Calculus and Probability
Assignment 3

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Exercise 6

a)

$$f(x) = \arccos(\cos(x^2))$$
$$f'(x) = \frac{2x\sin(x^2)}{\sqrt{1-\cos^2(x^2)}}$$

$f'(x) = \frac{2x\sin(x^2)}{\sqrt{1-\cos^2(x^2)}}$

Exercise 7

a) The numerator outweighs the denominator if we look to the limit towards infinity. Therefore:

$$\lim_{x\to\infty} \frac{x^2}{1+e^{-x}} = \infty$$

b) We get $\frac{0}{0}$ and therefore can apply L'Hopital which gives us $\lim_{x\to 0} \frac{\cos(x)+A+3Bx^2}{5x^4} = \frac{1+A}{0} \to \pm\infty$ unless $A = -1$

After applying L'Hopital repeatedly, we get $A \cdot B \cdot C = -1 \cdot \frac{1}{6} \cdot 120 = -20$

Exercise 8

a)
- (a) $g(x) = \cos(3x)$
- (b) $g^{(1)}(x) = -3\sin(3x)$
- (c) $g^{(2)}(x) = -3^2\cos(3x)$
- (d) $g^{(3)}(x) = 3^3\sin(3x)$
- (e) $g^{(4)}(x) = 3^4\cos(3x)$

Because the remainder of 2015 divided by 4 is 3, we get:

$$g^{2015}(x) = 3^{2015}\sin(3x)$$

Exercise 9

a) There are two roots which you can see right from the term without further calculation, namely:

$$x_0 = -1 \quad x_1 = 3$$

We get the y-intercept by putting a zero into the function, therefore:

$$f(0) = -3$$

Two roots: $x_0 = -1$ and $x_1 = 3$. y-intercept: $f(0) = -3$

b) $(x+1)^2$ goes into the positive direction, therefore the whole term depends on $(x-3)$. Therefore we get:

$$\lim_{x\ to-\infty} = -\infty \quad \text{and} \quad \lim_{x\ to+\infty} = +\infty$$

c) In order to find the local minima and maxima we need to calculate the first and the second derivative of the given function. After that we have to find the zeros of the first and second derivative in order to get the critical points. The critical points can then be tested for the requirements of being local maxima or minima.

$$\begin{aligned} f(x) &= (x+1)^2(x-3) \\ f'(x) &= ((x^2+2x+1)(x-3)) = (2x+2)(x-3) + x^2 + 2x + 1 = 3x^2 - 2x - 5 \\ f''(x) &= 6x - 2 \end{aligned}$$

The product rule has been applied to get the first derivative.

Zeros of of $f'(x) = 3x^2 - 2x - 5 = (x+1)(3x-5)$ are $x_0 = -1$ and $x_1 = \frac{5}{3}$
Zeros of $f''(x) = 6x - 2$ are $x = \frac{1}{3}$

We already got the x-coordinates of the critical points and only have to plug them in into the original function in order to get the y-coordinates:

$$\begin{aligned} f(-1) &= 0 \\ f(\frac{5}{3}) &\approx -9.481 \end{aligned}$$

Therefore, we get the critical points $(-1, 0)$ and $(\frac{5}{3}, -9.481)$

$f''(-1) = -14 < 0$, therefore it is a maximum and $f''(\frac{5}{3}) = 13 > 0$, therefore is is a minimum.

d) The function is concave $\Leftrightarrow f''(x) < 0 \Leftrightarrow 6x - 2 < 0 \Leftrightarrow x < \frac{1}{3}$. The function is convex $\Leftrightarrow f''(x) > 0 \Leftrightarrow 6x - 2 > 0 \Leftrightarrow x > \frac{1}{3}$. The point of inflection is at $x = \frac{1}{3}$

Exercise 10

a) $\frac{1}{6}\sin(x)(\cos(2x) + 5) + C$

b) $f_i(x) = \frac{1}{2}\sin^2(x) + C_i$ and pick three distinct value for C_i

Answer Form Assignment 3

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Question		**Answer**
6	(1pt)	$f'(x) = \frac{2x\sin(x^2)}{\sqrt{1-\cos^2(x^2)}}$
7a	(1pt)	$$\lim_{x\to\infty} \frac{x^2}{1+e^{-x}} = \infty$$
7b	(1pt)	After applying L'Hopital repeatedly, we get $A \cdot B \cdot C = -1 \cdot \frac{1}{6} \cdot 120 = -20$
8a	(1pt)	$$g^{2015}(x) = 3^{2015}\sin(3x)$$
9a	(1pt)	Two roots: $x_0 = -1$ and $x_1 = 3$. y-intercept: $f(0) = -3$
9b	(1pt)	$$\lim_{x\ to -\infty} = -\infty \quad \text{and} \quad \lim_{x\ to +\infty} = +\infty$$
9c	(1pt)	$f''(-1) = -14 < 0$, therefore it is a maximum and $f''(\frac{5}{3}) = 13 > 0$, therefore is is a minimum.
9d	(1pt)	The function is concave $\Leftrightarrow f''(x) < 0 \Leftrightarrow 6x - 2 < 0 \Leftrightarrow x < \frac{1}{3}$. The function is convex $\Leftrightarrow f''(x) > 0 \Leftrightarrow 6x - 2 > 0 \Leftrightarrow x > \frac{1}{3}$. The point of inflection is at $x = \frac{1}{3}$
10a	(1pt)	$\frac{1}{6}\sin(x)(\cos(2x) + 5) + C$
10b	(1pt)	$f_i(x) = \frac{1}{2}\sin^2(x) + C_i$ and pick three distinct value for C_i