Homework 13

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Contents

1 Write a program to compute the derivative of $f(x) = x^3 + 2x^2$ at any value of x.

```
deriv_limit <- function(func, x, h){
               f <- function(x) {eval(parse(text = func))}
               return((f(x + h) - f(x)) / h)
               }

deriv_limit(func = ('x^3 + 2*x^2'), x = 2, h = 0.000001)
```

```
## [1] 20.00001
```

```
deriv_limit(func = ('x^3 + 2*x^2'), x = 20, h = 0.000001)
```

```
## [1] 1280
```

Test using the analytic form

```
deriv_analytic <- function(func, val, var){
                      f_x <- D(parse(text = func), var)
                      assign(var, val)
                      return(eval(f_x))
                    }
deriv_analytic(func = 'x^3 + 2*x^2', val = 2, var = 'x')
```

```
## [1] 20
```

```
deriv_analytic(func = 'x^3 + 2*x^2', val = 20, var = 'x')
```

```
## [1] 1280
```

Your function should take in a value of x and return back an approximation to the derivative of $f(x)$ evaluated at that value. You should not use the analytical form of the derivative to compute it. Instead, you should compute this approximation using limits.

2 Now, write a program to compute the area under the curve for the function $3x^2 + 4x$ in the range x = $[1, 3]$.

```
auc <- function(func, range = seq(from = 1, to = 3, by = 0.000001)){
        return(sum((function(x) {eval(parse(text = func))})(range) * 0.000001))
       }
auc(func = '3*x^2+4*x')
```

```
## [1] 42.00002
```

You should first split the range into many small intervals using some really small δx value (say 1e-6) and then compute the approximation to the area under the curve.

3 Please solve these problems analytically (i.e. by working out the math) and submit your answers.

3.1 Use integration by parts to solve for $\int sin(x)cos(x)dx$

Substitute u = cos(X) and du = -sin(x)dx: $-\int udu$
u = $\frac{u^2}{2}$
Therefore: = $-\frac{u^2}{2} + C$
Substitute back u = cos(x) : = $-\frac{1}{2}cos^2(x) + C$

3.2 Use integration by parts to solve for $\int x^2 e^x dx$

For $e^x x^2$:
$\int fdg = fg - \int gdf$
f = x^2, dg = $e^x dx$, df = $2xdx$, g = e^x; =$e^x x^2 - 2\int e^x xdx$
For $e^x x$: $\int fdg = fg - \int gdf$ f = x, dg = $e^x dx$, df = dx, g = e^x: = $-2e^x x + e^x x^2 + 2\int e^x dx$ The integral of e^x is e^x: = $e^x x^2 - 2e^x x + 2e^x + C$

3.3 What is $\frac{d}{dx}(xcos(x))$?

Use the product rule, $\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{du}{dv}$ where u =x and v = $cos(x)$: = $cos(x)(\frac{d}{dx}(x)) + x(\frac{d}{dx}(cos(x)))$ The derivative of x is 1: = $cos(x) + x(\frac{d}{dx}(cos(x)))$ The derivative of cos(x) is -sin(x): = $cos(x) - sin(x)x$

3.4 What is $\frac{d}{dx}(e^{x^4})$?

Use the chain rule, $\frac{d}{dx}(e^{x^4}) = \frac{de^u}{du}\frac{du}{dx}$, where $u = x^4$ and $\frac{d}{du}(e^u) = e^u$ =$e^{x^4}(\frac{d}{du}(x^4))$ Use the power rule, $\frac{d}{dx}(x^n) = nx^{n-1}$, where n = 4: $\frac{d}{dx}(x^4) = 4x^3$: $= 4x^3 e^{x^4}$