# Calculus and Probability
# Assignment 1

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Group: Tutorial 5

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## Exercise 6

a)

$$\begin{aligned} x - x^3 &= 0 \\ &= x(1 - x^2) = 0 \\ &\rightarrow x_1 = -1, x_2 = 0, x_3 = 1 \end{aligned}$$

Values x for which $f(x) = 0 \rightarrow \{-1, 0, 1\}$

b)

$$\begin{aligned} x - x^3 &> 0 \\ &= x(1 - x^2) > 0 \\ &\rightarrow (0, 1), (-\infty, -1) \end{aligned}$$

Values x for which $f(x) > 0 \rightarrow (0, 1), (-\infty, -1)$

## Exercise 7

There are three cases in total.

1. When $a = b$, then $y = \{a\}$
2. When $b > a$, then $(b - a) \geq 0$. $x \in (0, 1) \rightarrow y = a + (b - a)x > a$ and $y = a + (b - a)x < a + (b - a) = b$. Determine if $y$ takes every value in the interval $(a, b)$: $c \in (a, b)$. $c = a + (b - a)x_c \Leftrightarrow x_c = \frac{c-a}{b-a}$. Therefore $y$ runs through all values in $(a, b)$ because for every element $c$ in $(a, b)$ there exists an element $x_c \in (0, 1)$ such that $c = a + (b - a)x_c$. Y runs through all values in $(a, b)$.
3. When $b < a$, then $(b - a) < 0$. Using $x \in (0, 1)$, there is $y = a + (b - a)x < a$ and $y = a + (b - a)x > a + (b - a) = b \rightarrow$ y takes values in $(b, a)$. Using the same procedure as above, $y$ runs through all values in $(b, a)$.

$y$ runs through all values in $(a, b)$, except when $a = b$, then $y = \{a\}$

## Exercise 8

a) $f(-x) = 3(-x) - (-x)^3 = -3x + x^3 = -(3x - x^3) = -f(x)$. The function is odd. The function is odd.

b) $f(-x) = \sqrt[3]{(1-(-x))^2} + \sqrt[3]{(1+(-x))^2} = \sqrt[3]{(1+x)^2} + \sqrt[3]{(1-x)^2} = f(x)$. The function is even. The function is even.

## Exercise 9

a) $f(x) = \sqrt{7 - x^2} + 1$
Assume that has to be $7 - x^2 \geq 0$. The only possible values lie in $-\sqrt{7} \leq x \leq \sqrt{7}$. So $D(f) = [-\sqrt{7}$

As $0 \leq 7 - x^2 \leq 7$, we have $0 \leq \sqrt{7 - x^2} \leq \sqrt{7}$, so $1 \leq f(x) \leq \sqrt{7} + 1$. So $R(f) = [1, \sqrt{7} + 1]$
$D(f) = [-\sqrt{7}, \sqrt{7}]$, $R(f) = [1, \sqrt{7} + 1]$

b) $f(x) = \frac{1}{|x|}$
The domain only excludes $x = 0$. So $D(f) = \mathbb{R} \setminus \{0\}$. The range is $R(f) = (0, \infty)$ $D(f) = \mathbb{R} \setminus \{0\}$, $R(f) = (0, \infty)$

## Exercise 10

a) $y(cx + d) = ax + b \rightarrow x(cy - a) = -dy + b$ so $x = \frac{-dy+b}{cy-a}$. So the inverse of y is $\frac{-dx+b}{cx-a}$

The inverse of y is $\frac{-dx+b}{cx-a}$

b) It is equal to $y$ when $d = -a$

## Exercise 11

a)

$$
\begin{aligned}
\lim_{x \to 2} \frac{x-2}{x^2 + x - 6} &= \lim_{x \to 2} \frac{x-2}{(x-2)(x+3)} \\
&= \lim_{x \to 2} \frac{1}{(x+3)} \\
&= \frac{1}{5}
\end{aligned}
$$

$\lim_{x \to 2} \frac{x-2}{x^2+x-6} = \frac{1}{5}$

b)

$$
\begin{aligned}
\lim_{x \to 1} \frac{x^2 - 4x + 3}{x^2 + x - 2} &= \lim_{x \to 1} \frac{(x-1)(x-3)}{(x+2)(x-1)} \\
&= \lim_{x \to 1} \frac{(x-3)}{(x+2)} \\
&= -\frac{2}{3}
\end{aligned}
$$

$\lim_{x \to 1} \frac{x^2-4x+3}{x^2+x-2} = -\frac{2}{3}$

## Answer Form Assignment 1

| | |
|---|---|
| **Name** | Christoph Schmidl |
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| **Question** | **Answer** |
|---|---|
| 6a (1pt) | Values x for which $f(x) = 0 \rightarrow \{-1, 0, 1\}$ |
| 6b (1pt) | Values x for which $f(x) > 0 \rightarrow (0, 1), (-\infty, -1)$ |
| 7 (1pt) | $y$ runs through all values in $(a, b)$, except when $a = b$, then $y = \{a\}$ |
| 8a (0.5pt) | The function is odd. |
| 8b (0.5pt) | The function is even. |
| 9a (1pt) | $D(f) = [-\sqrt{7}, \sqrt{7}]$, $R(f) = [1, \sqrt{7} + 1]$ |
| 9b (1pt) | $D(f) = \mathbb{R} \setminus \{0\}$, $R(f) = (0, \infty)$ |
| 10a (1pt) | The inverse of y is $\frac{-dx+b}{cx-a}$ |
| 10b (1pt) | It is equal to $y$ when $d = -a$ |
| 11a (1pt) | $\lim_{x \to 2} \frac{x-2}{x^2+x-6} = \frac{1}{5}$ |
| 11b (1pt) | $\lim_{x \to 1} \frac{x^2-4x+3}{x^2+x-2} = -\frac{2}{3}$ |