Untitled

Question 1

Question 2

Task 1

$$F(x) = \int_{-\infty}^{x} f(u)du$$

$$= \begin{cases} \frac{1}{2}\exp^{a(x-\mu_{DE})}, x < \mu_{DE} \\ \\ 1 - \frac{1}{2}\exp^{-a(x-\mu_{DE})}, x \geq \mu_{DE} \end{cases}$$

$$= \frac{1}{2} + \frac{1}{2}sgn(x-\mu_{DE})(1-\exp^{-\alpha|x-\mu_{DE}|})$$

- If $x \geq \mu_{DE}$:

$$\begin{aligned} y &= 1 - \frac{1}{2}\exp^{-\alpha(x-\mu_{DE})} \\ y - 1 &= -\frac{1}{2}\exp^{-\alpha(x-\mu_{DE})} \\ 2 - 2y &= \exp^{-\alpha(x-\mu_{DE})} \\ \ln(2-2y) &= \ln\exp^{-\alpha(x-\mu_{DE})} \\ \ln(2-2y) &= -\alpha(x-\mu_{DE}) \\ \frac{\ln(2-2y)}{\alpha} &= -x + \mu_{DE} \\ x &= \mu - \frac{\ln(2-2y)}{\alpha} \end{aligned}$$

- If $x < \mu_{DE}$:

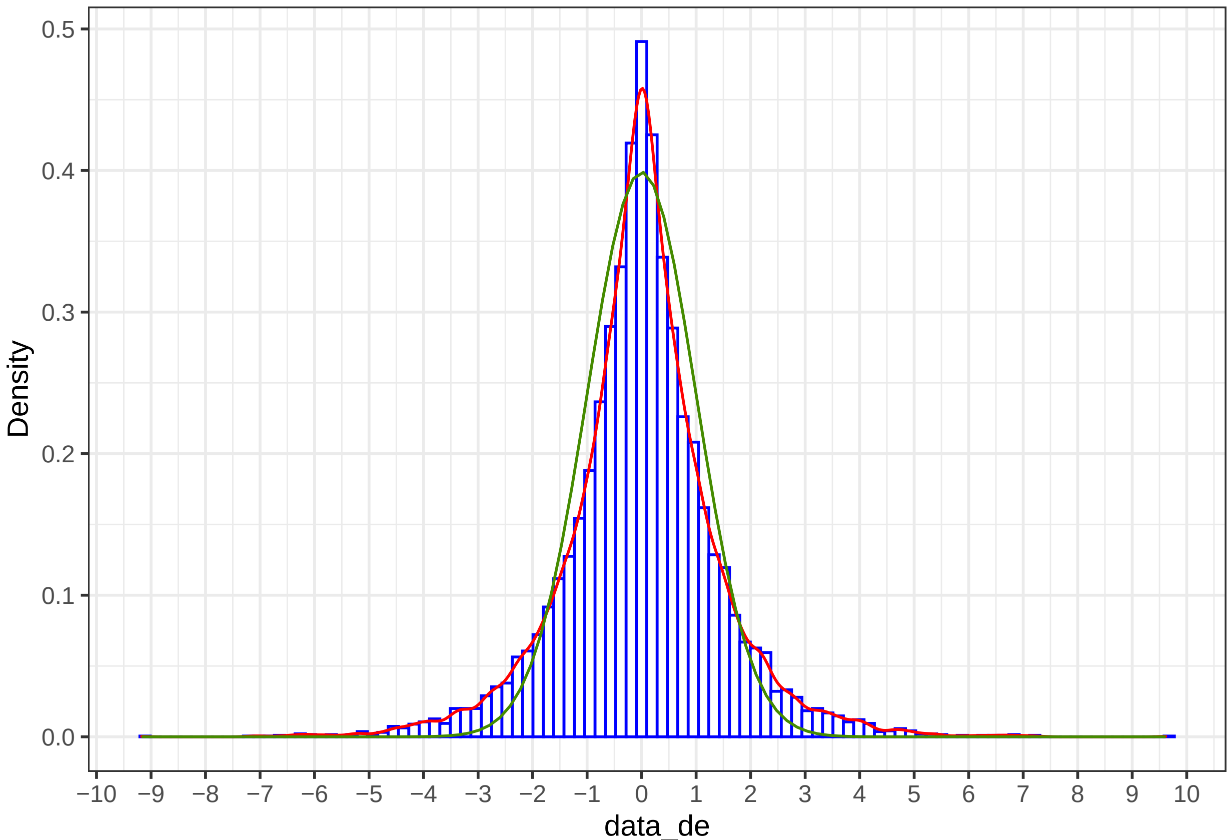
$$\begin{aligned} y &= \frac{1}{2}\exp^{\alpha(x-\mu_{DE})} \\ 2y &= \exp^{\alpha(x-\mu_{DE})} \\ \ln(2y) &= \ln\exp^{\alpha(x-\mu_{DE})} \\ \ln(2y) &= \alpha(x-\mu_{DE}) \\ \frac{\ln(2y)}{\alpha} &= x - \mu_{DE} \\ x &= \mu + \frac{\ln(2y)}{\alpha} \end{aligned}$$

$$
\begin{aligned}
F^{-1}(y) &= \int_{-\infty}^{y} f(u)du \\
&= \begin{cases} \mu - \frac{\ln(2-2y)}{\alpha}, x \geq \mu_{DE} \\ \\ \mu + \frac{\ln(2y)}{\alpha}, x < \mu_{DE} \end{cases} \\
&= \mu - \frac{1}{2} sgn(x-\mu) ln(1 + sgn(x-\mu) - sgn(x-\mu)2y) \\
&= \mu - \frac{1}{2} sgn(x-\mu) ln(1 - 2|x-\mu|)
\end{aligned}
$$

- Deduction: $P[F_y^{-1}(u) \leq y] = P(u \leq F_y(y) = F_u(F_y(y)) = F_y(y),\ U \sim U(0,1),\ \mu_u = \frac{1}{2}$

$$
\begin{aligned}
F^{-1}(u) &= \mu_{DE} - \frac{1}{2} sgn(u-\mu_u) ln(1 - 2|u-\mu_u|) \\
&= \mu_{DE} - \frac{1}{\alpha} sgn(u - \frac{1}{2}) ln(1 - |2u-1|)
\end{aligned}
$$

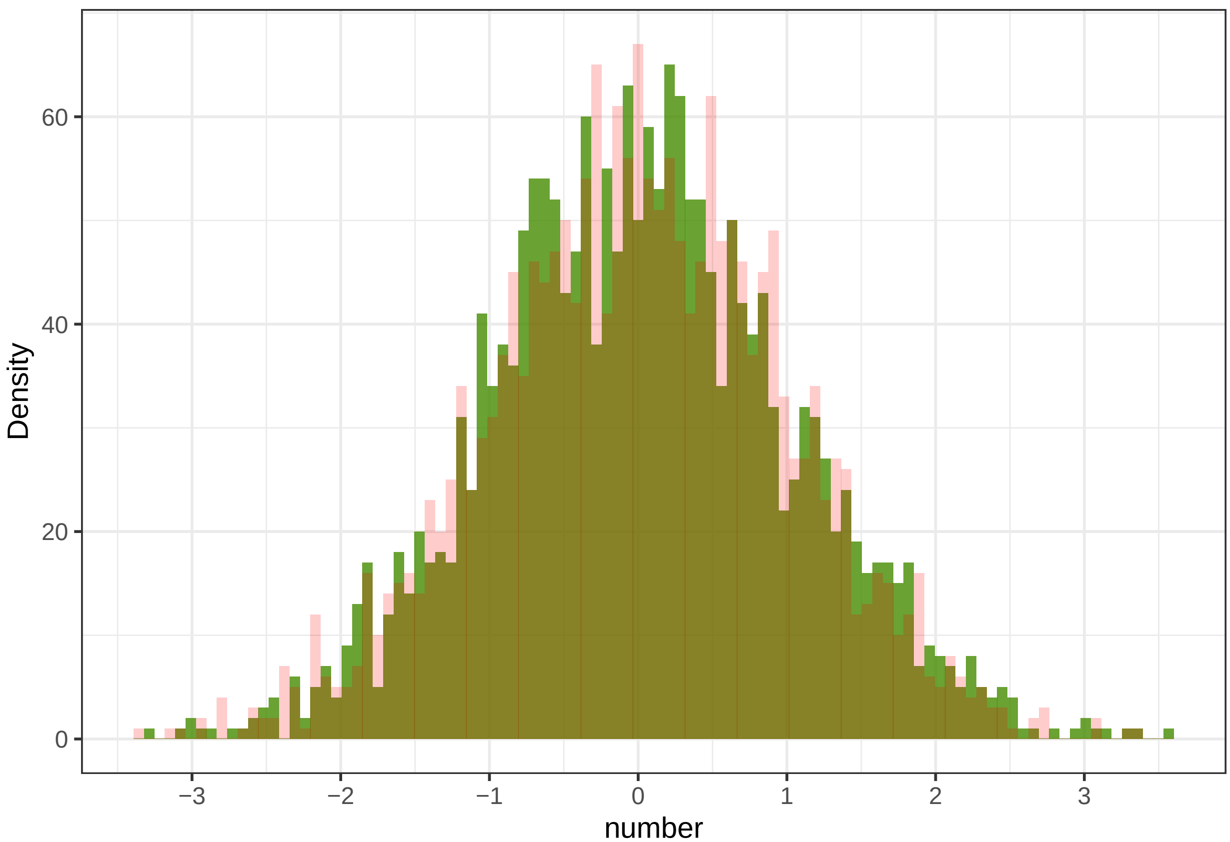
```r
set.seed(12345)
de_distribution = function(m = 0, a = 1){
  u = runif(1)
  result = m-(1/a)*sign(u-0.5)*log(1-2*abs(u-0.5))
  #result = m-sign(u-0.5)*1/a*log(1+sign(u-0.5)-sign(u-0.5)*2*u)
  return(result)
}

data_de = c()
for(i in 1:10000){
  data_de[i] <- de_distribution()
}

data_de = as.data.frame(data_de)

ggplot(data = data_de, aes(x = data_de)) +
  geom_histogram(bins = 100, color = "blue", fill = "white", aes(y=..density..))+
  geom_density(colour = "red")+
  stat_function(fun = dnorm, color = "chartreuse4")+
  ylab("Density")+
  scale_x_continuous(breaks = -10:10)+
  theme_bw()
```



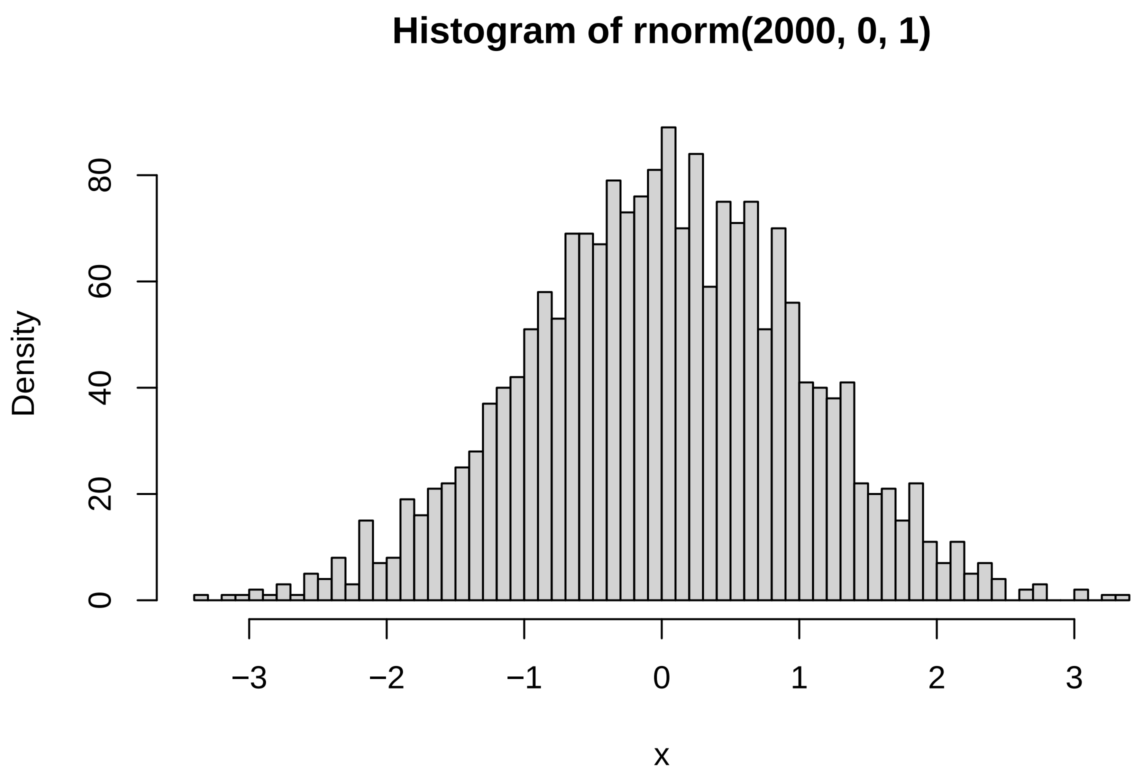
Task 2

```
de_pdf = function(x){
  (1/2)*exp(-abs(x))
}

c = (2*sqrt(exp(1))) / (sqrt(2*pi))

ar_method = function(c){
  x=NA
  rej_counter = 0
  while(is.na(x))
  {
    y = de_distribution()      # rv Y distributed as G
    U = runif(1)    # U(0,1)
    f_y = dnorm(y, mean = 0, sd = 1) # fx(y) density function
    g_y = de_pdf(y)    # g_y
    if(U < f_y / (c * g_y))
    {
      x=y # set x = y
    }
    else
    {
        rej_counter=rej_counter+1
    }
  }
```

```
  return(c(y, rej_counter))
}

ar_data <- data.frame(number = 1, rejections = 1)

for(i in 1:2000){
  ar_data[i,] <- ar_method(c=c)
}
set.seed(12345)
ar_data$norm = rnorm(2000,0,1)

ggplot(data = ar_data)+
  geom_histogram(bins = 100, fill = "chartreuse4", aes(x = number), alpha = 0.8)+
  geom_histogram(aes(x = norm), alpha = 0.2, fill = "red", bins = 100)+
  ylab("Density")+
  scale_x_continuous(breaks = -10:10)+
  theme_bw()
```




```
set.seed(12345)
hist(rnorm(2000,0,1), xlab = "x", ylab = "Density", breaks = 50 )
```




```r
reject_rate = 1- 2000/(2000+sum(ar_data$rejections))
reject_rate
```

```
## [1] 0.2472714
```

```r
exp_reject_rate = 1-1/c

print(paste("The observed rejection rate is: ", round(reject_rate, 5), " and the expected is: ", round(e
```

```
## [1] "The observed rejection rate is: 0.24727 and the expected is: 0.23983."
```