Calculus and Probability

Assignment 8

Note:

- You can hand in your solutions as a single PDF via the assignment module in Blackboard. Note that the document should be in English and typeset with LaTeX, Word or a similar program. It should not be a scan or picture of your handwritten notes.
- Make sure that your name, student number and group number are on top of the first page!
- **Note that your submission should be an individual submission because it can influence your final grade for this course. If we detect that your work is not completely your own work, we will ask the exam committee to investigate whether it is plagiarism or not!**

Exercises to be presented during the exercise hours

Exercise 1
We are going to sketch the graph of $y = f(x) = \frac{2x-1}{(x-1)^2}$. To do this, we first investigate first all the points required *i.e.* domain, parity, limits, extremes, monotonicity and asymptotes, points of inflection and convexity/concavity.

a) What is the domain of f? What are the zeros of f? Where does f intersect the y-axis?

b) What are the limits of the function?

c) What are the extremes of the function?

d) Where is the function convex/concave?

e) Sketch the function

Exercise 2
Compute the area bounded by the two parabolas: $y = \frac{x^2}{3}$ and $y = 4 - \frac{2}{3}x^2$. Sketch the area first.

Exercises to be handed in

> You are expected to explain your answers, even if this is not explicitly stated in the exercises themselves.

Exercise 3
State whether the following statements are T(rue) or F(alse) $\boxed{1 \text{ pt}}$

a) $\frac{d}{dx} \ln ax = \frac{1}{ax}$.

b) The most general antiderivative of $f(x) = x^{-2}$ is $F(x) = -\frac{1}{x} + C$.

c) The domain of $f(x) = \sqrt{1 + \frac{1}{x}}$ is $D_f = (-\infty, -1] \cup (0, \infty)$.

Exercise 4
The following function is given: $f(x) = e^{1-x^2}$. Find the tangent lines to the function f in the points where the graph of f intersects the line $y = 1$. $\boxed{2 \text{ pt}}$

Exercise 5
We are considering families with 4 children and we assume the probability of a male birth is 1/2. Answer the following questions:

a) Find the probability that that there will be at least 1 boy in a family. $\boxed{1 \text{ pt}}$

b) Find the probability that that there will be at least 1 boy and at least 1 girl in a family. $\boxed{1 \text{ pt}}$

c) Out of 2000 families with 4 children each, how many would you expect to have exactly 2 boys? $\boxed{1 \text{ pt}}$

Exercise 6
A random variable X has density function:

$$f(x) = \begin{cases} cx^2, & 1 \leq x \leq 2 \\ cx, & 2 < x < 3 \\ 0, & \text{otherwise} \end{cases}.$$

a) Find the constant c; $\boxed{1 \text{ pt}}$

b) Find $P(X > 2)$; $\boxed{1 \text{ pt}}$

c) Find $P(2 < X < 3/2)$. $\boxed{1 \text{ pt}}$

Exercise 7
Solve the following integrals $\boxed{2 \text{ pt}}$

a) $\int_0^\infty \left(\frac{1}{\sqrt{x}} - \frac{\sqrt{x}}{1+x} \right) \, \mathrm{d}x$

b) $\int_0^1 x \arctan x \, \mathrm{d}x$

Exercise 8

You are an ice cream vendor on a sunny beach day. Customers are uniformly distributed over a stretch of beach $\frac{1}{2}$ kilometer long. Suppose your government concession allows you to sell ice cream anywhere on this $\frac{1}{2}$ kilometer stretch of beach. 2 pt

a) Give the probability distribution $f(x)$ of customers

b) Let u be the position of the ice cream truck on the stretch of beach. Give a function $g(x)$ that computes the distance between a customer x and the ice cream truck.

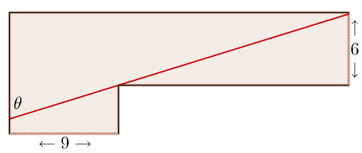
c) Compute the expected walking distance with respect to the position u of the truck. [Hint: Use 'The law of the unconscious statistician'.]

d) What location should you put your ice cream stand to minimize the average distance your customers have to walk along the beach?

e) How far does the average customer walk with the ice cream truck in the optimal position?

Exercise 9

A steel rod is carried down a hallway of 9 meter wide. At the end there is corner to the right into a narrower hallway of 6 meter wide. What is the maximum length of the steel rod that can be carried horizontally around the corner? 2 pt



(Hint: What happens at $\theta \to 0$ and $\theta \to \frac{1}{4}\pi$? Show that the angle at which the minimum is obtained is at $\theta = \arctan\left(\sqrt[3]{\frac{3}{2}}\right) \approx 0,853$.)

Your final grade is the sum of your scores divided by 1.5.