Homework 14

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This week, we'll work out some Taylor Series expansions of popular functions.

The Taylor Series of $f(x)$ defined as:

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 ...$$

1 $f(x) = \frac{1}{(1-x)}$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$f(x) = 1 + x + x^2 + x^3 + x^4 + x^5 + O(x^6)$$

2 $f(x) = e^x$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$1 + \frac{1}{1!}x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 ...$$

$$f(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + O(x^6)$$

3 $f(x) = ln(1+x)$

$$ln(1+x) = \sum_{n=0}^{\infty} (-1)^{n+1} \frac{x^n}{n}$$

$$= 0 + \frac{1}{1!}x + \frac{-1}{2!}x^2 + \frac{2}{3!}x^3 ...$$

$$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + O(x^7)$$