Drone Mechanics

Lutz Christophe

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1 Frames

R_g is terrestrial frame supposed Galilean.

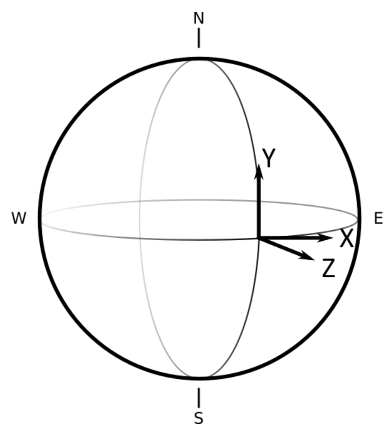

Figure 1: Terrestrial Galilean frame

R is drone frame.

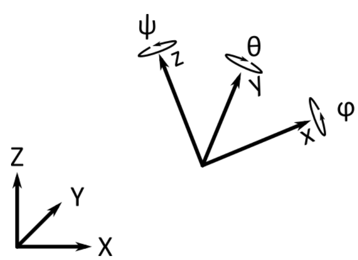

Figure 2: Drone frame

ψ is the yaw angle. θ is the roll angle. ϕ is the pitch angle.

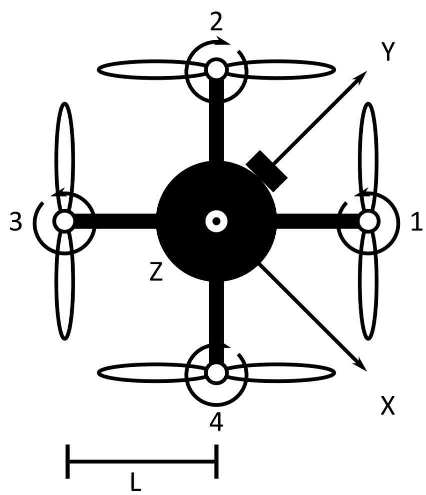

Figure 3: Drone frame Top View

2 Rotation Matrix

Matrices of rotation with change of basis.

$$R(x,\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & s\phi \\ 0 & -s\phi & c\phi \end{bmatrix}$$

$$R(y,\theta) = \begin{bmatrix} c\theta & 0 & -s\theta \\ 0 & 1 & 0 \\ s\theta & 0 & c\theta \end{bmatrix}$$

$$R(z,\psi) = \begin{bmatrix} c\psi & s\psi & 0 \\ -s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2.1 Galilean to Drone

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

with :

$$R = R(x,\phi)R(y,\theta)R(z,\psi) = \begin{bmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ -c\phi s\psi + s\phi s\theta c\psi & c\phi c\psi + s\phi s\theta s\psi & s\phi c\theta \\ s\phi s\psi + c\phi s\theta c\psi & -s\phi c\psi + c\phi s\theta s\psi & c\phi c\theta \end{bmatrix}$$

2.2 Drone to Galilean

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = R^T \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

with :

$$R^T = R(z,\psi)^T R(y,\theta)^T R(x,\phi)^T = \begin{bmatrix} c\theta c\psi & -c\phi s\psi + s\phi s\theta c\psi & s\phi s\psi + c\phi s\theta c\psi \\ c\theta s\psi & c\phi c\psi + s\phi s\theta s\psi & -s\phi c\psi + c\phi s\theta s\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix}$$

3 Equations of Motion

3.1 Acceleration

The equation for the acceleration of the center of gravity, expressed in the Galilean frame, is:

$$m \begin{bmatrix} \ddot{X}_G \\ \ddot{Y}_G \\ \ddot{Z}_G \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + R \begin{bmatrix} 0 \\ 0 \\ F_1 + F_2 + F_3 + F_4 \end{bmatrix}$$

3.2 Rotation

The rotation of Drone frame w.r.t. Galilean frame, expressed in the Drone frame, is:

$$\omega = \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + R(x,\phi) \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + R(x,\phi)R(y,\theta) \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -s\theta \\ 0 & c\phi & s\phi c\theta \\ 0 & -s\phi & c\theta c\phi \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

Due to the symmetry plans, the matrix of inertia at the center of gravity, expressed in the Drone frame, is:

$$I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$$

with: $I_{xx} = \int (y^2 + z^2)dm$, $I_{yy} = \int (x^2 + z^2)dm$ and $I_{zz} = \int (x^2 + y^2)dm$

The equation for the rotation, expressed in the Drone frame, is:

$$(\dot{I\omega}) = I \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} + \begin{bmatrix} p \\ q \\ r \end{bmatrix} \wedge I \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \frac{L}{\sqrt{2}}(-F_1 - F_2 + F_3 + F_4) \\ \frac{L}{\sqrt{2}}(F_1 - F_2 - F_3 + F_4) \\ -M_1 + M_2 - M_3 + M_4 \end{bmatrix}$$

4 Motors and Sensors

4.1 Motors

A DC motor is modelled as a series resistor R, an inductance L, and a voltage source V_{emf}. When supplied with a voltage V, we have:

$$V - RI + L\frac{dI}{dt} - V_{emf} \tag{1}$$

A DC motor driving a load of inertia J along the axis of the shaft with a torque M, submitted to vicious friction b must satisfy:

$$M - b\dot{\theta} = J\ddot{\theta} \tag{2}$$

By construction, the motor rotation speed is proportional to the e.m.f. voltage:

$$\dot{\theta} = k_v V_{emf} \tag{3}$$

$$V_{emf} = \frac{1}{k_e}\dot{\theta} \tag{4}$$

$$k_e = \frac{1}{k_v} \tag{5}$$

By construction, the motor torque is proportional to the flowing current:

$$M = k_t I \tag{6}$$

By energy conservation, the proportionality coefficients are linked:

$$k_e = k_t = \frac{1}{k_v} = K \tag{7}$$

The transfer function of the motor (Laplace transform) is hence:

$$\frac{\dot{\Theta}(s)}{V(s)} = \frac{K}{(Js+b)(Ls+R)+K^2} \tag{8}$$