

Homework 5

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1 Problem Set 1

Consider the unsolvable system $Ax = b$ as given below:

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 8 \\ 20 \end{bmatrix}$$

1.1 Write R Markdown script to compute $A^T A$ and $A^T b$

```
A <- matrix(c(1,1,1,1,0,1,3,4), ncol = 2)
b <- matrix(c(0,8,8,20))
```

```
ATA <- t(A) %*% A
ATb <- t(A) %*% b
```

```
results <- list("ATA" = ATA, "ATb" = ATb)
results
```

```
## $ATA
##      [,1] [,2]
## [1,]    4    8
## [2,]    8   26
##
## $ATb
##      [,1]
## [1,]   36
## [2,]  112
```

1.2 Solve for \hat{x} in R using the above computed matrices

```
x <- solve(ATA) %*% ATb
x
```

```
##      [,1]
## [1,]    1
## [2,]    4
```

1.3 What is the squared error of this solution?

```
p <- A %*% x
#b = p + e or e = p - b which we can substitute in our given values.
e <- p - b
# we then sum the square of errors.
e2 <- sum(e^2)
e2
```

```
## [1] 44
```

1.4 Find the exact solution with p instead of b

```
options(scipen = 999)
p <- matrix(c(1,5,13,17))
ATp <- t(A) %*% p
xp <- solve(ATA) %*% ATp
p2 <- A %*% xp
e <- p2-p
e
```

```
##      [,1]
## [1,] 0.000000000000000000000000
## [2,] 0.00000000000000008881784
## [3,] 0.00000000000000035527137
## [4,] 0.00000000000000035527137
```

Essentially, the error vector e is ≈ 0 .

```
e2p <- sum(e^2)
e2p
```

```
## [1] 0.0000000000000000000000002603241
```

Show that the error $e = b - p = [-1; 3; -5; 3]$.

```
b - p
```

```
##      [,1]
## [1,]   -1
## [2,]    3
## [3,]   -5
## [4,]    3
```

Show that the error e is orthogonal to p and to each of the columns of A .

As per the week 5 handout - We know that when two vectors are orthogonal, their dot product is zero.

```
e*p

##                [,1]
## [1,] 0.00000000000000000000
## [2,] 0.0000000000000004440892
## [3,] 0.0000000000000046185278
## [4,] 0.000000000000060396133

sum(e*A[,1])

## [1] 0.000000000000007993606
```

2 Problem Set 2

Write an R markdown script that takes in the auto-mpg data, extracts an A matrix from the first 4 columns and b vector from the fifth (mpg) column.

```
x <- as.matrix(read.table("https://raw.githubusercontent.com/ChristopheHunt/MSDA---Coursework/master/Data/
A <- as.matrix(x[,1:4])
b <- as.matrix(x[,5])
```

Using the least squares approach, your code should compute the best fitting solution

```
ATA <- t(A) %*% A
ATb <- t(A) %*% b
results <- list("ATA" = ATA, "ATb" = ATb)
results

## $ATA
##           V1           V2           V3           V4
## V1 19097634  9374647.0 259345480 1123011.9
## V2  9374647  4857524.0 132989885  607832.3
## V3 259345480 132989885.0 3757575489 17758103.6
## V4 1123012   607832.3  17758104   97656.9
##
## $ATb
##           [,1]
## V1 1529685.9
## V2  868718.8
## V3 25209061.4
## V4 146401.4
```

2.1 Solve for \hat{x} in R using the above computed matrices

```
x <- solve(ATA) %*% ATb
x

##           [,1]
## V1 -0.030037938
```

```
## V2 0.157115685
## V3 -0.006217883
## V4 1.997320955
```

Finally, calculate the fitting error between the predicted mpg of your model and the actual mpg

2.2 What is the squared error of this solution?

```
p <- A %*% x
# b = p + e or e = p - b which we can substitute in our given values.
e <- p - b
# we then sum the square of errors.
e2 <- sum(e^2)
e2

## [1] 13101.43
```