Calculus and Probability
Assignment 3

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Exercise 6

a)

$$f(x) = \arccos(\cos(x^2)) = x^2$$

Because arccos is the inverse of cos, we can rewrite the function to just $f(x) = x^2$. This makes finding the derivative rather simple and we get $f'(x) = 2x$.

The other option is to apply the chain rule:

$$\begin{aligned}
f(x) &= \arccos(\cos(x^2)) \\
f'(x) &= -\frac{1}{\sqrt{1-\cos^2(x^2)}} * (-2x\sin(x^2)) \\
&= \frac{2x\sin(x^2)}{\sqrt{1-\cos^2(x^2)}} \quad \text{Remember: } \sin^2(\theta)+\cos^2(\theta) = 1 \\
&= \frac{2x\sin(x^2)}{\sqrt{\sin^2(x^2)}} \\
&= \frac{2x\sin(x^2)}{\sin(x^2)} \\
&= 2x
\end{aligned}$$

$f'(x) = 2x$

Exercise 7

a) We do not need L'Hopital's rule here.

$$\lim_{x\to\infty} \frac{x^2}{1+e^{-x}} = \frac{\infty}{1} = \infty$$

$\lim_{x\to\infty} \frac{x^2}{1+e^{-x}} = \frac{\infty}{1} = \infty$

b) If we take the limit we get the undefined limit definition of $\frac{0}{0}$, therefore we can apply L'Hopital's rule. Answer 7b

Exercise 8

a)

$$\begin{aligned}
g(x) &= \cos(3x) \\
g'(x) &= 3 * (-\sin(3x)) \\
g''(x) &= 9 * (-\cos(3x)) \\
g'''(x) &= 24 * \sin(3x) \\
g^{(2015)} &= 3^{2015} * \sin(3x)
\end{aligned}$$

$g^{(2015)} = 3^{2015} * \sin(3x)$

Exercise 9

a) We can see the rools of f right from the function definition, namely:

$$\begin{aligned}
x_1 &= -1 \\
x_2 &= 3
\end{aligned}$$

We can get the y-intercept by putting in zero: $f(0) = -3$

Roots: $x_1 = -1$ and $x_2 = 3$. Y-intercept: $f(0) = -3$

b)

$$\begin{aligned}
\lim_{x \to -\infty} f(x) &= -\infty \\
\lim_{x \to +\infty} f(x) &= +\infty
\end{aligned}$$

$\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to +\infty} f(x) = +\infty$

c) Using the product rule to find the first derivative:

$$\begin{aligned}
f'(x) &= ((x^2 + 2x + 1)(x - 3))' \\
&= (2x + 2)(x - 3) + x^2 + 2x + 1 \\
&= 3x^2 - 2x - 5
\end{aligned}$$

The second derivative is therefore:

$$f''(x) = 6x - 2$$

To find the zeros of $f'(x)$ when can rewrite the term as $(x - 1)(3x - 5)$ which gives $x_1 = 1$ and $x_2 = \frac{5}{3}$. The only zero of $f''(x)$ is $x_1 = \frac{1}{3}$. We therefore found the x-coordinates for the critical points and insert them into the original function to get the actual points of the minima and maxima:

$$\begin{aligned}
f(-1) &= 0 \\
f(\frac{5}{3}) &= -\frac{2^8}{3^3} \approx 9.5
\end{aligned}$$

This gets us the points: $(-1, 0)$ and $(\frac{5}{3}, 9.5)$

$$f''(-1) = -14 < 0 \quad \text{maximum}$$
$$f''(\frac{5}{3}) = 13 > 0 \quad \text{minimum}$$

Maximum: $(-1, 0)$, Maximum: $(\frac{5}{3}, 9.5)$

d) The function is concave when: $f''(x) < 0$. Therefore, when $6x - 2 < 0$ which is the case when $x < \frac{1}{3}$ The function is convex when: $f''(x) > 0$. Therefore, when $6x - 2 > 0$ which is the case when $x > \frac{1}{3}$ Function f has a point of inflection at $x = \frac{1}{3}$ The function is concave when: $f''(x) < 0$. Therefore, when $6x - 2 < 0$ which is the case when $x < \frac{1}{3}$ The function is convex when: $f''(x) > 0$. Therefore, when $6x - 2 > 0$ which is the case when $x > \frac{1}{3}$ Function f has a point of inflection at $x = \frac{1}{3}$

Exercise 10

a)

$$\begin{aligned} h(x) &= \sin(x) - \frac{1}{3}\sin^3(x) \\ h'(x) &= \cos(x) - \sin^2(x)\cos(x) \\ h'(x) &= \cos(x) - (1 - \cos^2(x))\cos(x) \\ h'(x) &= \cos(x) - (\cos(x) - \cos^3(x)) \\ h'(x) &= \cos(x) - \cos(x) + \cos^3(x) \\ h'(x) &= \cos^3(x) \end{aligned}$$

$h(x) = \sin(x) - \frac{1}{3}\sin^3(x)$

b)

$$\begin{aligned} f_1(x) &= \frac{1}{2}\sin^2(x) + 1 \\ f_2(x) &= \frac{1}{2}\sin^2(x) + 2 \\ f_3(x) &= \frac{1}{2}\sin^2(x) + 3 \\ f_i'(x) &= \sin(x)\cos(x) \end{aligned}$$

$f_1(x) = \frac{1}{2}\sin^2(x) + 1, f_2(x) = \frac{1}{2}\sin^2(x) + 2, f_3(x) = \frac{1}{2}\sin^2(x) + 3$

Answer Form Assignment 3

Name	Christoph Schmidl
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Question	**Answer**
6 (1pt)	$f'(x) = 2x$
7a (1pt)	$\lim_{x\to\infty} \frac{x^2}{1+e^{-x}} = \frac{\infty}{1} = \infty$
7b (1pt)	Answer 7b
8a (1pt)	$g^{(2015)} = 3^{2015} * \sin(3x)$
9a (1pt)	Roots: $x_1 = -1$ and $x_2 = 3$. Y-intercept: $f(0) = -3$
9b (1pt)	$\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to+\infty} f(x) = +\infty$
9c (1pt)	Maximum: $(-1, 0)$, Maximum: $(\frac{5}{3}, 9.5)$
9d (1pt)	The function is concave when: $f''(x) < 0$. Therefore, when $6x - 2 < 0$ which is the case when $x < \frac{1}{3}$ The function is convex when: $f''(x) > 0$. Therefore, when $6x - 2 > 0$ which is the case when $x > \frac{1}{3}$ Function f has a point of inflection at $x = \frac{1}{3}$
10a (1pt)	$h(x) = \sin(x) - \frac{1}{3}\sin^3(x)$
10b (1pt)	$f_1(x) = \frac{1}{2}\sin^2(x) + 1, f_2(x) = \frac{1}{2}\sin^2(x) + 2, f_3(x) = \frac{1}{2}\sin^2(x) + 3$