Homework 6

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Contents

1 Page 251: problem 2

Use the model-building process described in Chapter 2 to analyze the following scenarios. After identifying the problem to be solved using the process, you may find it helpful to answer the following questions in words before formulating the optimization model.

a. Identify the decision variables: What decision is to be made?
b. Formulate the objective function: How does these decisions affect the objective?
c. Formulate the constraint set: What constraints must be satisfied? Be sure to consider whether negative values of the decision variables are allowed by the problem, and ensure they are so constrained if required.

Nutritional Requirements - A rancher has determined that the minimum weekly nutritional requirements for an average-sized horse include 40lb of protein, 20 lb of carbohydrates, and 45lb of roughage. These are obtain from the following sources in varying amounts at the prices indicated:

	Protein (lb)	Carbohydrate (lb)	Roughage (lb)	Cost
Hay (per table)	0.5	2.0	5.0	$1.80
Oats (per sack)	1.0	4.0	2.0	3.50
Feeding blocks (per block)	2.0	0.5	1.0	0.40
High-protein concentrate (per sack)	6.0	1.0	2.5	1.00
Requirements per horse (per week)	40.0	20.0	45.0	

Figure 1:

Formulate a mathematical model to determine how to meeting the minimum nutritional requirements at minimum cost.

a. The decision to be made is how best to meet the nutritional needs of an average-sized horse while maintaining costs a minimum.

b. There are 4 choices available with varying protein, carbohydrate, and roughage values that influence the decision. The decision to purchase one choice over the other impacts prices

c. Constraint set

$x_1 = Hay\ (per\ table)$
$x_2 = Oats (per\ sack)$
$x_3 = Feeding\ blocks (per\ block)$
$x_4 = High - protein\ concentrate (per\ sack)$

Cost = $1.8x_1 + 3.5x_2 + .4x_3 + 1x_4$

We want to minimize cost but we have the following constraints or subject to:

$$0.5x_1 + 1.0x_2 + 2.0x_3 + 6.0x_4 \geq 40.0$$
$$2.0x_1 + 4.0x_2 + 0.5x_3 + 1.0x_4 \geq 20.0$$
$$5.0x_1 + 2.0x_2 + 1.0x_3 + 2.5x_4 \geq 45.0$$
$$x_1, x_2, x_3, x_4 \geq 0$$

2 Page 264: problem 6

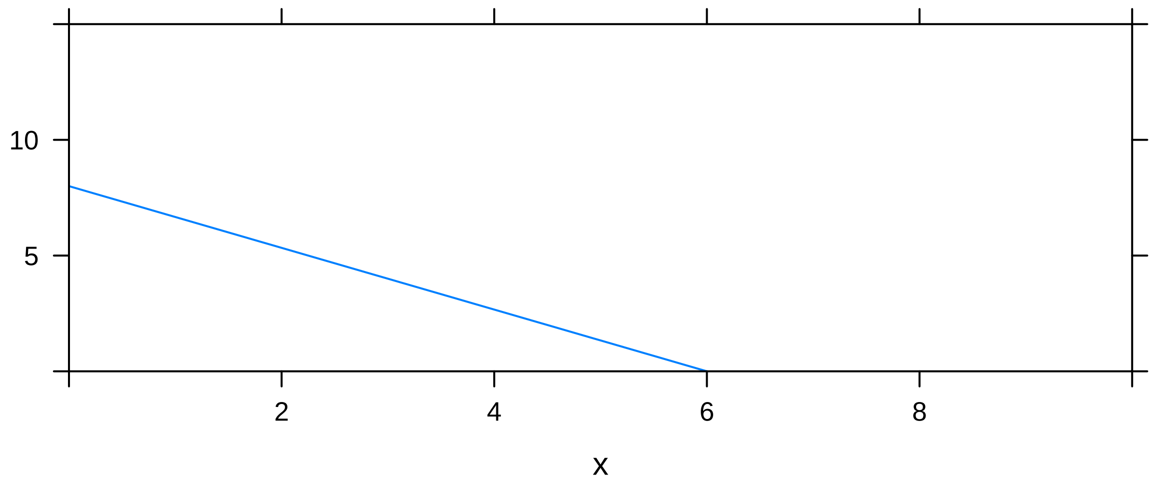
Solve using graphical analysis

Maximize 10x + 35y subject to

$8x + 6y \leq 48 \; (board - feet \, of \, lumber)$
$4x - y \leq 20 \; (hours \, of \, carpentry)$
$x \geq 5 (demand)$
$x, y \geq 0 (nonnegativity)$

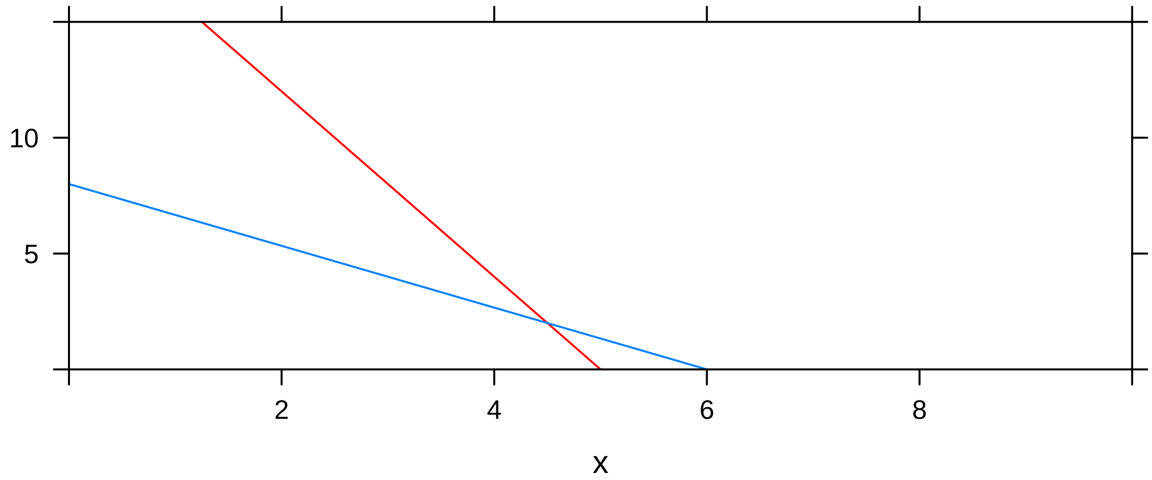
We first plot the constraint $8x + 6y \leq 48 \; (board - feet \, of \, lumber)$ as $y = -\frac{8}{6}x + \frac{48}{6}$, so anything below this line is acceptable.

```
suppressMessages(suppressWarnings(library(mosaic)))
plotFun(((-8/6)*x) + (48/6) ~ x, xlim = c(0, 10), ylim = c(0,15), ylab = "")
```



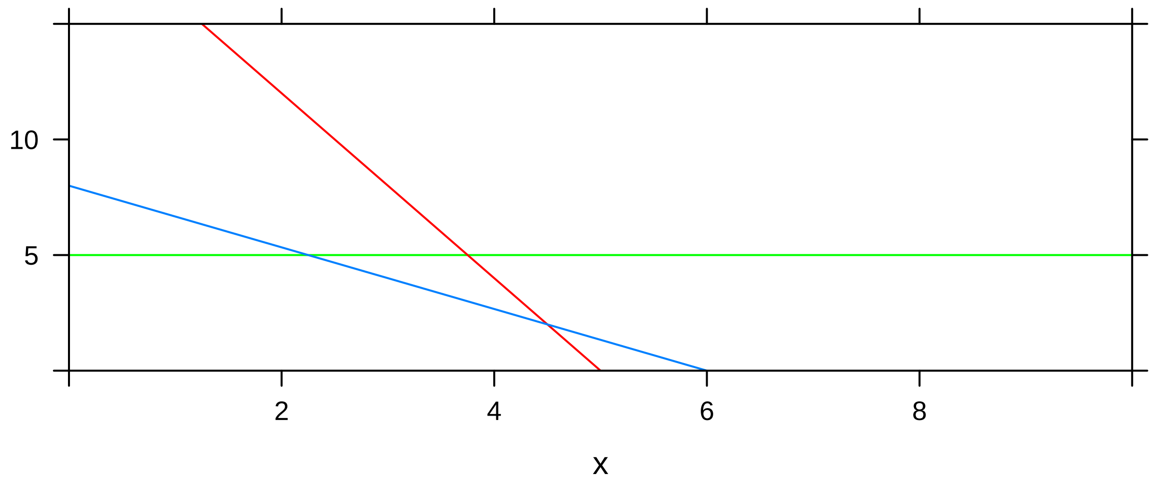
We now plot the constraint $4x - y \leq 20 \; (hours \, of \, carpentry)$ as $y = -4x + 20$. Anything below this line is acceptable.

```
plotFun((-4*x)+ 20 ~ x, xlim = c(0, 10), ylim = c(0,15),
        col = c("red"), add = TRUE, under = TRUE,  ylab = "")
```



We now plot the constraint $x \geq 5(demand)$ as $y = 5$. Anything above this line is acceptable.

```
plotFun(0*x+5 ~ x, xlim = c(0, 10), ylim = c(0,15),
        col = c("green"), add = TRUE, under = TRUE,  ylab = "")
```



As we are trying to maximize $10x + 35y$, we look at the graph above and the highest point within our feasibility range is (0,8). Which when applied is 280.

3 Page 268: problem 6 (i.e., only question #6 in section 7.2)

Using the $methods$ of 7.3 solve problems 6 from section 7.2

Maximize 10x + 35y subject to

$8x + 6y \leq 48\ (board - feet\ of\ lumber)$
$4x - y \leq 20\ (hours\ of\ carpentry)$
$x \geq 5 (demand)$
$x, y \geq 0 (nonnegativity)$

We add a "slack" variable

$8x + 6y + z_1 = 48\ (board - feet\ of\ lumber)$
$4x - y + z_2 = 20\ (hours\ of\ carpentry)$
$x = 5 (demand)$
$x, y, z_1, z_2 \geq 0 (nonnegativity)$

Lets begin by setting x and y to 0 for our first intersection.

$z_1 = 48$
$z_2 = 20\ 0 = 5$

This is unfeasible as x is constrained to 5.

For the second intersection lets set y to 0 and z_1 to 0.

$8x = 48$ = $x = 6$
$4x + z_2 = 20$ = $24 + z_2 = 20$ = $z_2 = -4$

This is unfeasible as z_2 is constrained to $\geq$ 0 for nonnegativity.

For the third intersection lets set y to 0 and z_2 to 0.

$8x + z_1 = 48$ = $40 + z_1 = 48$ = $z_1 = 8$
$4x = 20$ = $x = 5$
$x = 5$

This is feasible as the intersection (0,8) does not violate any constraints. We also know from our previous work this is the most optimal solution.

For the fourth intersection lets set x to 0 and z_1 to 0.

$6y = 48$ = $y = 8\ {-8} + z_2 = 20$ = $z_1 = 28\ 0 = 5$

This is not feasible as x is constrained at 5.

For the fifth intersection lets set x to 5 and z_1 to 0.

$40 + 6y = 48$ = $6y = 8$ = $y = $ $1\frac{1}{3}$
$20 - 1.3 + z_2 = 20$ = $ z_2 = $ $1\frac{1}{3}$
$5 = 5$

This is not feasible as ($1\frac{1}{3}$, $1\frac{1}{3}$) would not satisfy the x = 5 condition.

For the sixth intersection lets set x to 0 and z_2 to 0.

$6y + z_1 = 48$
$y = -20$

This is not feasible as -20 for y violates the nonnegativity.

For the seventh intersection lets set x to 5 and z_2 to 0

$40 + 6y + z_1 = 48$ = $6y + z_1 = 8$ = $z_1 = 8$
$20 - y = 20$ = $y = 0$

This is feasible and as previously seen is the most optimal solution. I am interested to know if my technique is correct here. I would have assumed that I would not get (0,8) twice using this method.

I am also concerned about this method or at least my attempt to solve this system using this method since (2.5, 5) is clearly a solution in the graphical solution but I did not arrive at it in my algebraically approach.

4 Page 284: problem 1

For the example problems in this section, determine the sensitivity of the optimal solution to a change in c_2 using the objective function $25x_1 + c_2x_2$.

First we determine the slope in the x_1, x_2-plane. $25x_1 + c_2x_2$
$c_2x_2 = -25x_1$
$x_2 = -\frac{25}{c_2}x_1$

The slope of the first constraint is $-\frac{2}{3}$. The slope of the second constraint is $-\frac{5}{4}$. Therefore, the inequality exists as:

$-\frac{2}{3} \ \leq \ -\frac{25}{c_2} \ \leq \ -\frac{5}{4}$

We further multiply by -1 to get:

$\frac{2}{3} \ \geq \ \frac{25}{c_2} \ \geq \ \frac{5}{4}$

We can then determine the upper and lower bounds by setting our $\frac{25}{c_2}$ = $\frac{2}{3}$ and $\frac{25}{c_2}$ = $\frac{5}{4}$

$\frac{25}{c_2}$ = $\frac{2}{3}$
Take the reciprocal of both sides:
$\frac{c_2}{25}$ = $\frac{3}{2}$
Multiply by both sides by 25
c_2 = $\frac{75}{2}$ = 37.5

$\frac{25}{c_2}$ = $\frac{5}{4}$ Take the reciprocal of both sides:
$\frac{c_2}{25}$ = $\frac{4}{5}$
Multiply by both sides by 25 c_2 = $\frac{75}{4}$ = 20

Therefore, our final inequality is:

$$20 \leq c_2 \leq 37.5$$