Calculus and Probability Theory

Assignment 1, February 2, 2017

Handing in your answers:

- submission via Blackboard (`http://blackboard.ru.nl`);
- one single pdf file (make sure that if you scan/photo your handwritten assignment, the result is clearly readable);
- all of your solutions are clearly and convincingly explained;
- please make sure that you write your name and your student number on the assignment.

Deadline: Friday, February 10, 14:30 sharp!

Goals: After completing these exercises successfully you should be confident with the following topics:

- Even and odd functions
- The domain and range of a function
- The limit of a function

Marks: You can score a total of 100 points.

1. **(10 points)** Let $f(x) = x - x^3$. Determine the values x for which

   $$1.\ f(x) = 0; \qquad 2.\ f(x) > 0.$$

2. **(10 points)** Let's assume that x runs through the interval $(0, 1)$. What values does y run through for $y = a + (b - a)x$, where $a, b \in \mathbb{R}$?

3. **(10 points)** Are the following functions even or odd? In your explanation use the definition.

   (a) $f(x) = 3x - x^3$;
   (b) $f(x) = \sqrt[3]{(1-x)^2} + \sqrt[3]{(1+x)^2}$;

4. **(10 points)** What is the inverse of

   $$y = \frac{ax + b}{cx + d} \qquad (ad - bc \neq 0)?$$

   When is it equal to the original function?

5. **(30 points)** Determine the domains and ranges of the following functions.

   (a) $f(x) = \sqrt{7 - x^2} + 1$;
   (b) $f(x) = \frac{x-5}{x^2-3x-10}$;
   (c) $f(x) = \frac{1}{|x|}$.

6. **(30 points)** Find the limits. (Hint: try to simplify as much as possible before applying the limit!)

   (a) $\lim_{x \to 0} \frac{3(x-1)+3}{x}$;
   (b) $\lim_{x \to 2} \frac{x-2}{x^2+x-6}$;
   (c) $\lim_{x \to 1} \frac{x^2-4x+3}{x^2+x-2}$.