Reasoning with Vague Spatial Information from Upper Mesopotamia (2000BC)

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Abstract

Historical records provide information regarding the location of ancient places. However, in many cases this information is incomplete and vague. In our research, we analyse historical records from Upper Mesopotamia (2000BC) from the HIGEOMES project. Our goal is to provide better understanding of the location of places, currently with unknown position. We combine cardinal statements found in the records to limit the possible search area, identifying the likehood for the object to be located in a specific point. Our results look promising we expect to improve our results by adding to our analysis, vague concepts other than cardinal directions.

1 Introduction

Humans think and express themselves using vague concepts. In the spatial domain these type of concepts take two forms: 1)*spatial relations*, for instance proximity and cardinal directions and 2) *regions*, for instance middle east, or downtown (Montello et al., 2003).

Computers and GIS software do a great job handling accurate spatial information, however there is still improvement needed for the handling of objects with vague descriptions. Information with vague spatial information abound, as mentioned before this kind of expressions are common in human

language. A suitable way to process these expressions would help researchers have access to a wealth of knowledge currently difficult to use.

In many cases the analysis of vague expressions would help us to discover the location of objects with an unknown/imprecise position. This is a common case in archaeology, where there are objects, cities, places, whose exact location is currently lost. In order to rediscover these objects, researchers can use vague spatial references that link the target object with objects with known positions. For instance, we might be looking for an object X with unknown location, but which position is defined by the expressions: *"X is located South-East of Paris and X is located North-West of Troyes"*. By analysing this expression, we would restrict our search area having a better use of our resources.

In this paper we present our research on the analysis of vague spatial expressions identified in the archaeological records of the HIGEOMES project. The goal of this project is to study the historic geography (2000 BC) of the Upper Mesopotamia. HIGEOMES is a French-German project financed by the ANR and the DFG (HIGEOMES, 2014).

In Section 2 we proceed to describe current research on the field of spatial relationships and vague concepts. In Section 3 we introduce the datasets we use. In Section 4 we describe our proposed model. Finally in Section **??** we discuss our findings, conclussions and possible future related work.

2   Related Research

In Fisher (1999), the author provides an overview of uncertainty problems that GIS users face: error, vagueness and ambiguity. According to Fisher (1999) if uncertainty is caused due to the object or the class to which it belongs are not well defined then we are facing a *vague classification* problem. This type of problems can be traditionally treated by a number of AI methods such as endorsement and/or fuzzy set theory (Fisher et al., 2006). Another type of uncertainy is *Ambiguity*, which arises when there are different perceptions regarding certain phenomenon. Fisher et al. (2006) identifies two types of ambiguity: *discord* and *non specificity*. In the first case, *discord* refers to conflicting views of certain topic, for instance when dealing a land cover using two different land cover classifications or taxonomies. In this case an object can be classified as member of two very different categories.Proposed solutions for this type of problems involve the use of expert

judgement for concept mapping, fuzzy logic, among others. In the second case, *non-specificity*, involves concepts with multiple equally valid interpretations. For instance *a is north of b*. Which could be interpreted as 1) the line *ab* makes an angle of 90 degrees with the horizontal positive axis, or 2) *a* has a latitude value, higher the one of *b*. The second interpretation would enclose concepts such as *a is nort − east of b* and *a is nort − west of b* (Fisher et al., 2006). Similar expressions are common in natural language, for instance proximity: *near by*, *far from* or directional information, *to the right/left*, etc.

Humans are able to extract knowledge from *non specific* expressions. However, computers do not have this capacity. Traditional development of Geographic Information Systems has focused on well specified geometries. The development of capabilities that would allow computers to handle uncertainty is an active research field.

In Montello et al. (2003), the authors follow an empirical approach to identify the concept behind the spatial vague object *Downtown Santa Barbara*. In order to achieve their goal, the researchers perform a survey. They asked people to draw the borders of the concept *"downtown"* in a map. The result showed a great array of understandings of the concept. However, the results suggest that there are core areas identified by all the surveyed people as part of downtown.

Traditional vector representation of spatial entities uses a boolean set approach for the spatial representation. A given point of the space is or is not a part of an object of interest $\{0, 1\}$. An alternative representation can be created using fuzzy sets, in which the spatial representation is based on a fuzzy set membership $[0, 1]$. In this case a point of interest could have a membership value of 0.5, which would mean that it has a smaller degree of belonging to the set compared to another point with a membership of 0.9 (Fisher et al., 2006).

An approach to handle objects with fuzzy boundaries was proposed by Cohn and Gotts (1996) with the *egg-yolk* model. In this work, a vague object with non crisp boundaries is represented by two boundaries: one internal (the yolk) and one external (the white). The internal boundary represents the spatial extension for which there is full certainty regarding the object existence. The area between the inner and outer boundary represents the area for which there is uncertainty. While the exterior of the outer boundary represent areas for which there is certainty about the non existence of the object.

In Charlier et al. (2008), the researchers extend the *yolk-egg* model in order to apply it, not only to spatial vagueness, but to attribute vagueness. The resulting approach is used to classify land cover from remote sensing images.

According to Schneider (1996), there are at least three alternatives to model spatial objects with undetermined boundaries: 1) fuzzy models, 2) probabilistic models, and 3) transfer of data models for sharp boundaries for use with objects with no clear boundaries. In Schneider (1996), the authors present the Realm/ROSE model which is based on the third approach. It aims to represent within a RDBMS objects with fuzzy boundaries elements using primitives such as points, lines or regions. To represent objects with fuzzy borders, the author creates two concentric boundaries similar in nature to the *yolk-egg* model. Schneider (1996) introduces formalisms required for spatial operations. A related approach is presented in Kraipeerapun (2004), using fuzzy set theory and vector data. The relationships between vague spatial regions have been studied in Schockaert et al. (2008), here the authors discuss a generalization of region connection calculus targeted to regions with fuzzy borders.

In the case of regions, a common approach for the formalisation of cardinal relationships involves the use of the bounding box (See Figure 1). In Goyal and Egenhofer (2000), the authors introduce a model to represent the spatial relations between objects using their bounding boxes taking into consideration the object shape. This model is extended on Skiadopoulos and Koubarakis (2004) and Sun and Li (2005). In Skiadopoulos and Koubarakis (2004), the authors present their work on the composition of cardinal directions. The goal of Skiadopoulos and Koubarakis (2004) is to develop mechanisms to detect possible inconsistencies in data with cardinal statements.

Later, Navarrete et al. (2013) extends the rectangular cardinal relation calculus (RCD calculus) to model cardinal relations between connected and disconnected regions.

In the case of points, a common approach is to divide the space into a number of non-overlapping sectors, each corresponding to a cardinal direction. The cardinal relations between the reference point and any other target object are defined by the intersection of the target object with a given sector (See Figure 1).

However, in reality spaces defined by cardinal statements have fuzzy boundaries, therefore it is necessary mechanisms to handly this uncertainty. An interesting work in this field is presented by Schockaert et al. (2008).
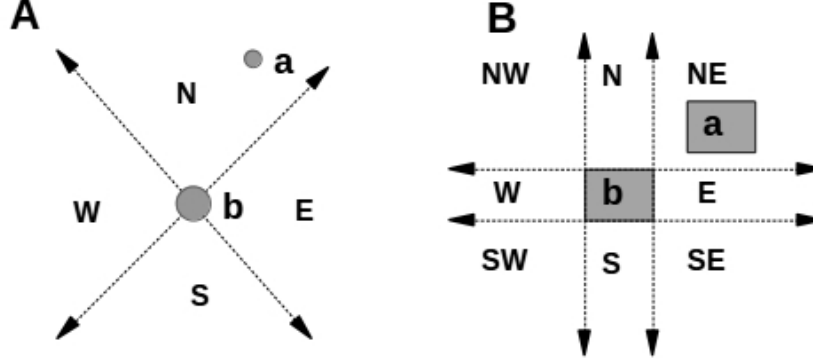
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Figure 1: Cardinal Directions defined for A) points and B) regions.

Here, the authors present a set of formal definitions to work with cardinal and proximity concepts with fuzzy boundaries. For instance, the concept *near to* is defined as:

$$
NearTo_{(\alpha,\beta)}(a,b)
\begin{cases}
1 & if\ d(a,b) < \alpha \\
0 & if\ d(a,b) > \alpha + \beta \\
\frac{\alpha+\beta-d(a,b)}{\beta} & otherwise(\beta \neq 0)
\end{cases}
\tag{1}
$$

where a, b are points in the space, while α, β are distances larger than 0. This is an implementation of the *yolk-egg* model, as presented by Cohn and Gotts (1996). The value of α defines the yolk area, which is the area that represents a full certitude for the vague concept. In the case of $NearTo_{(\alpha,\beta)}(a,b)$, it would be a common agreement that if the distance $d(a,b$ is smaller than *alpha*, then q is certainly near to a. If the distance is $(\alpha < d(a,b) < (\alpha+\beta))$ there is a certain degree of uncertainty. While if $((\alpha + \beta) < d(a,b))$, there is absolute certainty that b is not near a.

Similar formalisms can be used to define cardinal directions:

$$
CardinalDirection_{(\theta,\alpha,\beta)}(a,b)
\begin{cases}
1 & if\ ad(\theta_{ab},\theta) \leq \alpha \\
0 & if\ ad(\theta_{ab},\theta) > \alpha + \beta \\
\frac{\alpha+\beta-ad(\theta_{dp},\theta)}{\beta} & otherwise(\beta \neq 0)
\end{cases}
\tag{2}
$$

where θ is the angle that defines the cardinal direction, $ad(\theta_{pq},\theta)$ is the angular difference between the a line from ab a line with angle θ starting in a,
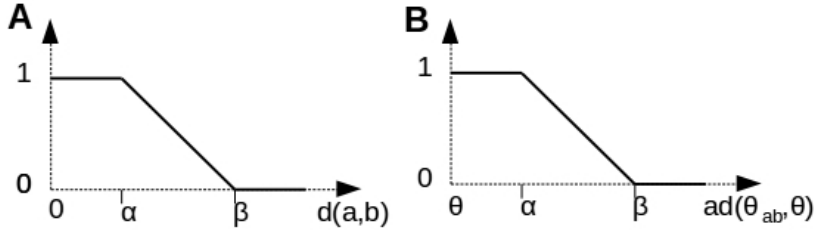
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Figure 2: Membership functions for for A) Proximity and B) Cardinal.

while α and β are two angles that define two concentric cones. The interior cone is constituted by points for which there is certainty they belong to the cardinal direction, while the area between the interior and the exterior cones constitutes the fuzzy boundary of the concept.

Figure 2 depicts the membership functions for Equations 1 and 2. In both cases there is an area in which there is full certainty regarding the membership (value 1). There is also an area in which there is full certitude regarding the non membership (value 0). Between both areas there is a transition representing the fuzzy membership areas. In both cases, cardinal and proximity, Schockaert et al. (2008) uses a linear function to express the gradual change from full to non membership.

An interesting use of epigraphical data in the search of lost places can be found in ?.Here the authors select a number of ancient texts and count the frequency of appearance of the names of places. The author argues that there is a relationship between the importance of the places and the frequency in the texts. Additionally it is suggested that the fact that places appear in the same text would indicate spatial proximity.

Research presented in Frank (1991) aims to infer new knowledge by combining previous cardinal statements. This research is later extended in and ? by including proximity statements. The approach is qualitative, in it the author aims to compose directions in order to have an approximate result.

In Lukasiewicz and Straccia (2009), the authors combine fuzzy description logics with stratified probabilistic logic programs. They also propose an interesting formalisms for the implementation of fuzzy objects using Description logics.

A spatial object defined by non-specific statements has undetermined boundaries. As stated by Gottsegen et al. (1999), this is the result from a less than perfect knowledge about a phenomena of interest. In this paper we

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focus on the concept of uncertainty regarding the location of ancient places. The real object has well defined coordinates. However, with the pass of time this information has been lost. The result is a set of places for which their location is based on spatial concepts with vague boundaries.

In the next section we proceed to describe the datasets we use and how the we can better understand the position of certain places using vague spatial concepts.

3 Datasets and methods

There is a great amount of written records from Mesopotamia between 2000 to 1600 BC. The project ARCHIBAB offers access to more than 31000 texts from this period. The ARCHIBAB project is hosted by the College de France, funded by the Agence Nationale de la Recherche (ANR) ((Archibab Project, 2012)). The information provided by ARCHIBAB is textual, it lacks semantic annotations.

In 2011 a new project called HIGEOMES started with the goal of analyze epigraphical data from ARCHIBAB, in order to study the socio-economic forms of space used in Upper Mesopotamia (HIGEOMES, 2014).This project is funded by ANR and Deutsche Forschungsgemeinschaft (DFG). The project HIGEOMES aims to add semantic annotations to the original data, allowing more sophisticated uses of the information. One of the goals of HIGEOMES is the integration of heterogeneous datasources using OGC standards such as Web Feature Services (WFS) and Web Map Services (WMS) (Karmacharya et al., 2013; Kohr et al., 2013).

One of the challenges of HIGEOMES is the handling of vague spatial information. Approaches like the one introduced in Schockaert et al. (2008) can be used for this task. However, the identification of suitable values for the parameters of Equations 1 and 2 , as well as the best transition/membership function needs further research. In this work, we analyse empirical data in order to construct a membership function for vague spatial objects based on cardinal statements. The results of this analysis would help archaeologist to better use their resources in their research.

Type	Center Angle (σ)
isEastOf	0°
isNorthOf	90°
isWestOf	180°
isSouthOf	270°

Table 1: Cardinal directions and angles with X positive axis.

3.1   Cardinal statements in HIGEOMES

In our records, we have a set of cardinal expressions, between places with known and unknown location. In this paper we present our work on the analysis of cardinal expressions in order to better understand their use by ancient Mesopotamians.

In our datasets we have entities called *toponyms*. These are places with names that have evolved along time. There is information related to the toponyms, like their various names along time, type of production, traffic routes for which they are components, etc. In some cases the actual coordinates of the place might be known while in other others they might have been lost in time.

Among the epigraphical information, we have cardinal statements that link *toponyms*, for instance *a isNorthOf b*, where *a* and *b* are toponyms. In some cases, these cardinal statements link two toponyms with known location. In some other cases, the cardinal statements are the only spatial information available regarding a toponym with unknown position.

For each of the cardinal relations we know the angle with the positive X axis that best represents the cardinal direction, we refer to those as center angles (σ). (See Table 1).

In our dataset the total number of toponyms is 1224. From those, we known with certitude the location of 864, while the position of the rest is unknown. In our dataset we can find 41 cardinal relations between toponyms with known location. Table 2 provides a summary of the relations identified.

For each cardinal statement between toponyms, we have a reference and a target toponym. For instance in t_t *isNorthOf* t_r, t_t is the target and t_r is the reference. If we know the position of both t_t and t_r, we can draw a line from t_r to t_t and measure the angle of this line with the center angle σ corresponding to the cardinal direction of the statement.

For instace, Figure 3 depicts the analysis of the statement *toponym-65 is*

Type	Count
isEastOf	7
isNorthOf	16
isWestOf	7
isSouthOf	11
Total	41

Table 2: Summary of relationships between toponyms with known location
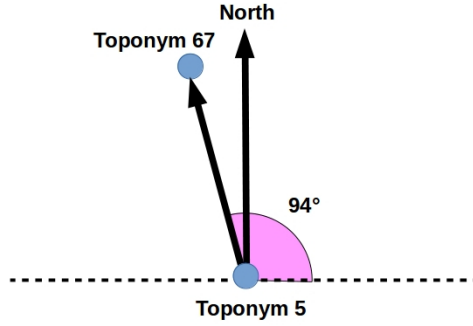

Figure 3: A cardinal relationship: *toponym-65 is North of toponym-5*

North of toponym-5. The line that links both toponyms has a 4° difference with the angle corresponding to North(90°).

By analysing all the available cardinal statements, we can estimate the vagueness of the cardinal statements as used by ancient inhabitants of upper Mesopotamia. Figure 4A depicts the frequency distribution of the angular differences. Our results indicate that the mean angular difference was close to zero degrees $(-0.04 \approx 0)$, with an standard deviation of 37.7. In the dataset the maximum number of standard deviations from the mean is 1.84. Figure 4B depicts the cardinal relationships with their number of standard deviations from the mean.

We intent to use the distribution of the angular differences as a tool to manage uncertainty in cardinal relations. In order to achieve this goal, we need to simplify the curve that represents the distribution. In statistics this procedure is called *curve fitting*. The shape of the distribution curve suggest that a suitable model for this task would be a *Gaussian* one. Figure 4A depicts the Gaussian curve fitted to the density distribution of the angular
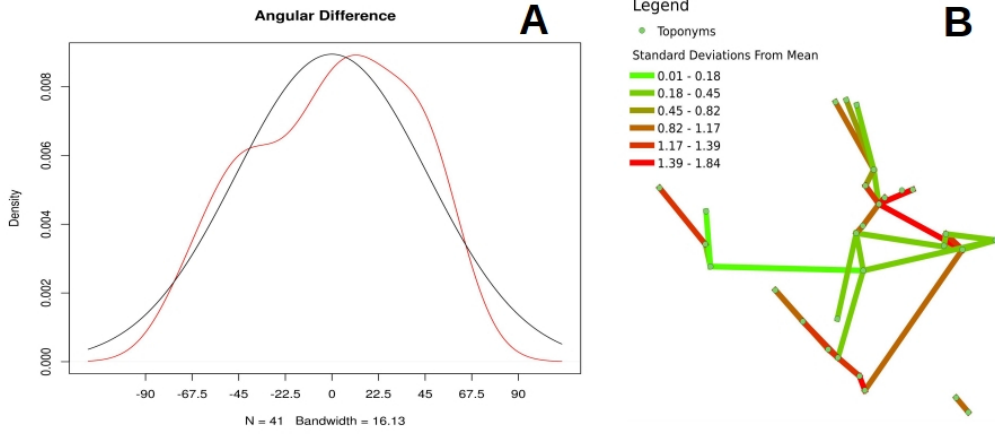
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Figure 4: Cardinal Statements between toponyms with known location: A) In red line, actual density distribution. In black line, a fitted Gaussian curve. B) The location of toponyms and cardinal relationships between them. The color depicts the standard deviations from the mean angular difference.

differences. The equation for the curve is:

$$density = p1 \; e^{\left(-\frac{\Delta ang^2}{p2}\right)} \qquad (3)$$

where $p1 = 0.008947$ and $p2 = 4313.35$.

We propose to use this curve in order to model a membership function for vague objects defined by the cardinal relationships. In order to to this, we would need to rescale the curve to match the membership range $[0, 1]$. We can accomplish this task by changing the value of parameter $p1$ to value 1. Then we have the membership function as:

$$m(\Delta ang) = e^{\left(-\frac{\Delta ang^2}{4313.35}\right)} \qquad (4)$$

Using this function it is possible to determine for any point in space its membership value for a vague object based on a cardinal statement. We can then, use this membership function in order to map the most likely location for *toponyms* which location is based only on vague cardinal statements. At the moment we have 25 *toponyms* with unknown coordinates linked to 17 *toponyms* with known ones. Figure 5 depicts the *toponyms* with known position.

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Figure 5: Toponyms with known location involved in the definition of vague objects

4 Model

In this section we present our model using Description Logics and First Order logic.

4.1 Spatial Points

The first concept that we need to define is an spatial point:

$$\mathcal{P} \equiv hasLongitude \sqcap hasLatitude \tag{5}$$

4.2 Toponym

This concept is a subset of Spatial Points, that represent places of human activities. These places had have several names along time, and are related to each other by cardinal statements. It is also possible to find them as part of commerce routes.

$$Toponym \equiv \mathcal{P} \sqcap hasModernName \sqcap hasOldBabylonianName \tag{6}$$

4.3 Vague spatial objects

A vague spatial object does not have crisp boundaries. It is represented as a function over space, in which the value of the function represents the different

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degrees of membership or certainty of existence of the object at the specific location. In order to discretize the function we propose the use of a lattice of points, each with a membership value. In this way a vague spatial object would be represented as:

$$\mathcal{V} \equiv hasVagueArea.\mathcal{M} \tag{7}$$

where $\mathcal{M}$ is a 2D matrix of points m. The lattice can be viewed as a grid of points that cover the study area. Each point in the grid has a location and a membership value in the range $[0,1]$. The membership value is given by a function $f(m_{pos})$ which evaluates the position of the point. The specific parameters of the function would depend on the domain of use.

4.4 Vague Cardinal regions

We define the cardinal regions as $\mathcal{C}$ which is a subclass of vague spatial objects:

$$\mathcal{C} \sqsubseteq \mathcal{V} \tag{8}$$

The class cardinal regions has two extra properties: $hasReferencePoint$ and $hasCenterAngle$.

$$
\begin{aligned}
\mathcal{C} \equiv & hasVagueArea.\mathcal{M} \sqcap \\
& (= 1 hasReferencePoint.\mathcal{P}) \sqcap \\
& (= 1 hasCenterAngle.Degrees) \sqcap
\end{aligned}
\tag{9}
$$

For a cardinal region, we need to set a matrix corresponding to the value of the property $hasVagueArea$. The lattice M is composed by a set of points m, each corresponding to a position in a regular grid. The membership value for each point is the result of the function $f(m_{po}s, r_{pos}, \sigma)$

$$
\begin{aligned}
\forall c \in \mathcal{C} | & hasVagueArea(c, M_c) \wedge \\
& hasReferencePoint(c, r) \wedge \\
& hasCenterAngle(c, \sigma) \\
& \rightarrow (\forall m \in M_c), f(m_{pos}, r_{pos}, \sigma) = e^{\left(-\frac{\Delta ang(m_{pos}, r_{pos}, \sigma)^2}{4313.35} \right)}
\end{aligned}
\tag{10}
$$

We evaluate the position of each point m in regard the position of the reference point r. We calculate the angle that the line between them makes with the horizontal using the geometric function arc tangent ($arcTan$). Then we calculate the difference between the angle and the center angle corresponding to the cardinal direction.

$$\Delta ang(m_{pos}, r_{pos}, \sigma) = arcTan\left(\frac{m_{lat} - r_{lat}}{m_{lng} - r_{lng}}\right) - \sigma \qquad (11)$$

where m_{lat} and m_{lng} represent the latitude and longitude of the point m, while r_{lat} and r_{lng} represent the same for the reference point.

4.5 Operations between vague objects

Operations between objects with vague boundaries have been studied previously by Zadeh (1965), Chakrabarty et al. (1997) and Schockaert et al. (2008). The common approach is the creation of a new vague set. The union operation is defined as:

$$c = a \cap b | f_c(x) = Max[f_a(x), f_b(x)], x \in X \qquad (12)$$

While the intersection operation is defined as:

$$c = a \cap b | f_c(x) = Min[f_a(x), f_b(x)], x \in X \qquad (13)$$

In our research we define the union of vague spatial concepts as:

$$\forall a, b \in \mathcal{C} | hasVagueArea(a, M_a) \wedge$$
$$hasVagueArea(b, M_b) \qquad (14)$$
$$\rightarrow M_a \cup M_b = Max(M_a, M_b)$$

While the intersection of vague spatial concepts would be:

$$\forall a, b \in \mathcal{C} | hasVagueArea(a, M_a) \wedge$$
$$hasVagueArea(b, M_b) \qquad (15)$$
$$\rightarrow M_a \cap M_b = Min(M_a, M_b)$$

Where M_a and M_b have the same size, and are composed by points with the same spatial position. The result of $Min(M_a, M_b)$ is a matrix in which

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we compare the points of both matrices. For each pair of points with the same spatial position, we keep the one with the lowest membership value. Using the operation it is possible to combine different vague spatial concepts.

4.6   Combining vague spatial objects

The dataset contains cardinal statements such as $t1$ $isNorthOf$ $t2$:

$$isNorthOf(t1, t2) \tag{16}$$

where both $t1$ and $t2$ are instances of the class $Toponym$.

From the previous statement we can deduce that there is a vague cardinal region $x1$ that is defined by the position of $t2$ and the angle associated to the expression $isNorthOf$:

$$
\begin{aligned}
\exists\ x1 | &\mathcal{C}(x1) \wedge \\
&hasReferencePoint(x, t2) \wedge \\
&hasCenterAngle(x1, 90) \wedge \\
&hasVagueArea(x1, M_{x1})
\end{aligned}
\tag{17}
$$

Then we can conclude that:

$$isWithin(t1, x1) \tag{18}$$

If there are more than one vague spatial concept that describes the position of $t1$, it would be easy to combine al the relevant spatial concepts using an intersection operator:

$$isWithin(t1, x1) \wedge isWithin(t1, x2) \ldots isWithin(t1, xn) \rightarrow isWithin(t1, x) \tag{19}$$

where:

$$hasVagueArea(x, M_x) | (M_x = M_{x1} \cap M_{x2} \cdots \cap M_{xn}) \tag{20}$$

Where $(x1, x2 \ldots xn)$ are instances of the class $Vague\ Spatial\ objects$ $(\mathcal{V})$. The result of the intersection operation is defined in Equation 15.
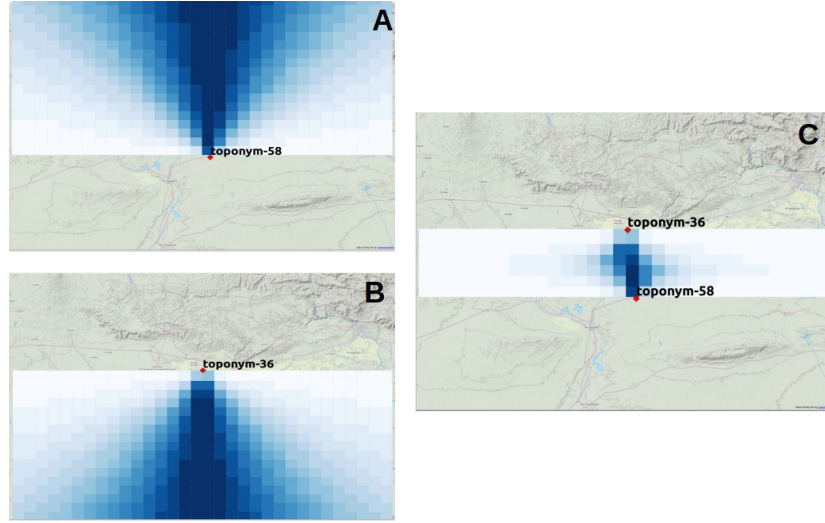
Figure 6: Location of *toponym-28* defined by: A)*toponym-28 isNorthOf toponym-58* B)*toponym-28 isSouthOf toponym-36* C) Vague location of *toponym-28*.

4.7 Implementation code

The model has been implemented using Stardog as a triple store. The spatial operations have been implemented in a JAVA application. The implementation of the properties and datatypes is straighforward. The lattices are stored in the triplestore as text. The operations between lattices are implemented in a Java program.

4.8 Example 1

The location of *toponym-28* is unknown. However, we know that: a)*toponym-28 isNorthOf toponym-58* and b)*toponym-28 isSouthOf toponym-36*. By creating the vague areas corresponding to *NorthOf toponym-58* and *SouthOf toponym-36* and intersecting them we would have created a vague object that represents the location of *toponym-28* (See Figure 6)

4.9 Example 2

To be defined!

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5 Conclussions

In Egenhofer and Al-taha (1992), the researchers aim to find the links between how people conceptualize their geographic environment and formal models aimed to GIS software. The authors identify elements that are influenced by cultural traits that affect people's perceptions of space. For instance, people commonly fail to estimate the third dimension of their environment, over-estimating slopes, depths and highs. Another example, is the perception of the flatness of the Earth, for example, when not considering the great circle when tracing straight lines between geographically distant points.

It would be interesting to consider how the gender of the author of the texts affected the way the vague spatial objects were encoded. There is research regarding how modern western educated humans provide directions. For instance Choi (2000) and Lawton (2001) studied the differences between males and females when providing directions. According to both studies while males tend to make more use of cardinal relations, females prefer the use of landmarks. Although Lawton (2001) concluded that in both genders the type of directions used was greatly affected by the environment. It is intriguing to compare the use of spatial directions between modern humans with ancient Mesopotamians.

In pre-metric cultures different measures are based on human capacity to do something. For instance acres, morgens or arpents are area units whose measure is based on the amount of surface that can be plow by a human in a certain amount of time. Similar units exist for distances based on how far would a man, an army or a hourse might walk in a given amount of time Egenhofer and Al-taha (1992). Future research on the analysis of ancient transcripts would require an accurate interpretation of distance units, as employed by ancient Mesopotamians.

In future work, we will include in our analysis distance statements as vague objects. However, to do this it is necessary to consider that distance as perceived by people is asymmetric. The distance from point A to point B might be perceived as the time it takes to go from $A - B$, while this time might be different in the opposite directionEgenhofer and Al-taha (1992).

It is more common to find errors in the perception of latitudes than longitudes Egenhofer and Al-taha (1992).

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