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Problem 1

Task a

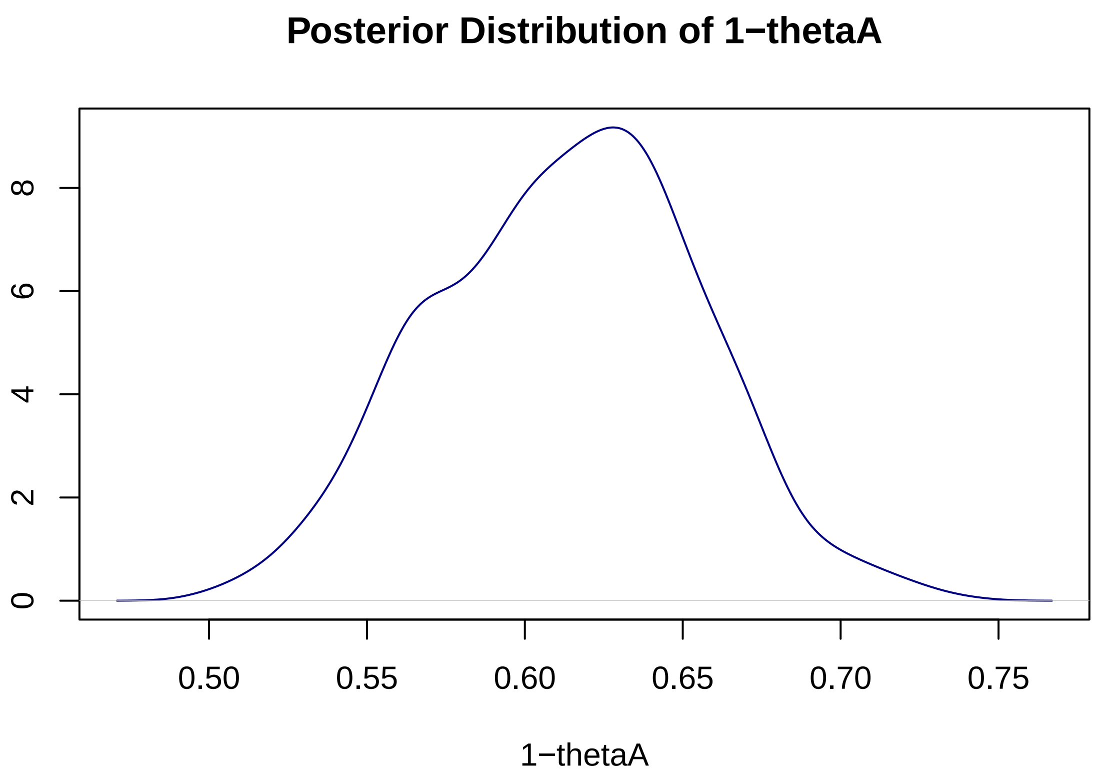
```r
#given values
n <- 100
s <- 38
f <- n-s
alpha0 <- 16
beta0 <- 24

#posterior parameters
post_alpha <- alpha0 + s
post_beta <- beta0 + f

#draw observations
nDraws <- 1000
thetaA <- rbeta(n=nDraws, shape1 = post_alpha, shape2 = post_beta)

#calculate posterior probability
prob <- pbeta(q=0.4, shape1 = post_alpha, shape2 = post_beta, lower.tail = FALSE)

plot(density(1-thetaA), type = "l",
     main = "Posterior Distribution of 1-thetaA",
     xlab = "1-thetaA", ylab = "", col = "navy")
```



The posterior probability that $\theta_A > 0.4$ is approximately 0.36.

Task b

```
ratio <- (1-thetaA)/thetaA

interval <-quantile(ratio, probs = c(0.025,0.975))

df_intervals <- data.frame(lower_bound = interval[1], upper_bound = interval[2])
colnames(df_intervals) <- c("lower bound", "upper bound")
rownames(df_intervals) <- c("95% Equal Tail Credible Interval")
knitr::kable(df_intervals)
```

	lower bound	upper bound
95% Equal Tail Credible Interval	1.138788	2.264101

The ratio shows the odds of not choosing brand A. The 95% equal tail credible interval for the ratio describes the values of the ratio with 95 % probability.

Task c

```
marginal_likelihood <- beta(post_alpha, post_beta)/beta(alpha0, beta0)
```

The marginal likelihood is approximately 7.55.

Task d

```
counts <- c(38,27,35)
alpha_const <- 20
alpha <- alpha_const*c(1,1,1)

K <- length(alpha)
xDraws <- matrix(0, nrow = nDraws, ncol = K)
thetaDraws <- matrix(0, nrow = nDraws, ncol = K)

for (i in 1:K) {
  xDraws[,i] <- rgamma(nDraws, shape = alpha[i] + counts[i], rate = 1)
}

for (j in 1:nDraws) {
  thetaDraws[j,] <- xDraws[j,]/sum(xDraws[j,])
}

prob <- mean(thetaDraws[1,] > thetaDraws[3,])
```

The posterior probability that $\theta_A > \theta_C$ is approximately 0.611.

Problem 2

Task a,b and c are hand written

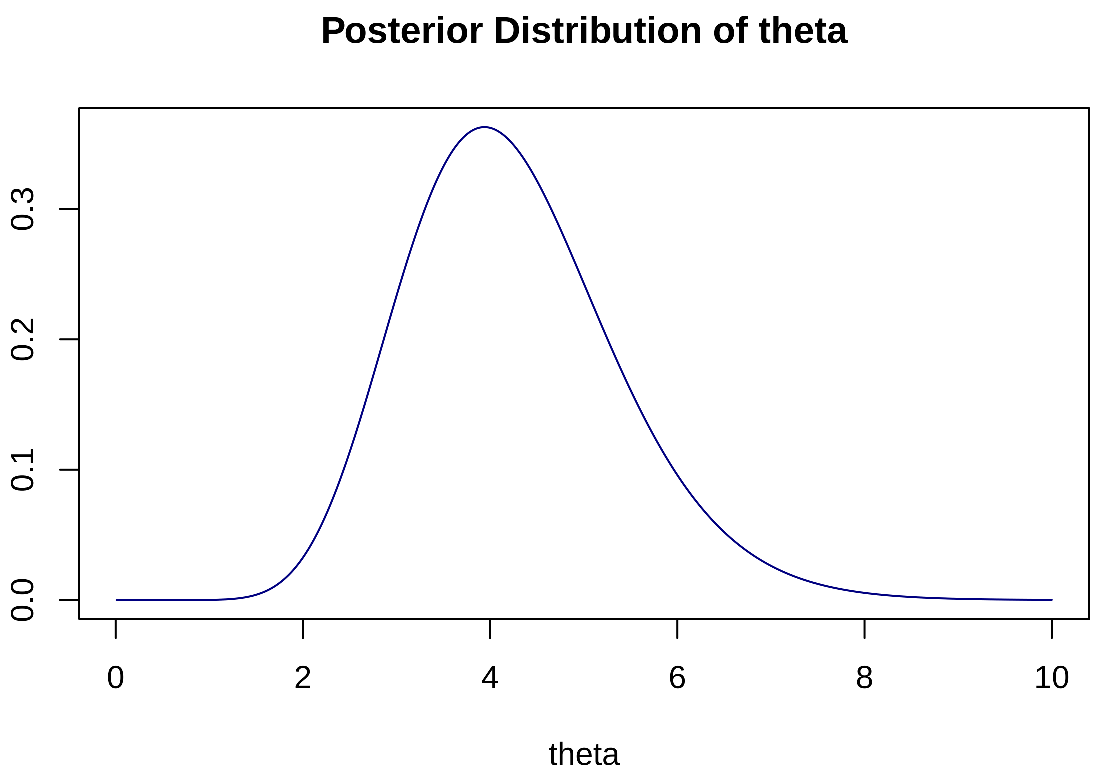
Task d

```
LogPost <- function(theta,n,sumx2) {
  res <- n*log(theta) - theta*(0.5 + sumx2)
  return(res)
}

n <- 13
thetaGrid <- seq(0.01,10,0.01)
sumx2 <- 2.8

postDens_propto <- exp(LogPost(thetaGrid,n,sumx2))
postDens <- postDens_propto/(0.01*sum(postDens_propto))

plot(thetaGrid, postDens, type = "l", col = "navy",
     main = "Posterior Distribution of theta",
     xlab = "theta", ylab = "")
```

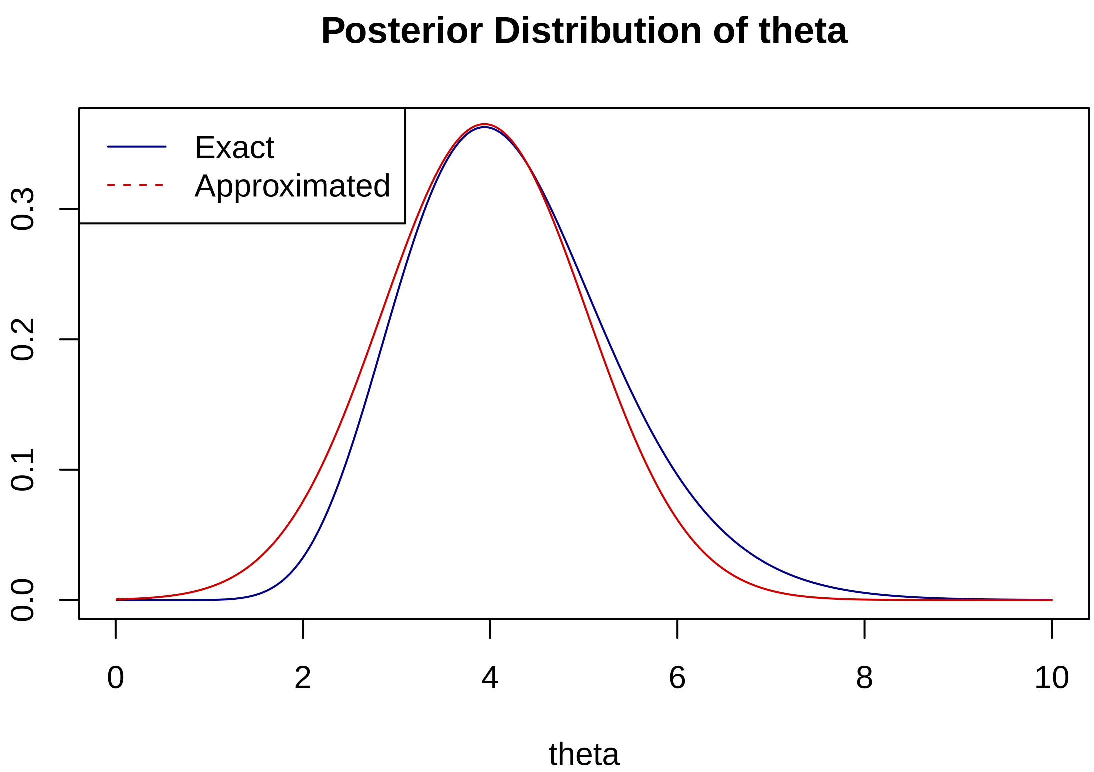


Task d

```
OptimRes <- optim(3,LogPost,gr=NULL,n,sumx2,
                  method=c("L-BFGS-B"),lower=0.1,
                  control=list(fnscale=-1),hessian=TRUE)

approx <- dnorm(thetaGrid, mean = OptimRes$par, sd = sqrt(-1/OptimRes$hessian))

plot(thetaGrid, postDens, type = "l", col = "navy",
     main = "Posterior Distribution of theta",
     xlab = "theta", ylab = "")
lines(thetaGrid, approx, type = "l", col = "red3")
legend("topleft", legend = c("Exact","Approximated"),
       col = c("navy","red3"), lty = 1:2)
```



The posterior's approximation is quite accurate, but the exact posterior distribution is skewed to the right.

Problem 3

Task a

```
library(mvtnorm)

mu_0 <- as.vector(rep(0,7))
Omega_0 <- (1/25) * diag(7)
v_0 <- 1
sigma2_0 <- 4
nIter <- 10000

PostDraws <- BayesLinReg(y, X, mu_0, Omega_0, v_0, sigma2_0, nIter)

Betas <- PostDraws$betaSample

BetasMean <- colMeans(Betas)

BetasMean <- as.data.frame(BetasMean)
rownames(BetasMean) <-c("Beta0","Beta1","Beta2","Beta3",
                        "Beta4","Beta5","Beta6")
knitr::kable(BetasMean)
```

	BetasMean
Beta0	1.3078660
Beta1	0.7004490

	BetasMean
Beta2	0.1572984
Beta3	0.4301263
Beta4	-0.1628046
Beta5	0.0762069
Beta6	-0.2402276

```
intervals <- matrix(0,7,2)

for (i in 1:7){
  intervals[i,] <- quantile(Betas[,i], probs = c(0.025,0.975))
}

intervals <- as.data.frame(intervals)
colnames(intervals) <- c("Lower Bound", "Upper Bound")
rownames(intervals) <-c("Beta0","Beta1","Beta2","Beta3",
                        "Beta4","Beta5","Beta6")
knitr::kable(intervals, caption = "95% Equal Tail Credible Interval")
```

Table 3: 95% Equal Tail Credible Interval

	Lower Bound	Upper Bound
Beta0	1.1471146	1.4645112
Beta1	0.5264450	0.8740771
Beta2	0.0492371	0.2642041
Beta3	0.0377224	0.8292395
Beta4	-0.3678454	0.0477160
Beta5	-0.2824960	0.4367703
Beta6	-0.4496628	-0.0279795

It is 95 % posterior probability that beta_1 is on the interval (0.528,0.876).

Task b

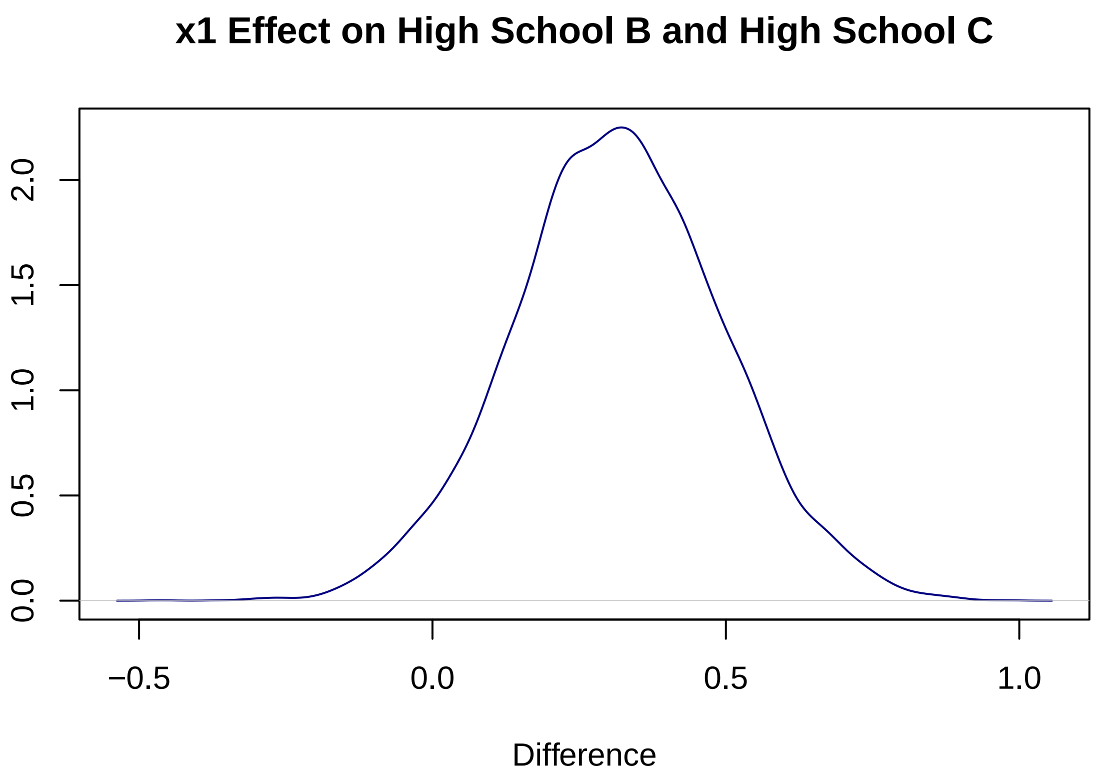
```
PostSigma2Median <- median(sqrt(PostDraws$sigma2Sample))
```

The posterior median of the standard deviation σ is 0.639.

Task c

```
effectB <- Betas[,2] + Betas[,6]
effectC <- Betas[,2] + Betas[,7]
diff <- effectB - effectC

plot(density(diff), type = "l", col = "navy",
     main = "x1 Effect on High School B and High School C",
     xlab = "Difference", ylab = "")
```




```
intervalDiff <- quantile(diff, probs = c(0.025, 0.975))
# table for the interval
intervalDiff <- data.frame(lower_bound = intervalDiff[1],
                           upper_bound = intervalDiff[2])
colnames(intervalDiff) <- c("lower bound", "upper bound")
rownames(intervalDiff) <- c("95% Equal Tail Credible Interval")
knitr::kable(intervalDiff)
```

	lower bound	upper bound
95% Equal Tail Credible Interval	-0.0364372	0.6757254

From the plot it seems that the effect on y from x1 is greater in high school B compared to high school C. However, the 95% equal tail credible interval for the difference of the slopes of x1 between the high schools reveals that the difference can be either negative or positive. Hence, the probability is not that high that this effect in high school B is larger than in high school C.

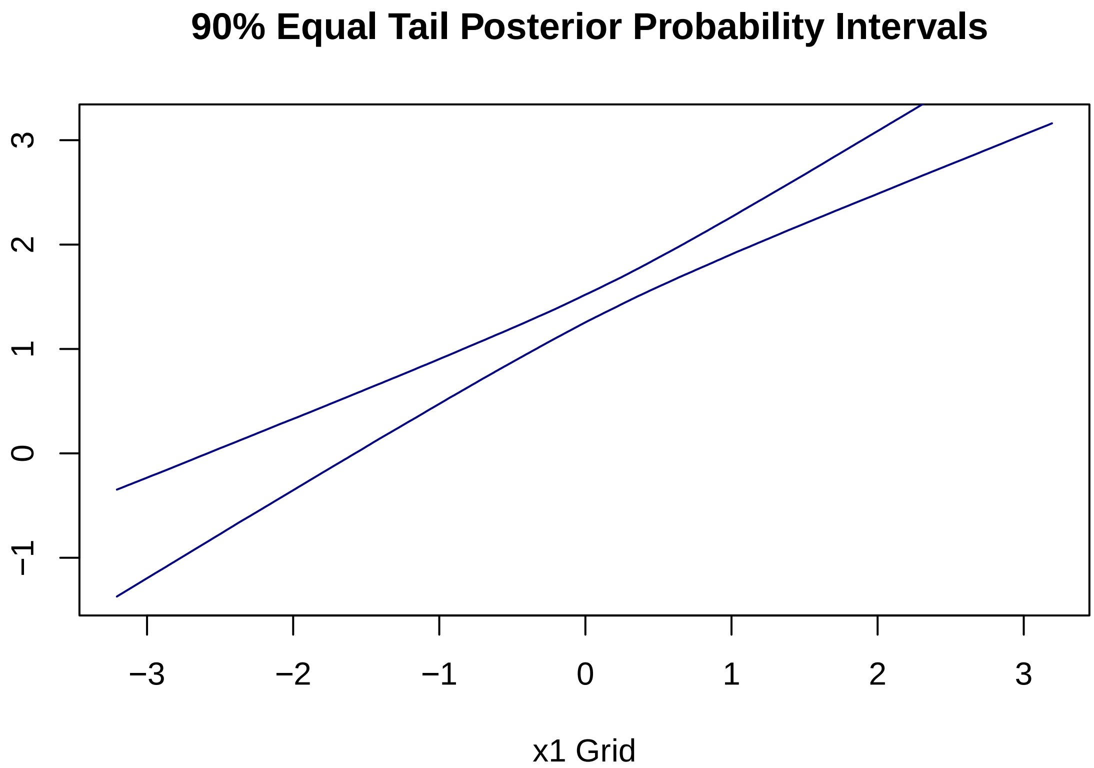
Task d

```
x1Grid <- seq(min(X[,2]), max(X[,2]),0.01)
intervalsMu <- matrix(0,length(x1Grid),2)

for (i in 1:length(x1Grid)) {
  mu <- Betas[,1] + Betas[,2] * x1Grid[i] + Betas[,3] * 0.5
  intervalsMu[i,] <- quantile(mu, probs=c(0.05,0.95))
}

plot(x1Grid, intervalsMu[,1], type = "l", col = "navy",
     main = " 90% Equal Tail Posterior Probability Intervals",
```

```r
     xlab = "x1 Grid", ylab = "")
lines(x1Grid, intervalsMu[,2], col = "navy")
```

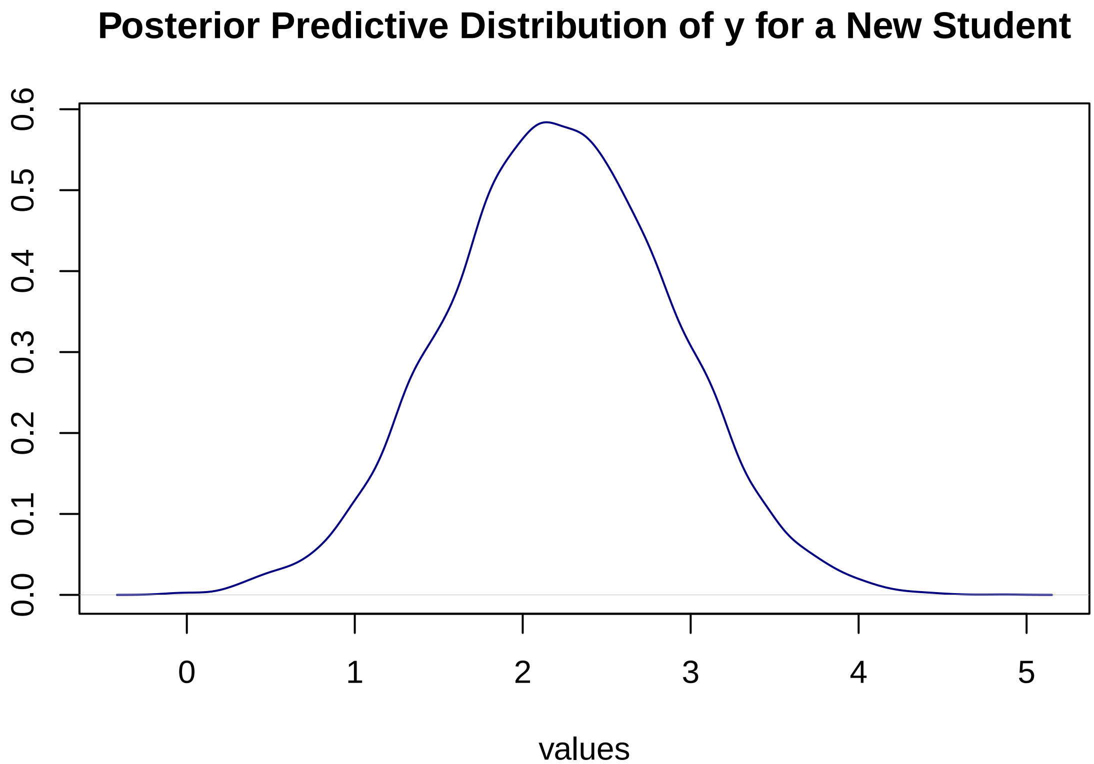


Task e

```r
mu <- Betas[,1] + Betas[,2] * 0.4 + Betas[,3] +
  Betas[,4] + Betas[,6] * 0.4
sigma <- sqrt(PostDraws$sigma2Sample)

values <- rnorm(nIter, mu, sigma)

plot(density(values), type = "l", col = "navy",
     main = "Posterior Predictive Distribution of y for a New Student",
     xlab = "values", ylab = "")
```


Posterior Predictive Distribution of y for a New Student
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
values