

# Homework 5

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## 1 Problem Set 1

Consider the unsolvable system  $Ax = b$  as given below:

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 8 \\ 20 \end{bmatrix}$$

### 1.1 Write R Markdown script to compute $A^T A$ and $A^T b$

```
A <- matrix(c(1,1,1,1,0,1,3,4), ncol = 2)
b <- matrix(c(0,8,8,20))
```

```
ATA <- t(A) %*% A
ATb <- t(A) %*% b
```

```
results <- list("ATA" = ATA, "ATb" = ATb)
results
```

```
## $ATA
##      [,1] [,2]
## [1,]    4    8
## [2,]    8   26
##
## $ATb
##      [,1]
## [1,]   36
## [2,]  112
```

## 1.2 Solve for $\hat{x}$ in R using the above computed matrices

```
x <- solve(ATA) %*% ATb
x
```

```
##           [,1]
## [1,]        1
## [2,]        4
```

## 1.3 What is the squared error of this solution?

```
p <- A %*% x
#b = p + e or e = p - b which we can substitute in our given values.
e <- p - b
# we then sum the square of errors.
e2 <- sum(e^2)
e2
```

```
## [1] 44
```

## 1.4 Find the exact solution with p instead of b

```
options(scipen = 999)
p <- matrix(c(1,5,13,17))
ATp <- t(A) %*% p
xp <- solve(ATA) %*% ATp
p2 <- A %*% xp
e <- p2-p
e
```

```
##           [,1]
## [1,] 0.000000000000000000000000
## [2,] 0.00000000000000008881784
## [3,] 0.00000000000000035527137
## [4,] 0.00000000000000035527137
```

Essentially, the error vector  $e$  is  $\approx 0$ .

```
e2p <- sum(e^2)
e2p
```

```
## [1] 0.0000000000000000000000002603241
```

Show that the error  $e = b - p = [-1; 3; -5; 3]$ .

```
b - p
```

```
##           [,1]
## [1,]       -1
## [2,]        3
## [3,]       -5
## [4,]        3
```

Show that the error  $e$  is orthogonal to  $p$  and to each of the columns of  $A$ .

As per the week 5 handout - We know that when two vectors are orthogonal, their dot product is zero.

```
e*p
```

```
##                                [,1]
## [1,] 0.000000000000000000000000
## [2,] 0.00000000000000004440892
## [3,] 0.0000000000000046185278
## [4,] 0.00000000000060396133
```

```
sum(e*A[,1])
```

```
## [1] 0.000000000000007993606
```

## 2 Problem Set 2

Write an R markdown script that takes in the auto-mpg data, extracts an A matrix from the first 4 columns and b vector from the fifth (mpg) column.

Apparently, an added column of 1 is necessary to obtain an intercept.

```
x <- as.matrix(read.table(paste0("https://raw.githubusercontent.com",
                                "/ChristopheHunt/MSDA---Coursework",
                                "/master/Data%20605/Assignment%205/",
                                "auto-mpg.data")))

A <- as.matrix(cbind(x[,1:4],1))
b <- as.matrix(x[,5])
```

Using the least squares approach, your code should compute the best fitting solution

```
ATA <- t(A) %*% A
ATb <- t(A) %*% b
results <- list("ATA" = ATA, "ATb" = ATb)
results
```

```
## $ATA
##           V1           V2           V3           V4
## V1 19097634.2  9374647.0 259345480 1123011.9 76209.5
## V2  9374647.0  4857524.0 132989885  607832.3 40952.0
## V3 259345480.0 132989885.0 3757575489 17758103.6 1167213.0
## V4  1123011.9   607832.3  17758104   97656.9   6092.2
##           76209.5   40952.0   1167213   6092.2   392.0
##
## $ATb
##           [,1]
## V1 1529685.9
## V2  868718.8
## V3 25209061.4
## V4  146401.4
##           9190.8
```

## 2.1 Solve for $\hat{x}$ in R using the above computed matrices

```
x <- solve(ATA) %*% ATb
x
```

```
##           [,1]
## V1 -0.006000871
## V2 -0.043607731
## V3 -0.005280508
## V4 -0.023147999
##      45.251139699
```

The least squares model using this method is:

$$mpg = -0.006*displacement + -0.04361*horsepower + -0.00528*weight + -0.02315*acceleration + 45.25114$$

Finally, calculate the fitting error between the predicted mpg of your model and actual mpg.

## 2.2 The fitting error to the predicted mpg and the actual mpg.

```
p <- A %*% x
#b = p + e or e = p - b which we can substitute in our given values.
e <- p - b
e
```

```
##           [,1]
## [1,]  0.95919198
## [2,]  1.18844197
## [3,]  0.40325031
## [4,]  2.47995603
## [5,]  1.87826895
## [6,] -4.11172901
## [7,] -4.26661947
## [8,] -3.73121329
## [9,] -4.88872305
## [10,] -0.90138187
## [11,]  1.49356259
## [12,]  2.99106999
## [13,]  1.22973585
## [14,]  2.18187690
## [15,]  3.55772236
## [16,]  2.60176023
## [17,]  6.82009386
## [18,]  5.31326672
## [19,]  2.24844730
## [20,]  6.49883380
## [21,]  1.28256448
## [22,]  3.51707079
## [23,]  2.53801867
## [24,]  1.51135631
## [25,]  4.80226596
```

```
## [26,] -0.97845147
## [27,]  1.23260402
## [28,] -0.26644392
## [29,]  0.59498197
## [30,]  2.24844730
## [31,]  0.17245834
## [32,]  3.34126348
## [33,]  6.28838301
## [34,]  4.80367168
## [35,]  4.45254442
## [36,]  3.11841090
## [37,]  3.77706091
## [38,] -0.54787405
## [39,] -2.61895040
## [40,]  0.22512378
## [41,]  0.87181916
## [42,] -3.32770358
## [43,] -2.90158894
## [44,] -5.20014767
## [45,]  4.95270250
## [46,]  6.11598637
## [47,]  2.71230228
## [48,]  5.00228167
## [49,]  5.72196928
## [50,]  1.09575284
## [51,]  0.32137057
## [52,]  0.16898091
## [53,]  1.18842304
## [54,] -2.12397951
## [55,]  4.92832807
## [56,]  4.85459255
## [57,]  4.04251609
## [58,]  4.55756074
## [59,]  6.86799519
## [60,]  7.31947322
## [61,]  7.63241624
## [62,]  0.10889294
## [63,] -2.21336428
## [64,] -0.34569464
## [65,]  0.36871047
## [66,]  0.22948866
## [67,] -2.11286260
## [68,] -0.69370753
## [69,]  0.33115718
## [70,] -2.07443304
## [71,]  8.98504767
## [72,]  1.04462895
## [73,]  2.77627435
## [74,]  1.28892587
## [75,]  0.94900081
## [76,]  5.81759309
## [77,]  5.53482769
## [78,]  3.55514388
## [79,]  3.69042710
```

```
## [80,] 5.75158442
## [81,] 0.18182614
## [82,] 3.73248675
## [83,] 1.40019701
## [84,] 2.36056653
## [85,] 0.56847599
## [86,] 3.22948866
## [87,] 2.46812479
## [88,] 1.78515902
## [89,] 1.56787515
## [90,] -4.37284127
## [91,] -0.54033113
## [92,] -0.08496701
## [93,] 0.09254556
## [94,] -5.02273809
## [95,] -5.68941815
## [96,] 2.02802495
## [97,] 4.45972516
## [98,] 5.66398032
## [99,] 5.57670109
## [100,] 5.57908559
## [101,] 1.21527018
## [102,] 5.88000140
## [103,] -1.40113778
## [104,] -2.62722100
## [105,] -2.19889539
## [106,] -0.74491128
## [107,] 6.42360830
## [108,] 8.35748564
## [109,] 7.14137593
## [110,] 5.55964889
## [111,] 11.37808640
## [112,] 9.18616531
## [113,] 5.27749023
## [114,] 2.41921441
## [115,] -0.02824295
## [116,] -5.98890515
## [117,] 3.39620761
## [118,] 5.53032903
## [119,] 6.64039377
## [120,] 5.13767810
## [121,] 3.59862910
## [122,] 1.35796118
## [123,] 3.85997728
## [124,] 4.69903473
## [125,] 3.15815563
## [126,] 4.80904343
## [127,] 6.38085886
## [128,] 0.11855073
## [129,] 1.70594837
## [130,] -0.19054495
## [131,] 2.32387113
## [132,] 3.03103289
## [133,] 3.31059628
```

```
## [134,] 1.86171532
## [135,] -0.85686044
## [136,] -1.53907691
## [137,] -1.04601815
## [138,] -1.52756882
## [139,] 0.04779960
## [140,] -0.05577611
## [141,] 5.13092213
## [142,] 2.78683828
## [143,] 2.43797215
## [144,] -0.92367328
## [145,] 1.88375643
## [146,] 5.95037707
## [147,] 3.10036634
## [148,] 2.81068139
## [149,] 1.56303853
## [150,] -0.07603066
## [151,] 3.15226385
## [152,] 2.53646575
## [153,] 7.00235458
## [154,] 8.48393572
## [155,] -5.47813534
## [156,] -1.94181274
## [157,] -3.28566697
## [158,] -2.21293299
## [159,] 0.95103615
## [160,] 2.16573333
## [161,] 3.76995852
## [162,] 1.33167529
## [163,] 1.67340492
## [164,] 1.56104750
## [165,] 7.80177417
## [166,] 0.56412498
## [167,] 3.46280003
## [168,] 3.74039683
## [169,] 3.89430055
## [170,] 1.68025076
## [171,] 4.49440159
## [172,] 2.47467782
## [173,] 5.90235961
## [174,] 2.10610455
## [175,] 3.58501532
## [176,] 2.84539710
## [177,] 1.68557732
## [178,] 2.36473523
## [179,] 0.09341094
## [180,] -0.48975078
## [181,] -0.51118356
## [182,] 3.90888396
## [183,] 1.47273527
## [184,] 2.90077893
## [185,] 2.04376751
## [186,] -2.74247263
## [187,] -1.62454857
```



```
## [188,] 1.45081813
## [189,] -0.53717587
## [190,] 0.11180974
## [191,] -0.86907843
## [192,] 0.20644503
## [193,] 0.73647051
## [194,] 2.21374471
## [195,] 5.60800083
## [196,] 2.10147495
## [197,] -0.48743598
## [198,] -0.14868292
## [199,] 2.99087609
## [200,] 0.83162304
## [201,] 4.78748476
## [202,] 2.15357400
## [203,] -1.21320200
## [204,] 1.63935390
## [205,] 0.91194585
## [206,] 3.02601477
## [207,] 2.69094878
## [208,] 3.91935318
## [209,] 4.77468705
## [210,] 1.95195429
## [211,] -4.60727165
## [212,] 2.13747876
## [213,] 3.98708648
## [214,] 3.64932432
## [215,] -1.02914779
## [216,] -0.62566015
## [217,] -3.81965702
## [218,] 2.32872935
## [219,] -2.47094955
## [220,] -1.17996718
## [221,] 0.01538920
## [222,] -0.65868813
## [223,] -0.25481453
## [224,] 2.48705692
## [225,] 0.30918636
## [226,] 0.96220775
## [227,] 1.92376235
## [228,] -3.53928595
## [229,] -2.01968157
## [230,] -4.05527938
## [231,] -2.57936521
## [232,] 1.68782109
## [233,] 1.16856851
## [234,] 3.01683165
## [235,] 0.11639234
## [236,] 0.19192982
## [237,] -3.78149419
## [238,] 0.88590208
## [239,] -1.12304666
## [240,] 2.94478720
## [241,] 4.20256925
```

```
## [242,] 3.79874042
## [243,] -11.46159974
## [244,] -4.15330112
## [245,] -1.21540941
## [246,] -9.07267943
## [247,] -3.89594464
## [248,] 0.86636012
## [249,] -2.19046981
## [250,] -1.97030516
## [251,] 0.97509011
## [252,] 2.32693538
## [253,] 4.12186437
## [254,] 0.75407704
## [255,] 0.52988330
## [256,] 3.18566623
## [257,] 0.47227201
## [258,] 2.94657785
## [259,] 0.95578752
## [260,] 2.01392090
## [261,] -0.49353973
## [262,] 0.46813035
## [263,] 2.09411698
## [264,] -2.12381900
## [265,] -0.06370765
## [266,] -1.04251300
## [267,] 0.62164263
## [268,] -1.66070995
## [269,] 5.58122105
## [270,] 1.65462659
## [271,] 1.35509650
## [272,] 3.36255971
## [273,] 4.36153906
## [274,] 4.92642409
## [275,] 2.78770237
## [276,] 3.90089954
## [277,] -0.73220242
## [278,] 0.53958771
## [279,] -0.14167754
## [280,] 4.33429647
## [281,] 2.61240952
## [282,] 2.07209030
## [283,] 0.37733034
## [284,] 0.11823990
## [285,] 0.23340464
## [286,] -0.56299481
## [287,] -1.32037540
## [288,] -3.87628262
## [289,] -0.28565880
## [290,] -0.38550981
## [291,] -3.05645820
## [292,] -0.76813621
## [293,] -2.98029020
## [294,] -4.97106775
## [295,] -0.80975997
```

```
## [296,] -3.71028228
## [297,] -6.29688711
## [298,] -3.31002226
## [299,] -2.70700479
## [300,] -4.55416366
## [301,] -4.62948987
## [302,] -1.00450418
## [303,] -7.19163019
## [304,] -2.44901141
## [305,] -3.56639016
## [306,] -2.15788028
## [307,] -6.88221912
## [308,] -11.81281751
## [309,] -6.82662340
## [310,] -2.04295734
## [311,] -6.34041015
## [312,] -2.10282947
## [313,] -1.40049872
## [314,] -0.20232730
## [315,] 2.58998350
## [316,] -4.95183725
## [317,] -3.95192342
## [318,] -3.86768545
## [319,] -9.67485116
## [320,] -3.17973394
## [321,] -16.25565835
## [322,] -3.28256094
## [323,] -10.47974988
## [324,] -13.19428012
## [325,] -13.66070306
## [326,] -10.83482685
## [327,] -6.21298216
## [328,] -12.90503935
## [329,] -3.79601597
## [330,] 2.11668159
## [331,] -9.84339199
## [332,] 3.70212678
## [333,] -7.86925116
## [334,] -3.41658897
## [335,] 0.06608472
## [336,] -0.21180645
## [337,] 0.33483096
## [338,] 1.23509008
## [339,] -2.11464757
## [340,] -6.51066987
## [341,] -7.33640929
## [342,] -2.61777122
## [343,] -1.86894575
## [344,] -5.97151093
## [345,] -6.91211851
## [346,] -3.21244123
## [347,] -4.86746876
## [348,] -3.74508420
## [349,] -1.11822031
```

```
## [350,] -3.49893751
## [351,] -4.36478674
## [352,] -3.86561390
## [353,] -6.87485532
## [354,] -4.63368334
## [355,] -4.71186095
## [356,] -6.77327957
## [357,] -1.82064081
## [358,] -0.84924541
## [359,] -1.73058441
## [360,] -8.13768042
## [361,]  3.45930053
## [362,]  3.91307029
## [363,] -1.46786177
## [364,] -0.62953154
## [365,] -6.32191833
## [366,] -4.09992014
## [367,] -1.92567746
## [368,] -1.43854041
## [369,]  0.89082449
## [370,] -5.41549367
## [371,] -6.37458714
## [372,] -0.07027041
## [373,] -7.68759346
## [374,] -6.01102628
## [375,] -7.04798797
## [376,] -6.64069900
## [377,] -4.69543672
## [378,] -6.94007534
## [379,] -0.95627894
## [380,] -7.12626818
## [381,] -1.56259100
## [382,] -14.34199262
## [383,]  0.31733394
## [384,]  1.66435658
## [385,] -6.19363836
## [386,] -8.03775475
## [387,] -2.55764599
## [388,] -1.43297264
## [389,] -13.41546919
## [390,] -3.60930947
## [391,] -1.20586132
## [392,] -4.85085028
```

## 2.3 What is the squared error of the solution?

```
e2 <- sum(e^2)
e2

## [1] 6979.413
```