Calculus and Probability

Assignment 2

Note:

- You can hand in your solutions as a single PDF via the assignment module in Blackboard. Note that the document should be in English and typeset with LaTeX, Word or a similar program. It should not be a scan or picture of your handwritten notes.
- Make sure that your name, student number and group number are on top of the first page!
- **Note that your submission should be an individual submission because it can influence your final grade for this course. If we detect that your work is not completely your own work, we will ask the exam committee to investigate whether it is plagiarism or not!**

Exercises to be presented during the exercise hours

Exercise 1
Find the limits. (Hint: try to simplify as much as possible before applying the limit!)

a) $\lim_{x\to\infty} \frac{3x^2+8}{x+1}$;

b) $\lim_{x\to a} \frac{x^n-a^n}{x-a}$ for some parameter $a \in \mathbb{R}$. (*Hint*: Do you recognize this limit? If not, you can always simplify the fraction using long division)

Exercise 2
Recall that if a function f is differentiable at a, then the derivative at the point a is defined as

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}.$$

Use this definition to find the derivative of the following functions at the point a:

a) $f(x) = \pi$, $a = 2$;

b) $f(x) = x^2 + 2x + 1$, $a = 3$;

Exercise 3
Given the equation $4y - x + 2(1 - \ln 2) = 0$ and the function $f(x) = a \ln x$, $a > 0$.

a) Find the slope of the line having the equation above.

b) There exists a value for a such that the line is the tangent line to $f(x)$ in the point $x = 2$. Find a.

c) Find the tangent line to $f(x)$ in the point $x = 2$ for every $a > 0$.

Exercise 4
Find the derivative on the domain of the following functions. You can freely use all the differentiation rules that were discussed in the lecture. Simplify the result as much as you can.

a) $f(x) = x^4 - 2x^3 + 7$;

b) $f(x) = \frac{x^2+5}{x-7}$;

c) $f(x) = \sin^2(\sqrt{x})$;

d) $f(x) = 1 - \cos^2(\sqrt{x})$;

e) $f(x) = \arcsin(1 - 2x)$;

f) $f(x) = 10^{x^2}$.

Exercise 5
Apply any rules (including chain or inverse rules) and the logarithmic differentiation as appropriate to compute the result. Try to solve each question in two different ways.

a) $f(x) = e^{\sin x}$, compute $f'(x)$;

b) $f(x) = e^{2x}$, compute $(f^{-1})'(x)$;

Exercises to be handed in

You are expected to explain your answers, even if this is not explicitly stated in the exercises themselves.

Exercise 6
Find the limits. (Hint: try to simplify as much as possible before applying the limit!)

a) $\lim_{x\to-\infty} \frac{x^3+2x^2+2}{3x^3+x+4}$; 1 pt

b) $\lim_{x\to\infty} \frac{2x+1}{x^2+x}$; 1 pt

Exercise 7
Recall that if a function f is differentiable at a, then the derivative at the point a is defined as

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}.$$

Use this definition to find the derivative of the following functions at the point a:

a) $f(x) = 2x + 3$, any a; 1 pt

b) $f(x) = \frac{5x-7}{4x+3}$, any $a \neq -\frac{3}{4}$. 1 pt

Exercise 8
Consider the function $f(x) = \frac{1}{(1+\frac{1}{x})}$.

a) Find the tangent line of $f(x)$ at $x = 2$. 1 pt

b) What is the tangent line of $f(x)$ as $x \to \infty$? 1 pt

Exercise 9
Find the derivative on the domain of the following functions. You can freely use all the differentiation rules that were discussed in the lecture. Simplify the result as much as you can.

a) $f(x) = \exp(\tan(x))$; 1 pt

b) $f(x) = -\ln(\cos(x))$; 1 pt

Exercise 10
Apply any rules (including chain or inverse rules) and the logarithmic differentiation as appropriate to compute the result. Try to solve each question in two different ways.

a) $f(x) = (\exp x)^{\exp x}$, compute $f'(x)$; (*Hint:* use logarithmic differentiation or the chain rule) 1 pt

b) $f(x) = \sqrt{x-2}$, compute $(f^{-1})'(x)$ (for $x > 2$). 1 pt

Your final grade is the sum of your scores divided by 1.0.