Homework 5

Christophe Hunt

March 4, 2017

Contents

1 Problem Set 1

Consider the unsolvable system Ax = b as given below:

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 8 \\ 20 \end{bmatrix}$$

1.1 Write R Markdown script to compute A^TA and A^Tb

```
A <- matrix(c(1,1,1,1,0,1,3,4), ncol = 2)
b <- matrix(c(0,8,8,20))

ATA <- t(A) %*% A
ATb <- t(A) %*% b

results <- list("ATA" = ATA, "ATb" = ATb)
results
```

```
## $ATA
##      [,1] [,2]
## [1,]    4    8
## [2,]    8   26
##
## $ATb
##      [,1]
## [1,]   36
## [2,]  112
```

1.2 Solve for $\hat{x}$ in R using the above computed matrices

```
x <- solve(ATA) %*% ATb
x
```

```
##      [,1]
## [1,]    1
## [2,]    4
```

1.3 What is the squared error of this solution?

```
p <- A %*% x
#b = p + e or e = p - b which we can substitute in our given values.
e <- p - b
# we then sum the square of errors.
e2 <- sum(e^2)
e2
```

```
## [1] 44
```

1.4 Find the exact solution with p instead of b

```
options(scipen = 999)
p   <- matrix(c(1,5,13,17))
ATp <- t(A) %*% p
xp  <- solve(ATA) %*% ATp
p2  <- A %*% xp
e   <- p2-p
e
```

```
##                          [,1]
## [1,] 0.00000000000000000000
## [2,] 0.00000000000000008881784
## [3,] 0.00000000000000035527137
## [4,] 0.00000000000000035527137
```

Essentially, the error vector e is = 0.

```
e2p <- sum(e^2)
e2p
```

```
## [1] 0.00000000000000000000000000002603241
```

Show that the error $e = b - p = [-1; 3; -5; 3]$.

```
b - p
```

```
##      [,1]
## [1,]   -1
## [2,]    3
## [3,]   -5
## [4,]    3
```

Show that the error e is orthogonal to p and to each of the columns of A.

As per the week 5 handout - We know that when two vectors are orthogonal, their dot product is zero.

```
e*p
```

```
##                          [,1]
## [1,] 0.00000000000000000000
## [2,] 0.00000000000004440892
## [3,] 0.00000000000046185278
## [4,] 0.00000000000060396133
```

```
sum(e*A[,1])
```

```
## [1] 0.00000000000007993606
```

2 Problem Set 2