Calculus and Probability

Assignment 1

Note:

- You can hand in your solutions as a single PDF via the assignment module in Brightspace. Note that the document should be in English and typeset with LaTeX, Word or a similar program. It should not be a scan or picture of your handwritten notes.
- Make sure that your name, student number and group number are on top of the first page!
- **Note that your submission should be an individual submission because it can influence your final grade for this course. If we detect that your work is not completely your own work, we will ask the exam committee to investigate whether it is plagiarism or not!**

Exercises to be presented during the exercise hours

Exercise 1
Let $f(x) = \frac{1}{2} - \frac{1}{1+(x-1)^2}$. Determine the values x for which

a) $f(x) = 0$

b) $f(x) > 0$

Exercise 2
Let's assume that x runs through the interval $(\frac{1}{2}, 1)$. What values does y run through for $y = \frac{a}{x}$, where $a \in \mathbb{R}$?

Exercise 3
Are the following functions even or odd? In your explanation use the definition.

a) $\sqrt{x^2}$

b) $f(x) = x^3 + 5x^2 + 2x + 10 + (5 - x)(x^2 + 2)$

Exercise 4
Determine the domains and ranges of the following functions. In your answer use $D(f)$ and $R(f)$ to be the domain respectively the range of the function f.

a) $f(x) = \frac{1}{1+\sqrt{x}}$;

b) $f(x) = \frac{x-5}{x^2-3x-10}$;

Exercise 5
Find the limits. (Hint: simplify as much as possible before applying the limit!)

a) $\lim_{x\to 0} \frac{3(x-1)+3}{x}$;

b) $\lim_{x\to -1} \frac{5x^2-5}{x^3+2x^2-x-2}$;

Exercises to be handed in

You are expected to explain your answers, even if this is not explicitly stated in the exercises themselves.

Exercise 6
Let $f(x) = x - x^3$. Determine the values x for which

a) $f(x) = 0$ 1 pt

b) $f(x) > 0$ 1 pt

Exercise 7
Let's assume that x runs through the interval $(0, 1)$. What values does y run through for $y = a + (b - a)x$, where $a, b \in \mathbb{R}$? 1 pt

Exercise 8
Are the following functions even or odd? In your explanation use the definition. 1 pt

a) $f(x) = 3x - x^3$;

b) $f(x) = \sqrt[3]{(1-x)^2} + \sqrt[3]{(1+x)^2}$;

Exercise 9
Determine the domains and ranges of the following functions. In your answer use $D(f)$ and $R(f)$ to be the domain respectively the range of the function f. 2 pt

a) $f(x) = \sqrt{7 - x^2} + 1$

b) $f(x) = \frac{1}{|x|}$

Exercise 10

a) What is the inverse of [1 pt]

$$y = \frac{ax + b}{cx + d} \qquad (ad - bc \neq 0)$$

b) When is it equal to the original function? [1 pt]

Exercise 11

Find the limits. (Hint: simplify as much as possible before applying the limit!) [2 pt]

a) $\lim_{x \to 2} \frac{x-2}{x^2+x-6}$;

b) $\lim_{x \to 1} \frac{x^2-4x+3}{x^2+x-2}$.

Exercise 12 (Old Exam Question (Bonus, 2pt))

This exercise combines several aspects of the theory of this week.

Let $f(x) = \frac{x}{2x+3}$ and let $g(x)$ be defined as follows

$$g(x) = \begin{cases} f^{-1}(x) & \text{if } x < -2 \text{ or } x > 3 \\ ax + b & \text{if } -2 \leq x \leq 3 \end{cases}$$

If $g(x)$ is continuous, then what is $\frac{b}{a}$?

Your final grade is the sum of your scores divided by 1.0.