# Calculus and Probability Theory: basic info

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Version: fall 2014





## Outline







## About this course I

### Lectures

- Weekly, 2 hours, on Tuesdays 10:45 (LIN 5)
- Presence not compulsory . . .
  - but active attitude expected, when present
- Covering the same material as in:
  - *Calculus* lecture notes by Bernd Souvignier ("LNBS Calculus")
  - *Kansrekening* lecture notes by Bernd Souvignier ("LNBS Kansrekening")
  - we use some slides (work in progress), but also the chalkboard
    - topics as in LNBS, sometimes different examples
- Course URL:

  www.ru.nl/ds/education/courses/analyse_2014/





## About this course II

### Exercise sessions

- Also weekly meetings, on Fridays, 8:45, three locations
  - Presence not compulsory
  - Questions about homework and solving exercises as well
- Handing in homeworks is compulsory (at least 5/8)
  - Homework exercises have to be done individually
- Assistants: Staff: Gergely Alpár, Ana Helena Sanchez; students: Safet Acifovic, Arjen Zijlstra
- Schedule:
  - New exercise on the web on Thursday (web page of the course), say in week $n$
  - You can try them yourself immediately and ask advice on Friday morning in week $n$
  - You can ask final questions, again on Friday in week $n+1$
  - You have to hand-in, via Blackboard, before Friday 13h30 sharp, in week $n+1$; late submissions will not be accepted.





## About this course III

### Exercise groups

- There will be three groups for the exercise classes, based on three levels of mathematical skills
- Rate your own skill honestly, according to:
  - strong, eg. $\geq 7\frac{1}{2}$ at secondary school
  - average
  - suboptimal, eg. little background (from HBO)
- Based on this input we will organise groups, and let you know, via Blackboard email
  - the classifications of the groups will not be used explicitly





## About this course IV

### Examination

- Final, written exam (4 Nov., 8:30-10:30, HAL 2)
- Final mark is the average of:
  1. 50% {Average homework grade}+ 50% Written exam
  2. (the exam mark must be $\geq 5$)

## About this course V

### How to pass this course . . .

- Practice, practice, practice . . .
- You don't learn it it by just staring at the slides - not a spectator sport!
- Exam questions will be in line with exercises





## About this course VI

### Some special points

- You can fail for this course!
  (Statistics tells that some of you will)
- 3ec means $3 \times 28 = 84$ hours in total
  - Let's say 20 hours for exam
  - 64 hours for 8 weeks means: 8 hours per week!
  - on average 4 hours for studying & making exercises
- Why computer scientists need math?
  - problem solving
  - programming, esp. for embedded/hybrid systems
  - computer hardware and architecture: computer networks, data encryption and compression, . . .
- Coming up-to-speed is your own responsibility
  - if you lack background knowledge, or have forgotten basic mathematics: Voorkennis site (via webpage), or Wikipedia





## Different numbers

$$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$$

- In the natural numbers $\mathbb{N}$ you can add and multiply: $x + y$ with 0, $x \cdot y$ with 1.
- In the integers $\mathbb{Z}$ you can also subtract: $x - y$
- In the rationals $\mathbb{Q}$ you can divide: $\frac{x}{y}$, for $y \neq 0$
- In the reals $\mathbb{R}$ you can take limits: $\lim\limits_{n\to\infty} r_n$, and thus also roots $\sqrt{x}$, for $x \geq 0$.
- In the complex numbers $\mathbb{C}$ one can take all roots, in particular $\sqrt{-1} = i$.





## Numbers: some basic properties

- associative laws, for addition and multiplication
  $$a + (b + c) = (a + b) + c \qquad a \cdot (b \cdot c) = (a \cdot b) \cdot c$$
- commutative laws, for addition and multiplication
  $$a + b = b + a \qquad a \cdot b = b \cdot a$$
- distributive law
  $$a \cdot (b + c) = a \cdot b + a \cdot c$$
- existence of an additive and multiplicative identities:
  $$a + 0 = a = 0 + a \qquad a \cdot 1 = a = 1 \cdot a$$
- existence of additive and multiplicative inverses
  $$a + (-a) = 0 = (-a) + a \qquad a \cdot \frac{1}{a} = 1 = \frac{1}{a} \cdot a, \text{ for } a \neq 0$$





## Basic definitions

### Definition (Functions)

A real function $f : D \to \mathbb{R}$, for $D \subseteq \mathbb{R}$, is a rule which assigns to each $x \in D$ precisely one $f(x) \in \mathbb{R}$.

- In this situation the subset $D \subseteq \mathbb{R}$ is called the domain of $f$. Sometimes we write $D(f)$ for $D$.
- $\mathbb{R}$ is the codomain of $f$, and the subset $R(f) = \{f(x) | x \in D\} \subseteq \mathbb{R}$ is called the range of $f$.

### Example

- $f(x) = |x|$,
  "absolute value" function, with $D(f) = \mathbb{R}$, $R(f) = [0, \infty)$
- $f(x) = \sqrt{9 - x^2}$
- $f(x) = sign(x)$





## More definitions

### Definition

A function $f : D \to \mathbb{R}$ is injective or one-to-one if $f(x) = f(y)$ implies $x = y$, for all $x, y \in D$.

A function $f : D \to \mathbb{R}$ is surjective or onto if its image is equal to its codomain

- This means: $R(f) = \mathbb{R}$, or: for each $y \in \mathbb{R}$ there is an $x \in D$ with $f(x) = y$. Symbolically: $\forall y \in \mathbb{R}\, \exists x \in D\, f(x) = y$.

A function $f : D \to \mathbb{R}$ is bijective if it is both injective and surjective. Then it is an isomorphism $f : D \xrightarrow{\cong} \mathbb{R}$.

### Definition (Graph of a real function)

For a function $f : D \to \mathbb{R}$, the graph $G(f) \subseteq D \times \mathbb{R}$ of $f$ contains all pairs $(x, f(x))$. So, we write: $G(f) = \{(x, f(x)) | x \in D\}$.

# More on functions

## Definition (Inverse and composition)

If a function $f : D \to \mathbb{R}$, is injective, we can define an inverse function $f^{-1}\colon R(f) \to D \subseteq \mathbb{R}$, namely:

- for $y \in R(f)$, say $y = f(x)$, define $f^{-1}(y) = x$
- this $x$ is uniquely determined: if $f(x) = y = f(x')$, then $x = x'$, since $f$ is injective
- by construction: $f(f^{-1}(y)) = y$ and also $f^{-1}(f(x)) = x$.

The composition of functions $f : X \to Y$ and $g : Y \to Z$ is the function $h = g \circ f : X \to Z$, for which $h(x) = g(f(x))$, for each $x \in X$.

A function $f : (-a, a) \to \mathbb{R}$ is even if $f(-x) = f(x)$, for all $x \in (-a, a)$, and odd if $f(-x) = -f(x)$, for all $x \in (-a, a)$.