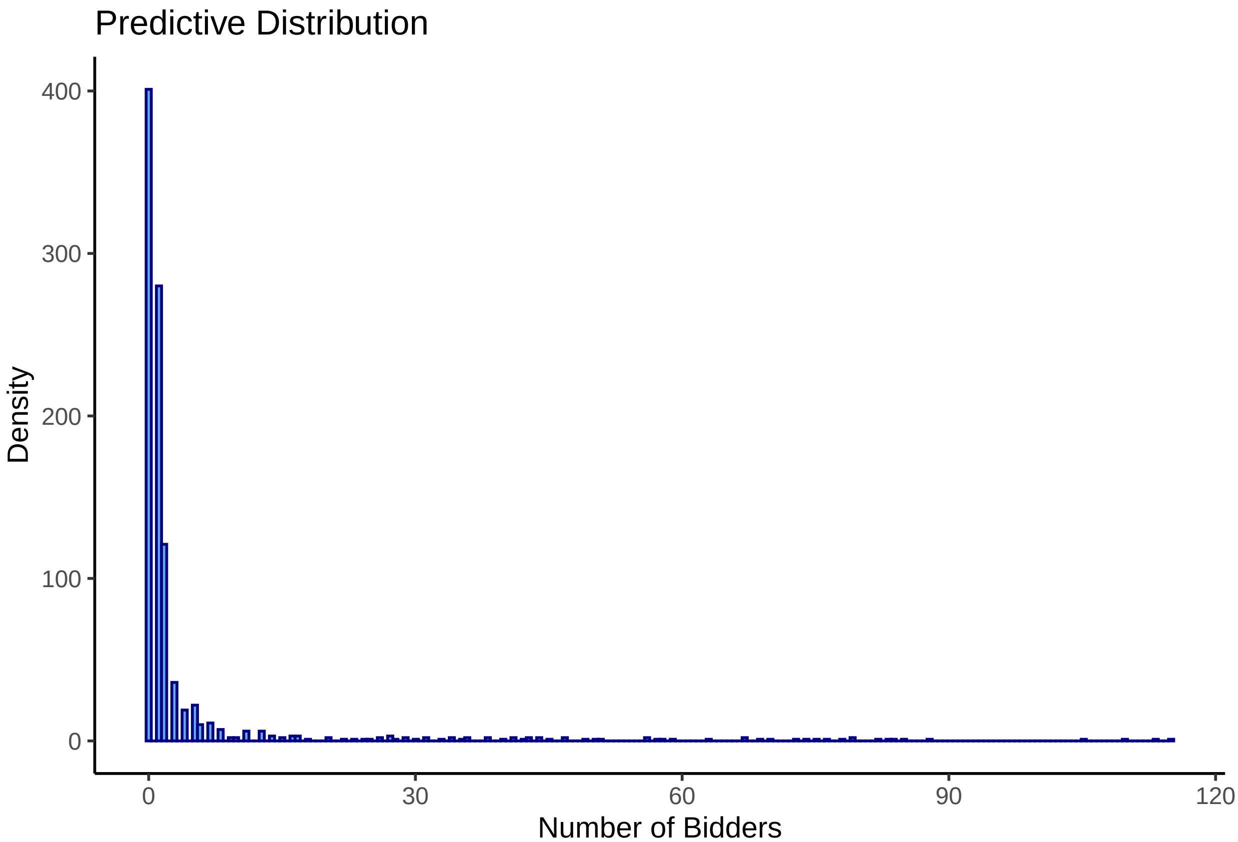
3) Compute the acceptance probability: $\alpha = min\Big(1, \frac{p(\theta_p|y)}{p(\theta^{(i-1)}|y)}$, where :

$\frac{p(\theta_p|y)}{p(\theta^{(i-1)}|y)} = exp\Big[log(p(\theta_p|y)) - log(p(\theta^{(i-1)}|y))$

4) With probability α set $\theta^{(i)} = \theta_p$ and $\theta^{(i)} = \theta^{(i-1)}$

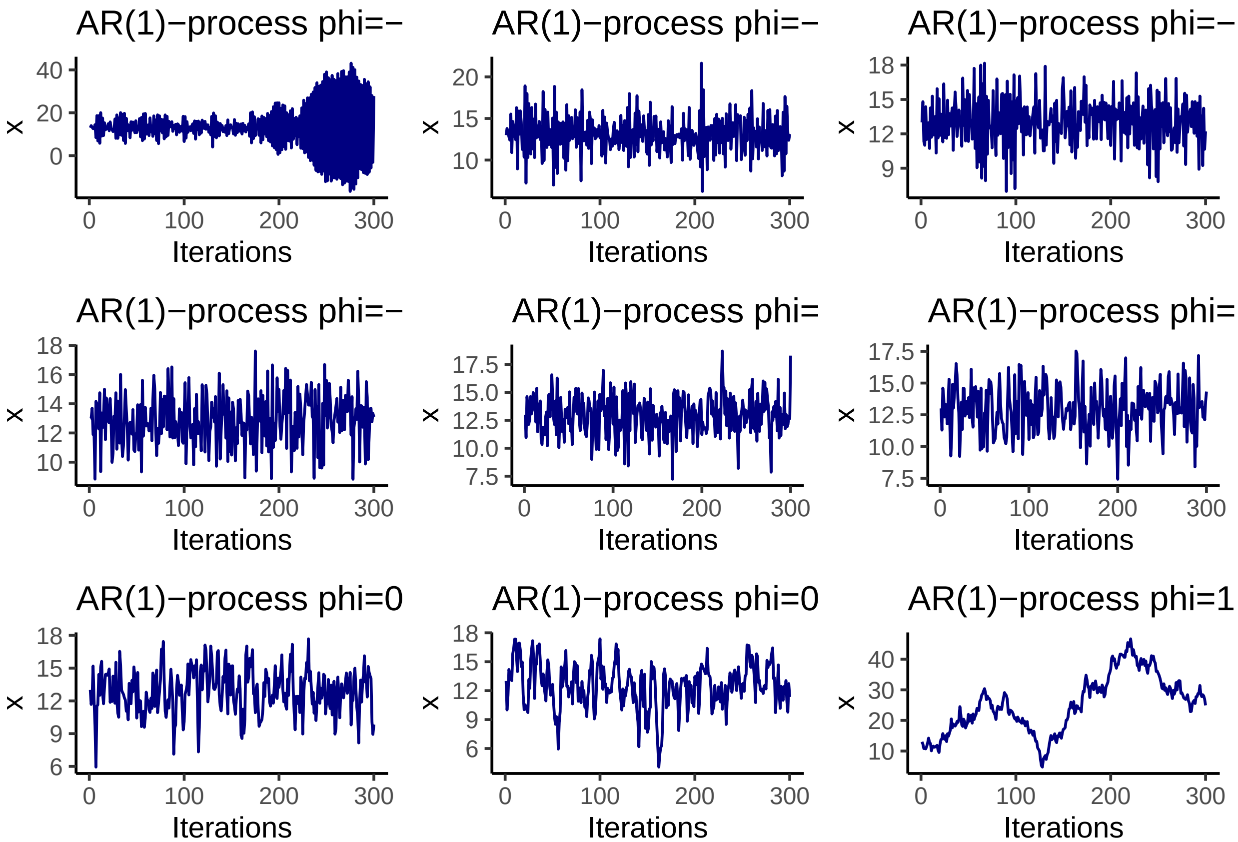

Convergence of b1
b1
1.1
1.0
0.9
0
250
500
750
1000
Iterations
Convergence of b2
b2
0.4
0.3
0.2
0.1
0.0
−0.1
0
250
500
750
1000
Iterations
Convergence of b3
b3
0.5
0.0
−0.5
0
250
500
750
1000
Iterations
Convergence of b4
b4
0.7
0.6
0.5
0.4
0.3
0
250
500
750
1000
Iterations
Convergence of b5
b5
0.6
0.4
0.2
0.0
0
250
500
750
1000
Iterations
Convergence of b6
b6
0.50
0.25
0.00
−0.25
0
250
500
750
1000
Iterations
Convergence of b7
b7
0.1
0.0
−0.1
−0.2
−0.3
0
250
500
750
1000
Iterations
Convergence of b8
b8
1.00
0.75
0.50
0.25
0.00
0
250
500
750
1000
Iterations
Convergence of b9
b9
1
0
−1
−2
0
250
500
750
1000
Iterations

Task d



Assignment 3 *Time series models in Stan*

Task a



Task b

```
## Trying to compile a simple C file

## Running /Library/Frameworks/R.framework/Resources/bin/R CMD SHLIB foo.c
## clang -mmacosx-version-min=10.13 -I"/Library/Frameworks/R.framework/Resources/include" -DNDEBUG   -I
## In file included from <built-in>:1:
## In file included from /Library/Frameworks/R.framework/Versions/4.1/Resources/library/StanHeaders/incl
## In file included from /Library/Frameworks/R.framework/Versions/4.1/Resources/library/RcppEigen/incluc
## In file included from /Library/Frameworks/R.framework/Versions/4.1/Resources/library/RcppEigen/incluc
## /Library/Frameworks/R.framework/Versions/4.1/Resources/library/RcppEigen/include/Eigen/src/Core/util/
## namespace Eigen {
## ^
## /Library/Frameworks/R.framework/Versions/4.1/Resources/library/RcppEigen/include/Eigen/src/Core/util/
## namespace Eigen {
##                ^
##                ;
## In file included from <built-in>:1:
## In file included from /Library/Frameworks/R.framework/Versions/4.1/Resources/library/StanHeaders/incl
## In file included from /Library/Frameworks/R.framework/Versions/4.1/Resources/library/RcppEigen/incluc
## /Library/Frameworks/R.framework/Versions/4.1/Resources/library/RcppEigen/include/Eigen/Core:96:10: fa
## #include <complex>
##          ^~~~~~~~~
## 3 errors generated.
## make: *** [foo.o] Error 1
##
```

```
## SAMPLING FOR MODEL '7a86c0d268cad593288653352f880fd9' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 8.4e-05 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.84 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 1: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 1: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 1: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 1: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 1: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 1: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 1: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 1: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 1: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 1: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 1: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0.935702 seconds (Warm-up)
## Chain 1:                0.127927 seconds (Sampling)
## Chain 1:                1.06363 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL '7a86c0d268cad593288653352f880fd9' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 2.6e-05 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.26 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 2: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 2: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 2: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 2: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 2: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 2: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 2: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 2: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 2: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 2: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 2: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 0.358506 seconds (Warm-up)
## Chain 2:                0.140073 seconds (Sampling)
## Chain 2:                0.498579 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL '7a86c0d268cad593288653352f880fd9' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 2.7e-05 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.27 seconds.
```

```
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 3: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 3: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 3: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 3: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 3: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 3: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 3: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 3: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 3: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 3: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 3: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 0.243799 seconds (Warm-up)
## Chain 3:                0.142307 seconds (Sampling)
## Chain 3:                0.386106 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL '7a86c0d268cad593288653352f880fd9' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 2.6e-05 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.26 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 4: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 4: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 4: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 4: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 4: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 4: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 4: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 4: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 4: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 4: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 4: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 4:
## Chain 4:  Elapsed Time: 0.40174 seconds (Warm-up)
## Chain 4:                0.149517 seconds (Sampling)
## Chain 4:                0.551257 seconds (Total)
## Chain 4:

##
## SAMPLING FOR MODEL '7a86c0d268cad593288653352f880fd9' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 4.3e-05 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.43 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
```

```
## Chain 1: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 1: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 1: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 1: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 1: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 1: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 1: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 1: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 1: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 1: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 1: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 1: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 0.213581 seconds (Warm-up)
## Chain 1:                0.147397 seconds (Sampling)
## Chain 1:                0.360978 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL '7a86c0d268cad593288653352f880fd9' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 2.4e-05 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.24 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 2: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 2: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 2: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 2: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 2: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 2: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 2: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 2: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 2: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 2: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 2: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 2:
## Chain 2:  Elapsed Time: 0.143245 seconds (Warm-up)
## Chain 2:                0.148322 seconds (Sampling)
## Chain 2:                0.291567 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL '7a86c0d268cad593288653352f880fd9' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 2.6e-05 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.26 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 3: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 3: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 3: Iteration:  600 / 2000 [ 30%]  (Warmup)
```

```
## Chain 3: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 3: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 3: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 3: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 3: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 3: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 3: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 3: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 0.232226 seconds (Warm-up)
## Chain 3:                0.129838 seconds (Sampling)
## Chain 3:                0.362064 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL '7a86c0d268cad593288653352f880fd9' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 2.6e-05 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.26 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 4: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 4: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 4: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 4: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 4: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 4: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 4: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 4: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 4: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 4: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 4: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 4:
## Chain 4:  Elapsed Time: 0.325386 seconds (Warm-up)
## Chain 4:                0.122835 seconds (Sampling)
## Chain 4:                0.448221 seconds (Total)
## Chain 4:
```

i)

Table 5: Table for phi=0.2

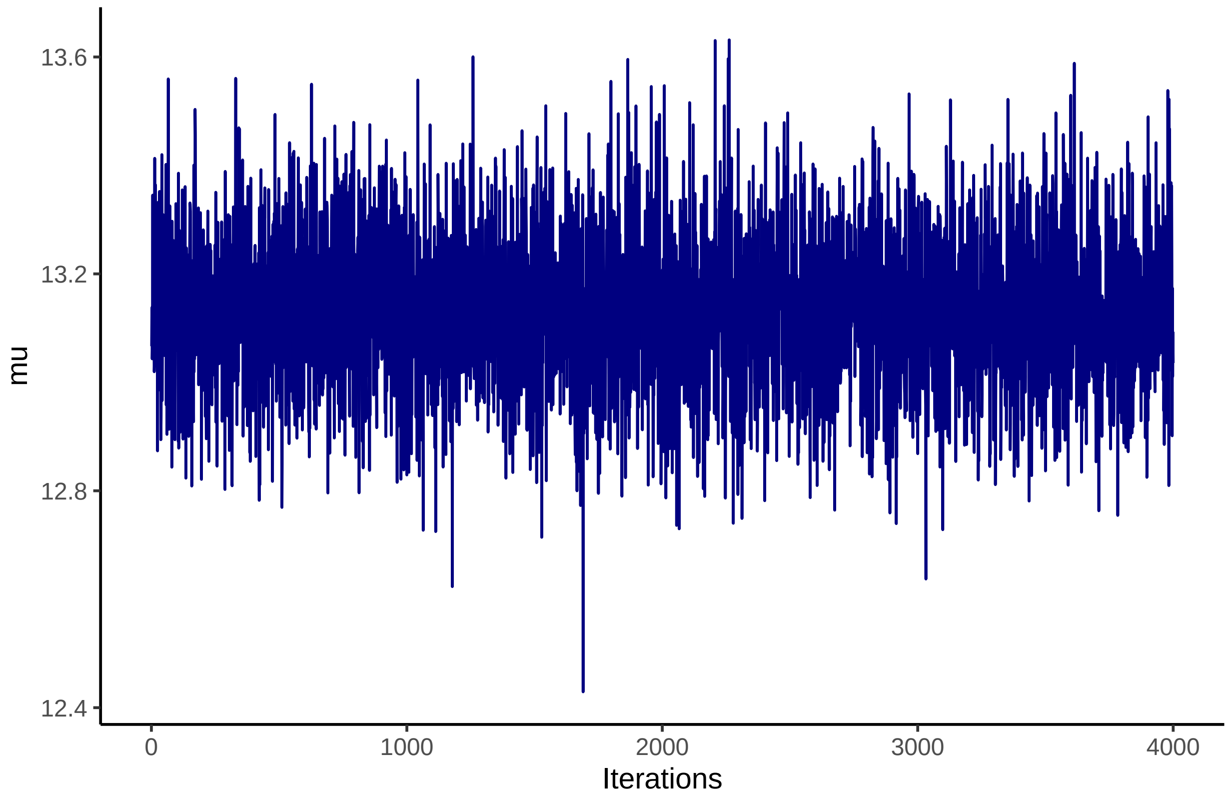
	mean	sd	2.5%	97.5%	n_eff
mu	13.1329660	0.1412896	12.8561068	13.4045697	3796.636
sigma2	2.9675904	0.2436688	2.5199519	3.4647268	3652.744
phi	0.2854719	0.0566617	0.1707683	0.3984766	3886.856

Table 6: Table for phi=0.95

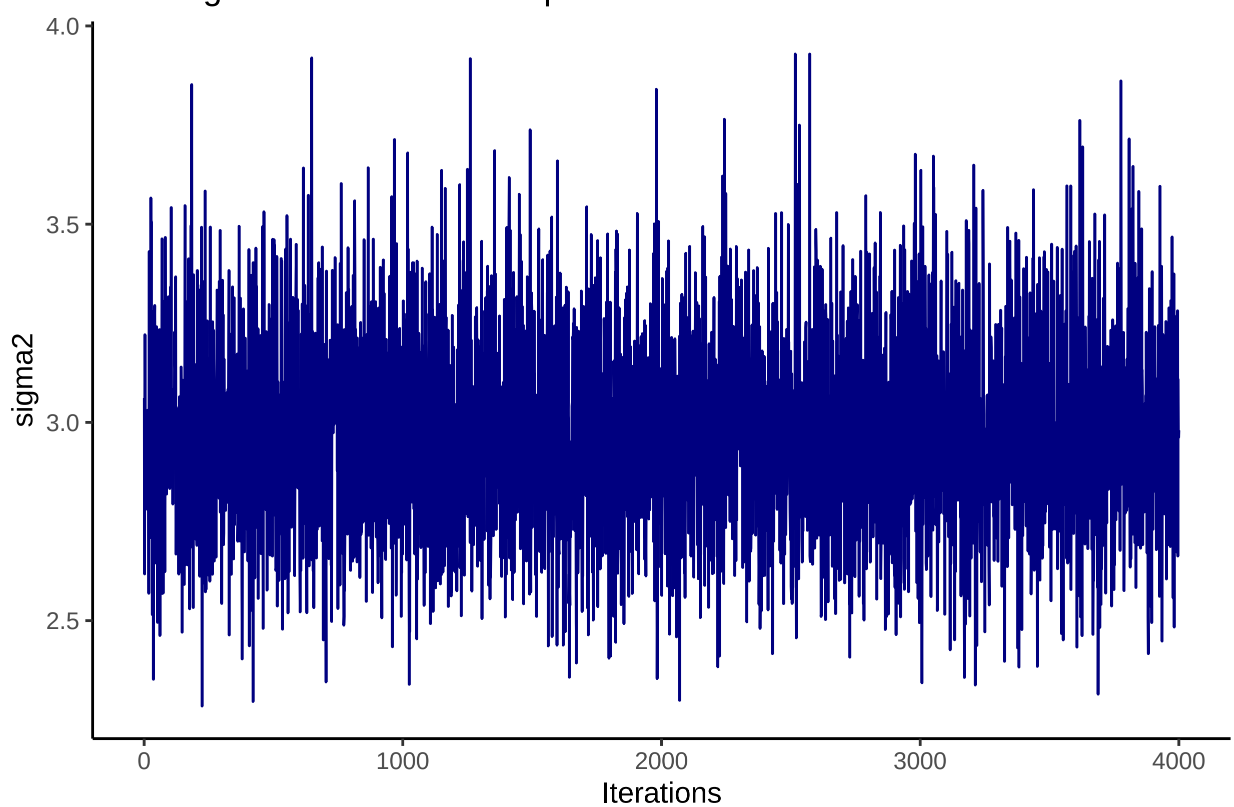
	mean	sd	2.5%	97.5%	n_eff
mu	12.9642423	0.1374885	12.6838053	13.236513	3587.043
sigma2	3.3359148	0.2713306	2.8430332	3.913049	3730.681
phi	0.2298855	0.0575086	0.1169206	0.341547	4297.623

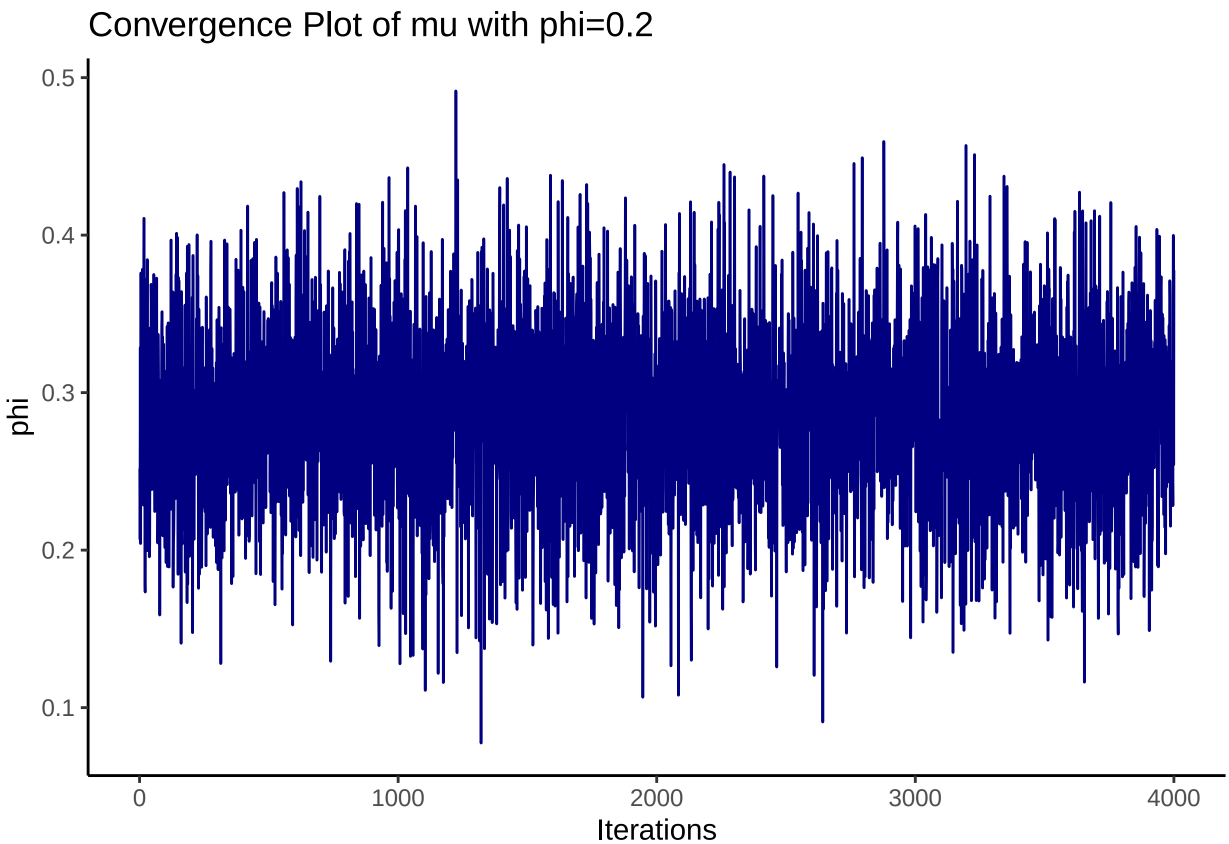
ii)

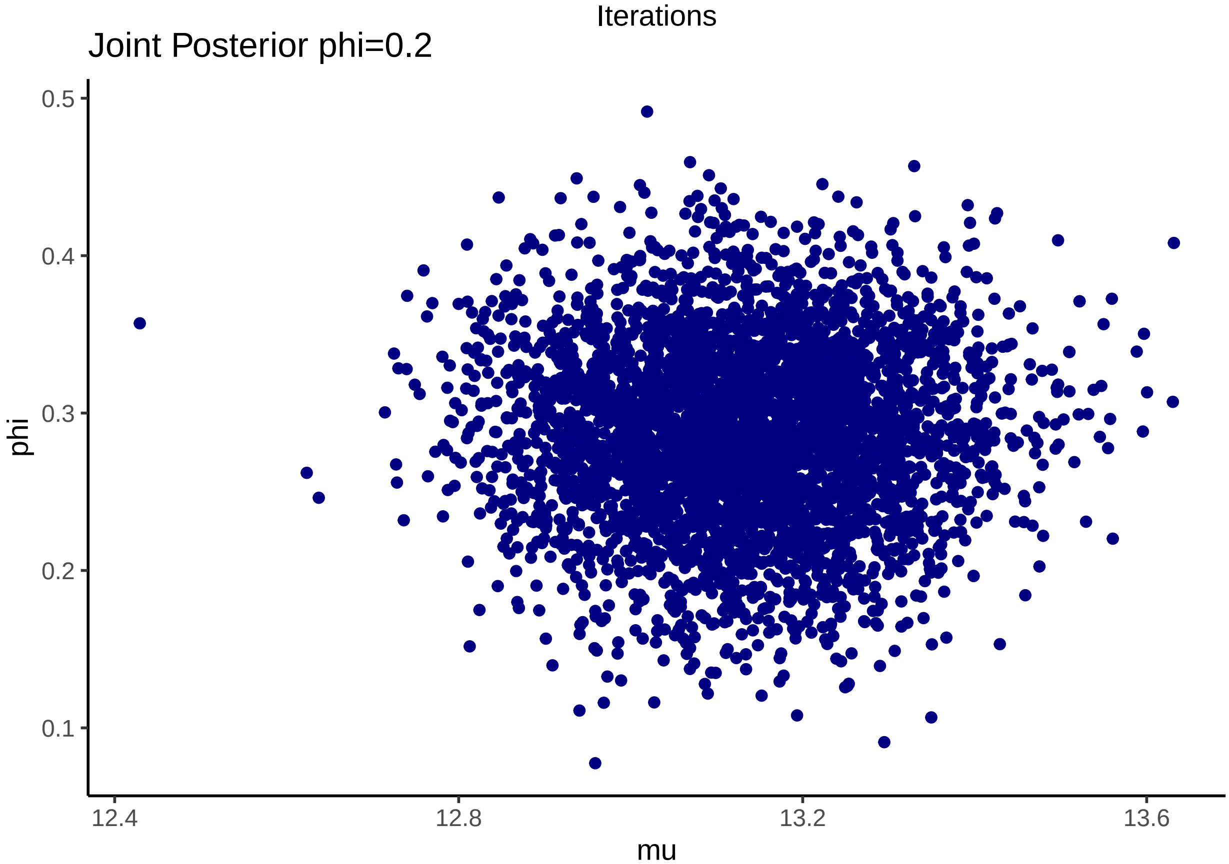
Convergence Plot of mu with phi=0.2

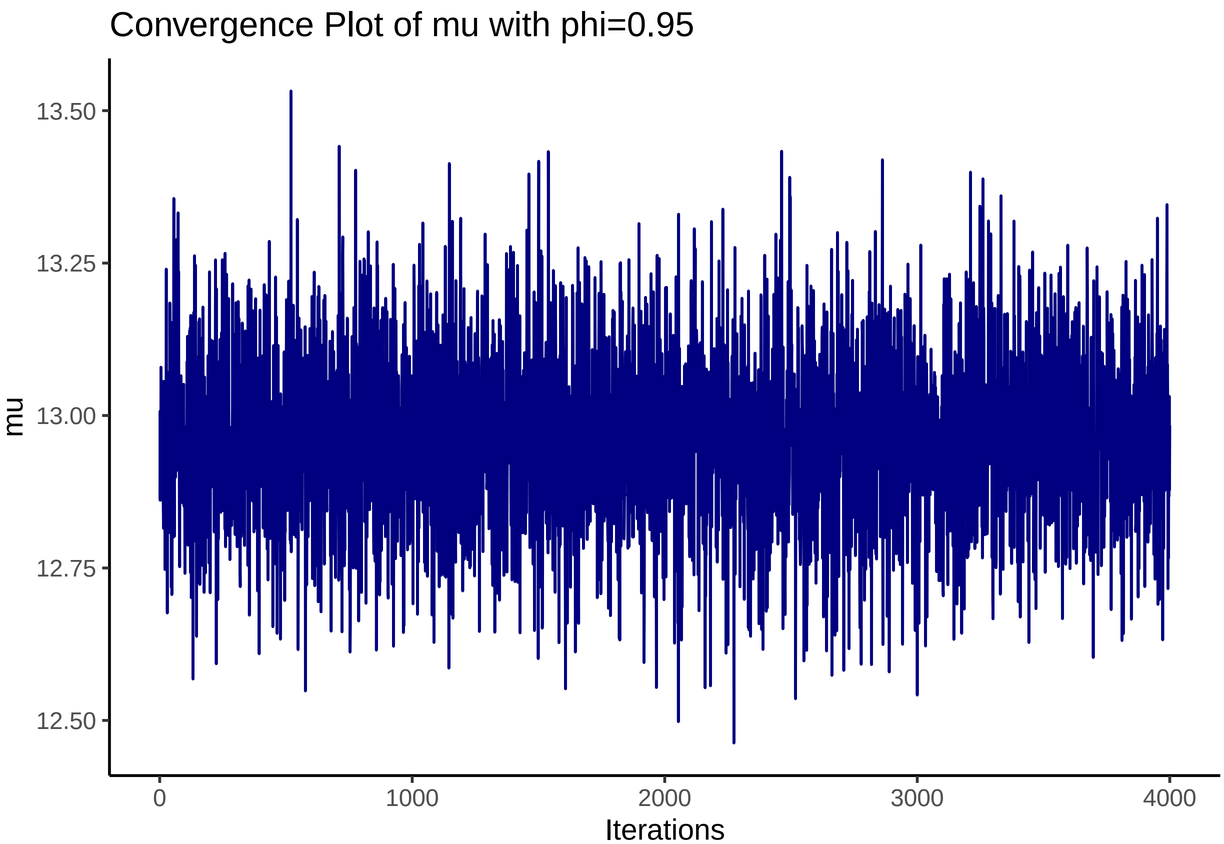


Convergence Plot of mu with phi=0.2

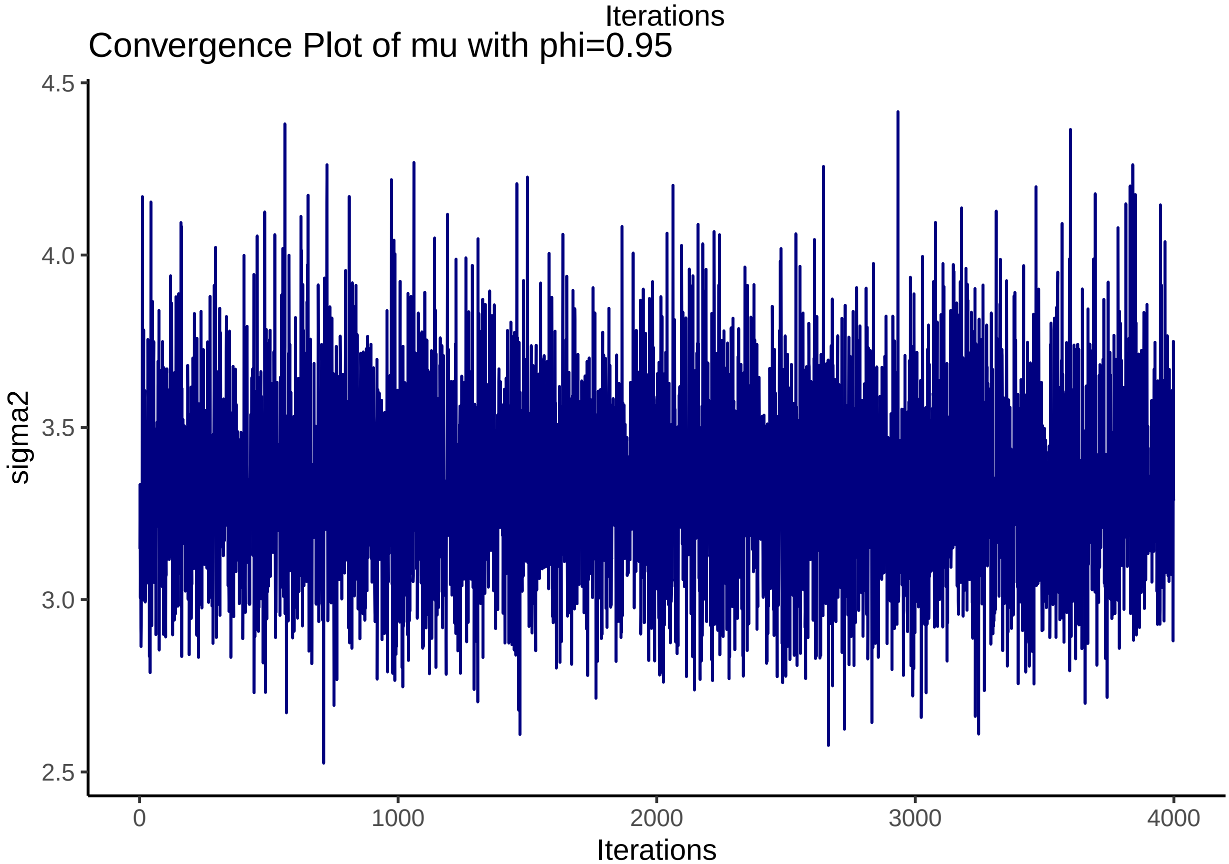



Convergence Plot of mu with phi=0.2
phi
0.5
0.4
0.3
0.2
0.1
0
1000
2000
3000
4000
Iterations

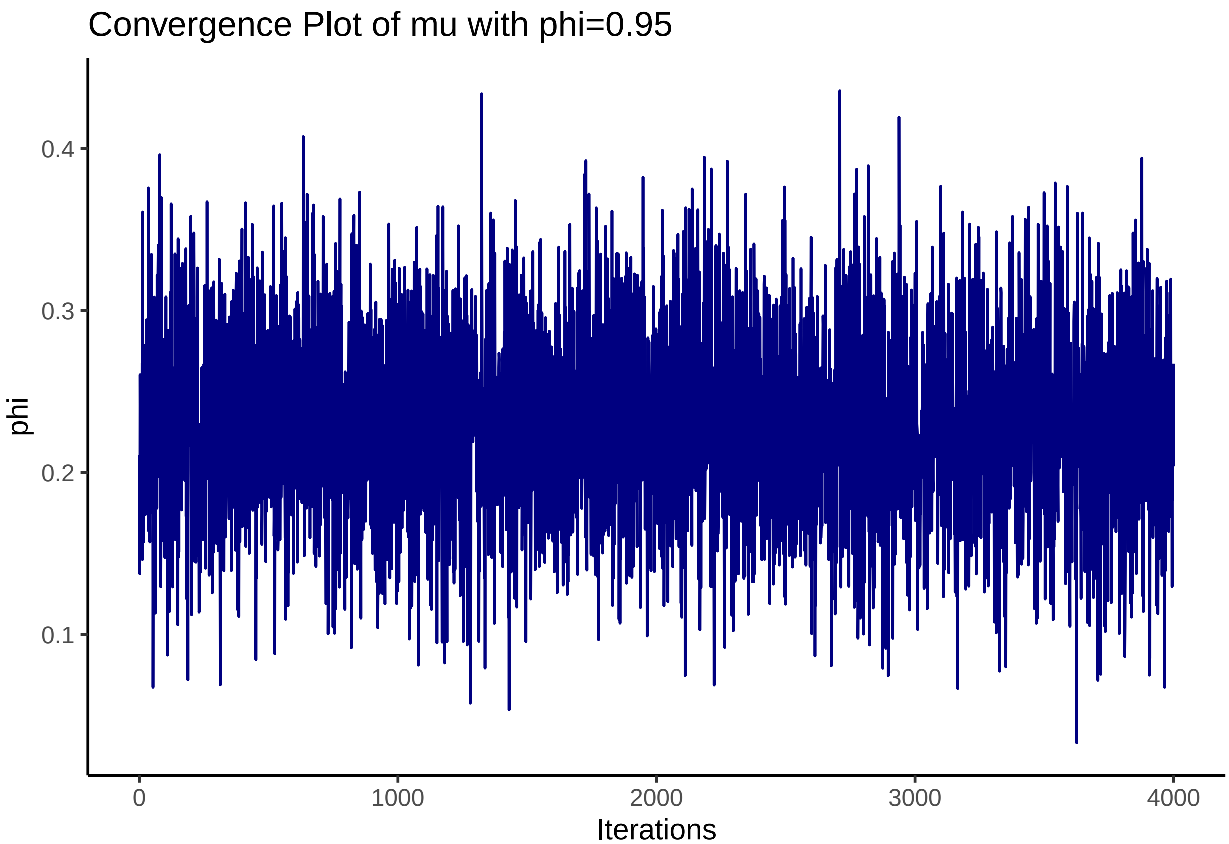

Joint Posterior phi=0.2
phi
0.5
0.4
0.3
0.2
0.1
12.4
12.8
13.2
13.6
mu

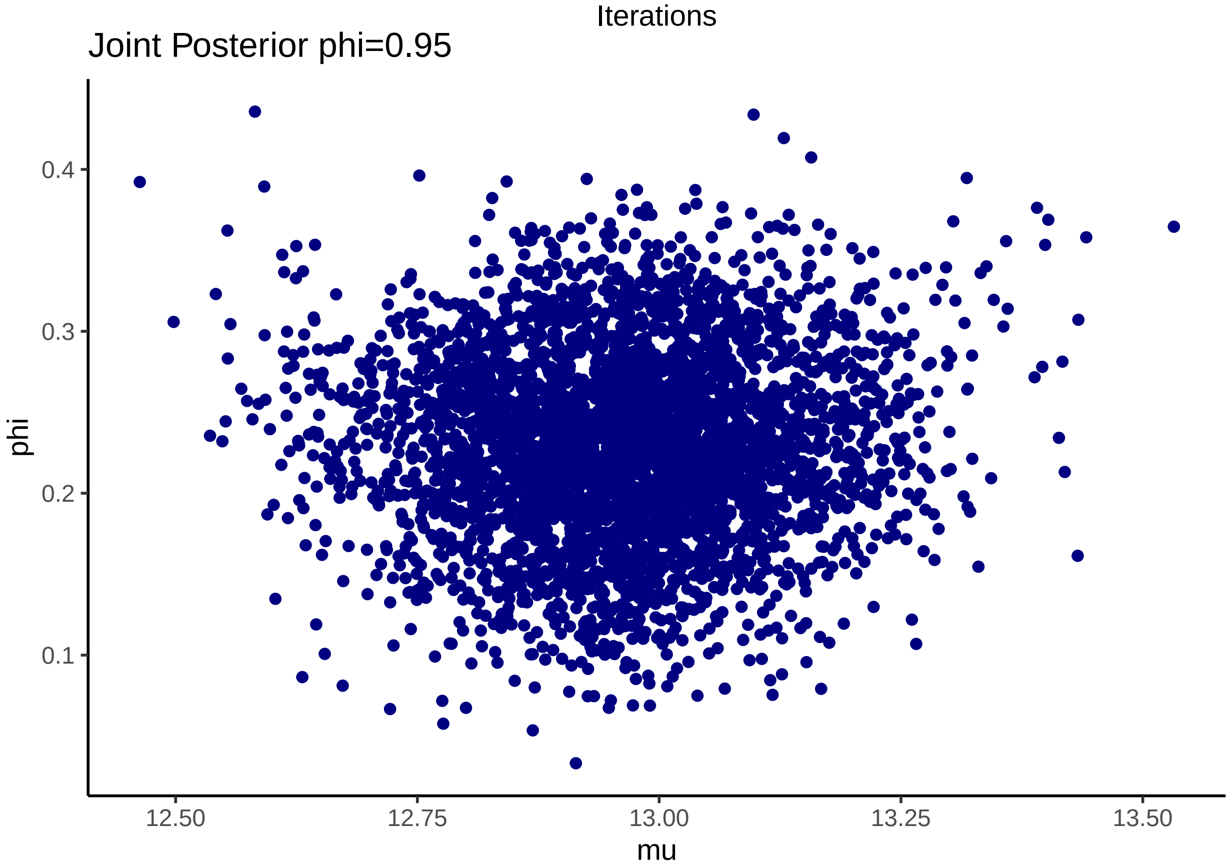


Convergence Plot of mu with phi=0.95
13.50
13.25
13.00
12.75
12.50
mu
0
1000
2000
3000
4000
Iterations



Convergence Plot of mu with phi=0.95
4.5
4.0
3.5
3.0
2.5
sigma2
0
1000
2000
3000
4000
Iterations


Convergence Plot of mu with phi=0.95
phi
0.1
0.2
0.3
0.4
0
1000
2000
3000
4000
Iterations


Joint Posterior phi=0.95
phi
0.1
0.2
0.3
0.4
12.50
12.75
13.00
13.25
13.50
mu

Appendix

```
knitr::opts_chunk$set(echo = TRUE)
knitr::opts_chunk$set(tidy.opts = list(width.cutoff = 60), tidy = TRUE)
library(ggplot2)
library(mvtnorm)
library(gridExtra)
library(rstan)
# ____________________________Assignment
# 1____________________________#

# Reading & preparing data
precipitation <- as.data.frame(readRDS("Precipitation.rds"))
colnames(precipitation) <- "records"
log_records <- log(precipitation$records)
# Task 1a
set.seed(1234567890)


n <- length(log_records)
n_0 <- 1
mu_0 <- mean(log_records)
sigma2_0 <- var(log_records)
tau2_0 <- 1

# mu_prop <- rnorm(n = 1, mean = mu_0, sqrt(tau2_0))
# sigma2_prop <- n_0*sigma2_0 / rchisq(1,n_0)

sigma2 <- sigma2_0
mu_gibbs <- c()
sigma2_gibbs <- c(sigma2)

N <- 1000
for (i in 1:N) {

    w <- (n/sigma2)/((n/sigma2) + (1/tau2_0))
    mu_n <- w * mean(log_records) + (1 - w) * mu_0
    tau2_n <- 1/(n/sigma2 + 1/tau2_0)

    mu <- rnorm(n = 1, mean = mu_n, sd = tau2_n)

    previous_sigma2 <- sigma2

    n_n <- n_0 + n

    sigma2 <- (n_n * ((n_0 * sigma2_0 + sum((log_records - mu)^2))/n_n))/rchisq(1,
        n_n)

    if (i == 1) {
        mu_gibbs <- c(mu)
        sigma2_gibbs <- c(sigma2)
    } else {
        mu_gibbs <- append(mu_gibbs, c(mu, mu))
```

```
        sigma2_gibbs <- append(sigma2_gibbs, c(previous_sigma2,
            sigma2))
    }
}

mu_rho <- acf(mu_gibbs)

IF_mu <- 1 + 2 * sum(mu_rho$acf[-1])

sigma2_rho <- acf(sigma2_gibbs)

IF_sigma2 <- 1 + 2 * sum(sigma2_rho$acf[-1])

IFs_df <- data.frame(mu = IF_mu, sigma2 = IF_sigma2)
rownames(IFs_df) <- c("Inefficiency Factors")
knitr::kable(IFs_df)

# df for plot
plot_df_traj <- data.frame(mu = mu_gibbs, sigma2 = sigma2_gibbs)

# first 200 points for plot
plot_df_traj <- plot_df_traj[1:200, ]  # test- just take the first x rows

# trajectories of the sampled Markov chains.
ggplot(plot_df_traj) + geom_path(aes(x = mu, y = sigma2), color = "navy") +
    geom_point(aes(x = mu[1], y = sigma2[1]), color = "red2",
        size = 2) + geom_point(aes(x = mu[200], y = sigma2[200]),
    color = "green2", size = 2) + ggtitle("Trajectories of the first 200 points") +
    ylab("sigma2") + xlab("mu") + theme_classic()

# Task b

set.seed(1234567890)

# predicted draws
predictions <- rnorm(nrow(precipitation), mu_gibbs, sqrt(sigma2_gibbs))

# kernel density estimation
density_actual <- density(precipitation$records)
density_pred <- density(exp(predictions))

# df for plot
plot_df_dens <- data.frame(actual_x = density_actual$x, actual_y = density_actual$y,
    pred_x = density_pred$x, pred_y = density_pred$y)

ggplot(plot_df_dens) + geom_line(aes(x = actual_x, y = actual_y),
    color = "navy") + geom_line(aes(x = pred_x, y = pred_y),
    color = "red2") + theme(legend.position = "right") + scale_color_identity(guide = "legend",
    name = "", breaks = c("navy", "red2"), labels = c("Actual Values",
        "Predicted Values")) + xlab("Value") + ylab("Density") +
    theme_classic()

# __________________Assignment 2__________________#
```

```r
# Reading data
ebay <- read.table("eBayNumberOfBidderData.dat", header = TRUE)
# Task a

# glm function, + 0 in order to do not input the covariate
# Const
model <- glm(formula = nBids ~ 0 + ., data = ebay, family = poisson)

# print the summary of the model
summary(model)
# the below code snippets from a demo of logistic
# regression in Lecture 6

# target value
y <- ebay$nBids

# prior inputs Select which covariates/features to include
X <- as.matrix(ebay[2:10])
Xnames <- colnames(X)

Npar <- dim(X)[2]

# Setting up the prior
mu <- as.matrix(rep(0, Npar))  # Prior mean vector
Sigma <- 100 * solve(t(X) %*% X)  # Prior covariance matrix

# Functions that returns the log posterior for the logistic
# and probit regression.  First input argument of this
# function must be the parameters we optimize on, i.e. the
# regression coefficients beta.

LogPostPoisson <- function(betas, y, X, mu, Sigma) {
    linPred <- X %*% betas
    # loglikelihood of Poisson regression
    logLik <- sum(linPred * y - exp(linPred))
    if (abs(logLik) == Inf)
        logLik = -20000  # Likelihood is not finite, stear the optimizer away from here!
    logPrior <- dmvnorm(betas, mu, Sigma, log = TRUE)

    return(logLik + logPrior)
}

# Select the initial values for beta
initVal <- matrix(0, Npar, 1)

# The argument control is a list of options to the
# optimizer optim, where fnscale=-1 means that we minimize
# the negative log posterior. Hence, we maximize the log
# posterior.
OptimRes <- optim(initVal, LogPostPoisson, gr = NULL, y, X, mu,
    Sigma, method = c("BFGS"), control = list(fnscale = -1),
    hessian = TRUE)
```

```r
names(OptimRes$par) <- Xnames  # Naming the coefficient by covariates
approxPostStd <- sqrt(diag(-solve(OptimRes$hessian)))  # Computing approximate standard deviations.
names(approxPostStd) <- Xnames  # Naming the coefficient by covariates
# print('The posterior mode is:') print(OptimRes$par)
# print('The approximate posterior standard deviation is:')
approxPostStd <- sqrt(diag(-solve(OptimRes$hessian)))
# print(approxPostStd)

invHessian <- data.frame(invhes = -solve(OptimRes$hessian))
colnames(invHessian) <- c("Constant", "PowerSeller", "VerifyID",
    "Sealed", "MinBlem", "MajBlem", "LargNeg", "LogBook", "MinBidShare")
knitr::kable(invHessian)

df_post_mode <- data.frame(post_mode = OptimRes$par)
colnames(df_post_mode) <- c("Value")
rownames(df_post_mode) <- c("Constant", "PowerSeller", "VerifyID",
    "Sealed", "MinBlem", "MajBlem", "LargNeg", "LogBook", "MinBidShare")
knitr::kable(df_post_mode)

df_approxPostStd <- data.frame(approxPostStd = sqrt(diag(-solve(OptimRes$hessian))))
colnames(df_approxPostStd) <- c("Value")
rownames(df_approxPostStd) <- c("Constant", "PowerSeller", "VerifyID",
    "Sealed", "MinBlem", "MajBlem", "LargNeg", "LogBook", "MinBidShare")
knitr::kable(df_approxPostStd)
set.seed(1234567890)

RWMSampler <- function(N, logPostFunc, init_beta, constant, cov_mat,
    target_val, features, mu) {

    init_sample = rmvnorm(1, init_beta, cov_mat)

    # matrix to store the betas
    betas <- matrix(nrow = N, ncol = length(init_sample))

    betas[1, ] <- init_sample

    for (i in 2:N) {
        # sample proposal
        sample_proposal <- as.vector(rmvnorm(1, betas[i - 1,
            ], constant * cov_mat))

        # LogPostPoisson <- function(betas,y,X,mu,Sigma)
        # calculating the new posterior
        post_new <- logPostFunc(sample_proposal, target_val,
            features, mu, cov_mat)

        # calculating the old posterior
        post_old <- logPostFunc(betas[i - 1, ], target_val, features,
            mu, cov_mat)

        # acceptance probability
        a <- min(1, exp(post_new - post_old))
```

```r
        u <- runif(1, 0, 1)

        if (u < a) {
            betas[i, ] <- sample_proposal
        } else {
            betas[i, ] <- betas[i - 1, ]
        }
    }
    return(betas)
}


RWMbetas <- RWMSampler(1000, LogPostPoisson, runif(9, 0, 1),
    0.35, -solve(OptimRes$hessian), ebay$nBids, as.matrix(ebay[2:10]),
    OptimRes$par)
plot_df_RWM <- data.frame(RWMbetas)
colnames(plot_df_RWM) <- c("b1", "b2", "b3", "b4", "b5", "b6",
    "b7", "b8", "b9")
plot_df_RWM$iterations <- 1:1000

p1 <- ggplot(plot_df_RWM) + geom_line(aes(x = iterations, y = b1),
    color = "navy") + ggtitle("Convergence of b1") + xlab("Iterations") +
    ylab("b1") + theme_classic()

p2 <- ggplot(plot_df_RWM) + geom_line(aes(x = iterations, y = b2),
    color = "navy") + ggtitle("Convergence of b2") + xlab("Iterations") +
    ylab("b2") + theme_classic()

p3 <- ggplot(plot_df_RWM) + geom_line(aes(x = iterations, y = b3),
    color = "navy") + ggtitle("Convergence of b3") + xlab("Iterations") +
    ylab("b3") + theme_classic()

p4 <- ggplot(plot_df_RWM) + geom_line(aes(x = iterations, y = b4),
    color = "navy") + ggtitle("Convergence of b4") + xlab("Iterations") +
    ylab("b4") + theme_classic()

p5 <- ggplot(plot_df_RWM) + geom_line(aes(x = iterations, y = b5),
    color = "navy") + ggtitle("Convergence of b5") + xlab("Iterations") +
    ylab("b5") + theme_classic()

p6 <- ggplot(plot_df_RWM) + geom_line(aes(x = iterations, y = b6),
    color = "navy") + ggtitle("Convergence of b6") + xlab("Iterations") +
    ylab("b6") + theme_classic()

p7 <- ggplot(plot_df_RWM) + geom_line(aes(x = iterations, y = b7),
    color = "navy") + ggtitle("Convergence of b7") + xlab("Iterations") +
    ylab("b7") + theme_classic()

p8 <- ggplot(plot_df_RWM) + geom_line(aes(x = iterations, y = b8),
    color = "navy") + ggtitle("Convergence of b8") + xlab("Iterations") +
    ylab("b8") + theme_classic()

p9 <- ggplot(plot_df_RWM) + geom_line(aes(x = iterations, y = b9),
```

```r
    color = "navy") + ggtitle("Convergence of b9") + xlab("Iterations") +
    ylab("b9") + theme_classic()

grid.arrange(p1, p2, p3, p4, p5, p6, p7, p8, p9, ncol = 3)
set.seed(1234567890)

auction <- c(1, 1, 0, 1, 0, 1, 0, 1.2, 0.8)

nbidders = c()
for (i in 1:nrow(RWMbetas)) {
    nbidders[i] <- rpois(1, exp(RWMbetas[i, ] %*% auction))
}

nbidders <- data.frame(nbidders)

prob <- sum(nbidders$nbidders == 0)/(nrow(nbidders))

ggplot(nbidders, aes(x = nbidders)) + geom_histogram(bins = 200,
    color = "navy", fill = "steelblue2") + ggtitle("Predictive Distribution") +
    xlab("Number of Bidders") + ylab("Density") + theme_classic()
# __________________Assignment 3__________________#

# Task 3a

set.seed(12345)

# AR(1)-process
ar_process <- function(t, mu, phi, sigma2) {
    x <- c()
    x[1] <- mu
    for (i in 2:t) {
        x[i] <- mu + phi * (x[i - 1] - mu) + rnorm(1, 0, sqrt(sigma2))
    }
    return(x)
}

# given parameters
mu <- 13
sigma2 <- 3
t <- 300
phi <- seq(-1, 1, 0.25)

res <- as.data.frame(sapply(phi, function(phi) ar_process(t,
    mu, phi, sigma2)))

res$iterations <- 1:t

# Time series plots of different phis

p1 <- ggplot(res) + geom_line(aes(x = iterations, y = V1), color = "navy") +
    ggtitle("AR(1)-process phi=-1") + xlab("Iterations") + ylab("x") +
    theme_classic()
```

```
p2 <- ggplot(res) + geom_line(aes(x = iterations, y = V2), color = "navy") +
    ggtitle("AR(1)-process phi=-0.75") + xlab("Iterations") +
    ylab("x") + theme_classic()

p3 <- ggplot(res) + geom_line(aes(x = iterations, y = V3), color = "navy") +
    ggtitle("AR(1)-process phi=-0.5") + xlab("Iterations") +
    ylab("x") + theme_classic()

p4 <- ggplot(res) + geom_line(aes(x = iterations, y = V4), color = "navy") +
    ggtitle("AR(1)-process phi=-0.25") + xlab("Iterations") +
    ylab("x") + theme_classic()

p5 <- ggplot(res) + geom_line(aes(x = iterations, y = V5), color = "navy") +
    ggtitle("AR(1)-process phi=0") + xlab("Iterations") + ylab("x") +
    theme_classic()

p6 <- ggplot(res) + geom_line(aes(x = iterations, y = V6), color = "navy") +
    ggtitle("AR(1)-process phi=0.25") + xlab("Iterations") +
    ylab("x") + theme_classic()

p7 <- ggplot(res) + geom_line(aes(x = iterations, y = V7), color = "navy") +
    ggtitle("AR(1)-process phi=0.5") + xlab("Iterations") + ylab("x") +
    theme_classic()

p8 <- ggplot(res) + geom_line(aes(x = iterations, y = V8), color = "navy") +
    ggtitle("AR(1)-process phi=0.75") + xlab("Iterations") +
    ylab("x") + theme_classic()

p9 <- ggplot(res) + geom_line(aes(x = iterations, y = V9), color = "navy") +
    ggtitle("AR(1)-process phi=1") + xlab("Iterations") + ylab("x") +
    theme_classic()

grid.arrange(p1, p2, p3, p4, p5, p6, p7, p8, p9, ncol = 3)

# Task 3b

set.seed(1234567890)

# stan_model
StanModel = "
data {
  int<lower=0> N;
  vector[N] x;
}
parameters {
  real mu;
  real<lower=0> sigma2;
  real phi;
}
model {
  mu ~ normal(0,100); // Normal with mean 0, st.dev. 100
  sigma2 ~ scaled_inv_chi_square(1,2); // Scaled-inv-chi2 with nu 1,sigma 2
  for(i in 2:N){
```

```
    x[i] ~ normal( mu + phi*(x[i-1]-mu), sqrt(sigma2));
  }
}"

# given parameters
phi1 <- 0.2
phi2 <- 0.95

# ar with given parameters
res1 <- ar_process(t, mu, phi1, sigma2)
res2 <- ar_process(t, mu, phi1, sigma2)

data1 <- list(N = t, x = res1)
data2 <- list(N = t, x = res2)

warmup <- 1000
niter <- 2000


fit1 <- stan(model_code = StanModel, data = data1, warmup = warmup,
    iter = niter, chains = 4)
fit2 <- stan(model_code = StanModel, data = data2, warmup = warmup,
    iter = niter, chains = 4)

fit1_summary <- summary(fit1)
phi1_stats <- fit1_summary$summary[1:3, c(1, 3, 4, 8, 9)]

knitr::kable(phi1_stats, caption = "Table for phi=0.2")

fit2_summary <- summary(fit2)
phi2_stats <- fit2_summary$summary[1:3, c(1, 3, 4, 8, 9)]

knitr::kable(phi2_stats, caption = "Table for phi=0.95")

postDraws1 <- data.frame(extract(fit1))
postDraws1$iterations <- 1:4000

ggplot(postDraws1) + geom_line(aes(x = iterations, y = mu), color = "navy") +
    ggtitle("Convergence Plot of mu with phi=0.2") + xlab("Iterations") +
    ylab("mu") + theme_classic()

ggplot(postDraws1) + geom_line(aes(x = iterations, y = sigma2),
    color = "navy") + ggtitle("Convergence Plot of mu with phi=0.2") +
    xlab("Iterations") + ylab("sigma2") + theme_classic()

ggplot(postDraws1) + geom_line(aes(x = iterations, y = phi),
    color = "navy") + ggtitle("Convergence Plot of mu with phi=0.2") +
    xlab("Iterations") + ylab("phi") + theme_classic()

ggplot(postDraws1) + geom_point(aes(x = mu, y = phi), color = "navy") +
    ggtitle("Joint Posterior phi=0.2") + theme_classic()
postDraws2 <- data.frame(extract(fit2))
postDraws2$iterations <- 1:4000
```

```r
ggplot(postDraws2) + geom_line(aes(x = iterations, y = mu), color = "navy") +
    ggtitle("Convergence Plot of mu with phi=0.95") + xlab("Iterations") +
    ylab("mu") + theme_classic()

ggplot(postDraws2) + geom_line(aes(x = iterations, y = sigma2),
    color = "navy") + ggtitle("Convergence Plot of mu with phi=0.95") +
    xlab("Iterations") + ylab("sigma2") + theme_classic()

ggplot(postDraws2) + geom_line(aes(x = iterations, y = phi),
    color = "navy") + ggtitle("Convergence Plot of mu with phi=0.95") +
    xlab("Iterations") + ylab("phi") + theme_classic()

ggplot(postDraws2) + geom_point(aes(x = mu, y = phi), color = "navy") +
    ggtitle("Joint Posterior phi=0.95") + theme_classic()
```