Calculus en Kansrekening

Assignment 3, September 18, 2014

Handing in your answers: To read the full story, see

`http://www.ru.nl/ds/education/courses/analyse_2014/`

Briefly,

- make sure to put
  - your name;
  - your student number and
  - the name of your TA (Safet and Arjen OR Ana Helena OR Gergely)

  on your solution sheet;
- submit via Blackboard (`http://blackboard.ru.nl`);
- it is one single pdf file.

Deadline: Friday, September 26, 13:30 sharp!

Goals: After completing these exercises successfully you should be confident with the following topics.

- Inverse trigonometric functions and their derivatives;
- Higher derivatives;
- Analysing a function and sketching its curve;
- Partial derivatives of a function of multiple variables.

Marks: You can score a total of 100 points. Note that you have to **explain your answers**, so it is clear how you have got the result. In order to get full points, you need to make sure that the reader can understand each step in your solution.

1. **(15 points)** As we learnt in the lecture (slide 33), the function arccos is the inverse function of cos.
   (a) What is the domain of the function $\arccos(x)$? Why?
   (b) Compute the following values and explain how you got the result:
   $$\arccos(1) =? \qquad \arccos(0) =? \qquad \arccos\left(\frac{\sqrt{3}}{2}\right) =?$$
   (c) Find the derivative of f:
   $$f(x) = \arccos\left(\frac{2x}{1-x}\right).$$
2. **(15 points)** Given the functions $f(x) = \log_2(4x)$ and $g(x) = \sin(2x)$.
   (a) What is $f'''(x)$?
   (b) What is $g^{(2014)}(x)$? (Hint: Start with finding the first few derivatives of f.)

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3. **(25 points)** Investigate function $f = (x-1)^2(x+2)$ by following the steps below. (Do not start with drawing a graph by means of a device or some web resource. Of course you may check your result when you are done.)

   (a) Determine the domain of function f.

   (b) What are the roots of f? Where does the graph of f intersect the y axis?

   (c) Determine the limits at the edges of the domain. In this case, there are only two edges:

$$\lim_{x\to-\infty} f(x) \qquad \text{and} \qquad \lim_{x\to+\infty} f(x).$$

   (d) Find f' and f''.

   (e) Find the zeros of f' and f''.

   (f) What are the critical points (determine their x and y coordinates)?

   (g) Find the local minimums and maximums.

   (h) Which parts of the function are convex and concave? Does function f have points of inflection? (Hint: Use the sign of the second derivative for answering both questions.)

   (i) Draw the graph of function f. (If you collect all intervals and special points in a table, it helps a lot in drawing the graph. Moreover, you get some extra points!)

4. **(25 points)** We will sketch the function

$$f(x) = \frac{x^2}{x-2}$$

   following similar steps as the ones in the previous problem. Additionally, we prove that the line $y = x+2$ is an asymptote on both sides. (Again, do not start with drawing a graph by means of a device or some web resource. Of course you may check your result when you are done.)

   (a) Determine the domain of function f.

   (b) What are the roots of f? Where does the graph of f intersect the y axis?

   (c) Determine the limits at the edges of the domain.

   (d) Find f' and f''.

   (e) Find the zeros of f' and f''.

   (f) What are the critical points (determine their x and y coordinates)?

   (g) Find the local minimums and maximums.

   (h) Which parts of the function are convex and concave? Does function f have points of inflection? (Hint: Use the sign of the second derivative for answering both questions.)

   (i) Show that the line $y = x+2$ is a slant asymptote of f. (Hint: Use the definition on slide 41 of the lecture and the following two limits.)

$$\lim_{x\to-\infty} (f(x) - (x+2)) = ? \qquad \text{and} \qquad \lim_{x\to+\infty} (f(x) - (x+2)) = ?$$

   (j) Draw the graph of function f.

5. **(12 points)** Given function f, find the partial derivatives. If it is necessary, simplify the result.

   (a) $f(x,y) = \sin(3x + 2xy)$; $\frac{\partial f(x,y)}{\partial x} = ?$ and $\frac{\partial f(x,y)}{\partial y} = ?$

   (b) $f(x,y) = \ln\left(\frac{y}{x}\right)$; $\frac{\partial f(x,y)}{\partial x} = ?$ and $\frac{\partial f(x,y)}{\partial y} = ?$

6. **(8 points)** If $f(x,y) = \frac{xy}{x-y}$, show that

$$x^2\frac{\partial^2 f(x,y)}{\partial x^2} + 2xy\frac{\partial^2 f(x,y)}{\partial x \partial y} + y^2\frac{\partial^2 f(x,y)}{\partial y^2} = 0.$$

   (Hint: First compute all the second partial derivatives of f, then substitute the results in the expression on the left-hand side.)