rmd_temp_tex

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The probability density fuction of Y~DE(0,1) is $f_Y(x) = \frac{1}{2}exp^{-|x|}$

The propability density function of N~(0,1) is $f_X(x) = \frac{1}{\sqrt{2\pi}}exp^{-\frac{x^2}{2}}$

$U \leq \frac{f_X(x)}{cf_Y(x)}$ and the ratio boundary is between 0 an 1 $0 < \frac{f_X(x)}{cf_Y(x)} \leq 1$, thus we have the following.

$$0 < \frac{f_X(x)}{cf_Y(x)} \leq 1$$
$$0 < \frac{f_X(x)}{f_Y(x)} \leq c$$

$$c \geq \frac{f_X(x)}{f_Y(x)}$$
$$c \geq \frac{\frac{1}{\sqrt{2\pi}}exp^{-\frac{x^2}{2}}}{\frac{1}{2}exp^{-|x|}}$$
$$c \geq \frac{2}{\sqrt{2\pi}}exp^{|x|-\frac{x^2}{2}}$$

We need to maximise $exp^{|x|-\frac{x^2}{2}}$.

$g(x) =| x | -\frac{x^2}{2}$

$g'(x) = 1 - \frac{2x}{2} = 1 - x$

$g'(x) = 0 \Leftrightarrow 1 - x = 0 \Leftrightarrow x = 1$

$$c \geq \frac{2}{\sqrt{2\pi}}e^{1-\frac{1}{2}} = \frac{2}{\sqrt{2\pi}}e^{\frac{1}{2}} = \frac{2\sqrt{e}}{\sqrt{2\pi}} \simeq 1.315$$