Calculus en Kansrekening

Assignment 4, September 25, 2014

Handing in your answers: To read the full story, see

`http://www.ru.nl/ds/education/courses/analyse_2014/`

Briefly,

- make sure to put
  - your name,
  - your student number and
  - the name of your TA (Safet and Arjen OR Ana Helena OR Gergely)

  on your solution sheet;
- submit via Blackboard (`http://blackboard.ru.nl`);
- it is one single pdf file.

Deadline: Friday, October 3, 13:30 sharp!

Goals: After completing these exercises successfully you should be confident with the following topics.

- Definite integral;
- Indefinite integral;
- Integration with substitution.

Marks: You can score a total of 100 points. Note that you have to **explain your answers**, so it is clear how you have got the result. In order to get full points, you need to make sure that the reader can understand each step in your solution.

1. **(26 points)** Evaluate the following definite integrals. (If necessary, round to two decimal places.)
   (a) $\int_1^4 (x + 3\sqrt{x} + 2)\, dx$;
   (b) $\int_1^{16} (4x^{7/3} - \sqrt[4]{x} + \frac{\pi}{x})\, dx$;
   (c) $\int_{-2}^{2} e^{5x-1}\, dx$. (Hint: Apply substitution.)
2. **(40 points)** Determine the indefinite integrals by applying the substitution method.
   (a) $\int \sqrt{x+1}\, dx$;*
   (b) $\int \sin(2x - 3)\, dx$;
   (c) $\int 5x \cdot \cos(3x^2 + 5)\, dx$;
   (d) $\frac{1}{3} \int 3^{x^4} \cdot x^3\, dx$;*
   (e) $\int \frac{2}{\sqrt{1-(2x+5)^2}}\, dx$; (Hint: arcsin.)
   (f) $\int \frac{4x-10}{\sqrt{1-(4x^2-20x+25)^2}}\, dx$; (Hint: again?)

* Additionally, verify the result at these exercises, that is, take the derivative of the result to check if you get back the original function.

Turn page

3. **(10 points)** Given the function $f(x) = -x^2 + 8x - 7$. What is the area under the curve of f between the zeros of the function?

4. **(24 points)** Let $f(x) = \ln x$. Solve the problems below in order.

   (a) Find $b \in \mathbb{R}$ such that $f(b) = 1$.

   (b) Determine the equation of the tangent line at the point $(b, 1)$.

   (c) Let this line intersect the x-axis at a. What is the area of the triangle with the following vertices: $(a, 0), (b, 1), (b, 0)$?

   (d) Someone tells you that $(x \ln x - x)' = \ln x$. Verify whether it is true.

   (e) Determine $\int_1^b \ln x \, dx$.