Homework 12

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Verify that the given function pair is a solution to the first-order system.

$x = -e^t, y = e^t$
$\frac{dx}{dt} = -y, \frac{dy}{dt} = -x$
$\frac{dx}{dt} = \frac{d}{dt}(-e^t) = e^t = y$; $\frac{dx}{dt} = -y$
$\frac{dy}{dt} = \frac{d}{dt}(e^t) = -e^t = x$; $\frac{dy}{dt} = -x$

2 Page 529: problem 6

Find and classify the rest points of the given autonomous system.

$\frac{dx}{dt} = -(y-1)$, $\frac{dy}{dt} = x - 2$

The rest point of the system is a point in the phase plane for which $f(x,y) = 0 and g(x,y) = 0$, then both the derivatives $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$.

when $y = 1$, $\frac{dx}{dt} = -(1-1)$; $\frac{dx}{dt} = 0$
when $x = 2$, $\frac{dy}{dt} = 2 - 2$; $\frac{dy}{dt} = 0$

$(2,1)$ is the rest point of the autonomous system $\frac{dx}{dt} = -(y-1)$, $\frac{dy}{dt} = x - 2$

3 Page 546: problem 1

Apply the first and second derivative tests to the function

$f(y) = y^a/e^{by}$ to show that $f(y) = y^a/e^{by}$

4 Page 566: problem 1

Use Euler's method to solve the first-order system subject to the specificed intial conditions. Use the given step size Δt and calculate the first three approximateions $(x_1, y_1), (x_2, y_2), and (x_3, y_3)$. Then repeat your calculations for $\Delta t/2$. Compare your approximations with the values of the given analytic solutions