August-2021-Exam

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Exercise 1

Task a

Hand written solution.

Task b

```r
set.seed(12345)

y <- c(2.32,1.82,2.4,2.08,2.13)
n <- 5

N <- 10000

#generate thetas from the posterior distribution
theta <- rgamma(N, shape = 2*n + 1, rate = 0.5 + sum(y))

#generate predictive values form the given gamma density
y_tilde <- rgamma(N, shape = 2, rate = theta)

#calculate the density
dens_y_tilde <- density(y_tilde)

df_plot1 <- data.frame("x" = dens_y_tilde$x, "y" = dens_y_tilde$y)

ggplot(df_plot1) +
  geom_line(aes(x=x,y=y), color = "navy") +
  ggtitle("Predictive Distribution") +
  xlab("Predictive Values") +
  ylab("Density") +
  theme_classic()
```
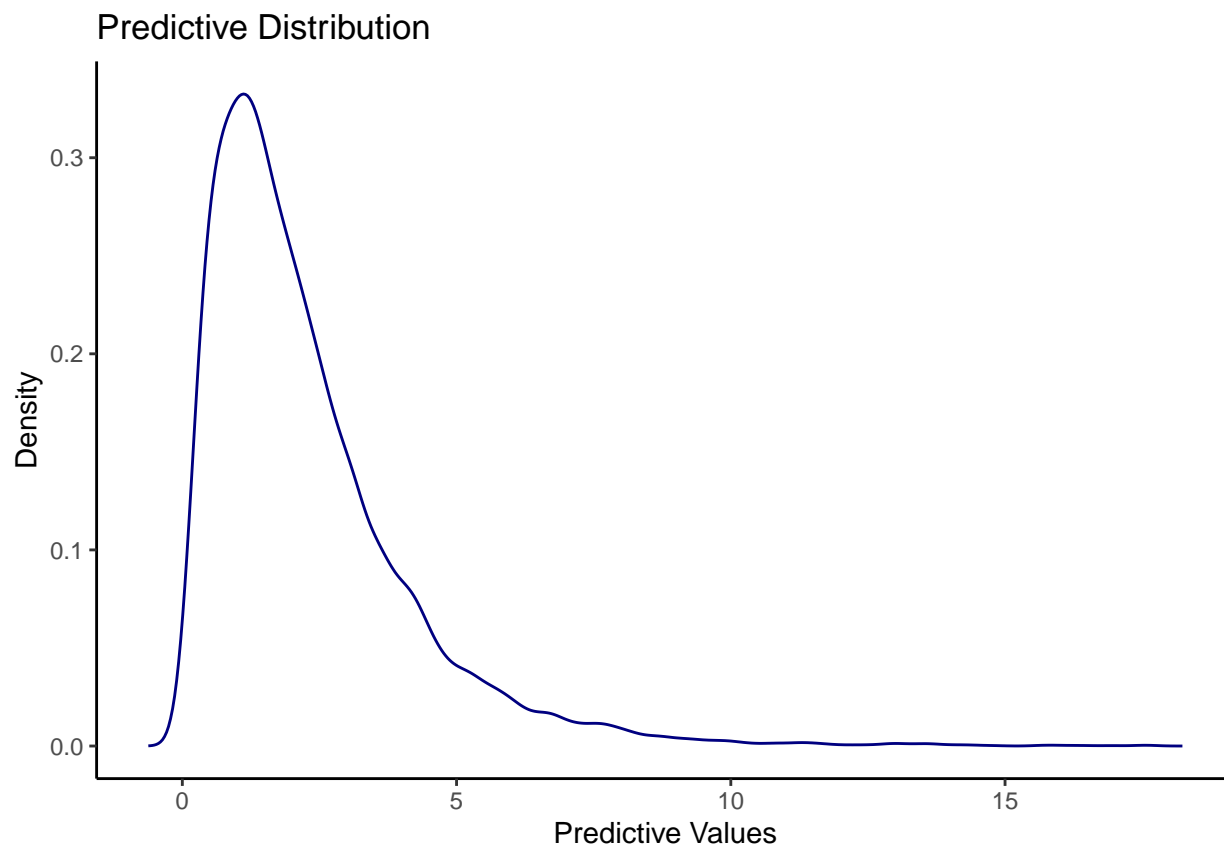
Predictive Distribution


```r
prob <- mean(y_tilde < 1.9)
```

The posterior predictive probability is roughly 0.53.

```r
set.seed(12345)

weeks <- 30
weights <- matrix(NA, nrow = N, ncol = weeks)

for (i in 1:N){
  #generate thetas from the posterior distribution
  theta <- rgamma(weeks, shape = 2*n + 1, rate = 0.5 + sum(y))

  #generate predictive values form the given gamma density
  weights[i,]<- t(rgamma(weeks, shape = 2, rate = theta))
}

overweight_weeks <- mean(rowSums(weights > 2.4))
```

The expected number of weeks out of the future 30 weeks in which the maximal weight will exceed 2.4 thousands of kilos about 10.5.

```r
#given loss function
loss_function <- function(a, weights){
  res <- a + mean(rowSums(weights > 0.9*log(a)))
  return(res)
}
```

```r
a <- runif(1000,1,10)
loss_results <- rep(NA,length(a),1)
count <- 0

for (i in a){
  count <- count + 1
  loss_results[count] = loss_function(i, weights)
}

optimal_a <- a[which.min(loss_results)]
```

The optimal build cost (a) is approximately 7.3 thousands of kilos.