Homework 12

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April 22, 2017

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Verify that the given function pair is a solution to the first-order system.

$x = -e^t$, $y = e^t$
$\frac{dx}{dt} = -y$, $\frac{dy}{dt} = -x$

$\frac{dx}{dt} = \frac{d}{dt}(-e^t) = e^t = y$; $\frac{dx}{dt} = -y$

$\frac{dy}{dt} = \frac{d}{dt}(e^t) = -e^t = x$; $\frac{dy}{dt} = -x$

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Find and classify the rest points of the given autonomous system.

$\frac{dx}{dt} = -(y-1)$, $\frac{dy}{dt} = x - 2$

The rest point of the system is a point in the phase plane for which $f(x,y) = 0 and g(x,y) = 0$, then both the derivatives $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$.

when $y = 1$, $\frac{dx}{dt} = -(1-1)$; $\frac{dx}{dt} = 0$
when $x = 2$, $\frac{dy}{dt} = 2 - 2$; $\frac{dy}{dt} = 0$

$(2, 1)$ is the rest point of the autonomous system $\frac{dx}{dt} = -(y-1)$, $\frac{dy}{dt} = x - 2$

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Apply the first and second derivative tests to the function $f(y) = y^a/e^{by}$ to show that $f(y) = y^a/e^{by}$ is a unique critical point that yields the relative maximum $f(a/b)$. Show also that $f(y)$ approaches zero as y tends to infinity.

first derivative:

$\frac{df(y)}{dy} = 0$
$\frac{d(\frac{y^a}{e^{by}})}{dy} = 0$

Use the product rule: $= y^a(\frac{d}{dy}(e^{-by})) + e^{-by}(\frac{d}{dy}(y^a))$

Use the chain rule: $= \frac{\frac{d}{dy}(y^a)}{e^{by}} + \frac{\frac{d}{dy}-(by)}{e^{by}}y^a$
Factor out constants: $= \frac{\frac{d}{dy}(y^a)}{e^{by}} + \frac{-b\frac{d}{dy}(y)}{e^{by}}y^a$
The derivative of y is 1: $= \frac{\frac{d}{dy}y^a}{e^{by}} - \frac{1by^a}{e^by}$

Use the power rule: $= \frac{-by^a}{e^{by}} + \frac{ay^{a-1}}{e^{by}}$

Answer: $y^{a-1}e^{-by}(a - by) = 0$

Since e^{by} cannot be zero :

$y = \frac{a}{b}\ or\ y = 0$

Second derivative

$\frac{d^2f(y)}{dy^2} = \frac{d\frac{ay^{a-1}-y^ab}{e^{by}}}{dy}$
$\frac{d^2f(y)}{dy^2} = \frac{e^{by}\{a(a-1)y^{a-2}-aya-1\}-(ay^{a-1}-y^a)be^{by}}{e^{2by}}$

Substitute the value of 0 :

$\frac{d^2 f(y)}{dy^2} = 0$

Substitute the value of $y = \frac{a}{b}$

$\frac{d^2 f(y)}{dy^2} = \frac{(\frac{a}{b})^{a-2} x - a}{e^a}$

As a and b are positive, the second derivative is less than zero and therefore it is proved that $y = \frac{a}{b}$ yields the relative maximum.