Calculus and Probability Theory

Assignment 2, February 9, 2017

Handing in your answers:

- submission via Blackboard (`http://blackboard.ru.nl`);
- one single pdf file (make sure that if you scan/photo your handwritten assignment, the result is clearly readable);
- all of your solutions are clearly and convincingly explained;
- make sure to write your name, your student number

Deadline: Friday, February 17, 14:30 sharp!

Goals: After completing these exercises successfully you should be confident with the following topics:

- Limits, possibly involving infinity
- The definition of the derivative
- The tangent line of a function
- Differentiation rules of (special) functions

Marks: You can score a total of 100 points.

1. **(15 points)** Find the limits. (Hint: try to simplify as much as possible before applying the limit!)
   (a) $\lim_{x\to-\infty} \frac{x^3+2x^2+2}{3x^3+x+4}$;
   (b) $\lim_{x\to\infty} \frac{3x^2+8}{x+1}$;
   (c) $\lim_{x\to\infty} \frac{2x+1}{x^2+x}$;
   (d) $\lim_{x\to a} \frac{x^n-a^n}{x-a}$ for some parameter $a \in \mathbb{R}$. (*Hint*: Do you recognize this limit? If not, you can always simplify the fraction using long division)
2. **(20 points)** Recall that if a function f is differentiable at a, then the derivative at the point a is defined as
$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}.$$
   Use this definition to find the derivative of the following functions at the point a:
   (a) $f(x) = \pi$, $a = 2$;
   (b) $f(x) = 2x + 3$, any a;
   (c) $f(x) = x^2 + 2x + 1$, $a = 3$;
   (d) $f(x) = \frac{5x-7}{4x+3}$, any $a \neq -\frac{3}{4}$.
3. **(15 points)** Given the equation $4y - x + 2(1 - \ln 2) = 0$ and the function $f(x) = a \ln x$, $a > 0$.
   (a) Find the slope of the line having the equation above.
   (b) There exists a value for a such that the line is the tangent line to $f(x)$ in the point $x = 2$. Find a.
   (c) Find the tangent line to $f(x)$ in the point $x = 2$ for every $a > 0$.
4. **(30 points)** Find the derivative on the domain of the following functions. You can freely use all the differentiation rules that were discussed in the lecture. Simplify the result as much as you can.
   (a) $f(x) = x^4 - 2x^3 + 7$;
   (b) $f(x) = \frac{x^2+5}{x-7}$;

(c) $f(x) = \sin^2(\sqrt{x})$;

(d) $f(x) = 1 - \cos^2(\sqrt{x})$;

(e) $f(x) = \exp(\tan(x))$;

(f) $f(x) = -\ln(\cos(x))$;

(g) $f(x) = \arcsin(1 - 2x)$;

(h) $f(x) = 10^{x^2}$.

5. **(20 points)** Apply any rules (including chain or inverse rules) and the logarithmic differentiation as appropriate to compute the result. If you can solve the problem in two different ways, you get *two extra points*.

   (a) $f(x) = e^{\sin x}$, compute $f'(x)$;

   (b) $f(x) = (\exp x)^{\exp x}$, compute $f'(x)$; (*Hint:* use logarithmic differentiation or the chain rule)

   (c) $f(x) = e^{2x}$, compute $(f^{-1})'(x)$;

   (d) $f(x) = \sqrt{x-2}$, compute $(f^{-1})'(x)$ (for $x > 2$).