August-2021-Exam

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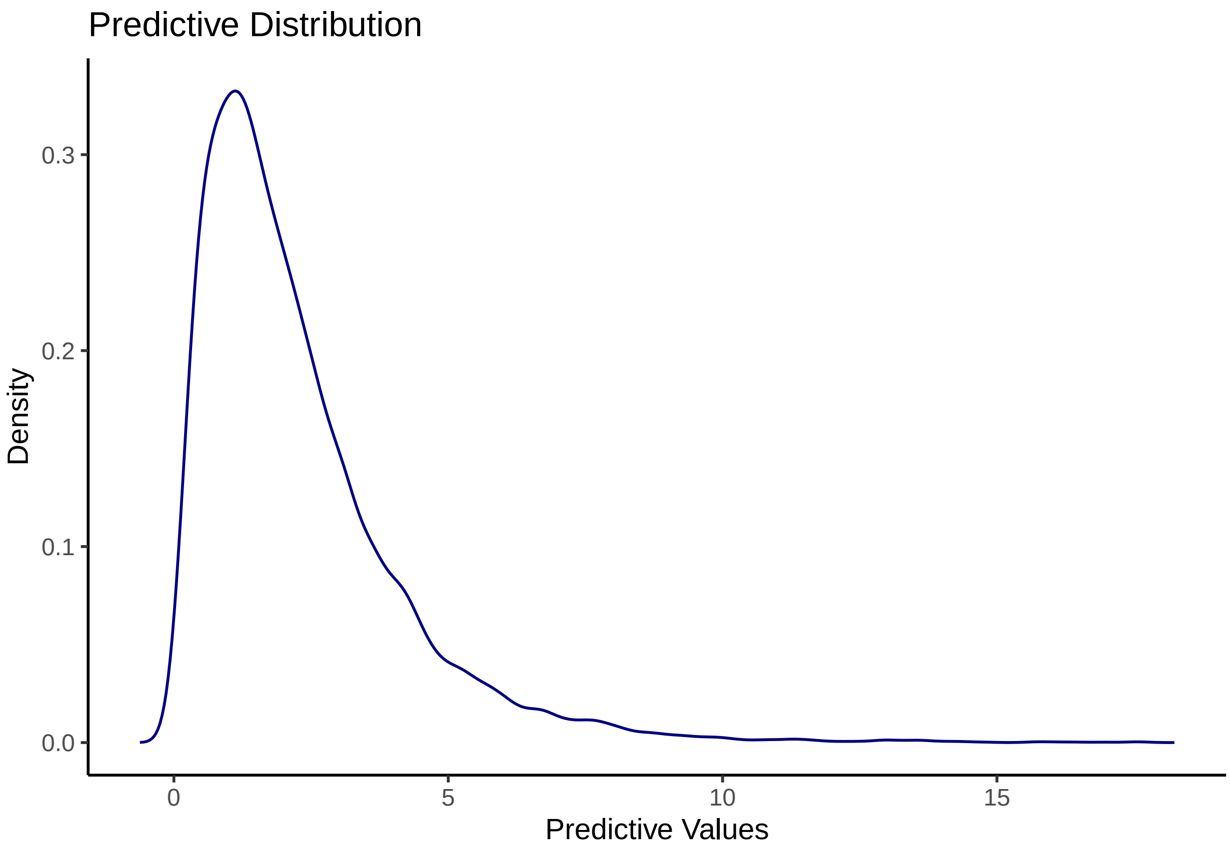
Exercise 1

Task a

Hand written solution.

Task b

```
set.seed(12345)

y <- c(2.32,1.82,2.4,2.08,2.13)
n <- 5

N <- 10000

#generate thetas from the posterior distribution
theta <- rgamma(N, shape = 2*n + 1, rate = 0.5 + sum(y))

#generate predictive values form the given gamma density
y_tilde <- rgamma(N, shape = 2, rate = theta)

#calculate the density
dens_y_tilde <- density(y_tilde)

df_plot1 <- data.frame("x" = dens_y_tilde$x, "y" = dens_y_tilde$y)

ggplot(df_plot1) +
  geom_line(aes(x=x,y=y), color = "navy") +
  ggtitle("Predictive Distribution") +
  xlab("Predictive Values") +
  ylab("Density") +
  theme_classic()
```




```
prob <- mean(y_tilde < 1.9)
```

The posterior predictive probability is roughly 0.53.

```
set.seed(12345)

weeks <- 30
weights <- matrix(NA, nrow = N, ncol = weeks)

for (i in 1:N){
  #generate thetas from the posterior distribution
  theta <- rgamma(weeks, shape = 2*n + 1, rate = 0.5 + sum(y))

  #generate predictive values form the given gamma density
  weights[i,]<- t(rgamma(weeks, shape = 2, rate = theta))
}

overweight_weeks <- mean(rowSums(weights > 2.4))
```

The expected number of weeks out of the future 30 weeks in which the maximal weight will exceed 2.4 thousands of kilos about 10.5.

```
#given loss function
loss_function <- function(a, weights){
  res <- a + mean(rowSums(weights > 0.9*log(a)))
  return(res)
}
```

```
a <- runif(1000,1,10)
loss_results <- rep(NA,length(a),1)
count <- 0

for (i in a){
  count <- count + 1
  loss_results[count] = loss_function(i, weights)
}

optimal_a <- a[which.min(loss_results)]
```

The optimal build cost (a) is approximately 7.3 thousands of kilos.

Exercise 2

```
#loading data & code
source("ExamData.R")
```

Task a

```
set.seed(12345)

#BayesLinReg <- function(y, X, mu_0, Omega_0, v_0, sigma2_0, nIter)

mu_0 <- as.vector(rep(0,8))
Omega_0 <- (1/9)*diag(8)
v_0 <- 1
sigma2_0 <- 9
nIter <- 10000
X <- as.matrix(X)

PosteriorDraws <- BayesLinReg(y,X,mu_0,Omega_0, v_0, sigma2_0, nIter)

betas <- PosteriorDraws$betaSample

interval <- quantile(betas[,2], probs =c(0.005,0.995))

df_interval <- data.frame("lower_bound" = interval[1], "upper_bound" = interval[2])
colnames(df_interval) <- c("lower bound", "upper bound")
rownames(df_interval) <- c("99% Equal Tail Credible Interval")
knitr::kable(df_interval)
```

	lower bound	upper bound
99% Equal Tail Credible Interval	-0.3714471	1.825996

Task b

```
x <- c(1,1,1,0.5,0,1,0,1)
mu <- betas %*% x
sigma <- sqrt(PosteriorDraws$sigma2Sample)
```

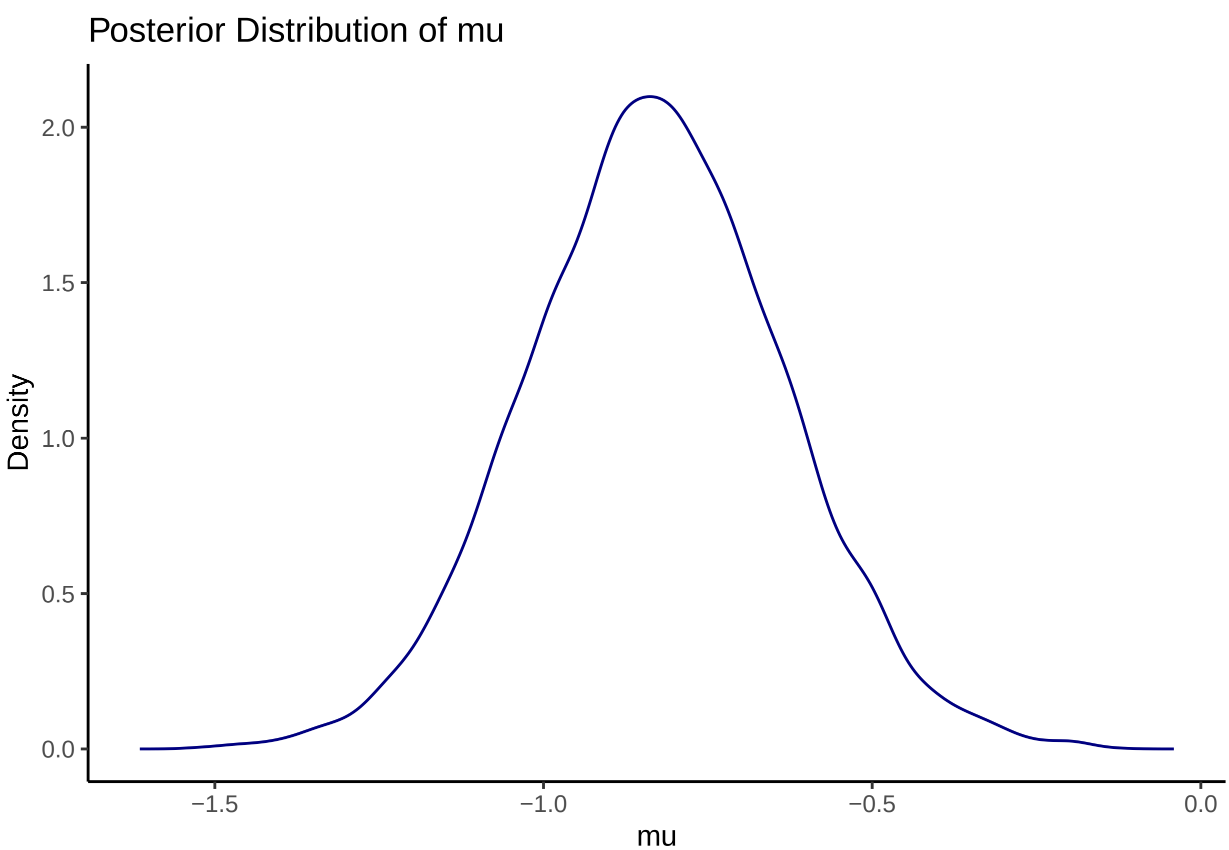
```
CV <- median(sigma/mu)
```

The median is about 1.816.

Task c

Task d

```
x <- c(1,-0.5,-0.5,0,0,1,0,-0.5)
mu <- betas %*% x

mu_density <- density(mu)

plot_df <- data.frame("x"=mu_density$x, "y"=mu_density$y)

ggplot(plot_df) +
  geom_line(aes(x=x, y=y), color = "navy") +
  ggtitle("Posterior Distribution of mu") +
  xlab("mu") +
  ylab("Density") +
  theme_classic()
```

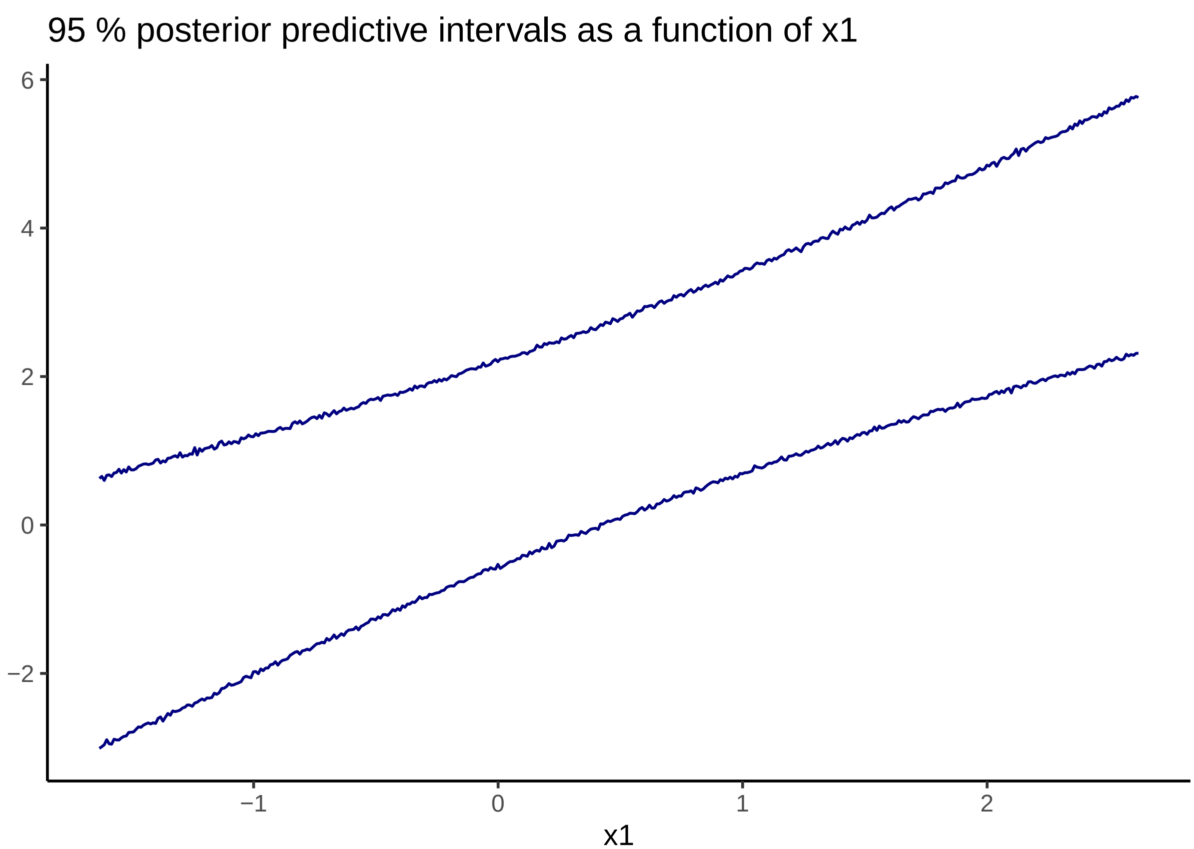



```
postprob <- mean(mu > 0)
```

The posterior probability that $\mu > 0$ is approximately 0.

Task e

```r
set.seed(12345)

x1 <- seq(min(X[,2]), max(X[,2]), 0.01)
intervals <- matrix(0, nrow = length(x1), ncol = 2)

for (i in 1:length(x1)){
  mu <- betas %*% c(1,x1[i],1,0.5,1,0,x1[i]*1,0)
  y_pred <- rnorm(nIter, mu, sigma)
  intervals[i,] <- quantile(y_pred, probs = c(0.025,0.975))
}

df_plot <- data.frame("x1"= x1, "low" = intervals[,1], "upper" = intervals[,2])

ggplot(df_plot) +
  geom_line(aes(x=x1, y=low), color= "navy") +
  geom_line(aes(x=x1, y=upper), color= "navy") +
  ggtitle("95 % posterior predictive intervals as a function of x1")+
  ylab("") +
  theme_classic()
```



Exercise 3

Task a,b and c are hand written.

Task d

```
set.seed(1)

logPost <- function(theta,n,sumlogx){

  logLik <- -n*theta**2 + 2*theta*sumlogx

  return(logLik)
}

theta <- runif(100,0,1)
n <- 5
sumlogx <- 2

posterior <- exp(logPost(theta,n,sumlogx))

posterior <- posterior/(0.01*sum(posterior))

df_plot2 <- data.frame("x"=posterior)

ggplot(df_plot2, aes(x=x)) +
  geom_density(colour = "red4", size = 1) +
  ggtitle("Posterior Distribution of Theta") +
  xlab("Theta Values") +
  ylab("Density") +
  theme_classic()
```