Calculus and Probability

Assignment 6

Note:

- You can hand in your solutions as a single PDF via the assignment module in Blackboard. Note that the document should be in English and typeset with LaTeX, Word or a similar program. It should not be a scan or picture of your handwritten notes.
- Make sure that your name, student number and group number are on top of the first page!
- **Note that your submission should be an individual submission because it can influence your final grade for this course. If we detect that your work is not completely your own work, we will ask the exam committee to investigate whether it is plagiarism or not!**

Exercises to be presented during the exercise hours

Exercise 1

Compute the following indefinite integrals. You can use *substitution* or *integration by parts*.

In each problem *verify* your result, and don't forget about the constant term. You may need some of the following, well-known trigonometric identities:

$$\sin(2x) = 2\sin(x)\cos(x), \quad \cos(2x) = \cos^2(x) - \sin^2(x), \quad \sin^2(x) + \cos^2(x) = 1.$$

Also, it is highly recommended to consult with the lecture slides and solve the problems there before you start with these ones.

a) $\int \cos^2(x)\,dx$

$\int \cos^2(x)\,dx = \int \frac{1}{2} + \frac{1}{2}\cos(2x)\,dx = \frac{1}{2}x + \frac{1}{4}\sin(2x) + C$. (Another format: $\frac{1}{2}(\sin x\cos x + x) + C$.)

b) $\int \frac{1}{\sqrt{1-4x^2}}\,dx$

$\int \frac{1}{\sqrt{1-(2x)^2}}dx = \frac{1}{2}\arcsin(2x) + C.$

c) $\int e^{3x}\sin(x)\,dx$

We compute using partial integration

$$\begin{aligned}
\int e^{3x}\sin(x)\,dx &= -\int e^{3x}\frac{d\cos(x)}{dx}\,dx \\
&= \left(-e^{3x}\cos(x)\right) - \left(-\int \frac{de^{3x}}{dx}\cos(x)\,dx\right) \\
&= -e^{3x}\cos(x) + 3\int e^{3x}\cos(x)\,dx \\
&= -e^{3x}\cos(x) + 3\int e^{3x}\frac{d\sin(x)}{dx}\,dx \\
&= -e^{3x}\cos(x) + 3e^{3x}\sin(x) - 9\int e^{3x}\sin(x)\,dx.
\end{aligned}$$

Hence we see that

$$\int e^{3x}\sin(x)\,dx = \frac{1}{10}e^{3x}\left(3\sin(x) - \cos(x)\right) + C.$$

Exercise 2
Compute the definite integral $\int_{-1}^{1}\sqrt{1-x^2}\,dx$

a) using calculus, and

We compute:

$$
\begin{aligned}
\int_{-1}^{1} \sqrt{1-x^2}\,dx &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1-\sin^2 u}\ \frac{d\sin u}{\mathrm{d}u}\ \ \mathrm{d}u && \text{substitute } x \text{ by } \sin u \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos u \cdot \cos u\ \ \mathrm{d}u && \int \cos^2(u) \text{ from 1(a)} \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2} + \frac{1}{2}\cos(2u)\ \ \mathrm{d}u \\
&= \left[\frac{1}{2}u + \frac{1}{4}\sin(2u)\right]_{u=-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
&= \left(\frac{\pi}{4} + \frac{1}{4}\sin(\pi)\right) - \left(-\frac{\pi}{4} + \frac{1}{4}\sin(-\pi)\right) \\
&= \frac{\pi}{2} && \text{since } \sin(\pi) = \sin(-\pi) = 0
\end{aligned}
$$

Another form of the primitive (with which some students may work): $\int \cos^2 u\ \ \mathrm{d}u = \frac{1}{2}(\sin u \cos u + u)$.

b) using a geometric argument.

Since the area under the function $\sqrt{1-x^2}$ on $[-1,1]$ is half of the unit disk, the answer is $\frac{\pi}{2}$.

[Hint: (a) instead of substituting a function of x by u, now substitute $x = \sin(u)$. Hint: (b) what is the shape of $\sqrt{1-x^2}$?]

Exercise 3
Compute the following improper integrals.

a) $\int_0^\infty re^{-r^2}\,dr$;

$\int_0^\infty re^{-r^2}\,dr \;=\; \lim_{t\to\infty}\left[-\frac{1}{2}e^{-r^2}\right]_{r=0}^{t} \;=\; 0-(-\frac{1}{2}) \;=\; \frac{1}{2}.$

b) $\int_0^{2\pi}\left(\int_0^\infty re^{-r^2}\,dr\right)dt$;

$\int_0^{2\pi}\left(\int_0^\infty re^{-r^2}\,dr\right)dt \;=\; \int_0^{2\pi} dt \,\cdot\, \int_0^\infty re^{-r^2}\,dr \;=\; 2\pi\cdot\frac{1}{2} \;=\; \pi.$

c) Prove that $\int_{-\infty}^{\infty} e^{-z^2}\,dz = \sqrt{\pi}$.

You may use that $\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\,dx\right)dy = \int_0^{2\pi}\left(\int_0^{\infty} re^{-r^2}\,dr\right)dt$.

We have:

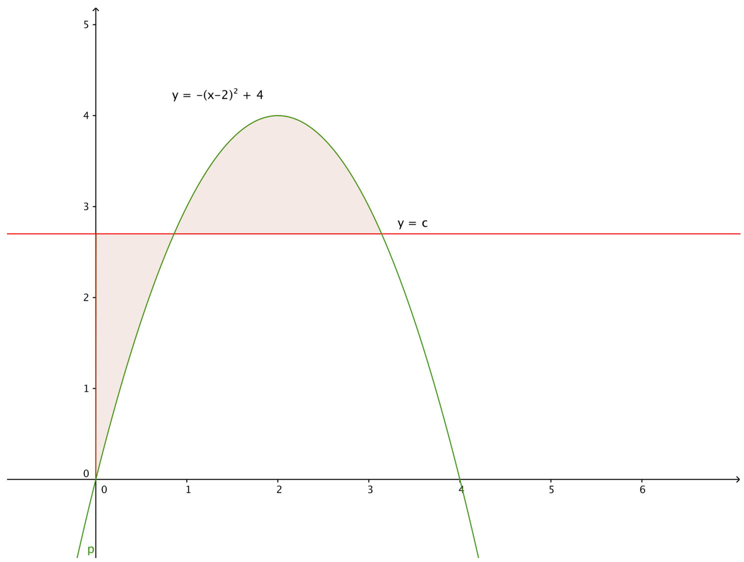
$$\begin{aligned}\int_{-\infty}^{\infty} e^{-z^2}\,dz &= \left(\int_{-\infty}^{\infty} e^{-x^2}\,dx \cdot \int_{-\infty}^{\infty} e^{-y^2}\,dy\right)^{\frac{1}{2}} \\ &= \left(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\,dxdy\right)^{\frac{1}{2}} \\ &= \left(\int_0^{2\pi}\int_0^{\infty} re^{-r^2}\,drdt\right)^{\frac{1}{2}} = \sqrt{\pi}\end{aligned}$$

d) $\int_0^{\infty} e^{-z^2}\,dz$.

We have $\int_{-\infty}^{\infty} e^{-z^2}\,dz = \sqrt{\pi}$. Solution 1: Since e^{-z^2} is even (*i.e.* $e^{-z^2} = e^{-(-z)^2}$), $\int_0^{\infty} e^{-z^2}\,dz = \frac{1}{2}\sqrt{\pi}$. Solution 2: $\int_{-\infty}^{\infty} e^{-z^2}\,dz = \int_0^{\infty} e^{-z^2}\,dz + \int_{-\infty}^{0} e^{-z^2}\,dz$. By substituting z by $-w$, we see that $\int_{-\infty}^{0} e^{-z^2}\,dz = \int_{\infty}^{0} e^{-(-w)^2}\frac{d(-w)}{dw}\,dw = -\int_{\infty}^{0} e^{-w^2}\,dw = \int_0^{\infty} e^{-w^2}\,dw$. Thus $\int_{-\infty}^{\infty} e^{-z^2}\,dz = 2\int_0^{\infty} e^{-z^2}\,dz$. Hence $\int_0^{\infty} e^{-z^2}\,dz = \frac{1}{2}\sqrt{\pi}$.

Exercise 4

The figure shows a horizontal line $y = c$ having an intersection with the curve $y = -(x-2)^2 + 4$. Find the number c such that the areas of the shaded regions are equal.



$f(x) = -(x-2)^2 + 4 = -x^2 + 4x$ and $g(x) = c$. Let the x-coordinates of the two intersection points are a, b where $0 < a < b < 4$. Then the two areas are $\int_0^a (g - f) = \int_a^b (f - g)$. Moreover, $b = 4 - a$ and $f(b) = c$, that is, $-b^2 + 4b = c$.

$\int_0^a (g-f) = \int_0^a (c - (-x^2+4x))dx = \int_0^a (c + x^2 - 4x)dx = cx + \frac{1}{3}x^3 - 2x^2]_0^a = ca + \frac{1}{3}a^3 - 2a^2$

$\int_a^b (f-g) = \int_a^b ((-x^2+4x) - c)dx = -\frac{1}{3}x^3 + 2x^2 - cx]_a^b = -\frac{1}{3}b^3 + 2b^2 - cb + \frac{1}{3}a^3 - 2a^2 + ca$.

Therefore, we have three unknowns (a, b, c) and three equations:

$$-b^2 + 4b = c \tag{1}$$

$$b = 4 - a$$

$$-\frac{1}{3}b^3 + 2b^2 - cb + \frac{1}{3}a^3 - 2a^2 + ca = ca + \frac{1}{3}a^3 - 2a^2 \tag{2}$$

Equation (2) becomes $-\frac{1}{3}b^3 + 2b^2 - cb = 0$. Since $b \neq 0$ (as by the initial assumption $b > a > 0$) $-\frac{1}{3}b^2 + 2b - c = 0$ or $b^2 - 6b + 3c = 0$. Applying Equation (1), we get that $-c - 2b + 3c = 0$ or $b = c$. Using Equation (1) again, we get that $-c^2 + 3c = 0$, that is, $(c \neq 0)$ $c = 3$.

Indeed, if $c = 3$, $a = 1, b = 3$ both shaded areas are $1 + 1/3$.

Exercise 5

If a value is requested, give the exact value, not some decimal approximation. Simplify your answer as much as possible.

a) $\int_{-2}^{2} (x^2 + 2)e^x \, \mathrm{d}x$

Using partial integration twice we obtain

$$\begin{aligned}\int_{-2}^{2}(x^2+2)e^x \,\mathrm{d}x &= (x^2+2)e^x|_{-2}^{2} - \int_{-2}^{2} 2xe^x \,\mathrm{d}x \\ &= (x^2+2)e^x|_{-2}^{2} - \left(2xe^x|_{-2}^{2} - \int 2e^x \,\mathrm{d}x\right) \\ &= (x^2+2)e^x|_{-2}^{2} - \left(2xe^x|_{-2}^{2} - 2e^x|_{-2}^{2}\right) \\ &= (x^2-2x+4)e^x|_{-2}^{2} \\ &= \underline{\underline{4e^2-12e^{-2}}}\end{aligned}$$

b) $\int_1^7 \frac{2x-4}{\sqrt[3]{3x+6}} \,\mathrm{d}x$

We use substitution $u = 3x+6$ giving $2x-4 = \frac{2}{3}u - 8$ and $\frac{1}{3}\,\mathrm{d}u = \,\mathrm{d}x$

$$\begin{aligned}\int_1^7 \frac{2x-4}{\sqrt[3]{3x+6}} \,\mathrm{d}x &= \frac{1}{3}\int_9^{27} \frac{\frac{2}{3}u-8}{\sqrt[3]{u}} \,\mathrm{d}u \\ &= \frac{1}{3}\int_9^{27} \left(\frac{2}{3}u-8\right) u^{-\frac{1}{3}} \,\mathrm{d}u \\ &= \frac{1}{3}\int_9^{27} \frac{2}{3}u^{\frac{2}{3}} - 8u^{-\frac{1}{3}} \,\mathrm{d}u \\ &= \frac{1}{3}\left(\frac{2}{5}u^{\frac{5}{3}} - 12u^{\frac{2}{3}}\right)|_9^{27} \\ &= \underline{\underline{\frac{14}{5}9^{\frac{2}{3}} - \frac{18}{5}}}\end{aligned}$$

c) $\int_e^8 \frac{1}{x\ln\sqrt{x}} \,\mathrm{d}x$

Let $u = \ln x,\ \mathrm{d}u = \frac{1}{x}\,\mathrm{d}x$. Then

$$\begin{aligned}\int_e^8 \frac{1}{x\ln\sqrt{x}} \,\mathrm{d}x &= 2\int_e^8 \frac{1}{x\ln x} \,\mathrm{d}x \\ &= 2\int_1^{\ln 8} \frac{1}{u} \,\mathrm{d}u \\ &= 2\ln|u||_1^{\ln 8} \\ &= \underline{\underline{2\ln(\ln 8)}}\end{aligned}$$

d) $\int_0^{\ln\sqrt{3}} \cosh^2 x \; \mathrm{d}x$ [Hint: $\cosh x = \frac{1}{2}\left(e^x + e^{-x}\right)$]

$$\begin{aligned}
\int_0^{\ln\sqrt{3}} \cosh^2 x \; \mathrm{d}x &= \int_0^{\ln\sqrt{3}} \left(\frac{1}{2}\left(e^x + e^{-x}\right)\right)^2 \; \mathrm{d}x \\
&= \frac{1}{4}\int_0^{\ln\sqrt{3}} \left(e^{2x} + 2 + e^{-2x}\right) \; \mathrm{d}x \\
&= \frac{1}{4}\left(\frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x}\right)\Big|_0^{\ln\sqrt{3}} \\
&= \frac{1}{4}\left(\frac{1}{2}\cdot 3 + 2\ln(\sqrt{3}) - \frac{1}{2}\cdot\frac{1}{3}\right) - \frac{1}{4}\left(\frac{1}{2} + 0 - \frac{1}{2}\right) \\
&= \underline{\underline{\frac{1}{3} + \frac{1}{4}\ln(3)}}
\end{aligned}$$

e) $\int_1^2 \ln^2 x \; \mathrm{d}x$

Partial integration can be used to show that $\int \ln x \; \mathrm{d}x = x\ln(x) - x$. Partial integration on the original formula gives

$$\begin{aligned}
\int_1^2 \ln^2 x \; \mathrm{d}x &= \int_1^2 \ln x \cdot \ln x \; \mathrm{d}x \\
&= \ln x (x\ln(x) - x)|_1^2 - \int \frac{1}{x}(x\ln(x) - x) \; \mathrm{d}x \\
&= \ln x (x\ln(x) - x)|_1^2 - \left(\int_1^2 \ln(x) \; \mathrm{d}x - \int_1^2 1 \; \mathrm{d}x\right) \\
&= \ln x (x\ln(x) - x) - (x\ln(x) - x) + x|_1^2 \\
&= x(\ln(x))^2 - 2x\ln(x) + 2x|_1^2 \\
&= \underline{\underline{2(\ln(2))^2 - 4\ln 2 + 2}}
\end{aligned}$$

Exercises to be handed in

You are expected to explain your answers, even if this is not explicitly stated in the exercises themselves.

Exercise 6
Compute the following indefinite integrals. You can use *substitution* or *integration by parts.*

In each problem *verify* your result, and don't forget about the constant term. You may need some of the following, well-known trigonometric identities:

$$\sin(2x) = 2\sin(x)\cos(x), \quad \cos(2x) = \cos^2(x) - \sin^2(x), \quad \sin^2(x) + \cos^2(x) = 1.$$

a) $\int \sin(x)\cos(x)\,dx$ 1 pt

$\int \sin(x)\cos(x) = \frac{1}{2}\sin^2(x) + C.$

b) $\int \ln(ax)\,dx$ where $a > 0$ 1 pt

Substituting x by $\frac{y}{a}$ we see that $\int \ln(ax)\,dx = \int \ln(y)\frac{d(a^{-1}y)}{dy}\,dy = \int a^{-1}\ln(y)\,dy = a^{-1}y(\ln(y) - 1) + C = a^{-1}ax(\ln(ax) - 1) + C = x(\ln(ax) - 1) + C.$

Exercise 7
Compute the length of the curve $f(x) = \sqrt{1 - x^2}$ where $x \in [-1, 1]$

a) using calculus, and 1 pt

First note that $f'(x) = -\frac{x}{\sqrt{1-x^2}}$. Thus the length of the curve is

$$\begin{aligned}
\int_{-1}^{1} \sqrt{1 + (f'(x))^2}\,dx &= \int_{-1}^{1} \sqrt{1 + \frac{x^2}{1 - x^2}}\,dx \\
&= \int_{-1}^{1} \sqrt{\frac{1 - x^2 + x^2}{1 - x^2}}\,dx \\
&= \int_{-1}^{1} \frac{1}{\sqrt{1 - x^2}}\,dx \\
&= [\arcsin(x)]_{-1}^{1} \\
&= \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi.
\end{aligned}$$

b) using a geometric argument. 1 pt

The length of the curve is half of the unit circle, the answer is π.

[Hint: (b) what is the shape of $\sqrt{1-x^2}$?]

Exercise 8
Compute the following improper integrals.

a) $\int_0^\infty e^{-x}\,dx$; 1 pt

$\int_0^\infty e^{-x}\,dx = [-e^{-x}]_0^\infty = 0-(-1) = 1.$

b) $\int_0^\infty xe^{-x}\,dx$ using integration by parts; 1 pt

$\int_0^\infty e^{-x}\,dx = \int_0^\infty \frac{dx}{dx}e^{-x}\,dx = [xe^{-x}]_0^\infty - \int_0^\infty x\frac{de^{-x}}{dx}\,dx = (0-0) - (-\int_0^\infty xe^{-x}\,dx) = \int_0^\infty xe^{-x}\,dx$. Thus $\int_0^\infty xe^{-x}\,dx = 1$.

c) $\int_0^\infty x^n e^{-x}\,dx$ for all $n \in \{0,1,\dots\}$; 1 pt

Writing $\gamma_n = \int_0^\infty x^n e^x\,dx$, we have, for all $n \in \mathbb{N}$,

$$\begin{aligned}
\gamma_n &= \int_0^\infty x^n e^{-x}\,dx \\
&= \frac{1}{n+1}\int_0^\infty \frac{dx^{n+1}}{dx}e^{-x}\,dx \\
&= \frac{1}{n+1}\left[x^{n+1}e^{-x}\right]_0^\infty - \frac{1}{n+1}\int_0^\infty x^{n+1}\frac{de^{-x}}{dx}\,dx \\
&= \frac{0-0}{n+1} + \frac{1}{n+1}\int_0^\infty x^{n+1}e^{-x}\,dx \\
&= \frac{\gamma_{n+1}}{n+1}.
\end{aligned}$$

Thus $\gamma_0 = 1$ by (a), and $\gamma_{n+1} = (n+1)\gamma_n$ for all $n \in \mathbb{N}$. Hence $\gamma_n = n!$.

[Hint: (c) Try first for $n = 0,1,2,3$]

Exercise 9
Compute the area of the region bounded by $y = (x-1)^3$ and $y = (x-1)^2$. 1 pt

The intersections are $1, 2$ because $(x-1)^3 = (x-1)^2$ iff $(x-1)^3 - (x-1)^2 = 0$ iff $(x-1)^2(x-1-1) = (x-1)^2(x-2) = 0$. (In fact, at $x = 1$ the two curves tangent to each other.)

If $x \in [1, 2]$ $x - 1 \in [0, 1]$ and $(x-1)^3 \le (x-1)^2$. Therefore, the area can be computed as

$$\int_1^2 ((x-1)^2 - (x-1)^3)\, dx = -\frac{x^4}{4} + \frac{4x^3}{3} - \frac{5x^2}{2} + 2x]_1^2 = \frac{1}{12}.$$

Exercise 10

Make two from the integrals below, or get upto 3 bonus poins for doing more. If a value is requested, give the exact value, not some decimal approximation. Simplify your answer as much as possible. $\boxed{2 \text{ pt}}$

a) $\int \frac{1}{1+e^{2x}}\, \mathrm{d}x$

We add zero followed by the substitution $u = 1 + e^{2x},\ \ \mathrm{d}u = 2e^{2x}\, \mathrm{d}x$ or equivalently $\frac{1}{2}\, \mathrm{d}u = e^{2x}\, \mathrm{d}x$.

$$\begin{aligned}
&\int \frac{1}{1+e^{2x}}\, \mathrm{d}x \\
&\qquad = \int \frac{1 + e^{2x} - e^{2x}}{1+e^{2x}}\, \mathrm{d}x \\
&\qquad = \int 1 - \frac{e^{2x}}{1+e^{2x}}\, \mathrm{d}x \\
&\qquad = x - \int \frac{1}{u}\frac{1}{2}\, \mathrm{d}u \\
&\qquad = x - \frac{1}{2}\ln|u| \\
&\qquad = \underline{\underline{x - \frac{1}{2}\ln|1+e^{2x}| + C}}
\end{aligned}$$

b) $\int \sqrt{4 - \sqrt{x}}\, \mathrm{d}x$

We use substitution twice. First with $u = \sqrt{x},\ \ \mathrm{d}u = \frac{1}{2\sqrt{x}}\, \mathrm{d}x = \frac{1}{2u}\, \mathrm{d}x$ or equivalently $2u\, \mathrm{d}u =\ \mathrm{d}x$. Second with $v = 4 - u,\ \ \mathrm{d}v = -\, \mathrm{d}u$ or

$-\,\mathrm{d}v = \,\mathrm{d}u$, and $2u = 8 - 2v$.

$$\begin{aligned}
\int \sqrt{4-\sqrt{x}}\,\mathrm{d}x &= \int \sqrt{4-u}2u\,\mathrm{d}u \\
&= \int \sqrt{v}(8-2v)(-\,\mathrm{d}v) \\
&= \int \sqrt{v}(2v-8)\,\mathrm{d}v \\
&= \int \sqrt{v}(2v-8)\,\mathrm{d}v \\
&= \int 2v^{\frac{3}{2}} - 8v^{\frac{1}{2}}\,\mathrm{d}v \\
&= \frac{4}{5}v^{\frac{5}{2}} - \frac{16}{3}v^{\frac{3}{2}} \\
&= \underline{\underline{\frac{4}{5}(4-\sqrt{x})^{\frac{5}{2}} - \frac{16}{3}(4-\sqrt{x})^{\frac{3}{2}} + C}}
\end{aligned}$$

c) $\int_0^{\pi^2} \sin(\sqrt{x})\,\mathrm{d}x$

We use substitution with $u = \sqrt{x}$, $\mathrm{d}u = \frac{1}{2\sqrt{x}}\,\mathrm{d}x = \frac{1}{2u}\,\mathrm{d}x$ or $2u\,\mathrm{d}u = \,\mathrm{d}x$, followed by partial integration.

$$\begin{aligned}
\int_0^{\pi^2} \sin(\sqrt{x})\,\mathrm{d}x &= \int_0^{\pi} 2u\sin u\,\mathrm{d}u \\
&= 2u(-\cos(u))|_0^{\pi} - \int_0^{\pi} 2(-\cos u)\,\mathrm{d}u \\
&= -2u\cos u + 2\sin u|_0^{\pi} \\
&= \underline{\underline{2\pi}}
\end{aligned}$$

d) $\int_0^1 \frac{1}{\sqrt{x}+\sqrt[3]{x}}\,\mathrm{d}x$ [Hint: substitute the common factor in $\sqrt{x}$ and $\sqrt[3]{x}$]

We use substitution twice. Let $u = x^{\frac{1}{6}}$, then $\mathrm{d}u = \frac{1}{6}x^{-\frac{5}{6}}\,\mathrm{d}x = \frac{1}{6}\frac{1}{u^5}\,\mathrm{d}x$ or equivalently $6u^5\,\mathrm{d}u = \,\mathrm{d}x$. In addition let $v = u+1$, then $\mathrm{d}v = \,\mathrm{d}u$ and

$u = v - 1$.

$$\begin{aligned}
\int_0^1 \frac{1}{\sqrt{x}+\sqrt[3]{x}}\,\mathrm{d}x &= \int_0^1 \frac{1}{(x^{\frac{1}{6}})^3+(x^{\frac{1}{6}})^2}\,\mathrm{d}x \\
&= \int_0^1 \frac{1}{u^3+u^2}6u^5\,\mathrm{d}u \\
&= 6\int_0^1 \frac{u^3}{u+1}\,\mathrm{d}u \\
&= 6\int_1^2 \frac{(v-1)^3}{v}\,\mathrm{d}v \\
&= 6\int_1^2 \frac{v^3-3v^2+3v-1}{v}\,\mathrm{d}v \\
&= 6\int_1^2 v^2-3v+3-\frac{1}{v}\,\mathrm{d}v \\
&= 2v^3-9v^2+18v-6\ln(v)|_1^2 \\
&= \underline{\underline{5-6\ln(2)}}
\end{aligned}$$

The primitive of the original function can be found by translating the next to last function in terms of x. After some algebra one would obtain

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\ln|6\sqrt[6]{x}+1|+C$$

e) $\int \frac{1}{1+\sin x}\,\mathrm{d}x$ [Hint: you can use a Weierstrass substitution]

Method 1: A Weierstrass substitution with $u = \tan\left(\frac{x}{2}\right)$, $\mathrm{d}x = \frac{2\,\mathrm{d}u}{1+u^2}$, $\sin x = \frac{2u}{1+u^2}$. The indefinite integral is

$$\begin{aligned}
\int \frac{1}{1+\sin x}\,\mathrm{d}x &= \int \frac{1}{1+\frac{2u}{1+u^2}}\cdot\frac{2\,\mathrm{d}u}{1+u^2} \\
&= \int \frac{2}{(1+u^2)+2u}\,\mathrm{d}u \\
&= \int \frac{2}{(u+1)^2}\,\mathrm{d}u \\
&= \frac{-2}{u+1} \\
&= \frac{-2}{\tan\left(\frac{x}{2}\right)+1}+C'
\end{aligned}$$

Method 2: Multiply by one, followed by substitution of $u = \cos x$, and

$\mathrm{d}u = -\sin x \, \mathrm{d}x$. Then

$$\begin{aligned}
\int \frac{1}{1+\sin x} \, \mathrm{d}x &= \int \frac{1}{1+\sin x} \cdot \frac{1-\sin x}{1-\sin x} \, \mathrm{d}x \\
&= \int \frac{1-\sin x}{1-\sin^2 x} \, \mathrm{d}x \\
&= \int \frac{1-\sin x}{\cos^2 x} \, \mathrm{d}x \\
&= \int \frac{1}{\cos^2 x} + \frac{-\sin x}{\cos^2 x} \, \mathrm{d}x \\
&= \tan x + \int \frac{1}{u^2} \, \mathrm{d}u \\
&= \tan x + \frac{-1}{u} \\
&= \tan x - \frac{1}{\cos x} + C
\end{aligned}$$

Although the functions in method 1 and 2 look different there is only a constant difference between these two functions.

Your final grade is the sum of your scores divided by 1.0.