Synchrotron-emission of the Crab Nebula

Report about a practical training in astrophysics by Christoph Wendel

This work is about synchrotron-radiation from a region with the radius $r \approx 0.034\,\mathrm{pc}$ in the central part of the Crab Nebula. The relativistic electrons, that are distributed according to a power-law, owe an energy of the order of magnitude of $10^{42}\,\mathrm{erg}$. The assumption of a magnetic flux density of $B = 10^{-7}\,\mathrm{T}$ and a number-density of the electrons of $5 \times 10^{-8}\,\mathrm{cm}^{-3}$ seems reasonable. The synchrotron radiation power of all these electrons was determined to approximately $10^{33}\,\mathrm{erg\,s^{-1}}$. In contrast, the strict optical synchrotron emission power is about $10^{32}\,\mathrm{erg\,s^{-1}}$. In the sequel the filter-function of filter F550M of Hubble's ACS which cuts a small range of visible light, was considered and the emitted synchrotron power received by this filter was computed to approximately $10^{31}\,\mathrm{erg\,s^{-1}}$.

1 Introduction

figure 1: Image of the Crab Nebula taken by the Subaru Telescope. [7]

In the year 1054 A.D. Chinese astronomers observed a „guest star“, that was brighter than planet Venus, appearing in the constellation of Taurus [1] for some weeks and fading away afterwards. Today we know that this event was a core-collapse supernova, i.e. an explosion of a massive star ($M_0 > 8M_\odot$) after running out of fuel. By this, an amount of kinetic energy of the order of magnitude $10^{44}\,\mathrm{J}$ is released and most of the matter previously constituting the star is ejected into the interstellar space ($v_{\text{eje}} \approx 2\,000 - 30\,000\,\mathrm{km/s}$) while a smaller fraction of the progenitor star collapses to form a neutron star [3].
In the case of SN 1054 the neutron star is spinning and thus observable as a pulsar, that emits pulses over the entire EM-spectrum with a period of approximately $33\,\mathrm{ms}$ [2]. The ejected matter spread into space and formed the supernova remnant, that we call Crab Nebula (for the look it had, when Lord Rosse drew it in 1844 [4]). The distance of the slightly ellipsoidal Crab Nebula is $\approx 2\,\mathrm{kpc}$ [5] and its radius is about $3\,\mathrm{pc}$ [6] in the radio regime [1]. In figure 1 the wholly Nebula is shown in optical light which reveals two major constituents of it:
Firstly the filamentary structure is made from the debris of the progenitor star. It contains several elements, predominantly hydrogen, but also neutral oxygen, singly ionised sulphur and doubly ionised oxygen [8].
Secondly the bluish diffuse haze is, according to commonly accepted theory, due to synchrotron radiation, i.e. EM-radiation emitted by relativistic electrons, which gyrate around the magnetic field lines located in the nebula.

Figure 2 shows an X-ray image of the Nebula, corresponding to the synchrotron part. The bright dot is the central pulsar. About $5''$ at the lower left of the pulsar there is a also bright area called „anvil“. This region is the object of my practical training. From figure 3 one can approximately estimate the angular radius of the anvil-region to $3.5''$. Taken the above distance of the Nebula one gets the radius:

$$r = 3.5'' \cdot \overbrace{\frac{1'}{60''} \cdot \frac{1^\circ}{60'}}^{\text{conversion to degree}} \cdot \overbrace{\frac{2\pi\ \text{rad}}{360^\circ}}^{\text{conversion to rad}} \cdot \overbrace{2000\,\text{pc}}^{d} \cdot \overbrace{3.1 \times 10^{18}\,\text{cm/pc}}^{\text{conversion to cm}} \approx 1.05 \times 10^{17}\,\text{cm}$$

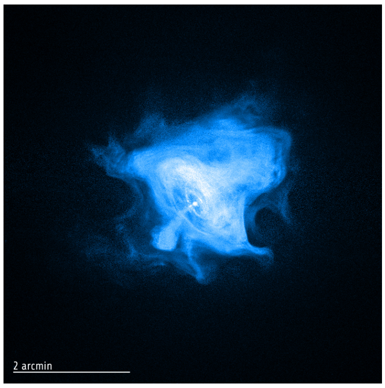


figure 2: Chandra-ACIS-image of the Crab Nebula. [9]

One can assume that the anvil-region is roughly of spherical shape. In this case its volume is

$$V = \frac{4\pi}{3} \cdot r^3 \approx 4.83 \times 10^{51}\,\mathrm{cm}^3. \tag{1}$$

In several HST-ACS-images (see figure 3) a rise in the brightness was observed in the anvil-region. The ultimate aim of the astrophysicists of Würzburg is to find out, whether the energy, that was released in this brightness-raising can be associated with a deceleration of the angular velocity of the central pulsar, that is believed to power the whole nebula. The aim of my practical training was to determine the synchrotron-power-output of the considered region.

[1] It is $1\,\mathrm{pc} \approx 3.3\,\mathrm{Ly} \approx 3.1 \times 10^{16}\,\mathrm{m}$.

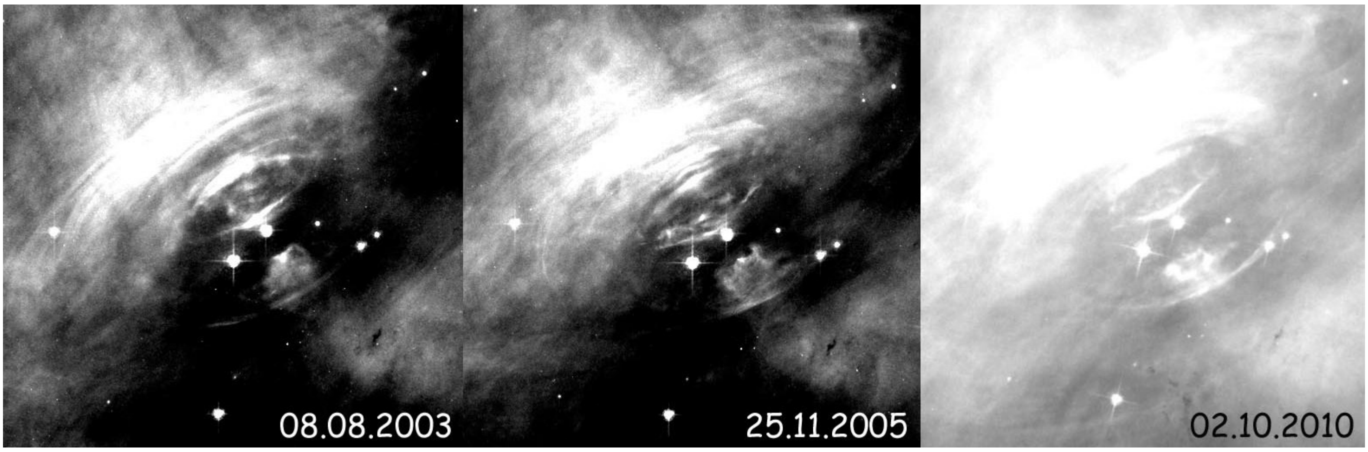


figure 3: Series of HST-ACS-images taken with filter F550M of the inner Crab Nebula of different dates. The upper right bright star is the Crab pulsar. To the lower right of the two bright stars is the anvil-region. Be aware that the image of 2010 was recorded with another aperture than those of 2003 and 2005. Compared with figure 2 this image is rotated about $90°$ to the left [10]

2 Theoretical model of synchrotron radiation

The first step was to acquire the mechanism, responsible for the emission of synchrotron radiation, which is described in the following. It is general consensus, that the crab pulsar is permanently emitting electrons and positrons (henceforth just called electrons) from its surface outwards. It is not clear in all details today, how these electrons are accelerated to relativistic velocities but a widespread idea is that they get their energy by the strong electric field, that is caused by Lorentz-transforming the magnetic dipole-field ($B \approx 10^8\,\mathrm{T}$) of the pulsar into the rest frame of the electrons. Not all electrons have the same energy E. Actually the electrons' energy covers a wide range. This energy-distribution can be described by a power-law

$$n(E)dE = KE^{-q}dE, \tag{2}$$

where n is the number-density per unit energy-intervall (unit: $\mathrm{m}^{-3}\mathrm{J}^{-1}$), E is the energy of the electrons (unit: J), K is a normalisation-coefficient (unit: $\mathrm{m}^{-3}\mathrm{J}^{\mathrm{q}-1}$) and q is a dimensionless power-law-index. So $n(E)dE$ is the number of electrons whose energy lies within the interval $[E, E + dE]$ per unit volume.
Alternatively the energy-distribution can be described, using the Lorentz-factor γ instead of E

$$n'(\gamma)d\gamma = K'\gamma^{-q}d\gamma \tag{3}$$

As $n(E)dE \stackrel{!}{=} n'(\gamma)d\gamma$, $E = \gamma \cdot m_e c^2$ and $dE = d\gamma \cdot m_e c^2$ one can find the relation

$$K = K' \cdot (m_e c^2)^{q-1} \tag{4}$$

This time the unit of n' is m^{-3}, γ is without unit and K' has the unit m^{-3}, too.
Throughout the whole energy-range q and K is not constant but has specific values for the radio-regime (index 1), the intermediate regime (index 2) and for the high-energy regime (index 3). By an investigation over several publications I found values for q and for the total, energy-integrated number-density n_{tot} of electrons. The following values are used in all subsequent calculations:

Radio-regime:	$q_1 = 1.52$	for	$10^8\,\mathrm{eV} < E < 10^{10.75}\,\mathrm{eV}$	$196 < \gamma < 110\,047$	from [11]	(5a)
Intermediate regime:	$q_2 = 1.8$	for	$10^{10.75}\,\mathrm{eV} < E < 10^{12.5}\,\mathrm{eV}$	$110\,047 < \gamma < 6\,188\,410$	from [11]	(5b)
High-energy regime:	$q_3 = 2.4$	for	$10^{12.5}\,\mathrm{eV} < E$	$6\,188\,410 < \gamma$	from [11]	(5c)

$$n_{\text{tot}} \approx 5 \times 10^{-8}\,\mathrm{cm}^{-3} \quad \text{(exactly } 2 \times 10^{-8}\,\mathrm{cm}^{-3}\text{) from [12]} \tag{6}$$

These electrons travel out into the nebula, where a magnetic field is present. By the study of various publications I concluded, that an appropriate value for the magnetic flux density in the considered region of the Crab Nebula is

$$B \approx 10^{-7}\,\mathrm{T}. \tag{7}$$

Values of this order of magnitude have been determined from observational data e.g. by [12] and [13].
Moving through this magnetic field the electrons are deflected by the Lorentz-force. Imagine an electron is moving with a

velocity $\vec{v} \perp \vec{B}$. Consequently this electron moves in circles around the magnetic field lines. Let now α be the so-called pitch angle, which is the angle between $\vec{v}$ and $\vec{B}$. In the case $\alpha \neq \pi/2$ the electrons move in spiral paths along the magnetic field lines. This motion is called gyration and the corresponding frequency is the gyro-frequency $f_{\mathsf{gyr}} = eB/(2\pi\gamma m_e)$, sometimes called Larmor-frequency or cyclotron-frequency, too. Here e is the elementary charge, B the magnetic flux density, and m_e the electron's mass.

According to electrodynamics every accelerated charge emits EM-radiation. In the case of a nonrelativistically gyrating electron the emitted radiation is called cyclotron radiation. If an observer examines the emitted radiation he will find a sinusoidally oscillating field strength, which means that this radiation has exactly the frequency f_{gyr}, thus is monochromatic. If an electron moves with relativistic velocities the radiation is emitted within a small cone of half-angle γ^{-1} around $\vec{v}$ due to relativistic effects. Hence, if such a gyrating electron is observed, the recorded field strength shows a recurring narrow peak as the beam sweeps over the line-of-sight. According to Fourier-transformation this pulses correspond to EM-radiation of a wide frequency range.

A complete analysis of the frequency-dependent synchrotron-power can be obtained from [14] or [15]: Starting point is the acceleration- and frequency-dependent energy distribution of the radiation which stems from electrodynamics. After switching into a convenient system of coordinates and a number of calculations and argumentations, taking relativistic effects like aberration, doppler shifting and time retardation into account, one can derive a formula for the power-spectrum of a single relativistcally gyrating electron, i.e. the emitted power per unit frequency as a function of frequency.[2]

$$P(f,\gamma) = 2\pi \frac{\sqrt{3}e^3 B \sin\alpha}{8\pi^2 \epsilon_0 c m_e} \cdot \frac{f}{f_{\mathsf{crit}}(\gamma)} \cdot \int_{\frac{f}{f_{\mathsf{crit}}(\gamma)}}^{\infty} K_{5/3}(z) dz \tag{8}$$

where ϵ_0 is the vacuum permittivity, c the speed of light, $K_{5/3}(z)$ the modified Bessel function of order $5/3$ and $f_{\mathsf{crit}}(\gamma) = 3/2\gamma^2 f_{\mathsf{gyr}} \sin\alpha$. The unit of $P(f,\gamma)$ is $\mathrm{W\,Hz^{-1}}$, so $P(f,\gamma)df$ is the power, emitted in the frequency interval $[f, f+df]$ by one single electron with energy γ.

In case of the Crab, there is not only one electron gyrating in the magnetic field but a large number of electrons with different values of E, which are distributed with equation (2). To obtain the total amount of power, which is emitted by all these electrons one should add each power emitted by a certain electron up. To be more precise, one should add each power emitted by an electron of energy E multiplied with the numberdensity of electrons with energy E up, hence add $P(f,\gamma(E)) \cdot KE^{-q}dE$ up, to get the total amount of power $p(f)$, which is emitted by all these electrons per unit volume. This results in an integral

$$p(f) = \int_0^\infty P(f,\gamma(E)) \cdot KE^{-q} dE$$

which can be computed to yield the synchrotron-power-density output per unit frequency (unit: $\mathrm{W\,cm^{-3}Hz^{-1}}$):

$$p(f) = 2\pi \frac{\sqrt{3}e^3 BK}{16\pi^2 \epsilon_0 c m_e (q+1)} \cdot \left(\frac{2\pi f m_e^3 c^4}{3eB}\right)^{-(q-1)/2} \cdot \frac{\sqrt{\pi}\Gamma\left(\frac{q}{4}+\frac{19}{12}\right)\Gamma\left(\frac{q}{4}-\frac{1}{12}\right)\Gamma\left(\frac{q}{4}+\frac{5}{4}\right)}{\Gamma\left(\frac{q}{4}+\frac{7}{4}\right)} \tag{9}$$

Here, $\Gamma(z)$ is the Euler-Gamma-function. Additionally, in the last step of transformation it was averaged over α of an isotropic distribution of pitch angles α.

We now have an equation which gives us the emitted power per unit frequency per unit volume. Notice that $p(f)$ is still a function of the frequency. This formula will be evaluated during the next part of the report.

3 Computational analysis

To use the formula (9) one needs to know B, q and K as all the other values are well-known. For B one can assume the value from (7) and for q one can assume the values from (5). K is not known yet, but it can be computed from (5) and (6). This will be done in the following, using the computer program *Mathematica*.

First of all let us exemplarily plot the power spectrum of one single electron from equation (8). This is done in figure 4 for $\alpha = \pi/2$ and for different electron energies. One will remark that not only the amount of emitted energy (i.e. the area below each curve) but also the frequency of maximum emittivity increases with increasing γ, which is a typical characteristic for synchrotron radiation. For latter use I have plotted the equation (8) for the border values of γ from $(5\mathrm{a})$ - $(5\mathrm{c})$ (look appendix figures 9(a), 9(b) and 9(c)). From this plots one can roughly assign a border frequency to each border-γ. The following values are corresponding:

$$\gamma = 196 \quad \Leftrightarrow \quad f = 10^8\,\mathrm{Hz} \tag{10a}$$

$$\gamma = 110\,047 \quad \Leftrightarrow \quad f = 3 \times 10^{13}\,\mathrm{Hz} \tag{10b}$$

$$\gamma = 6\,188\,410 \quad \Leftrightarrow \quad f = 10^{17}\,\mathrm{Hz} \tag{10c}$$

[2]In the following, all formulae are out of [14] unless otherwise stated.

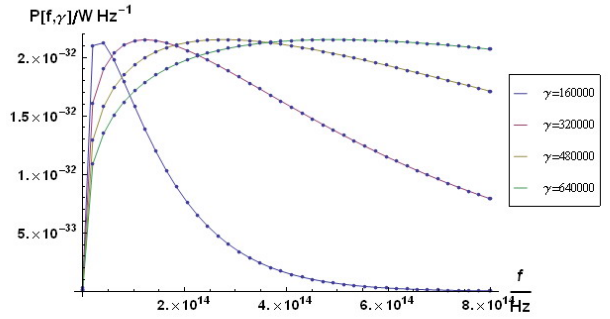


figure 4: Power per unit frequency of synchrotron radiation of one single electron for various values of γ.

Likewise for latter use, we have to get the border-γ values for electrons, that emit synchrotron radiation in optical wavelengths. In figure 9(d) and 9(e) in the appendix there are synchrotron-spectra for different values of γ. From these plots one can get the approximate correspondence:

$$\lambda = 700\,\mathrm{nm} \quad \Leftrightarrow \quad f \approx 4 \times 10^{14}\,\mathrm{Hz} \quad \Leftrightarrow \quad \gamma = 320\,000 \tag{11a}$$

$$\lambda = 400\,\mathrm{nm} \quad \Leftrightarrow \quad f \approx 7.5 \times 10^{14}\,\mathrm{Hz} \quad \Leftrightarrow \quad \gamma = 2\,000\,000 \tag{11b}$$

Now we are able to tackle the electrons' energy distribution according to equation (3). For the present we have to omit K', the reason is the following: Remember that the energy distribution consists of three parts with different slopes i.e. with differing values of q. So we could write

$$n'(\gamma)/K' = \begin{cases} n'_1(\gamma)/K' = C_1\gamma^{-q_1} & \text{for} \quad 196 < \gamma < 110\,047 \\ n'_2(\gamma)/K' = C_2\gamma^{-q_2} & \text{for} \quad 110\,047 < \gamma < 6\,188\,410 \\ n'_3(\gamma)/K' = C_3\gamma^{-q_3} & \text{for} \quad 6\,188\,410 < \gamma \end{cases} \tag{12}$$

what can be done, using the *Mathematica*-function `If[condition,t,f]`, for example. As the energy distribution should be continuous our first aim is to adjust the coefficients so that there are no leaps in the distribution. I set $C_2 = 1$ and then I found the following values

$$C_1 = 0.038\,757\,7$$
$$C_3 = 11\,883.585$$

Using these coefficients the energy distribution is shown in a logarithmic plot in figure 5, in which one can see that the distribution is indeed continuous.
Now we can compute K' by using the knowledge that the sum of all differential number-densities is exactly the value n_{tot} from equation (6). Hence, the area below the electrons' energy distribution should be equal to n_{tot}. So we have

$$n_{\mathsf{tot}} = \int_{196}^{\infty} n'(\gamma)d\gamma = \int_{196}^{\infty} K'C_{1,2,3}\gamma^{-q_{1,\,2,\,3}}d\gamma \tag{13}$$

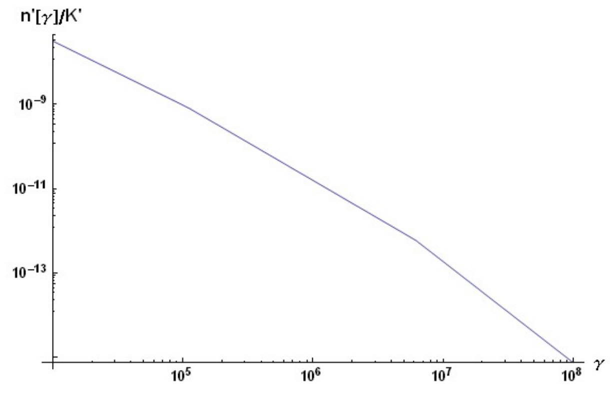


figure 5: Energy distribution of relativistic electrons in the Crab Nebula. Notice that in this plot $n'(\gamma)$ is divided by the normalisation-coefficient K'.

Resolving this for K' and computing the integral one obtains

$$K' = 1058.53 \times 10^{-8}\,\mathrm{cm}^{-3} \tag{14}$$

Now we have all values appearing in the electrons' energy-density. As an useful application we can get the total energy-density (unit: $\mathrm{eV\,cm^{-3}}$) conserved in all the EM-radiation emitting electrons by simply determine the integral

$$\varepsilon_{\mathsf{e,\,EM}} = \int_{196}^{\infty} \gamma n'(\gamma)d\gamma \cdot 511\,\mathrm{keV} \approx 759\,\mathrm{eV\,cm}^{-3} \tag{15}$$

Here we had to add the factor $511\,\mathrm{keV}$ because γ is the energy of an electron in units of m_ec^2. Secondly, the energy-density of those electrons emitting optical synchrotron radiation can be determined by using the border-γ values from equation $(11\mathrm{a})$ and $(11\mathrm{b})$.

$$\varepsilon_{\mathsf{e,\,opt}} = \int_{320\,000}^{2\,000\,000} \gamma n'(\gamma)d\gamma \cdot 511\,\mathrm{keV} \approx 151\,\mathrm{eV\,cm}^{-3} \tag{16}$$

Now it is staightforward to get the total energy that is stored in either all or in the optically emitting electrons by multipling with the radiating volume V (from (1)) of the anvil:

$$E_{\mathsf{e,\,EM}} = \varepsilon_{\mathsf{e,\,EM}} \cdot V \cdot 1.602\,2 \times 10^{-19}\,\mathrm{J\,eV}^{-1} \cdot 10^{7}\,\mathrm{erg\,J}^{-1} \approx \underline{\underline{5.88 \times 10^{42}\,\mathrm{erg}}} \tag{17}$$

$$E_{\mathsf{e,\,opt}} = \varepsilon_{\mathsf{e,\,opt}} \cdot V \cdot 1.602\,2 \times 10^{-19}\,\mathrm{J\,eV}^{-1} \cdot 10^{7}\,\mathrm{erg\,J}^{-1} \approx \underline{\underline{1.17 \times 10^{42}\,\mathrm{erg}}} \tag{18}$$

The cgs-unit $1\,\mathrm{erg} = 10^{-7}\,\mathrm{J}$ is used here to be able to compare these values with other elderly astrophysical resources. These quantities can be compared with the energy conserved in the magnetic field, which is

$$E_{\mathsf{B}} = \frac{B^2}{2\mu_0} \cdot V \approx 1.92 \times 10^{44}\,\mathrm{erg} \tag{19}$$

where μ_0 is the vacuum permeability. One can see, that E_{B} is bigger than $E_{\mathsf{e,\,EM}}$. This should be the case because if it was the other way round, the electrons would radiate much more intensively, thus lose much more energy and this would again lead to $E_{\mathsf{B}} > E_{\mathsf{e,\,EM}}$.

From now on we deal with the synchrotron radiation of the ensemble of relativistic electrons. To do this we use equation (9) but we have to split it again into three parts with different values for q and K like we have done in equation (12). Corresponding to equations (5) as well as (10) we separate into Radio-, intermediate and high-energy regime while making use of equation (4):

$$K_1 = C_1 \cdot K' \cdot (m_e c^2)^{q_1 - 1} \quad \text{with} \quad p_1 = 1.52 \quad \text{for} \quad 10^8\,\mathrm{Hz} < f < 3 \times 10^{13}\,\mathrm{Hz} \tag{20a}$$
$$K_2 = C_2 \cdot K' \cdot (m_e c^2)^{q_2 - 1} \quad \text{with} \quad p_2 = 1.8 \quad \text{for} \quad 3 \times 10^{13}\,\mathrm{Hz} < f < 10^{17}\,\mathrm{Hz} \tag{20b}$$
$$K_3 = C_3 \cdot K' \cdot (m_e c^2)^{q_3 - 1} \quad \text{with} \quad p_3 = 2.4 \quad \text{for} \quad 10^{17}\,\mathrm{Hz} < f \tag{20c}$$

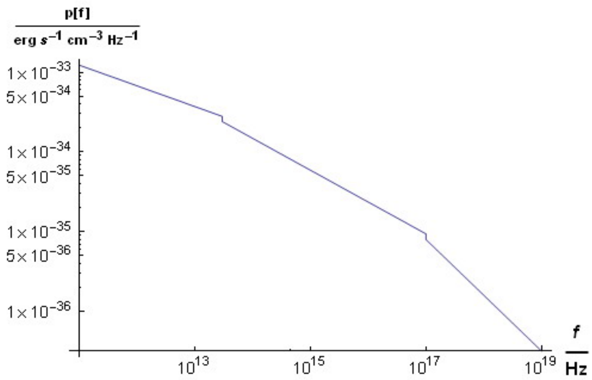


figure 6: Power-density $p(f)$ per unit frequency of synchrotron radiation of the whole ensemble of electrons, i.e. synchrotron spectrum over the entire EM-range. Notice that $\mathrm{erg/s}$ is used instead of W.

The equation (9) can now be displayed with the help of the *Mathematica*-function `If[condition,t,f]`, which allows us to apply the different q and K values. This is done in figure 6, where the spectral power-density $p(f)$ is shown for the whole EM-spectrum in a logarithmic manner, as well as in figure 8 on page 6, where the spectral power-density $p(f)$ is shown for the range of visible light (blue curve).

As one can see, the synchrotron spectrum of a power-law-distribution of electrons is again a power-law, although the synchrotron spectrum of one single electron is quite different from a power law. The small discontinuities in $p(f)$ are relics from the adjustment of the C-values, which are again results of the abrupt change of the q-values. One should keep in mind, that the division of the energy-distribution into three parts with different but within one part constant q-values is just a rough approximation. In real, the q doesn't change its value abruptly at one specific value of γ or f respectively, but changes slowly and steadily.

Now one can again take the area below the curve in figure 6 to get the total power-density, i.e. computing the integral

$$p_{\mathsf{tot,\,EM}} = \int\limits_{10^8\,\mathrm{Hz}}^{\infty} p(f) df \approx 1.56 \times 10^{-18}\,\mathrm{erg\,s^{-1}\,cm^{-3}} \tag{21}$$

Alternatively we can alter the integral-borders to the values from equation $(11a)$ and $(11b)$ to obtain the synchrotron power-density output, that is strictly visible light.

$$p_{\mathsf{tot,\,opt}} = \int\limits_{4 \times 10^{14}\,\mathrm{Hz}}^{7.5 \times 10^{14}\,\mathrm{Hz}} p(f) df \approx 2.61 \times 10^{-20}\,\mathrm{erg\,s^{-1}\,cm^{-3}} \tag{22}$$

Multipling these quantities with the volume V of the considered region of the Crab, yields the emitted power.

$$P_{\mathsf{tot,\,EM}} = p_{\mathsf{tot,\,EM}} \cdot V \approx \underline{\underline{7.55 \times 10^{33}\,\mathrm{erg\,s^{-1}}}} \tag{23}$$
$$P_{\mathsf{tot,\,opt}} = p_{\mathsf{tot,\,opt}} \cdot V \approx \underline{\underline{1.26 \times 10^{32}\,\mathrm{erg\,s^{-1}}}} \tag{24}$$

Because of $P_{\mathsf{tot,\,opt}}/P_{\mathsf{tot,\,EM}} \approx 1/60$ the whole EM-synchrotron power is approximately 60 times as big as the optical synchrotron power.

Let us come back now to figure 3, which displays the anvil-region. This picture, which shows just the intensity but no dependency of the wavelength of the incoming photons, was recorded with the filter F550M of the Advanced Camera for Surveys (ACS) of the Hubble Space Telescope. The transmittance of F550M as a function of the wavelength from [16] can be imported into *Mathematica* as a list using the function
`filter=Import["D:/folder/subfolder/f550m.dat","Table"]`.

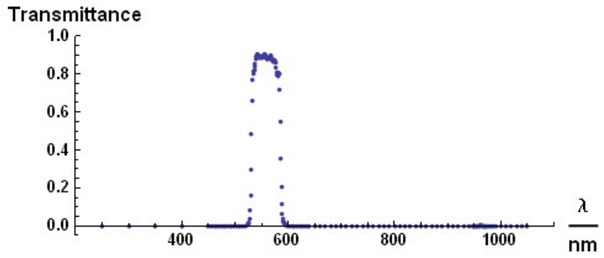


figure 7: The filter-function of F550M. The transmittance is plotted as a function of wavelength.

It is visualized in figure 7.
Obviously the ACS equipped with F550M only receives photons with $520\,\mathrm{nm} \lessapprox \lambda \lessapprox 600\,\mathrm{nm}$. Using the *Mathematica*-functions `wavelengths=filter[[All,1]]`, `transmittances=filter[[All,2]]`, `frequencies=N[299792458/(wavelengths*10^(-9))]` and `filteroffreq=Partition[Riffle[frequencies,transmittances],2]` one after the other, the list can be manipulated so that the transmittance is a function of the frequency. Now we want to determine the amount of power-density that is released in the frequency range that can be received by the filter F550M. To do this, we make a discrete function, i.e. a list, out of the continuous function $p(f)$ of equation (9) by using the command
`powdens=Table[{f,p[f]},{f,frequencies}]`.
As we want to have the received power-density, we have to multiply the emitted spectral power-density with the filter-function. This can be done by
`recpowdens=Table[{frequencies[[i]],filteroffreq[[i,2]]*powdens[[i,2]]},{i,Length[frequencies]}]`.

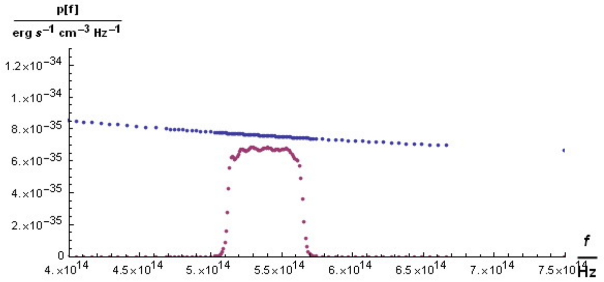


figure 8: The blue curve shows the spectral power-density $p(f)$ per unit frequency of synchrotron radiation of the whole ensemble of electrons, i.e. the emitted synchrotron spectrum over the visible range of f. The lilac curve represents the spectral power-density received with filter F550M. Notice that $\mathrm{erg/s}$ is used instead of W.

This quantity is shown in figure 8 as the lilac curve. The synchrotron-light, that is described by this curve is responsible for the exposed sites in figure 3.
For following purpose we calculate the distances between every single pair of list-dots of the frequency-axis with the function `frequencyintervals=Table[frequencies[[i]]-frequencies[[i+1]],{i,Length[frequenzen]-1}]`.
The above quantity `recpowdens` is still a spectral one. This means that it is the power-density emitted per unit frequency as a function of the frequency. To compute the total received power-density, we have to determine the area below the lilac curve in figure 8. We do this by adding many rectangles of height `recpowdens[[i+1,2]]` and breadth `frequencyintervals[[i]]` up. The necessary command is
`totrecpowdens=Sum[frequencyintervals[[i]]*recpowdens[[i+1,2]],i,1,Length[frequencyintervals]]`.
This value $p_{\text{tot, rec}}$ is now multiplied with the volume V of the anvil-region. This gives us the entire power received[3] by the filter.

$$P_{\text{tot, rec}} = p_{\text{tot, rec}} \cdot V \approx \underline{\underline{1.71 \times 10^{31}\,\mathrm{erg\,s^{-1}}}} \tag{25}$$

This is approximately $1/7$ of the total optical emitted synchrotron-power, which seems to be reasonable if one compares the areas under the lilac and blue curve in figure 8.

The same procedure as the one described until now has been executed, using the following formula out of [17] instead of equation (9):

$$p(f) = 1.7 \times 10^{-21} \cdot a(q) \cdot K \cdot B^{(q+1)/2} \cdot \left(\frac{6.26 \times 10^{18}\,\mathrm{Hz}}{f}\right)^{(q-1)/2} \mathrm{erg\,s^{-1}cm^{-3}Hz^{-1}} \tag{26}$$

The values $a(q)$ are listed in [17]: It is used $(a,q) = (1.5, 0.147), (2.0, 0.103), (2.5, 0.085\,2)$ here. It seems reasonable to fill in B in the unit G[4] as cgs-units are used throughout [17]. By this one gets the following results:

$$P_{\text{tot, EM}} \approx \underline{\underline{6.66 \times 10^{33}\,\mathrm{erg\,s^{-1}}}} \tag{27}$$

$$P_{\text{tot, opt}} \approx \underline{\underline{1.11 \times 10^{32}\,\mathrm{erg\,s^{-1}}}} \tag{28}$$

$$P_{\text{tot, rec}} \approx \underline{\underline{1.50 \times 10^{31}\,\mathrm{erg\,s^{-1}}}} \tag{29}$$

Obviously this values are close to those, that we have obtained by using (9) The slight differences might be due to rounding errors by deriving the equation (26). Furthermore it is not clear how [17] treated the pitch angle α of the electrons' trajectory. Since it doesn't appear in equation (26) it was either integrated over α, like it was done by [14], too, or it was filled in a certain value. So the pitch angle is a second source of inaccuracy.
Nevertheless the orders of magnitude of the values received by equation (9) and by (26) are in good agreement.

[3]Remark that in this case „received" does not denote that the quantity $p_{\text{tot, rec}}$ is really recorded by the filter F550M. It rather means that this quantity is emitted of the anvil-region in all directons in those wavelengths that are receivable by the filter. If the ACS equipped with F550M would occupy a whole sphere surrounding the anvil-region, it would really record $p_{\text{tot, rec}}$.
[4]It is $1\,\mathrm{G} = 10^{-4}\,\mathrm{T}$.

4 Summary

The synchrotron emission power of a region (radius $r \approx 0.034\,\mathrm{pc}$) in the Crab Nebula near the central pulsar was considered. The electrons, distributed by a power-law, conserve an energy of the order of magnitude of $10^{42}\,\mathrm{erg}$. The synchrotron radiation power of all these electrons is, assumed a magnetic flux density of $B = 10^{-7}\,\mathrm{T}$ and a number-density of the electrons of $5 \times 10^{-8}\,\mathrm{cm}^{-3}$, about $10^{33}\,\mathrm{erg\,s^{-1}}$, while the strict optical synchrotron emission power is of the order of magnitude of $10^{32}\,\mathrm{erg\,s^{-1}}$. Taken the filter-function of filter F550M of Hubble's ACS which cuts a small range of visible light, the emitted synchrotron power is approximately $10^{31}\,\mathrm{erg\,s^{-1}}$. This filter was used for a series of images covering the so-called anvil-region of the Crab, in which a rise of brightness of this region was observed.
In future work, it would be interesting to examine these ACS-images more thoroughly and to adjust the theoretical model to the observations. By this one could, for example, infer more accurate values for the electrons' number-density or for the magnetic flux density. Another future aim is to examine, if the energy released in the rise of brightness can be recognized in a slow down of the pulsar's rotational velocity which is, in the end, the energy source of the whole nebula.

References

[1] J. J. L. Duyvendak. *Further data bearing on the identification of the crab nebula with the supernova of 1054 A.D.*. Bibcode: 1942PASP...54...91D

[2] R. N. Manchester et al. *The australia telescope national facility pulsar catalogue*. Bibcode: 2005AJ....129.1993M

[3] S. Rosswog, M. Brüggen. *Introduction to High-Energy Astrophysics*. Cambridge University Press. 2007

[4] http://www.seds.org/messier/more/m001_rosse.html

[5] V. Trimble. *The Distance to the Crab Nebula and NP 0532*. Bibcode: 1973PASP...85..579T

[6] A. A. Abdo et al. *Gamma-ray flares from the Crab Nebula*. arXiv:1011.3855v3 [astro-ph.HE]

[7] http://www.naoj.org/Pressrelease/2007/03/12/index.html

[8] http://www.spacetelescope.org/news/heic0515/

[9] http://chandra.harvard.edu/photo/2008/crab/

[10] The HST online archive. http://archive.stsci.edu/hst/search.php

[11] A. S. Wilson. *The structure of the Crab Nebula II - The spatial distribution of the relativistic electrons*. Bibcode: 1972MNRAS.160..355W

[12] V. J. Moroz. *The radiation flux from the Crab Nebula at λ 2 μ and some conclusions on the spectrum and magnetic field*. Bibcode: 1960SvA.....4..250M

[13] J. H. Oort et al. *Polarization and composition of the Crab Nebula*. Bibcode: 1956BAN....12..285O

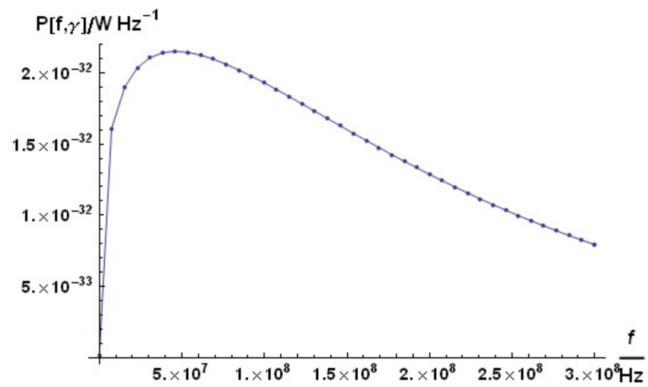
[14] M. S. Longair. *High energy astrophysics - volume 2*. Cambridge University Press. 2002

[15] G. B. Rybicki, A. P. Lightman. *Radiative processes in astrophysics*. John Wiley & Sons. 1979

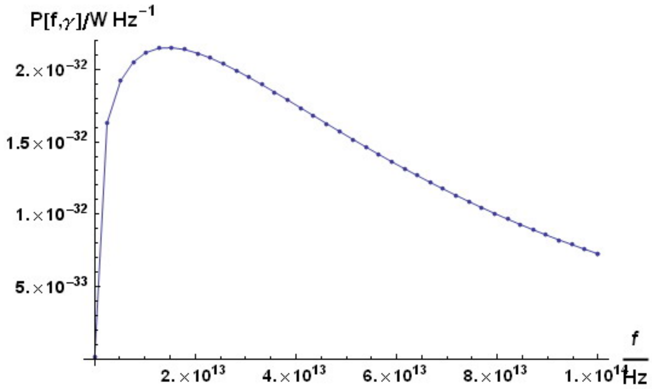
[16] http://acs.pha.jhu.edu/instrument/filters/data/f550m.dat

[17] F. Melia. *High-energy astrophysics*. Princeton University Press, 2009

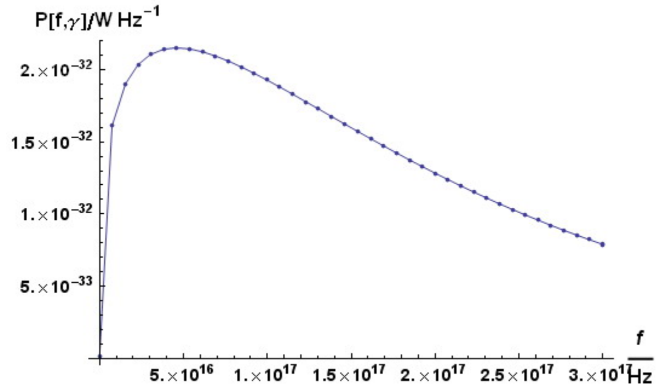
5 Appendix



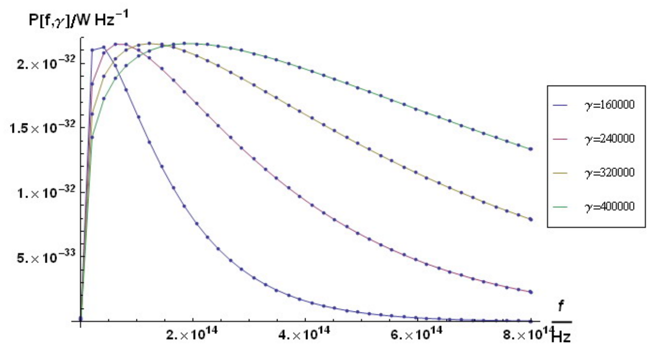
(a) Power per unit frequency of synchrotron radiation of one single electron with $\gamma = 196$.



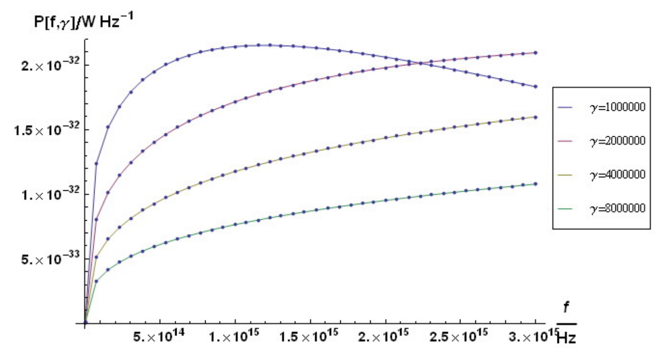
(b) Power per unit frequency of synchrotron radiation of one single electron with $\gamma = 110\,047$.



(c) Power per unit frequency of synchrotron radiation of one single electron with $\gamma = 6\,188\,410$.



(d) Power per unit frequency of synchrotron radiation of one single electron for various values of γ. The maxima are arranged around $f \approx 4 \times 10^{14}\,\mathrm{Hz}$ to get the corresponding value for γ.



(e) Power per unit frequency of synchrotron radiation of one single electron for various values of γ. The maxima are arranged around $f \approx 7.5 \times 10^{14}\,\mathrm{Hz}$ to get the corresponding value for γ.

figure 9