

Calculus en Kansrekening

Assignment 5, September 29, 2015

Handing in your answers:

- submission via Blackboard (<http://blackboard.ru.nl>);
- one single pdf file (make sure that if you scan/photo your handwritten assignment, the result is clearly readable);
- all of your solutions are clearly and convincingly explained;
- make sure to write your name, your student number and **the name of your TA** (Joost or Bram).

Deadline: Wednesday, October 7, 14:30 sharp!

Goals: After completing these exercises successfully you should be:

- familiar with definite and indefinite integrals;
- able to apply the most important integration methods; more specifically, substitution and integration by parts;
- confident about switching between different representations of a function;
- able to compute area of a finite or infinite region;
- able to apply the formula for the arc length of a function over a finite interval.

Marks: You can score a total of 100 points. (Additionally, you can collect +10 bonus points.)

1. **(20 points)** Compute the following indefinite integrals. You can use *substitution* or *integration by parts*. In each problem *verify* your result, and don't forget about the constant term. You may need some of the following, well-known trigonometric identities:

$$\sin(2x) = 2 \sin(x) \cos(x), \quad \cos(2x) = \cos^2(x) - \sin^2(x), \quad \sin^2(x) + \cos^2(x) = 1.$$

Also, it is highly recommended to consult with the lecture slides and solve the problems there before you start with these ones.

- (a) $\int \sin(x) \cos(x) dx$
- (b) $\int \ln(ax) dx$ where $a > 0$
- (c) $\int \cos^2(x) dx$
- (d) $\int \frac{1}{\sqrt{1-4x^2}} dx$
- (e) $\int e^{3x} \sin(x) dx$

2. **(20 points)** Compute the length of the curve $f(x) = \sqrt{1-x^2}$ where $x \in [-1, 1]$

- (a) using calculus, and
- (b) using a geometric argument.

[Hint: (b) what is the shape of $\sqrt{1-x^2}$?]

3. **(20 points)** Compute the definite integral $\int_{-1}^1 \sqrt{1-x^2} dx$

- (a) using calculus, and
- (b) using a geometric argument.

[Hint: (a) instead of substituting a function of x by u , now substitute $x = \sin(u)$.]

4. **(15 points)** Compute the following improper integrals.

(a) $\int_0^\infty r e^{-r^2} dr$;

(b) $\int_0^{2\pi} \left(\int_0^\infty r e^{-r^2} dr \right) dt$;

(c) (bonus, +3 points) Prove that $\int_{-\infty}^\infty e^{-z^2} dz = \sqrt{\pi}$.

You may use the fact that $\int_{-\infty}^\infty \left(\int_{-\infty}^\infty e^{-(x^2+y^2)} dx \right) dy = \int_0^{2\pi} \left(\int_0^\infty r e^{-r^2} dr \right) dt$.

(d) $\int_0^\infty e^{-z^2} dz$.

5. **(15 points)** Compute the following improper integrals.

(a) $\int_0^\infty e^{-x} dx$;

(b) $\int_0^\infty x e^{-x} dx$ using integration by parts;

(c) (bonus, +2 points) $\int_0^\infty x^n e^{-x} dx$ for all $n \in \{0, 1, \dots\}$;

(d) $\int_0^\infty x^{-\frac{1}{2}} e^{-x} dx$.

[Hint: (c) Try first for $n = 0, 1, 2, 3$; (d) substitute $u = \sqrt{x}$ and, at the end, some information from a previous exercise turns out to be useful.]

6. **(10 points)**

(a) Given three lines, $y = x + 2$, $y = -x + 6$ and $y = 2x - 3$, enclosing a triangle. Determine the *coordinates* of the three vertices and the *area* of the triangle.

(b) Compute the area of the region bounded by $y = (x - 1)^3$ and $y = (x - 1)^2$.

7. **(bonus, 5 points)** The figure shows a horizontal line $y = c$ intersecting the curve $y = -(x - 2)^2 + 4$. Find the number c such that the areas of the shaded regions are equal.

