Calculus and Probability
Assignment 2

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Group: Tutorial 5

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Exercise 6

a) By dividing the numerator and denominator by x^3 we get $\frac{x^3+2x^2+2}{3x^3+x+4} = \frac{1+\frac{2}{x}+\frac{2}{x^3}}{3+\frac{1}{x^2}+\frac{4}{x^3}}$. By ignoring the last terms of both the numerator and the denominator because they do not contribute that much in the very end, we get

$$\lim_{x \to -\infty} \frac{x^3 + 2x^2 + 2}{3x^3 + x + 4} = \frac{1}{3}$$

b) By dividing the numerator and denominator by x^2 we get $\frac{2x+1}{x^2+x} = \frac{\frac{2}{x}+\frac{1}{x^2}}{1+\frac{1}{x}}$. The numerator converges towards 0 and the denominator towards 1. Therefore we get

$$\lim_{x \to \infty} \frac{2x + 1}{x^2 + x} = 0$$

Exercise 7

a)

$$f(a + h) = 2(a + h) + 3 = 2a + 2h + 3$$

$$f(a) = 2a + 3$$

$$f(a + h) - f(a) = 2a + 2h + 3 - 2a - 3 = 2h$$

$$f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} = \lim_{h \to 0} \frac{2h}{2} = 2$$

b)

$$f(a+h) = \frac{5(a+h)-7}{f(a+h)+3} = \frac{5a-7+5h}{4a+3+4h}$$

$$\begin{aligned}
f(a+h)-f(a) &= \frac{5a-7+5h}{4a+3+4h} - \frac{5a-7}{4a+3} \\
&= \frac{(5a-7+5h)(4a+3)-(5a-7)(4a+3+4h)}{(4a+3+4h)(4a+3)} \\
&= \frac{43h}{(4a+3+4h)(4a+3)}
\end{aligned}$$

$$\begin{aligned}
\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} &= \lim_{h\to 0} \frac{43h}{(4a+3+4h)(4a+3)} \frac{1}{h} \\
&= \lim_{h\to 0} \frac{43}{(4a+3+4h)(4a+3)} \\
&= \frac{43}{(4a+3)^2}
\end{aligned}$$

$f'(a) = \frac{43}{(4a+3)^2}$ for every a in $\mathbb{R} \setminus \{-\frac{3}{4}\}$

Exercise 8

a) Getting the slope of the tangent line at $x = 2$:

$$\begin{aligned}
f'(x) &= \frac{1}{(x+1)^2} \\
f'(2) &= \frac{1}{9}
\end{aligned}$$

Getting a point on the tangent line to be able to formulate the equation:

$$f(x) = \frac{1}{1+\frac{1}{2}} \to y = \frac{2}{3}$$

Therefore, we now have found the coordinate $(2, \frac{2}{3})$ for the point shared by $f(x)$ and the line to $f(x) = 2$. The only step left is to use the point $(2, \frac{2}{3})$ and slope $\frac{1}{9}$ in the point-slope formula for a line:

$$\begin{aligned}
y - y_1 &= (m(x - x_1)) \\
y &= \frac{x}{9} + \frac{4}{9}
\end{aligned}$$

$$y = \frac{x}{9} + \frac{4}{9}$$

b) ... Answer 8b

Exercise 9

a) $(e^x)' = e^x$, $(tan(x))' = \frac{1}{cos^2 x}$ Using the chain rule we get

$$f'(x) = exp(tan(x))\frac{1}{cos^2 x} = \frac{exp(tan(x))}{cos^2 x}$$

b) $(\ln x)' = \frac{1}{x}$, $(\cos x))' = -\sin x$ Using the chain rule we get

$$f'(x) = -\frac{1}{\cos x}(-\sin x) = \frac{\sin x}{\cos x} = tan(x)$$

Exercise 10

a) 1. Using the chain rule given that $g(x) = x^x$ and $h(x) = exp\ x \rightarrow f(x) = (g \circ h)(x)$. $g'(x) = x^x(log\ x + 1)$, $h'(x) = exp\ x$. Therefore:

$$f'(x) = g'(h(x))h'(x) = (exp\ x)^{exp\ x}(x+1)exp\ x = (exp\ x)^{exp\ x+1}(x+1) = e^{x(e^x+1)}(x+1)$$

2. Using logarithmic differentiation: $\ln((exp\ x)^{exp\ x}) = x\ exp\ x \rightarrow \frac{f'(x)}{f(x)} = exp\ x(x+1)$.

$$f'(x) = (exp\ x)^{exp\ x+1}(x+1) = e^{x(e^x+1)}(x+1)$$

$$f'(x) = (exp\ x)^{exp\ x+1}(x+1) = e^{x(e^x+1)}(x+1)$$

b) 1. Using the inverse: $f^{-1}(x) = x^2 + 2 \rightarrow (f^{-1})'(x) = 2x$
2. $f^{-1}(x) = x^2 + 2$, $f'(x) = \frac{1}{2\sqrt{x-2}} \rightarrow (f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} ? \frac{1}{\frac{1}{2\sqrt{(x^2+2)-2}}} = \frac{1}{\frac{1}{2x}} = 2x$

$$(f^{-1})'(x) = 2x$$

Answer Form Assignment 2

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Question	**Answer**
6a (1pt)	$$\lim_{x \to -\infty} \frac{x^3 + 2x^2 + 2}{3x^3 + x + 4} = \frac{1}{3}$$
6b (1pt)	$$\lim_{x \to \infty} \frac{2x + 1}{x^2 + x} = 0$$
7a (1pt)	$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{2h}{2} = 2$$
7b (1pt)	$f'(a) = \frac{43}{(4a+3)^2}$ for every a in $\mathbb{R} \setminus \{-\frac{3}{4}\}$
8a (1pt)	$$y = \frac{x}{9} + \frac{4}{9}$$
8b (1pt)	Answer 8b
9a (1pt)	$$f'(x) = exp(tan(x)) \frac{1}{cos^2 x} = \frac{exp(tan(x))}{cos^2 x}$$
9b (1pt)	$$f'(x) = -\frac{1}{\cos x}(-\sin x) = \frac{\sin x}{\cos x} = tan(x)$$
10a (1pt)	$$f'(x) = (exp\ x)^{exp\ x + 1}(x+1) = e^{x(e^x+1)}(x+1)$$
10b (1pt)	$$(f^{-1})'(x) = 2x$$