Statistical Machine Learning
Assignment 1

Inez Wijnands (s4149696) & Guido Zuidhof (s4160703)

Radboud University Nijmegen

29/09/2015

Exercise 1

The entire code listing is in a separate file. The listings shown here are merely code snippets.

1. The function $t = f(x)$ we have created is $f(x) = 1 + sin(8x + 1)$. This function is neither even nor odd. An even function is a function such as $cos(x)$, and an odd function is a function such as $sin(x)$, since we changed the phase and intercept of $sin(x)$ our function is neither. See Figure 1 for the function $f(x)$ and observations $\mathcal{D}$.

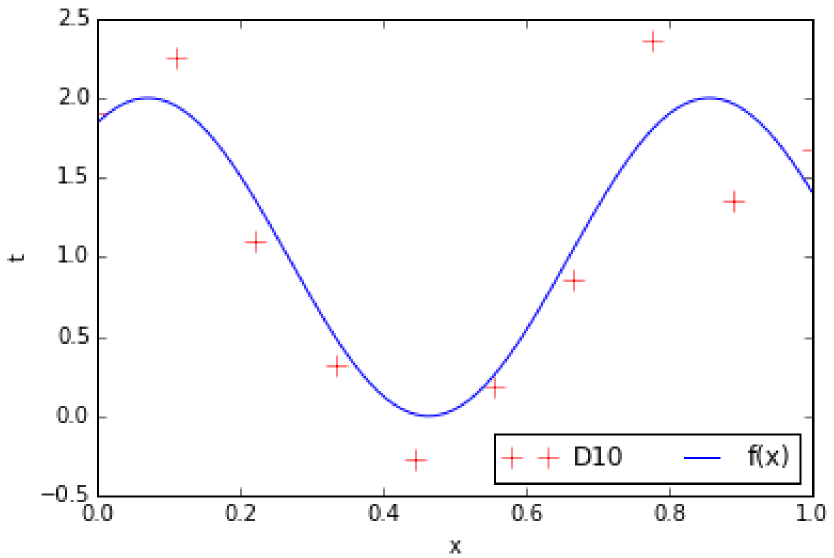

Figure 1: The function $f(x)$ and observations $\mathcal{D}$ are plotted (similar to Bishop, Fig.1.2)

2. See Listing 1 for the function $\boldsymbol{w} = PolCurFit(\mathcal{D}, M)$.

   Listing 1: Python code for function PolCurFit – Input are the observations $\mathcal{D}$ and the results t of function $f(x)$ and the order of the polynomial M. The functions calculates the A-matrix and T-vector and solves this equation to find the weights.

```
def PolCurFit(D,M):
    x = D[0]
    t = D[1]
    M = M + 1
    A = np.array([[Aij(i,j,x) for j in xrange(M)] for i in xrange(↩
        M)])
    T = np.array([Ti(i,t,x) for i in xrange(M)])
    return np.linalg.solve(A,T)
```

3. See Figure 2 & 3

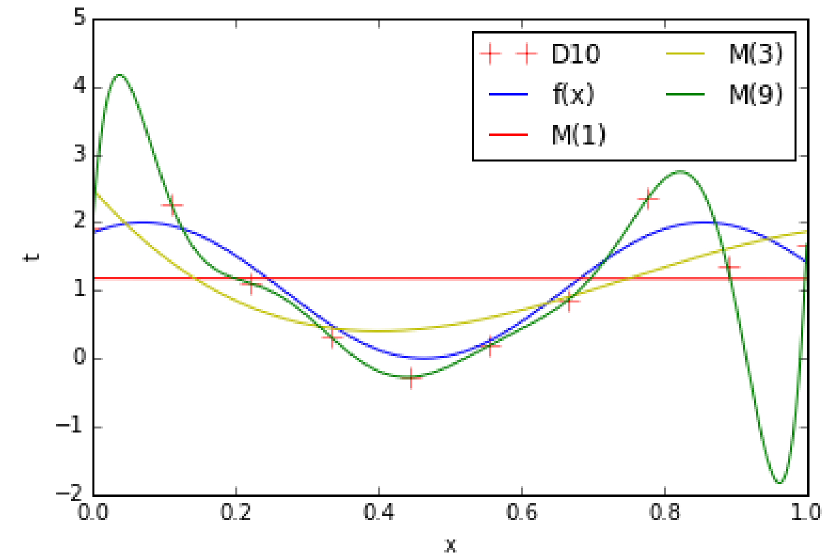

Figure 2: In this figure, the observations $\mathcal{D}$, the function $f(x)$ and polynomials of different orders of M are shown.

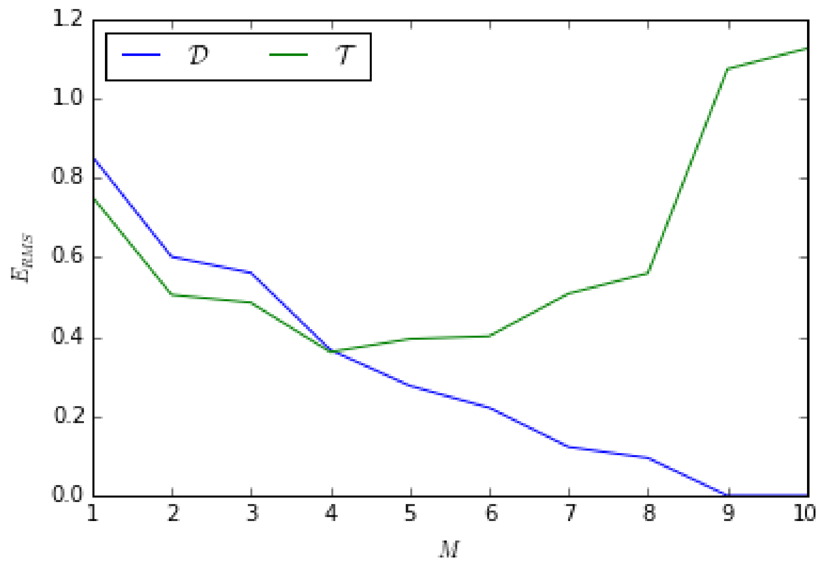

Figure 3: For the various orders $M = [1, \ldots, 10]$ and for each solution $\boldsymbol{w}^*$ the root-mean-quare error $E_{RMS} = \sqrt{2E(\boldsymbol{w}^*)/N}$ is computed of the corresponding polynomial, evaluated on both the training set $\mathcal{D}$ and the testset $\mathcal{T}$ (similar to Bishop, Fig.1.5)

4.

5.

Exercise 2

1. Simplify:

$h(x, y) = 100(y - x^2)^2 + (1 - x)^2 = 100(y^2 - 2x^2y + x^4) + x^2 - 2x + 1 =$
$100y^2 - 200x^2y + 100x^4 + x^2 - 2x + 1$

Derive:

$\frac{dh(x,y)}{dx} = -400xy + 400x^3 + 2x - 2$
$\frac{dh(x,y)}{dy} = 200y - 200x^2$

Set derivative of $h(x, y)$ equal to zero:

$200y - 200x^2 = 0 \rightarrow 200y = 200x^2 \rightarrow y = x^2$
$-400xy + 400x^3 + 2x - 2 = 0 \rightarrow$ use $y = x^2 \rightarrow 400x^3 = 400x^3 + 2x - 2 \rightarrow 2x = 2 \rightarrow x = 1$
$y = x^2$ and $x = 1 \rightarrow y = 1^2 = 1$
(1,1) is the minimum of $h(x, y)$

2.

Exercise 3