

Calculus en Kansrekening

Assignment 3, September 15, 2015

Handing in your answers:

- submission via Blackboard (<http://blackboard.ru.nl>);
- one single pdf file (make sure that if you scan/photo your handwritten assignment, the result is clearly readable);
- all of your solutions are clearly and convincingly explained;
- make sure to write your name, your student number and **the name of your TA** (Joost or Bram).

Deadline: Wednesday, September 23, 14:30 sharp!

Goals: After completing these exercises successfully you should be able to:

- apply all differentiation rules on elementary and transcendental functions;
- solve problems including higher-order derivatives;
- apply l'Hôpital's rules when applicable;
- analyse graphs of a given real function.

Marks: You can score a total of 100 points.

1. **(10 points)** As we learnt in the lecture (slide 38), the function \arcsin is the inverse function of \sin .

- (a) What is the domain of the function $\arcsin(x)$? Why?
(b) Compute the following values and explain how you got the result:

$$\arcsin(1) = ? \qquad \arcsin(0) = ? \qquad \arcsin\left(\frac{\sqrt{3}}{2}\right) = ?$$

- (c) Find the derivative of f :

$$f(x) = \arcsin\left(\frac{2x}{1-x}\right).$$

2. **(20 points)** Find the limits of the following functions. (Note that before you can apply L'Hôpital's rule, you have to verify whether it is possible.)

(a) $\lim_{x \rightarrow \infty} \frac{\ln(2015x)}{x^3};$

(b) $\lim_{a \rightarrow -3} \frac{\sin(a \cdot \pi)}{a^2 - 9};$

(c) $\lim_{x \rightarrow -\infty} \frac{e^{3-x}}{7x^2}.$

3. **(10 points)** Given the functions $f(x) = \log_3(2x)$ and $g(x) = \cos(3x)$.

- (a) What is $f'''(x)$?
(b) What is $g^{(2015)}(x)$? (Hint: Start with finding the first few derivatives of g .)

4. **(25 points)** Investigate function $f = (x+1)^2(x-3)$ by following the steps below. (Do not start with drawing a graph. Of course, you may check your solution with GeoGebra or with some other tool.)

- (a) Determine the domain of function f .
(b) What are the roots of f ? What is the y -intercept, that is, where is the intersection of the graph of f and the y -axis?

- (c) Determine the limits at the edges of the domain. In this case, there are only two edges:

$$\lim_{x \rightarrow -\infty} f(x) \quad \text{and} \quad \lim_{x \rightarrow +\infty} f(x).$$

- (d) Find f' and f'' .
 (e) Find the zeros of f' and f'' .
 (f) What are the critical points (determine their x and y coordinates)?
 (g) Find the local minima and maxima.
 (h) Which parts of the function are convex and concave? Does function f have points of inflection? (Hint: Use the sign of the second derivative for answering both questions.)
5. **(25 points)** We will investigate the function

$$f(x) = \frac{(x-2)^2}{x+2}.$$

following similar steps as the ones in the previous problem. Additionally, we prove that the line $y = x - 6$ is a slant asymptote on both sides.

- (a) Determine the domain of function f .
 (b) What are the roots of f ? Where does the graph of f intersect the y axis?
 (c) Determine the limits at the edges of the domain.
 (d) Find f' and f'' .
 (e) Find the zeros of f' and f'' .
 (f) What are the critical points (determine their x and y coordinates)?
 (g) Find the local minima and maxima.
 (h) Which parts of the function are convex and concave? Does function f have points of inflection? (Hint: Use the sign of the second derivative for answering both questions.)
 (i) Show that the line $y = x - 6$ is a slant asymptote of f . (Hint: Use the definition on slide 47 of the lecture and the following two limits.)

$$\lim_{x \rightarrow -\infty} (f(x) - (x - 6)) = ? \quad \text{and} \quad \lim_{x \rightarrow +\infty} (f(x) - (x - 6)) = ?$$

6. **(10 points)**

- (a) Find the derivative of $f(x) = \ln(\cos(\ln(\cos(x))))$.
 (b) Find a function $g(x)$ such that $g'(x) = \tan(2x)$.
 (c) Find three functions f_1, f_2, f_3 such that $f_1'(x) = f_2'(x) = f_3'(x) = \sin(x) \cos(x)$.