Homework 5

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Contents

1 Problem Set 1

Consider the unsolvable system Ax = b as given below:

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 8 \\ 20 \end{bmatrix}$$

1.1 Write R Markdown script to compute A^TA and A^Tb

```
A <- matrix(c(1,1,1,1,0,1,3,4), ncol = 2)
b <- matrix(c(0,8,8,20))

ATA <- t(A) %*% A
ATb <- t(A) %*% b

results <- list("ATA" = ATA, "ATb" = ATb)
results
```

```
## $ATA
##      [,1] [,2]
## [1,]    4    8
## [2,]    8   26
##
## $ATb
##      [,1]
## [1,]   36
## [2,]  112
```

1.2 Solve for $\hat{x}$ in R using the above computed matrices

```
x <- solve(ATA) %*% ATb
x
```

```
##      [,1]
## [1,]    1
## [2,]    4
```

1.3 What is the squared error of this solution?

```
p <- A %*% x
#b = p + e or e = p - b which we can substitute in our given values.
e <- p - b
# we then sum the square of errors.
e2 <- sum(e^2)
e2
```

```
## [1] 44
```

1.4 Find the exact solution with p instead of b

```
options(scipen = 999)
p   <- matrix(c(1,5,13,17))
ATp <- t(A) %*% p
xp  <- solve(ATA) %*% ATp
p2  <- A %*% xp
e   <- p2-p
e
```

```
##                          [,1]
## [1,] 0.00000000000000000000
## [2,] 0.00000000000000008881784
## [3,] 0.00000000000000035527137
## [4,] 0.00000000000000035527137
```

Essentially, the error vector e is = 0.

```
e2p <- sum(e^2)
e2p
```

```
## [1] 0.00000000000000000000000000002603241
```

Show that the error $e = b - p = [-1; 3; -5; 3]$.

```
b - p
```

```
##      [,1]
## [1,]   -1
## [2,]    3
## [3,]   -5
## [4,]    3
```

Show that the error e is orthogonal to p and to each of the columns of A.

As per the week 5 handout - We know that when two vectors are orthogonal, their dot product is zero.

```
e*p
```

```
##                          [,1]
## [1,] 0.00000000000000000000
## [2,] 0.00000000000004440892
## [3,] 0.00000000000046185278
## [4,] 0.00000000000060396133
```

```
sum(e*A[,1])
```

```
## [1] 0.00000000000007993606
```

2 Problem Set 2

Write an R markdown script that takes in the auto-mpg data, extracts an A matrix from the first 4 columns and b vector from the fifth (mpg) column.

Apparently, an added column of 1 is necessary to obtain an intercept.

```
x <- as.matrix(read.table("https://raw.githubusercontent.com/ChristopheHunt/MSDA---Coursework/master/Da

A <- as.matrix(cbind(x[,1:4],1))
b <- as.matrix(x[,5])
```

Using the least squares approach, your code should compute the best fitting solution

```
ATA <- t(A) %*% A
ATb <- t(A) %*% b
results <- list("ATA" = ATA, "ATb" = ATb)
results
```

```
## $ATA
##             V1          V2         V3         V4        
## V1  19097634.2   9374647.0  259345480  1123011.9   76209.5
## V2   9374647.0   4857524.0  132989885   607832.3   40952.0
## V3 259345480.0 132989885.0 3757575489 17758103.6 1167213.0
## V4   1123011.9    607832.3   17758104    97656.9    6092.2
##        76209.5     40952.0    1167213     6092.2     392.0
##
## $ATb
##          [,1]
## V1  1529685.9
## V2   868718.8
## V3 25209061.4
## V4   146401.4
##        9190.8
```

2.1 Solve for $\hat{x}$ in R using the above computed matrices

```
x <- solve(ATA) %*% ATb
x
```

```
##            [,1]
## V1 -0.006000871
## V2 -0.043607731
## V3 -0.005280508
## V4 -0.023147999
##    45.251139699
```

The least squares model using this method is:

$$mpg = -0.006 * displacement + -0.04361 * horsepower + -0.00528 * weight + -0.02315 * acceleration + 45.25114$$

Finally, calculate the fitting error between the predicted mpg of youur model and actual mpg.

2.2 The fitting error to the predicted mpg and the actual mpg.

```
p <- A %*% x
#b = p + e or e = p - b which we can substitute in our given values.
e <- p - b
e
```

```
##               [,1]
##   [1,]   0.95919198
##   [2,]   1.18844197
##   [3,]   0.40325031
##   [4,]   2.47995603
##   [5,]   1.87826895
##   [6,]  -4.11172901
##   [7,]  -4.26661947
##   [8,]  -3.73121329
##   [9,]  -4.88872305
##  [10,]  -0.90138187
##  [11,]   1.49356259
##  [12,]   2.99106999
##  [13,]   1.22973585
##  [14,]   2.18187690
##  [15,]   3.55772236
##  [16,]   2.60176023
##  [17,]   6.82009386
##  [18,]   5.31326672
##  [19,]   2.24844730
##  [20,]   6.49883380
##  [21,]   1.28256448
##  [22,]   3.51707079
##  [23,]   2.53801867
##  [24,]   1.51135631
##  [25,]   4.80226596
```

```
##  [26,]  -0.97845147
##  [27,]   1.23260402
##  [28,]  -0.26644392
##  [29,]   0.59498197
##  [30,]   2.24844730
##  [31,]   0.17245834
##  [32,]   3.34126348
##  [33,]   6.28838301
##  [34,]   4.80367168
##  [35,]   4.45254442
##  [36,]   3.11841090
##  [37,]   3.77706091
##  [38,]  -0.54787405
##  [39,]  -2.61895040
##  [40,]   0.22512378
##  [41,]   0.87181916
##  [42,]  -3.32770358
##  [43,]  -2.90158894
##  [44,]  -5.20014767
##  [45,]   4.95270250
##  [46,]   6.11598637
##  [47,]   2.71230228
##  [48,]   5.00228167
##  [49,]   5.72196928
##  [50,]   1.09575284
##  [51,]   0.32137057
##  [52,]   0.16898091
##  [53,]   1.18842304
##  [54,]  -2.12397951
##  [55,]   4.92832807
##  [56,]   4.85459255
##  [57,]   4.04251609
##  [58,]   4.55756074
##  [59,]   6.86799519
##  [60,]   7.31947322
##  [61,]   7.63241624
##  [62,]   0.10889294
##  [63,]  -2.21336428
##  [64,]  -0.34569464
##  [65,]   0.36871047
##  [66,]   0.22948866
##  [67,]  -2.11286260
##  [68,]  -0.69370753
##  [69,]   0.33115718
##  [70,]  -2.07443304
##  [71,]   8.98504767
##  [72,]   1.04462895
##  [73,]   2.77627435
##  [74,]   1.28892587
##  [75,]   0.94900081
##  [76,]   5.81759309
##  [77,]   5.53482769
##  [78,]   3.55514388
##  [79,]   3.69042710
```

```
##  [80,]   5.75158442
##  [81,]   0.18182614
##  [82,]   3.73248675
##  [83,]   1.40019701
##  [84,]   2.36056653
##  [85,]   0.56847599
##  [86,]   3.22948866
##  [87,]   2.46812479
##  [88,]   1.78515902
##  [89,]   1.56787515
##  [90,]  -4.37284127
##  [91,]  -0.54033113
##  [92,]  -0.08496701
##  [93,]   0.09254556
##  [94,]  -5.02273809
##  [95,]  -5.68941815
##  [96,]   2.02802495
##  [97,]   4.45972516
##  [98,]   5.66398032
##  [99,]   5.57670109
## [100,]   5.57908559
## [101,]   1.21527018
## [102,]   5.88000140
## [103,]  -1.40113778
## [104,]  -2.62722100
## [105,]  -2.19889539
## [106,]  -0.74491128
## [107,]   6.42360830
## [108,]   8.35748564
## [109,]   7.14137593
## [110,]   5.55964889
## [111,]  11.37808640
## [112,]   9.18616531
## [113,]   5.27749023
## [114,]   2.41921441
## [115,]  -0.02824295
## [116,]  -5.98890515
## [117,]   3.39620761
## [118,]   5.53032903
## [119,]   6.64039377
## [120,]   5.13767810
## [121,]   3.59862910
## [122,]   1.35796118
## [123,]   3.85997728
## [124,]   4.69903473
## [125,]   3.15815563
## [126,]   4.80904343
## [127,]   6.38085886
## [128,]   0.11855073
## [129,]   1.70594837
## [130,]  -0.19054495
## [131,]   2.32387113
## [132,]   3.03103289
## [133,]   3.31059628
```

```
## [134,]   1.86171532
## [135,]  -0.85686044
## [136,]  -1.53907691
## [137,]  -1.04601815
## [138,]  -1.52756882
## [139,]   0.04779960
## [140,]  -0.05577611
## [141,]   5.13092213
## [142,]   2.78683828
## [143,]   2.43797215
## [144,]  -0.92367328
## [145,]   1.88375643
## [146,]   5.95037707
## [147,]   3.10036634
## [148,]   2.81068139
## [149,]   1.56303853
## [150,]  -0.07603066
## [151,]   3.15226385
## [152,]   2.53646575
## [153,]   7.00235458
## [154,]   8.48393572
## [155,]  -5.47813534
## [156,]  -1.94181274
## [157,]  -3.28566697
## [158,]  -2.21293299
## [159,]   0.95103615
## [160,]   2.16573333
## [161,]   3.76995852
## [162,]   1.33167529
## [163,]   1.67340492
## [164,]   1.56104750
## [165,]   7.80177417
## [166,]   0.56412498
## [167,]   3.46280003
## [168,]   3.74039683
## [169,]   3.89430055
## [170,]   1.68025076
## [171,]   4.49440159
## [172,]   2.47467782
## [173,]   5.90235961
## [174,]   2.10610455
## [175,]   3.58501532
## [176,]   2.84539710
## [177,]   1.68557732
## [178,]   2.36473523
## [179,]   0.09341094
## [180,]  -0.48975078
## [181,]  -0.51118356
## [182,]   3.90888396
## [183,]   1.47273527
## [184,]   2.90077893
## [185,]   2.04376751
## [186,]  -2.74247263
## [187,]  -1.62454857
```

```
## [188,]   1.45081813
## [189,]  -0.53717587
## [190,]   0.11180974
## [191,]  -0.86907843
## [192,]   0.20644503
## [193,]   0.73647051
## [194,]   2.21374471
## [195,]   5.60800083
## [196,]   2.10147495
## [197,]  -0.48743598
## [198,]  -0.14868292
## [199,]   2.99087609
## [200,]   0.83162304
## [201,]   4.78748476
## [202,]   2.15357400
## [203,]  -1.21320200
## [204,]   1.63935390
## [205,]   0.91194585
## [206,]   3.02601477
## [207,]   2.69094878
## [208,]   3.91935318
## [209,]   4.77468705
## [210,]   1.95195429
## [211,]  -4.60727165
## [212,]   2.13747876
## [213,]   3.98708648
## [214,]   3.64932432
## [215,]  -1.02914779
## [216,]  -0.62566015
## [217,]  -3.81965702
## [218,]   2.32872935
## [219,]  -2.47094955
## [220,]  -1.17996718
## [221,]   0.01538920
## [222,]  -0.65868813
## [223,]  -0.25481453
## [224,]   2.48705692
## [225,]   0.30918636
## [226,]   0.96220775
## [227,]   1.92376235
## [228,]  -3.53928595
## [229,]  -2.01968157
## [230,]  -4.05527938
## [231,]  -2.57936521
## [232,]   1.68782109
## [233,]   1.16856851
## [234,]   3.01683165
## [235,]   0.11639234
## [236,]   0.19192982
## [237,]  -3.78149419
## [238,]   0.88590208
## [239,]  -1.12304666
## [240,]   2.94478720
## [241,]   4.20256925
```

```
## [242,]   3.79874042
## [243,] -11.46159974
## [244,]  -4.15330112
## [245,]  -1.21540941
## [246,]  -9.07267943
## [247,]  -3.89594464
## [248,]   0.86636012
## [249,]  -2.19046981
## [250,]  -1.97030516
## [251,]   0.97509011
## [252,]   2.32693538
## [253,]   4.12186437
## [254,]   0.75407704
## [255,]   0.52988330
## [256,]   3.18566623
## [257,]   0.47227201
## [258,]   2.94657785
## [259,]   0.95578752
## [260,]   2.01392090
## [261,]  -0.49353973
## [262,]   0.46813035
## [263,]   2.09411698
## [264,]  -2.12381900
## [265,]  -0.06370765
## [266,]  -1.04251300
## [267,]   0.62164263
## [268,]  -1.66070995
## [269,]   5.58122105
## [270,]   1.65462659
## [271,]   1.35509650
## [272,]   3.36255971
## [273,]   4.36153906
## [274,]   4.92642409
## [275,]   2.78770237
## [276,]   3.90089954
## [277,]  -0.73220242
## [278,]   0.53958771
## [279,]  -0.14167754
## [280,]   4.33429647
## [281,]   2.61240952
## [282,]   2.07209030
## [283,]   0.37733034
## [284,]   0.11823990
## [285,]   0.23340464
## [286,]  -0.56299481
## [287,]  -1.32037540
## [288,]  -3.87628262
## [289,]  -0.28565880
## [290,]  -0.38550981
## [291,]  -3.05645820
## [292,]  -0.76813621
## [293,]  -2.98029020
## [294,]  -4.97106775
## [295,]  -0.80975997
```

```
## [296,]  -3.71028228
## [297,]  -6.29688711
## [298,]  -3.31002226
## [299,]  -2.70700479
## [300,]  -4.55416366
## [301,]  -4.62948987
## [302,]  -1.00450418
## [303,]  -7.19163019
## [304,]  -2.44901141
## [305,]  -3.56639016
## [306,]  -2.15788028
## [307,]  -6.88221912
## [308,] -11.81281751
## [309,]  -6.82662340
## [310,]  -2.04295734
## [311,]  -6.34041015
## [312,]  -2.10282947
## [313,]  -1.40049872
## [314,]  -0.20232730
## [315,]   2.58998350
## [316,]  -4.95183725
## [317,]  -3.95192342
## [318,]  -3.86768545
## [319,]  -9.67485116
## [320,]  -3.17973394
## [321,] -16.25565835
## [322,]  -3.28256094
## [323,] -10.47974988
## [324,] -13.19428012
## [325,] -13.66070306
## [326,] -10.83482685
## [327,]  -6.21298216
## [328,] -12.90503935
## [329,]  -3.79601597
## [330,]   2.11668159
## [331,]  -9.84339199
## [332,]   3.70212678
## [333,]  -7.86925116
## [334,]  -3.41658897
## [335,]   0.06608472
## [336,]  -0.21180645
## [337,]   0.33483096
## [338,]   1.23509008
## [339,]  -2.11464757
## [340,]  -6.51066987
## [341,]  -7.33640929
## [342,]  -2.61777122
## [343,]  -1.86894575
## [344,]  -5.97151093
## [345,]  -6.91211851
## [346,]  -3.21244123
## [347,]  -4.86746876
## [348,]  -3.74508420
## [349,]  -1.11822031
```

```
## [350,]  -3.49893751
## [351,]  -4.36478674
## [352,]  -3.86561390
## [353,]  -6.87485532
## [354,]  -4.63368334
## [355,]  -4.71186095
## [356,]  -6.77327957
## [357,]  -1.82064081
## [358,]  -0.84924541
## [359,]  -1.73058441
## [360,]  -8.13768042
## [361,]   3.45930053
## [362,]   3.91307029
## [363,]  -1.46786177
## [364,]  -0.62953154
## [365,]  -6.32191833
## [366,]  -4.09992014
## [367,]  -1.92567746
## [368,]  -1.43854041
## [369,]   0.89082449
## [370,]  -5.41549367
## [371,]  -6.37458714
## [372,]  -0.07027041
## [373,]  -7.68759346
## [374,]  -6.01102628
## [375,]  -7.04798797
## [376,]  -6.64069900
## [377,]  -4.69543672
## [378,]  -6.94007534
## [379,]  -0.95627894
## [380,]  -7.12626818
## [381,]  -1.56259100
## [382,] -14.34199262
## [383,]   0.31733394
## [384,]   1.66435658
## [385,]  -6.19363836
## [386,]  -8.03775475
## [387,]  -2.55764599
## [388,]  -1.43297264
## [389,] -13.41546919
## [390,]  -3.60930947
## [391,]  -1.20586132
## [392,]  -4.85085028
```

```
e2 <- sum(e^2)
e2
```

```
## [1] 6979.413
```