Homework 12

Christophe Hunt

April 29, 2017

Contents

1 Page 576: problem 2

Consider a company that allows back ordering. That is, the company notifies customers that a temporary stock-out exists and that their order will be filled shortly. What considerations might argue for such a policy? What effect does such a policy have on storage costs? Should costs be assigned to stock-outs? Why? How would you make such an assignment? What assumptions are implied by the model in Figure 13.7? Suppose a “loss of goodwill cost” of w dollars per unit per day is assigned to each stock-out. Compute the optimal order quantity Q* and interpret your model.

2 Page 585: problem 2

Find the local minimum value of the function

$f(x,y) = 3x^2 + 6xy + 7y^2 - 2x + 4y$

d/dx:

$\frac{\partial}{\partial x}(3x^2 + 6xy + 7y^2 - 2x + 4y)$
$= 3(\frac{\partial}{\partial x}(x^2)) + 6y(\frac{\partial}{\partial x}(x)) + \frac{\partial}{\partial x}(7y^2) - 2(\frac{\partial}{\partial x}(x)) + \frac{\partial}{\partial x}(4y)$

Derivative of x is 1:

$= 3(\frac{\partial}{\partial x}(x^2)) + 6y + \frac{\partial}{\partial x}(7y^2) - 2 + \frac{\partial}{\partial x}(4y)$

Use the power rule, $\frac{\partial}{\partial x}(x^n) = nx^{n-1}$, where n = 2: $\frac{\partial}{\partial x}(x^2) = 2x$ and derivative of $4y$ = 0:

$= 3(2x) + 6y + \frac{\partial}{\partial x}(7y^2) - 2$

The derivative of $7x^2$ = 0 :

$\frac{\partial}{\partial x}(3x^2 + 6xy + 7y^2 - 2x + 47) = 6x + 6y - 2$

d/dy:

$\frac{\partial}{\partial y}(3x^2 + 6xy + 7y^2 - 2x + 47)$
$= (\frac{\partial}{\partial y}(3x^2)) + 6x(\frac{\partial}{\partial y}(y)) + 7\frac{\partial}{\partial y}(y^2) - (\frac{\partial}{\partial y}(-2x)) + 4\frac{\partial}{\partial y}(y)$

The derivative of -2x, and $3x^2$ = 0:

$= 6x(\frac{\partial}{\partial y}(y)) + 7\frac{\partial}{\partial y}(y^2) + 4\frac{\partial}{\partial y}(y)$

The derivative of y = 1:

$= 6x + 7\frac{\partial}{\partial y}(y^2) + 4$

Using the power rule :

$0 = 6x + 7 * 2y + 4 = 6x + 14y + 4$

Finding the local minima:
$0 = 6x + 6y - 2$
$-6y = 6x - 2$
$y = -x + \frac{1}{3}$

$0 = 6x + 14y + 4$
$-6x = 14y + 4$
$x = -\frac{14}{6}y - \frac{4}{6}$
$x = -\frac{14}{6}(-x + \frac{1}{3}) - \frac{4}{6}$
$x = \frac{13}{12}$

$y = -\frac{13}{12} + \frac{1}{3}$

min at (x,y) = $(\frac{13}{12}, -\frac{1}{4})$

3 Page 591: problem 5

Find the hottest point (x, y, z) along the elliptical orbit

$$4x^2 + y^2 + 4z^2 = 16$$

Where the temperature function is:

$$T(x, y, z) = 8x^2 + 4yx - 16z + 600$$

Maximize $T(x, y, z)8x^2 + 4yx - 16z + 600$ such that $4x^2 + y^2 + 4z^2 = 16$.

$L(x, y, z, \lambda) = 8x^2 + 4yx - 16z + 600 + \lambda 4x^2 + \lambda y^2 + \lambda 4z^2 - \lambda 16)$

d/dx:

$= 8(\frac{\partial}{\partial x}(x^2)) + 4y\frac{\partial}{\partial x}(x)) + \frac{\partial}{\partial x}(-16z) + \frac{\partial}{\partial x}(600) + 4\lambda\frac{\partial}{\partial x}(x^2) + \frac{\partial}{\partial x}(y^2\lambda) + \frac{\partial}{\partial x}(4z^2\lambda) + \frac{\partial}{\partial x}(-16\lambda)$

The derivatives of 600, $-16z$, -16λ, λy^2, and $4\lambda z^2$

$= 8(\frac{\partial}{\partial x}(x^2)) + 4y\frac{\partial}{\partial x}(x) + 4\lambda\frac{\partial}{\partial x}(x^2)$

Using the power rule:

$= 16x + 4y + 8x\lambda$

d/dy :

$= \frac{\partial}{\partial y}(8x^2) + 4x(\frac{\partial}{\partial y}(y)) + \frac{\partial}{\partial y}(-16z) + \frac{\partial}{\partial y}(600) + \frac{\partial}{\partial y}(4x^2\lambda) + \lambda(\frac{\partial}{\partial y}(y^2)) + \frac{\partial}{\partial y}(\lambda 4z^2) + \frac{\partial}{\partial y}(-16\lambda)$

The derivative of 600, $8x^2$, -16z, -16λ, $4\lambda x^2$ and $4\lambda z^2$:

$= 4x(\frac{\partial}{\partial y}(y)) + \lambda(\frac{\partial}{\partial y}(y^2))$

Using the power rule:

$= 4x + 2y\lambda$

d/dz:

$\frac{\partial}{\partial z}(8x^2) + \frac{\partial}{\partial z}(4yx) + 16\frac{\partial}{\partial z}(z) + \frac{\partial}{\partial z}\frac{\partial}{\partial z}(600) + \frac{\partial}{\partial z}(4x^2\lambda) + \frac{\partial}{\partial z}(y^2\lambda) + 4\lambda(\frac{\partial}{\partial z}(z^2)) + \frac{\partial}{\partial z}(-16\lambda)$

The derivative of 600, $8x^2$, $4xy$, -16λ, $4\lambda x^2$, and λy^2 are 0:

$16\frac{\partial}{\partial z}(z) + 4\lambda(\frac{\partial}{\partial z}(z^2))$

The derivative of x is 1 and using the power rule :

$= -16 + 8z\lambda$

$0 = 16x + 4y + 8x\lambda$
$y = -4x + 2x\lambda$

$0 = 4x + 2y\lambda$
$0 = 4x - 8x + 4x\lambda$
$0 = -4x + 4x\lambda$
$4x = 4x\lambda$
$\lambda = 1$

$0 = -16 + 8z\lambda$ $0 = -16 + 8z$ $z = 2$

$y = -2x$

$0 = 4x + 2(-2x)$
$0 = 0$

$x = x, y = -2x, z = 2, \lambda = 1$

$8x^2 + 4(-2x)x - 16(2) + 600$
$8x^2 + (-8x^2) + 568$

Hottest point = 568

4 Page 599: problem 5

One of the key assumptions underlying the models developed in this section is that the harvest rate equals the growth rate for sustainable yield. The reproduction sub-models in Figures 13.19 and 13.22 suggests that if the current population levels are known, it is possible to estimate the growth rate. The implications of this knowledge is that if a quota for the season is established based on the estimated growth rate, then the fish population can be maintained, increased, or decreased as desired. This quota system might be implemented by requiring all commercial fishermen to register their catch daily and then closing the season when the quota is reached. Discuss the difficulties in determining reproduction models precise enough to be used in this manner. How would you estimate population level? What are the disadvantages of having a quota that varies from year to year? Discuss the practical political difficulties in implemented such a procedure.