Homework 1

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An annuity increases each month by an automatic deposit of 1% interest on the previous month's balance. Your grandparents withdraw $1000 at the beginning of each month for living expenses. Currently, they have $50,000 in the annuity. Model the annuity with a dynamic system.

$$\Delta\, b_n = \Delta\, b_{n+1} - b_n = .01b_n - 1,000$$
$$b_{n+1} = b_n + .01b_n - 1,000$$
$$b_o = 50,000$$

Will the annuity run out of money?

```
a_n   <- original_amount <- 50000
x     <- 1000
count <- 0

while (1.01*a_n > 1000){
a_n <- a_n + (.01 * a_n) - x
count <- count + 1
if (a_n >= original_amount){
  print("the annunity will not run out")
  break
 }
}
```

Yes, the code above runs without indicating that the annuity will not run out. This is because our initial amount is decreasing with each sequence so it has a termination point.

When? The annuity will run out at 69 months.

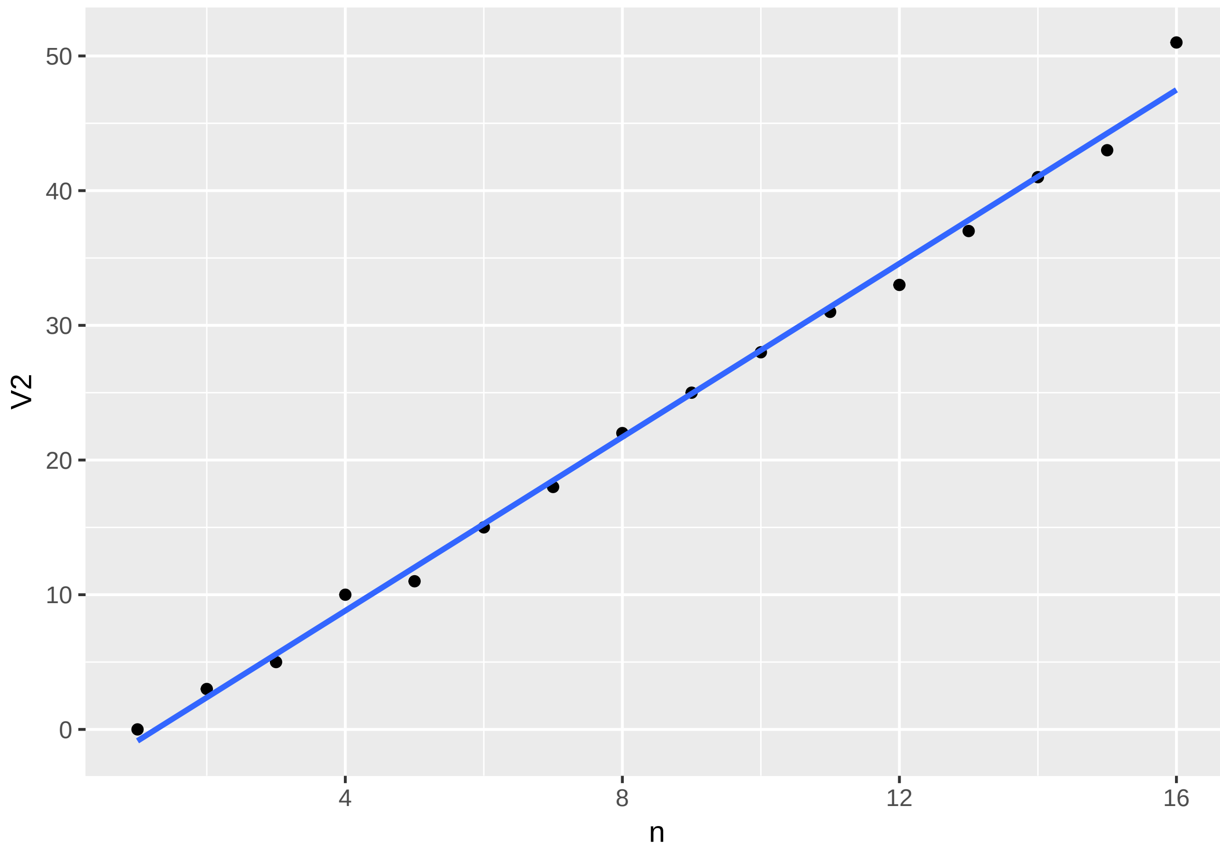
2 Page 17: problem 9

The data in the accompanying table show the speed n (in increments of 5 mph) of an automobile and the associate distance a_n in feet required to stop it once the breaks are applied. For instance, $n = 6$ (representing 6 x 5 = 30 mph) requires a stopping distance of a_6 = 47ft.

a. Calculate and plot the change $\Delta\ a_n$ versus n. Does the graph reasonably approximate a linear relationship?

The graph of $\Delta\ a_n$ does approximate a linear relationship.

```
library(ggplot2)
n <- c(1:16)
a_n <- c(3, 6, 11, 21, 32, 47, 65, 87, 112, 140, 171, 204, 241, 282, 325, 376)
ggplot(data = as.data.frame(cbind(n, c(0, diff(a_n)))), aes(x = n, y = V2)) +
        geom_point() +
        geom_smooth(method = "lm", se = FALSE)
```

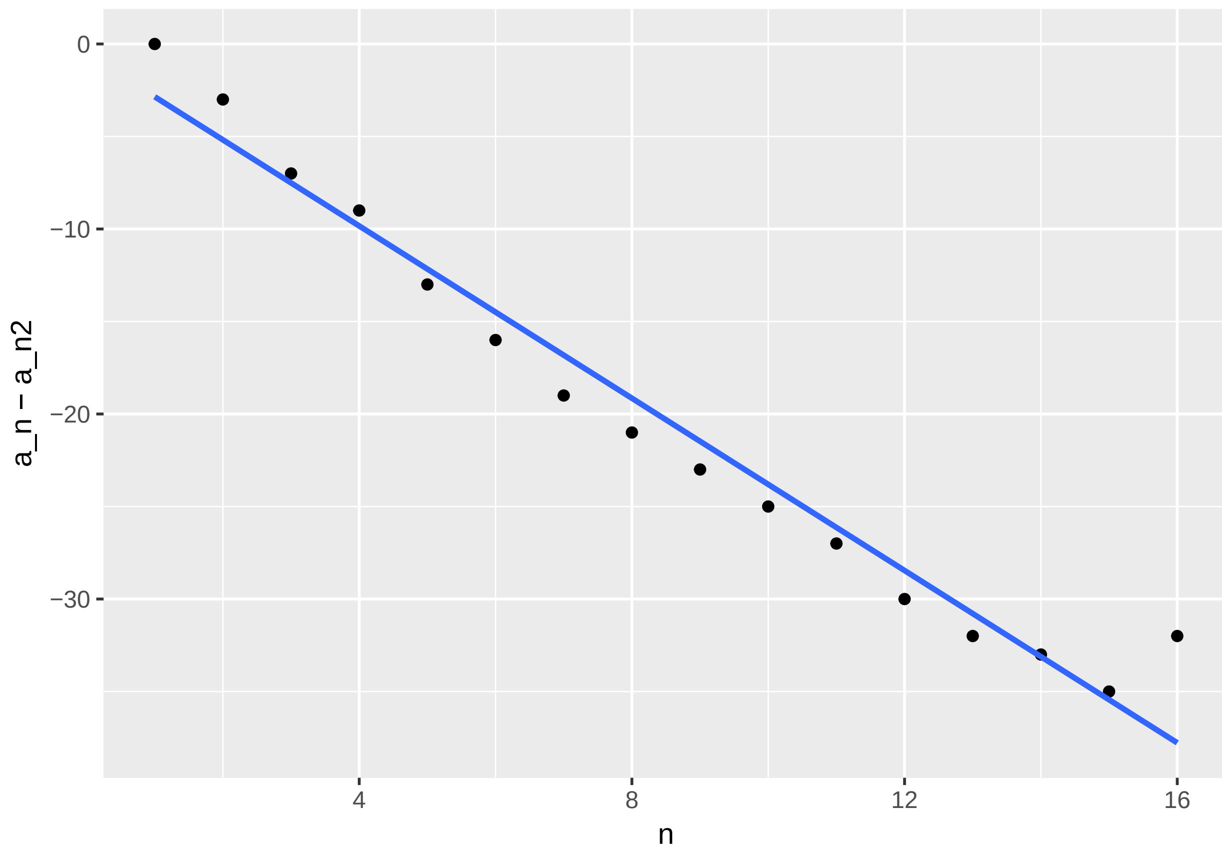


b. Based on your conclusion in part(a), find a difference equation model for the stopping distance data. Test your model by plotting the errors in the predicted values against n. Discuss the appropriateness of the model.

The plotting of errors shows a linear relationship which will only grow over time. Therefore, this model is not appropriate.

$$a_{n+1} = 3n + a_n$$

```
df <- as.data.frame(cbind(n, a_n, diff = c(0, diff(a_n))))

for (i in 1:length(a_n)){
    if (i == 1){
     df$a_n2[[i]] <- a_n[[i]]
    } else {
     df$a_n2[[i]] <-  (df$n[[i]] *(df$diff[2] - df$diff[1]) + df$a_n2[[i-1]])
    }
}

ggplot(data = df, aes(x = n, y = a_n - a_n2)) +
        geom_point() +
        geom_smooth(method = "lm", se = FALSE)
```



3 Page 34: problem 13

Considering the spreading of a rumor through a company of 1000 employees, all working in the same building. We assume that the spreading of a rumor is similar to the spreading of a contagious disease in that the number of people hearing the rumor each day is proportional to the product of the number who have heard the rumor previously and the number who have not heard the rumor. This is given by

$$r_{n+1} = r_n + kr_n(1000 -_n)$$

where k is a parameter that depends on how fast the rumor spreads and n is the number of days. Assume k = 0.001 and further assume that four people initially have heard the rumor. How soon will all 1000 employees have heard the rumor?

```
r_n <- 4
k <- .001
count <- 0

while (r_n <= 1000){
r_n <- r_n + ((.001*r_n)*(1000-count))
count <- count + 1
if (r_n <= 0){
  print("no one will have heard the rumor")
  break
 }
}
```

All 1000 employees will have heard the rumor by the 8 day

4 Page 55: problem 6

An economist is interested in the variation of the price of a single product. It is observed that a high price for the product in the market attracts more suppliers. However, increasing the quantity of the product supplied tends to drive the price down. Over time, there is an interaction between price and supply. The economist has proposed the following model, where P_n represents the price of the product at year n, and Q_n represents quantity. Find the equilibrium values for the system.

$$P_{n+1} = P_n - 0.1(Q_n - 500)$$
$$Q_{n+1} = Q_n + .2(P_n - 100)$$

Solve for P

$$0 = P - .1(Q - 500)$$
$$0 = P - .1Q - 50$$
$$P = .1Q - 50$$

Now Solve for Q

$$0 = Q + .2(P - 100)$$
$$0 = Q + .2p - 20$$
$$Q = -.2P + 20$$

Substitue and Solve for Value of Q

$$Q = -.2(.1Q - 50) + 20$$
$$Q = -.02Q + 30$$
$$1.02Q = 30$$
$$Q = 29.41$$

Substitue and Solve for Value of P

$$P = (.1 * 29.41) - 50$$
$$P = -47.06$$

The equilibrium of the systems is P = -47.06; Q = 29.41.

a. Does the model make sense intuitively? What is the significance of the constants 100 and 500? Explain the significance of the significance of the constants -0.1 and 0.2.

The model does not make sense intuitively because the equilibrium has a negative price, therefore, there must be a price floor that is not explained in this model where we will no longer sell a product for a certain price. The significanes of the constants of 100 and 500 is to reduce the quantity and price for their influence, you'll notice that -500 is used for quantity which makes sense as our price will vary less than the quantity so we will want to reduce its impact. The constants of -0.1 and 0.2 is also to normalize our value, you will notice that the -.1 is for the price because we expect a decrease in price as the quantity increase.

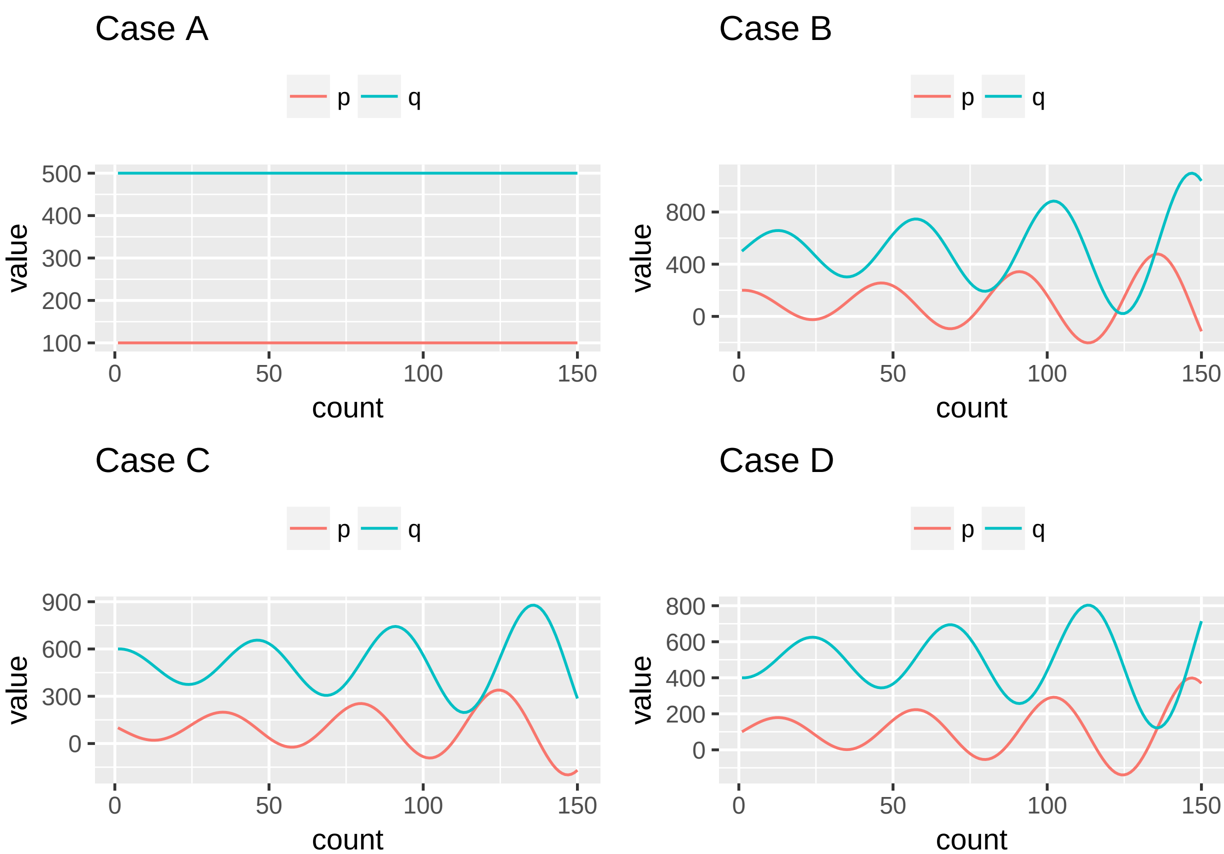
b. Test the initial conditions in the following table and predict the long-term behavior

```
library(knitr)
price <- c(100, 200, 100, 100)
quantity <- c(500, 500, 600, 400)
row_names <- c("Case A", "Case B", "Case C", "Case D")
x <- as.data.frame(cbind(price, quantity))
row.names(x) <- row_names
kable(x)
```

	price	quantity
Case A	100	500
Case B	200	500
Case C	100	600
Case D	100	400

```
library(tidyverse)
library(gridExtra)

lt_behav <- function(df){
for (i in 1:149){
  p <- df$p[[i]] - .1*(df$q[[i]] - 500)
  q <- df$q[[i]] + .2*(df$p[[i]] - 100)
  result <- cbind(p, q)
  df <- rbind(df, result)
  }
df$count <- as.numeric(row.names(df))
df <- as.data.frame(gather(df, count))
colnames(df) <- c('count','cat','value')
return(df)
}

p <- x[1,1]
q <- x[1,2]
df <- as.data.frame(cbind(p,q))
df.1 <- lt_behav(df)

p <- x[2,1]
q <- x[2,2]
df <- as.data.frame(cbind(p,q))
df.2 <- lt_behav(df)

p <- x[3,1]
q <- x[3,2]
```

```
df <- as.data.frame(cbind(p,q))
df.3 <- lt_behav(df)

p <- x[4,1]
q <- x[4,2]
df <- as.data.frame(cbind(p,q))
df.4 <- lt_behav(df)


plot1 <- ggplot(df.1) +
         geom_line(aes(x = count, y = value, color = as.factor(cat))) +
         theme(legend.position="top", legend.direction="horizontal", legend.title=element_blank()) +
         ggtitle("Case A")

plot2 <- ggplot(df.2) +
         geom_line(aes(x = count, y = value, color = as.factor(cat))) +
         theme(legend.position="top", legend.direction="horizontal", legend.title=element_blank()) +
         ggtitle("Case B")

plot3 <- ggplot(df.3) +
         geom_line(aes(x = count, y = value, color = as.factor(cat))) +
         theme(legend.position="top", legend.direction="horizontal", legend.title=element_blank()) +
         ggtitle("Case C")

plot4 <- ggplot(df.4) +
         geom_line(aes(x = count, y = value, color = as.factor(cat))) +
         theme(legend.position="top", legend.direction="horizontal", legend.title=element_blank()) +
         ggtitle("Case D")

grid.arrange(plot1, plot2, plot3, plot4, ncol=2)
```


Case A
p
q
500
400
300
200
100
value
0
50
100
150
count
Case B
p
q
800
400
0
value
0
50
100
150
count
Case C
p
q
900
600
300
0
value
0
50
100
150
count
Case D
p
q
800
600
400
200
0
value
0
50
100
150
count