BayesExam

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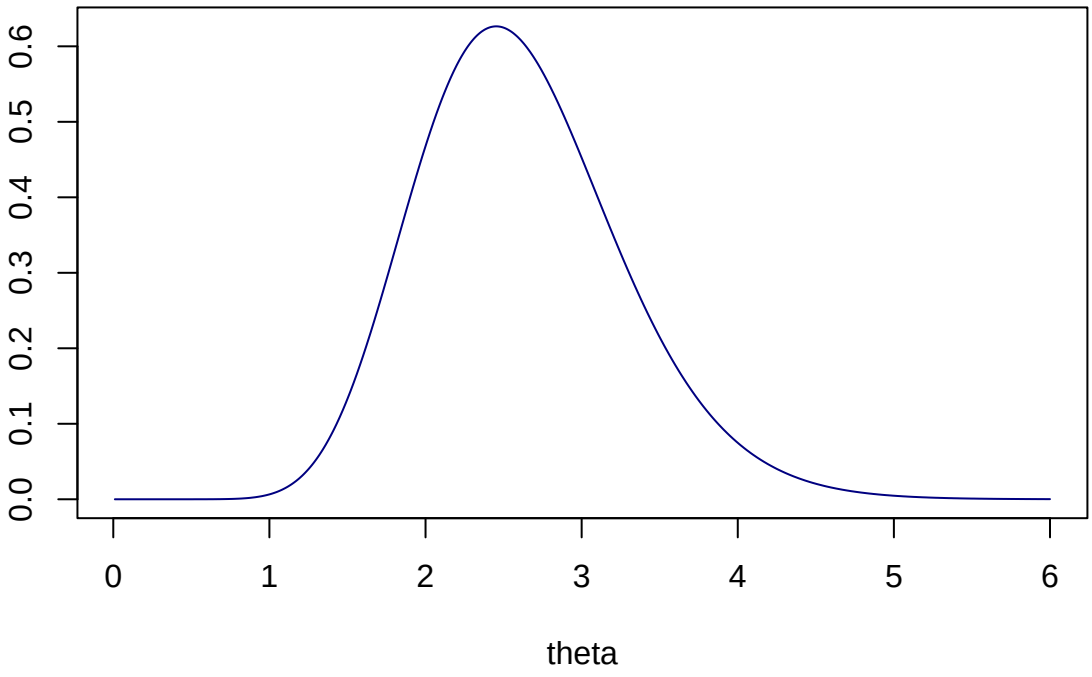
Problem 1

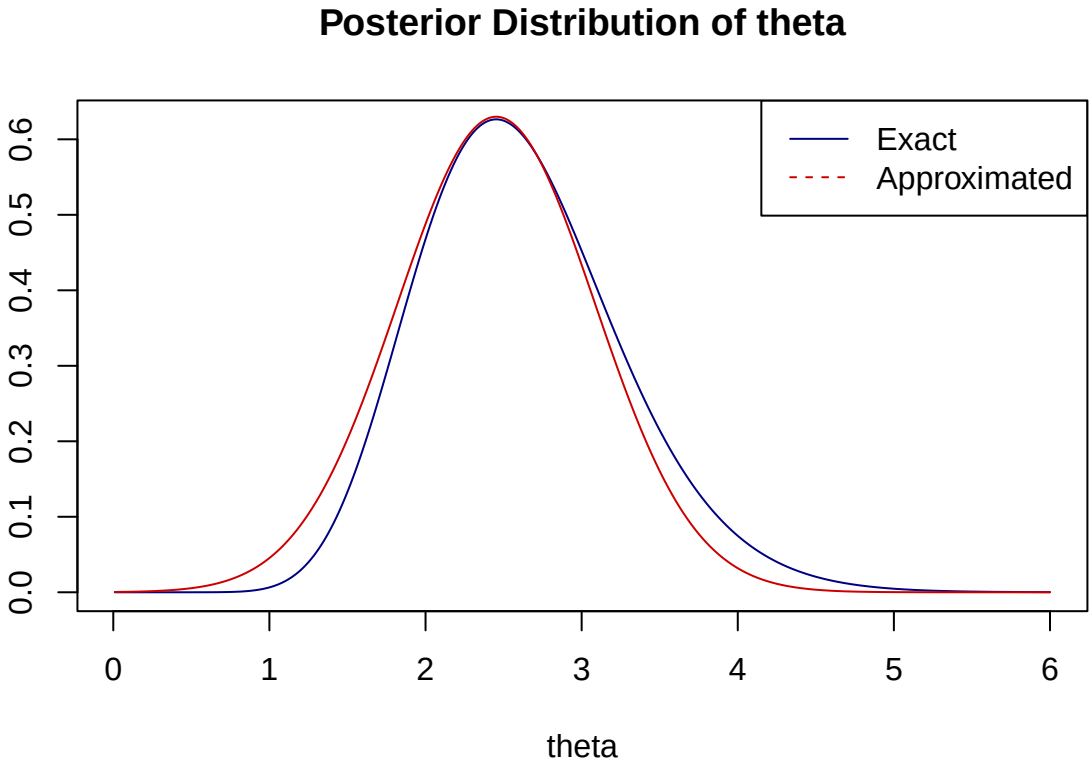
Task d

```
LogPost <- function(theta,n,sumx){
  res <- (3*n+3)*log(theta) - theta*(2+sumx)
  return(res)
}

thetaGrid <- seq(0.01,6,0.01)
n <- 4
vals <- c(0.7, 1.1, 0.9, 1.5)
sumx <- sum(1/vals)

LogPost_propto <- exp(LogPost(thetaGrid,n,sumx))
LogPost_dens <- LogPost_propto/(0.01*sum(LogPost_propto))

plot(thetaGrid,LogPost_dens, type = "l", col = "navy",
     main = "Posterior Distribution of theta",
     xlab = "theta", ylab = "")
```



Task e

```
OptimRes <- optim(2, LogPost, gr = NULL, n, sumx,
                  method = c("L-BFGS-B"), lower = 0.1,
                  control = list(fnscale = -1), hessian = TRUE)

approx <- dnorm(thetaGrid, mean = OptimRes$par, sd = sqrt(diag(-solve(OptimRes$hessian))))

plot(thetaGrid,LogPost_dens, type = "l", col = "navy",
     main = "Posterior Distribution of theta",
     xlab = "theta", ylab = "")
lines(thetaGrid,approx, type = "l", col = "red3")
legend("topright", legend = c("Exact", "Approximated"),
       col = c("navy", "red3"), lty = 1:2)
```



The posterior approximation is not that accurate, the exact posterior is skewed to the right.

Problem 2

```
source("ExamData.R")
```

Task a

```
set.seed(12345)

mu_0 <- as.vector(rep(0,3))
Sigma2_0 <- 100*diag(3)
nIter <- 20000

PostDraws <- BayesLogitReg(y, X, mu_0, Sigma2_0, nIter)
```

```
Betas <- PostDraws$betaSample
```

```
intervalB1 <- quantile(Betas[,2], probs = c(0.025,0.975))

intervalB1 <- data.frame(lower_bound = intervalB1[1], upper_bound = intervalB1[2])
colnames(intervalB1) <- c("Lower Bound", "Upper Bound")
rownames(intervalB1) <- c("95% Equal Tail Credible Interval")
knitr::kable(intervalB1)
```

	Lower Bound	Upper Bound
95% Equal Tail Credible Interval	0.0146446	0.1813111

The 95% posterior probability that β_1 is on this interval.

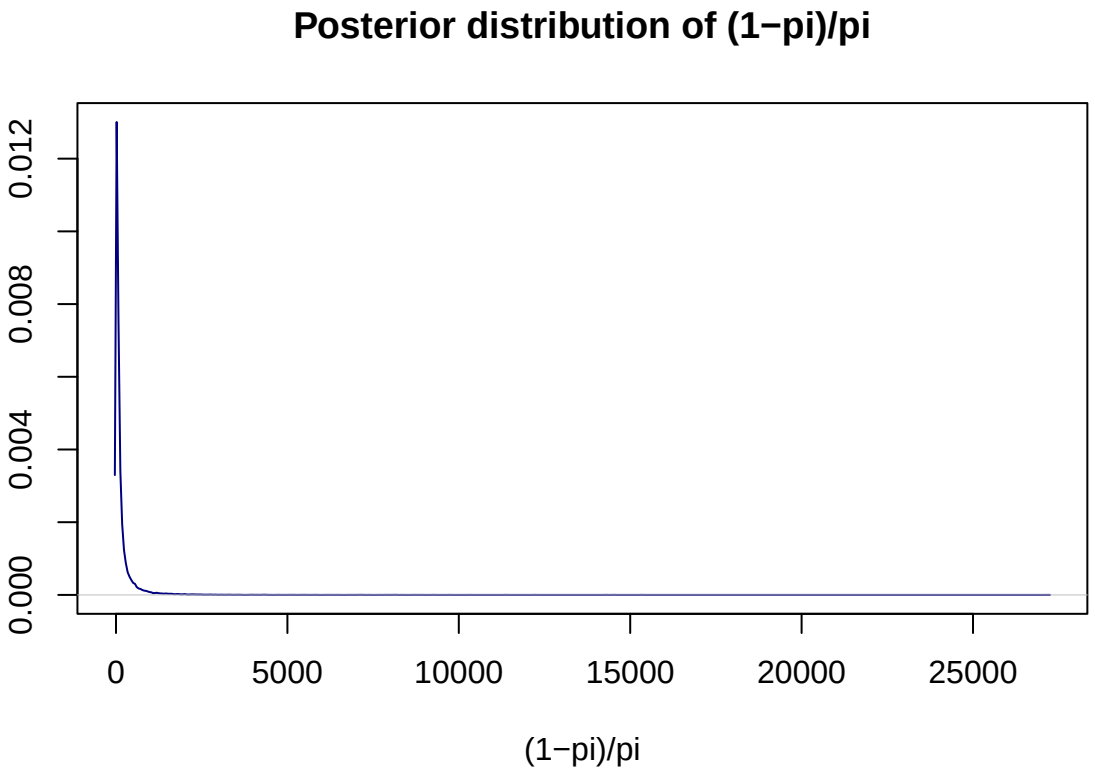
Task b

```
prob2b <- mean(Betas[,2]>0 & Betas[,3]>0)
```

The joint posterior probability that both $\beta_1 > 0$ and $\beta_2 > 0$ is approximately 0.92.

Task c

```
numerator <- exp(Betas[,1]+Betas[,2]*5+Betas[,3])
pi <- numerator/(1+numerator)

ratio <- (1-pi)/pi

plot(density(ratio), type = "l", col = "navy",
     main = "Posterior distribution of (1-pi)/pi",
     xlab = "(1-pi)/pi", ylab = "")
```

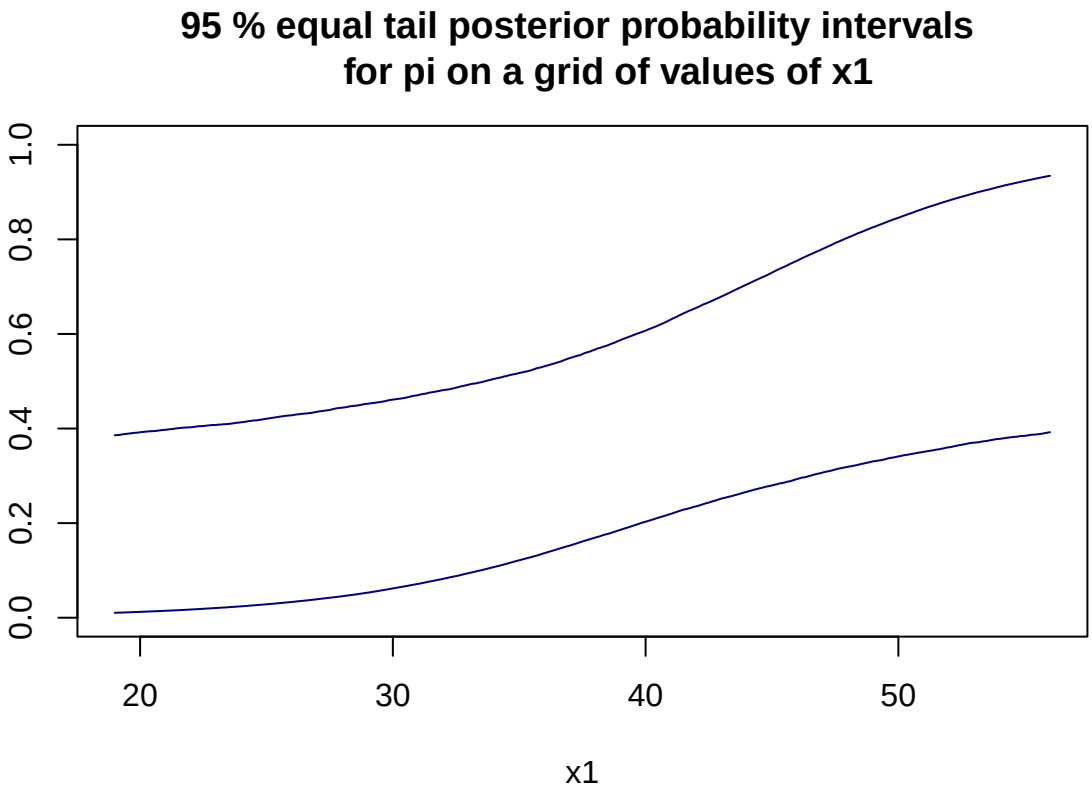


The ratio represents the odds that a bridge does not need repair within the next five years. The plot seems reasonable, because it is a 5 year old and it should not need repair. The reliability of results for such bridges should be question because it is a very new bridge.

Task d

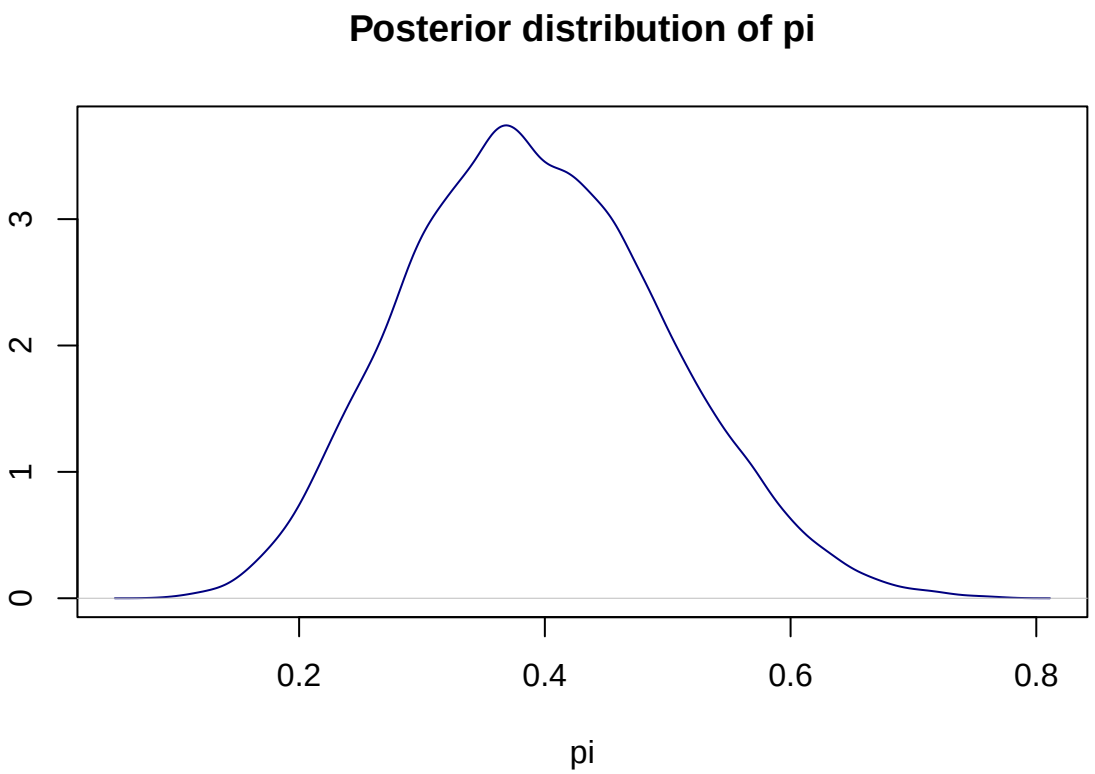
```
x1Grid <- seq(min(X[,2]), max(X[,2]), 0.1)
intervals <- matrix(0,length( x1Grid), 2)

for (i in 1:length(x1Grid)){
```

```
  numerator <- exp(Betas[,1]+Betas[,2]*x1Grid[i]+Betas[,3])
  pi <- numerator/(1+numerator)
  intervals[i,] <- quantile(pi, probs = c(0.025,0.975))
}

plot(x1Grid, intervals[,1], type = "l", col = "navy",
     main = "95 % equal tail posterior probability intervals
     for pi on a grid of values of x1",
     xlab = "x1", ylab = "", ylim = c(0,1))
lines(x1Grid, intervals[,2], type = "l", col = "navy")
```



Task e

```
numerator <- exp(Betas[,1]+Betas[,2]*40+Betas[,3])
pi <- numerator/(1+numerator)

plot(density(pi), col = "navy",
     main = "Posterior distribution of pi",
     xlab = "pi", ylab = "")
```




```
prob2e <- mean(pi>0.5)
```

The posterior probability that $p_i > 0.5$ is approximately 0.16

Problem 3

Task c

```
buy <- 80*(19/30) - 20
nobuy <- 420*(19/30) - 240
```

The bank should buy the option.