Homework 3

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1 Problem Set 1

1.1 Problem 1

What is the rank of the matrix A?

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 1 & 3 \\ 0 & 1 & -2 & 1 \\ 5 & 4 & -2 & -3 \end{bmatrix}$$

```
A <- t(matrix(c(1 , 2 ,3 , 4,
              -1 , 0 , 1 , 3,
              0 , 1 , -2 , 1,
              5 , 4 , -2 , -3), nrow = 4, ncol = 4))
```

Since A is a square matrix of (4 x 4) and the determinate is -9 which is $\neq 0$, the rank is simply 4

Below is my attempt to create a function to rank matrix A and matrix B in problem 3 programmatically. I think that the part of the function that calculates the determinates of the submatrices could be improved.

```
A <- t(matrix(c(1 , 2 ,3 , 4,
              -1 , 0 , 1 , 3,
              0 , 1 , -2 , 1,
              5 , 4 , -2 , -3), nrow = 4, ncol = 4))

subdet <- function(A, t){
            subdet <- 0
           for (i in 1:t){
             subdet <- rbind(subdet,det(A[-i,-i]))
           }
           if (sum(subdet) !=0){
             return(as.integer(ncol(A))-1)
           } else {
             0
           }
```

```
}

Matrix.Rank <- function(A){
               sq.matrix <- (as.integer(ncol(A)) == as.integer(nrow(A)))
               if (all(A == 0)){
                 return(0)
               } else if (sq.matrix == TRUE){
                 det.0 <- det(A) != 0
                 if (det.0 == TRUE) {
                 return(as.integer(ncol(A)))
                 } else if (subdet(A,ncol(A)) != 0) {
                  return(as.integer(ncol(A))-1)
                 } else {
                     return(1)
                   }
                 }
               }

Matrix.Rank(A)
```

```
## [1] 4
```

1.2 Problem 2

Given an mxn matrix where $m > n$, what can be the maximum rank? The minimum rank, assuming that the matrix is non-zero?

> The maximum rank of a rectangular matrix is the maximum columns or rows for the lesser value. Therefore, given an mxn matrix where $m > n$, the maximum rank is n.
>
> Assuming the rectangular matrix has at least one non-zero element, it's minimum rank must be greater than zero, therefore the minimum rank would be 1.

1.3 Problem 3

What is the rank of matrix B?

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 3 \\ 2 & 4 & 2 \end{bmatrix}$$

```
B <- t(matrix(c( 1 , 2 , 1 ,  3 ,  6 , 3 ,  2 ,  4 , 2 ), ncol = 3, nrow = 3))
```

The matrix rows are linearly dependent, as R2 = 3, 6, 3 and R3 = 2, 4, 2 are mulitples of R1 = 1, 2, 1. This made my function more challenging because the determinates of the submatrices of B are also $= 0$. However, as long as there is at least one non-zero element in the matrix the minimum rank will be $= 1$.

```
Matrix.Rank(B)
```

```
## [1] 1
```

2 Problem Set 2

Compute the eigenvalues and eigenvectors of the matrix A. You'll need to show your work. You'll need to write out the characteristic polynomial and show your solution.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

$$det(\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}) = 0$$

$$det(\begin{bmatrix} 1-\lambda & 2 & 3 \\ 0 & 4-\lambda & 5 \\ 0 & 0 & 6-\lambda \end{bmatrix}) = 0$$

which reduces to:

$(1-\lambda)(4-\lambda)(6-\lambda) = 0$

Therefore, our Eigen Values are:

$\lambda_1 = 1; \lambda_2 = 4; \lambda_3 = 6$

Our Eigen Vectors are as follows:

$\lambda_1 = 1$

$$det(\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}) = 0$$

$$\begin{bmatrix} 0 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

$$v_1 = 1;\ v_2 = 0;\ v_3 = 0$$

Therefore:

$$E\lambda_{=1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda_2 = 4$$

$$det(\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}) = 0$$

$$\begin{bmatrix} -3 & 2 & 3 \\ 0 & 0 & 5 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

Which we can reduce to $-3v_1 + 2v_2 + 3v_3 = 0 \; and \; v_3 = 0$; substituting back we have $-3v_1 + 2v_2 = 0$ or $3v_1 = 2v_2$ then $v_1 = \frac{3}{2}v_2$ where $v_1 = 1$.

Therefore:

$$E\lambda_{=4} = \begin{bmatrix} 1 \\ \frac{3}{2} \\ 0 \end{bmatrix}$$

$$\lambda_3 = 6$$

$$det(\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} - \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}) = 0$$

$$\begin{bmatrix} -5 & 2 & 3 \\ 0 & -2 & 5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

Which we can reduce to $-5v_1 + 2v_2 + 3v_3 = 0 \; and \; -2v_1 + 5v_2 = 0$; solving for $-2v_2 + 5v_3 = 0$, we reduce to $5v_3 = 2v_2$ or $v_3 = \frac{5}{2}v_2$. Substituting back $v_3 = 1$ we get $1 = \frac{5}{2}v_2$ or $\frac{2}{5} = v_2$. Now, subsituting back to $-5v_1 + 2v_2 + 3v_3 = 0$ or $-5v_1 + 2(\frac{2}{5}) + 3(1) = 0$ then $-5v_1 + 3\frac{4}{5} = 0$ then $5v_1 = 3\frac{4}{5}$ then $5v_1 = 3\frac{4}{5}$ then $v_1 = \frac{19}{25}$

Therefore:

$$E\lambda_{=6} = \begin{bmatrix} \frac{19}{25} \\ \frac{2}{5} \\ 1 \end{bmatrix}$$