Calculus en Kansrekening

Assignment 4, September 22, 2015

Handing in your answers:

- submission via Blackboard (`http://blackboard.ru.nl`);
- one single pdf file (make sure that if you scan/photo your handwritten assignment, the result is clearly readable);
- all of your solutions are clearly and convincingly explained;
- make sure to write your name, your student number and **the name of your TA** (Joost or Bram).

Deadline: Wednesday, September 30, 14:30 sharp!

Goals: After completing these exercises successfully you should be able to:

- analyse and sketch real functions;
- apply differentiation rules to determine higher-order partial derivatives;
- find primitives of well-known functions;
- compute definite integrals when the primitive function is known;
- compute improper integrals.

Marks: You can score a total of 100 points. There are three bonus exercises.

1. **(20 points)** Investigate the function $f(x) = \frac{e^x}{x+1}$ as follows. (Do not start with drawing a graph by means of a device or some web resource. Of course you may check your result when you're done.)
   (a) Determine the domain of the function f.
   (b) What are the roots of f?
   (c) Determine the limits at the edges of the domain. (Hint: there are 4 cases, use l'Hôpital!)
   (d) Find f' and f''.
   (e) Find the zeros of f' and f''.
   (f) What are the critical points (determine their x and y coordinates)?
   (g) Find the local minimums and maximums.
   (h) Which parts of the function are convex and concave? Does function f have points of inflection? (Hint: Use the sign of the second derivative for answering both questions.)
   (i) Draw the graph of function f. (If you collect all intervals and special points in a table, it helps a lot in drawing the graph. Moreover, you get some extra points!)
2. **(bonus, +1 point)** Write your name, student number, and the name of your TA on the first page.
3. **(bonus, +4 points)** Consider the function $f(x) = e^x \sin(x)$.
   (a) Determine the domain of the function f.
   (b) What are the roots of f? Where does the graph of f intersect the y axis?
   (c) Find f' and f''.
   (d) Find all the zeros of f' and f''.
4. **(20 points)** Given function f, find the partial derivatives. If it is necessary, simplify the result.
   (a) i. $f(x,y) = \cos(4y - xy)$; $\frac{\partial}{\partial x} f(x,y) =?$ and $\frac{\partial}{\partial y} f(x,y) =?$
   ii. $f(x,y) = e^{\frac{x}{y}}$; $\frac{\partial}{\partial x} f(x,y) =?$ and $\frac{\partial}{\partial y} f(x,y) =?$

(b) For the two functions above, show that $\frac{\partial}{\partial x}\big(\frac{\partial}{\partial y}f(x,y)\big) = \frac{\partial}{\partial y}\big(\frac{\partial}{\partial x}f(x,y)\big)$.

5. **(20 points)** If $f(x,y) = \frac{xy}{x+y}$, show that

$$x^2 \cdot \tfrac{\partial}{\partial x}\big(\tfrac{\partial}{\partial x}f(x,y)\big) \;+\; 2xy \cdot \tfrac{\partial}{\partial x}\big(\tfrac{\partial}{\partial y}f(x,y)\big) \;+\; y^2 \cdot \tfrac{\partial}{\partial y}\big(\tfrac{\partial}{\partial y}f(x,y)\big) \;=\; 0.$$

(Hint: First compute all the second partial derivatives of f, then substitute the results in the expression on the left-hand side.)

6. **(20 points)** Evaluate the following definite integrals. (Hint: use slide 38 of the lectures about derivatives, and slide 13 of the lectures about primitives)

(a) $\int_{-1}^{1}(x^3 + x - 1)\,dx$;

(b) $\int_{1}^{2}(3\sqrt{x} + \frac{3}{x^2})\,dx$;

(c) $\int_{0}^{\pi}(\sin(x) + \cos(x))\,dx$;

(d) $\int_{-1}^{1}\frac{-5}{\sqrt{1-x^2}}\,dx$.

7. **(20 points)** Evaluate the following improper integrals.

(a) $\int_{1}^{\infty}\frac{1}{x^n}\,dx$, n an integer such that $n \geq 2$; (Hint: this generalizes an example solved in the lecture on slide 8)

(b) $\int_{-\infty}^{-\pi/2}\frac{x\cos(x)-\sin(x)}{x^2}\,dx$; (Hint: use the quotient rule for derivation to find the primitive)

(c) $\int_{2}^{\infty}\frac{-1}{x\ln^2(x)}\,dx$. (Hint: use a fraction of well known functions to find the primitive)

8. **(bonus, +4 points)** Find primitives of the following functions f. That is, find F such that $F'(x) = f(x)$.

(a) $f(x) = \frac{1}{2\sqrt{x}} - \frac{1}{x^2}$;

(b) $f(x) = 2\sin(x)\cos(x)$;

(c) $f(x) = \frac{2}{1+4x^2}$;

(d) $f(x) = \frac{1-\ln(x)}{x^2}$.