Homework 3

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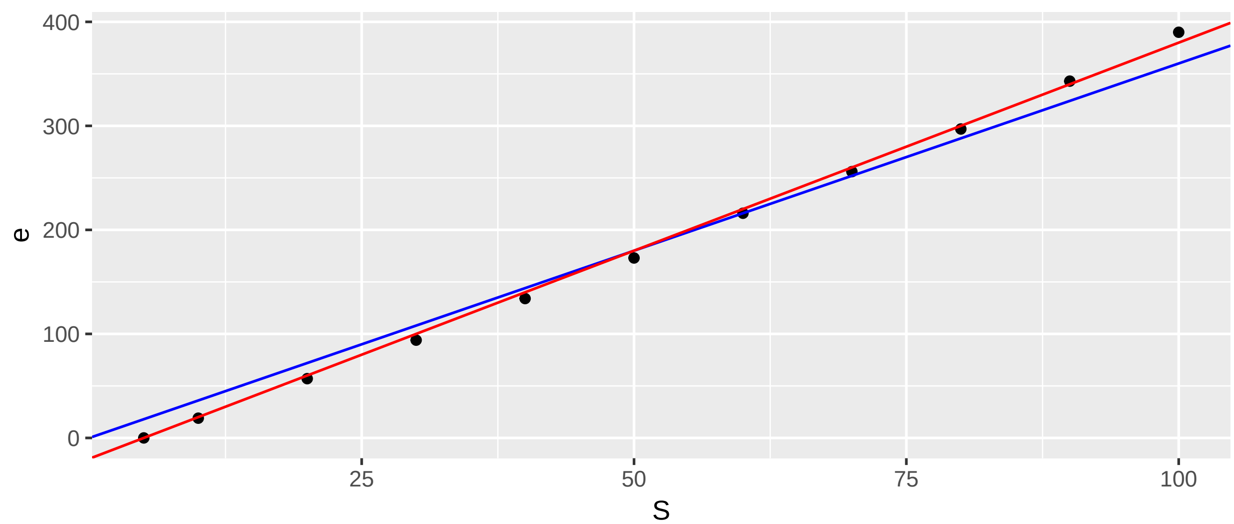
1 Problem : Page 113: 2

The following table gives the elongation e in inches (in./in.) for a given stress S on a steel wire measured in pounds per square inch (lb/in.^{2}). Test the models $e = c_1 S$ by plotting the data. Estimate c_1 graphically.

Table 1:

$S(x10^{-3})$	5	10	20	30	40	50	60	70	80	90	100
$e(x10^5)$	0	19	57	94	134	173	216	256	297	343	390

```r
library(ggplot2)
S <- c(5,10,20,30,40,50,60,70,80,90,100)
e <- c(0,19,57,94,134,173,216,256,297,343,390)
ggplot(data = as.data.frame(cbind(S,e)), aes(x = S, y = e)) +
  geom_point() +
  geom_abline(slope = 3.6, color = 'blue') +
  geom_abline(intercept = -20, slope = 4, color = 'red')
```



Above is the graph of the elongation $e% versus stress S x 10^{-1}. By eyeballing the results of several plots we can give the estimate of ~3.6 for c_1 for the model $e = c_1 S$ (this is the blue line). However, do see a much better fit with ~4 for c_1, if we provide an intercept of -20. These are simply best guesses.

2 Problem : Page 121: 2.a

For each of the following data sets, formulate the mathematical model that minimizes the largest deviation between the data and the line y = ax + b. If a computer is available solve for the estimates of a and b.

Table 2:

x	1	2.3	3.7	4.2	6.1	7.0
y	3.6	3.0	3.2	5.1	5.3	6.8

```r
x <- c(1,2.3,3.7,4.2,6.1,7.0)
y <- c(3.6, 3.0, 3.2, 5.1, 5.3, 6.8)
```

```
mean.x <- mean(x)
mean.y <- mean(y)

x.i <- (x - mean.x)
y.i <- (y - mean.y)

x.i.y.i <- (x.i * y.i)
x.i.2   <- (x.i^2)

m <- sum(x.i.y.i) / sum(x.i.2)
b <- mean.y - m*mean.x

y2 <- y - (m*x + b)
```

The model y = ax + b for this date = y = 0.56x+2.21.

3 Problem : Page 127: 10

Data For planets

Body	Period (sec)	Distance from sun (m)
Mercury	7.60 x 10^6	5.79 x 10^10
Venus	1.94 x 10^7	1.08 x 10^11
Earth	3.16 x 10^7	1.5 x 10^11
Mars	5.94 x 10^7	2.28 x 10^11
Jupiter	3.74 x 10^8	7.79 x 10^11
Saturn	9.35 x 10^8	1.43 x 10^12
Uranus	2.64 x 10^9	2.87 x 10^12
Neptune	5.22 x 10^9	4.5 x 10^12

Fit the model $y = ax^{3/2}$

```
period <- c(( 7.60 * 10^6 ), ( 1.94 * 10^7 ), ( 3.16 * 10^7 ),
            ( 5.94 * 10^7 ), ( 3.74 * 10^8 ), ( 9.35 * 10^8 ),
            ( 2.64 * 10^9 ), ( 5.22 * 10^9 ))

distances <- c(( 5.79 * 10^10 ), ( 1.08 * 10^11 ), ( 1.5 * 10^11  ),
               ( 2.28 * 10^11 ), ( 7.79 * 10^11 ), ( 1.43 * 10^12 ),
               ( 2.87 * 10^12 ), ( 4.5 * 10^12  ))
```

Least square solution to the formula $y = An^x$, for the model $y = an^{3/2}$.

```
a <- sum(period^(3/2) * distances)/sum((period^2)^(3/2))
a
```

```
## [1] 0.01320756
```

Resulting in the form $y = 0.0132n^{3/2}$.

4 Problem : Page 136: 7

a. In the following data, W represents the weight of a fish (bass) and l represents its length. Fit the model $W = kl^3$ to the data using the least-squares criterion.

Length, l (in.)	14.5	12.5	17.25	14.5	12.625	17.75	14.125	12.635
Weight, W (oz)	27	17	41	26	17	49	23	16

```
x <- length.in <- c(14.5, 12.5, 17.25 , 14.5 , 12.625 , 17.75 , 14.125 , 12.625)
y <- weight.oz <- c(27 , 17   , 41    , 26   , 17     , 49    , 23     , 16 )

a <- sum(x^3*y)/(sum((x^2)^3))
y2 <- a*(x^3)
y.y2 <- (y - y2)
D <- (sum(y.y2^2)/8)^(1/2)
```

The least-squares fit of $W = kl^3$ is $W = 0.008437l^3$. The sum of the squares of the deviations as 12.1683418 so $D = 1.2333056$. As the largest absoulte deviation is 2.305, c_{max} can be bound as follows:

$$D = 1.2333056 \leq c_{max} \leq 2.305 = d_{max}$$

b. In the following data, g represents the girth of a fish. Fit the model $W = klg^2$ to the data using the least squares criterion

Length, l (in.)	14.5	12.5	17.25	14.5	12.625	17.75	14.125	12.625
Girth g (in)	9.75	8.375	11	9.75	8.5	12.5	9.0	8.5
Weight, W (oz)	27	17	41	26	17	49	23	16

```
x <- length.in <- c(14.5, 12.5 , 17.25, 14.5, 12.625, 17.75 , 14.125 , 12.625)
y <- weight.oz <- c(27   , 17   , 41    , 26   , 17    , 49    , 23     , 16 )
z <- girth.in  <- c(9.75, 8.375, 11    , 9.75, 8.5   , 12.5  , 9.0    , 8.5)

a <- sum((x*z^2)*y)/(sum((x*z^2)^2))
y2 <- a*(x*z^2)
y.y2 <- (y - y2)
D <- (sum(y.y2^2)/8)^(1/2)
```

The least-squares fit of $W = klg^2$ is $W = 0.018675lg^2$. The sum of the squares of the deviations as 17.6710973 so $D = 1.4862325$. As the largest absoulte deviation is 2.794, c_{max} can be bound as follows:

$$D = 1.4862325 \leq c_{max} \leq 2.794 = d_{max}$$

5 Problem : Page 146: 5

Solve Problems 1 - 4 with the model V = m(log P) + b. Compare the errors with those computed in Problem 4. Compare the two models. Which is better?

6 Problem : Page 157: 4

7 Problem : Page 169: 11

8 Problem : Page 181: 5