

Calculus en Kansrekening

Assignment 2, September 11, 2014

Handing in your answers: the full story, see

http://www.ru.nl/ds/education/courses/analyse_2014/

Briefly,

- submission via Blackboard (<http://blackboard.ru.nl>);
- one single pdf file;
- make sure to write your name and your student.
 - your name;
 - your student number and
 - the name of your TA (Safet and Arjen OR Ana Helena OR Gergely)

Deadline: Friday, September 19, 13:30 sharp!

Goals: After completing these exercises successfully you should be confident with the following topics.

- Definition of differentiation (derivative).
- Knowledge of basic properties of differentiation.
- Application of multiple rules in combination to differentiate functions.

Marks: You can score a total of 100 points. Note that you have to explain your answers, so it is clear how you have got the result. In order to get full points, you need to make sure that the reader can understand each step in your solution.

1. **(18 points)** Given function f and $a \in D(f)$. Recall that the definition¹ of the *derivative of function f at a* is

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}.$$

This is the slope of function f at point $(a, f(a))$. Compute the derivative of the function at the given a using the definition.

- (a) $f(x) = 3x^2$ at point $a = -\frac{1}{2}$.
(b) $f(x) = \frac{1}{x+2}$ at point $a = 1$.
(c) $f(x) = \sin 2x$ at point $a = 0$. (*Hint:* Use the fact that $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$)
2. **(50 points)** Find the derivative of the following functions. You can freely use all the differentiation rules that were discussed on the lecture. If it is possible, simplify the result.

(a) $f(x) = x^5 + 5x^4 - 10x^2 + 6$

(b) $f(x) = 5x^{\frac{1}{2}} - x^{\frac{3}{2}} + 2x^{-\frac{1}{2}}$

(c) $f(t) = \frac{1}{2t^2} + \frac{4}{\sqrt{t}}$

(d) $y = (1 - 4x)^5$

(e) $f(x) = (x+1)(x+2)$

(f) $f(x) = \frac{3x+1}{2x+4}$

(g) $f(x) = \left(\frac{x^2-1}{2x^3+1} \right)^4$

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¹Assuming that the given lim exists.

3. **(32 points)** Apply any rules (including chain or inverse rules) and the logarithmic differentiation as appropriate to compute the result. If you can solve a problem in two different ways, you get *two extra points*.

(a) $f(x) = \sin^2 x + \cos^2 x$; $f'(x) = ?$

(b) $f(x) = 3^{2x^2-1}$; $f'(x) = ?$

(c) $f(x) = 2^{\ln(\tan x)}$; $f'(x) = ?$

(d) $f(x) = \frac{5x^2-3}{\sqrt{x+1}}$; $f'(x) = ?$

(e) $f(x) = x^3 - 2$; $(f^{-1})'(x) = ?$