Homework 9

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April 1, 2017

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Using the definition provided for the movement diagram, determine whether the following zero-sum games have a pure strategy Nash equilibrium. If the game does have a pure strategy Nash equilibrium, state the Nash equilibrium. Assume the row player is maximizing his playoffs which are showing in the matrices below.

		Colin	
		C1	C2
Rose	R1	10	10
	R2	5	0

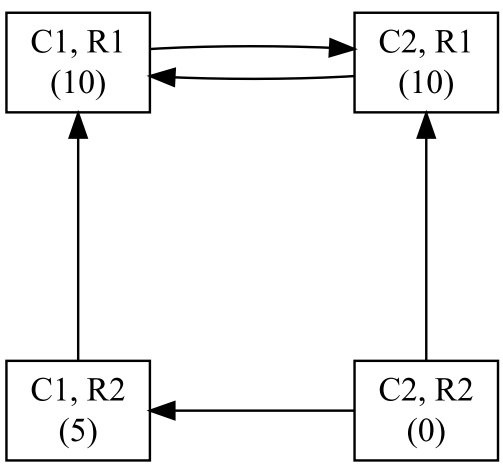
```r
library(DiagrammeR)
grViz("digraph boxes {

  graph [layout = neato, overlap = true, outputorder = edgefirst]

  node [shape = box]

  A [pos = '-1, 1!', label = 'C1, R1 \n (10)'];
  B [pos = ' -1, -1!', label = 'C1, R2 \n (5)'];
  C [pos = ' 1, 1!',label = 'C2, R1 \n (10)'];
  D [pos = ' 1, -1!', label = 'C2, R2 \n (0)'];

  # several 'edge' statements
  B->A C->A A->C D->B D->C
}
")
```



We do have a pure strategy Nash equilibrium of 10 as our arrows all point to one value, when Rose plays strategy 1 and Colin plays either strategy 1 or 2. For graphing simplicity I set the strategy to Rose 1 and Colin 1.

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Using the definition provided for the movement diagram, determine whether the following zero-sum games have a pure strategy Nash equilibrium. If the game does have a pure strategy Nash equilibrium, state the Nash equilibrium. Assume the row player is maximizing his playoffs which are showing in the matrices below.

		Pitcher	
		Fastball	Knuckleball
Batter	GFast	.400	.100
	Gknuckle	.300	.250

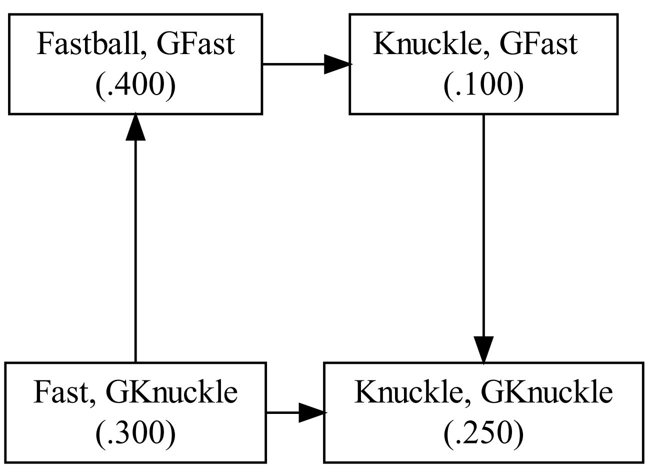
```
grViz("digraph boxes {

  graph [layout = neato, overlap = true, outputorder = edgefirst]

  node [shape = box]

  A [pos = '-1, 1!', label = 'Fastball, GFast \n (.400)'];
  B [pos = ' 1, 1!', label = 'Knuckle, GFast  \n (.100)'];
  C [pos = ' -1, -1!', label = 'Fast, GKnuckle \n (.300)'];
  D [pos = ' 1, -1!', label = 'Knuckle, GKnuckle \n (.250)'];

  # several 'edge' statements
  C->D A->B C->A B->D
}
")
```



The EV = .250 when the pitcher pitches a knuckle and the batter guesses a knuckle ball.

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For problems a-g build a linear programming model for each player's decisions and solve it both geometrically and algebraically. Assume the row player is maximizing his playoffs which are showing in the matrices below.

		Colin	
		C1	C2
Rose	R1	10	10
	R2	5	0

Geometrically:

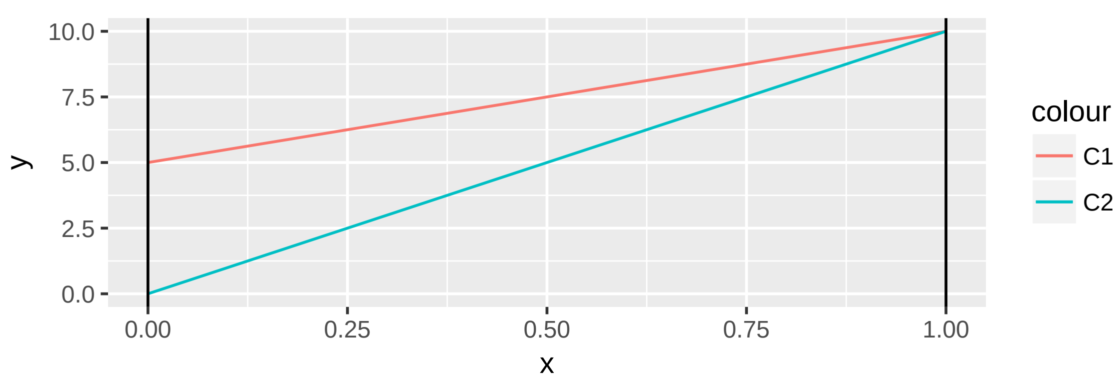
R1 has probability $p = x$, therefore the probability of R2 is $p = 1 - x$.

$V_{C1} \leq$ 10x + 5(1-x) = 5x + 5
$V_{C2} \leq$ 10x + 0(1-x)= 10x
$x \leq 1$
$x \geq 0$

```
library(knitr)
x <- c(0, 1, 0, 1)
A <- c(5, 10, 0, 10)
kable(as.data.frame(cbind(x,A)))
```

x	A
0	5
1	10
0	0
1	10

```
library(ggplot2)
plotc <- ggplot(data.frame(x=c(0,1)), aes(x)) +
          stat_function(fun=function(x)5*x+5, geom="line", aes(colour = "C1")) +
          stat_function(fun=function(x)10*x, geom="line", aes(colour = "C2")) +
            geom_vline(xintercept = 1) +
          geom_vline(xintercept = 0)
plotc
```



Both are maxed at x = 1, V_{C1} = 10, V_{C2} = 10.

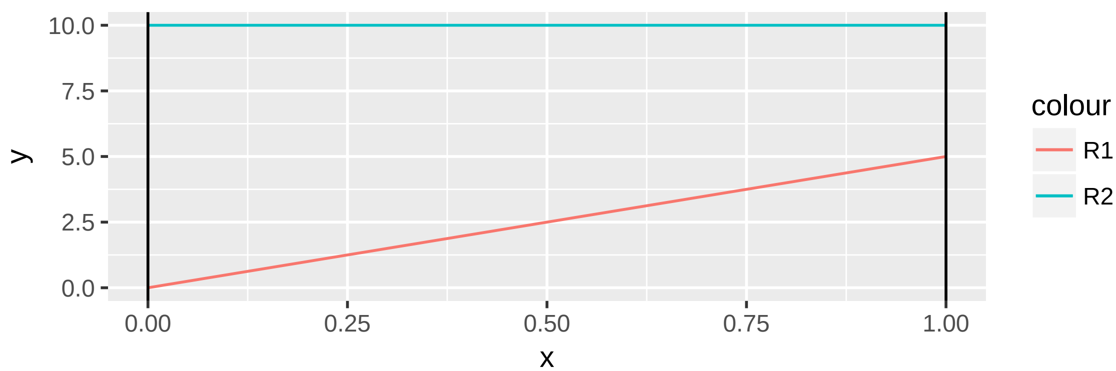
R1 has probability $p = x$, therefore the probability of R2 is $p = 1 - x$.

$V_{R1} \leq$ 10x + 10(1-x) = 10 $V_{R2} \leq$ 5x + 0(1-x)= 5x $x \leq 1$ $x \geq 0$

```
library(knitr)
x <- c(0, 1, 0, 1)
A <- c(10, 10, 0, 5)
kable(as.data.frame(cbind(x,A)))
```

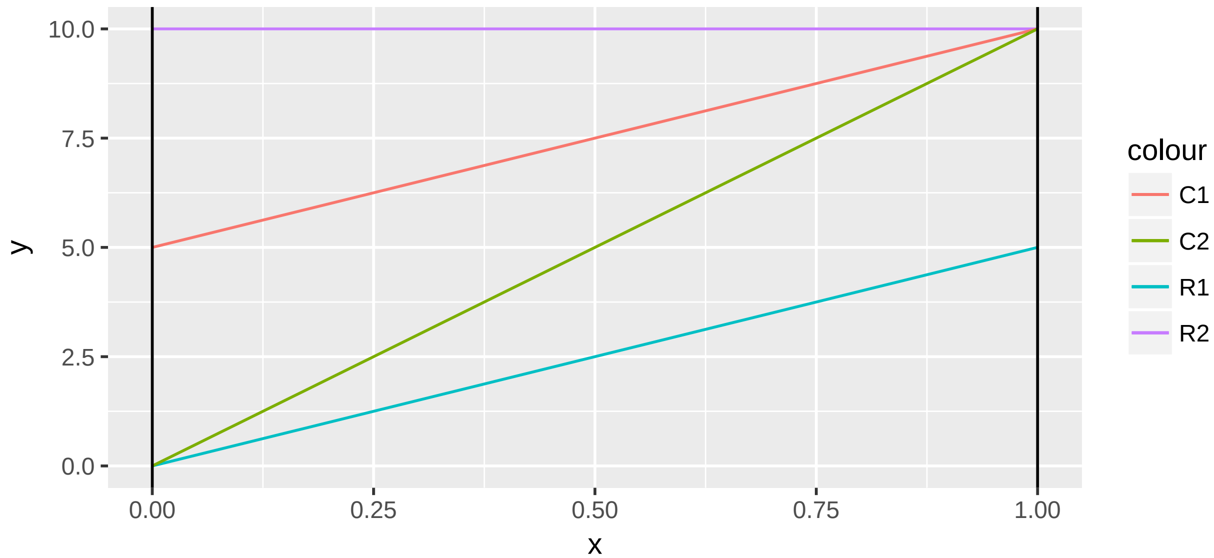
x	A
0	10
1	10
0	0
1	5

```
plotr <- ggplot(data.frame(x=c(0,1)), aes(x)) +
          stat_function(fun=function(x)5*x, geom="line", aes(colour = "R1")) +
          stat_function(fun=function(x)10, geom="line", aes(colour = "R2")) +
            geom_vline(xintercept = 1) +
          geom_vline(xintercept = 0)
plotr
```



Now we plot both strategies for Rose and Colin.

```
library(ggplot2)
plotrc <- ggplot(data.frame(x=c(0,1)), aes(x)) +
          stat_function(fun=function(x)5*x, geom="line", aes(colour = "R1")) +
          stat_function(fun=function(x)10, geom="line", aes(colour = "R2")) +
          stat_function(fun=function(x)5*x + 5, geom="line", aes(colour = "C1")) +
          stat_function(fun=function(x)10*x, geom="line", aes(colour = "C2")) +
          geom_vline(xintercept = 1) +
          geom_vline(xintercept = 0)
plotrc
```



V_{R1} is a constant at 10, whereas, V_{R2} is maxed at x = 1; V_{R2} = 5.

We can therefore determine that Rose will play strategy R1 and Colin can play either strategy C1 or C2.

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In the following problems, use the maximim and minimax method and movement diagram to determine if any pure strategy solution exist. Assume the row player is maximizing his payoffs which are shown in the matrices below.

		Colin		Row Minimum
		C1	C2	
Rose	R1	10	10	**10**
	R2	5	0	**0**

		Colin	
		C1	C2
Rose	R1	10	10
	R2	5	0
Column Maximum		**10**	**0**

As we can see from the row minimum that Colin can play either C1 or C2; and the column maximum indicates that Rose plays R1 which would be a pure strategy.