# Calculus and Probability Theory
# Assignment 1

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**After completing these exercises successfully you should be confident with the following topics:**

- Even and odd functions
- The domain and range of a function
- The limit of a function

1. (**10 points**) Let $f(x) = x - x^3$. Determine the values $x$ for which

   1. $f(x) = 0$
      **Solution:**

      $$\begin{aligned} x - x^3 &= 0 \\ x &= x^3 \end{aligned}$$

      Therefore,

      $$\begin{aligned} x_1 &= -1 \\ x_2 &= 0 \\ x_3 &= 1 \end{aligned}$$

   2. $f(x) > 0$
      **Solution:**

      $$\begin{aligned} x - x^3 &> 0 \\ x &> x^3 \end{aligned}$$

      Therefore,
      $0 < x < 1$ and $x < -1$ or stated in another way: $(0, 1)$ and $(-\infty, -1)$

2. (**10 points**) Let's assume that $x$ runs through the interval $(0, 1)$. What values does $y$ run through for $y = a + (b - a)x$, where $a, b \in \mathbb{R}$

   **Solution:**

   The most convenient form of straight-line equations is the "slope-intercept" form

   $$y = mx + b$$

   where $m$ is the slope and $b$ gives the y-intercept.

   In the given equation

   $$y = (b - a)x + a$$

   $(b - a)$ is the slope and $a$ is the y-intercept. Because we are dealing with variables here, I can't give definitive values for which y would run through. Nevertheless, the y-intercept would be $a$ and the slope would be $(b - a)$.

3. (**10 points**) Are the following functions even or odd? In your explanation use the definition.

   > *Definition: Parity of function*
   >
   > A function $f : (-a, a) \to \mathbb{R}$ is **even** if $f(-x) = f(x)$, for all $x \in (-a, a)$, and **odd** if $f(-x) = -f(x)$, for all $x \in (-a, a)$.

   (a) $f(x) = 3x - x^3$

   **Solution:**

   Let's check if the function is odd by checking if $f(-x) = -f(x)$

   $$\begin{aligned} f(x) &= 3x - x^3 \\ f(-x) &= 3(-x) - (-x)^3 \\ f(-x) &= -3x + x^3 \\ &= -(3x - x^3) \\ &= -f(x) \end{aligned}$$

   Therefore, the function is odd.

   (b) $f(x) = \sqrt[3]{(1-x)^2} + \sqrt[3]{(1+x)^2}$

   **Solution:**

Let's check if the function is even by checking if $f(-x) = f(x)$

$$\begin{aligned}
f(x) &= \sqrt[3]{(1-x)^2} + \sqrt[3]{(1+x)^2} \\
f(-x) &= \sqrt[3]{(1-(-x))^2} + \sqrt[3]{(1+(-x))^2} \\
f(-x) &= \sqrt[3]{(1+x)^2} + \sqrt[3]{(1-x)^2} \\
f(-x) &= f(x)
\end{aligned}$$

Therefore, the function is even.

4. (**10 points**) What is the inverse of

$$y = \frac{ax+b}{cx+d} \quad (ad - bc \neq 0)?$$

**Solution:**

- Switch the x's for y's

$$\frac{ay-b}{cy+d} = x$$

- Multiply both sides by $cy + d$

$$\begin{aligned}
\frac{ay-b}{cy+d} \cdot cy + d &= x \cdot cy + d \\
ay - b &= x(cy + d)
\end{aligned}$$

- Expand the x through the paranthesis

$$ay - b = cxy + dx$$

- Move y's to one side

$$ay - cxy = b + dx$$

- Factor out the x

$$y(a - cx) = b + dx$$

- Divide both sides by $a - cx$ and you get the inverse

$$y = \frac{b+dx}{a-cx}$$

5. (**30 points**) Determine the domains and ranges of the following functions.

   (a) $f(x) = \sqrt{7 - x^2} + 1$
   **Solution:**

- $D(f) = \{x \in \mathbb{R} \mid -\sqrt{7} \leq x \leq \sqrt{7}\}$
- $R(f) = \{f(x) \in \mathbb{R} \mid 1 \leq f(x) \leq 1 + \sqrt{7}\}$

(b) $f(x) = \frac{x-5}{x^2-3x-10}$
**Solution:**

Trying to simplify the term:

$$\begin{aligned} f(x) &= \frac{x-5}{x^2-3x-10} \\ &= \frac{x-5}{(x-5)(x+2)} \\ &= \frac{1}{x+2} \end{aligned}$$

- $D(f) = \{x \in \mathbb{R} \mid x \neq -2 \wedge x \neq 5\}$
- $R(f) = \{f(x) \in \mathbb{R} \mid f(x) \neq 0 \wedge f(x) \neq \frac{1}{7}\}$

(c) $f(x) = \frac{1}{|x|}$
**Solution:**

- $D(f) = \{x \in \mathbb{R} \mid x \neq 0\}$
- $R(f) = \{f(x) \in \mathbb{R} \mid f(x) > 0\}$

6. (**30 points**) Find the limits. (Hint: try to simplify as much as possible before applying the limit!)

(a) $\lim_{x\to 0} \frac{3(x-1)+3}{x}$
**Solution:**

$$\begin{aligned} \lim_{x\to 0} \frac{3(x-1)+3}{x} &= \lim_{x\to 0} \frac{3x-3+3}{x} \\ &= \lim_{x\to 0} \frac{3x}{x} \\ &= 3 \end{aligned}$$

(b) $\lim_{x\to 2} \frac{x-2}{x^2+x-6}$
**Solution:**

$$
\begin{aligned}
\lim_{x\to 2} \frac{x-2}{x^2+x-6} &= \lim_{x\to 2} \frac{x-2}{(x-2)(x+3)} \\
&= \lim_{x\to 2} \frac{1}{x+3} \\
&= \frac{1}{5}
\end{aligned}
$$

(c) $\lim_{x\to 1} \frac{x^2-4x+3}{x^2+x-2}$
**Solution:**

$$
\begin{aligned}
\lim_{x\to 1} \frac{x^2-4x+3}{x^2+x-2} &= \lim_{x\to 1} \frac{(x-1)(x-3)}{(x-1)(x+2)} \\
&= \lim_{x\to 1} \frac{x-3}{x+2} \\
&= -\frac{2}{3}
\end{aligned}
$$