Calculus and Probability
Assignment 2

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Group: not assigned? Where can I find this?

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Exercise 7

a) Dividing by x^3 makes the limit more obvious:

$$\lim_{x\to-\infty} \frac{x^3+2x^2+2}{3x^3+x+4} = \lim_{x\to-\infty} \frac{1+\frac{2}{x}+\frac{2}{x^3}}{3+\frac{1}{x^2}+4\frac{4}{x^3}} = \frac{1}{3}$$

The numerator approaches 1 and the denominator 3.
$\lim_{x\to-\infty} \frac{x^3+2x^2+2}{3x^3+x+4} = \frac{1}{3}$

b) Dividing by x^2 makes the limit more obvious:

$$\lim_{x\to\infty} \frac{2x+1}{x^2+x} = \lim_{x\to\infty} \frac{\frac{2}{x}+\frac{1}{x^2}}{1+\frac{1}{x}} = 0$$

The numerator approaches 0 and the denominator 1.
$\lim_{x\to\infty} \frac{2x+1}{x^2+x} = 0$

Exercise 8

a) Derivative of the function $f(x) = 2x+3$ for any point a in $\mathbb{R}$:

$$\begin{aligned}
f'(a) &= \lim_{h\to0} \frac{f(a+h)-f(a)}{h} \\
f'(a) &= \lim_{h\to0} \frac{2(a+h)+3-(2a+3)}{h} \\
&= \lim_{h\to0} \frac{2a+2h+3-2a-3}{h} \\
&= \lim_{h\to0} \frac{2h}{h} \\
&= 2
\end{aligned}$$

$f'(a) = \lim_{h\to0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to0} \frac{2h}{h} = 2$, $f'(a) = 2$ for every a in $\mathbb{R}$

Exercise 9

a) Applying the chain rule:

$$f'(x) = -\frac{1}{(1+\frac{1}{x})^2} * (-\frac{1}{x^2}) = \frac{1}{x^2(1+\frac{1}{x})^2}$$

Therefore we already got our slope m in the known formula: y = mx + b
The tangent line at x = a is therefore:

$$y = \frac{1}{a^2(1+\frac{1}{a})^2}x + b$$

Answer 9a

b) ... Answer 9b

Exercise 10

a) We already know that $(e^x)' = e^x$ and $(tan(x))' = \frac{1}{\cos^2(x)}$. By using the chain rule we get:

$$f(x) = exp(tan(x))$$
$$f'(x) = exp(\tan(x))\frac{1}{\cos^2(x)} = \frac{exp(\tan(x))}{\cos^2(x)}$$

$f'(x) = exp(\tan(x))\frac{1}{\cos^2(x)} = \frac{exp(\tan(x))}{\cos^2(x)}$

b) We already know that $(\ln(x))' = \frac{1}{x}$ and $(\cos(x))' = -\sin(x)$. By using the chain rule we get:

$$f(x) = -\ln(\cos(x))$$
$$f'(x) = -\frac{1}{\cos(x)} * (-\sin(x)) = \frac{\sin(x)}{\cos(x)} = \tan(x)$$

$f'(x) = -\frac{1}{\cos(x)} * (-\sin(x)) = \frac{\sin(x)}{\cos(x)} = \tan(x)$

Exercise 11

a)

$$f(x) = (exp(x))^{exp(x)}$$

- Logarithmic differentation: $\ln((exp(x)^{exp(x)})) = x * exp(x)$. $\frac{f'(x)}{f(x)} = exp(x)(x+1)$. $f'(x) = (exp(x)^{exp(x+1)})(x+1) = exp(x^{(exp(x)+1)})(x+1)$
- Chain rule: $g(x) = x^x$ and $h(x) = exp(x)$. $f(x) = (g \circ h)$. $g'(x) = x^x(log(x+1))$ and $h'(x) = exp(x)$. $f'(x) = g'(h(x))h'(x) = (exp(x))^{exp(x)(x+1)}exp(x) = exp(x^{(exp(x)+1)})(x+1)$

$f'(x) = (exp(x)^{exp(x+1)})(x+1) = exp(x^{(exp(x)+1)})(x+1)$

b)

$$f(x) = \sqrt{x-2}, \text{ compute}(f^{-1})'(x) \text{ (for } x > 2)$$

- Computing the inverse and differentiate: $f^{-1}(x) = x^2 + 2$. $(f^{-1})'(x) = 2x$
- Inverse rule: $f^{-1}(x) = x^2 + 2$. $f'(x) = \frac{1}{2\sqrt{x-2}}$. Therefore: $(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} = \frac{1}{\frac{1}{2\sqrt{(x^2+2)-2}}} = \frac{1}{\frac{1}{2x}} = 2x$

$f^{-1}(x) = x^2 + 2$. $(f^{-1})'(x) = 2x$

Exercise 12

a) text

Answer Form Assignment 2

Name	Christoph Schmidl
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Question		**Answer**
7a	(1pt)	$\lim_{x \to -\infty} \frac{x^3+2x^2+2}{3x^3+x+4} = \frac{1}{3}$
7b	(1pt)	$\lim_{x \to \infty} \frac{2x+1}{x^2+x} = 0$
8	(1pt)	$f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h \to 0} \frac{2h}{h} = 2$, $f'(a) = 2$ for every a in $\mathbb{R}$
9a	(1pt)	Answer 9a
9b	(1pt)	Answer 9b
10a	(1pt)	$f'(x) = exp(\tan(x))\frac{1}{\cos^2(x)} = \frac{exp(\tan(x))}{\cos^2(x)}$
10b	(1pt)	$f'(x) = -\frac{1}{\cos(x)} * (-\sin(x)) = \frac{\sin(x)}{\cos(x)} = \tan(x)$
11a	(1pt)	$f'(x) = (exp(x)^{exp(x+1)})(x+1) = exp(x^{(exp(x)+1)})(x+1)$
11b	(1pt)	$f^{-1}(x) = x^2 + 2.\ (f^{-1})'(x) = 2x$
12	(1pt)	