Hierarchical architectures for spiking Winner-Take-All networks

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SCIENCE
PASSION
TECHNOLOGY

Outline

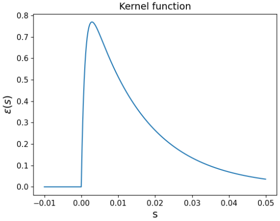
1 Introduction

2 Experiments

3 Results

Biological background

- Spiking neural networks
  - Resemble biological neural networks closely
  - Generate and propagate neural spikes
- Winner-Take-All networks
- Probabilistic brain
- Synaptic plasticity



Biological background

- Networks are organized in hierarchical structure
- Feedback used for attention / biased competition
- Lee and Mumford found that feedback could let neurons see illusory contour

Kanizsa square, Source: Lee and Mumford

Source: Lee T.S., Mumford D. (2003), “Hierarchical Bayesian inference in the visual cortex.”, In: J Opt Soc Am A Opt Image Sci Vis

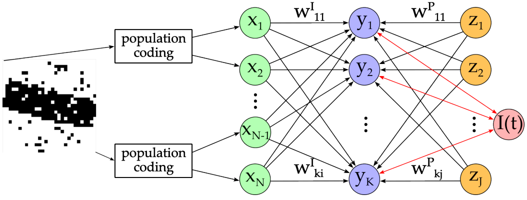
Theoretical background

- Bayesian inference gives the probability of an hypothesis given related evidence
- $P(Y = k|X, Z) = \frac{P(X|Y=k)P(Y=k|Z)}{\Sigma_{k'} P(X|Y=k')P(Y=k'|Z)}$
- Model of Nessler et al. expanded by prior neuron layer
  - Proved mathematically that expansion is valid
- Nessler et al. claimed that synaptic input weights converge towards the log of likelihood, $w^l_{ki} = log(P(x_i = 1|Y = k))$

Source: Nessler et al. (2013), “Bayesian Computation Emerges in Generic Cortical Microcircuits through Spike-Timing-Dependent Plasticity.”, In: PLOS Computational Biology 9.4

The network

- $u_k(t) = \sum_{i=1}^{N} w^I_{ki} \cdot x_i(t) + \sum_{j=1}^{J} w^P_{kj} \cdot z_j(t)$
- $p(y_k \text{ fires at time t}) \propto e^{u_k(t) - I(t)}$
- $w^I_{ki} = log(P(x_i = 1 | Y = k))$
- $w^P_{kj} = log(P(Y = k | Z = j))$



Goals

- Increase the understanding of the network model
- Simulate feedback found in the visual cortex
- Show connection between Bayesian inference and network model

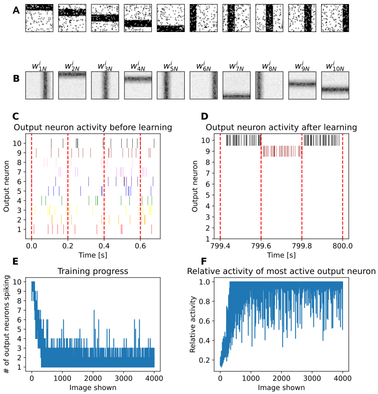
Methodology

- Simulation was performed in Python
- Simulation step size was 1 ms
- Pixels of input images and the prior had a noise level of 10%
- Kullback-Leibler divergence was chosen to evaluate performance of model
  - Compared firing rates of output neurons to analytical Bayesian posterior

Ambiguous visual stimuli 1

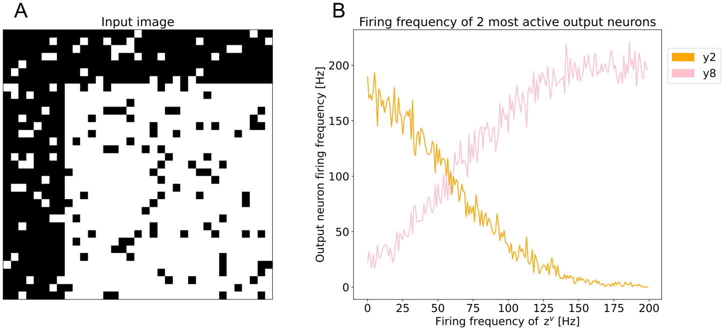
- Network learned to group horizontal and vertical bars into 10 groups
- After training ambiguous images with 1 horizontal and 1 vertical bar were shown
- Network was able to focus on individual bars, due to prior neurons

Ambiguous visual stimuli 2



Training plot

Ambiguous visual stimuli 3

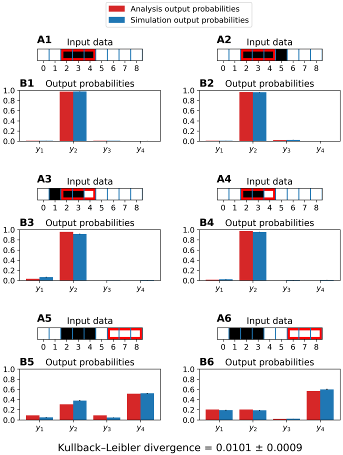


Variable prior activity

Analysis and simulation of the network 1

- Usage of smaller 1-D images to make network easier to analyse
- Mathematical derivation of Bayesian likelihood, prior and posterior
- Derived synaptic weights from Bayesian likelihood and prior
- Simulated network with those weights and fitted hyperparameters
- Compared Bayesian posterior to output of the simulation

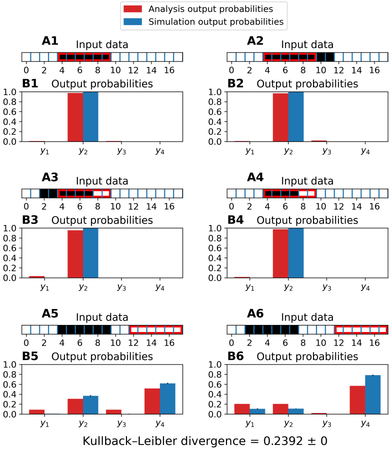
Analysis and simulation of the network 2



Transferability of hyperparameters 1

- Network was simulated with same hyperparameters of smaller network, to check if they are applicable to any network size
- Input size and prior neuron firing rate was doubled

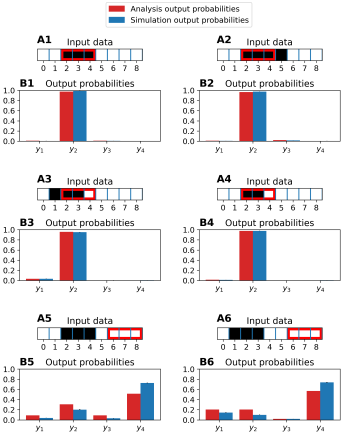
Transferability of hyperparameters 2



Training with predetermined hyperparameters 1

- Determined hyperparameters were used to train weights
- Trained weights were compared to analytically determined weights

Training with predetermined hyperparameters 2



Kullback–Leibler divergence = 0.0342 ± 0.0016

Training with predetermined hyperparameters 3

A $e^{\text{learned input weights}}$

0.52	0.49	0.53	0.06	0.1	0.08	0.12	0.11	0.06
0.09	0.07	0.49	0.49	0.51	0.09	0.09	0.09	0.06
0.07	0.08	0.08	0.06	0.51	0.5	0.52	0.07	0.1
0.11	0.1	0.12	0.07	0.09	0.06	0.53	0.46	0.51

B $e^{\text{calculated input weights}}$

0.9	0.9	0.9	0.1	0.1	0.1	0.1	0.1	0.1
0.1	0.1	0.9	0.9	0.9	0.1	0.1	0.1	0.1
0.1	0.1	0.1	0.1	0.9	0.9	0.9	0.1	0.1
0.1	0.1	0.1	0.1	0.1	0.1	0.9	0.9	0.9

C $e^{\text{learned prior weights}}$

1.14	0.02	0.02	0.02
0.03	1.17	0.03	0.01
0.02	0.02	1.15	0.02
0.02	0.03	0.03	1.17

D $e^{\text{calculated prior weights}}$

0.9	0.03	0.03	0.03
0.03	0.9	0.03	0.03
0.03	0.03	0.9	0.03
0.03	0.03	0.03	0.9

Results

- Connection between model and Bayesian inference was shown
  - Network outputs spikes according to Bayesian posterior
  - Trained weights converge towards the log of their respective probabilities
- Importance of neural feedback was shown for
  - Attention / Ambiguity resolution
  - Illusory contour effect

Results

- Optimal hyperparameters are dependent on network size
- Training process could not achieve perfectly trained weights

Conclusion

- Thesis provided insight to hierarchical spiking Winner-Take-All network model
- Showed that the network model can simulate effects like attention and changing beliefs through feedback
- Provided ideas on how to further analyse and improve the model

Sources

- Lee T.S., Mumford D. (July 2003). “Hierarchical Bayesian inference in the visual cortex.” In: J Opt Soc Am A Opt Image Sci Vis. DOI: doi:10.1364/josaa.20.001434
- Nessler, Bernhard et al. (Apr. 2013). “Bayesian Computation Emerges in Generic Cortical Microcircuits through Spike-Timing-Dependent Plasticity.” In: PLOS Computational Biology 9.4, pp. 1–30. doi: 10.1371/journal. pcbi.1003037. url: https://doi.org/10.1371/journal.pcbi.1003037