Calculus and Probability
Assignment 1

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Exercise 6

a) $f(x) = x - x^3 = x(1 - x^2) = x(1 - x)(1 + x)$
Therefore, $f(x) = 0$ if $x \in \{-1, 0, 1\}$

b) $(1 - x^2) > 0 \Leftrightarrow |x| < 1 \Leftrightarrow x \in (-1, 1)$ So we have to distinguish between four cases:

- Case 1: $x < -1$. Then $x < 0$ and $(1 - x^2) < 0$. Therefore, $f(x) > 0$
- Case 2: $x \in (-1, 0)$. Then $x < 0$ and $(1 - x^2) > 0$. Therefore, $f(x) < 0$
- Case 3: $x \in (0, 1)$. Then $x > 0$ and $(1 - x^2) > 0$. Therefore, $f(x) > 0$
- Case 4: $x > 1$.Then $x > 0$ and $(1 - x^2) < 0$. Therefore, $f(x) < 0$

Exercise 7

- Case 1: When $a = b$, we get $y = \{a\}$
- Case 2: When $b > a$ then $(b - a) \geq 0$. y runs through all values in (a, b)
- Case 3: When $b < a$ then $(b - a) < 0$. y runs through all values in (b, a)

Exercise 8

a) $f(x) = 3x - x^3$

If we fill in $-x$, we get:
$f(-x) = 3(-x) - (-x)^3 = -3x + x^3 = -(3x - x^3) = -f(x)$.
Therefore, the function is odd.

b) $f(x) = \sqrt[3]{(1 - x)^2} + \sqrt[3]{(1 + x)^2}$

If we fill in $-x$, we get:
$f(-x) = \sqrt[3]{(1 - (-x))^2} + \sqrt[3]{(1 + (-x))^2} = \sqrt[3]{(1 + x)^2} + \sqrt[3]{(1 - x)^2} = f(x)$
Therefore, the function is even.

Exercise 9

a) $f(x) = \sqrt{7 - x^2} + 1$
To be able to compute the square root, the following property has to hold: $7 - x^2 \geq 0$. So $D(f) = [-\sqrt{7}, \sqrt{7}]$ and $R(f) = [1, \sqrt{7} + 1]$

b) $f(x) = \frac{1}{|x|}$
The only value which is excluded is zero because you cannot divide by it. Therefore, $D(f) = \mathbb{R}\backslash\{0\}$ and $R(f) = (0, \infty)$

Exercise 10

a) $y(cx+d) = ax+b \to x(cy-a) = -dy+b$, so that $x = \frac{-dy+b}{cy-a}$. Therefore the inverse function g is given by $g(x) = \frac{-dx+b}{cx-a}$

b) It is equal to the original function when $d = -a$

Exercise 11

a) $\lim_{x\to 2} \frac{x-2}{x^2+x-6} = \lim_{x\to 2} \frac{x-2}{(x-2)(x+3)} = \lim_{x\to 2} \frac{1}{x+3}$
Therefore, $\lim_{x\to 2} \frac{x-2}{x^2+x-6} = \frac{1}{5}$

b) $\lim_{x\to 1} \frac{x^2-4x+3}{x^2+x-2} = \lim_{x\to 1} \frac{(x-1)(x-3)}{(x+2)(x-1)} = \lim_{x\to 1} \frac{(x-3)}{(x+2)}$
Therefore, $\lim_{x\to 1} \frac{x^2-4x+3}{x^2+x-2} = -\frac{2}{3}$

Exercise 12

First we need to find the inverse $f^{-1}(x)$:

$$\begin{aligned} x &= \frac{y}{2y+3} \\ x(2y+3) &= y \\ 2xy+3x &= y \\ 2xy &= y-3x \\ 2xy-y &= -3x \\ y(2x-2) &= -3x \\ y &= -\frac{3x}{2x-1} \end{aligned}$$

Hence, the inverse $f^{-1}(x) = -\frac{3x}{2x-1}$, and therefore the endpoints we seek are $p_1 = f^{-1}(-2) = -\frac{6}{5} = -1.2$ and $p_2 = f^{-1}(3) = -\frac{9}{5} = -1.8$. We then need to compute the line $y = m \cdot x + b$ of the line going through points p_1 and p_2 where m is the slope of the function and b the intercept.

Finding the slope:

$$\begin{aligned} m &= \frac{y_2-y_1}{x_2-x_1} \\ &= \frac{3+2}{-1.8+1.2} \\ &\approx -8.3 \end{aligned}$$