Bayesian Learning (732A91) Lab2 Report

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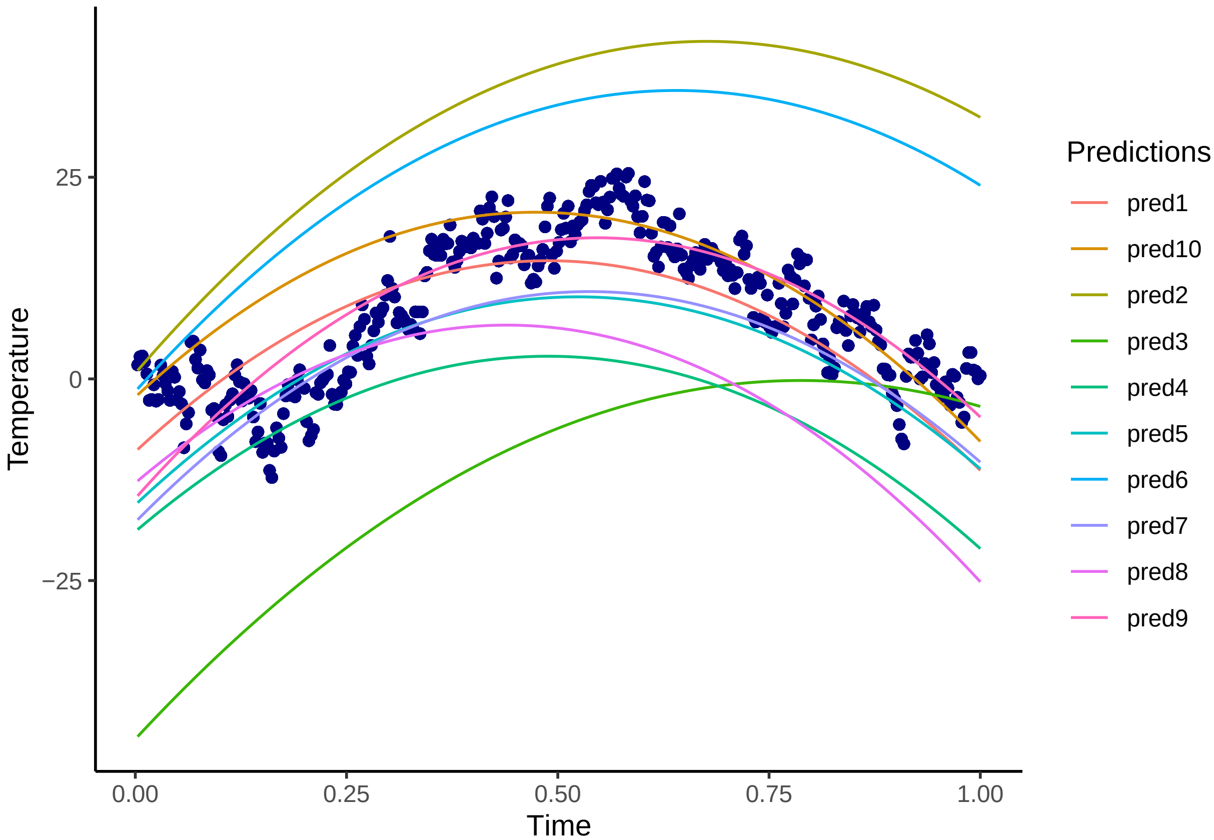
2022-04-19

Assignment 1 *Linear and polynomial regression*

Task 1a



The above graph illustrates the actual temperatures (blue points) and a collection of regression curves; one for each draw from the prior. The graph provides mixed results as half of the curves are not in the region of the actual temperatures. Thus, the the prior hyperparameters must be changed.

Task 1b

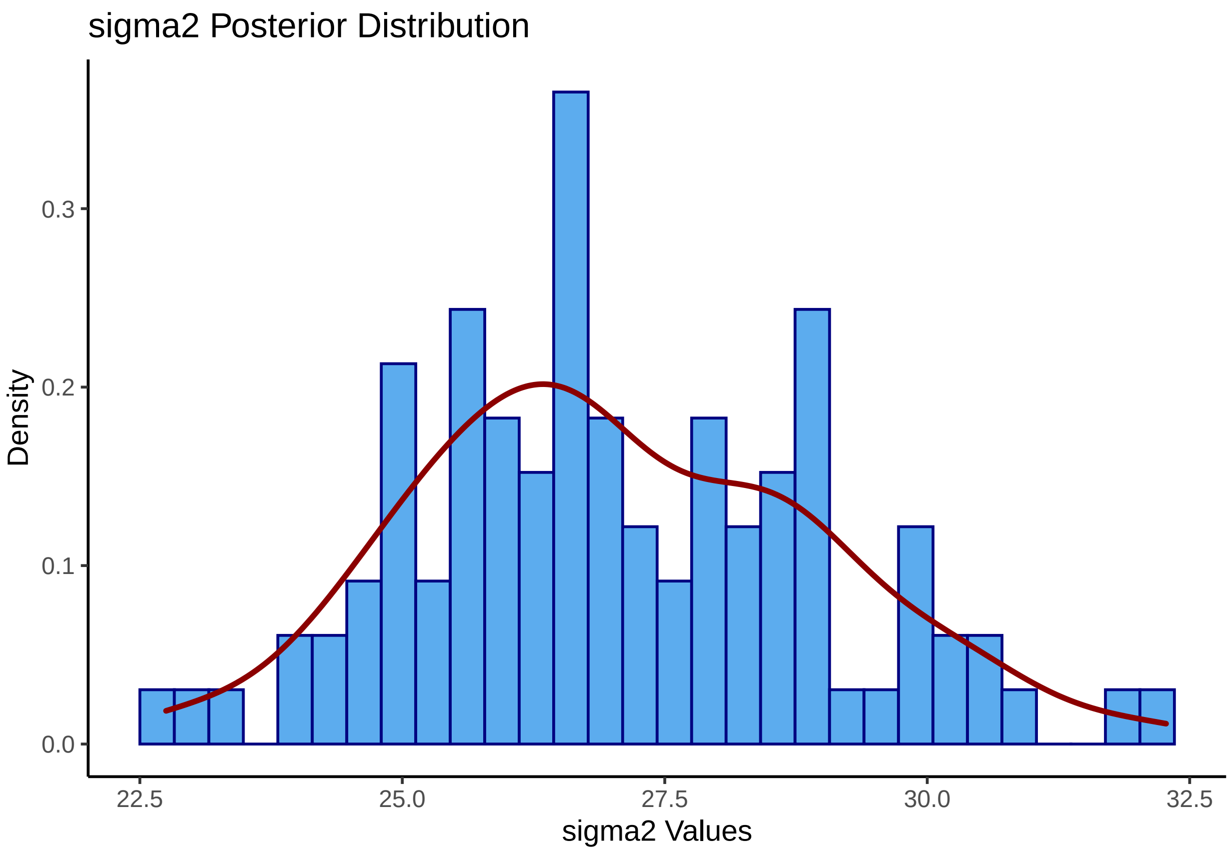
The joint posterior for β and σ^2 is given by:

$\beta|\sigma^2, y \sim N(\mu_n, \sigma^2\Omega_n^{-1})$
$\sigma^2|y \sim Inv - \chi^2(\nu_n, \sigma_n^2)$

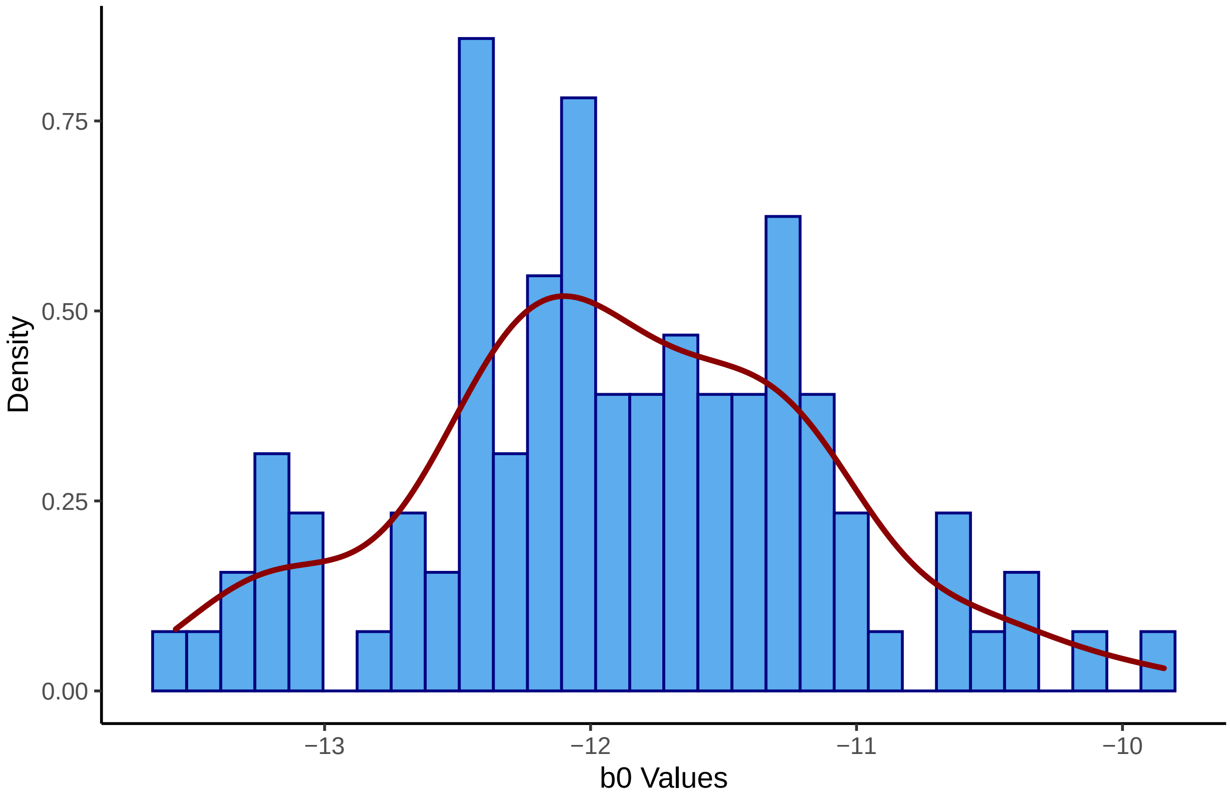
Where:

$\widehat{\beta} = (X'X)^{-1}X'y$
$\mu_n = (X'X + \Omega_0)^{-1}(X'X\widehat{\beta} + \Omega_0\mu_0)$
$\Omega_n = X'X + \Omega_0$
$\nu_n = \nu_0 + n$
$\sigma_n^2 = \frac{\nu_0\sigma_0^2 + (y'y + \mu_0'\Omega_0\mu_0 - \mu_n'\Omega_n\mu_n)}{\nu_n}$

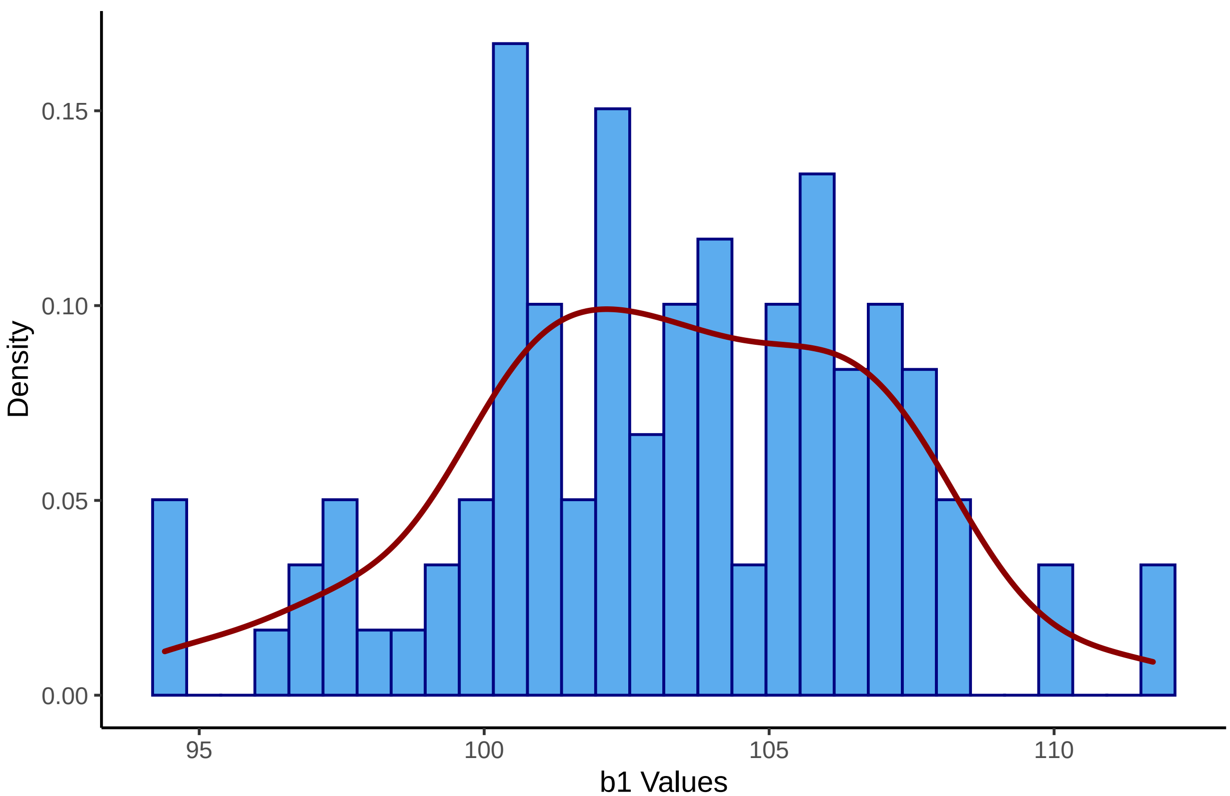
i)

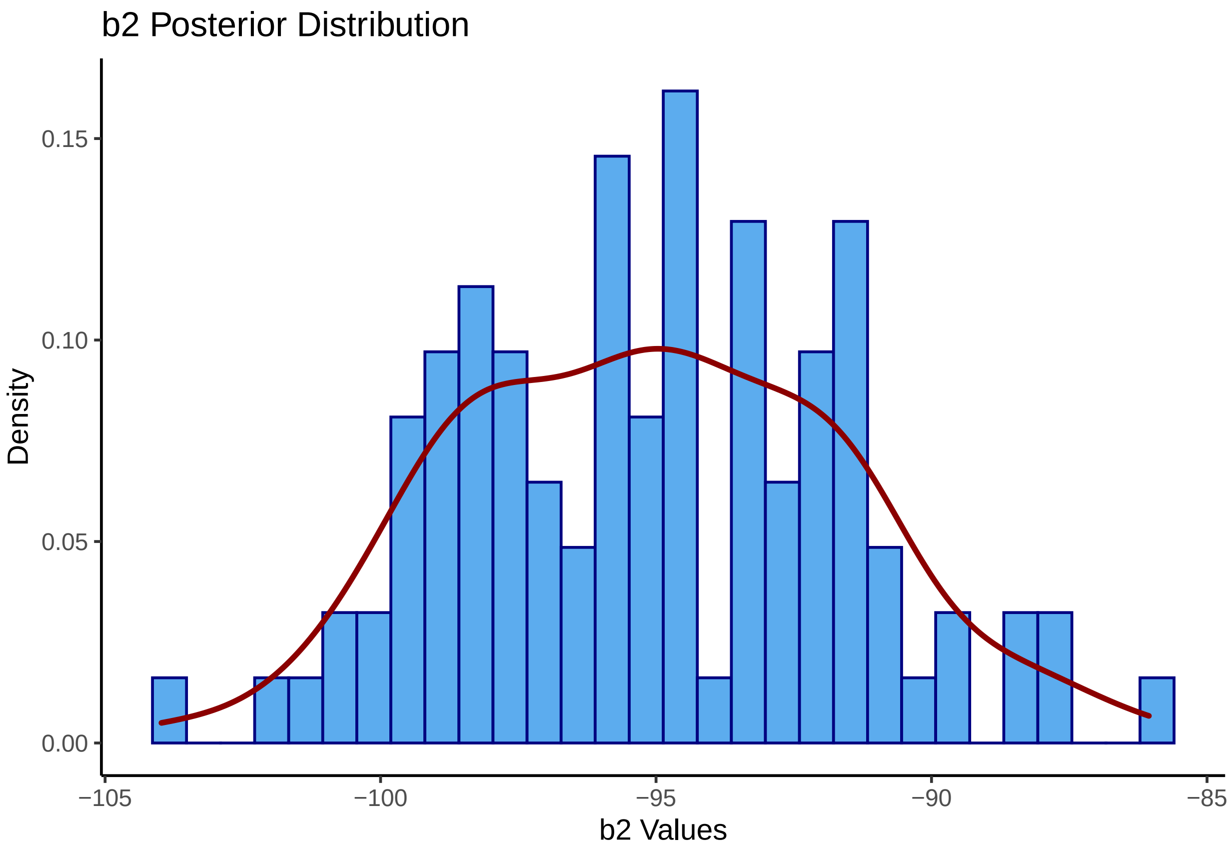


b0 Posterior Distribution

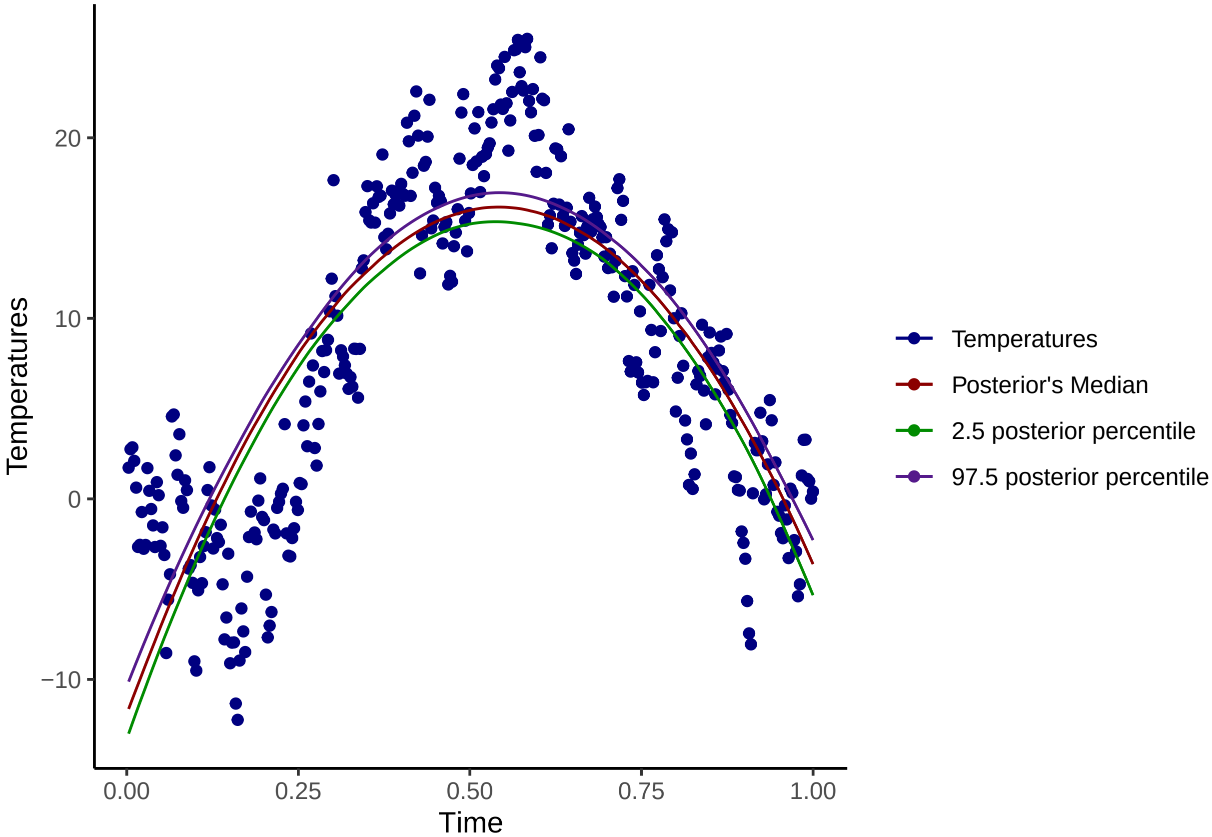


b1 Posterior Distribution




b2 Posterior Distribution
Density
0.15
0.10
0.05
0.00
−105
−100
−95
−90
−85
b2 Values

ii)



It is evident that 95% equal tail posterior probability intervals do not contain most of the data points. It should not contain most of the data points, because the interval illustrates the region of the beta parameter where the true parameter value lies with 95% certainty.

Task 1c

It is given that the regression function is $f(time) = \beta_0 + \beta_1 \cdot time + \beta_2 \cdot time^2$. In order to locate the time with the highest expected temperature; the time, where f(time) is maximal, is needed to be found. Thus, the derivative of $f(time)$ is needed to be found and set it equal to zero to find the maximal.

Set $time = x$; thus, $f(x) = \beta_0 + \beta_1 \cdot x + \beta_2 \cdot x^2$

Calculate the derivative, $f'(x) = \beta_1 + 2 \cdot \beta_2 \cdot x$

Find the maximal: $f'(x) = 0 \Leftrightarrow \beta_1 + 2 \cdot \beta_2 \cdot x = 0 \Leftrightarrow x = \frac{-\beta_1}{2 \cdot \beta_2}$

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# Task 1c

# getting the time with the highest expected temperature by
# using the above expression
max_temp <- max(-b[, 2]/(2 * b[, 3]))
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The time with the highest expected temperature is approximately 0.56.

Task 1d

Estimating a polynomial regression of order 8 with the suspicion that higher order terms may not be needed, might lead to overfitting the data. The main problem that could lead to overfitting is the number of knots. A solution to this problem is to introduce a regularised prior, $\beta_j|\sigma^2 \sim N(\mu_0, \frac{\sigma^2}{\lambda})$, where $Omega_0 = \lambda \cdot I$.