Homework 10

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The following data were obtained for the growth of a sheep population introduced into a new environment on the island of Tasmania (adapted from J. Davidson “On the Growth of the Sheep of Tasmania” Trans. R. Soc. S. Australia.)

Table 1: My caption

t (year)	1814	1824	1834	1844	1854	1864
P (t)	125	275	830	1200	1750	1650

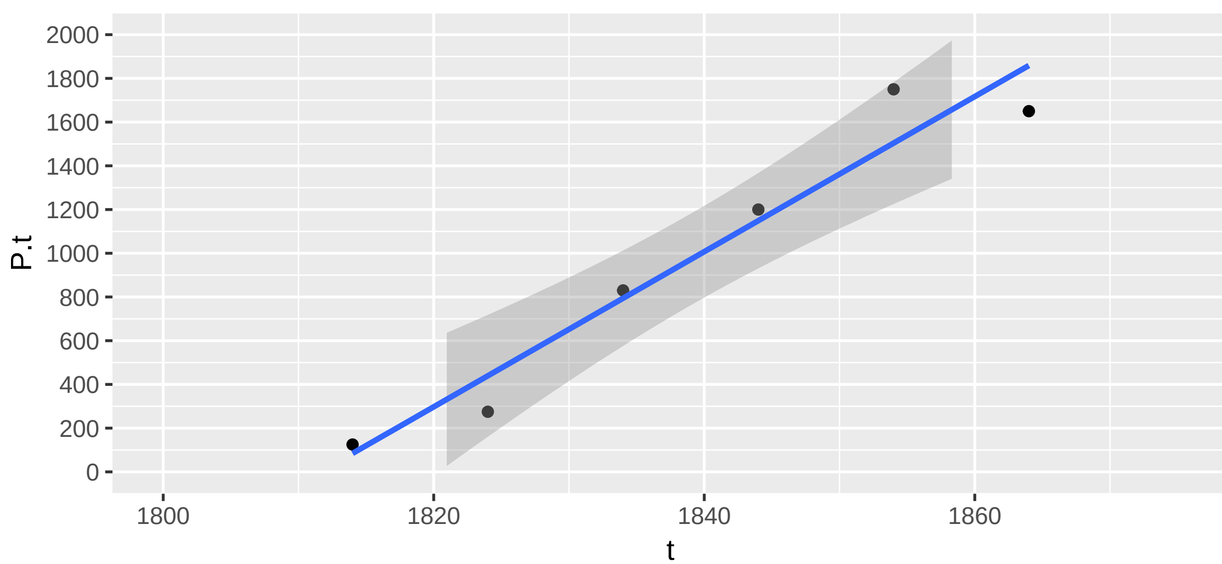
1.1 a. Make an estimate of M by graphing $P(t)$.

```
t <- c(1814, 1824, 1834, 1844, 1854, 1864)
P.t <- c(125, 275, 830, 1200, 1750, 1650)
df <- as.data.frame(cbind(t, P.t))
```

```
library(tidyverse)

number_ticks <- function(n) {
  function(limits) pretty(limits, n)
  } # http://stackoverflow.com/a/17257422/4153261

ggplot(df, aes(x = t, y = P.t)) +
  geom_point() +
  scale_y_continuous(breaks = number_ticks(15), limits = c(0, 2000)) +
  xlim(1800, 1875) +
  stat_smooth(method = "lm")
```

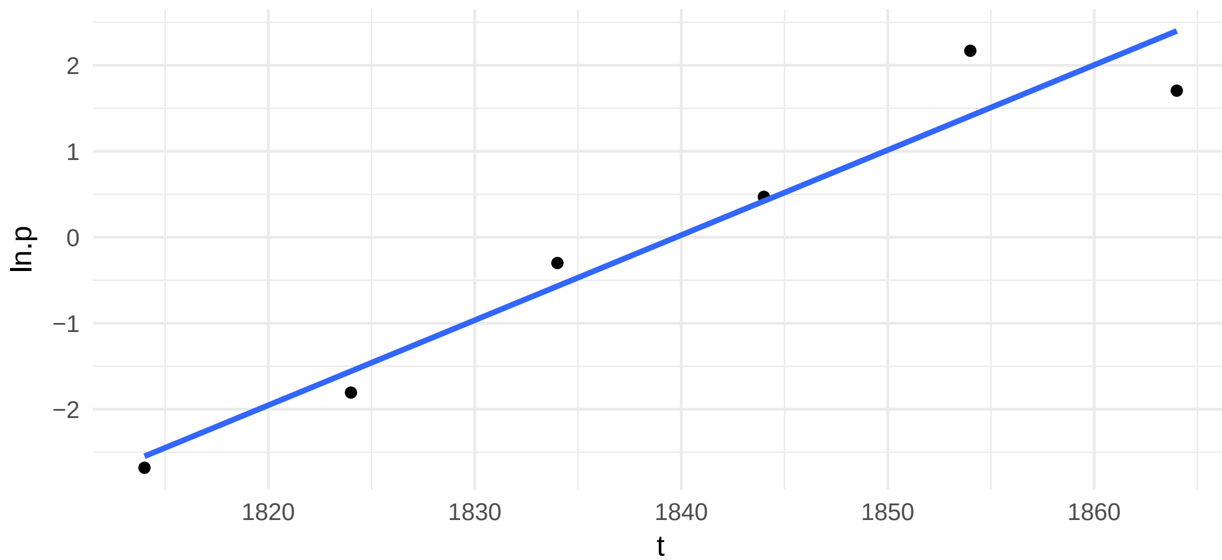


Based on the above graph I would estimate the population limit at M = 1900.

1.2 b. Plot ln[P/(M-P)] against t. If a logistic curve seems reasonable, estimate rM and t*.

```
M <- 1900
ln.p <- log(P.t/(M - P.t))
df2 <- as.data.frame(cbind(t,ln.p))
```

```
ggplot(df, aes(x = t, y = ln.p)) +
  geom_point() +
  stat_smooth(method = "lm", se = FALSE) +
  theme_minimal()
```



This graph result does approximate a straight line, thus we accept the assumptions of logistic growth. Therefore, lets estimate estimate rM and t*.

```
rM <- lm(ln.p ~ t)$coefficients["t"]
```

rM = 0.1034121

t^* follows this equation: $t* = \frac{-C}{rM}$

```
C <- log(P.t[1]/(M - P.t[1]))-rM
`t*` <- as.numeric(-C/rM) + t[1]
`t*`
```

```
## [1] 1840.657
```

$t^* = 1841$

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Suggest other phenomena for which the model described in the text might be used.

> Ancedontally, I understand that a major utility cost of a building is the heating and cooling. Administering the correct “dosage” of heated air or cooled air to maintain a constant comfortable temparture would be a fiscal and pyschological benefit. By using the model it seems possible to tune the air volume to correct the internal temperature in the most effecient manner.

3 Page 522: problem 21

Oxygen flows through a tube into a liter flask filled with air, and the mixture of oxygen and air (considered well stirred) escapes through another tube. Assuming that air contains 21% oxygen, what percentage of oxygen will the flask contain after 5 L have passed through the intake tube?

> Reframing the question, 100% Oxygen is flowing into a liter flask that is currently filled at 21% oxygen and 79% air, as the mixture leaves the liter flask the air is being replaced by oxygen so we are interested in the change in the volume of oxygen in the liter flask. We are further interested in the total oxygen volume after 5L has been passed through the 1L flask.

$V'C_r = V'C + V_r + V_r \frac{dC}{dt}$

$Cr = 1, C_0 = .21, V_r = 1$

$\frac{V^1}{V_r} t = ln \frac{C_r - C}{C_r - C_0} = ln \frac{1-C}{1-0.21}$

$C = 1 - 0.79 e^{\frac{-v^1 t}{v_r}}$

```
C <- 1 - 0.79*exp(-5)
C
```

```
## [1] 0.994677
```

The total volume of oxygen after 5L has passed through a 1L flask is 99.5%