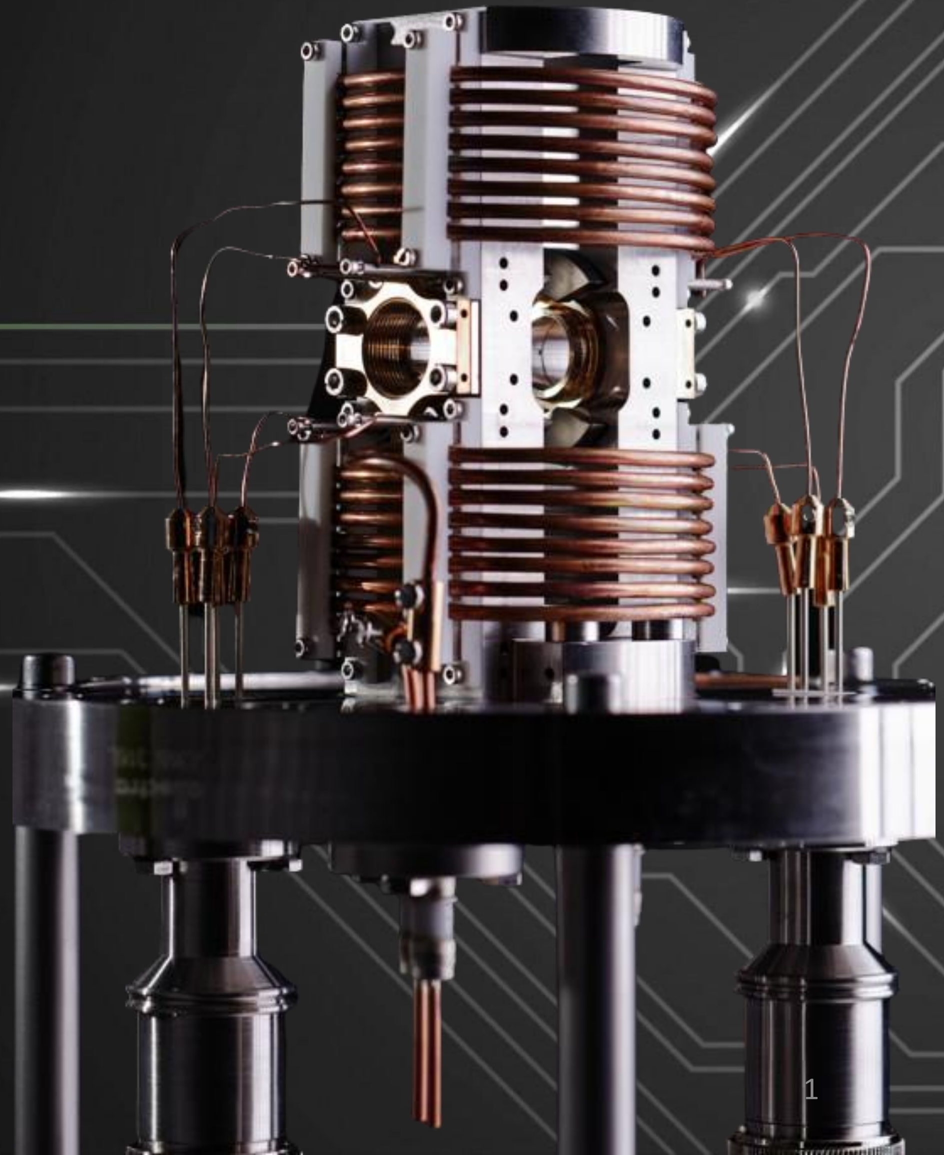
Introduction to Optimization using an Analog Quantum Computer



Victor Drouin-Touchette, 21 mars 2024

Table of Content

PASQAL and the Neutral Atom QPU

PASQAL's neutral atom QPU: harnessing light-matter interactions

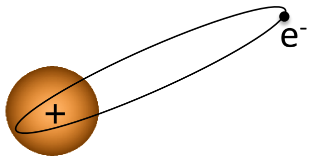
Atomic Qubits: Need a <u>controllable</u>, <u>two-level</u> system.

We use as a $|0\rangle$ the ground state of Rubidium:

$1S^2\ 2S^2 2P^6 3S^2 3P^6 3D^{10} 4S^2 4P^6\ \underbrace{5S^1}$

Rydberg state

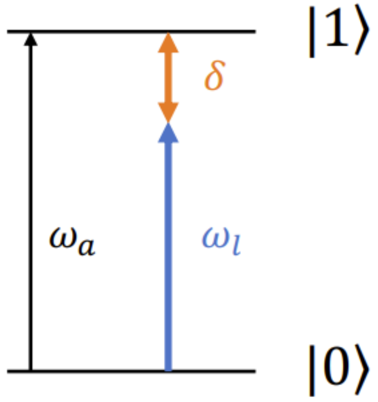
$|r\rangle \rightarrow$ Excited; $|1\rangle = |60S\rangle$



$|g\rangle \rightarrow$ Not excited, $|0\rangle = |5S\rangle$

Ground state

We address it by shining on the atom a laser beam very close to the transition energy between $|0\rangle$ and $|1\rangle$



The bottom line is that the system of valence electron coupled to light can be described by a simple two-level Hamiltonian:

$$H = \begin{bmatrix} -\delta & \Omega \\ \Omega & 0 \end{bmatrix}$$

- δ is the detuning of the laser
- Ω is the Rabi frequency, related to the intensity of the laser

See Rabi oscillations…

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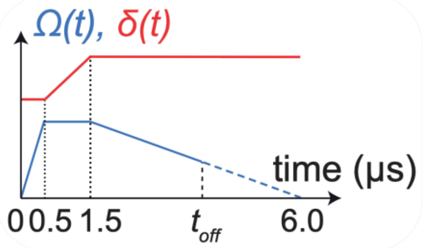
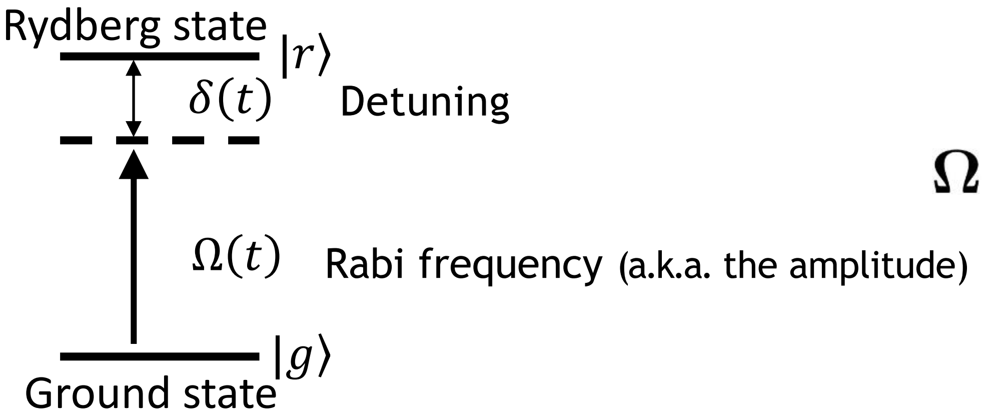
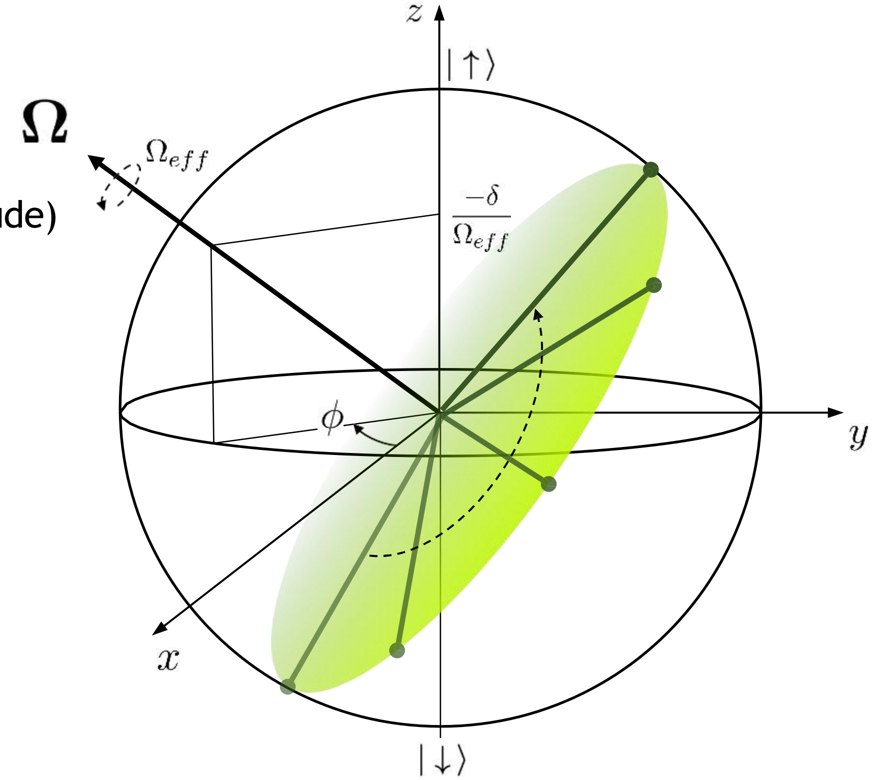
Global laser pulses: single qubit rotations

Global laser pulses

$$\mathcal{H}_1(t) = \frac{\hbar}{2}\Omega(t)\sum_j \sigma_j^x - \frac{\hbar}{2}\delta(t)\sum_j \sigma_j^z$$







$$H^D(t) = \frac{\hbar}{2}\mathbf{\Omega}(t)\cdot\boldsymbol{\sigma}$$

Driving Hamiltonian

$$\mathbf{\Omega}(t) = \begin{pmatrix} \Omega(t)\cos(\phi) \\ -\Omega(t)\sin(\phi) \\ -\delta(t) \end{pmatrix}$$

"Angular velocity" vector

$$\Omega_{eff} = |\mathbf{\Omega}| = \sqrt{\Omega^2 + \delta^2}$$

Effective Rabi frequency

A laser is tuned to drive a coherent transition between the two energy levels.

Describes a rotation around $\mathbf{\Omega}$ *with angular velocity* Ω_{eff}

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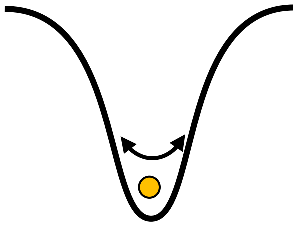
Arbitrary configurations

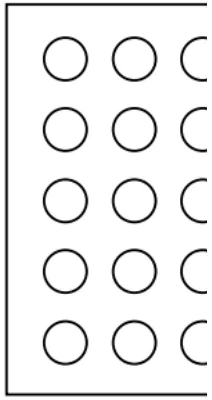


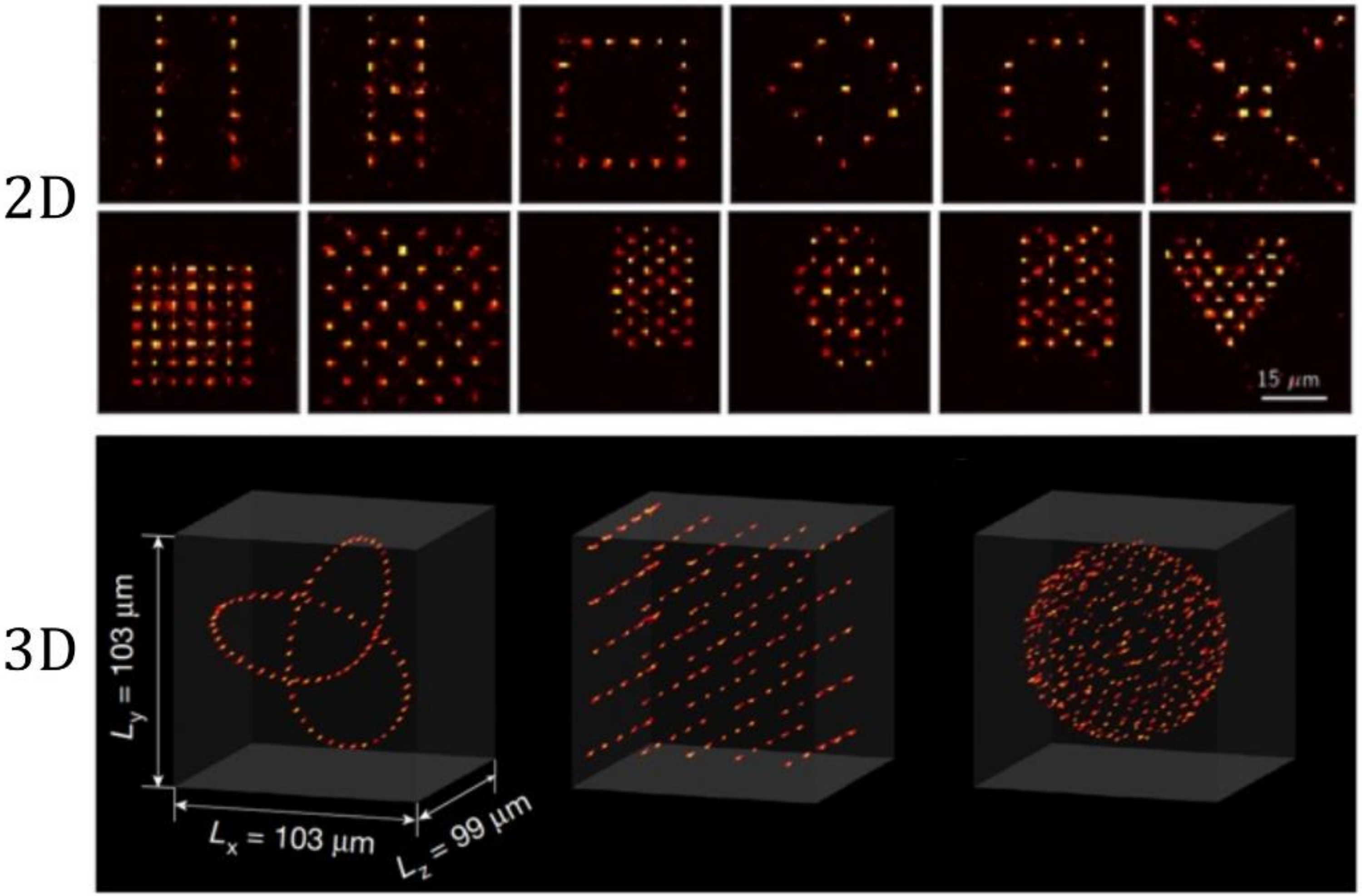
Optic

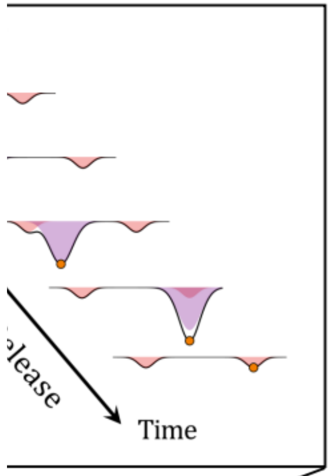
Th

1 atom trapped
per tweezer

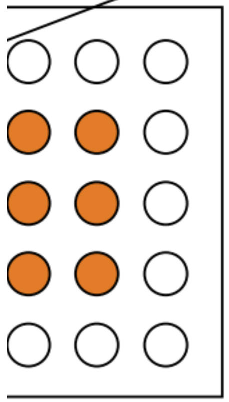
Prepare son





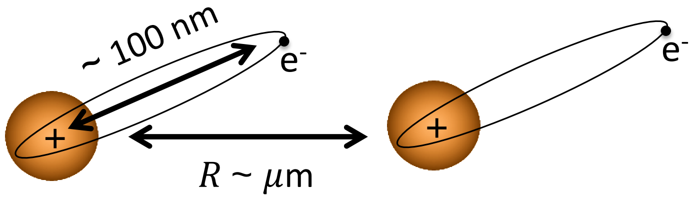


layout



Producing entanglement

Rydberg Blockade

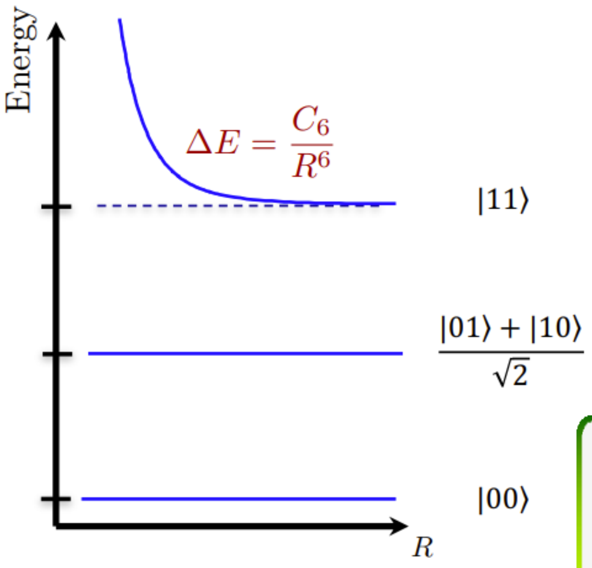


What's the physics behind the Rydberg blockade?

Two Rydberg states interact with a van der Waals interaction decaying as R^{-6} (with R distance between the atoms)

The interaction shifts upwards the energy of the $|11\rangle$ level, favouring the excitation of the entangled state $|01\rangle + |10\rangle$ instead

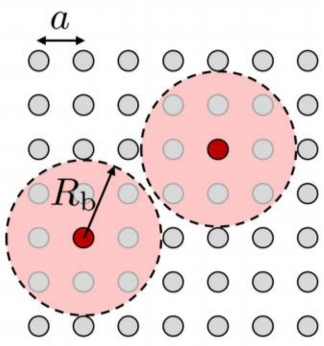
$$H_{nn} = \frac{C_6}{r_{ij}^6} n_i n_j$$



The interaction strength is only meaningful between Rydberg atoms at a distance smaller than

$$R_b = \left(\frac{C_6}{\hbar\Omega}\right)^{1/6}$$

called the Rydberg blockade radius.



The phenomena can be harnessed to create multi-qubit CZ gates, or complex/frustrated AF Ising hamiltonians.

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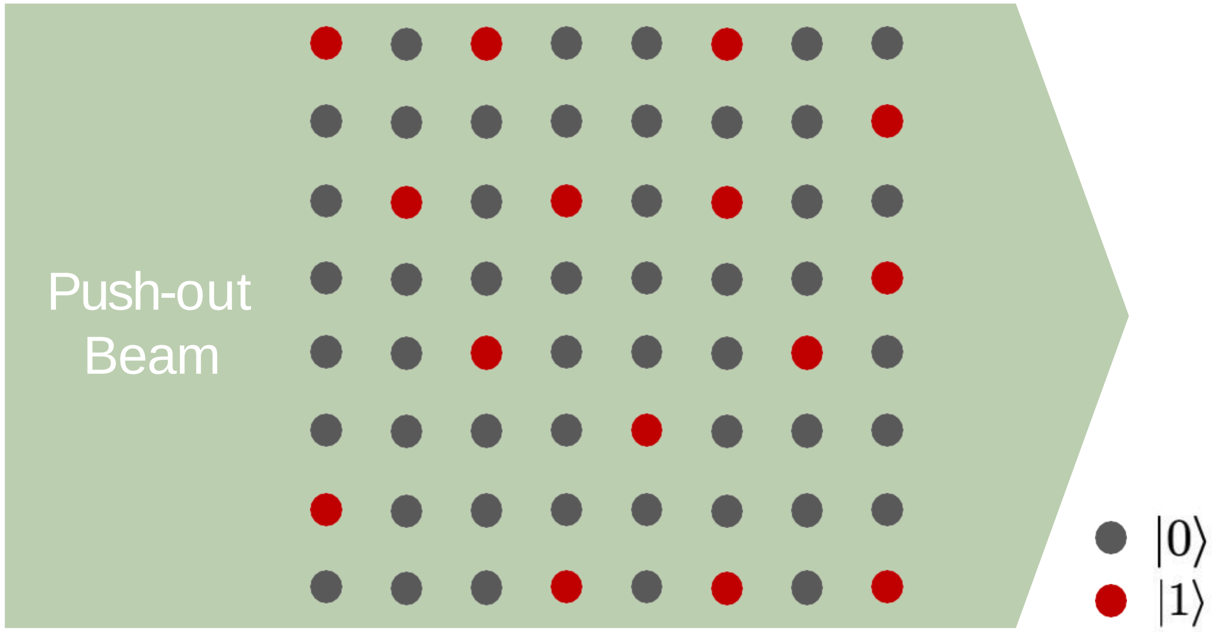
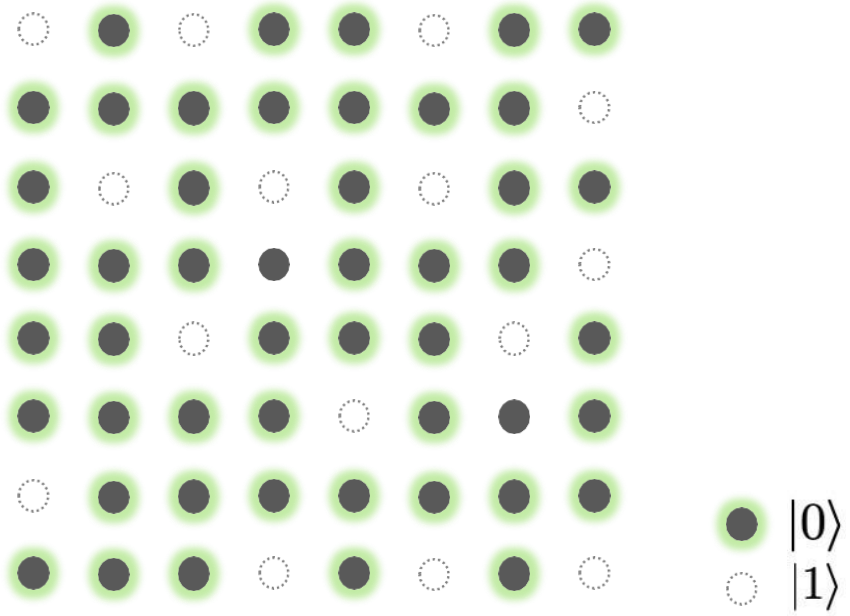
Measurement



State at the end of a run



Fluorescence image

Checklist of a quantum computer

We have:

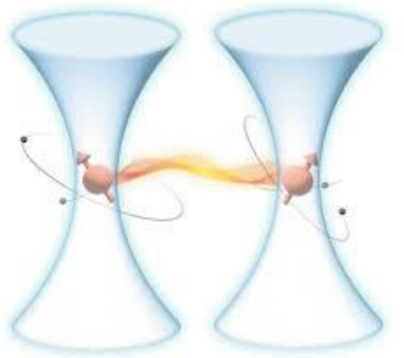
1. We have a $|0\rangle$ and a $|1\rangle$ state: Ground and Rydberg state of Rubidium

2. We can address transitions between $|0\rangle$ and $|1\rangle$: Laser beams

3. We know where each atom is: Optical tweezers

4. We can produce entanglement between the atoms: Rydberg blockade

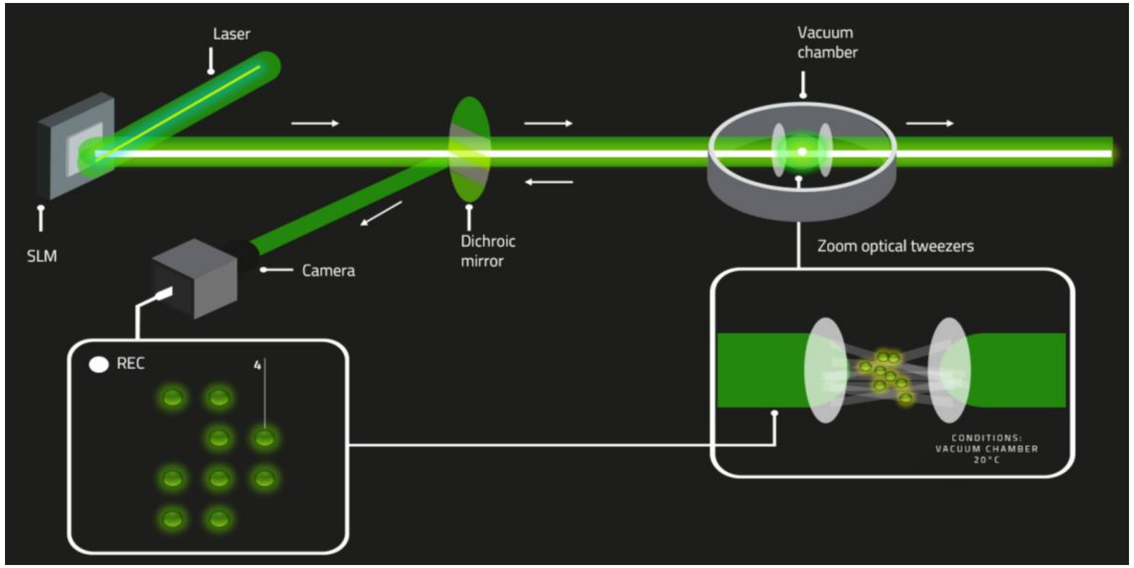
5. We can measure the state of the system: Fluorescence imaging



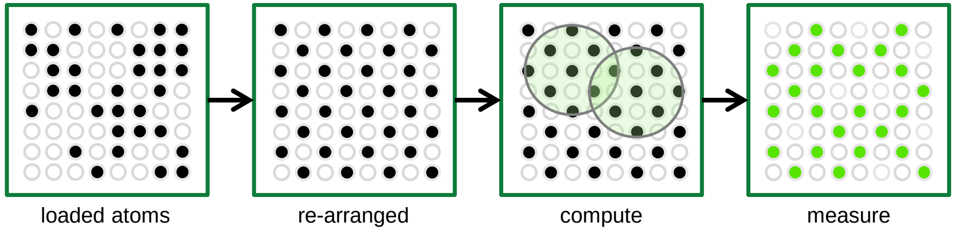
Final *time-dependent* Hamiltonian

$$\mathcal{H}(t) = \frac{\hbar}{2}\Omega(t)\sum_j \sigma_j^x - \hbar\delta(t)\sum_j n_j + \sum_{i\neq j}\frac{C_6}{r_{ij}^6} n_i n_j,$$

PASQAL's neutral atom QPU



- **Scalability**: today's commercial system 100 qubits, 1000 qubits showcase soon, no physical limit to scale to 10.000 qubits
- **Performance**: analog computation provides best chances for near-term quantum (industrial) advantage
- **Fidelity**: using nature's pristine qubit means there are no fabrication issues
- **Flexibility**: ability to arrange qubits in reprogrammable 2D array



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PASQAL approach to Quantum Computing

Building a full-stack analog quantum computing solution

Quantum stack

Metrics

Applications

- **Computational problems** Optimization, Machine Learning, partial differential equations, …
- **Quantum Simulation** Chemistry, Material science, Drug discovery …

Quality, speed or reliability of the procedure in comparison with state-of-the art classical methods

Quantum software

- **Digital control** Sequence of gates in a Quantum circuit
- **Analog control** Sequences of Quantum Hamiltonians

Complexity of the procedure, and/ or gate count

Sequence of laser pulses

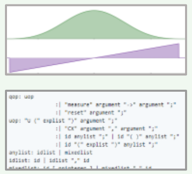
Quantum Hardware

- Laser controls
- Electronics
- Quantum register
- Detection system

Number of atoms, gate fidelity $\mathscr{F}$ quality factor $\mathscr{Q}$, and repetition rate of the QPU

Turn-key solutions

Coding environment



Quantum Processor:
Optical tweezers to control neutral atoms and engineer full-stack processors with high connectivity and scalability

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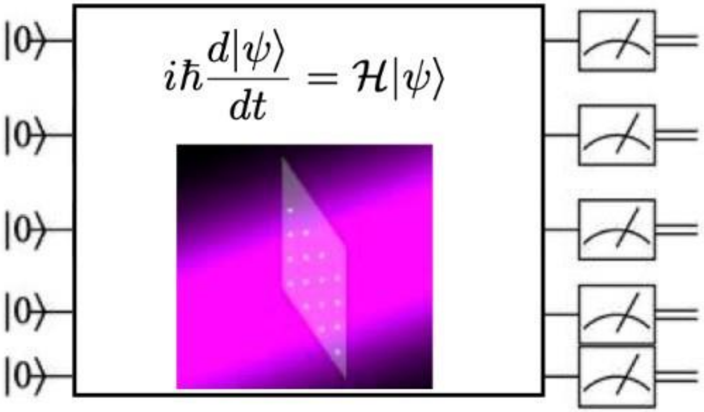
Analog vs Digital

Overview: Analog vs. Digital

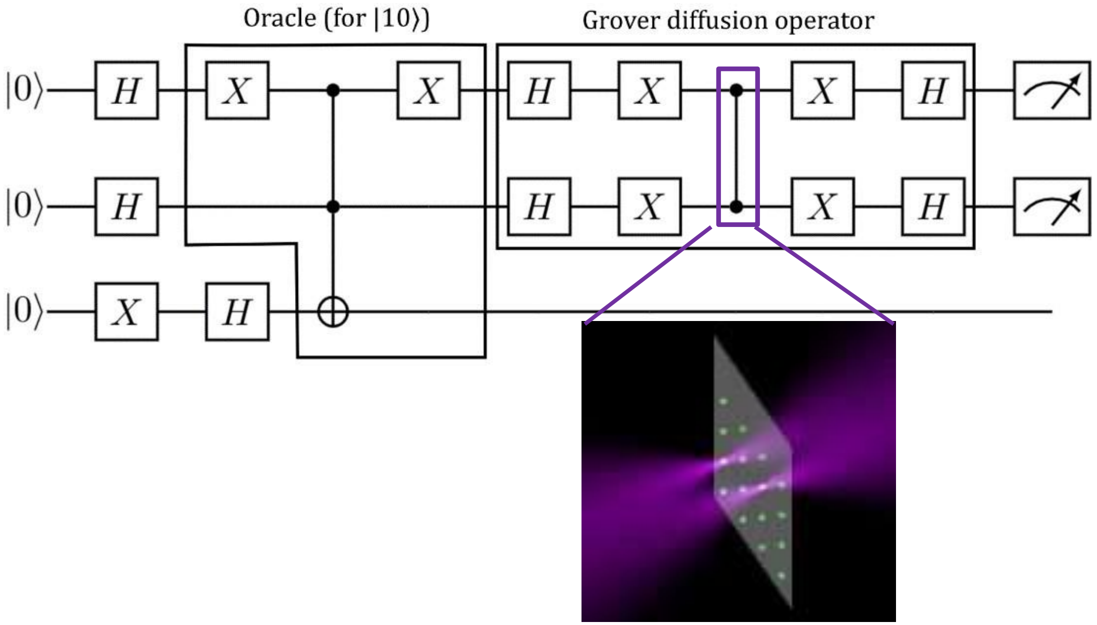
Analog Approach

$$\mathcal{H}(t) = \sum_i \left(\frac{\hbar\Omega(t)}{2}\sigma_i^x - \hbar\delta(t)\hat{n}_i + \sum_{j<i} \frac{C_6}{(R_{ij})^6}\hat{n}_i\hat{n}_j \right)$$



One Hamiltonian with continuously controlled parameters dictates the dynamics of the whole system.

Digital Approach



Each gate is done in isolation, by acting locally only on the involved qubits.

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Analog Approach

A tunable Ising Hamiltonian

Transverse field

Ising couplings: $J_{ij} \propto 1/R_{ij}^6$

$$\mathcal{H}(t) = \sum_i \left(\frac{\hbar\Omega(t)}{2}\sigma_i^x - \hbar\delta(t)\underbrace{\hat{n}_i}_{\frac{1+\sigma_i^z}{2}} + \sum_{j<i} \frac{C_6}{(R_{ij})^6}\hat{n}_i\hat{n}_j \right)$$

Quantum Simulation

Permits the simulation of quantum many-body systems far beyond the limits of classical computers, allowing, for example, the:

- Observation of out-of-equilibrium dynamics
- Adiabatic preparation of ground states

Solving hard combinatorial optimization problems

Some of these problems don't have a satisfactory classical solution but are mappable to the Ising Hamiltonian. Examples include:

- The Max-k-Cut problem
- The Maximum Independent Set problem

Our resource!

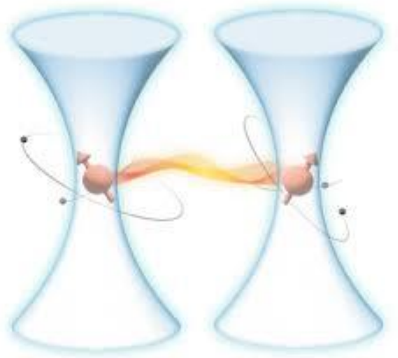
Final *time-dependent* Hamiltonian

$$\mathcal{H}(t) = \frac{\hbar}{2}\Omega(t)\sum_j \sigma_j^x - \hbar\delta(t)\sum_j n_j + \sum_{i\neq j}\frac{C_6}{r_{ij}^6} n_i n_j,$$

- Positive detuning on one atom decreases the energy of the system (excitation is energetically favoured)

- Two excitations close to each other increase the energy of the system (close excitations are energetically disfavoured)

- It’s a transverse field quantum Ising model, where the interaction strength depends on the distance between atoms



What can we do with a time dependent quantum TFIM on arbitrary lattices?

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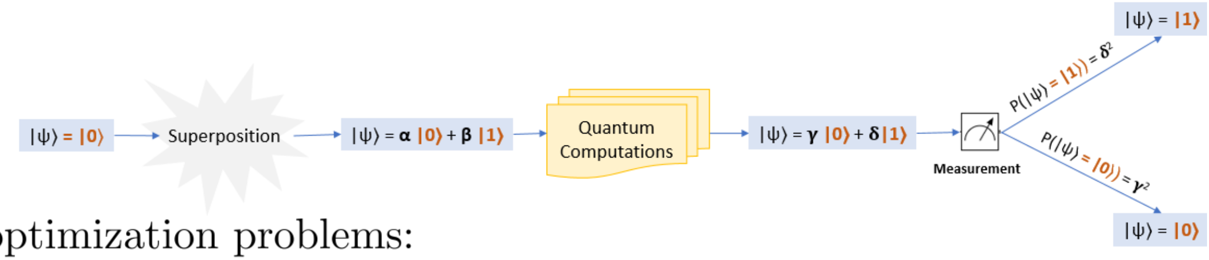
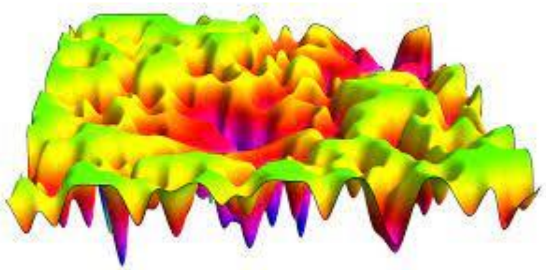
Analog quantum computing approaches

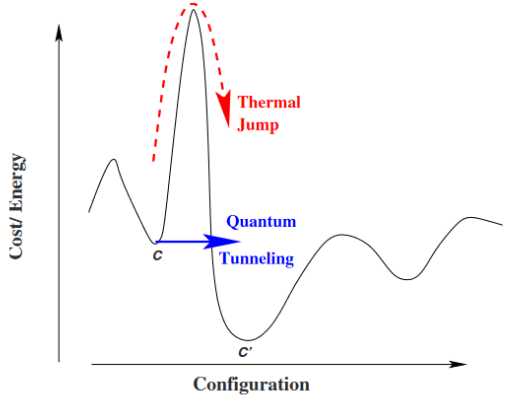


$$|\psi_0\rangle \quad \xrightarrow{\mathcal{H}(t)\ldots} \quad |\psi(T)\rangle$$

Very versatile in tackling optimization problems:

- Encode your problem in such a way that the file state $|\psi(T)\rangle$ optimizes some cost function. For example, the ground state of a given hamiltonian.
- Example: $|\psi(T)\rangle = c_1|01001010\rangle + c_2|01110101\rangle + \ldots$ where $|c_1| \ll |c_n|$, and $|\psi_1\rangle$ represents the solution.
- See: The adiabatic quantum algorithm.





S state

$\mathcal{H}$

if $T > \Delta$

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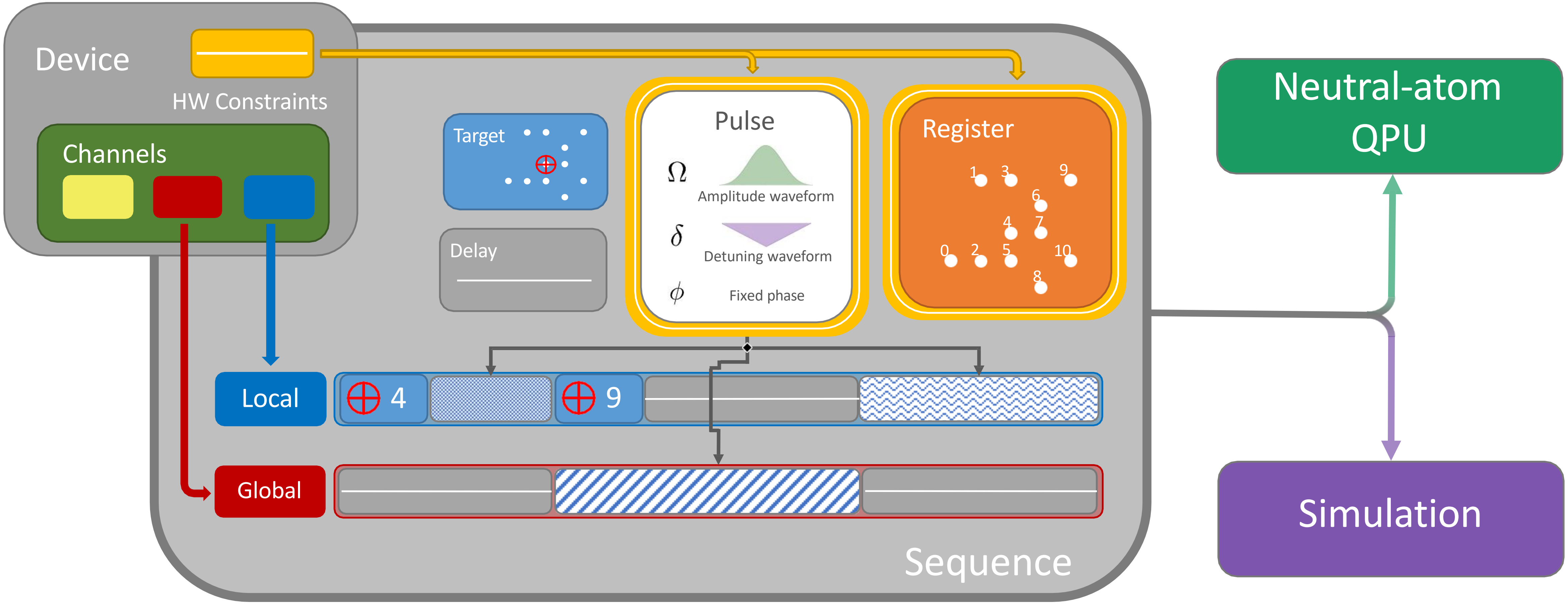
Pulser

How is an analog neutral atom QPU operated?

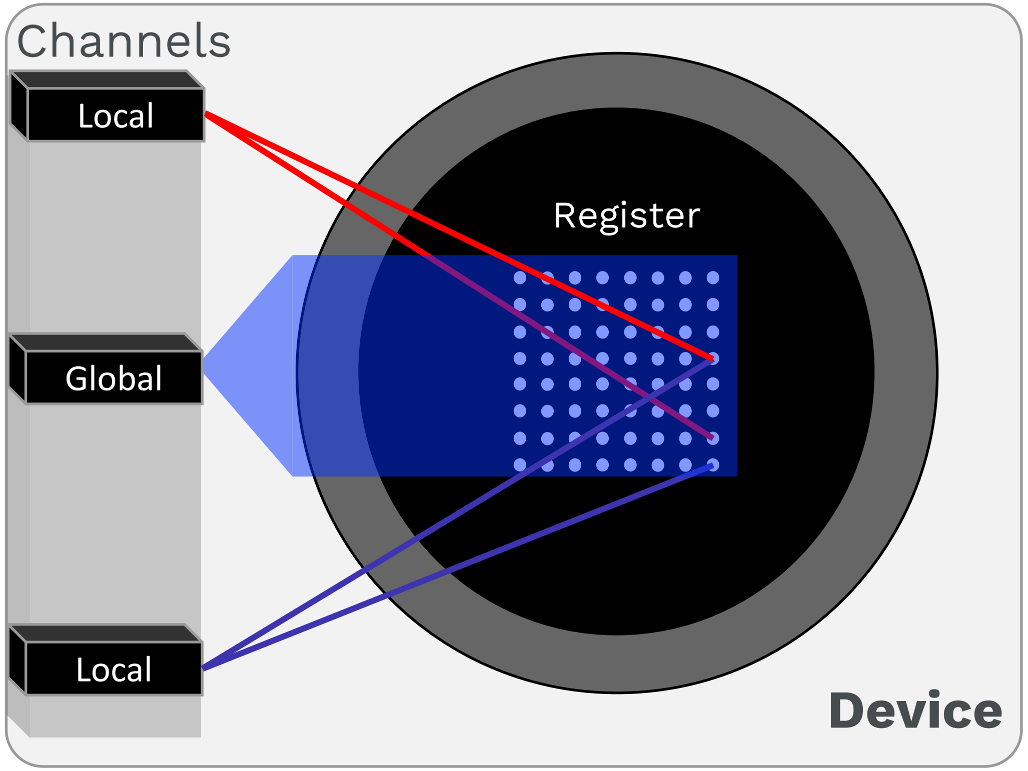


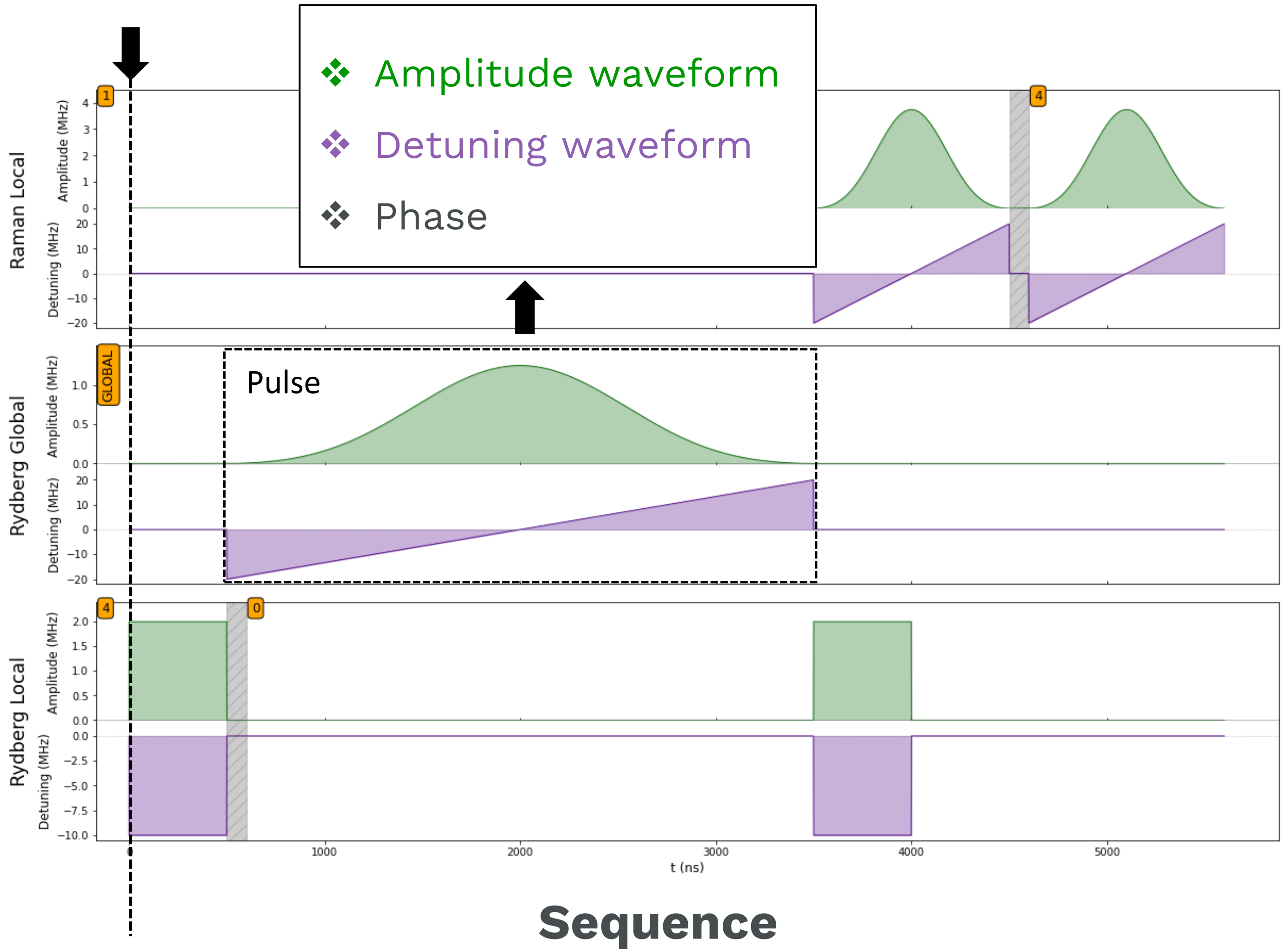
A Sequence is formed of a register and pulses, which is then sent to a device.

Where atoms are == what interactions/blockade are present?

What dynamical process do we want the qubits to undergo?

Controlling a QC at the pulse level with Pulser

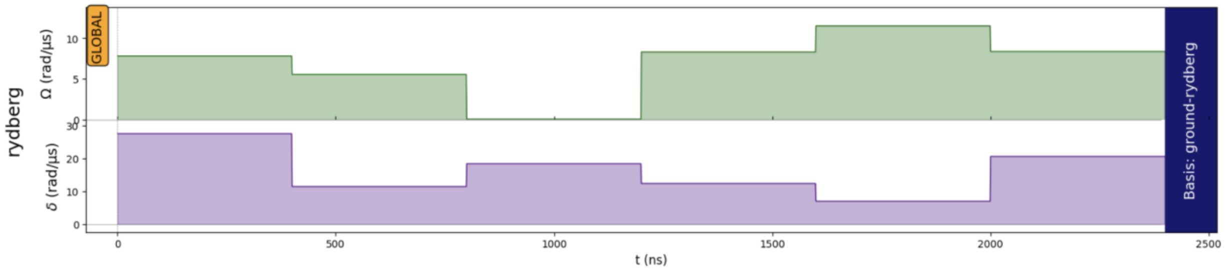




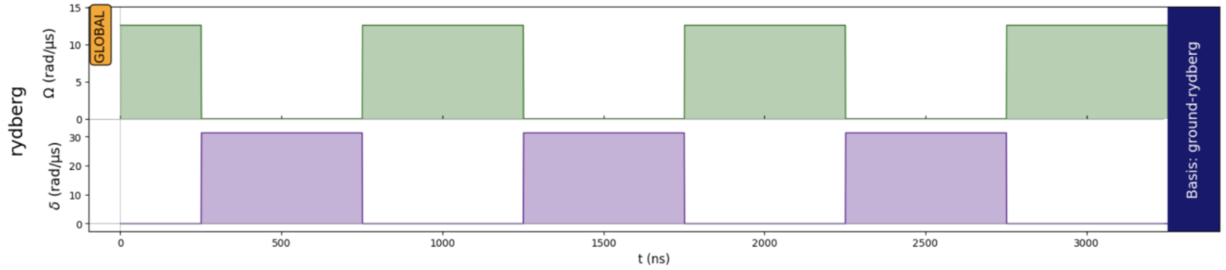
Henrique Silvério, et al. *Pulser: An open-source package for the design of pulse sequences in programmable neutral-atom arrays*, *Quantum* 6. (2022): 629.
Check **pulser.readthedocs.io**

Controls parameterization and constraints

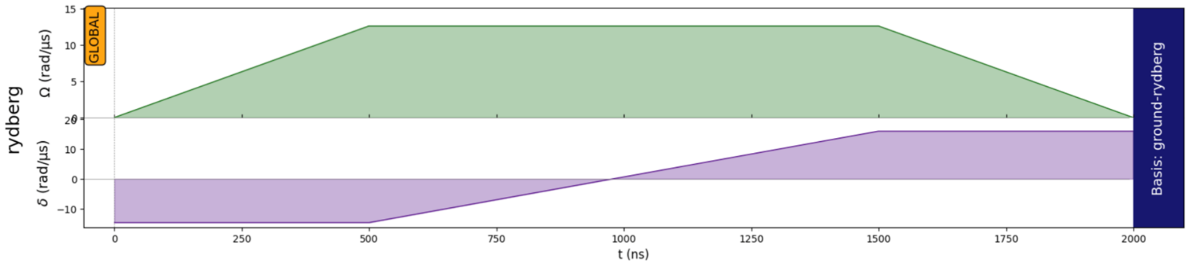
Pcw-cst pulses for GRAPE-like algos $\rightarrow \theta = [\Omega_i, \delta_i]$



Pulse train for QAOA-like sequences $\rightarrow \theta = [t_i]$



Interpolated pulses for smooth sequences $\rightarrow \theta = [t_i, \Omega_i, \delta_i]$



Effective parameterized controls

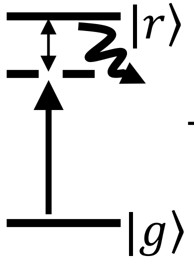
$\Omega_\theta(t_\theta), \delta_\theta(t_\theta)$ with $\theta \in \Theta$

$+$

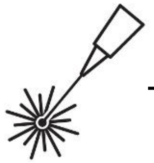
Hardware constraints

Constraining parameter space

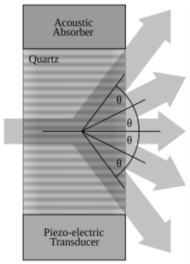
Black box in closed loop



$\rightarrow T_{max}$



$\rightarrow \Omega_{max}, \delta_{min/max}$



$\rightarrow T_{min}$, Bandwidth

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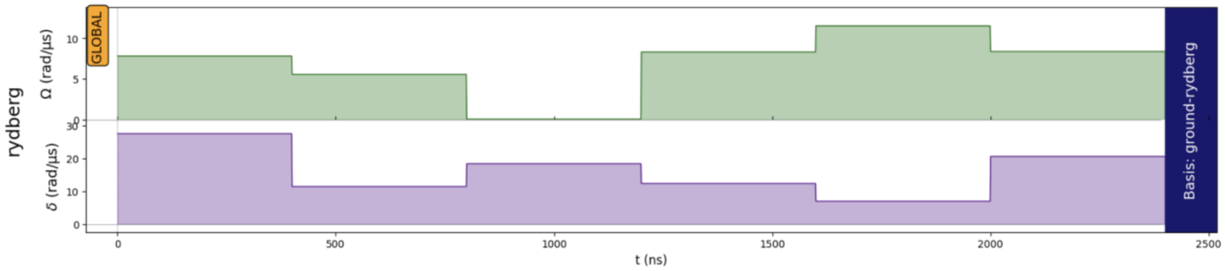
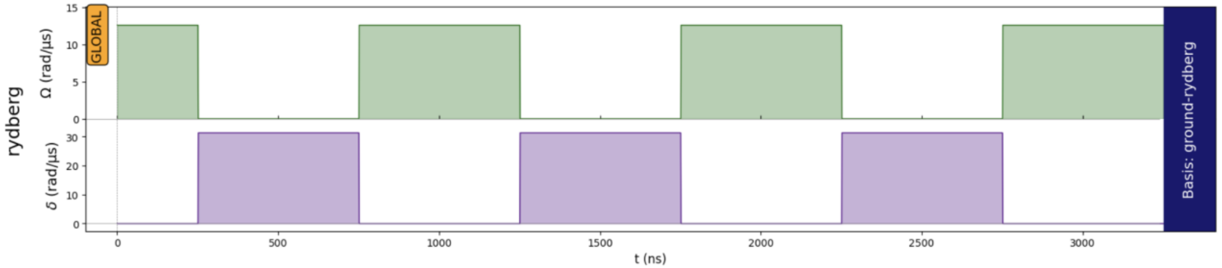
Controls parameterization and constraints

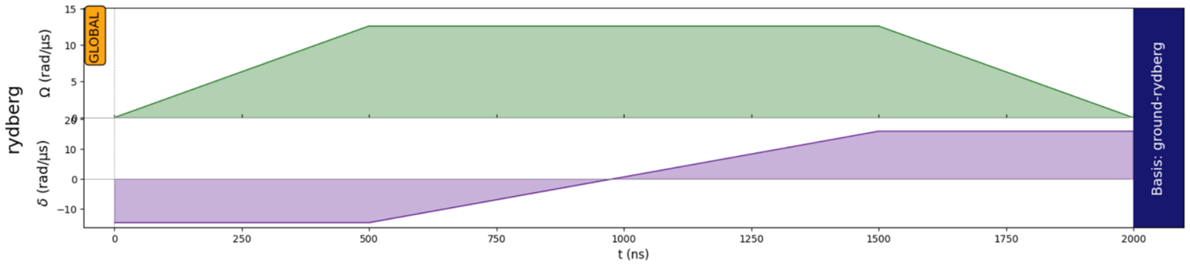
Pcw-cst pulses for GRAPE-like algos → $\theta = [\Omega_i, \delta_i]$

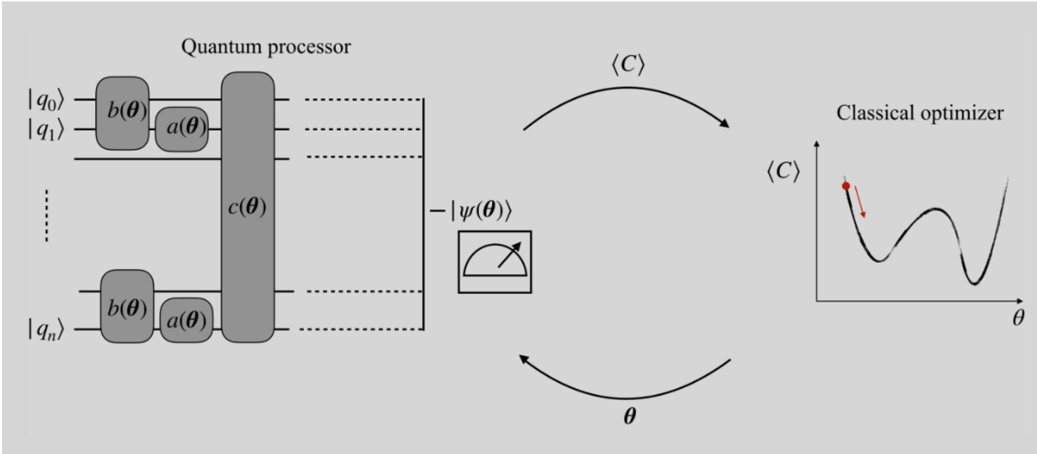


Pulse train for QAOA-like sequences → $\theta = [t_i]$



Interpolated pulses for smooth sequences → $\theta = [t_i, \Omega_i, \delta_i]$





Hybrid quantum-classical approaches may provide a path for near-term quantum advantage: leverage QPU for increased accuracy/TTS/energy demand.

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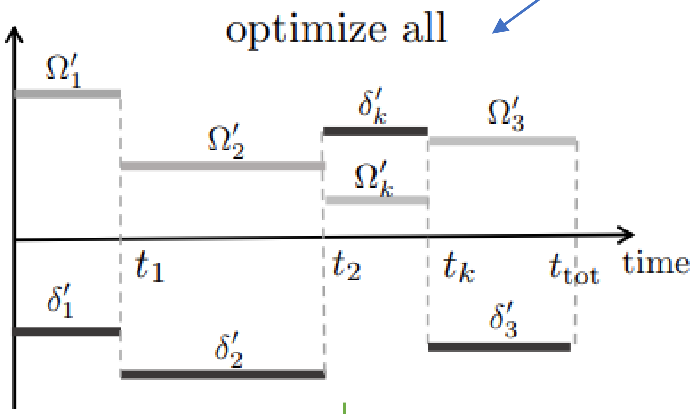
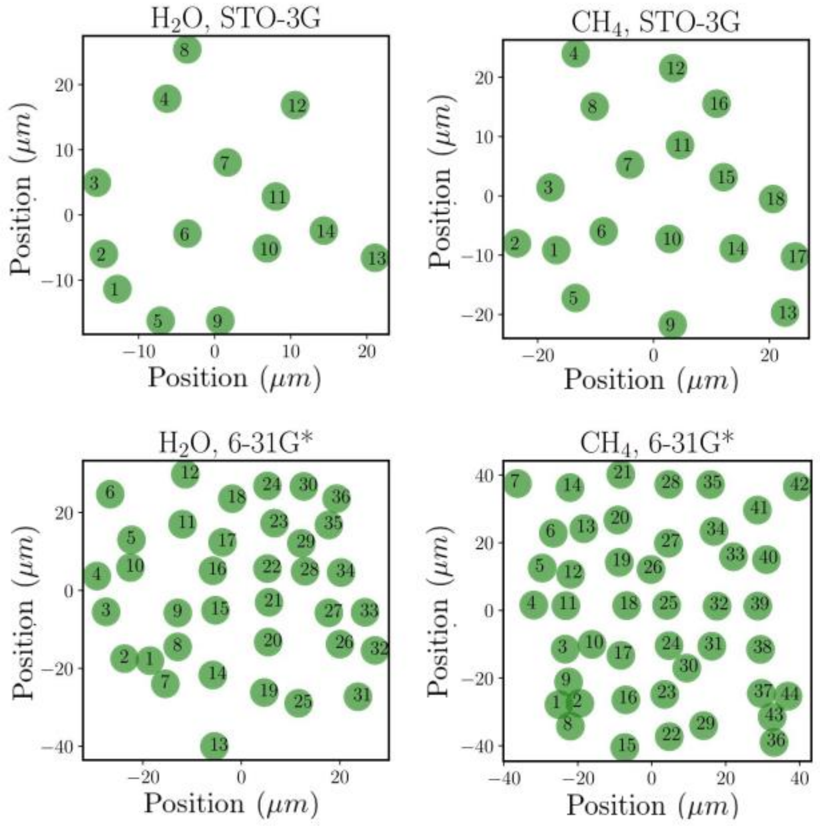
Example: VQE for Quantum Chemistry

Pulse optimization + register optimization





Objective:
Highest fidelity of sampling the ground state

Objective:
Capturing the right interactions

Michel, A., Grijalva, S., Henriet, L., Domain, C., & Browaeys, A. (2023). Blueprint for a digital-analog variational quantum eigensolver using Rydberg atom arrays. *Physical Review A*, *107*(4), 042602.

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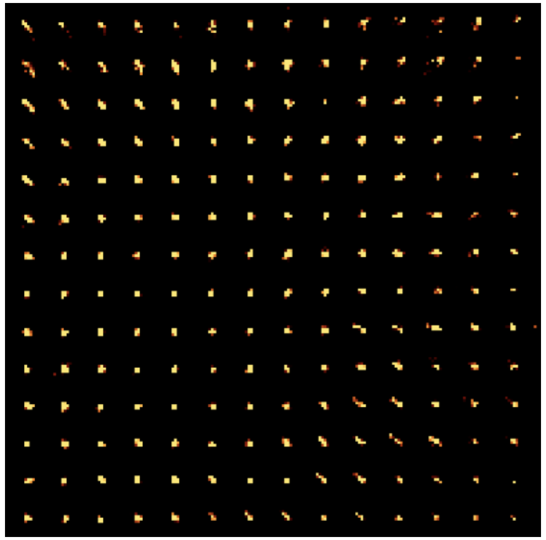
Example: 2D antiferromagnet

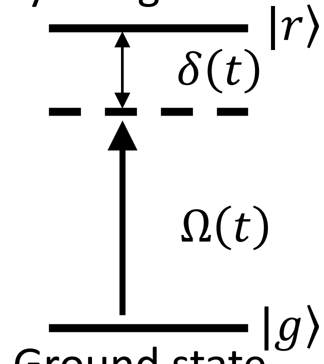
Laser controlled particles + Many body interactions = Quantum dynamics

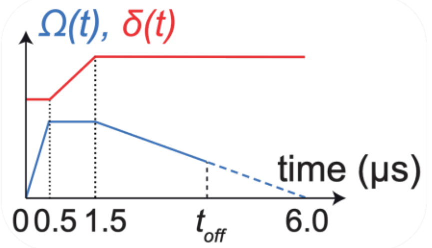


$|\psi(t=0)\rangle = |g \dots g\rangle$

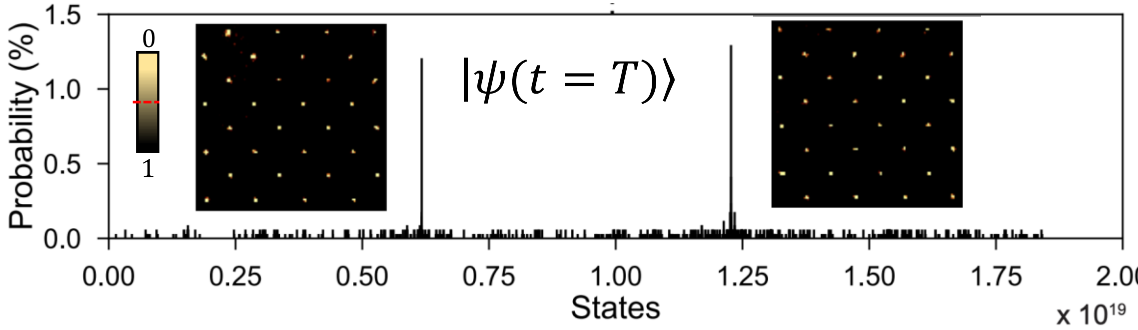
Laser addressed transitions

$|\psi(t=T)\rangle$

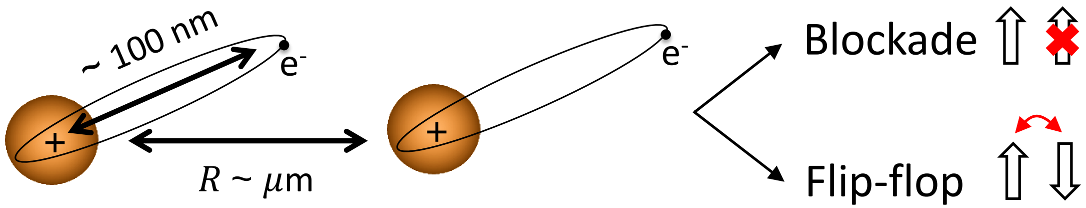




P. Scholl *et al.* Programmable quantum simulation of 2D antiferromagnets with hundreds of Rydberg atoms, Nature.



Dipole-dipole interactions



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Graphs and optimization

Introduction to combinatorial optimization

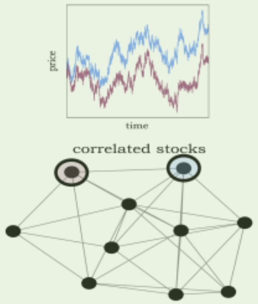
Goal: tackle combinatorial optimization problems

- Consists of finding the ***best*** out of a finite, but exponentially large, set of options
- Extensively studied by both academic and industrial communities
- Vast range of application in real-world systems
- Too many options to solve exactly! Many heuristic (approx.) approaches
- Can naturally be defined on graphs G with **vertices** V and **edges** E [weight of connections]. Optimization problem consists to selecting a subset of vertices *while optimally satisfying some rules.*

Delivering

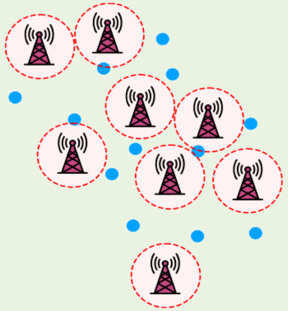
Find an itinerary which visits every location while minimizing total distance

Portfolio Optimization



Select the best stocks to invest

Antenna placement



Minimize interference

Introduction to combinatorial optimization

Graph theory problem	Application example
Maximum independent set	Traffic optimization Antenna placement Channel allocation Windmill optimization
Maximum clique	Portfolio optimization Biological and social networks
Minimum vertex cover	Fraud detection Immunization strategies
Graph coloring	Task scheduling Route optimization

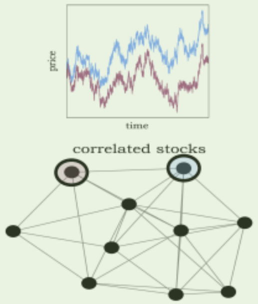
Approximate solutions are hard to find using classical computers = a great opportunity to quantum speedup (or better solution quality) using novel techniques!

In quantum computer: Harness superposition, entanglement and interference to generate new heuristic approaches that (try to) rival SotA.

Delivering

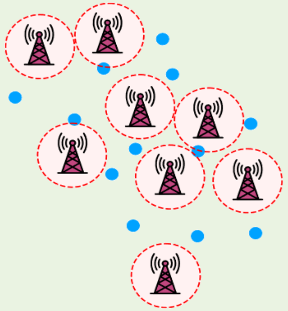
Find an itinerary which visits every location while minimizing total distance

Portfolio Optimization



Select the best stocks to invest

Antenna placement



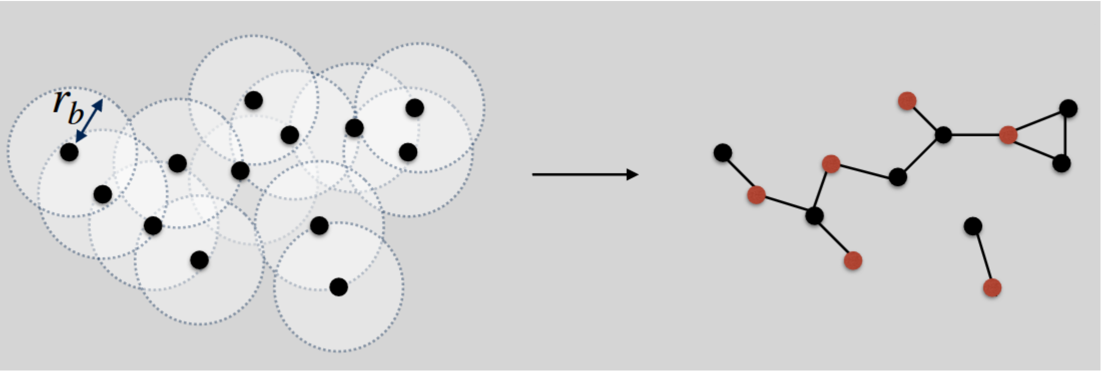
Minimize interference

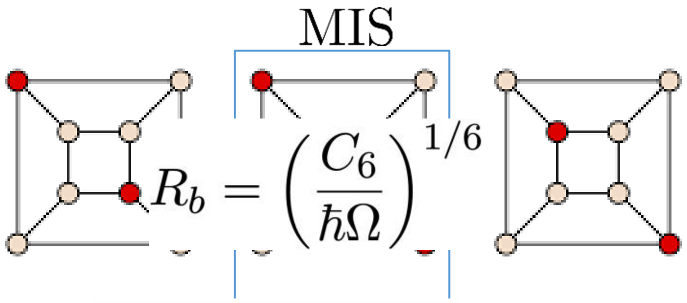
Example: the Maximal Independent Set (MIS)

$$\mathcal{H}(t) = \frac{\hbar}{2}\Omega(t)\sum_j \sigma_j^x - \hbar\delta(t)\sum_j n_j + \sum_{i\neq j}\frac{C_6}{r_{ij}^6} n_i n_j,$$

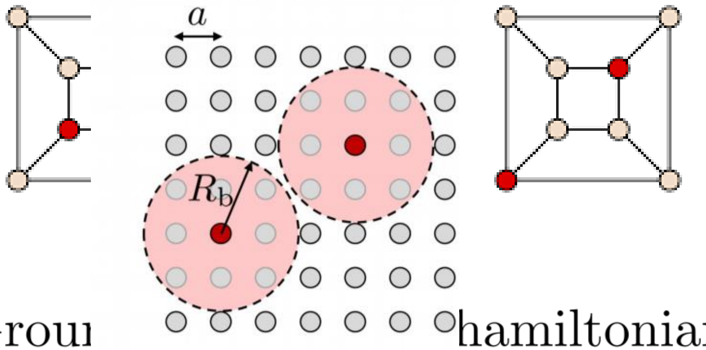
Van-der-Walls interaction is very strong at small distance ($r < R_b$) but decays fast after. Essentially a hard-core repulsion, where states $|01>$ and $|10>$ are favored.

This is **exactly** the setting of MIS problems on unit-disk graphs, where nodes closer than R_b from one another are connected.





$$R_b = \left(\frac{C_6}{\hbar\Omega}\right)^{1/6}$$



Grou hamiltonian with Rydberg blockade!

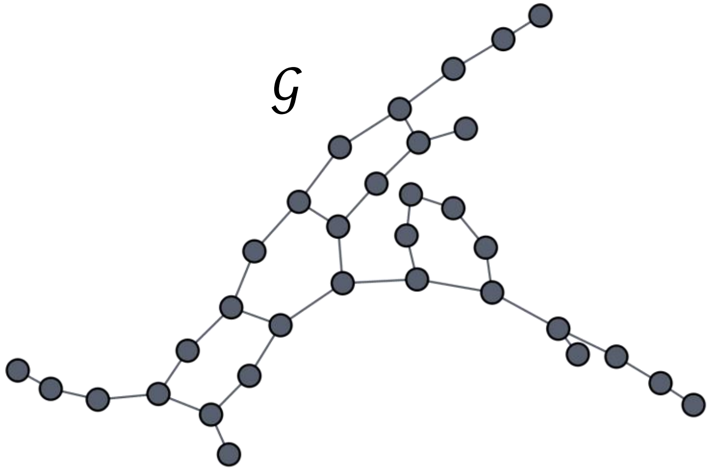
Ideal pl

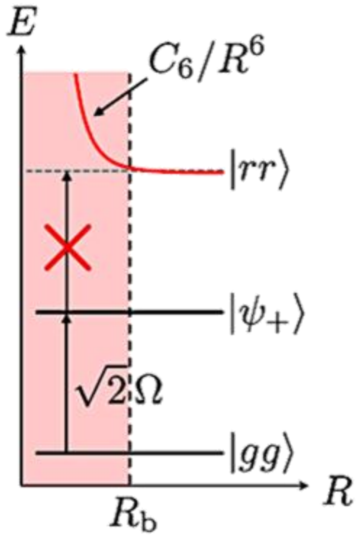
Our QPU

This makes the platform compatible with graph optimization problems (MIS, MAXCUT) with strong independence constraints

Embedding: Mapping UD graphs to atoms with interactions

1 node → 1 atom , edges → interactions





Dipole-dipole

$|r_S\rangle \leftrightarrow |r_S\rangle$

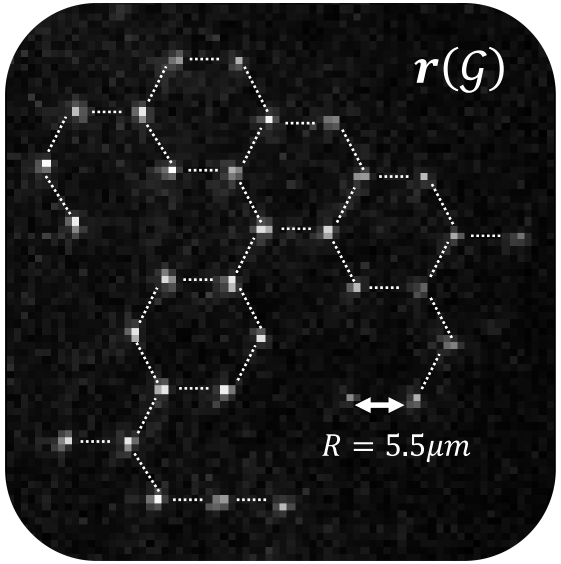
↓

Van der Waals interactions

$\propto 1/R^6$

↓

NN Blockade effect



Graph topology

$$H_{\mathcal{G}} = \sum_{(i,j)\in E(\mathcal{G})} n_i n_j$$

Interaction Hamiltonian

$$H_{dd}\big(\boldsymbol{r}(\mathcal{G})\big) \propto \sum_{i>j} \left(\frac{R}{R_{ij}}\right)^6 n_i n_j$$

Nearest Neighbour

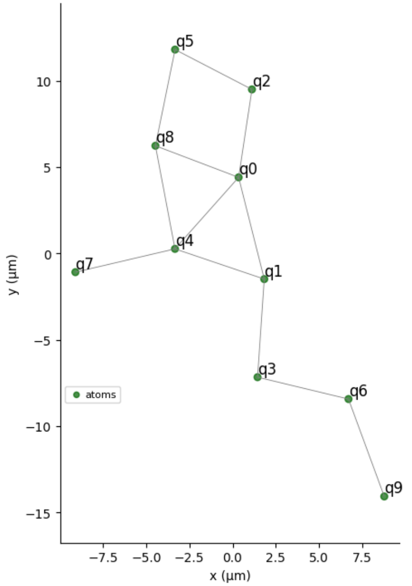
approximation

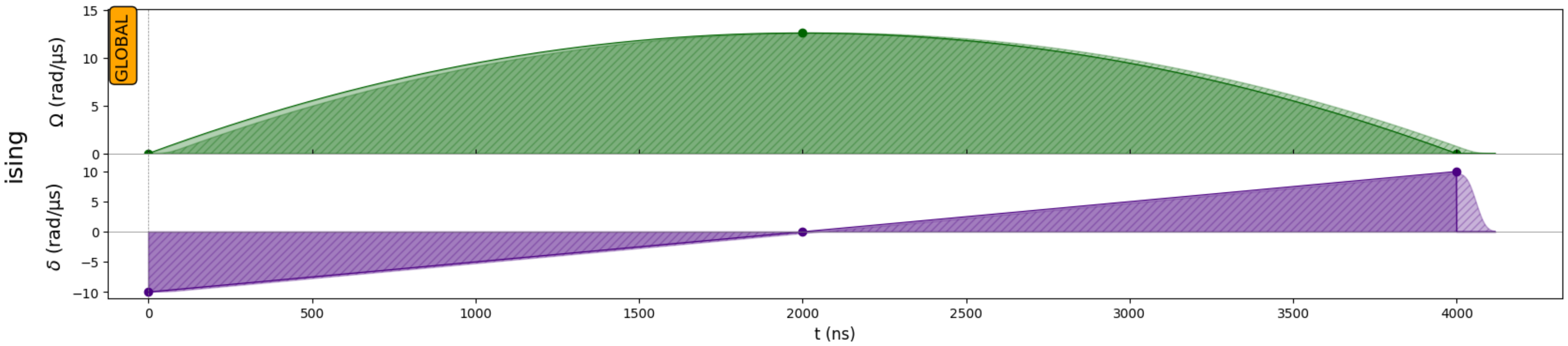
✓

$$\boxed{H_{dd}\big(\boldsymbol{r}(\mathcal{G})\big) \propto H_{\mathcal{G}}}$$

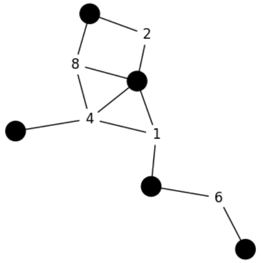
Pulser example: MIS solver

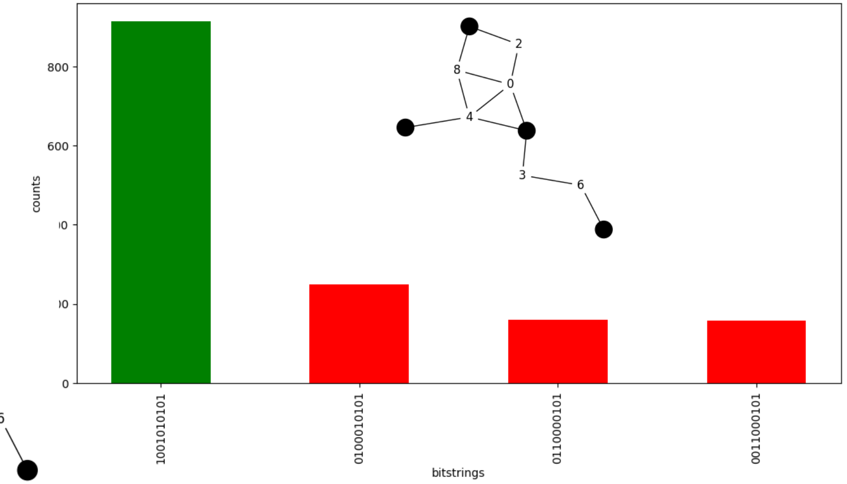
What does QPU execute? A quasi-adiabatic pulse that returns IS samples





- Slow excitation of the system (through Ω) from a known state at $t = 0$ ($|000...\rangle$) to the solution(?) at $t = T$
- Ω_{max} sets the blockade radius (the graph edges).
- Result: sample many IS.





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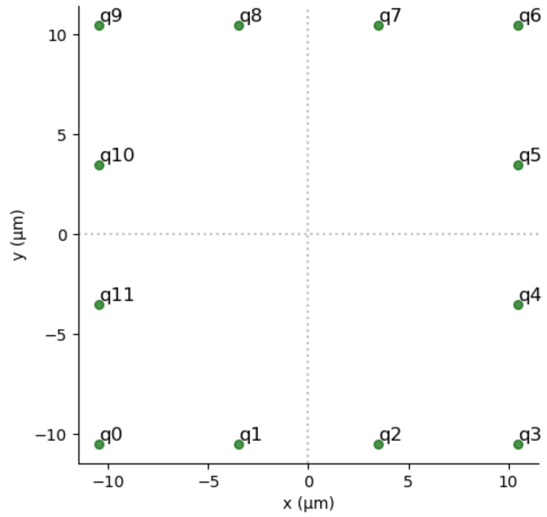
Pulser example: crafting an optimal Pulse



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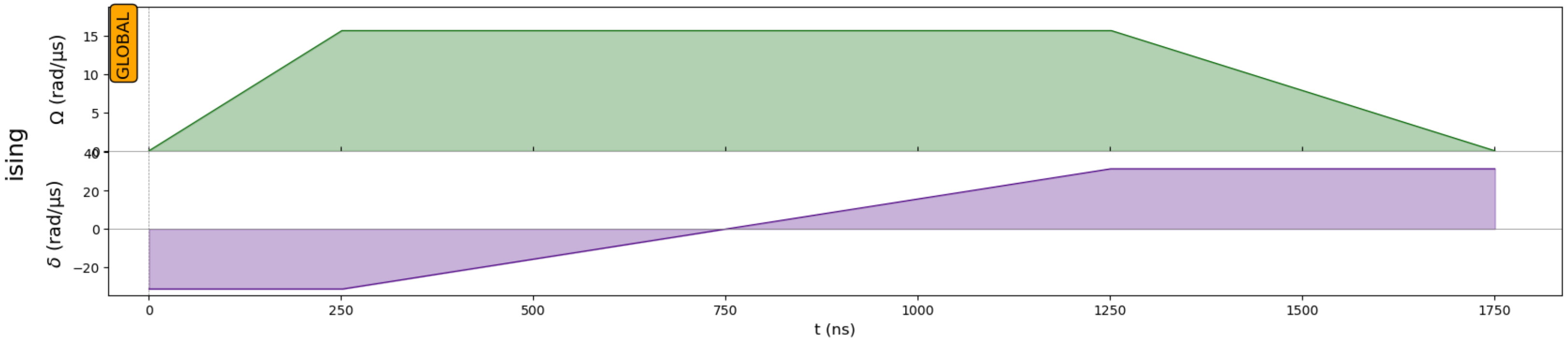
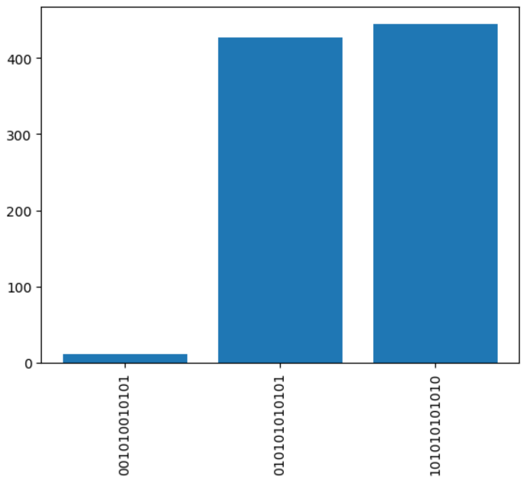


Preparation of an AF state:

$$|\Psi_f\rangle = \frac{1}{\sqrt{2}}[|010101010...\rangle + |101010101...\rangle]$$

Good but not perfect fidelity…We can see the superposition of Neel states



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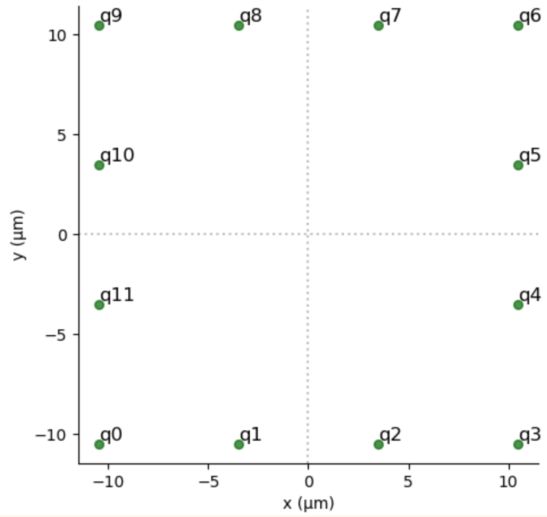
Pulser example: crafting an optimal Pulse



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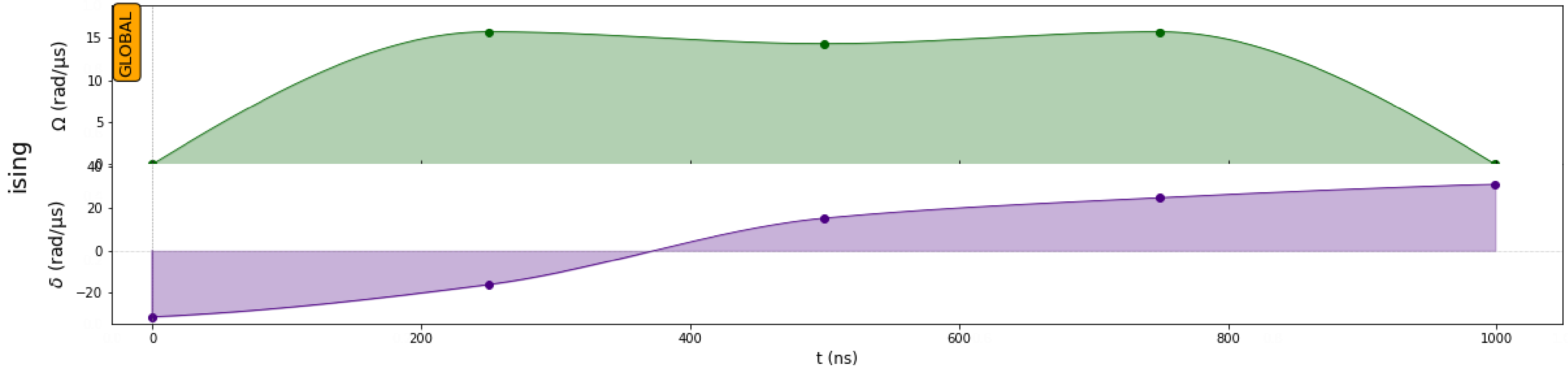
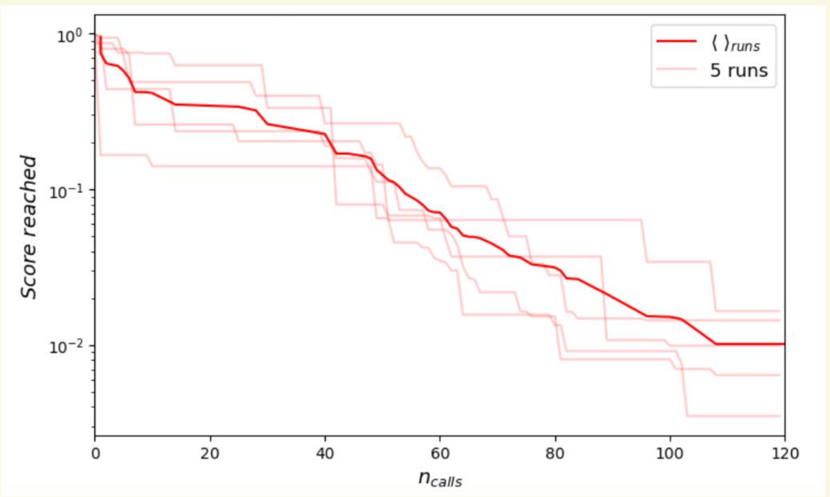
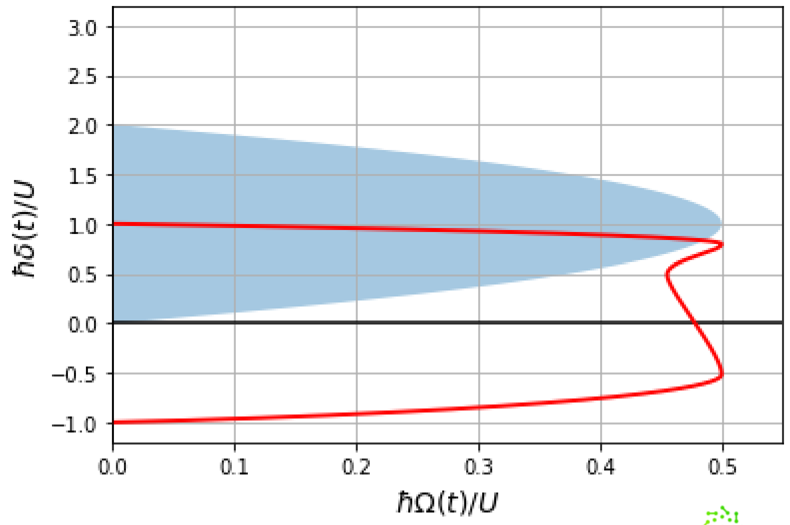




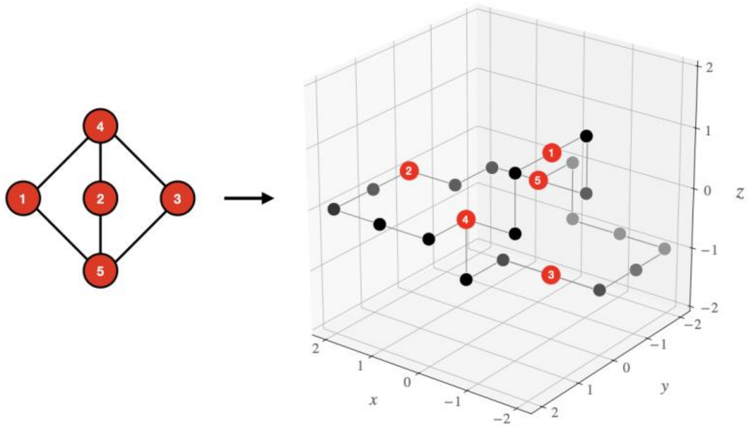
Up to 99% fidelity in generating the correct state.

Approaches like this <u>(fast then slow near minimum gap)</u> are explored more and more in the community to improve performance



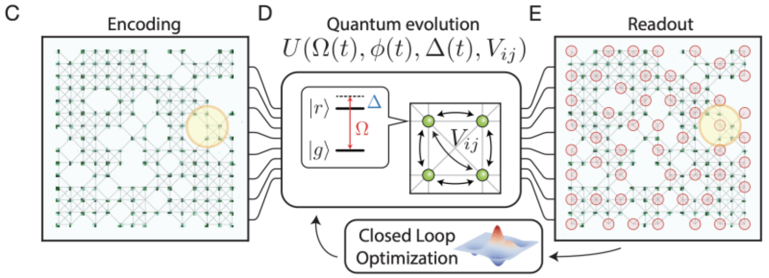
Use of pulse-level hybrid algorithms to tackle tough problems

Harder problems, closed loop on large instances & ML training

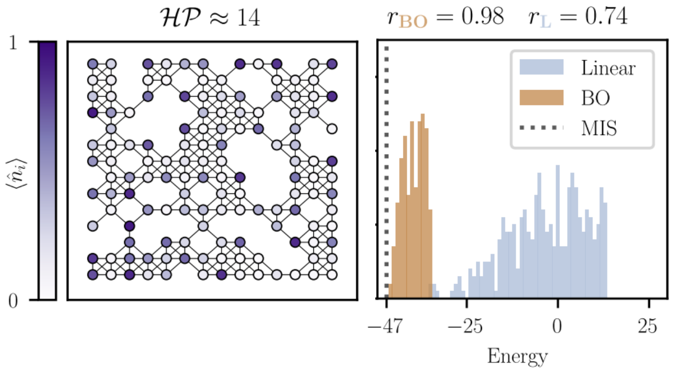


Embedding planar non-UD graphs using 3D quantum wires.

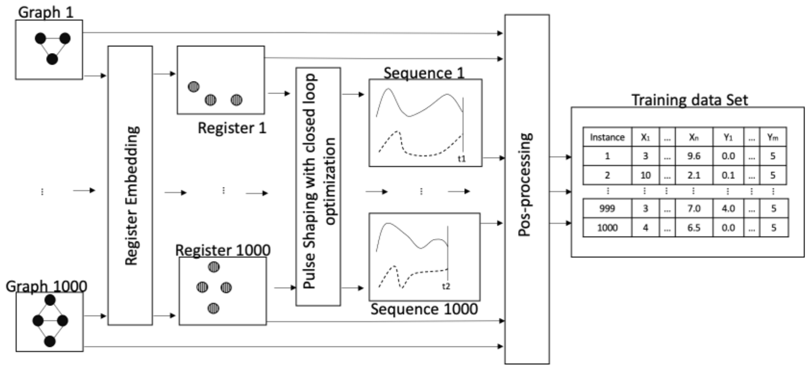
C. Dalyac *et al. Embedding the MIS problem for non-local graphs with bounded degree using 3D arrays of atoms*, arXiv:2209.05164



Ebadi, S *et al, Quantum optimization of maximum independent set using Rydberg atom arrays. Science, 376*(6598), 1209-1215.



J. Rudi Finžgar *et al, Designing Quantum Annealing Schedules using Bayesian Optimization*, arXiv:2305.13365



Map-and-shape ML algo using dataset of registers and pulses produced by VQA

W. Da Silva Coehlo *et al, Efficient protocol for solving combinatorial graph problems on neutral-atom quantum processors*, arXiv:2207.13030

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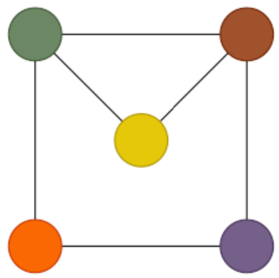
Application: graph coloring

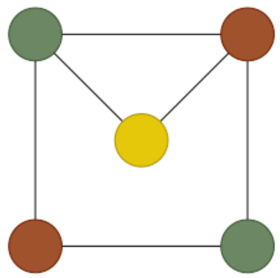
Graph coloring problem

Let G = (V,E) be a graph with a set V of nodes and a E of edges.
Also, let C be a set of available colors.

The **Minimum Vertex Coloring Problem (MVCP)** consists in coloring the vertices of G with exactly one color from C in a such way that the number of used colors is minimized while ensuring that no two adjacent vertices have the same color.



(a) Trivial coloring.



(b) Optimal coloring.

Graph coloring enjoys many practical applications such as
- Scheduling problems
- Resources assignment

da Silva Coelho, Wesley, Loïc Henriet, and Louis-Paul Henry. "Quantum pricing-based column-generation framework for hard combinatorial problems." *Physical Review A* 107, no. 3 (2023): 032426.

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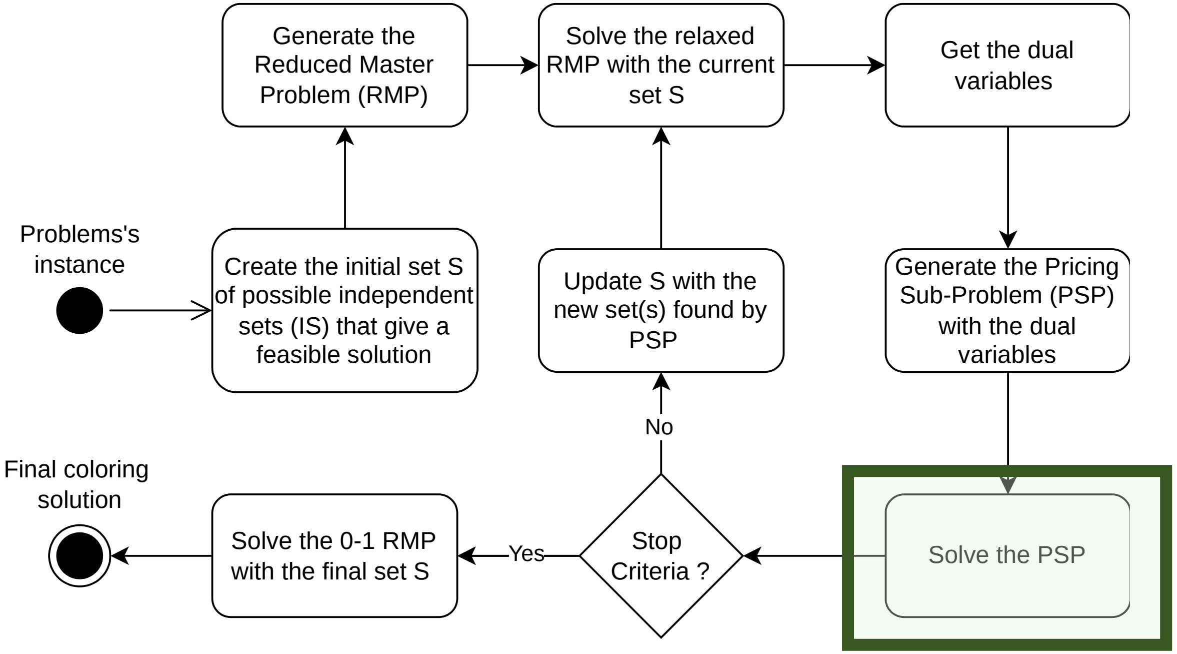
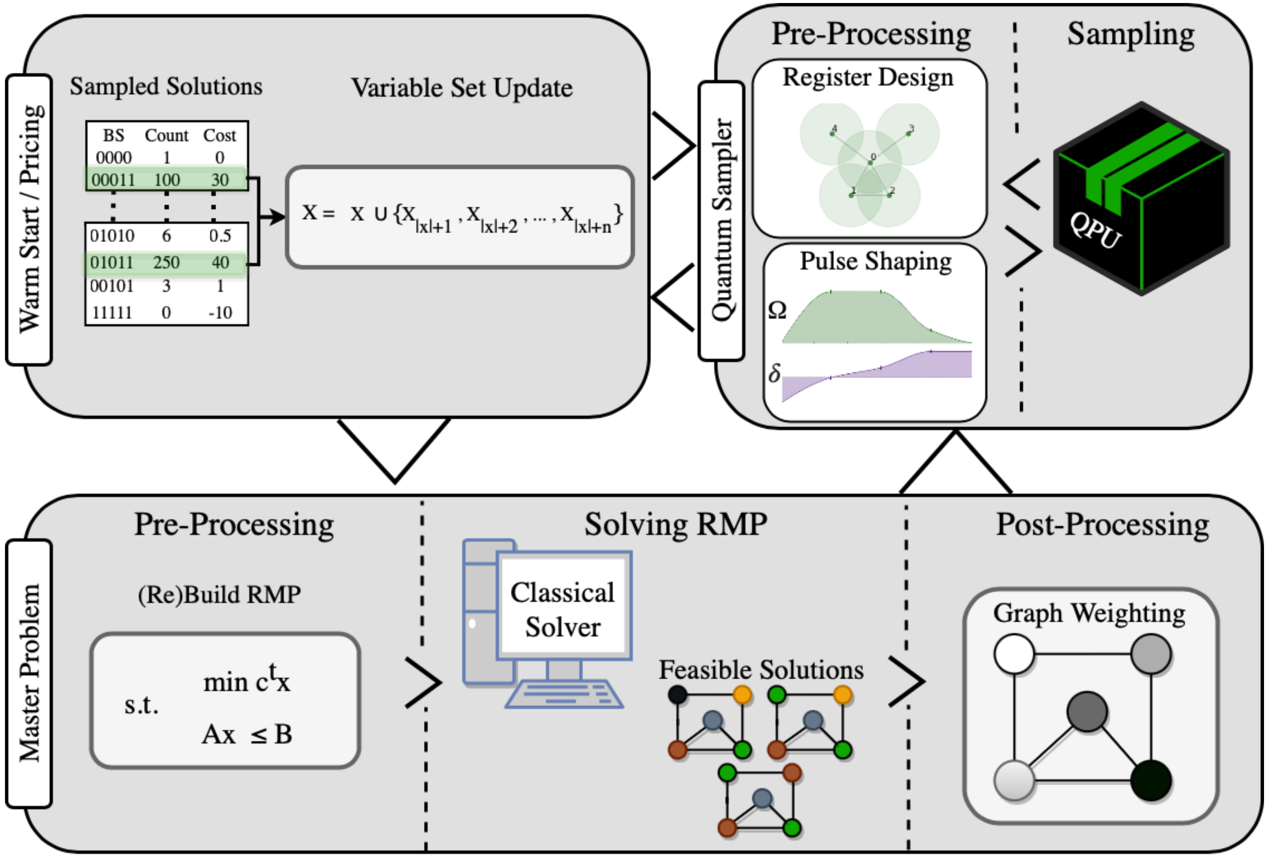
Hybrid column generation

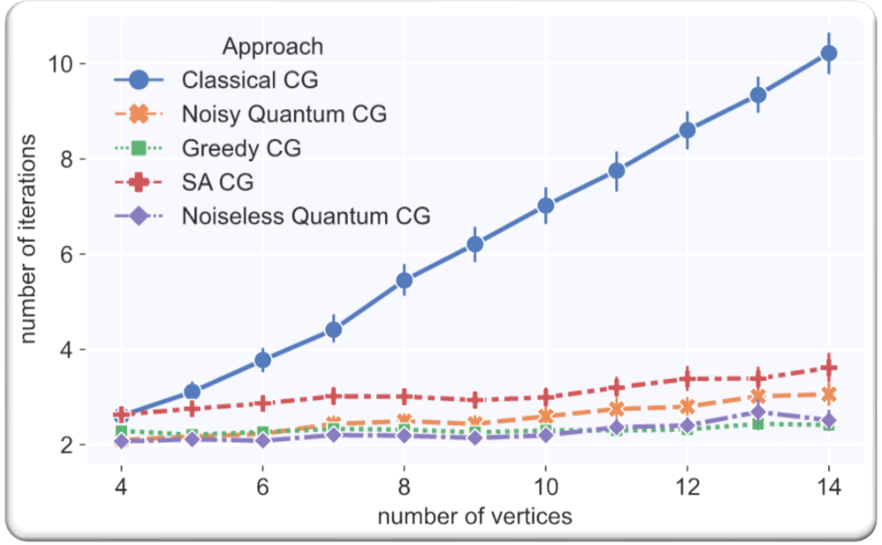




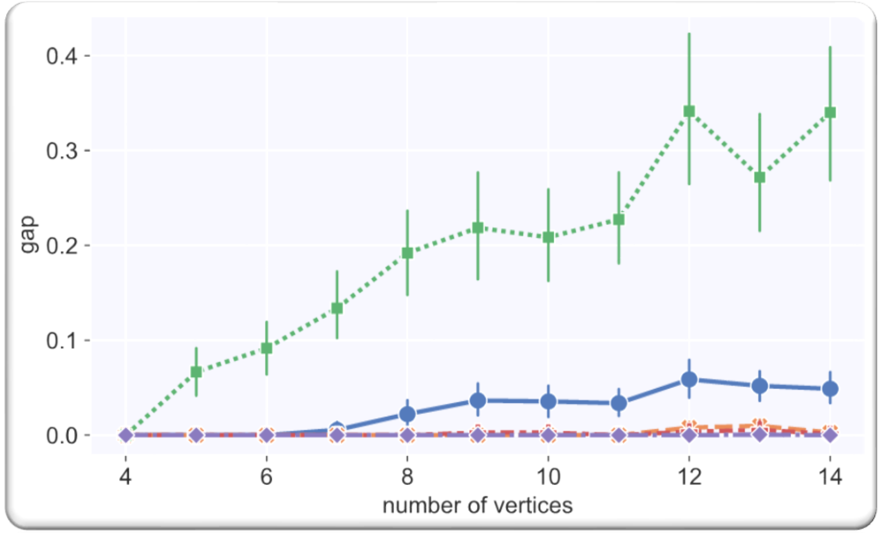
Use QPU as sampler of feasible solutions (i.e. **IS == color**)

Hybrid column generation

Number of iterations generating new options



Distance from the global optimal solution



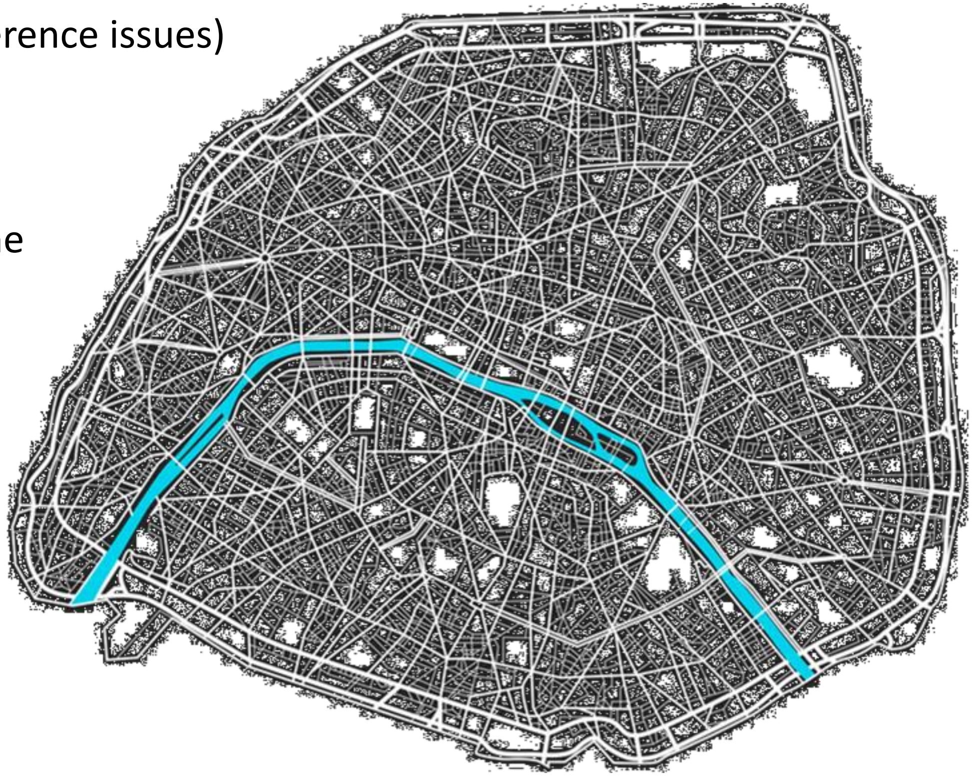
- Hybrid quantum-classical method has the best overall performance
  - It can find better columns than state-of-the-art heuristics : up to 80% less colours
  - It is faster than fully classical methods : up to 6 times faster than the exact classical framework
- However: limited to small graphs for now due to qubit #

Example: Designing the 5G network in Paris

- The frequency assignment problem consists in
  - Set a specific range of radio frequency to each antenna
  - Antennas close to each other cannot emit the same frequency (due to interference issues)

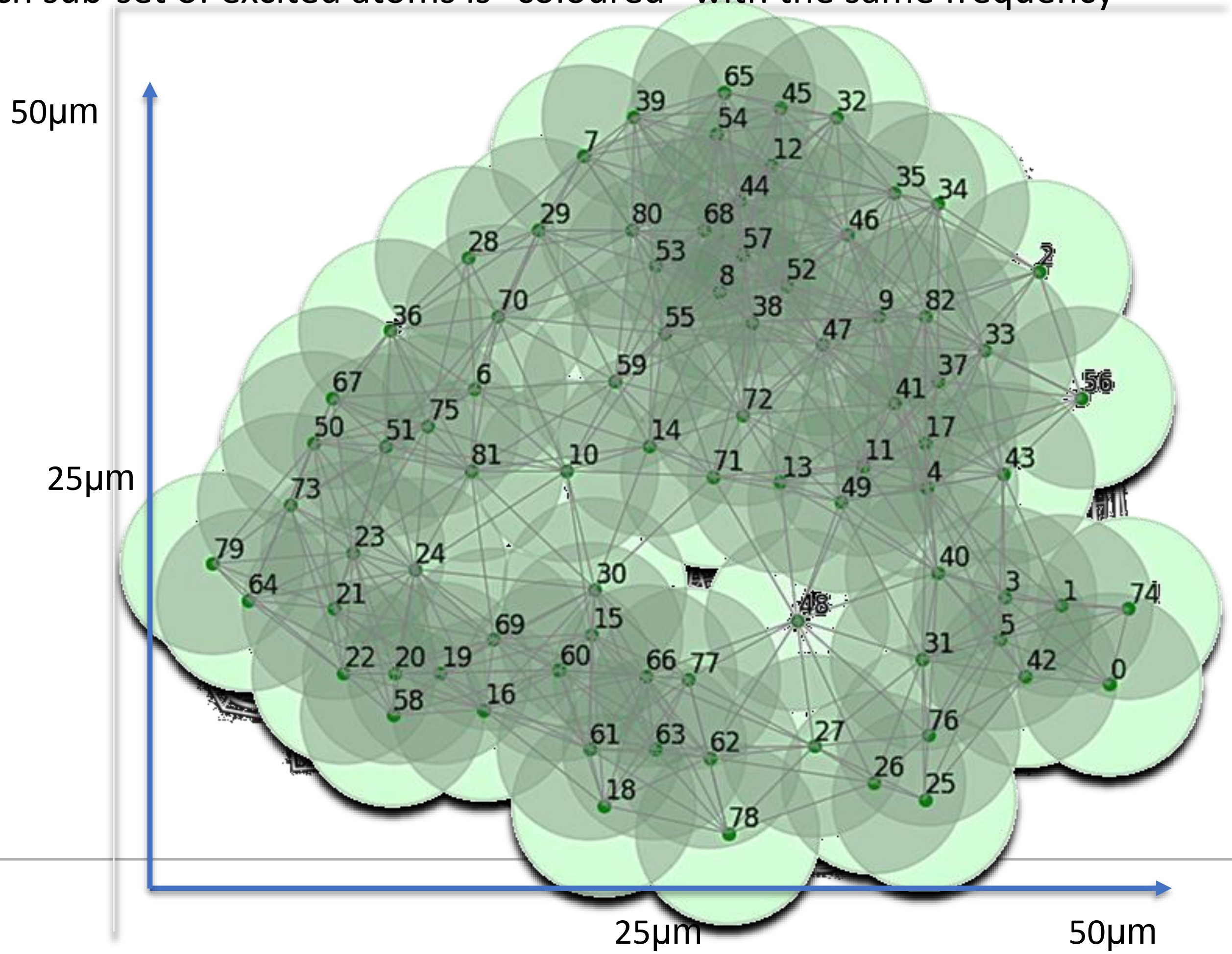
- Since the right to use frequencies are got from auctions
  - We want to minimize the number of different frequencies to cover all the zone
  - While ensuring quality of the services to the clients
    - Maximum coverage
    - Minimum interference

- We can solve the problem as a colouring one
  - Each antenna is a dot
  - Each frequency is a colour



Example: Designing the 5G network in Paris

- We can create a micro-Paris inside PASQAL QPUs
  - Each atom represents an antenna
  - Atoms and Antennas' positions are the same (after rescaling)
  - Atoms too close to each other cannot be excited at the same time
  - On each iteration, each sub-set of excited atoms is "coloured" with the same frequency



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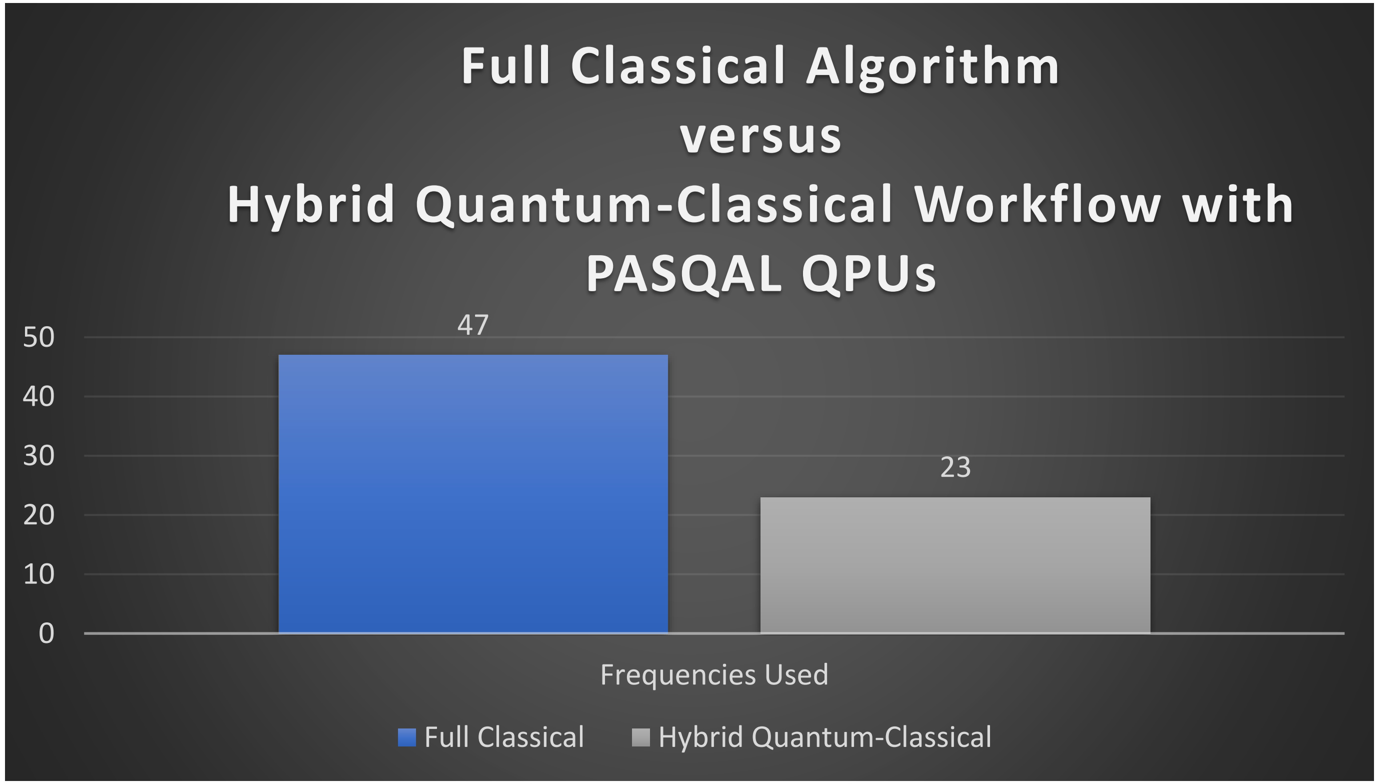
Example: Designing the 5G network in Paris



- Had to stop the classical algorithm after 4 days due to memory consumption.
  - It could not reach the optimal solution.
  - Did not yet use the state of the art classical approach (benchmark is a difficult field!)
  - Hybrid QPU (on emulator) run would take an estimated 3.7 hours

- Each additional frequency range costs millions of euros[1].

[1] ARCEP : Independent French agency in charge of regulating telecommunications.

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Conclusion

Future applications

Vehicle routing problem (deliveries, logistics, transportation…)

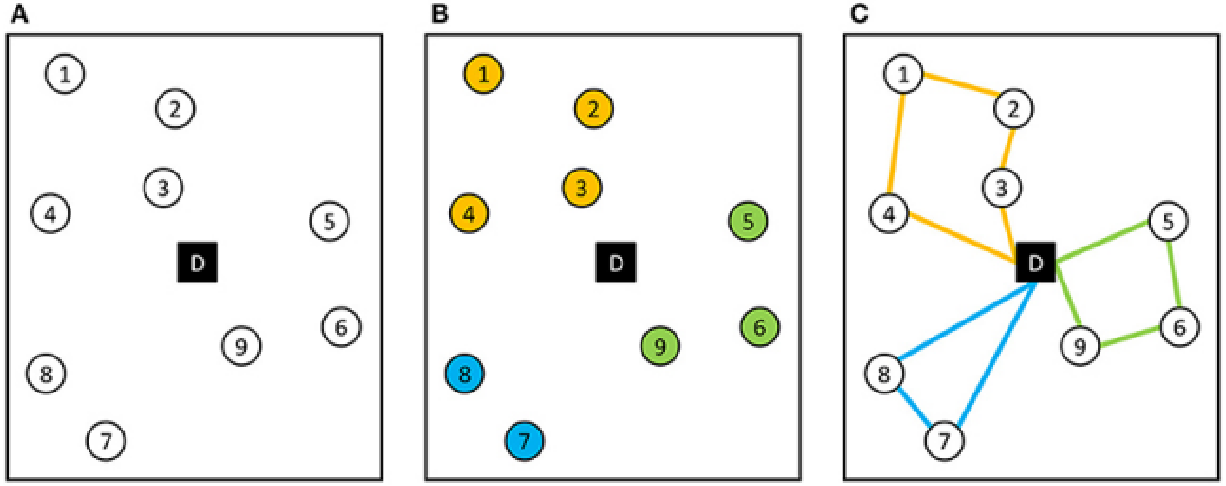

Figure 14: Graphical description of the VRP Problem [27]

Feld et al, A hybrid solution method for the capacitated vehicle routing problem using a quantum annealer. Frontiers in ICT, 6, 6 2019.

Major challenges:

- *Embedding dense graphs on QPU*
- *Ensuring high fidelity pulses*
- *Finding the classical bottleneck to exploit*

More generally: QUBO formalism for combinatorial optimization problems, versatile but with deep challenges

$$QUBO = \mathrm{argmin}\,\vec{x}^T Q \vec{x} = \begin{bmatrix} x_i & \cdots & x_j \end{bmatrix} \begin{bmatrix} Q_{ii} & \cdots & Q_{ij} \\ \vdots & \ddots & \vdots \\ Q_{ji} & \cdots & Q_{jj} \end{bmatrix} \begin{bmatrix} x_i \\ \vdots \\ x_j \end{bmatrix}$$

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The chasm

Applied quantum computing: taking *actual* optimization problems and solving them using QPUs/hybrid

What we are trying to do

MIS

MaxCut

QUBO

VRP

Graph Coloring

Scheduling

Bin Packing

Fundamentally, they are the same class of problem, but applied use-cases have many more constraints and (for now) require specific engineering of the QPU capabilities

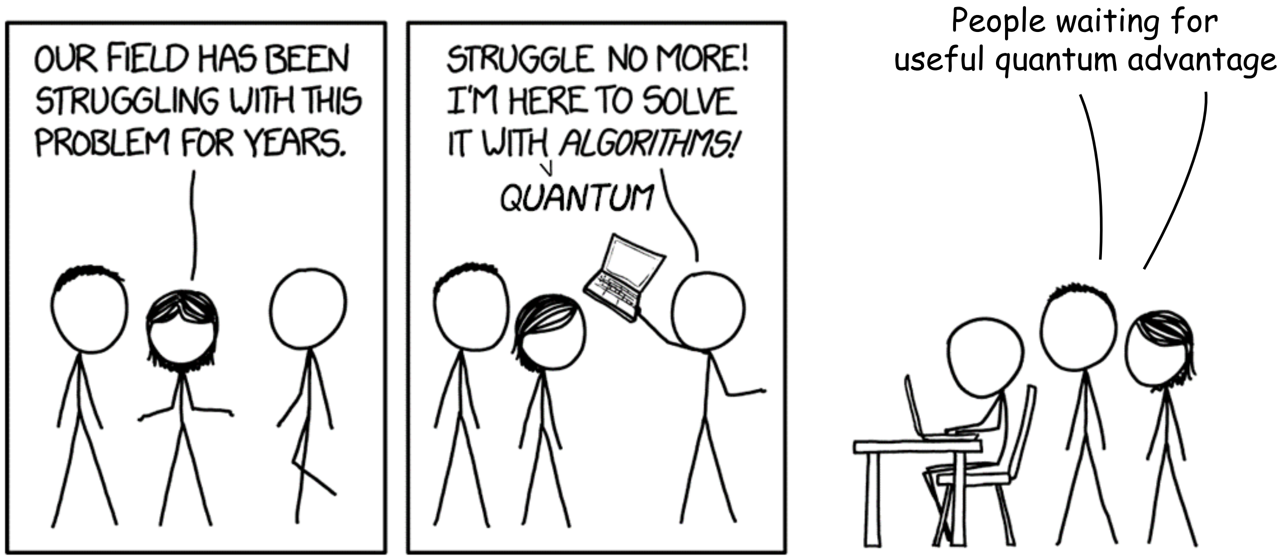
Conclusion

- Rydberg atoms are a promising route for analog quantum computing, with re-programmability and a high degree of control of the quantum states.

- Pulse control + an embedding algorithm provides the versatility to tackle various graph-based combinatorial optimization problems.

THANK YOU

QUESTIONS ?



If different fields don't talk to each other, we can wait a long time…

PASQAL