Calculus and Probability Theory

Assignment 7, March 16, 2017

Handing in your answers:

- submission via Blackboard (`http://blackboard.ru.nl`);
- one single pdf file (make sure that if you scan/photo your handwritten assignment, the result is clearly readable);
- all of your solutions are clearly and convincingly explained;
- make sure to write your name, your student number

Deadline: **Friday, March 24, 14:30 sharp!**

Goals: After completing these exercises successfully you should be able to:

- recognize and use common distributions of discrete and continuous random variables;
- compute the expectation and variance of discrete and continuous random variables.

Marks: You can score a total of 100 points.

1. **(20 points)** A shooter has exactly 6 bullets and shoots on a target. A random variable X is the number of bullets used *until he/she hits it for the first time.* The probability of a bullet hitting the target is 0.4 for every attempt.

   (a) Find the probability distribution of X; that is, give the probabilities for all possible values.
   (b) What is the expected value for X?
   (c) What is the the variance?
   (d) What is the the standard deviation?

2. **(20 points)** Consider a class where students have to hand in exercises every week. They have to hand in eight assignments in total and have to pass at least five to be able to attend the exam. Student A does not study very hard, so for each assignment he/she has a probability of 0.5 to pass. Student B studies very hard, so for each assignment he/she has a probability of 0.8 to pass. The random variable X_A is the number of passes of student A and the random variable X_B is the number of passes of student B.

   (a) Find $P(X_A = 5)$;
   (b) Find $P(X_A \geq 5)$;
   (c) Find $P(X_B \geq 5)$.

3. **(20 points)** A continuous random variable X has the following probability density function:

   $$f(x) = \begin{cases} a \cdot (1 - 4x^2) & \text{if } -\frac{1}{2} < x < \frac{1}{2}, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find the constant a.
   (b) Find the cumulative distribution function $F(x)$.
   (c) Compute the probability $P(X = \frac{1}{4})$.
   (d) Compute the probability $P(0 < X < \frac{1}{4})$.

4. **(20 points)** TV sets with various defects are brought to the service for reparation. The time of reparation is a continuous random variable T. The cumulative distribution function of T is given as:

   $$F(t) = \begin{cases} 0 & \text{if } t < 0, \\ 1 - e^{-kt} & \text{if } t \geq 0, \end{cases}$$

   where $k > 0$.

(a) Find the probability density function f of the random variable.

(b) Find the expectation and variance.

5. **(20 points)** A normal random variable X has probability density function

$$f(x) = \frac{1}{3} \exp\left(-\frac{\pi}{9}(x^2 - 4x + 4)\right).$$

(a) Find the mean μ and the variance σ.

(b) Let Y be the random variable defined by $Y = \frac{X-\mu}{\sigma}$. For a real number a show that

$$P(Y \leq -a) = 1 - P(Y \leq a).$$

(Hint: use that $\int_\alpha^\beta \phi(x)\, dx = -\int_\beta^\alpha \phi(x)\, dx$.)