Calculus and Probability

Assignment 3

Note:

- You can hand in your solutions as a single PDF via the assignment module in Blackboard. Note that the document should be in English and typeset with LaTeX, Word or a similar program. It should not be a scan or picture of your handwritten notes.
- Make sure that your name, student number and group number are on top of the first page!
- **Note that your submission should be an individual submission because it can influence your final grade for this course. If we detect that your work is not completely your own work, we will ask the exam committee to investigate whether it is plagiarism or not!**

Exercises to be presented during the exercise hours

Exercise 1
The function arcsin is the inverse function of sin.

a) What is the domain of the function $\arcsin(x)$? Why?

b) Compute the following values and explain how you got the result:

$$\arcsin(1) =? \qquad \arcsin(0) =? \qquad \arcsin\left(\frac{\sqrt{3}}{2}\right) =?$$

c) Find the derivative of f:

$$f(x) = \arcsin\left(\frac{2x}{1-x}\right).$$

Exercise 2
Find the limits of the following functions. (Note that before you can apply L'Hôpital's rule, you have to verify whether it is possible.)

a) $\lim\limits_{x\to\infty} \frac{e^{n-x}}{x^{-m}}$ with $m, n \in \mathbb{N}$; (Hint: if unclear first solve a particular case, e.g., $n = 0, m = 3$.)

b) If $\lim\limits_{x\to 0} \frac{\sqrt[3]{(a\cdot x+b)}-2}{x} = \frac{5}{12}$ with $a, b \in \mathbb{N}$ then $a \cdot b = ?$;

Exercise 3
Given the function $f(x) = \log_3(2x)$: What is $f'''(x)$?

Exercise 4
We will investigate the function

$$f(x) = \frac{(x-2)^2}{x+2}.$$

Additionally, we prove that the line $y = x - 6$ is a slant asymptote on both sides.

a) Determine the domain of function f.

b) What are the roots of f? Where does the graph of f intersect the y axis?

c) Determine the limits at the edges of the domain.

d) Find f' and f''.

e) Find the zeros of f' and f''.

f) What are the critical points (determine their x and y coordinates)?

g) Find the local minima and maxima.

h) Which parts of the function are convex and concave? Does function f have points of inflection? (Hint: Use the sign of the second derivative for answering both questions.)

i) Show that the line $y = x - 6$ is a slant asymptote of f. (Hint: Use the definition on slide 47 of the lecture and the following two limits.)

$$\lim_{x\to-\infty} (f(x) - (x-6)) = ? \qquad \text{and} \qquad \lim_{x\to+\infty} (f(x) - (x-6)) = ?$$

Exercise 5

a) Find the derivative of $f(x) = \ln(\cos(\ln(\cos(x))))$.

b) Find a function $g(x)$ such that $g'(x) = \tan(2x)$.

Exercises to be handed in

You are expected to explain your answers, even if this is not explicitly stated in the exercises themselves.

Exercise 6
The function arccos is the inverse function of cos. Find the derivative of f (hint: for some ways to derive this, it can be useful to remember that $\sin^2 x + \cos^2 x = 1$):

$$f(x) = \arccos\left(\cos x^2\right).$$

1 pt

Exercise 7
Find the limits of the following functions. (Note that before you can apply L'Hôpital's rule, you have to verify whether it is possible.)

a) $\lim_{x\to\infty} \frac{x^2}{1+e^{-x}}$; 1 pt

b) If $\lim_{x\to 0} \frac{\sin(x)+Ax+Bx^3}{x^5} = \frac{1}{C}$ with $A, B, C \in \mathbb{Q}$, then $A \cdot B \cdot C = ?$. 1 pt

Exercise 8
Given the function $g(x) = \cos(3x)$: What is $g^{(2015)}(x)$? (Hint: Start with finding the first few derivatives of g.) 1 pt

Exercise 9
Investigate function $f = (x+1)^2(x-3)$ by following the steps below. (Do not start with drawing a graph. Of course, you may check your solution with GeoGebra or with some other tool.)

a) What are the roots of f? What is the y-intercept, that is, where is the intersection of the graph of f and the y-axis? 1 pt

b) Determine the limits at the edges of the domain. In this case, there are only two edges:

$$\lim_{x\to-\infty} f(x) \qquad \text{and} \qquad \lim_{x\to+\infty} f(x).$$

1 pt

c) Find the local minima and maxima. 1 pt

d) Which parts of the function are convex and concave? Does function f have points of inflection? (Hint: Use the sign of the second derivative for answering both questions.) 1 pt

Exercise 10

a) Find a function $h(x)$ such that $h'(x) = \cos^3(x)$ (hint: recall that $\cos^2(x) + \sin^2(x) = 1$) 1 pt

b) Find three functions f_1, f_2, f_3 such that $f_1'(x) = f_2'(x) = f_3'(x) = \sin(x)\cos(x)$.

1 pt

Your final grade is the sum of your scores divided by 1.0.