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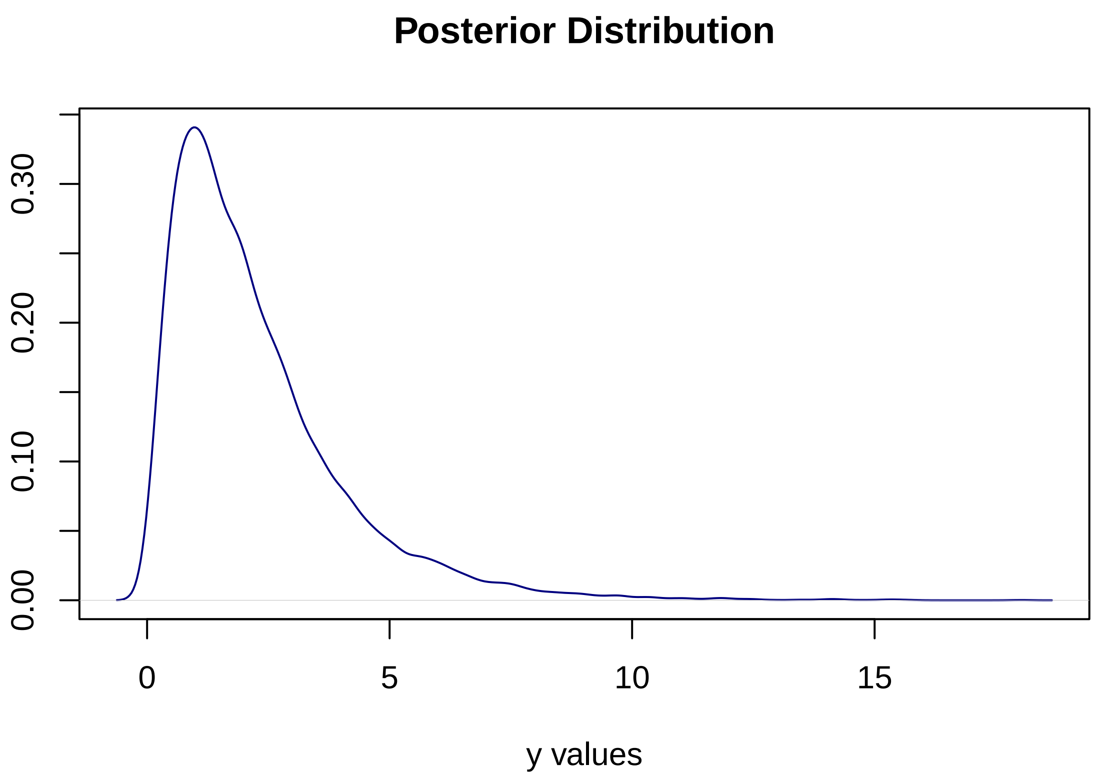
Problem 1

Task a

Hand written solution

Task b

```
obs <- c(2.32,1.82,2.4,2.08,2.13)
n <- length(obs)
Ndraws <- 10000

thetaDraws <- rgamma(n = Ndraws, shape = 2*n + 1, rate = 0.5 + sum(obs))
y <- rgamma(n = Ndraws, shape = 2, rate = thetaDraws)

plot(density(y), type = "l", col = "navy",
     main = "Posterior Distribution",
     xlab = "y values", ylab = "")
```




```
prob <- mean(y<1.9)
```

The $Pr(Y_6 < 1.9|y1, ..., y5)$ is 0.5313.

Task c

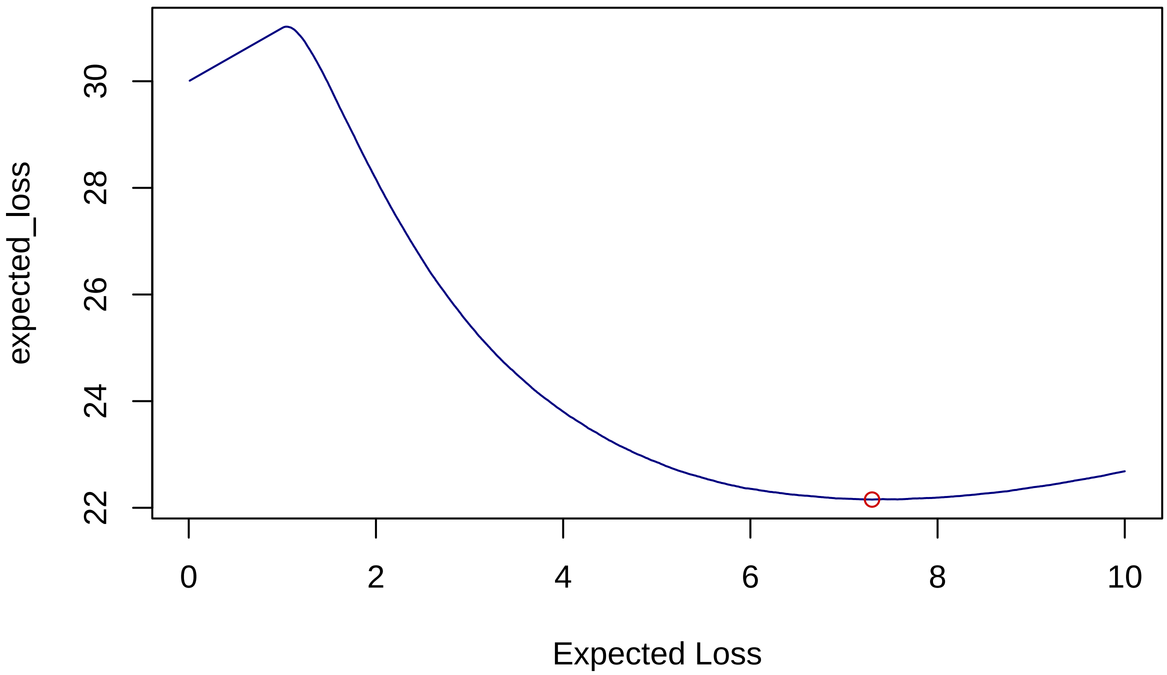
```
Nweeks <- 30
weights <- matrix(0, Ndraws, Nweeks)

for (i in 1:Ndraws) {
  thetaDraws <- rgamma(n = Nweeks, shape = 2*n + 1, rate = 0.5 +sum(obs))
  weights[i,]<- rgamma(n = Nweeks, shape = 2, rate = thetaDraws)
}

ExceedWeights <- mean(rowSums(weights>2.4))
```

The expected number of weeks out of the future 30 weeks in which the maximal weight will exceed 2.4 thousands of kilos is approximately 10.5.

Task d

```
loss_function <- function(a,weights){
  res <- a + mean(rowSums(weights>0.9*log(a)))
  return(res)
}

aGrid <- seq(0.01,10,0.01)

expected_loss <- matrix(0,length(aGrid),1)
```

```r
for (i in 1:length(aGrid)){
  expected_loss[i,] <- loss_function(aGrid[i],weights)
}

aOpt <- aGrid[which.min(expected_loss)]

plot(aGrid,expected_loss, type = "l", col = "navy",
     xlab = "Expected Loss")
points(aOpt, loss_function(aOpt,weights),col = "red3")
```



The optimal build cost is 7.39.

Problem 2

```r
source("ExamData.R")
```

Task a

```r
library(mvtnorm)

nIter <- 10000
mu_0 <- as.vector(rep(0,8))
Omega_0 <- 1/9 * diag(8)
v_0 <- 1
sigma2_0 <- 9
X <- as.matrix(X)

PostDraws <- BayesLinReg(y, X, mu_0, Omega_0, v_0, sigma2_0, nIter)
```

```r
BetaDraws <- PostDraws$betaSample

interval <- quantile(BetaDraws[,2], probs = c(0.005,0.995))


interval <- data.frame(lower_bound = interval[1], upper_bound = interval[2])
colnames(interval) <- c("lower bound", "upper bound")
rownames(interval) <- c("99% Equal Tail Credible Interval")
knitr::kable(interval)
```

	lower bound	upper bound
99% Equal Tail Credible Interval	-0.3712723	1.88609

It is 99 % posterior probability that β_1 is on the interval (-0.33,1.84).

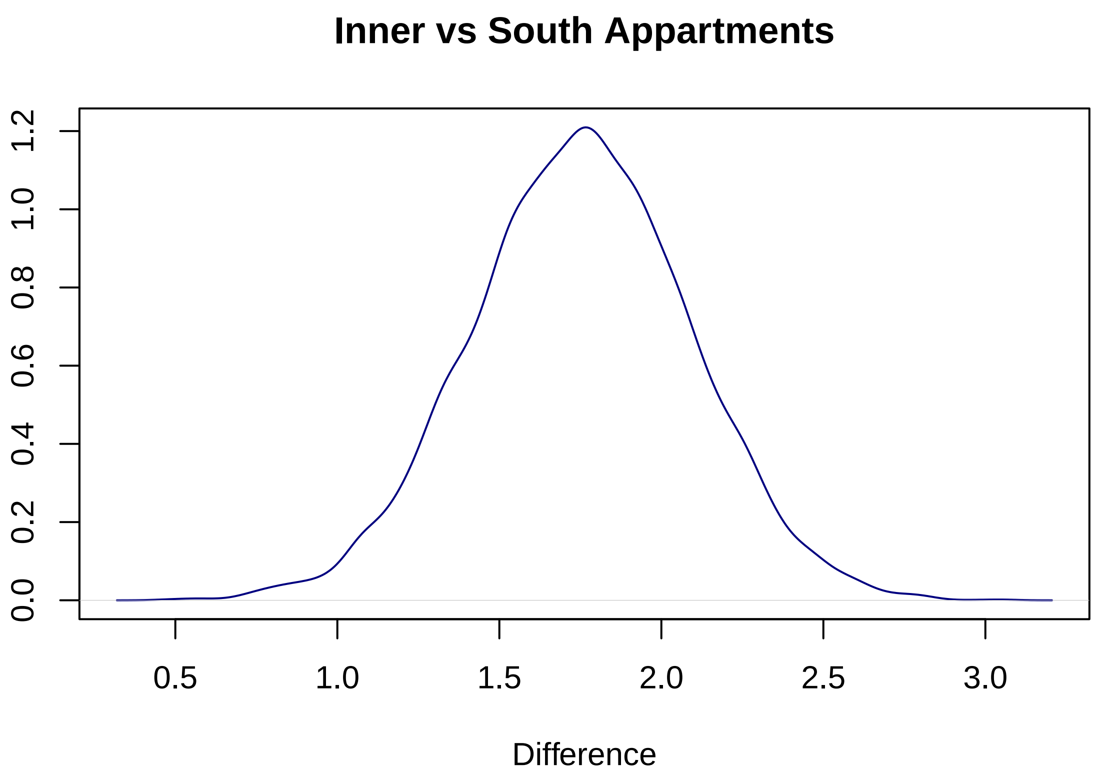
Task b

```r
mu <- BetaDraws[,1] + BetaDraws[,2] + BetaDraws[,3] + BetaDraws[,4] * 0.5 +
  BetaDraws[,6] + BetaDraws[,8]

CV <- sqrt(PostDraws$sigma2Sample)/mu

MedianCV <- median(CV)
```

The median of the coefficient of variation is approximately 1.83.

Task c

```r
Effect_Inner <- BetaDraws[,1] + BetaDraws[,2] +
  BetaDraws[,5] + BetaDraws[,7]
Effect_South <- BetaDraws[,1] + BetaDraws[,2] +
  BetaDraws[,6] + BetaDraws[,8]

Diff <- Effect_Inner - Effect_South
plot(density(Diff), type = "l", col = "navy",
     main = "Inner vs South Appartments",
     xlab = "Difference", ylab = "")
```

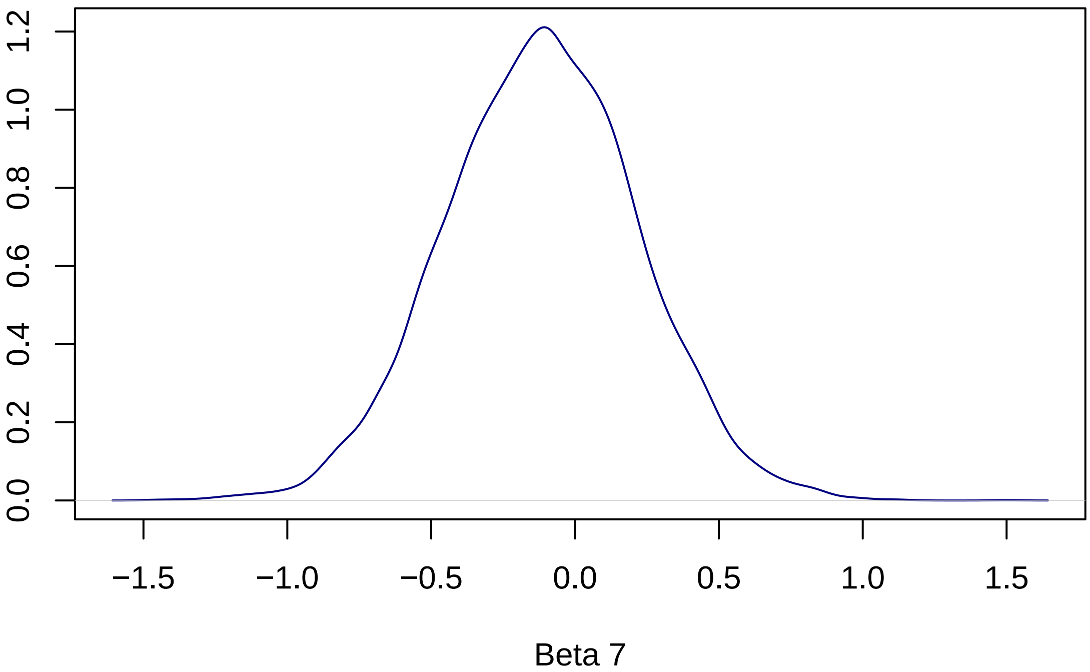



```
intervalDiff <- quantile(Diff, probs = c(0.025,0.975))
intervalDiff <- data.frame(lower_bound = intervalDiff[1],
                           upper_bound = intervalDiff[2])
colnames(intervalDiff) <- c("lower bound", "upper bound")
rownames(intervalDiff) <- c("95% Equal Tail Credible Interval")
knitr::kable(intervalDiff)
```

	lower bound	upper bound
95% Equal Tail Credible Interval	1.085007	2.409792

There is a high probability that the apartments in the inner city have a higher price than the apartments on the south side of the city. The 95% equal tail credible interval has positive bounds which strengthens the assumption.

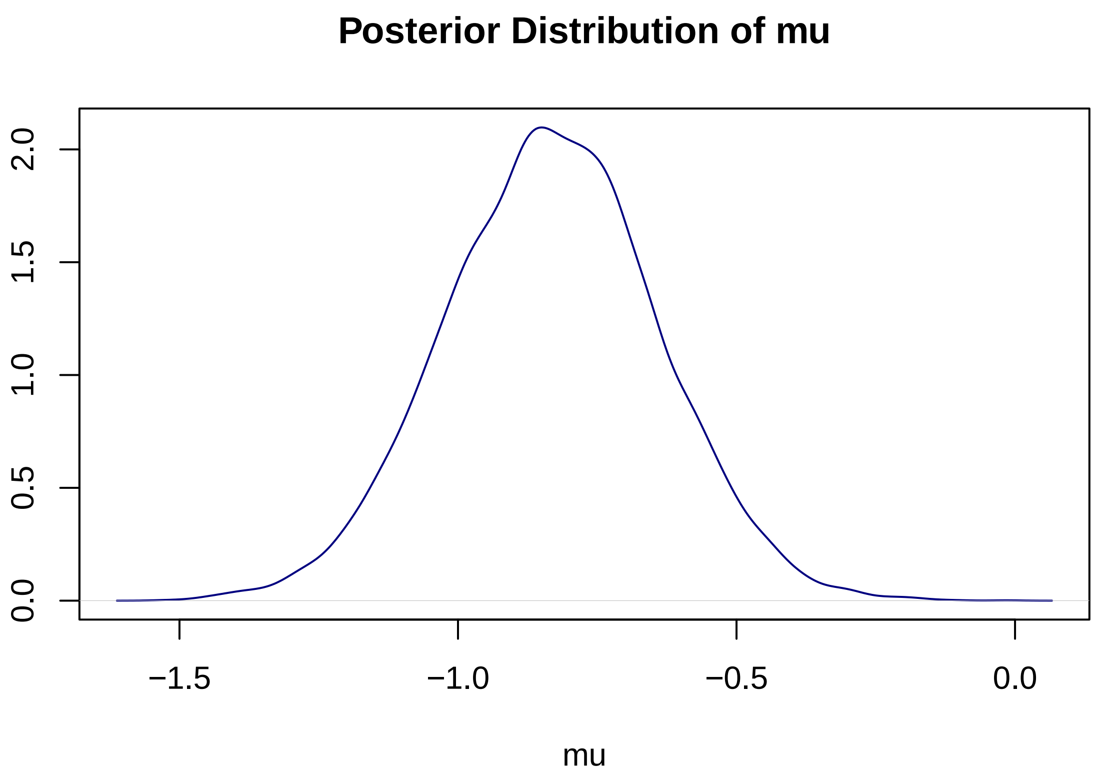
```
plot(density(BetaDraws[,8]), type = "l", col = "navy",
     main = "", xlab = "Beta 7", ylab = "")
```



There is a substantial probability mass on both sides of 0. Thus, the effect on the selling price y from x1 is not different for apartments on the south side of the city compared to apartments which are neither in the inner city nor on the south side of the city.

Task d

```
mu <- BetaDraws[,1] + BetaDraws[,2] * (-0.5) + BetaDraws[,3] *(-0.5) +
  BetaDraws[,6] + BetaDraws[,8] * (-0.5)

plot(density(mu), type = "l", col = "navy",
     main = "Posterior Distribution of mu",
     xlab = "mu", ylab = "")
```

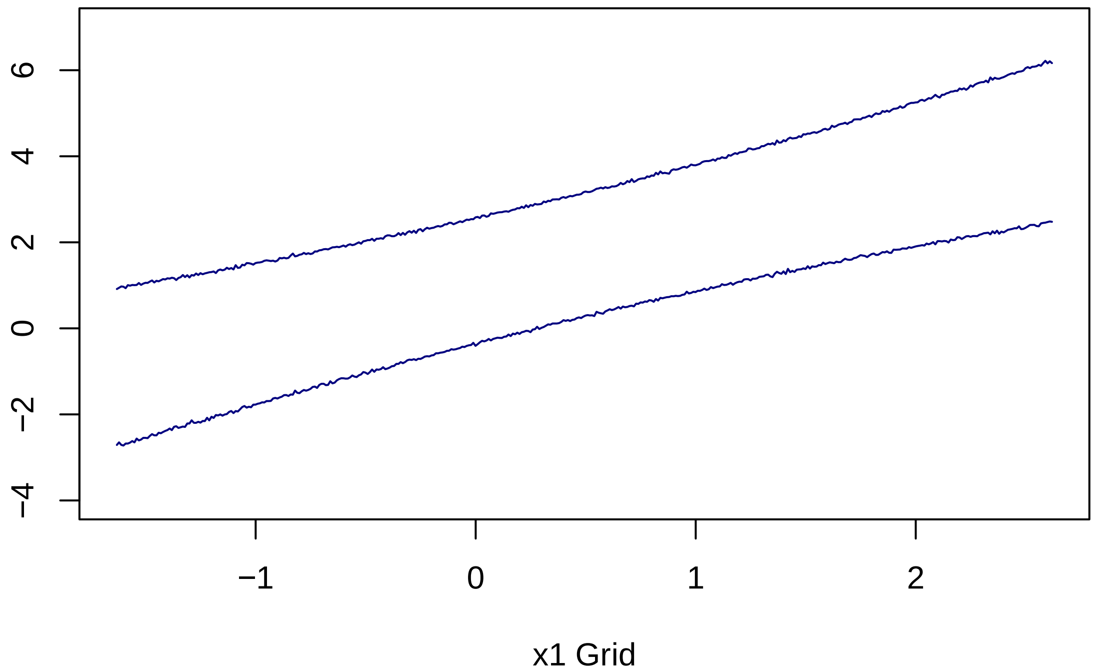



```
prob <- mean(mu>0)
```

The posterior probability that $\mu > 0$ is 0.

Task e

```
x1Grid <- seq(min(X[,2]), max(X[,2]), 0.01)
intervals <- matrix(0,length(x1Grid),2)

for (i in 1:length(x1Grid)) {
  mu <- BetaDraws[,1] + BetaDraws[,2] * x1Grid[i] + BetaDraws[,3] +
    BetaDraws[4,] * 0.5 + BetaDraws[,5] + BetaDraws[,7] * x1Grid[i]
  intervals[i,] <- quantile(rnorm(nIter, mu, sqrt(PostDraws$sigma2Sample)),
                            probs = c(0.025,0.975))
}

plot(x1Grid,intervals[,1], type = "l", col = "navy",
     main = "", xlab = "x1 Grid", ylab = "", ylim=c(-4,7))
lines(x1Grid, intervals[,2], type = "l", col = "navy")
```



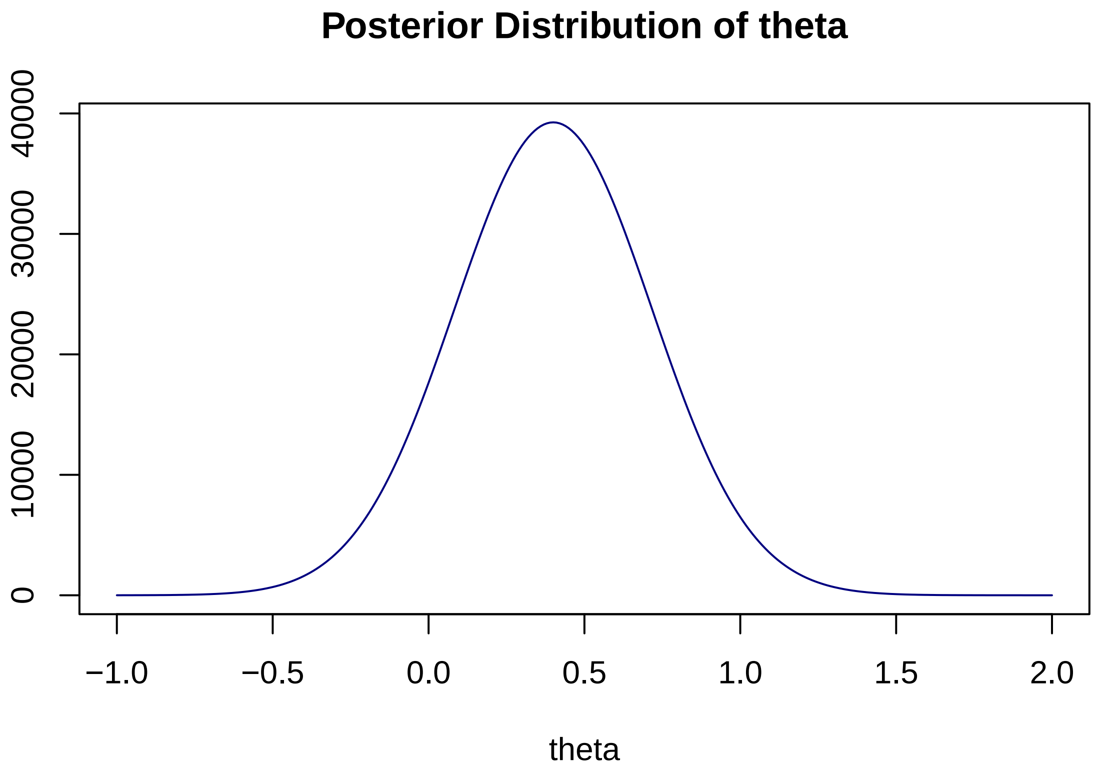
Problem 3

Task a,b,c

Hand written solution.

Task d

```
LogPost <- function(theta, n, sumlogx){
  res <- 2*theta*sumlogx - n*(theta^2)
  return(res)
}

thetaGrid <- seq(-1,2,0.01)
n <- 5
sumlogx <- 2

PostDens_propto <- exp(LogPost(thetaGrid,n,sumlogx))
PostDens <- PostDens_propto/0.01*sum(PostDens_propto)

plot(thetaGrid,PostDens, type = "l", col = "navy",
     main = "Posterior Distribution of theta",
     xlab = "theta", ylab = "")
```


Posterior Distribution of theta
0
10000
20000
30000
40000
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
theta