Calculus and Probability Theory
Assignment 1

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After completing these exercises successfully you should be confident with the following topics:

- The domain and range of a function
- The limit of a function

1. (**10 points**) Let $f(x) = x - x^3$. Determine the values x for which

    1. $f(x) = 0$
       Solution:

       $\{-1, 0, 1\}$
    2. $f(x) > 0$
       Solution:

       $(0, 1)$ and $(-\infty, -1)$

2. (**10 points**) Let's assume that x runs through the interval $(0, 1)$. What values does y run through for $y = a + (b - a)x$, where $a, b \in \mathbb{R}$

    Solution:

    The most useful form of straight-line equations is the "slope-intercept" form

    $$y = mx + b$$

    where m is the slope and b gives the y-intercept.

    In the given equation

    $$y = (b - a)x + a$$

$(b-a)$ is the slope and a is the y-intercept. Because we are dealing with variables here, I can't give definitive values for which y would run through. Nevertheless, the y-intercept would be a and the slope would be $(b-a)$.

3. (**10 points**) Are the following function even or odd? In your explanation use the definition.

Definition: Parity of function
A function $f : (-a, a) \to \mathbb{R}$ is **even** if $f(-x) = f(x)$, for all $x \in (-a, a)$, and **odd** if $f(-x) = -f(x)$, for all $x \in (-a, a)$.

(a) $f(x) = 3x - x^3$
Solution:

Let's check if the function is odd by checking if $f(-x) = -f(x)$

$$\begin{aligned} f(x) &= 3x - x^3 \\ f(-x) &= 3(-x) - (-x)^3 \\ f(-x) &= -3x + x^3 \\ &= -(3x - x^3) \\ &= -f(x) \end{aligned}$$

Therefore, the function is odd.

(b) $f(x) = \sqrt[3]{(1-x)^2} + \sqrt[3]{(1+x)^2}$
Solution:

Let's check if the function is even by checking if $f(-x) = f(x)$

$$\begin{aligned} f(x) &= \sqrt[3]{(1-x)^2} + \sqrt[3]{(1+x)^2} \\ f(-x) &= \sqrt[3]{(1-(-x))^2} + \sqrt[3]{(1+(-x))^2} \\ f(-x) &= \sqrt[3]{(1+x)^2} + \sqrt[3]{(1-x)^2} \\ f(-x) &= f(x) \end{aligned}$$

Therefore, the function is even.

4. (**10 points**) What is the inverse of

$$y = \frac{ax+b}{cx+d} \ \ (ad - bc \neq 0)?$$

Solution:

- Switch the x's for y's

$$\frac{ay-b}{cy+d} = x$$

- Multiply both sides by $cy + d$

$$\frac{ay-b}{cy+d} \times cy + d = x \times cy + d$$
$$ay - b = x(cy + d)$$

- Expand the x through the paranthesis

$$ay - b = cxy + dx$$

- Move y's to one side

$$ay - cxy = b + dx$$

- Factor out the x

$$y(a - cx) = b + dx$$

- Divide both sides by $a - cx$

$$y = \frac{b + dx}{a - cx}$$

The inverse is

$$y = \frac{b + dx}{a - cx}$$

5. (**30 points**) Determine the domains and ranges of the following functions.

   (a) $f(x) = \sqrt{7 - x^2} + 1$
   Solution:

   - $D(f) = \{x \in \mathbb{R} | -\sqrt{7} \leq x \leq \sqrt{7}\}$ or in Interval-Notation: $[-\sqrt{7}, \sqrt{7}]$

   - $R(f) = \{f \in \mathbb{R} | 1 \leq f \leq 1 + \sqrt{7}\}$ or in Interval-Notation: $[1, 1 + \sqrt{7}]$

   (b) $f(x) = \frac{x-5}{x^2-3x-10}$
   Solution:

   Trying to simplify the term:

$$\begin{aligned} f(x) &= \frac{x-5}{x^2 - 3x - 10} \\ &= \frac{x-5}{(x-5)(x+2)} \\ &= \frac{1}{x+2} \end{aligned}$$

- $D(f) = \{x \in \mathbb{R} | x \neq -2\}$ or in Interval-Notation: $(-\infty, -2) \cup (-2, \infty)$

- $R(f) = \{f \in \mathbb{R} | f \neq 0\}$ or in Interval-Notation: $(-\infty, 0) \cup (0, \infty)$

(c) $f(x) = \frac{1}{|x|}$
Solution:

- $D(f) = \{x \in \mathbb{R} | x \neq 0\}$ or in Interval-Notation: $(-\infty, 0) \cup (0, \infty)$

- $R(f) = \{f \in \mathbb{R} | f > 0\}$ or in Interval-Notation: $(0, \infty)$

6. (**30 points**) Find the limits. (Hint: try to simplify as much as possible before applying the limit!)

(a) $\lim_{x \to 0} \frac{3(x-1)+3}{x}$

Solution:

$$\begin{aligned}\lim_{x \to 0} \frac{3(x-1)+3}{x} &= \lim_{x \to 0} \frac{3x-3+3}{x} \\ &= \lim_{x \to 0} \frac{3x}{x} \\ &= 3\end{aligned}$$

(b) $\lim_{x \to 2} \frac{x-2}{x^2+x-6}$

Solution:

$$\begin{aligned}\lim_{x \to 2} \frac{x-2}{x^2+x-6} &= \lim_{x \to 2} \frac{x-2}{(x-2)(x+3)} \\ &= \lim_{x \to 2} \frac{1}{x+3} \\ &= \frac{1}{5}\end{aligned}$$

(c) $\lim_{x \to 1} \frac{x^2-4x+3}{x^2+x-2}$

Solution:

$$
\begin{aligned}
\lim_{x \to 1} \frac{x^2 - 4x + 3}{x^2 + x - 2} &= \lim_{x \to 1} \frac{(x-1)(x-3)}{(x-1)(x+2)} \\
&= \lim_{x \to 1} \frac{x-3}{x+2} \\
&= -\frac{2}{3}
\end{aligned}
$$