

# Homework 3

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## 1 Problem Set 1

### 1.1 Problem 1

What is the rank of the matrix A?

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 1 & 3 \\ 0 & 1 & -2 & 1 \\ 5 & 4 & -2 & -3 \end{bmatrix}$$

```
A <- t(matrix(c(1 , 2 ,3 , 4,
               -1 , 0 , 1 , 3,
               0 , 1 , -2 , 1,
               5 , 4 , -2 , -3), nrow = 4, ncol = 4))
```

Since A is a square matrix of (4 x 4) and the determinate is -9 which is  $\neq 0$ , the rank is simply 4

Below is my attempt to create a function to rank matrix A and matrix B in problem 3 programmatically. I think that the part of the function that calculates the determinates of the submatrices could be improved.

```
A <- t(matrix(c(1 , 2 ,3 , 4,
               -1 , 0 , 1 , 3,
               0 , 1 , -2 , 1,
               5 , 4 , -2 , -3), nrow = 4, ncol = 4))

subdet <- function(A, t){
  subdet <- 0
  for (i in 1:t){
    subdet <- rbind(subdet,det(A[-i,-i]))
  }
  if (sum(subdet) !=0){
    return(as.integer(ncol(A))-1)
  } else {
    0
  }
}
```

```

}

Matrix.Rank <- function(A){
  sq.matrix <- (as.integer(ncol(A)) == as.integer(nrow(A)))
  if (all(A == 0)){
    return(0)
  } else if (sq.matrix == TRUE){
    det.0 <- det(A) != 0
    if (det.0 == TRUE) {
      return(as.integer(ncol(A)))
    } else if (subdet(A,ncol(A)) != 0) {
      return(as.integer(ncol(A))-1)
    } else {
      return(1)
    }
  }
}

```

```
Matrix.Rank(A)
```

```
## [1] 4
```

## 1.2 Problem 2

Given an  $m \times n$  matrix where  $m > n$ , what can be the maximum rank? The minimum rank, assuming that the matrix is non-zero?

The maximum rank of a rectangular matrix is the maximum columns or rows for the lesser value. Therefore, given an  $m \times n$  matrix where  $m > n$ , the maximum rank is  $n$ .

Assuming the rectangular matrix has at least one non-zero element, it's minimum rank must be greater than zero, therefore the minimum rank would be 1.

## 1.3 Problem 3

What is the rank of matrix B?

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 3 \\ 2 & 4 & 2 \end{bmatrix}$$

```
B <- t(matrix(c( 1 , 2 , 1 , 3 , 6 , 3 , 2 , 4 , 2 ), ncol = 3, nrow = 3))
```

This matrix rows that are linearly dependent,  $R_2 = 3, 6, 3$  and  $R_3 = 2, 4, 2$  are multiples of  $R_1 = 1, 2, 1$ . This made my function more challenging because the determinates of the submatrices of B are also = 0. However, as long as there is at least one non-zero element in the matrix the minimum rank will be = 1.

```
Matrix.Rank(B)
```

```
## [1] 1
```

## 2 Problem Set 2

Compute the eigenvalues and eigenvectors of the matrix A. You'll need to show your work. You'll need to write out the characteristic polynomial and show your solution.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

$$\det\left(\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} 1-\lambda & 2 & 3 \\ 0 & 4-\lambda & 5 \\ 0 & 0 & 6-\lambda \end{bmatrix}\right) = 0$$

which reduces to:

$$(1-\lambda)(4-\lambda)(6-\lambda) = 0$$

Therefore, our Eigen Values are:

$$\lambda_1 = 1; \lambda_2 = 4; \lambda_3 = 6$$

Our Eigen Vectors are as follows:

$$\lambda_1 = 1$$

$$\det\left(\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) = 0$$

$$\begin{bmatrix} 0 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

$$v_1 = 1; v_2 = 0; v_3 = 0$$

Therefore:

$$E\lambda_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

---

$$\lambda_2 = 4$$

$$\det\left(\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}\right) = 0$$

$$\begin{bmatrix} -3 & 2 & 3 \\ 0 & 0 & 5 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

Which we can reduce to  $-3v_1 + 2v_2 + 3v_3 = 0$  and  $v_3 = 0$ ; substituting back we have  $-3v_1 + 2v_2 = 0$  or  $3v_1 = 2v_2$  or  $v_1 = \frac{2}{3}v_2$  where  $v_1 = 1$ .

Therefore:

$$E\lambda_4 = \begin{bmatrix} 1 \\ \frac{3}{2} \\ 0 \end{bmatrix}$$


---

$$\lambda_3 = 6$$

$$\det\left(\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} - \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}\right) = 0$$

$$\begin{bmatrix} -5 & 2 & 3 \\ 0 & -2 & 5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

Which we can reduce to  $-5v_1 + 2v_2 + 3v_3 = 0$  and  $-2v_1 + 5v_2 = 0$ ; solving for  $-2v_2 + 5v_3 = 0$ , we reduce to  $5v_3 = 2v_2$  or  $v_3 = \frac{2}{5}v_2$ . Substituting back  $v_3 = 1$  we get  $1 = \frac{5}{2}v_2$  or  $\frac{2}{5} = v_2$ . Now, substituting back to  $-5v_1 + 2v_2 + 3v_3 = 0$  or  $-5v_1 + 2(\frac{2}{5}) + 3(1) = 0$  or  $-5v_1 + 3\frac{4}{5} = 0$  or  $5v_1 = 3\frac{4}{5}$  or  $5v_1 = 3\frac{4}{5}$  or  $v_1 = \frac{19}{25}$

Therefore:

$$E\lambda_6 = \begin{bmatrix} \frac{19}{25} \\ \frac{2}{5} \\ 1 \end{bmatrix}$$

```
library(MASS)
fractions(.8)
```

```
## [1] 4/5
```

```
A <- t(matrix(c(1 , 2 , 3, 0 , 4 , 5 , 0 , 0 , 6 ), nrow = 3, ncol=3))
```